-3 or 4 questions

Chapter-4

Logic, Induction & Reasoning (25-30 marks)

Proposition

Any statement that is either true or false but not both at the same time is known as proposition.

Eg: (a) Today is Friday (T)

(b) Pokhara is the ~~capoc~~ capital city of Nepal (F).

The statement that are not proposition are:

(a) $x>3$

(b) Open the door (This is command)

We denote proposition by lower case alphabets such as p, q, r, s etc.

Eg:

p: "It is raining Today."

Logical connectives / operators

Logical connectives are used to construct compound proposition. The diffeient types of Logical operators are

(a) Negation (¬) (NOT gate)

p: "Today is Friday."

¬p: "Today is not Friday."

Truth table:

p	¬p
T	F
F	T

(b) Conjunction ($\wedge$) (AND)

p: "Today is Friday."

q: "It is raining Today."

$p \wedge q$: "Today is Friday and it is raining today."

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(c) Disjunction ($\vee$) [OR]

p: "Today is Friday."

q: "It is raining today."

$p \vee q$: "Today is Friday or it is raining today."

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

d. Exclusive OR ($\oplus$) [XOR]

Let p & q be the two proposition then the Exclusive OR of p and q is the proposition "Either p or q but not both

p: "Today is Friday"

q: "It is raining tody.

$p \oplus q$: "Either Today is Friday or it is raining Today but not both."

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

e. Implication / conditional statement : $(\rightarrow)$

Let p and q be the two proposition and the conditional statement of p & q is the proposition.

$$p \rightarrow q : \text{" If } p \text{ then } q \text{."}$$

p : "There is combustion."
q : "There is oxygen."

$p \rightarrow q$: "If there is combustion, then there is oxygen."

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The proposition p is called hypothesis or premise and the proposition ~~p is~~ q is called conclusion.

$$\boxed{P \rightarrow Q}$$

Sufficient condition. Necessary condition

(f) Biconditional statement ($\leftrightarrow$):

Let p & q be the two proposition then the Biconditional statement is:

$p \leftrightarrow Q$: " P if and only if Q"

" P iff Q"

p: "I am alive."

q: "I am breathing."

$p \leftrightarrow q$: I am alive if and only if I'm breathing.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Translating from English sentences:

Qn.1 You can access internet from NCIT if You are a student or if you are faculty member.

Solution

Let p: You can access internet from NCIT.

q: You are a student.

r: You are a faculty member.

$$(q \vee r) \longrightarrow p$$

Qn.2 You cannot have voting right if you are below 18 years and you are mentally unfit.

Solⁿ

Let p: You can have voting right.

q: You are below 18 years.

r: You are mentally unfit.

$$(q \wedge r) \longrightarrow \neg p$$

Qn.3 leaders will make correct decision only if you vote right leader and raise your voice against corruption.

Note: यदी statement मा only if छ भने

hypothesis only if conclusion.

Let p: leaders will make correct decision.

q: You vote right leader

r : Raise your voice against corruption.

$p \rightarrow \cancel{(q \vee r)}\ (q \wedge r)$.

Qn 4. The message was sent from unknown system but not scanned for virus.

Soln

Note: AND ≡ But

Let

p: "The message was sent from unknown system.

q: " Message was not scanned for virus."

$p \wedge q$

If q: " Message was scanned for virus"

$\boxed{p \wedge \neg q}$

Imp

Conve

p

(a) Cor

q →

(b) Ir

7p

(c) C

p

Converse, Inverse and contrapositive ~~and~~ of conditional statement.

$p \rightarrow Q$

p: "Today is saturday."

q: "college is closed."

$p \rightarrow q$: "If Today is saturday then college is closed."

(a) Converse

$$p \rightarrow Q \xRightarrow{\text{converse}} Q \rightarrow p$$

$q \rightarrow p$: "If college is closed then today is saturday."

(b) Inverse

$$p \rightarrow Q \xRightarrow{\text{Inverse}} \neg P \rightarrow \neg Q$$

$\neg p \rightarrow \neg q$: "If Today is not saturday then college is not closed."

(c) Contrapositive

$$P \rightarrow Q \xRightarrow{\text{contraposite}} \neg q \rightarrow \neg P$$

~~p~~ $\neg q \rightarrow \neg p$: "If college is not closed then today is not saturday."

Imp

Find the converse, inverse and contrapositive of following conditional statement.

(7 marks)

(a) "When I stay up late it is necessary that I sleep until noon."

Solution

p: I stay up late.

q: I sleep until noon.

$p \to q$: "If stay up late then I sleep until noon."

(i) converse

$q \to p$: "If I sleep until noon then I stay up late."

(ii) Inverse

$\neg p \to \neg q$: "If I donot ~~sleep un~~ stay up late then I donot sleep until noon."

(iii) contrapositive

$\neg q \to \neg p$: "If I donot sleep untile noon then I donot stay up late."

A positive integer is prime only if it has no divisor other than 1 and itself.

Solution

Let

p : A positive integer is prime.

q : It has no divisor other than 1 and itself.

$p \to q$: If a positive integer is prime ~~only~~ then it has no divisor other than 1 and itself.

(i) converse.

$q \to p$: "If a positive integer has no divisor other than 1 and itself then it is prime.

(ii) Inverse

$\neg p \to \neg q$: If a positive integer is not prime then it has divisor other than 1 and itself.

(iii) Contrapositive.

$\neg q \to \neg p$: If a positive integer has divisor other than 1 and itself then it ~~has~~ is not prime.

Truth Table of Compound Proposition

ⓐ $(p \to q) \oplus (p \wedge q)$ ~~$(p \to q) \oplus (p \wedge q)$~~

p	q	$p \to q$	$p \wedge q$	$p \to q \oplus (p \wedge q)$
T	T	T	T	F
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

ⓑ $(p \to q) \oplus (p \wedge q)$

p	q	$(p \to q) \oplus (p \wedge q)$
T	T	F
T	F	F
F	T	T
F	F	T

ⓒ $(p \leftrightarrow q) \vee (q \to p)$

p	q	$p \leftrightarrow q$	$q \to p$	$(p \leftrightarrow q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	T	T	T

c. $(p \vee r) \rightarrow (q \rightarrow p)$

p	q	r	$p \vee r$	$q \rightarrow p$	$(p \vee r) \rightarrow (q \rightarrow p)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	F	T	T

Propositional Equivalences.

a. Tautology

A compound proposition that is always true no matter the truth value of its proposition value that occur in it is known as tautology.

$$(p \rightarrow q) \vee (q \rightarrow p)$$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(b) Contradiction

A compound proposition that is always False no matter the truth value of its proposition value that occur in it is known as contradiction.

$$\neg(p \wedge q) \leftrightarrow (p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \leftrightarrow (p \wedge q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

e. Contingency:

A compound proposition that is neither tautology nor contradiction is known as contingency.

Logical Equivalences

Two compound proposition are said to be logically equivalent if they have same Truth value in every possible case ($A \equiv B$)

(a) $(p \to q)$ and $(\neg p \vee q)$

p	q	$(p \to q)$	$\neg p$	$(\neg p \vee q)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore (p \to q) \equiv (\neg p \vee q)$

Verify de-Morgan's law:

a. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

b. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

a. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(b)

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

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Show that implication and its contraposition are logically equivalent.

Implification : $p \to q$

contrapositive: $\neg q \to \neg p$

p	q	$p \to q$	$\neg q$	$\neg p$	$(\neg q \to \neg p)$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$$\therefore (p \to q) \equiv (\neg q \to \neg p)$$

~~** Some Rules **~~

** Some Rules **

	Rules	Name
a.	$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Laws
b.	$p \wedge F \equiv F$ $p \vee T \equiv T$	Domination Law
c.	$\neg(\neg p) \equiv p$	Double Negation.
d.	$(p \vee q) \equiv (q \vee p)$ $(p \wedge q) \equiv (q \wedge p)$	Commutative Law
e.	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Law
f.	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's law
g.	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Law
h.	$p \wedge \neg p \equiv F$ $p \vee \neg p \equiv T$	Negation Law.
i. Imp →	$p \rightarrow q \equiv \neg p \vee q$	Implication Law
j.	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Double Implication Law.
k.	$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent law
l.	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive law.

* Show that the following compound proposition are logically equivalent without using truth ta

Solution,

(a) ~~Taking~~ $\neg(p \to q) \equiv p \wedge \neg q$

Taking LHS,

$\equiv \neg(p \to q)$

$\equiv \neg(\neg p \vee q)$ [∵ Implication law]

$\equiv \neg(\neg p) \wedge \neg q$ [∵ DeMorgan's law].

$\equiv p \wedge \neg q$ [∵ Double Negation].

$\equiv$ RHS

(b) $(p \to q) \vee \neg p \equiv \neg p \vee q$

Solution,

Taking LHS:

$\equiv (p \to q) \vee \neg p$

$\equiv (\neg p \vee q) \vee \neg p$ [∵ Implication Law]

$\equiv$ ~~$\neg p \vee$~~ $(\neg p \vee \neg p) \vee q$ [∵ Associat law]

$\equiv \neg p \vee q$ [∵ Idempotant law]

$\equiv$ RHS

proved

c. $\neg(p \to q) \vee (p \wedge q) \equiv p$

Soln

Taking LHS,

$\equiv \neg(p \to q) \vee (p \wedge q)$

$\equiv \neg(\neg p \vee q) \vee (p \wedge q)$ [∵ Implication Law]

$\equiv (p \wedge \neg q) \vee (p \wedge q)$ [∵ De Morgan's law]

$\equiv p \wedge (\neg q \vee q)$ [∵ Distributive law].

$\equiv p \wedge T$ [∵ Negation law]

$\equiv p$ [∵ Identity law].

$\equiv$ RHS proved

d. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Soln

Taking LHS,

$\equiv \neg(p \vee (\neg p \wedge q))$

$\equiv \cancel{\neg((p \vee \neg p) \wedge (p \vee q))}$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ [∵ De Morgan's law]

$\equiv \neg p \wedge (p \vee \neg q)$ [∵ De Morgan's law]

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ [∵ Distributive law]

$\equiv F \vee (\neg p \wedge \neg q)$ [∵ Negation]

$\equiv \neg p \wedge \neg q$ [∵ Identity law].

$\equiv$ RHS

proved

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Rules of Inference

We can always check the validitity of an argument by constructing a truth table. However, if the numbers of the variables is large it can be tedious / leenthy approach.

Eg: If we have 10 variables, we need $20^{10} = 1024$ combination.

Instead, we can estabilish the validity of some relatively simple argument known as Rules of inference.

a. Modus Ponens

Truth Table:

P	Q	P→Q	(P→Q) ∧ q	((P→Q) ∧ P) → Q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

b. Modus Tollens

P	Q	P→Q	7P	7Q	P→Q ∧ 7Q	(P→Q)∧7Q→7
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	F	T	T	T	T	T

c. Hypothetical syllogism

$$\begin{array}{l} P \rightarrow Q \\ \underline{Q \rightarrow R} \\ \therefore P \rightarrow R \end{array}$$

d. DisJunctive syllogism

$$\begin{array}{l} P \vee Q \\ \underline{\neg P} \\ \therefore Q \end{array}$$

(Since, $\neg$ is negation '–')

e. Simplification

$$\begin{array}{l} \underline{P \wedge Q} \\ \therefore P/Q \end{array}$$

(Either P is true or Q is true)

f. Addition

$$\begin{array}{l} \underline{P} \\ \therefore P \vee Q \end{array}$$

g. Resolution

$$\begin{array}{l} P \vee Q \\ \underline{\neg P \vee R} \\ \therefore Q \vee R \end{array}$$

h. conjuction

$$\begin{array}{l} P \\ \underline{Q} \\ \therefore P \wedge Q \end{array}$$

Qn.1 Show that the premise: "If I play football then I am tired the next day. I will take rest if I am tired. I didnot take rest" leads to the conclusion: I did not play football.

Solution.

Let

P: I play football.

Q: I am tired next day.

R: I will take rest.

Hypothesis:

ⓐ $P \to Q$

ⓑ $Q \to R$

ⓒ $\neg R$

Conclusion: $\therefore \neg P$

	Statement		Reasons:
1.	$P \to Q$	1.	Given hypothesis.
2.	$Q \to R$	2.	Given hypothesis
3.	$P \to R$	3.	Hypothetical syllogism on 1 and 2.
4.	$\neg R$	4.	Given Hypothesis.
5.	$\neg P$	5.	Modus tollens on 3 and 4.

Show that the premise: "If the interest rate drops then the housing market will improved. The federal discount rate will drop or the housing market will not improved. Interest rate will drop. Therefore, the federal discount rate will drop."

Solution

Let

p: Interest rate drops

q: Housing market will improve.

r: The federal discount rate drops.

Hypothesis:

(a) $p \to q$

(b) $r \vee \neg q$

(c) p

conclusion: $\therefore r$

Statement	Reasons
$p \to q$	1. Given Hypothesis.
p	2. Given Hypothesis
q	3. Modus ponens on 1 and 2.
$r \vee \neg q$	4. Given Hypothesis.
r	5. Disjunction syllogism 3 and 4.

Verify the following arguments using rules of inference.

"If clinton doesn't live in France then he doesn't speak French. Clinton doesn't drive datson. If clinton live in France then he ride a motorcycle. Either clinton speaks French or he drives a datson. Hence, a clinton rides a motorcycle.

Solution

Let,

P: clinton live in France.

Q: He speak Frech

R: Clinton drive datson.

S: clinton rides a motorcycle.

ⓐ Hypothesis:

a. $\neg P \rightarrow \neg Q$

b. $\neg R$

c. $P \rightarrow S$

d. $Q \vee R$

conclusion: S

$$\begin{array}{l} \neg P \rightarrow \neg Q \\ \neg(\neg Q) \\ \hline \neg(\neg P) \Rightarrow P \end{array}$$

Statement	Reasons
1. $\neg R$	1. Given hypothesis
2. $Q \vee R$	2. Given hypothesis
3. Q	3. Disjunction syllogism on 1 and 2.
4. $\neg P \rightarrow \neg Q$	4. Given hypothesis
5. P	5. Modus Tollen's on 3 and 4.
6. $P \rightarrow S$	6. Given hypothesis.
7. S	7. Modus tollen's on 5 and 6.

"If I get the Job and Worked hard ~~than~~ then I will get promoted. If I get promoted then I will be happy. I will not be happy. Therefore, Either I will not get Job or I will not worked hard.

Solution

Let,

P: I get the Job.

Q: I worked hard.

R: I will get promoted.

S: I will be happy.

Hypothesis:

(a) $(p \land Q) \rightarrow R$

(b) $R \rightarrow S$

(c) $\neg S$

Conclusion: $\neg p \lor \neg Q$

Statement	Reasons
$R \rightarrow S$	1. Given hypothesis.
$\neg S$	2. " "
$\neg R$	3. Modus Tollens on 1 & 2.
$P \land Q \rightarrow R$	4. Given hypothesis.
$\neg(p \land Q)$	5. De-Morgan's law.
$\neg P \lor \neg Q$	6.

$p \to q \equiv \neg p \vee q$

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Qn. Show that a premise : If you send me an email message then I will finish writing the program. If you donot send me an email message then I will go to sleep early. If I ~~will~~ go to sleep early then I will wake up feeling refreshed. Leads to the conclusion if I donot finish writing the program then I will wake up feeling refreshed.

Solution

Let

p: You send me an email message.
q: I'll finish writing the program
r: I'll go to sleep early.
s: I'll wake up feeling refreshed.

Hypothesis: (a) $p \to q$
(b) $\neg p \to r$
(c) $r \to s$

Conclusion $\therefore \neg q \to s$

S.N	Statements	Reasons
1.	$\neg p \to r$	1. Given Hypothesis.
2.	$r \to s$	2. " "
3.	$\neg p \to s$	3. Hypothetical syllogism on 1 and 2.
4.	$p \to q$	4. Given Hypothesis.
5.	$\neg p \vee q$	5. Implication Law on 4
6.	$p \vee s$	6. Implication Law on 5
7.	$q \vee s$	7. Resolution in 5 & 6.
8.	$\neg q \to s$	8. Implication Law on 7.

Board

Qn.2. It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it's sunny. If we donot go swimming then we will take road trip. If we take road trip then we will be home by sunset. Therefore, we will be home by sunset.

Solution

Let

p: It is sunny this afternoon.

q: It is colder than yesterday.

r: we will go swimming

s: we will take road trip

t: we will be home by sunset.

Hypothesis:

ⓐ $\neg p \wedge q$

ⓑ $r \rightarrow p$

ⓒ $\neg r \rightarrow s$

ⓓ $s \rightarrow t$

conclusion : $\therefore t$

S.N	Statements	S.N	Reasons.
1.	$\neg r \rightarrow s$	1.	Given Hypothesis.
2.	$s \rightarrow t$	2.	" "
3.	$\neg r \rightarrow t$	3.	Hypothetical syllogism on 1 & 2.
4.	$\neg p \wedge q$	4.	Given hypothesis.
5.	$\neg p$	5.	Simplification on 4.
6.	$r \rightarrow p$	6.	Given hypothesis.
7.	$\neg r$	7.	Modus Tollens on 5 & 6
8.	t	8.	Modus Ponens on 3 & 7.

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Qn3. If it doesn't rain or if it isn't foggy then the sailing race will be held and life saving demonstration will go on. If the sailing race ~~wit~~ is held then the trophy will be awarded. The trophy was not awarded. Therefore, it rained.

Solution

Let

p: It rains

q: It is foggy

r: The sailing race will be held.

s: life saving demonstration will go on

t: Trophy will be awarded.

Hypothesis:

(a) $(\neg p \vee \neg q) \rightarrow (r \wedge s)$

(b) $r \rightarrow t$

(c) $\neg t$

Conclusion: $\therefore p$

S.N	Statements	Reasons.
1	$r \rightarrow t$	1. Given hypothesis.
2.	$\neg t$	2. " "
3.	$\neg r$	3. Modus Tollens on 1 & 2.
4	$\neg r \vee \neg s$	4. Addition on 3.
5.	$\neg r \wedge s$	5. De-morgan's law
6.		

VVI

2. Predicate logic

p: "All kings are men"
q: "All men are mortal"
r: ∴ "All kings are mortal"

Above the above statement cannot be true using Propositional Logic. Therefore, we need more powerful type of logic called predicate logic or first order logic or propositional logic. To understand Predicate logic, we need need to understand following topics;

Domain
Subject
Predicate
Quantifiers

Subject is the topic under discussion.
Predicate refers to the property of an statement

p: "Josh is Tall"

subject — Josh
Predicate — is Tall

$\boxed{\text{Tall (Josh)}}$

Quantifiers:
"Every cats Drinks Milk"

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(a) Universal Quantifier:

The universal Quantification of $P(x)$, is

$$\forall_x \; P(x)$$

"For Every x, $P(x)$"

OR

"For all x, $P(x)$"

Example: All student in this class has done.

Qn) Consider the following statement $Q(x): x < 5$ What is the truth value of Quantification ~~for~~ $\forall_x Q(x)$ when domain is all real numbers?

Solution

"x"

Domain : real numbers.

$\forall_x \; Q(m)$: "$x < 5$"

For $x = 6$,

$Q(6): 6 < 5 \; (F)$

$\forall_x \; \cancel{Q(m)} \; Q(x) = \text{False}.$

Qn 2. $Q(x) : x \geq 0$

$\forall_x Q(x)$

x
(0 to ∞)

Domain : +ve Number only.

Consider the above statement $Q(x) : x \geq 0$. What is the truth value of $\forall_x Q(x)$? Where domain is positive numbers.

Solution

Given,

$Q(x) : x \geq 0$

$\forall_x Q(x) = ?$

x
(0 to ∞)

Domain : +ve Number only.

Since, $Q(x)$ is True for all value of x in domain.

$\therefore \forall_x Q(x) = \text{True}$

(b) Existential Quantitie.

(Some, Few, There is)

~~$\forall_x Q(m)$: "~~

The existential quantification of $P(x)$, $\boxed{\exists_x P(x)}$, (For some x, $P(x)$ or There exists x, such that $P(x)$)

Eg: Some lions Drinks Milk.

milk(x) : " X drinks milk"

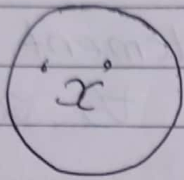
'x'

Domain: lion

Qn.1 Translate the following statement using logical connectives, quantifiers and predicates

a. "Every students in this class visited Pokhara"
[Domain : student in this class]

Ans

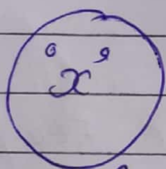


Domain : students in this class

Ans: $P(x)$: " x has visited Pokhara"

$$\forall_x \, p(x)$$

b. "Every student in this class visited Pokhara"



Domain : All students

Ans: Let

$S(x)$: " x is student in this class"
$p(x)$: " x has visited Pokhara"

$$\forall_x \, [s(x) \rightarrow p(x)]$$

[Note: If universal quantification implication $\rightarrow$]

in this class
Every student has done their homework or attend the lecture.

(i) Domain : All students in this class

x

P(x) : x has done homework.
Q(x) : x has attend Lecture"

$$\forall_x [P(x) \cup Q(x)]$$

ii) Domain : All student

x

∴ Let
S(x) : x is student in this class
P(x) : x has done homework.
Q(x) : x has attend Lecture"

$$\forall_x [S(x) \to P(x) \vee Q(x)]$$

Some student have long hair

x

Domain : All student

i) P(x) : "x has long hair"

$$\exists_x P(x).$$

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d. (ii) x Domain: All people.

Ans: Let

$S(x)$: " x is student "
$P(x)$: " x has long hair "

$$\exists_x [S(x) \wedge P(x)]$$

[Note: Existential Quantification AND ($\wedge$)]

e. There is a student in this class who can speak Russian and who know C++.
[Domain: All student in this class].

Let
Ans: $P(x)$: " x can speak Russian "
$Q(x)$: " x knows C++ "

$$\exists_x P(x) \wedge Q(x)$$

Consider the following statement:
No Professor are ignorant.
All Ignorant People are in Vain.
Some professor are ignorant.
Express above statement using quantifiers where domain consists of all people.

x

Domain : People

Solution
Here,
$P(x)$: " x is Professor".
$Q(x)$: " x is Ignorant"
$R(x)$: " x is in Vain".

(a) $\forall x\,[P(x) \rightarrow \neg Q(x)]$

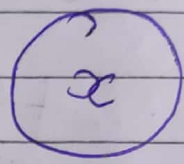
(b) $\forall x\,[Q(x) \rightarrow R(x)]$

(c) $\exists x\,[P(x) \wedge Q(x)]$

Qn 3. Consider the following statements.

a. All humming birds are richly coloured.
b. No large birds lives on honey.
c. Birds that donot lives on honey all dull in colour.
d. Humming birds are small.

[Domain : All birds]



Domain : All birds.

Ans: let

$P(x)$: "x is a humming birds"
$Q(x)$: "x is richly coloured"
$R(x)$: "x is large bird."
$S(x)$: "x lives on honey."

(a) $\forall x\,[P(x) \rightarrow Q(x)]$

(b) $\forall x\,[R(x) \rightarrow \neg S(x)]$

(c) $\forall x\,[\neg S(x) \rightarrow \neg Q(x)]$

(d) $\forall x\,[P(x) \rightarrow \neg R(x)]$

Negating Quantities

"Every student has passed First Semester"

'x'

Domain : student

ation

$P(x)$: "x has passed 1^{st} semester"

$$\forall_x \; P(x)$$

→ "Some student has failed First semester"

$$\exists_x [\neg P(x)]$$

$$\neg[\forall_x P(x)] \equiv \exists_x [\neg P(x)]$$

$$\neg[\exists_x (P(x))] \equiv \forall_x [\neg (P)]$$

De-Morgan's Law For Quantities

Nested Quantifiers.

"The sum of any two Positive Number is Positive."

x, y
$-\infty \cdots \infty$

Domain: Real numbers.

$$\forall_x \forall_y (x > 0) \wedge (y > 0) \longrightarrow (x+y) > 0$$

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Qn.1 "For Every real number, there exists a real number such that their sum is zero".

x, y
$-\infty \ldots \infty$

Domain: real numbers.

Ans: $\forall_x \exists_y (x+y=0)$

Qn.2 "The product of any two ~~rea~~ negative number is always positive."

$-\infty \ldots \infty$
"x, y"

Domain: real numbers

$\forall_x \forall_y ((x<0) \wedge (y<0) \rightarrow xy>0)$

Qn.3 Every real number except 0 has a multiplicative inverse.

$-\infty \ldots \infty$
"x, y"

Ans: $\forall_x \exists_y (xy = 1)$

"If a person is Female & is a Parent then this person is some one's mother"

"x, y"

Domain : All people

$P(x)$: "x is Female"

$Q(x)$: "x is Parent

$R(x,y)$: " x is mother of y"

$$\forall x\,[(P(x) \wedge Q(x)) \rightarrow \exists_y\, R(x,y)]$$

Let, $P(x)$: $x+y = xy$. What is the Truth value of Quantification : (Domain = all real numbers)

$\forall_x \exists_y P(x,y)$ (b) $\exists_y \forall_x P(x,y)$

Solution

Here,

$-\infty \cdots \infty$

Domain : real numbers.

(a) $\forall_x \exists_y P(x,y)$: $x+y=0$

(True)

(b) $\exists_y \forall_x P(x,y)$: $x+y=0$

(False)

$$\boxed{\forall_m \exists_y P(x,y) \neq \exists_y \forall_x P(x,y)}$$

Let $P(x,y): x+y=y+x$. What is the Truth value of Quantification: (Domain = all real numbers).

ⓐ $\forall_x \forall_y P(x,y)$ ⓑ $\forall_y \forall_x P(x,y)$

Soln

Here,

ⓐ $\forall_x \forall_y P(x,y): x+y=y+x$

(True)

$-\infty \ldots \infty$

Domain: real nos.

ⓑ $\forall_y \forall_x P(x,y): x+y=y+x$.

(True)

$\forall_x \forall_y P(x,y) = \forall_y \forall_x P(x,y)$

$\exists_x \exists_y P(x,y) = \exists_y \exists_x P(x,y)$

Rules of Inference for Quantifiers

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(a) Universal Instantiation

$\forall_x P(x)$

$\therefore P(c)$; for ~~abit~~ arbitary value of c

(b) Universal Generalization

$P(c)$; For arbitary value of c.

$\therefore \forall_x P(x)$

(c) Existential Instantiation

$\exists_x P(x)$

$P(c)$; For Some Value of c.

(d) Existential Generalization

$P(c)$; For Some value of C

$\therefore \exists_x P(x)$

Qn. 1

"All kings are men"

"All Men are Mortal"

$\therefore$ All kings are Mortal

① Domain : x

All people

Soln

Here,

Let, $P(x)$: "x is king"

$Q(x)$: "x is men"

$R(x)$: "x is mortal"

(a) Hypothesis : (i) $\forall_x [P(x) \rightarrow Q(x)]$

(ii) $\forall_x [Q(x) \rightarrow R(x)]$

(b) Conclusion : $\therefore \forall_x [P(x) \rightarrow R(x)]$

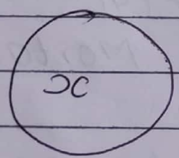
P.T.O

Note: * Always Remove quantifiers,
* Rules of Inference of Proposition, * Generalization

Statements	Reasons
(i) $\forall x [P(x) \rightarrow Q(x)]$	(i) Given hypothesis
(ii) $P(c) \rightarrow Q(c)$	(ii) Universal Instantiation
(iii) $\forall x [Q(x) \rightarrow R(x)]$	(iii) Given Hypothesis
(iv) $Q(c) \rightarrow R(c)$	(iv) Universal Instantiation.
(v) $P(c) \rightarrow R(c)$	(v) Hypothetical Syllogism on 2 & 4.
(vi) $\forall x [P(x) \rightarrow R(x)]$	(v) Universal Generalization on (V)

Qn.2. Show that the premise: "Everyone in the class has taken course in computer science." "Sita is a student in this class." Lead to the conclusion: "Sita has taken course on computer science
[Domain: All student]

Soln



Domain: All student.

Let,

$P(x)$: "x is in this class."

$Q(x)$: "x has taken course on computer science."

(a) Hypothesis:

(i) $\forall x [P(x) \rightarrow Q(x)]$

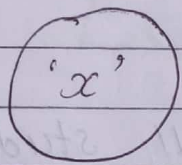
(ii) $P(sita)$

(b) Conclusion : $\therefore Q(sita)$

Statements	Reasons
$\forall x\,[P(x) \rightarrow Q(x)]$	(i) Given hypothesis.
$P(sita) \rightarrow Q(sita)$	(ii) Universal Instantiation
$P(sita)$	(iii) Given hypothesis.
$Q(sita)$	(iv) Modus Ponens on 2 & 3.

"~~All~~ students in this class has not read the book."
"Everyone in this class has passed 1st exam."
leads to the conclusion: "Someone who passed the first ~~the~~ exam has not read the book."

Sol^n



Domain: All students.

Let, $P(x)$: "x is in this class."
$Q(x)$: "x has read the book"
$R(x)$: "x has passed first exam".

(a) Hypothesis:
(a) $\exists x\,[P(x) \wedge \neg Q(x)]$
(b) $\forall x\,[P(x) \rightarrow R(x)]$

Conclusion: $\therefore \exists x\,[R(x) \wedge \neg Q(x)]$

Statements	Reasons
$\exists x\,[P(x) \wedge \neg Q(x)]$	(i) Given Hypothesis
$P(c) \wedge \neg Q(c)$	(ii) Existential ~~Universal~~ Instantiation.
$\forall x\,[P(x) \rightarrow R(x)]$	(iii) Given Hypothesis
$P(c) \rightarrow R(c)$	(iv) universal Instantiation.
$P(c)$	(v) Simplification on 2.
$R(c)$	(vi) Modus Ponens on (iv) & (v)
$\neg Q(c)$	(vii) Simplification on 2.

(viii) $R(c) \wedge \neg Q(c)$	(viii) Conjunction on vi & vii
(ix) $\exists_x [R(x) \wedge \neg Q(x)]$	(ix) Existential Generalization on viii.

Qn.4. "There is someone in this class who has been to Pokhara." Everyone who goes to Pokhara visits Sharangkot." leads to the conclusion: someone in this class has visited Sharangkot."

'x'

Domain: All students.

Soln

Here,

Let, $P(x)$: "x is in this class."

$Q(x)$: "x has been to Pokhara."

$R(x)$: "x visits sharangkot."

(a) Hypothesis

(i) $\exists_x [P(x) \wedge Q(x)]$

(ii) $\forall_x [Q(x) \rightarrow R(x)]$

Conclusion: $\therefore \exists_x [P(x) \wedge R(x)]$

Statements	Reasons.
(i) $\exists_x [P(x) \wedge Q(x)]$	(i) Given Hypothesis.
(ii) $P(c) \wedge Q(c)$	(ii) Existential Initialization
(iii) $\forall_x [Q(x) \rightarrow R(x)]$	(iii) Given Hypothesis
(iv) $Q(c) \rightarrow R(c)$	(iv) Universal Initialization
(v) $Q(c)$	(v) Simplification on (ii).
(vi) $R(c)$	(vi) Modus Ponens on (iv) & v.

$P(c)$	(vii)	Simplification on (i).
$R(c) \wedge P(c)$	(viii)	Conjunction on (vi) & (vii).
$\exists x\,[P(x) \wedge R(x)]$	(ix)	Existential Generalization on (viii).

1 question

Proofs

1. Direct Proof
2. Indirect Proof
   - Proof by contraposition.
   - Proof by contradiction.
3. Mathematical Induction.

Direct Proofs.

A direct proof of a conditional statements $(p \to q)$ is constructed by assuming hypothesis is true & subsiquent steps are used show that conclusion is also true.

Show that if 'n' is odd then 'n^2' is also odd.

<u>Soln</u>

p: "n is odd" (Hypothesis)

q: "n^2 is odd (conclusion)

Assume,

n is odd

By defination of odd integer,

$n = 2k+1$ where

k = some integer.

Now,

Squaring both sides,

$n^2 = (2k+1)^2$

$\Rightarrow n^2 = 4k^2 + 1 + 4k$

$\Rightarrow n^2 = 4k^2 + 4k + 1$

$\Rightarrow n^2 = 2(2k^2 + 2k) + 1$

$\Rightarrow n^2 = 2m + 1$

where,

$m = 2k^2 + 2k$

$= \text{Integer}$.

$\therefore n^2$ is odd.

Qn.2. Prove that sum of two odd number is always even.

Solution

If m and n are odd, then m+n is even.

p: "m and n are odd" (Hypothesis)

q: "m+n" is even (conclusion).

Assume,

n is odd.

By defination of odd integer,

$m = 2k+1$; where k = some integer

$n = 2l+1$; " l = " "

Now,

$m+n = 2k+1+2l+1$

$= 2k+2l+2$

$= 2(k+l+1)$

$m+n = 2k'$

where, $k' = k+l+1$

$\therefore$ $m+n$ is even.

Let a, b, c are integers prove that if a divides b & a divides c then a divides (b+c)

Soln

"If a divides b & a divides c, a divides (b+c)"

p: "a divides b and a divides c" (Hypothesis)
q: "a divides b+c" (conclusion).

Now,

If a divides b. Then,

$$\frac{b}{a} = k \;;\; \text{where } k = \text{integer}$$

$$b = ak \quad \text{--- (I)}$$

If a divides c. Then,

$$\frac{c}{a} = l \;,\; \text{where } l = \text{integer}.$$

$$c = al \quad \text{--- (II)}$$

Now,

$$b+c = ak + al$$

$$b+c = a(k+l)$$

$$\frac{b+c}{a} = k+l.$$

$$= \frac{b+c}{a} = m \;; \text{ where } m = k+l = \text{integer}.$$

$\therefore$ a divides (b+c).

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Qn.4. Show that if m & n are perfect square then mn is also a perfect square. 2.

Solution

"If m & n are perfect square, mn is also a perfect square."

Then,

p: "m & n are perfect square"

q: "mn is perfect square"

Now,

If 'm' is perfect square. Then,

$m = k^2$ —(i) where, k = some integer

If 'n' is perfect square. Then,

$n = l^2$ —(ii) where l = some integer

Now,

$mn = k^2 l^2$

$mn = (kl)^2$

$mn = (k')^2$ where,

$k' = kl$ = integer.

Qn.1

Indirect Proof:

Sometimes direct proof leads to dead end. in such case we need to apply some other methods of proof that doesn't begin with hypothesis.

(a) Proof by contraposition:

Proof by contraposition makes use of fact that implication & its contraposition are logically equivalent. ($p \to q \equiv \neg q \to \neg p$)

Prove that if n is integer & "$3n+2$" is odd then n is odd.

Soln

"If $\underbrace{3n+2 \text{ is odd}}_{p}$, then $\underbrace{n \text{ is odd}}_{q}$"

p: $3n+2$ is odd

q: n is odd.

(a) Direct proof:

let, p is true,

$3n+2$ is odd.

$3n+2 = 2k+1$; k = integer

$n = \frac{2k-1}{3}$

$\times$

dead end.

(b) Proof by contraposition ($\neg q \to \neg p$)

$\neg q$: n is even (Hypothesis)

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$\neg p$: $3n+2$ is even (conclusion)

Now, if n is even then,

$n = 2k$ — (1)

where k = integer.

Now,

$3n+2 = 3\cdot(2k)+2$

$= 6k+2$

$= 2(3k+1)$

$3n+2 = 2l$ where,

$l = 3k+1$ (integer)

$\therefore$ $3n+2$ is ~~even, odd~~. even.

(b) Proof by contradiction.

The basic idea is to assume that the statement we want to prove is false and then show this assumption leads contradiction

Qn.1 ~~Proof th~~ Prove that there exists no integer a & b such that $5a+15b=1$.

Soln

Here,

Lets assume that there are integers 'a' and 'b'. such that,

$5a+15b=1$

$\Rightarrow 5(a+3b)=1$

$\Rightarrow a+3b = 1/5$

(contradiction)

$a+3b$ is not integer.

$\therefore$ $5a+15b=1$ is not integer.

Imp

Qn.2. Proof that $\sqrt{2}$ is a irrational number.

Qn.3 Proof that if four teams play 7 games then at least some pair of teams play at least two times.

Soln

Let the four teams are A, B, C and D. lets assume that every pair of teams play at most one time.

Now,

possible pair of games are.

(a) A - B
(b) A - C
(c) A - D
(d) B - C
(e) B - D
(f) C - D

Since, 7 games will be played this leads to contradiction.

Therefore, if four teams play 7 games then at least some pair of teams play at least two times.

Qn.4. If 100 balls are placed in 9 boxes. Then, some box contains 12 or more ball.

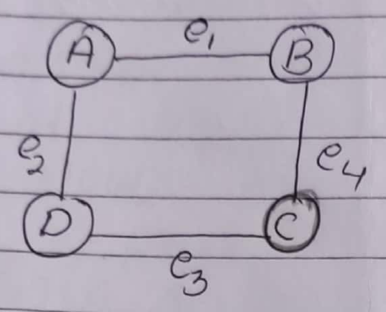
Solⁿ

chapter-5 (3 or 4 questions)

Graph Theory

Date ____ Page ____

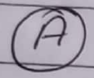
A graph is defined as $\boxed{G = (V, E)}$ where
V = non-empty set of vertices
E = set of edges.



$V = \{A, B, C, D\}$

$E = \{e_1, e_2, e_3, e_4\}$

(i)



(ii)

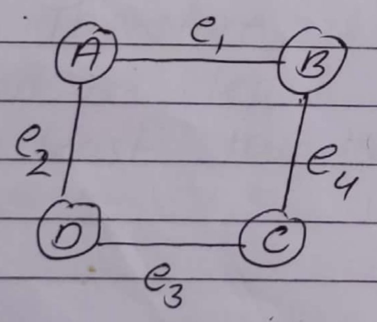
Trival Graph

$V = 1$

$E = 0$

Types of Graph.

(1) Undirected Graph



The graph whose edges is defined by unordered pair of vertices is called undirected graph.

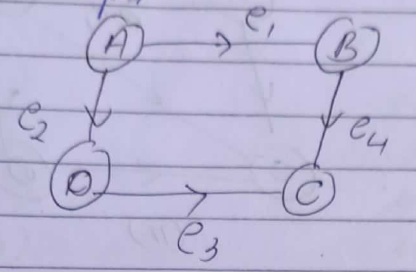
$e_1 = (A, B)$ or (B, A)

$e_2 = (A, D)$ or (D, A)

$e_3 = (D, C)$ or (C, D)

$e_4 = (B, C)$ or (C, B)

(2) Directed Graph

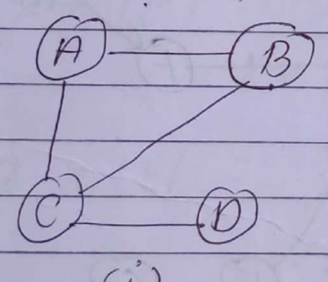


The graph whose edge is defined by ordered pair of vertices is called directed graph.

$$e_1 = (A, B) \neq (B, A)$$
$$e_2 = (A, D) \neq (D, A)$$
$$e_3 = (D, C) \neq (C, D)$$
$$e_4 = (B, C) \neq (C, B).$$

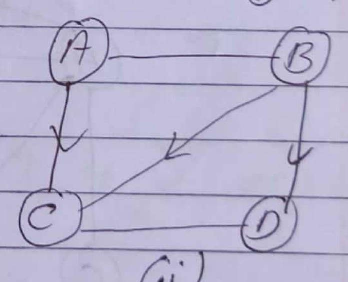
(3) Simple Graph

A graph that doesn't contain parallel edges and loops is called simple graph.



(i)

simple graph undirected.



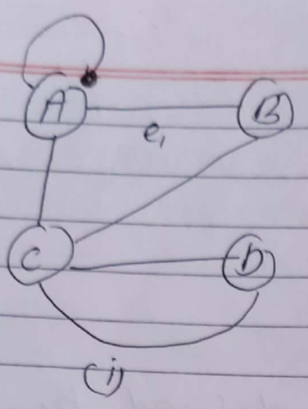
(ii)

simple graph directed.

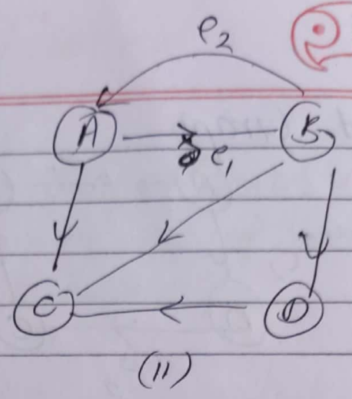
(4) Multi Graph/ Pseudo Graph.

The graph that contains parallel edges or loops is known as multi graph.

→



(i)

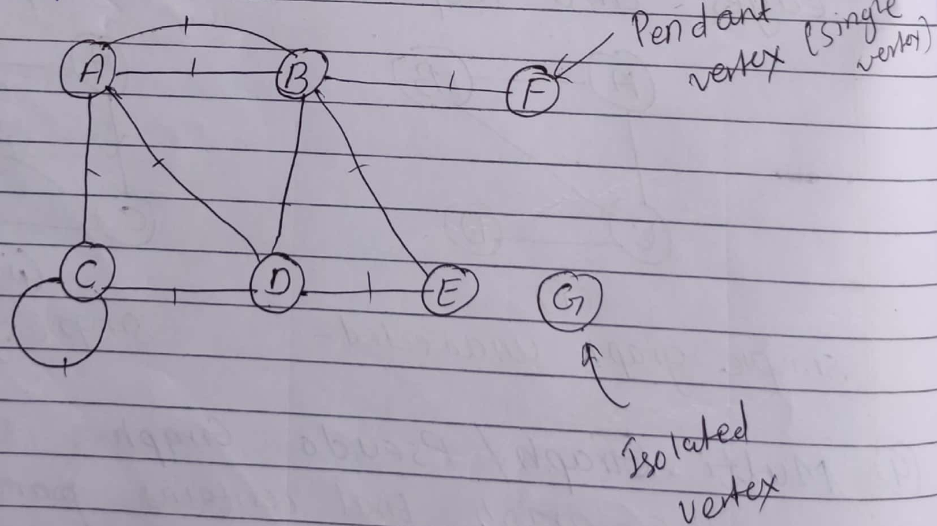


(ii)

Degree of Vertex in Undirected Graph

The degree of vertex in an undirected graph is sum of all the edges incident to that vertex. A loop at a vertex contributes degree 2.

Example:



deg (V)

deg (A)	4
deg (B)	5
deg (C)	4
deg (D)	4
deg (E)	2
deg (F)	F
deg (G)	0
$\sum$ deg(V)	20

$|E| = 10$

Theorem 1: Hand-Shaking Theorem

Let, $G = (V, E)$ be an undirected graph then

$$2 * |E| = \sum_{v \in V} deg(v)$$

$$2 * |E| = \sum_{v \in V_1} deg(v) + \sum_{v \in V_2} deg(v)$$

Even = (even + even)

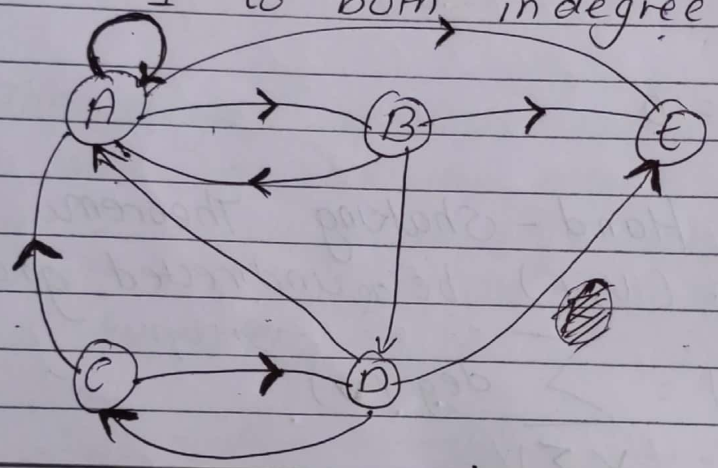
Theorem 2:

Undirected graph has even no. of vertices with odd degree.

① Degree of vertex in directed Graph

① Degree of vertex in directed graph is sum of indegree & outdegree.

- loop at a vertex contributes degree 1 to both indegree & outdegree.



In-degree ($deg^-(v)$)	Out-degree ($deg^+(v)$)	Total ($deg(v)$)
$deg^-(A) = 4$	$deg^+(A) = 3$	7
$deg^-(B) = 1$	$deg^+(B) = 3$	4
$deg^-(C) = 1$	$deg^+(C) = 2$	3
$deg^-(D) = 2$	$deg^+(D) = 3$	5
$deg^-(E) = 3$	$deg^-(E) = 0$	3
$\Sigma\, deg^-(v) = 11$	$\Sigma\, deg^+(v) = 11$	$\Sigma\, deg(v) = 22$

Theorem 3:

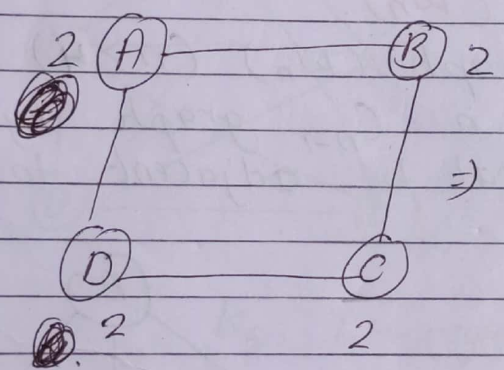
Let $G = (V, E)$ be a directed graph.

$$|E| = \sum_{v \in V} deg^+(v) = \sum_{v \in V} deg^-(v)$$

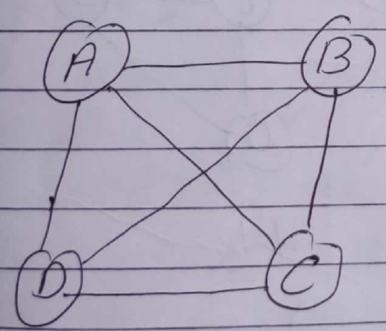
Some Special Type of Graph

1. Regular Graph

A graph is said to be regular if degree of ~~each~~ every vertex is equal or same.



⇒ 2-regular Graph.

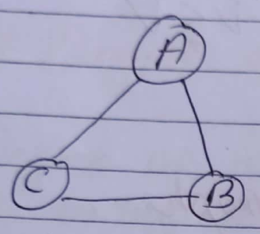


⇒ 3-regular Graph.

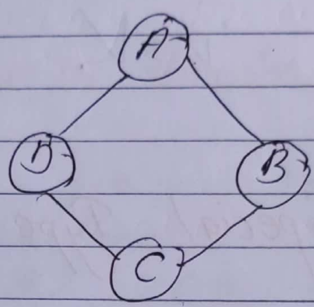
4.

2. Cycle Graph (C_n):

A graph with 'n' vertices ($n \geq 3$) is called cycle graph, if all edges forms a cycle of length 'n'.



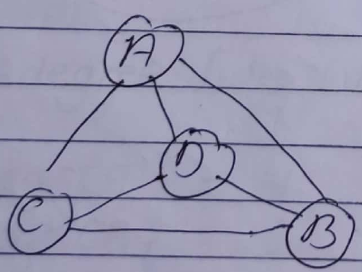
C_3



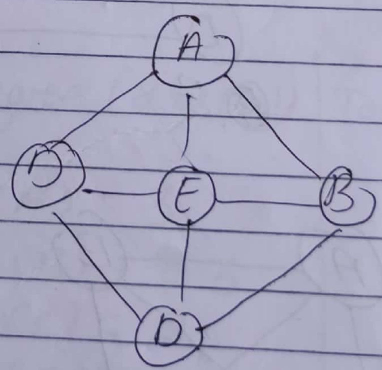
C_4

3. Wheel Graph (W_n):

A wheel graph (W_n) ($n \geq 4$) is formed ~~from a~~ from a C_{n-1} graph by adding one vertex that is adjacent to all other vertices C_{n-1}.



W_4



W_5

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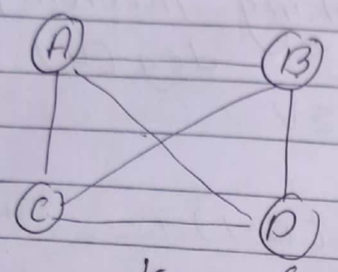
4. Complete Graph (K_n):

A complete graph with n vertices is a type of graph that contains exactly 1 pair ~~of~~ is betn each pair of distinct vertices.

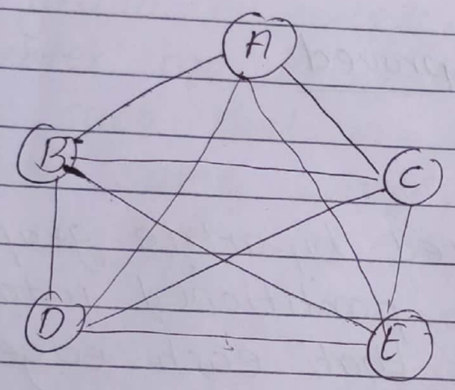


K_4 (3-regular)

$n = 4$

$E = \frac{4 \times 3}{2}$

$= 6$



K_5 (4-regular)

Properties:

- A complete graph K_n is always $n-1$ regular graph.
- A complete graph is a simple graph with max^m no. of edges.
- The total edges in a complete graph is given by;

$$\boxed{E = \frac{n * (n-1)}{2}}$$

Prove that total no. of edges in a complete graph is $E = \frac{n \times (n-1)}{2}$

Soln

From Handshaking Theorem;

$$2 \times E = \sum_{v \in V} \deg(v)$$

$$2E = (n-1) + (n-1) + (n-1) + \cdots + (n-1)$$

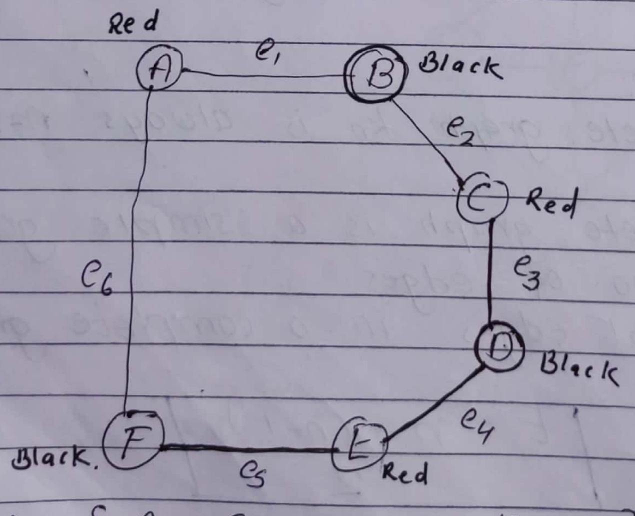
$$2E = n \cdot (n-1)$$

$$E = \frac{n \times (n-1)}{2}$$

Hence proved

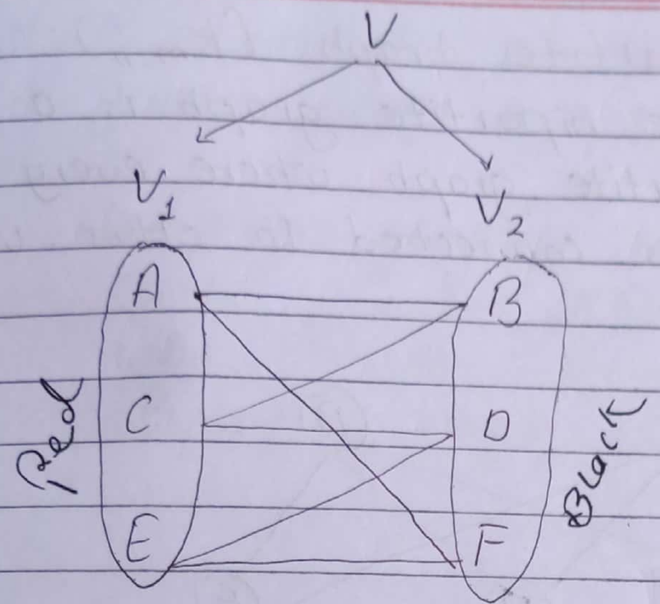
5. Bipartite Graph:

A graph is called bipartite graph if its vertex set can be partitioned into two disjoint sets such that each edge in a graph connects a vertex in one set to a vertex in another set.



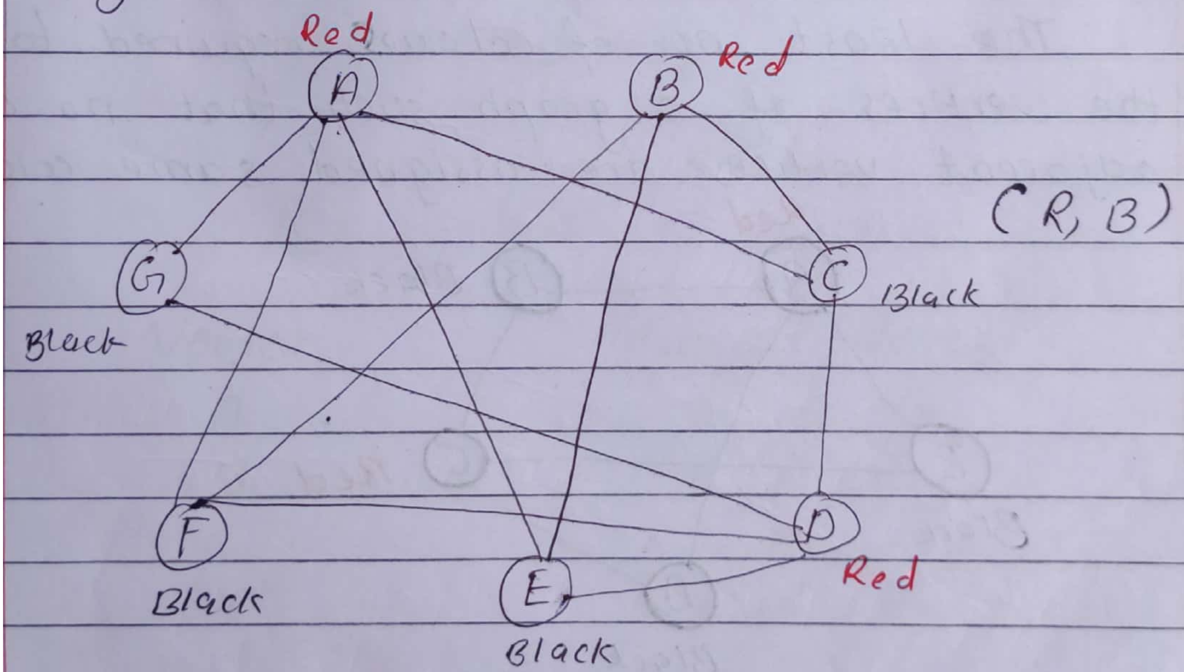
$V = \{A, B, C, D, E, F\}$

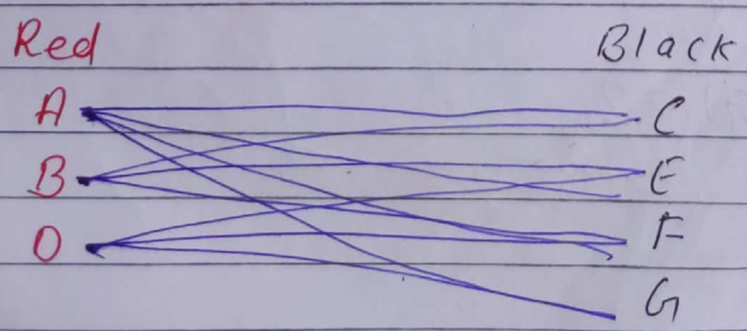
Theo
A
its
colour
assig
Black



Theorem 4:

A graph is called bipartite if and only if its vertex can be coloured by using exactly two colours such that no any adjacent vertices are assigned same colour.

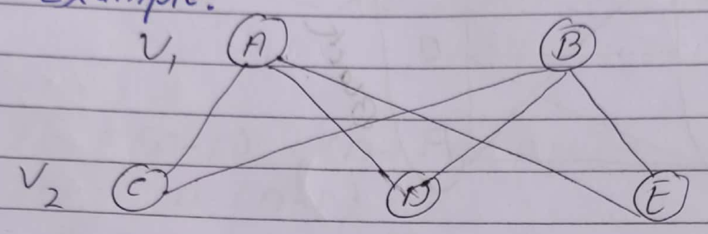




f. complete Bipartite Graph ($K_{m,n}$):

A complete bipartite graph is a special type of bipartite graph where every vertex in one set is connected to other vertex in another set.

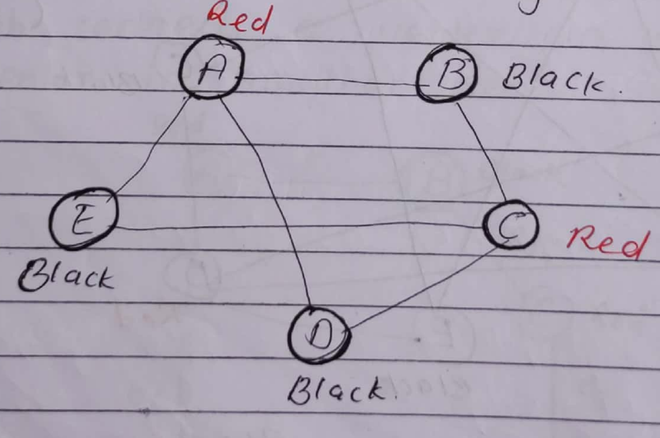
Example:



$\therefore K_{2,3}$

Chromatic Number:

The least no. of colours required to colour the vertices of a graph such that no any adjacent vertices are assigned same colour.



$\chi(G) = 2$

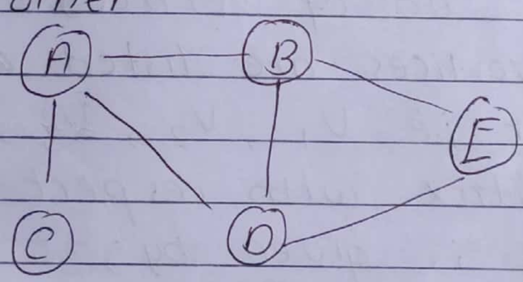
What is chromatic no. of complete graph, cycle graph, wheel graph & bipartite graph?

Complete graph = $\chi(G) = n$
Cycle graph = $\chi(G) =$
Wheel graph = $\chi(G) =$
Bipartite graph = $\chi(G) = 2$

Graph Representation

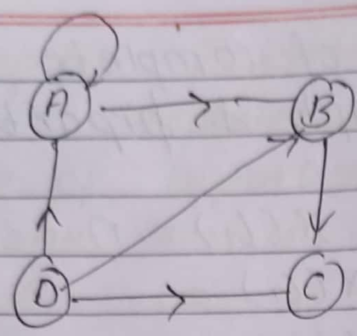
Adjacency List:

It specifies the vertices that are adjacent to each other.



Vertex	Adjacent Vertex
A	C, B, D
B	A, D, E
C	A
D	A, B, E
E	B, D

for undirected



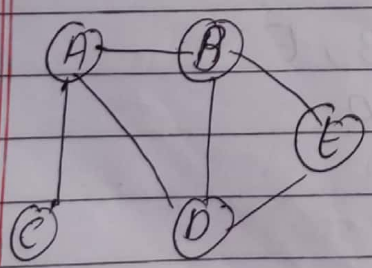
Initial Vertex	Terminal Vertex
A	A, B
B	C
C	—
D	A, B, C

b. Adjacency Matrix:

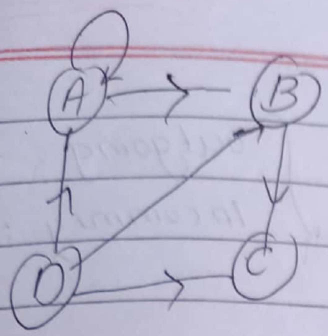
Let $G = (V, E)$ be a simple graph where no. of vertices $= |V|$ i.e $n = |V|$. Suppose the vertices are listed as in some arbitary order i.e $V_1, V_2, V_3, \ldots, V_n$. The adjacency matrix with respect to this order in a vertices is given by;

$$M = [m_{ij}]_{n \times n} \text{ where,}$$

$$m_{ij} = \begin{cases} 1; & \text{if } (V_i, V_j) \text{ is an edge} \\ 0; & \text{otherwise} \end{cases}$$



$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}_{5 \times 5} \end{array}$$



$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ M = B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{4\times4} \end{array}$$

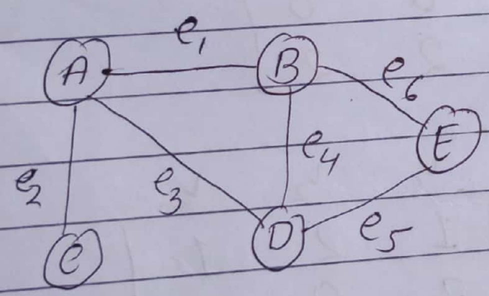
- The adjacency matrix for undirected graph is always symmetrical but not the case for directed graph.

c. Incidence Matrix:

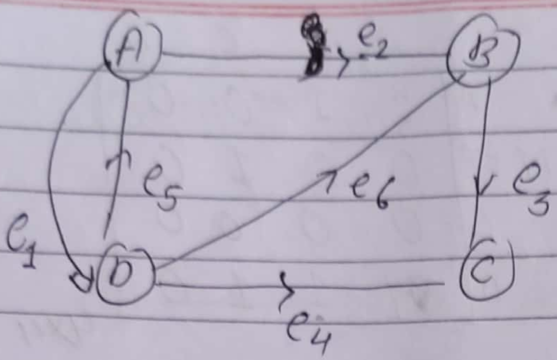
Let $G = (V, E)$ be a graph suppose the vertices are listed as $v_1, v_2, \ldots v_n$ and The edges are listed as $e_1, e_2, \ldots e_m$ & incidence matrix wrt this ordering of edges & vertices is given by;

$$I = [I_{ij}] \text{ where}$$

$$I_{ij} = \begin{cases} 1; & \text{if } e_j \text{ is incident on } v_i \\ 0; & \text{otherwise.} \end{cases}$$



$$I = \begin{array}{c c} & \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{5\times6} \end{array}$$



outgoing : +1
Incomming : -1

$$
I = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
+1 & +1 & 0 & 0 & -1 & 0 \\
0 & -1 & +1 & 0 & 0 & -1 \\
0 & 0 & -1 & -1 & 0 & 0 \\
-1 & 0 & 0 & +1 & +1 & +1
\end{bmatrix}
\end{array}
$$

4×6.

Qn.1. Draw the graph represented by following adjacency matrix.

$$
A = \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array}
\begin{bmatrix}
1 & 2 & 1 \\
2 & 0 & 0 \\
0 & 2 & 2
\end{bmatrix}
$$

Solution,

Here,

$$
A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array}
\begin{array}{c}
\begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\
\begin{bmatrix}
1 & 2 & 1 \\
2 & 0 & 0 \\
0 & 2 & 2
\end{bmatrix}
\end{array}
$$

Then,

$$
|A| = 1 \begin{vmatrix} 0 & 0 \\ 2 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}
$$

$$
= 0 - 2 \times 4 + 1 \times 4
$$

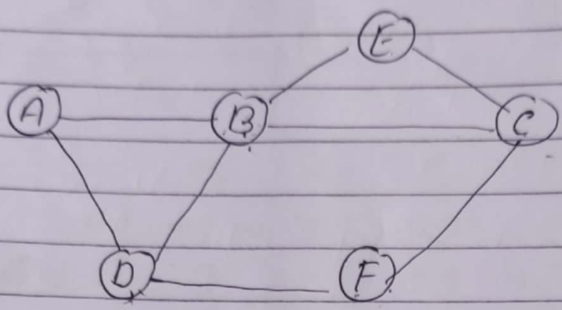
$$
= -8 + 4
$$

$$
= -4 \neq 0
$$

$$A^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

Graph Connectivity

a. Walk.



Both vertices & edges can repeat in a walk.
It is defined as finite alternating sequence of vertices and edges. It are two types of walk. They are:

1. Open walk:
If starting and ending vextex of walk is different, it is called open walk.
Example: " A – B – C – E – B – A – D "

2. Closed walk:
If starting and ending vextex of a walk is same it is called closed walk.
Example: " A – B – C – F – D – A "

b. Trial
It is an open walk in which vertices can repeat but not the edges.

Example. " A – B – C – E – B – D ".

Circuit:

It is defined as closed trial.

Eg: "A-B-C-E-B-D-A"

Path

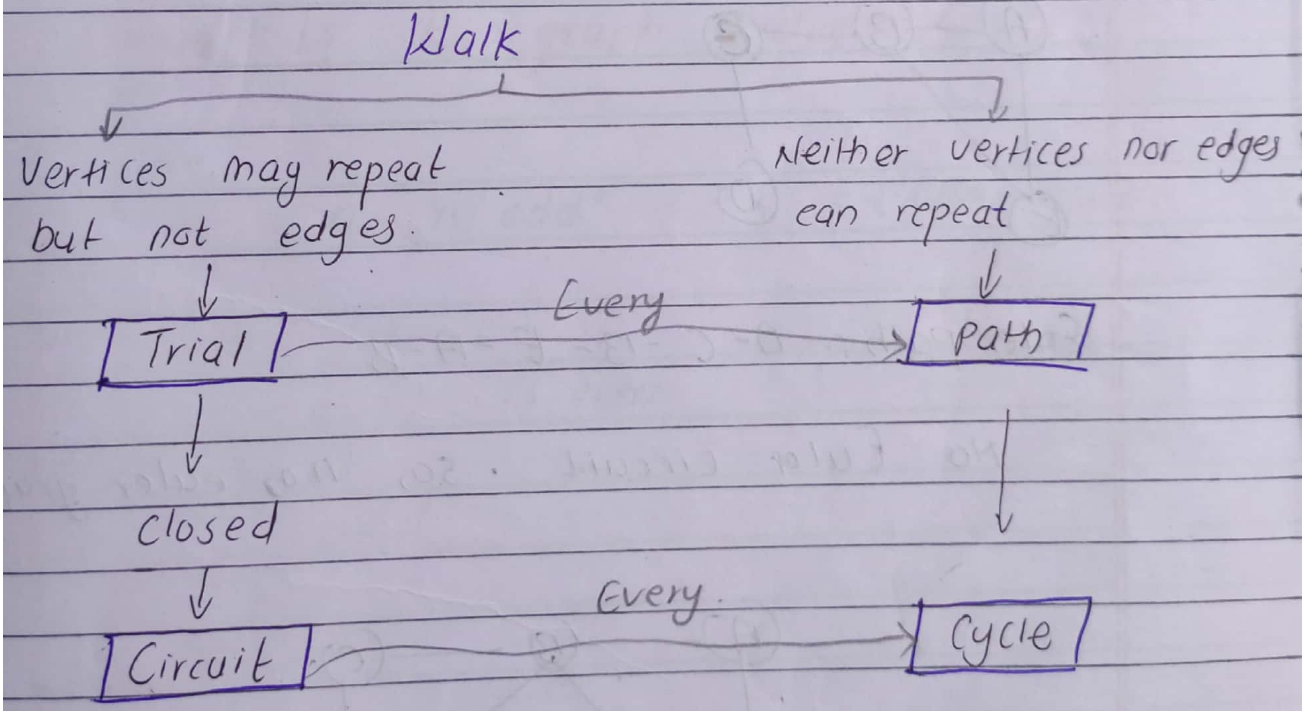
It is an open walk in which neither vertices nor any edges are allowed to repeat.

Eg: "A-B-C-F-D"

Cycle:

It is defined as closed path.

Example : "A-B-D-A"



V.V.Imp.

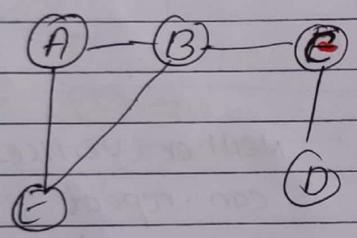
Euler Graph

- Euler path
  It is defined as the ~~path~~ trial that visits every edges in the graph exactly once.

- Euler circuit.
  A closed euler path is known as Euler circuit.

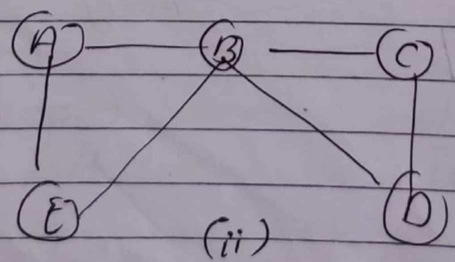
A graph that contains Euler circuit that is known as Euler graph or Eulerian graph.

V.imp Qn.1



Euler path: D-C-B-E-A-B

No Euler circuit. so, no euler graph.



Euler circuit: A-B-C-D-B-E-A

Theorem 1: Necessary & sufficient conditions for Euler's circuit.

A graph contains Euler's circuit if & only if degree of every vertices is even.

Theorem 2: Necessary & sufficient conditions for Euler's path.

A graph contains Euler's path if & only if it contains exactly two vertices with odd degree and this vertices are starting and ending vertices of a path.

Where is K_n graph Eulerian?

$\downarrow$

Eulerian circuit

$\downarrow$

all vertices even degree.

Soln

Here,

n is odd

K_n,

$(n-1)$ is even

$\uparrow$

odd.

√ - imp

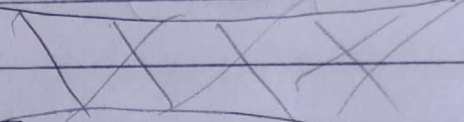
Date
Page

Qn.2 When is $K_{m,n}$ graph Eulerian ?

Soln

Here,

$K_{m,n}$ = Total 'm' vertices V_1



Total 'n' vertices V_2.

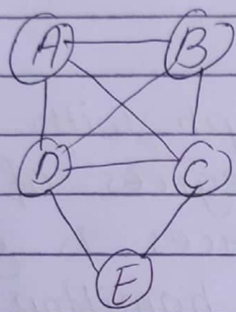
V_1 = degree 'n'

V_2 = degree 'm'

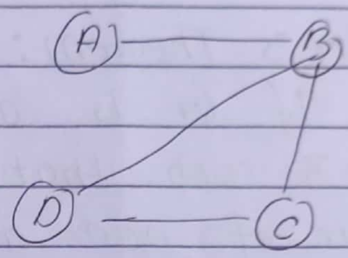
Hamilton Graph:

Hamilton Path ⇒ It is a path by which we can visit every vertex exactly once.

Hamilton Circuit ⇒ It is a closed hamilton path.



(i)



(ii)

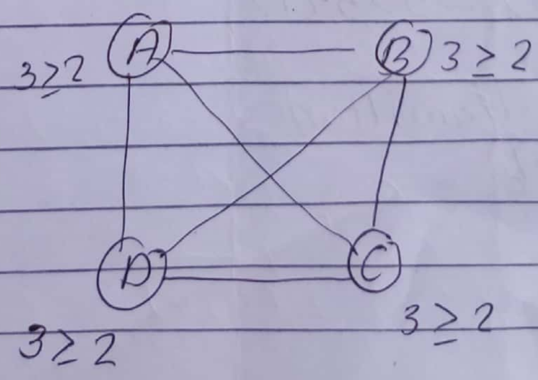
Hamilton ckt
= A − B − C − E − D − A

Hamilton path = A − B − C − D

Sufficient Condition for Hamilton Graph:

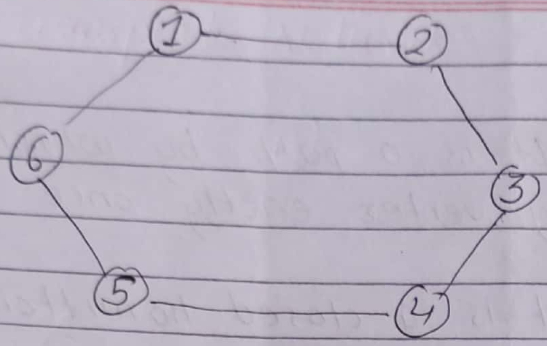
Dirac's Theorem:

If G is a simple graph with 'n' vertices and $n \geq 3$ such that degree of every vertex in G is atleast $n/2$, Then 'G' has hamilton circuit.



$n = 4$

⇒ A − B − D − C − A



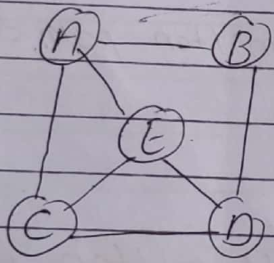
$n = 6/_2$

$2 \geq 3$ (F)

$\Rightarrow 1-2-3-4-5-6-1$

2\. Ore's Theorem:

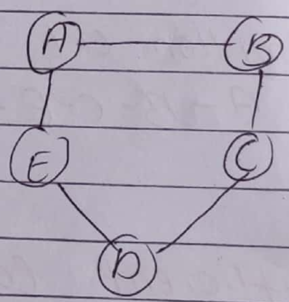
If G is a simple graph with n vertices $n \geq 3$ such that sum of degrees of every pair of non-adjacent vertices is greater or equal to n. Then, G has hamilton circuit.



Total vertices $(n) = 5$

- $A+B = 3+2 = 5 \geq 5$ (T)
- $C+B = 3+2 = 5 \geq 5$ (T)
- $E+B = 3+2 = 5 \geq 5$ (T)

$\therefore$ It has Hamilton circuit.



Total vertices $(n) = 5$

- $A+C = 2+2 = 4 \geq 5$ (F)

When is $K_{m,n}$ graph Hamilton Graph?

Soln

$K_{m,n}$ graph is hamilton if & only if $m = n \geq 2$

Proof:

Transport Network

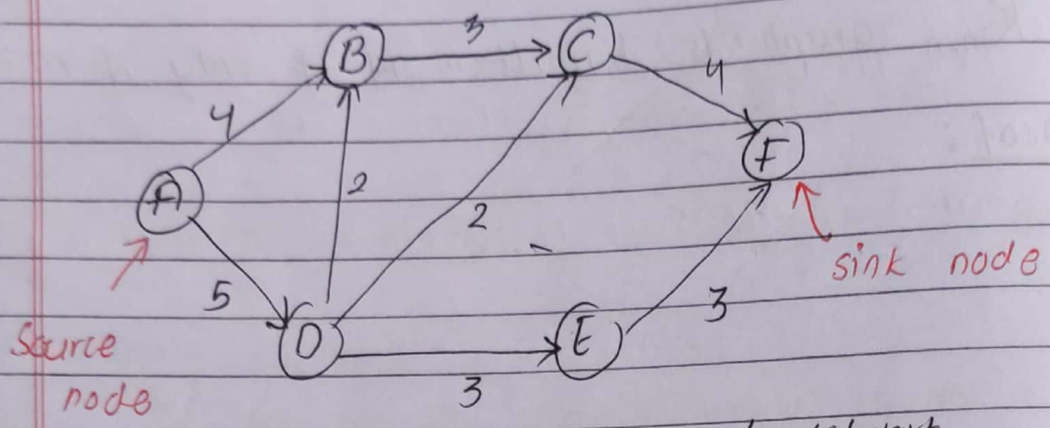


fig: Transport Network

A transport network is a connected diagraph with following properties.

(a) There is a node with indegree 0. known as source
(b) There is a node with outdegree ~~is~~ 0 known as sink.
(c) Each edge has a label that represents maximum capacity of a flow that can be sent from one node to another.

Flow

A flow in a network is a function that assigns to each edge a non-negative number subject to following constraints.

(a) Capacity Constraint

For each edge (u,v), the flow $F(u,v)$ is less than or equal to capacity $C(u,v)$.

ie $\boxed{0 \leq F(u,v) \leq C(u,v)}$

(b) Conservation Constraint:

For each node, except source node & sink node, the total flow incoming to that node should be equal to outgoing flow.

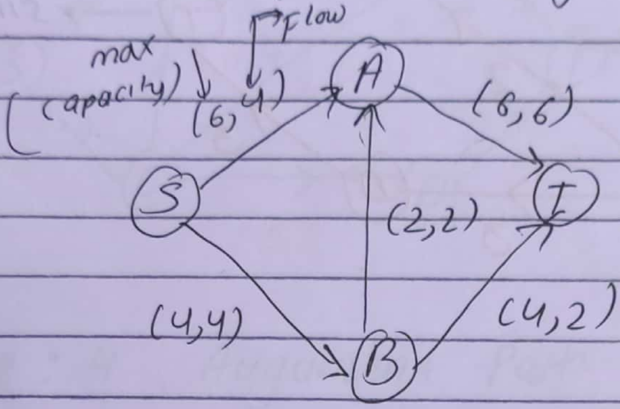


fig: Flow network.

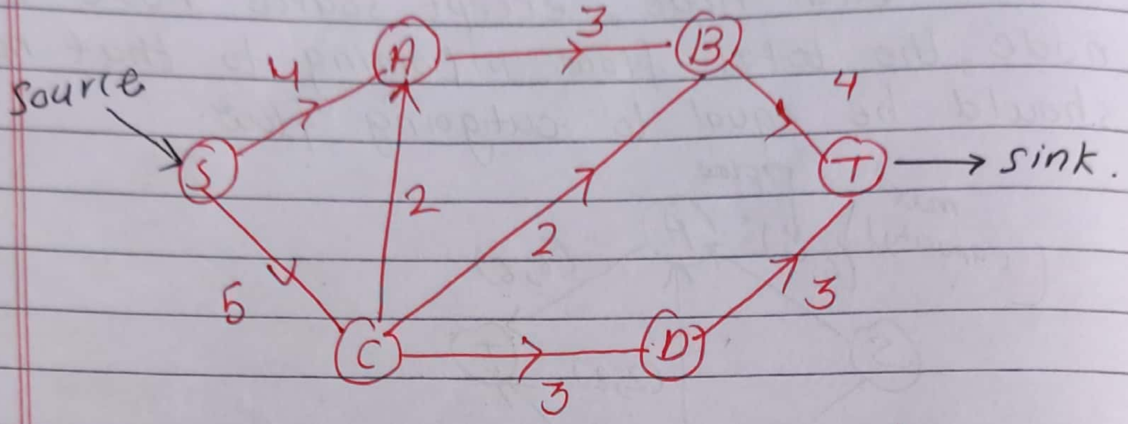
V. Imp

Ford - Fulkerson's Algorithm.

Initialize all the flow to zero.
constructor residual graph.
Find augmenting path in the residual graph.
If no augumenting path excists the algorithm terminates.
① Determine bottleneck capacity of the ~~augen~~ augmenting path.
~~Upgr~~ Update the flow along augmenting path by increasing the flow on each foward edge with bottleneck capacity.
Update the residual graph to reflect net flow.
Repeat step 3 to 6 until no more augmenting path is formed.
The maxm flow is the sum of flow of all edges leaving source node

8 -marks.

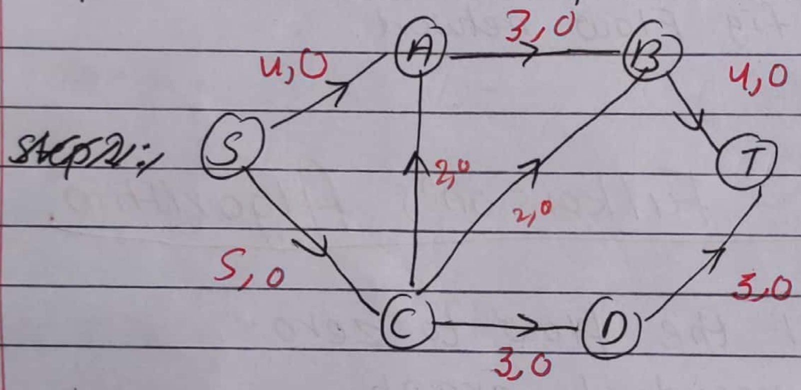
Qn. Find maximum flow in given network.



Solution

Here

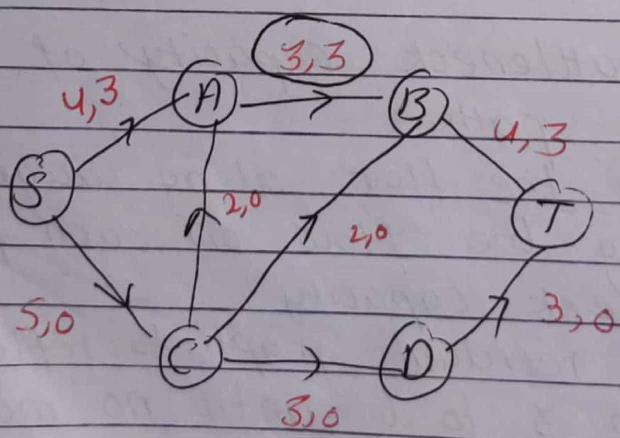
step 1: set all the flow to zero.



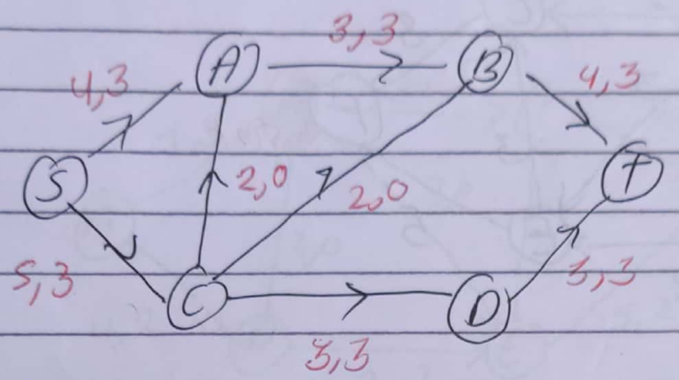
step ~~2~~: Augumenting path:

S — A — B — T : 3

~~step 3:~~

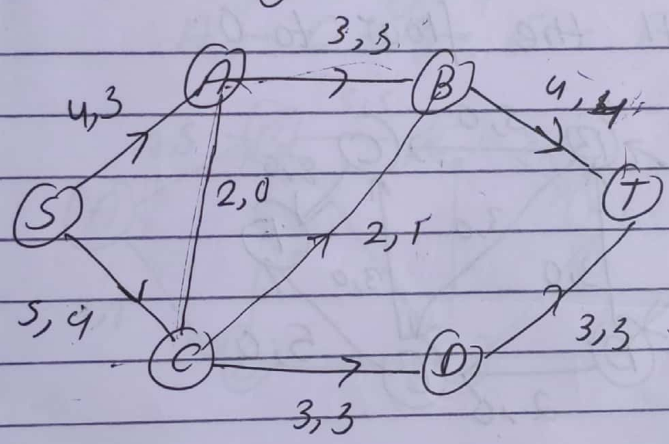


Step 3: Augumenting Path:
S - C - D - T : 3



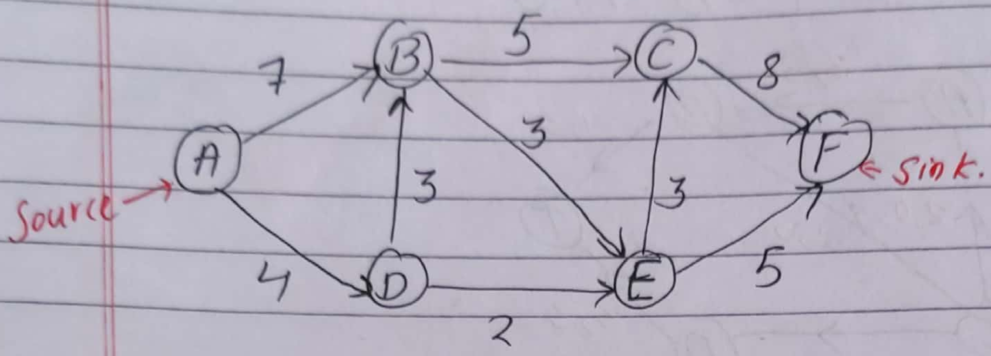
Step : 4 Augument Path:

S - C - B - T : 1



Step : 5 max flow : 7

Qn.2 Find maximum flow in given network.

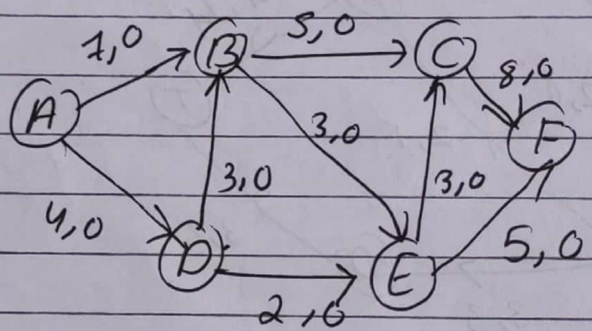


Solution

Here,

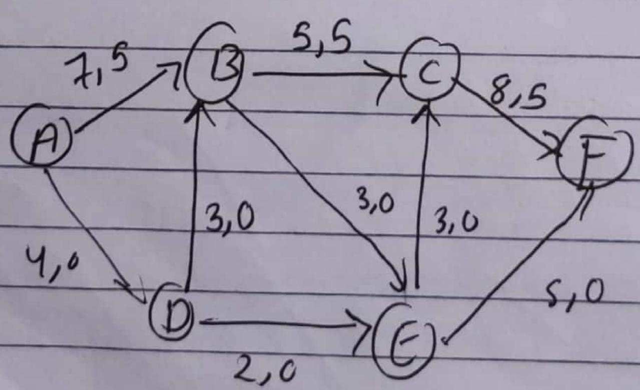
~~Step~~ 1: set all the flow to 0.

~~step 2~~:

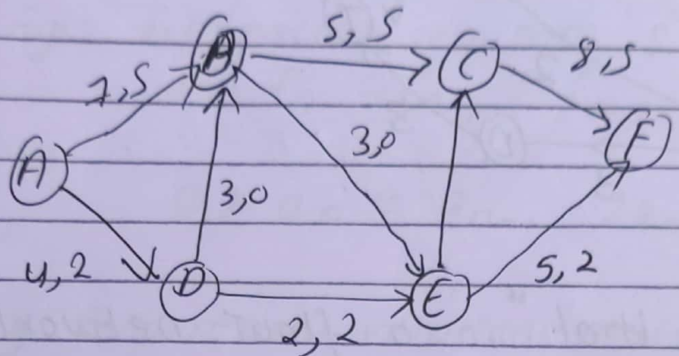


Step 2: Augumenting path:

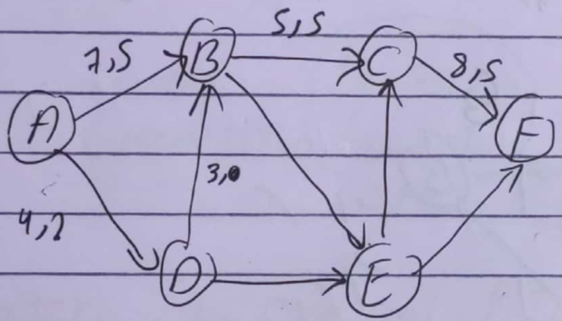
A − B − C − F : 5



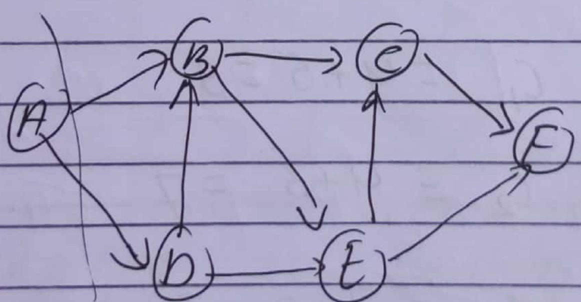
Step 3: Aumenting path:
A - D - E - F : 2



Step 4: Augumenting path:
A - D - B - E - F : 2

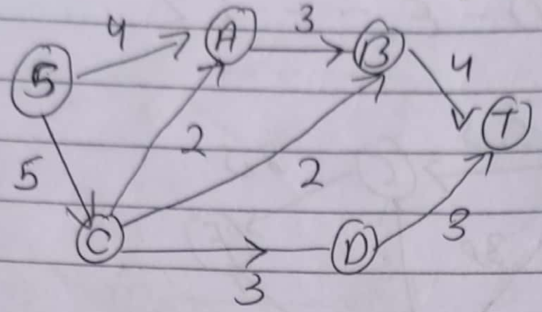


Step 5: Augumenting path:
A - B - C - E : 1

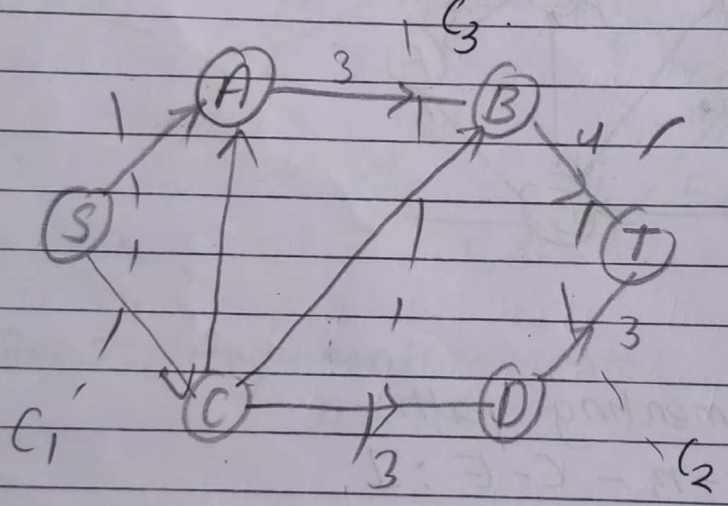


Step 6: maximum flow : 5 + 2 + 2 + 1 = 10.

Max flow Min-cut Theorem.



It states that "in a flow network, the maximum amount of flow that can be sent from source to sink is equal to the minimum capacity of the cut seperating the source & sink node."



cut : $C_1 = 4+5 = 9$

$C_2 = 4+3 = 7$

$C_3 = 3+2+3 = 8$

chapter 3 (2 questions)

Recurrence Relation

A sequence that can be expressed in terms of its previous element is called recurrence relation.

Eg: fibonacci sequence

(a) $f_n = f_{n-1} + f_{n-2}$

(b) $a_n = a_{n-1} + 2a_{n-2}$

Every Recurrence relation must have initial condition.

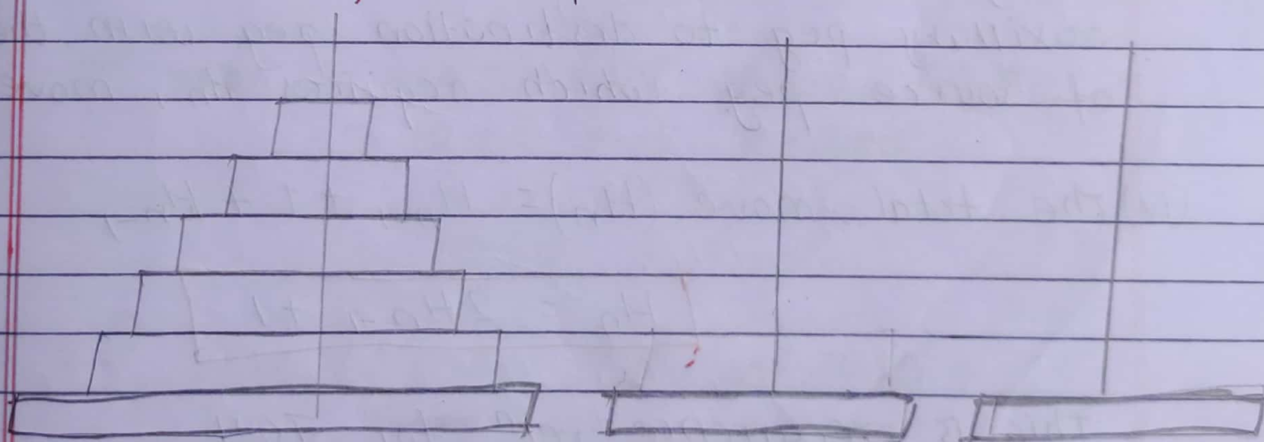
fibonacci sequence

$f_n = f_{n-1} + f_{n-2}$

initial condition. $f_1 = 0$, $f_2 = 1$

V.imp

Tower of Hanoi (TOH):



Source - Peg (A) | Auxillary Peg (B) | Destination peg (C)

TOH is the mathematical puzzle where we have 3 Peg (rod) and 'n' disk'.

The problem is to move n-disk from source-peg to destination peg with the help of auxillary peg by following below rules.

a. Only one disk can be moved at a time.
b. Only topmost disk can be moved.
c. larger disk cannot placed over smaller disk.

The steps for solving TOH puzzle are [Let H_n be the total move]

(i) First we have to move n-1 disk from source peg to auxillary peg with the help of destination peg which requires H_{n-1} move.

(ii) Now, we have to move the largest disk from source peg to destination peg which requires 1 move.

(iii) Lastly, we have to move n-1 disk from auxillary peg to destination peg with the help of source peg which requires H_{n-1} move.

(iv) The total move $(H_n) = H_{n-1} + 1 + H_{n-1}$

$$\boxed{H_n = 2H_{n-1} + 1}$$

This is recurrence relⁿ for TOH.

$$H_n = 2[2H_{n-2} + 1] + 1$$

$$H_n = 2^2 H_{n-2} + 2 + 2^0$$

$$H_n = 2^2 (2H_{n-3} + 1) + 2^1 + 2^0$$

source-peg
auxillary-
ne.
disk.
Let H_n be the total moves
ource -
destination
k from
quires
help

$$H_n = 2^3(H_{n-3}) + 2^2 + 2^1 + 2^0$$

$$H_n = 2^{n-1} H_1 + \cdots + 2^2 + 2^1 + 2^0$$

$$H_n = 2^{n-1} + \cdots + 2^2 + 2^1 + 2^0$$

$$H_n = 2^0 \frac{[2^n - 1]}{(2-1)}$$

$$\boxed{H_n = 2^n - 1}$$

Qn.1 Suppose you deposited rupees 1000 at an interest rate of 5% compounded annually. What is the value of investment after 4 years.

Soln

Principal (P) = Rs 1000
rate (R) = 5%
Time (T) = 4 years.

1st year = 1000 + (5% of 1000)
= $1000 + \frac{5}{100} \times 1000$
= 1050

2nd year = 1050 + 5% of 1050
= $1050 + \frac{5}{100} \times 1050$
= 1102.5

3rd year = 1102.5 + 5% of 1102.5
= 1157.625

4^{th} year = 11.57.625 + 5% of 11.57.625

$= 11.57.625 + 0.05 \times 11.57.625$

= Rs 1215.50

$\therefore$ Rs 1215.50 is the value of investment after 4 years.

Qn2. Suppose that a person invest Rs 2000 at 14% compounded annually.
(a) Find the recurrence reln.
(b) Find initial condition.
(c) Find A_1, A_2 & A_3
(d) Find ~~exclus~~ explicit formula.
(e) How long will it take for a person to ~~inve~~ double the intial investment.

Soln

Given,

Principal (P) = Rs 2000

Rate (R) = 14%.

(a) $A_1 = A_0 + 0.14 A_0$

$= 1.14 A_0$

$A_2 = A_1 + 0.14 A_1$

$= 1.14 A_1$

$\therefore \boxed{A_n = 1.14 A_{n-1}}$

(b) A_0 (intial condition) = ~~1.14x~~

Rs 2000

$A_1 = 1.14 A_0 = 1.14 \times 2000 = 2280$ Rs.

$A_2 = 1.14 A_1 = 1.14 \times 2280 =$ Rs 2599.2

$A_3 = 1.14 A_2 = 1.14 \times \cancel{2963.088}$ Rs 2963.088

2599.2.

)

$$A_2 = 1.14 A_1$$

$$A_2 = 1.14 (1.14 A_0)$$

$$A_2 = (1.14)^2 A_0$$

⋮

$$\boxed{A_n = (1.14)^n A_0}$$

By question.

$$A_n = (1.14)^n A_0$$

$$\Rightarrow 2 A_0 = (1.14)^n A_0$$

$$\Rightarrow 2 = (1.14)^n$$

$$\Rightarrow n = 5.2 \approx 5 \text{ years.}$$

Qn3. A patient is injected with 160 mL of a drug. Every 6 hours 25% of the drug passes out of a blood stream. To compensate further 20 mL of dose is given every 6 hours.

(a) Find recurrence relation for the amount of drug in the blood stream.

(b) Use the reln to find the amount of drug remaining after 24 hours.

Soln

(a) Initial dose (U_0) = 160 ml

After 6 hrs, the amount of drug,

$$U_1 = (75\% \text{ of } U_0) + 20$$

$$U_1 = 0.75\, U_0 + 20$$

$$\vdots$$

$$U_n = (0.75)U_{n-1} + 20$$

(b)

After 6 hrs, $U_1 = 0.75 \times U_0 + 20$
$= 0.75 \times 160 + 20$
$= 140$ ml.

After 12 hrs, $U_2 = 0.75 \times U_1 + 20$
$= 0.75 \times 140 + 20$
$= 125$ ml.

After 18 hrs, $U_3 = 0.75 \times U_2 + 20$
$= 0.75 \times 125 + 20$
$= 113.75$ ml

After 24hrs $(U_4) = 0.75 \times U_3 + 20$

$= 0.75 \times 113.75 + 20$

$= 105.31 ml.$

V.v.imp

Linear Homogeneous Recurrence Relation

$a_n = C_1 a_{n-1} + C_2 a_{n-2} + C_3 a_{n-3} + \dots + C_k a_{n-k}$ —(i)

$C_k \neq 0$ is called Linear homogeneous Recurrence Relation of degree 'K'.

Solution

~~Example: $a_n = a_{n-1}$~~

Here,

Let $a_n = r^n$ ---(ii) be the soln of (i).

From (ii) & (i);

$$\frac{r^n}{r^{n-k}} = \frac{C_1 r^{n-1} + C_2 r^{n-2} + C_3 r^{n-3} + \dots + C_k r^{n-k}}{r^{n-k}}$$

$\Rightarrow r^k = C_1 r^{k-1} + C_2 r^{k-2} + C_3 r^{k-3} + \dots + C_k$ —(iii)

Eqn (iii) is called characteristic eqn. and its root is called characteristic root.

Example:

$a_n = 3a_{n-1} + 2a_{n-2}$ (2)

$r^2 \quad 3r^1 + 3r^0$

$r^2 - 3r - 2 = 0$

$a_n = 4a_{n-2} + 6a_{n-3}$

$r^3 = 4r + 6.$

$a_n - 4a_{n-1} + 2a_{n-2} = 0$

$r^2 - 4r + 2 = 0$

Theorem I: Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be LHRR of degree 2. The characteristic Eqn,

$$r^2 - c_1 r - c_2 = 0$$

Let r_1 and r_2 be the characteristic root.

Case I: All roots distinct:

$$\boxed{a_n = \alpha_1 r_1^n + \alpha_2 r_2^n}$$

Case II: All roots same,

$$\boxed{a_n = [\alpha_1 + n\alpha_2] r^n.}$$

Case III: Imaginary

$$r = \alpha \pm i\beta$$

$$\boxed{a_n = [\alpha_1 \sin(n\theta) + \alpha_2 \cos(n\theta)] R^n}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

$$R = \sqrt{\alpha^2 + \beta^2}$$

CamScanner

Solve the following Recurrence relation.

$$a_n = 5a_{n-1} - 6a_{n-2} \; ; \; a_0 = 3, \; a_1 = 5$$

<u>Sol</u>

Sol^n

Here,

The given recurrence relation,

$$a_n - 5a_{n-1} + 6a_{n-2} = 0 \quad \text{---(i)}$$

The characteristic eq^n is;

$$r^2 - 5r + 6 = 0$$

$$r_1 = 2$$

$$r_2 = 3$$

Since all roots are distinct,

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$\boxed{a_n = \alpha_1 2^n + \alpha_2 3^n} \quad \text{---(ii)}$$

Using initial condition,

For $n=0$, $a_0 = \alpha_1 2^0 + \alpha_2 3^0$

$$\alpha_1 + \alpha_2 = 3 \quad \text{---(a)}$$

For $n=1$, $a_1 = \alpha_1 2^1 + \alpha_2 3^1$

$$2\alpha_1 + 3\alpha_2 = 5 \quad \text{---(b)}$$

Solving (a) & (b) we get,

$$\alpha_1 = 4$$

$$\alpha_2 = -1$$

Putting value of α_1 and α_2 in eq^n (ii).

$\boxed{a_n = 4 \cdot 2^n - 3^n}$ This is required sol^n.

Qn 2. $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 5$, $a_1 = 7$

Soln

Here the given recurrence relation,

$$a_n - 6a_{n-1} + 9a_{n-2} = 0 \quad \text{—(i)}$$

The characteristic eqn is

$$r^2 - 6r + 9 = 0$$

$$r_1 = 3$$
$$r_2 = 3$$

Since all roots are same,

$$a_n = [\alpha_1 + n\alpha_2] r^n$$
$$a_n = [\alpha_1 + n\alpha_2] 3^n \quad \text{—(ii).}$$

Using Initial condition,

For $n = 0$

$$a_0 = [\alpha_1 + n\alpha_2] 3^0$$
$$\alpha_1 = 5 \quad \text{—(a)}$$

For $n = 1$,

$$a_1 = [\alpha_1 + n\alpha_2] 3$$
$$3\alpha_1 + 3\alpha_2 = 7 \quad \text{—(b)}$$

Solving (a) & (b).

$$\alpha_1 = 5$$
$$\alpha_2 = -8/3$$

Putting value of α_1 & α_2 in eqn (ii).

$$a_n = \left[5 - \frac{8n}{3}\right] 3^n$$

This is required soln.

Qn 3

V.V. imp

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Find explicit formula for fibonacci series.

$$a_n = a_{n-1} + a_{n-2} \quad a_0 = 0, \; a_1 = 1$$

soln

Here,

The given recurrence relⁿ

$$a_n - a_{n-1} - a_{n-2} = 0 \quad \text{— (i)}$$

The characteristic eqn is

$$r^2 - r - 1 = 0$$

$$r_1 = \frac{1+\sqrt{5}}{2} \qquad r_2 = \frac{1-\sqrt{5}}{2}$$

Since, all roots are distinct,

$$a_n = [\alpha_1 r_1^n + \alpha_2 r_2^n]$$

$$a_n = \alpha_1\left[\frac{1+\sqrt{5}}{2}\right]^n + \alpha_2\left[\frac{1-\sqrt{5}}{2}\right]^n \quad \text{— (ii)}$$

Using Initial condition,

For $n=0$

$$0 = \alpha_1 + \alpha_2$$

For $n=1$

$$1 = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)$$

solving (a) & (b)

$$\alpha_1 = 0.44 = 1/\sqrt{5}$$

$$\alpha_2 = -0.44 = -1/\sqrt{5}$$

Putting value of α_1 & α_2 in eqn (ii)

$$a_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n + \frac{-1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

This is req solution.

1. $a_n + 2a_{n-1} + 2a_{n-2} = 0$ $a_0 = 0$, $a_1 = -1$

Soln Here, the given recurrence reln;

$$a_n + 2a_{n-1} + 2a_{n-2} = 0 \quad \text{---(1)}$$

The characteristic eqn is.

$$r^2 + 2r + 2 = 0$$

$$r = -1 \pm i \quad (\alpha \pm i\beta)$$

$$\alpha = -1$$

$$\beta = 1$$

since, all roots imaginary,

$$a_n = [\alpha_1 \sin(n\theta) + \alpha_2 \cos(n\theta)] R^n$$

$$R = \sqrt{\alpha^2 + \beta^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right) = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{-\pi}{4}$$

$$a_n = \left[\alpha_1 \sin\left(\frac{-\pi n}{4}\right) + \alpha_2 \cos\left(\frac{-\pi n}{4}\right)\right](\sqrt{2})^n.$$

$$n = 1$$

$$a_n = \left[\sin\left(\frac{-\pi n}{4}\right)\right](\sqrt{2})^n$$

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Theorem II : $a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3}$

characteristic Eqn :

$$r^3 - c_1 r^2 + c_2 r - c_3 = 0$$

Let r_1, r_2 and r_3 be the roots.

Case I : All different

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \alpha_3 r_3^n$$

Case II : All same.

$$a_n = [\alpha_1 + n\alpha_2 + n^2 \alpha_3] r^n$$

Case III : Any two same.

$$a_n = [\alpha_1 + n\alpha_2] r^n + \alpha_3 r_2^n$$

1. $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; with $a_0 = 2$, $a_1 = 5$, $a_2 = 15$.

Soln

The given recurrence reln

$$a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0 \quad \text{—(1)}$$

The characteristic eqn is ;

$$r^3 - 6r^2 + 11r - 6 = 0$$

$$r_1 = 1$$
$$r_2 = 2$$
$$r_3 = 3$$

Since, all roots are distinct.

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \alpha_3 r_3^n$$

Linear Non-homogenous recurrence relation:

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + \dots + c_k a_{n-k} + f(n) \text{ —(i)}$$

Homogenous part

non-homogenous part

Total solution (a_n) = Homogenous solution $[a_n(h)]$ + Particular solution $[a_n(p)]$

Theorem:

When $f(n) = \phi(n) \cdot b^n$ where,
$\phi(n)$ is a polynomial of degree 'r'

case I: If 'b' is not characteristic root.

$$a_n(p) = [A_0 + A_1 n + A_2 n^2 + \dots + A_r n^r] b^n$$

case II: If 'b' is characteristic root with multiplicity 'm'.

$$a_n(p) = n^m [A_0 + A_1 n + A_2 n^2 + \dots + A_r n^r] b^n$$

Qn.1 $a_n - 3a_{n-1} + 2a_{n-2} = 2^n$ with $a_0 = 0$, $a_1 = 1$.

Solution

Given,

$$a_n - 3a_{n-1} + 2a_{n-2} = 2^n \text{ —(i)}$$

Part I: The homogeneous part,

$$a_n - 3a_{n-1} + 2a_{n-2} = 0 \text{ —(ii)}$$

The characteristic eqn is

$$r^2 - 3r + 2 = 0$$

$$r_1 = 1$$

$$r_2 = 2$$

Since the roots are distinct,

$$a_n(h) = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$\boxed{a_n(h) = \alpha_1 + \alpha_2 2^n} \quad \text{——— (iii)}$$

Part II : Non-homogenous part:

$$f(n) = 2^n$$

$$f(n) = 1 \cdot 2^n = \phi(n) \cdot b^n$$

where,

$$\phi(n) = 1 n^0 \text{ ; degree } 0.$$

$$b^n = 2^n \text{ ; } b = 2$$

Since, 'b' is characteristic root with multiplicity '1'.

$$a_n(P) = n^1 [A_0] 2^n$$

$$a_n(P) = n A_0 2^n \quad \text{——— (iv)}$$

Now,

Putting $a_n(P)$ in eqn (i),

$$n A_0 2^n - 3(n-1) A_0 2^{n-1} + 2(n-2) A_0 2^{n-2} = 2^n$$

Dividing both sides by 2^n.

$$n A_0 - 3(n-1) A_0 2^{-1} + 2(n-2) A_0 2^{-2} = 1$$

$$\Rightarrow n A_0 - 3n A_0 2^{-1} + 3 A_0 2^{-1} + 2 n A_0 2^{-2} - 2^2 \cdot A_0 2^{-2} = 1$$

$$\Rightarrow n A_0 - \frac{3 n A_0}{2} + \frac{3}{2} A_0 + \frac{n A_0}{2} - A_0 = 1.$$

$$\Rightarrow 2n A_0 - 3n A_0 + 3 A_0 + n A_0 - 2 A_0 = 2$$

$$\Rightarrow -\cancel{n A_0} + 3 A_0 + \cancel{n A_0} - 2 A_0 = 2$$

$$\Rightarrow A_0 = 2$$

Now

Putting value of A_0 in eqn (iv)

$$a_n(p) = n \cdot 2^{n+1} \quad \text{—(v)}$$

For $n=0$,

$a_0 = \alpha_1 + \alpha_2$

$\alpha_1 + \alpha_2 = 0$ —(a)

For $n=1$,

$a_1 = \alpha_1 + 2\alpha_2 + 2$

$\alpha_1 + 2\alpha_2 + 4 = 1$

$\alpha_1 + 2\alpha_2 = -3$ —(b)

Solving (a) & (b)

$\alpha_1 = 3$

$\alpha_2 = -3$

Now,

$$a_n = 3 - 3 2^n + n 2^{n+1}$$

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Qn.2 $a_n - 6a_{n-1} + 8a_{n-2} = 3$; $a_0 = 10$, $a_1 = 25$

Soln

Given,

$$a_n - 6a_{n-1} + 8a_{n-2} = 3 \quad \text{—— (i)}$$

Part I: The homogenous part is

$$a_n - 6a_{n-1} + 8a_{n-2} = 0 \quad \text{—— (ii)}$$

The characteristic eqn is

$$r^2 - 6r + 8 = 0$$
$$r_1 = 2$$
$$r_2 = 4$$

Since, the roots are distinct,

$$\boxed{a_n(h) = \alpha_1 2^n + \alpha_2 4^n} \quad \text{—— (a)}$$

Part II : non-homogenous part,

$$f(n) = 3 \qquad \phi(n) \cdot b^n$$
$$= 3 \cdot 1^n$$

$\phi(n) = 3$; degree $= 0$

$b^n = 1^n$; $b = 1$

Since b is characteristic root with multiplicity '0'.

$$a_n(p) = [A_0] b^n$$
$$\boxed{a_n(p) = A_0} \quad \text{—— (b)}$$

From (i) & (b)

$$A_0 - 6A_0 + 8A_0 = 3$$
$$A_0 = 1$$

Putting $A_0 = 1$ in eqn (b)

$$\boxed{a_n(p) = 1}$$

Therefore,

$$a_n = a_n(p) + a_n(h)$$

$$a_n = \alpha_1 2^n + \alpha_2 4^n + 1 \quad \text{— (c)}$$

putting $n = 0$

$$a_0 = \alpha_1 2^0 + \alpha_2 4^0 + 1$$

or, $10 = \alpha_1 + \alpha_2 + 1$

or, $\alpha_1 + \alpha_2 = 9 \quad \text{— (iii)}$

Putting $n = 1$

$$a_1 = \alpha_1 2^1 + \alpha_2 4^1 + 1$$

or $25 = 2\alpha_1 + 4\alpha_2 + 1$

or, $\alpha_1 + 2\alpha_2 = 24/2$

or $\alpha_1 + 2\alpha_2 = 12 \quad \text{— (iv)}$

solving (iii) & (iv).

$$\alpha_1 = 6$$

$$\alpha_2 = 3.$$

Then,

$$a_n = 6 \cdot 2^n + 3 \cdot 4^n + 1$$

Qn.3. $a_n - 5a_{n-1} + 6a_{n-2} = (2+n)$; $a_0 = 1$, $a_1 = 1$

Sol^n

Given,

$$a_n - 5a_{n-1} + 6a_{n-2} = 2+n \quad \text{—(a)}$$

Homogenous eq^n of given eq^n is.

$$a_n - 5a_{n-1} + 6a_{n-2} = 0 \quad \text{—(b)}$$

Its characteristic eq^n is:

$$r^2 - 5r + 6 = 0$$

By solving,

$$r_1 = 2$$

$$r_2 = 3$$

$$a_n(h) = \alpha_1 2^n + \alpha_2 3^n \quad \text{—(1)}$$

$$f(n) = (2+n)\cdot 1^n$$

$$\phi(n) = 2+n \;;\; \text{degree} = 1$$

$$b^n = 1^n \;;\; b = 1.$$

$$a_n(p) = [A_0 + A_1 n] 1^n$$

$$a_n(p) = A_0 + A_1 n \quad \text{—(11)}$$

From (a) & (11).

$$[A_0 + A_1 n] - 5[A_0 + A_1(n-1)] + 6[A_0 + A_1(n-2)] = n^1 + 2n^0.$$

$$\Rightarrow [A_0 - 5A_0 + 6A_0]n^0 + [A_1 - 5A_1(n-1) + 6A_1(n-2)] = n^1 + 2n^0$$

$$\Rightarrow 2A_0 n^0 + A_1 n - 5A_1 n + 5A_1 + 6A_1 n - 12A_1 = n^1 + 2n^0$$

$$\Rightarrow 2A_0 n^0 + (A_1 - 5A_1 + 6A_1)n - 7A_1 = n^1 + 2n^0$$

$(2A_0 - 7A_1]n^0 + [2A_1]n^1 = 2n^0 + n^1.$

Now,

comparing coff of like terms;

$2A_0 - 7A_1 = 2$ — (iii)

$2A_1 = 1$ —— (iv).

or $A_1 = 1/2$

Putting value of A_1 in eqn (iii)

$$2A_0 - \frac{7 \times 1}{2} = 2.$$

$$\Rightarrow 2A_0 = 2 + \frac{7}{2}$$

$$\Rightarrow A_0 = 11/4$$

Then,

$$a_n(p) = 11/4 + \frac{n}{2}$$

∴ The required soln is

$$a_n = \alpha_1 2^n + \alpha_2 3^n + 11/4 + n/2.$$

Chapter-2

Relation & Functions

Cartesian Product

The cartesian product of two sets A and B is denoted by $A \times B$, which is set of all ordered pair (a, b) where $a \in A$ and $b \in B$

Mathematically,

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Relation

Let A and B be two sets. A relation from A ~~to~~ B is a subset of $A \times B$ ie

"$R \leq A \times B$"

Qn-1 let $A = \{1, 2, 3\}$ $B = \{6, 7, 8\}$

$R = \{(a, b) : a \text{ divides } b\}$

Solution

$A = \{1, 2, 3\}$

$B = \{6, 7, 8\}$

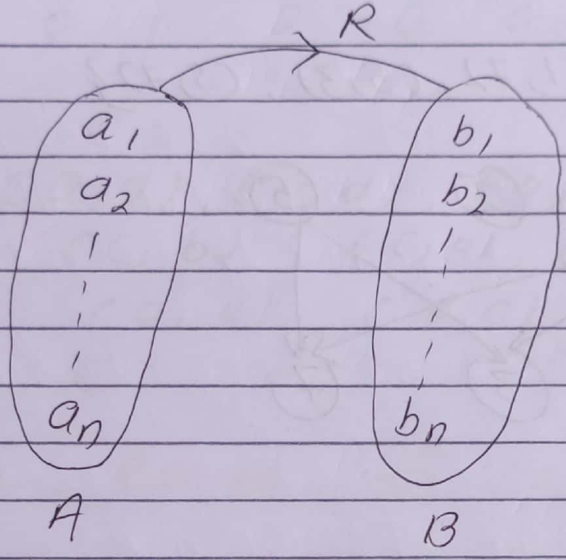
$A \times B = \{(1, 6), (1, 7), (1, 8), (2, 6), (2, 7), (2, 8), (3, 6), (3, 7), (3, 8)\}$

$R = \{(1, 6), (1, 7), (1, 8), (2, 6), (2, 8), (3, 6)\}$

Representing Relations:

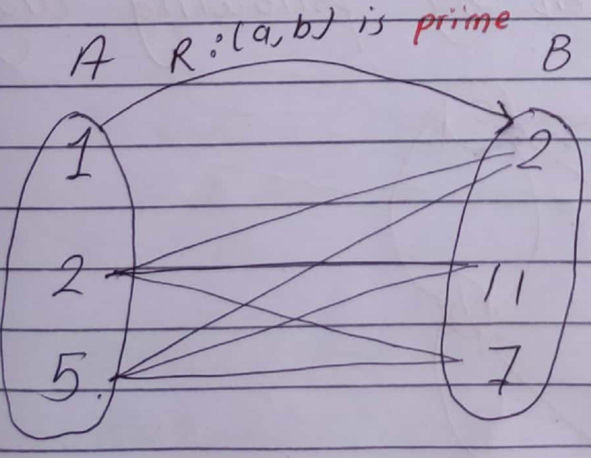
(a) Matrix of relation

Suppose we have a relation R betⁿ ~~set~~ to set A and B.



We can represent the relation R as a matrix M_R with n row and m column where (i, j) entry of a matrix is 1 if (a_i, b_j) is in R, 0 otherwise.

Qn. 2. Let $A = \{1, 2, 5\}$, $B = \{2, 11, 7\}$
$R = \{(a, b) : \text{'a' and 'b' is prime}\}$



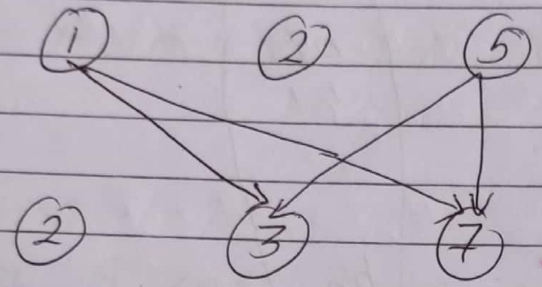
$$M_R = \begin{array}{c|ccc} R & 2 & 11 & 7 \\ \hline 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 \end{array}$$

b. Diagraph of relation

Qn.1 Let $A = \{1, 2, 5\}$ $B = \{2, 3, 7\}$
$R = \{(a, b) : 'a' \text{ and } 'b' \text{ is odd}\}$

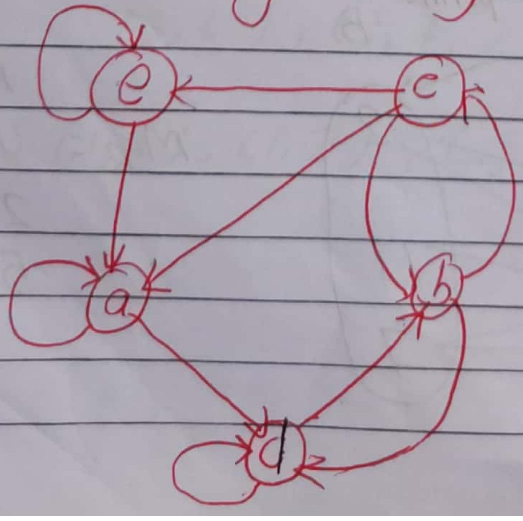
Soln

$R = \{(1, 3), (1, 7), (5, 3), (5, 7)\}$



To construct a diagraph for a relation R betn sets A and B, first draw a node for each element in A & B. Draw a directed edge from a node corresponding to a and to the node corresponding to b.

Qn 2. Let $A = \{a, b, c, d, e\}$ Find M_R & R. The relation R is given by following diagraph.



$$M_R = \begin{array}{c|ccccc} R & a & b & c & d & e \\ \hline a & 1 & 0 & 0 & 1 & 0 \\ b & 0 & 0 & 1 & 1 & 0 \\ c & 1 & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 0 & 1 & 0 \\ e & 1 & 0 & 0 & 0 & 1 \end{array}$$

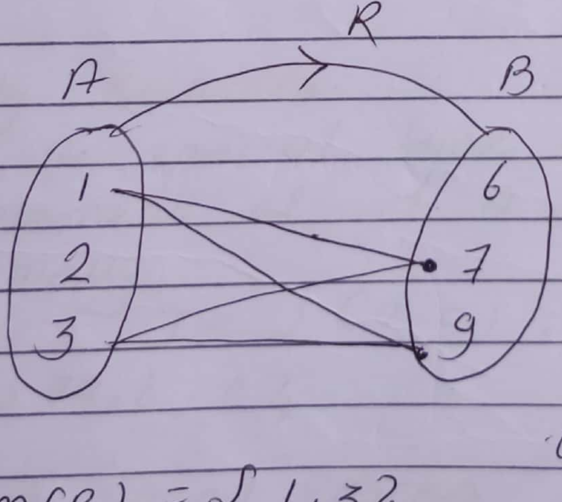
$R = \{(a,a), (a,d), (b,c), (b,d), (c,a), (c,b), (c,e), (d,b), (d,d), (e,a) \ (e,e)\}$

Domain, Range & co-domain

let, $A = \{1, 2, 3\}$
$B = \{6, 7, 9\}$

$R = \{(a,b) \mid$ 'a' is odd & 'b' is odd$\}$

$R = \{(1,7), (1,9), (3,7), (3,9)\}$



Co-domain $= \{6, 7, 9\}$

$Dom(R) = \{1, 3\}$

$Ran(R) = \{7, 9\}$

(a) Domain

The set which contains all the first element of all ordered pair of a relation R.

$$Dom(R) = \{a \in A : (a, b) \in R, \text{ for some } b \in B\}$$

(b) Range

The set which contains all the second element of all ordered pair of a relation R.

$$Ran(R) = \{b \in B : (a, b) \in R, \text{ for some } a \in A\}$$

(c) Co-Domain

The co-domain of relation R is set of all element in 'b' in B such that there is at least one element 'a' in A such that $(a, b) \in R$

Types of Relations :

Inverse Relations :

The inverse of relation R is denoted by R^{-1} which is defined as .

$$R^{-1} = \{ (b, a) : (a, b) \in R \}$$

$$R = \{ (1,2), (2,4), (3,2) \}$$

$$R^{-1} = \{ (2,1), (4,2), (2,3) \}$$

Complement Relations.

The complement of the relation R is denoted by R' or $\bar{R}$.

$$R' = \{ (a, b) : a \in A, b \in B \text{ and } (a, b) \notin R \}$$

$$A \times B = \{ (1,2), (1,3), (1,4), (2,2), (2,3), (2,4) \}$$

$$R = \{ (1,2), (1,4), (2,4) \}$$

$$R' = \{ (1,3), (2,2), (2,3) \}$$

Identity Relations

It is a special type of relation that relates every element of set A to itself.

Formally,

$$I = \{ (a, a) : a \in A \}$$

$$A = \{ a, b, c \}$$

$$I = \{ (a, a), (b, b), (c, c) \}$$

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d. Universal Relations

It is a special type of relation that relate every element of set A to every element of A itself.

$$U = \{(a,b) : a, b \in A\}$$

$$A = \{a, b, c\}$$

$A \times A = \{(a,b), (a,c), (a,a), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$

$\Rightarrow U = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$

e. Void Relation:

$$A = \{2, 4, 6\}$$
$$B = \{8, 2, 4\}$$

$$R = \{(a,b) : a + b = 0\}$$
$$R = \{\ \}$$

Imp Property of Relations

1. Reflexive and Irreflexive Relation

a. Reflexive relation

A relation R on set A is said to be reflexive if for all $a \in A$, $(a, a) \in R$

$$A = \{1, 2, 3\}$$

A X A	1	2	3
1	1, 1	1 2	1 3
2	2 1	2 2	2 3
3	3 1	3 2	3 3

$R_1 = \{(1,1), (2,2) (3,3)\}$ (reflexive)

$R_2 = \{(3,1), (1,2), (1,1), (2,1), (2,2), (3,3)\}$

$R_3 = \{(1,1), (2,2), (3,2)\}$

$R_4 = \phi$

$R_5 = A \times A$

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b. Irreflexive relation

A relation R on set A is said to be irreflexive if for all $a \in R$, $(a,a) \notin R$

$A = \{1, 2, 3\}$

$A \times A$	1	2	3
1	11	12	13
2	21	22	23
3	31	32	33

× $R_1 = \{(1,1), (2,2), (3,3)\}$ (not irreflexive)

✓ $R_2 = \{(3,1), (1,2), (1,3)\ (2,1)\}$ (irreflexive)

× $R_3 = \{(1,1), (2,3)\ (3,2)\}$ (not irreflexive)

✓ $R_4 = \phi$ (irreflexive).

× $R_5 = A \times A$ (not irreflexive)

$R_6 = \{(1,2), (3,1), (3,2)\}$ (irreflex..

2. Sym

(a) Sym
A
for

(b) ...
re

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Symmeteric, Assymmetric & Anti-Symmetric Relation.

Symmetric relation

A relation R on set A is said to be symmetric if for all $a \in A$ and $(a,b) \in R$, $(b,a) \in R$

~~Example~~ :

✓ $R_1 = \{(1,2),(2,1)\}$ (symmetric)

✓ $R_2 = \{(2,3),(3,2),(2,2),(3,3)\}$ (symmetric)

✓ $R_3 = \{(1,1),(2,2),(3,3)\}$ (symmetric)

✓ $R_4 = \phi$ (symmetric)

✓ $R_5 = A \times A$ (symmetric)

✗ $R_6 = \{(1,2)\ (2,3)\ (1,3)\}$ (not symmetric)

Asymmetric relation

A relation R on set A is said to be asymmetric relation if for all $a \in A$ and $(a,b) \in R$, $(b,a) \notin R$

Example: $n = \{1,2,3\}$

✗ $R_1 = \{(1,2),(2,1),(3,1)\}$ (not-Asymmetric)

✗ $R_2 = \{(2,3),(3,2),(2,2),(3,1),(1,3)\}$ (not asymmetric)

✗ $R_3 = \{(1,1),(2,2),(3,3)\}$ (not asymmetric)

✓ $R_4 = \phi$ (asymmetric)

✗ $R_5 = A \times A$ (not asymmetric

✓ $R_6 = \{(1,2),(2,3),(1,3)\}$ Asymmetric)

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c. Anti- Symmetric relation:

A relation R on set A is said to be anti-symmetric if for all $(a,b) \in A$, $(a,b) \in R$ and $(b,a) \in R$, then $a = b$

✓ $R_1 = \{(1,2), (2,2), (3,1)\}$ (Anti-symmetric)

× $R_2 = \{(2,3), (3,1), (2,2), (3,1), (1,3)\}$ (not anti-symmetric)

✓ $R_3 = \{(1,1), (2,2), (3,3)\}$ (anti-symmetric)

✓ $R_4 = \phi$ (anti-symmetric)

× $R_5 = A \times A$

✓ $R_6 = \{(1,2), (2,3), (1,3)\}$ (anti-symmetric)

~~V. imp~~

~~imp~~

Note:

Symmetric : $(a,b) = (b,a)$

Asymmetric : $(a,b) \neq (b,a)$

Anti- symmetric : $(a,b) \neq (b,a)$ but (a,a)

(8-marks)

Let A be Z (set of all integer) and a relation R is defined as

$$R = \{(a,b) \in A \times A : a < b\}$$

Determine whether the relation R is asymmetric, symmetric, anti-symetric

Soln

Here,

$$A = \{-\infty \ldots -2, -1, 0, 1, 2, \ldots +\infty\}$$

(a) Symmetric

A relation R on set A is said to be symmetric if for all ~~(a,b)~~ $(a \in A)$ and $(a,b) \in R$ then $(b,a) \in R$.

Suppose, $(a,b) \in R$.

$a < b$ (True)

Now, if (b,a) were to be in R then $b < a$ which is contradiction to $a < b$.

ie if $(a,b) \in R$ then $(b,a) \notin R$

$\therefore$ R is not symmetric

(b) Asymmetric

A relation R on set A is said to be asymmetric if for all $a \in A$ and $(a,b) \in R$ then $(b,a) \notin R$.

Suppose : $(a,b) \in R$

$a < b$ (True)

Now, if (b,a) were to be in R then, $b<a$ which is contradiction to $a<b$.

ie if $(a,b) \in R$, then $(b,a) \notin R$.

$\therefore$ R is Asymmetric.

(c.) Anti Symmetric:

A relation R on set A is said to be anti symmetric if for all $(a,b) \in R$ and $(b,a) \in R$, then $a=b$.

Suppose : $(a,b) \in R$

$a<b$ (True.

Now, if (b,a) were to be in R then $b<a$ which is contradiction to $a=b$.

i.e if $(a,b) \in R$, then $(b,a) \notin R$ and $(a,a) \notin R$

$\therefore$ R is Anti symmetric.

Date ____
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Q.n 2. Let A be Z (set of all integer) and relation R is defined as

$R = \{ (a,b) \in A \times A : (a/b) \text{ (a divides b)} \}$

Determine whether relation R is symmetric; asymmetric or anti symmetric?

Soln

Here,

$R = \{ (a,b) \in A \times A : a/b \}$

$A = \{ -\infty, \ldots -2, -1, 0, 1, 2, 3 \ldots, +\infty \}$

(a) Symmetric:

A relation ~~is said~~ R on set A is said to be symmetric if for all $a \in A$ and $(a,b) \in R$ then $(b,a) \in R$.

Suppose : $(a,b) \in R$ then ~~a/b~~

b/a = integer

$a \cdot/\cdot b = 0$

Now, if (b,a) were to be in R then

b/a = integer which is false.

i.e if $(a,b) \in R$ then $(b,a) \notin R$

$\therefore$ R is not - symmetric.

(b) Asymmetric.

A relation R on set A is said to be asymmetric if for all $a \in A$ and $(a,b) \in R$ and $(b,a) \notin R$

Suppose $(a,b) \in R$

$$b/a = \text{integer}$$

$$a \% b = 0$$

Now, if (b,a) wer to be in R then

b/a = integer which is false.

ie if $(a,b) \in R$ then $(b,a) \in R$

$\therefore$ R is not symmetric.

(c) Anti-asymmetric

A relation R on set A is said to be anti-asymmetric if for all $(a,b) \in A$, $(a,b) \in R$ and $(b,a) \in R$ then $a=b$.

Suppose $(a,b) \in R$ then $a \% b = 0$

Now if (b,a) were to be in R then $b \% a = 0$ which is contradiction.

$$a=b$$

$\therefore$ R is anti-symmetric.

V.V. Imp

Qn.3. Let A be Z^+ (set of all positive integer) and relation R is defined as.

$$R = \{(a,b) \in A \times A : a+b = \text{even}\}$$

Determine whether R is symmetric, asymmetric or anti symmetric?

Sol

Given

$$R = \{(a,b) \in A \times A : a+b = \text{even}\}$$

Transitive Relation

A relation R is on set A is said to be transitive if for all $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Let $A = \{1, 2, 3\}$

✓ $R_1 = \{(1,2), (2,1), (1,1), (2,2)\}$

✓ $R_2 = \{(1,1), (2,2), (3,3)\}$

✓ $R_3 = \{(1,2), (1,3)\}$

✓ $R_4 = \{(3,1), (2,3)\}$

✓ $R_5 = \phi$

✓ $R_6 = A \times A$

$R_7 = \{(1,3), (3,2), (2,2)\}$

× (not transisitive)

V. imp

Closure of a Relation

Closure of a relation refers to the process of adding all the necessary elements to the relation in order to make it satisfy a certain property.

Let R be the relation on set A. The closure of R with respect to property P is a new relation R' that satisfies following condition.

(a) R' contains all the ordered pair of R.
(b) R' must satisfy property P.
(c) R' is the smallest relation that satisfies condition (a) & condition (b).

(A) Reflexive closure

The reflexive closure of a Relation 'R' is given by,

$R' = R \cup D$; where $D = \{(a,a) / a \in A\}$ (diagonal element)

let $A = \{1,2,3\}$

$R = \{(1,2), (2,3) (2,2)\}$

$D = \{(1,1), (2,2), (3,3)\}$.

$R' = R \cup D = \{(1,2) (2,3) (2,2) (1,1) (3,3)\}$

(Reflexive closure)

(B) Symmetric Closure:

The symmetric closure of a Relation R' is given by,

$$R' = R \cup R^{-1} \text{ ; where } R' = \{(b, a) \mid (a,b) \in R\}$$

(Inverse of Relation R)

let $A = \{1, 2, 3\}$

$R = \{(1,2), (2,3), (2,2), (3,1), (2,1)\}$

$R' = \{(2,1), (3,2), (2,2), (1,3), (1,2)\}$

R' (symmetric closure) $= R \cup R' = \{(1,2), (2,1), (2,3), (3,2), (2,2), (3,1), (1,3)\}$

(C) Transitive Closure:

The transitive closure of a Relation R' is given by:

$$R^{\infty} = R^1 \cup R^2 \cup R^3 \cup \ldots R^n \text{ ; } n = |A|$$

imp

Warshall's Algorithm for Transitive closure

Qn1. Find the Transitive closure of Relation.

let $A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,3), (3,4), (2,1)\}$

Solution

Here,

step I: Construct a relation matrix, W_0

$W_0 =$

R	1	2	3	4
1	0	1	0	0
2	1	0	1	0
3	0	0	0	1
4	0	0	0	0

step II: First column of W_0 First Row of W_0

- W_0 has 1 entry on position '2' at column one $C = \{2\}$

- W_0 has 1 entry on position '2' at row one. $R = \{2\}$

$C \times R = \{(2,2)\}$

$W_1 =$

R	1	2	3	4
1	0	1	0	0
2	1	1	1	0
3	0	0	0	1
4	0	0	0	0

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step III : Second column of W_1, second Row of W_1,

- W_1 has 1 entry on position at column two : $C = \{1, 2\}$

- W_1 has 1 entry on position at row two : $R = \{1, 2, 3\}$

$C \times R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\}$

$W_2 =$

R	1	2	3	4
1	1	1	1	0
2	1	1	1	0
3	0	0	0	1
4	0	0	0	0

Step : IV
Third column of W_2 Third row of W_2.

- W_2 has 1 entry on position at column ~~thr~~ three : $C = \{1, 2\}$

- W_2 has 1 entry on position at row three : $R = \{4\}$

$C \times R = \{(1,4), (2,4)\}$.

$\longrightarrow$

$$
W_3 = \begin{array}{c|cccc}
R & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 & 1 \\
3 & 0 & 0 & 0 & 1 \\
4 & 0 & 0 & 0 & 0
\end{array}
$$

Step V : Fourth column of W_3 fourth row of W_3.

- W_3 has 1 entry on position (1, 2, 3) at column four : $C = \{1, 2, 3\}$
- W_3 has 1 entry on position ( ) at row four $R = \{ \}$

$C * R = \{ \quad \}$

Transitive closure $= \{(1,1), (1,2), (1,3)\ (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\}$

Q.n2. Let $A = \{a, b, c, d\}$ and $R = \{(a,b), (b,c), (c,d), (b,a)\}$. Find the transitive closure.

<u>Solution</u>

Here,

$$A = \{a, b, c, d\}$$
$$R = \{(a,b), (b,c), (c,d), (b,a)\}$$

Step I: construct a relation matrix W_0.

R	a	b	c	d
a	0	1	0	0
b	1	0	1	0
c	0	0	0	1
d	0	0	0	0

$W_0 =$ (matrix above)

Step II: First column of W_0 First Row of W_0.

- W_0 has 1 entry on position 2 at column one $C = \{b\}$
- W_0 has 1 entry on position 2 at row one $R = \{b\}$

$$C \times R = \{(b,b)\}$$

$W_1 =$

R	a	b	c	d
a	0	1	0	0
b	1	1	1	0
c	0	0	0	1
d	0	0	0	0

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step III: Second column of kl_1, second row of kl_1

- kl_1 has 1 entry on position at column two:
$C = \{a, b\}$

- kl_1 has 1 entry on position at row two:
$R = \{a, b, c\}$

$C \times R = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c)\}$

$$
\begin{array}{c|cccc}
R & a & b & c & d \\
\hline
a & & & & \\
b & & & & \\
c & & & & \\
d & & & &
\end{array}
$$

~~for~~ $kl_2 =$ (the table above)

Qn.3 Let $A = \{0, 1, 2, 3\}$ and $R = \{(0,1), (0,2), (1,1), (1,3)\ (2,2)\ (3,0)\}$

$S = \{(0,0), (0,3), (1,0), (1,2), (2,0), (3,2)\}$

Find Transitive closure of R and S using Warshall's Algorithm.

Solution

Here,

$A = \{0, 1, 2, 3\}$

$R = \{(0,1), (0,2), (1,1), (1,3), (2,2), (3,0)\}$

$S = \{(0,0), (0,3), (1,0), (1,2), (2,0), (3,2)\}$

V. imp

Equivalence Relations

A relation R on set A is said to be equivalence relation if it is reflexive, symmetric and transitive.

Let $A = \{a, b, c\}$

× $R_1 = \{\ \}$ (not equivalence)

✓ $R_2 = \{(a,a), (b,b), (c,c)\}$ (smallest possible equivalence reln)

× $R_3 = \{(a,a), (b,b), (c,c), (b,a)\}$

× $R_4 = \{(a,a), (a,c), (b,a), (c,a)\}$

✓ $R_5 = \{(a,a), (b,b), (c,c), (a,b), (a,c), (b,a), (c,a)\}$ (equivalence reln).

✓ $R_6 = A \times A$ (largest equivalence reln)

Qn 1. Let 'R' be the relation on set of real numbers such that aRb if and only if a-b is an integer. i.e.

$R = \{(a,b) : a-b \in Z\}$ Is 'R' is an equivalence relation?

Solution

$A = \{-\infty, \ldots\ldots +\infty\}$

$R = \{(a,b) : a-b \in Z\}$

(a) reflexive

For a relation to be reflexive, aRa must hold for any real numbers 'a'

i.e $a - a = 0$; which is integer.

$\therefore$ R is Reflexive.

(b) Symmetric

For a relation to be symmetric, if aRb holds than bRa must hold.

Let $(a,b) \in R$ then $a-b =$ integer
Now, for

$(b,a) \in R$, $b-a = -(a-b)$
$= \text{integer}$

$\therefore$ R is Symmetric.

(c) Transitive

For a relation to be Transitive, if aRb and bRc then aRc.

Let $(a,b) \in R$ then $a-b = \text{integer}$
$(b,c) \in R$ then $b-c = \text{integer}$
Now,

$(a-b)+(b-c) = \text{integer}$
$a-c = \text{integer}$

$\therefore$ R is Transitive

Since, R satisfies all property reflexsive symmetric and Transitive. It is equivalence Relation.

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Qn 2. Let 'R' be the relation on set of positive integers such that aRb if and only if a+b = even i.e

$R = \{(a,b) : a+b = even\}$. Is 'R' an equivalence relation?

Solution

$A = \{0, 1, 2, 3, ..., \infty\}$

$R = \{(a,b) : a+b = even\}$

(a) reflexive:-

For a relation to be reflexive, aRa must hold for any real number 'a'.

let $(a,a) \in R$

$a+a$

$= 2a$

$= even$

$\therefore$ R is reflexive.

(b) Symmetric

For a relation to be symmetric, if aRb holds than bRa must hold.

let $(a,b) \in R$ then $a+b = even$

From commutative property, $b+a = even$

i.e $(b,a) \in R$

$\therefore$ R is symmetric.

$\longrightarrow$

(c) Transitive

For a relation R to ~~set~~ be Transitive, if aRb and bRc then aRc.

let $(a,b) \in R$ then $a+b = even$

$(b,c) \in R$ then $b+c = even$

Now,

$$(a+b) + (b+c) = Even$$

$$a + 2b + c = Even$$

$$a + c = Even - 2b$$

$$a + c = Even$$

$\therefore (a,c) \in R$

Since, R satisfies all property reflexive, symmetric & Transitive. It is equivalence relation

3. Let 'R' be the relation on set of all integer such that aRb if and only if (a-b) is divisible by 5.

$R = \{(a,b) : (a-b) \text{ is divisible by } 5\}$

Is R is an equivalence relation?

Solution

$$A = \{\infty, \ldots\ldots, -\infty\}$$

$$R = \{(a,b) : 5 | (a-b)\}$$

(a) reflexive

For a relation to be reflexive, aRa must hold for any real number 'a'

Let $(a,a) \in R$

$a-a$
$= 0$
which is divisible by 5.
$\therefore$ R is reflexive.

(b) Symmetric

For a relation to be symmetric, if aRb holds then bRa must hold.

Let $(a, b) \in R$ then $a-b$ is divisible by 5

Let $(b, a) \in R$
$= b-a$
$= -(a-b)$ which is divisible by 5.

$\therefore$ R is symmetric.

(c) Transitive

For a relation to be Transitive, if aRb holds and bRc holds then aRc must hold

Let $(a, b) \in R$ then, $(a-b)/5$ = integer.
$(b, c) \in R$ then, $(b-c)/5$ = integer.

Now,
$$\frac{a-b}{5} + \frac{b-c}{5} = \text{Integer}$$
$$\frac{a-c}{5} = \text{Integer}$$

$\therefore (a, c) \in R.$

Since, R satisfies all property reflexive, Symmetric & Transitive. It is equivalence Relⁿ.

4. Let R' be the relation on set of all integers such that aRb if and only if (a+2b) is multiple of 3.

$R = \{(a,b) : (a+2b) = \text{multiple of } 3\}$.

Is R an equivalence relation?

Soln

let $A = \{a, c, d, e\}$

$R = \{(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (b,a), (d,e), (e,d)\}$.

reflexive : $a + 2a = 3a$.

Symmetric : $a + 2b = 3k$

(b,a)

$= b + 2a$

$= b + 2(3k - 2b)$

$= b + 6k - 4b$

$= 6k - 3b$

$= 3(2k - b)$

= multiple of 3.

Equivalence class

Let $A = \{a, b, c, d, e\}$

$R = \{(a,a), (b,b), (c,c), (d,d)\ (e,e)\ (a,b),$
$(b,a), (d,e), (e,d)\}$

Equivalence class

$[a]_R = \{a, b\}$

$[b]_R = \{a, b\}$ $\quad P_1 = \{a, b\}$

$[c]_R = \{c\}$ $\quad P_2 = \{c\}$

$[d]_R = \{d, e\}$ $\quad P_3 = \{d, e\}$

$[e]_R = \{d, e\}$

partition

Let R be the equivalence relation of the set. The equivalent class of 'a' with respect to R is given by

$$[a]_R = \{ s / s \in R \text{ and } (a, s) \in R \}$$

Qn 1 Let $A = \{1, 2, 3, 4, 5\}$ $R = \{(a, b) : a+b = \text{even}\}$
Find equivalence class?

Solution

Given

$$A = \{1, \cancel{4}, 2, 3, 4, 5\}$$

~~(1,1), (2,2)~~

~~R = {(1, 3), (1,5), (2,4), (3,5)}~~

$$R = \{(1,1), (2,2), (4,4), (3,3), (5,5), (1,3), (1,5), (2,4), (3,5)\}$$

$$[1]_R = \{1, 3\}$$

$$[2]_R = \{2, 4\}$$

$$[3]_R = \{1, 3, 5\}$$