Partial Order Relation
(POSET)

Let $A = \{1, 2, 3\}$

✓ $R_1 = \{(1,1), (2,2) (3,3)\}$
✗ $R_2 = \{(1,2), (1,1), (2,1), (2,2), (3,3)\}$
✓ $R_3 = \{(1,1), (1,2), (3,1), (2,2), (3,3)\}$
✗ $R_4 = \phi$
✗ $R_5 = \{(1,1), (1,2), (2,3), (3,3)\}$
✗ $R_6 = A \times A$

A relation R on set A is said to be partial order relation if it is reflexive, anti-symmetric and transitive.

reflexive : (a, a)
transitive : $(a, b) \wedge (b, c) \rightarrow (a, c)$
anti-symmetric : $\Delta \{(a, b) \wedge (b, a) \rightarrow a = b\}$

~~Ant Let~~

A set A together with the ~~boi~~ partial order Relation 'R' is known as posET (partially order set). (A, R)

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1. Let A be the set of integers show that greater or equal relation on the set A is partial order relation $(Z, \geq)$ (poset)

Solution $Z = \{-\infty, \cdots +2, +1, 0, -1, -2 \cdots +\infty\}$

$$R = \{(a,b) : a \geq b\}$$

(a) reflexive :

For a relation R to be reflexive aRa must hold i.e

$$(a,a) \in R$$

For $(a,a) \in R$, $a \geq a$ (True)

$\therefore$ R is reflexive.

(b) Anti-symmetric:-

For a relation "R" to be anti-symmetric if a R b and b Ra holds the a=b.

For $(a,b) : a \geq b$ — (i)

$(b,a) : b \geq a$ — (ii)

For (i) and (ii) to be true,

$a = b$

$\therefore$ R is anti-symmetric.

(c) Transitive

For a relation 'R' to be transitive if aRb and bRc hold then aRc must hold.

For $(a,b) : a \geq b$ — (i)

$(a,c) : b \geq c$ — (ii)

From transitive property of number,

$a \geq c$

$\therefore (a,c) \in R$.

$\therefore$ R is Transitive.

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2. Show that divisibility relation on the set of integers. $(Z, /)$.

Soln

$Z = \{-\infty, \ldots \bar{2}, \bar{1}, 0, +1, +2 \ldots +\infty\}$.

$R = \{(a,b) : a/b\}$ $\quad b/a = \text{integer}$.

(a) reflexive: For a relation R to be reflexive aRa must hold ie $(a,a) \in R$

For (a,a), $a/a = \text{integer}$ (True)

$\therefore$ R is reflexive.

(b) Anti-symmetric:

For a relation 'R' to be anti-symmetric if aRb and bRa holds then $a=b$.

For $(a,b) = a/b = \text{integer}$ — (i)

$(b,a) = b/a = \text{integer}$ — (ii)

For (i) and (ii) to be true, $a=b$.

$\therefore$ R is anti-symmetric.

(c) Transitive

For a relation 'R' to be transitive if aRb and bRc holds then aRc must hold.

For

$(a,b) : a/b = k$ — (i)

$(b,c) : b/c = l$ — (ii)

k, l are integer

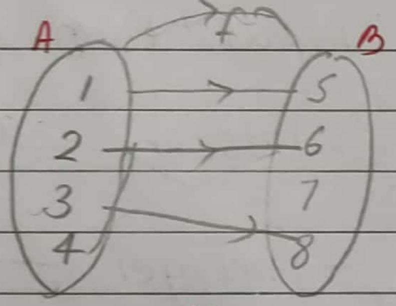
$$\frac{a}{b} \times \frac{b}{c} = kl$$

$$\frac{a}{c} = \text{Integer} \quad \text{ie } (a,c) \in R$$

$\therefore$ R is transitive.

FUNCTIONS

Let A and B be two non-empty sets. A function F defined from set A to set B denoted by $f: A \to B$ is a rule that assigns every element of A $(a \in A)$ to the unique element in set B.



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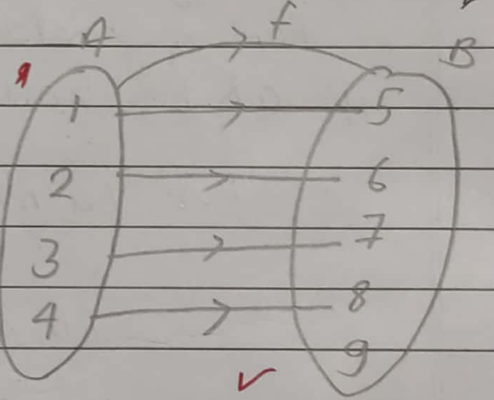
Reflexive but not function



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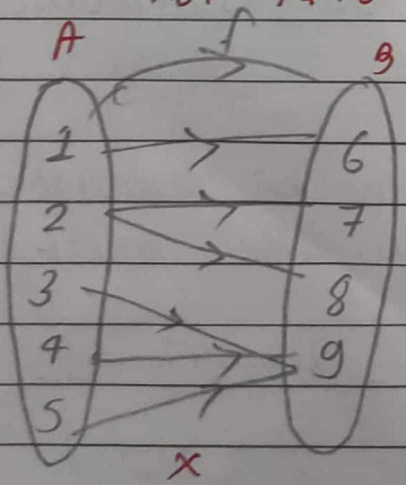
Function



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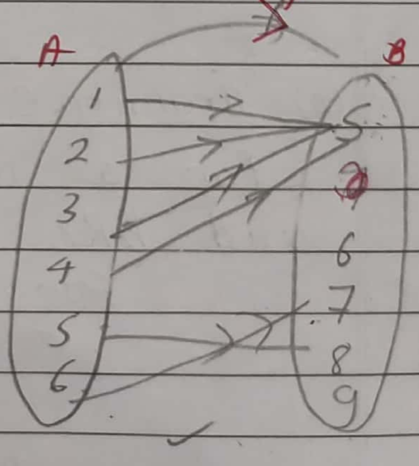
not function.

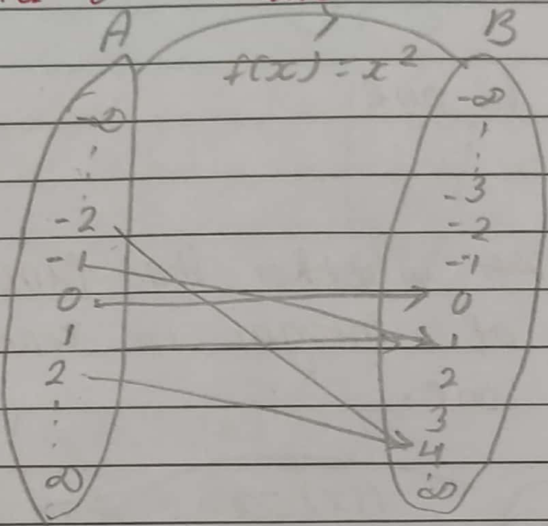


✓

function.

Domain = {1, 2, 3, 4, 5, 6}
Range = {5, 7, 8}
Co-domain = {5, 6, 7, 8, 9}

Qn. Let $f(x) = x^2$ where domain is set of integer, Find range and co-domain.



Range = {0, 1, 4, 8, 16 ...}
= perfect square.

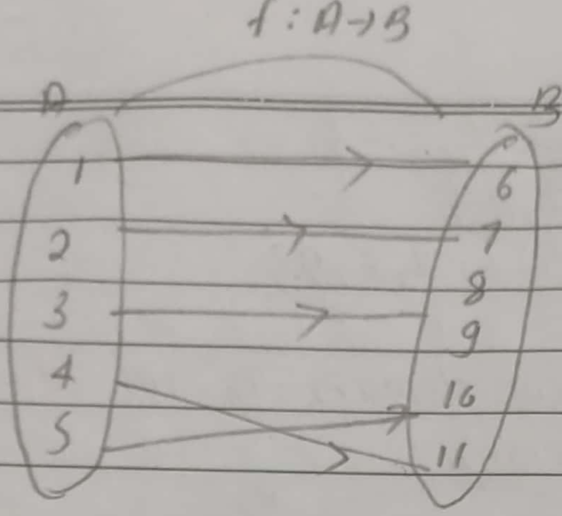
Co-domain = {$-\infty$, ... -2, -1, 0, 1, 2 ..., ∞}

Types of Functions.

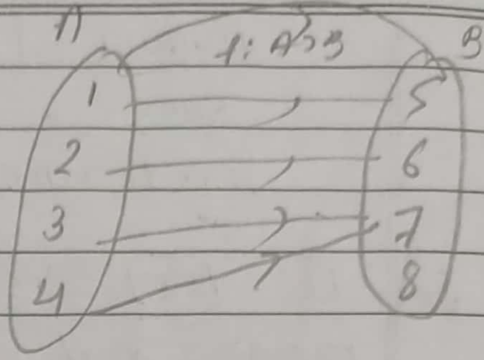
(1) One to one (Injective) Function:

A function f from set A to B is said to be one to one function if no two elements in set A are mapped to same element in set B.
i.e

$$\forall_{a \in A} \forall_{b \in A} \left[(a \neq b) \rightarrow f(a) \neq f(b) \right]$$

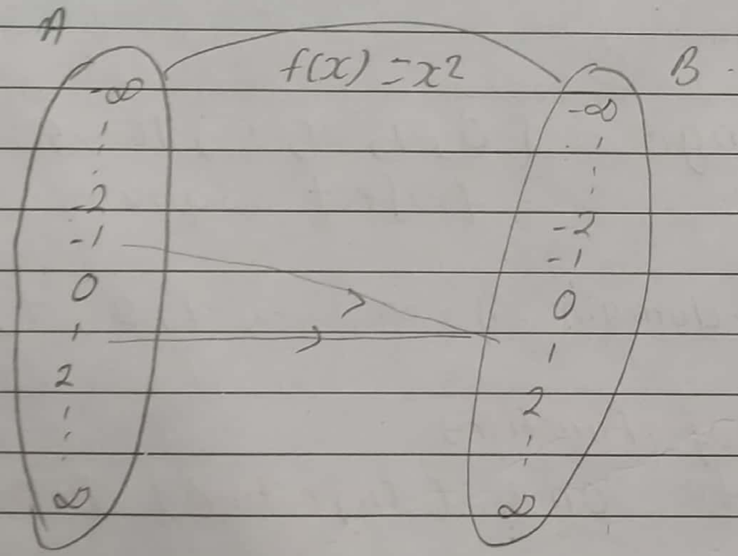


one to one



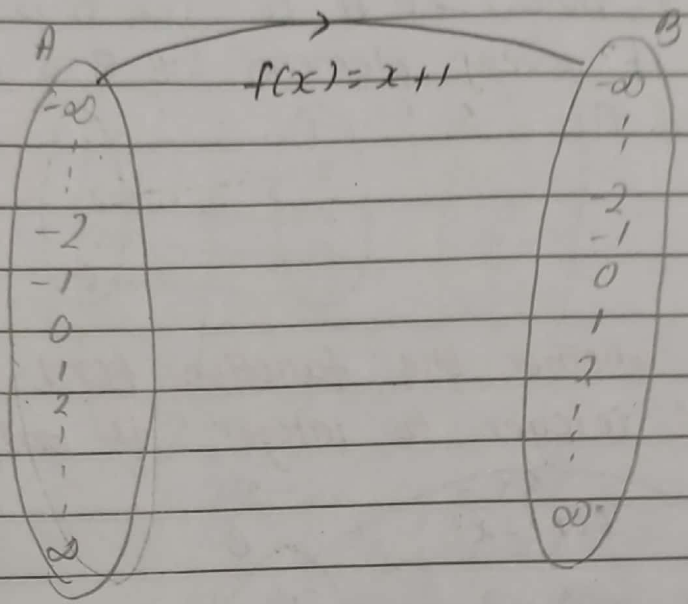
Not one to one.

Q. Determine ~~the~~ whether the function $f(x) = x^2$ from one set of integer to another set of integer is one to one.



Since, $f(-1) = f(1)$ not one to one func^n.

For $f(x) = x+1$



$$f(a) = a+1$$
$$f(b) = b+1$$

lets suppose

$$f(a) = f(b)$$
$$a+1 = b+1$$
$$a = b.$$

$\therefore f(x) = x+1$ one to one function

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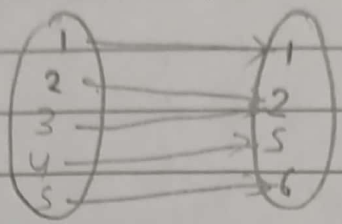
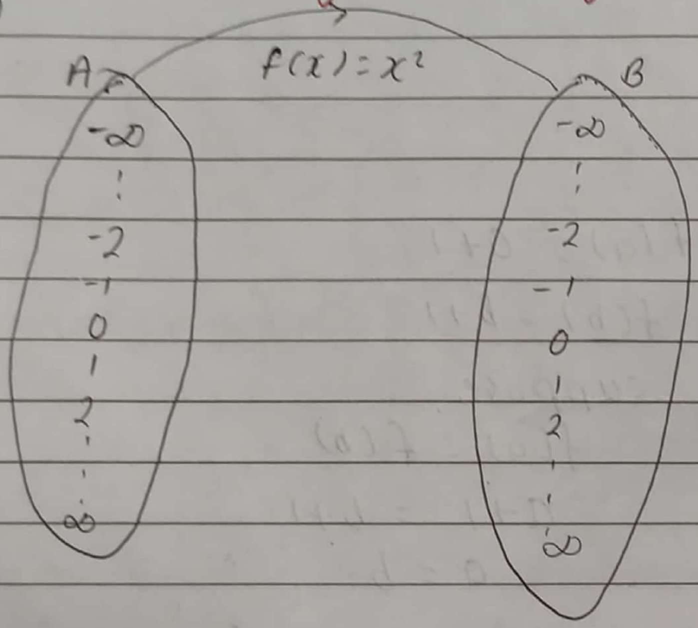
Onto function (SURJECTIVE function)

A function f from set A to set B is said to be onto function if every elements of B is mapped by elements of A.



Qn.1. Determine whether the function $f(x) = x^2$ from the set of integer to integer is onto function.

Soln



"Not onto"

Since the square of a number is always positive, the negative numbers in set B is not mapped by any elements in set A.

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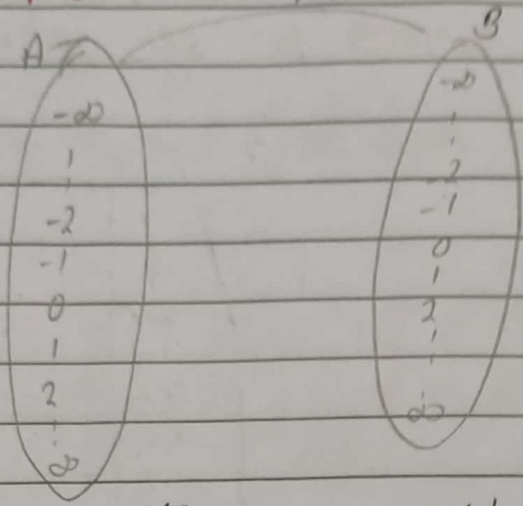
For $f(x) = x+1$ $f(x) = x+1$



Here, $y = x+1$

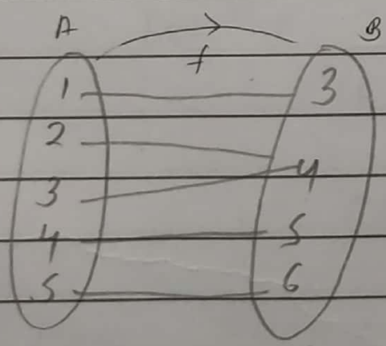
$y-1 = x$

$\therefore$ It is onto-function.

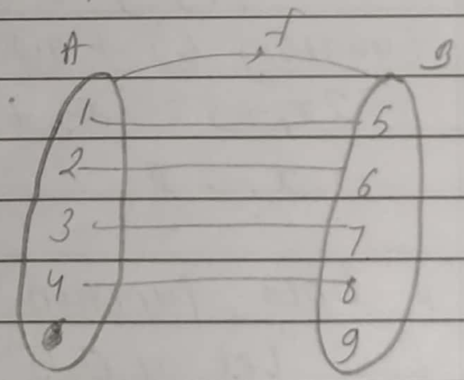
v.imp

3. Bijective function (one-to one Correspondance)

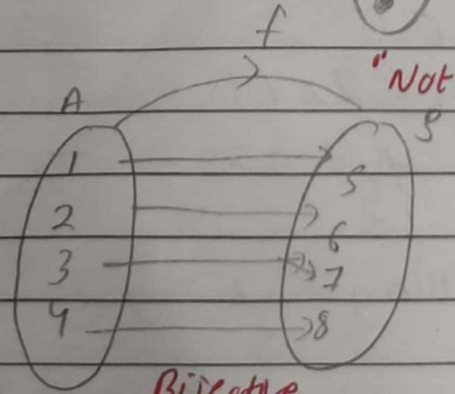
A function f from set A to set B is said to be ~~bij~~ bijective function if and only if it is both one to one and onto.



"Not Bijective"



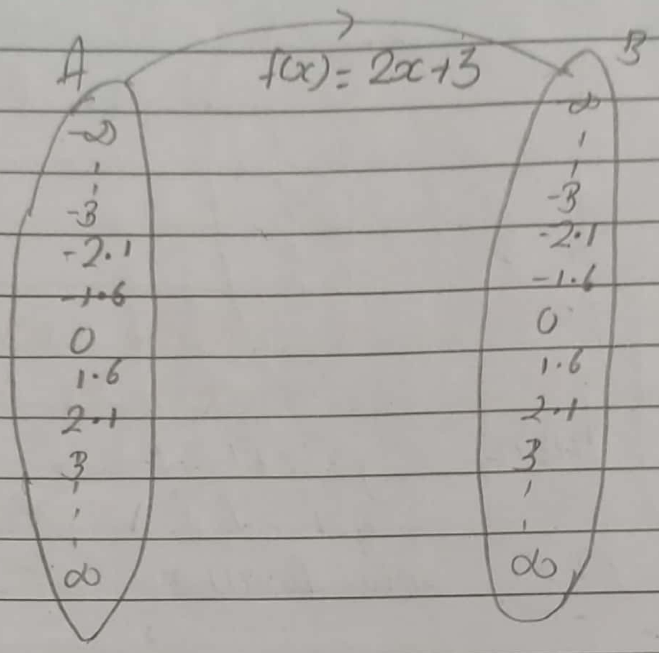
"Not Bijective"



Bijective

For a function to be bijective ; $|A| = |B|$

1. Determine the whether the function $f(x) = 2x+3$ from set of real numbers to set of real number is bijective.



1. One to one function.

Let x_1 and $x_2 \in A$

$$f(x_1) = 2x_1 + 3 \quad \text{---(i)}$$

$$f(x_2) = 2x_2 + 3 \quad \text{---(ii)}$$

Equating (i) & (ii)

$$2x_1 + 3 = 2x_2 + 3$$

$$\Rightarrow x_1 = x_2$$

2. Onto function

Let $y \in B$

$$y = 2x+3$$

or $$\frac{y-3}{2} = x \quad \text{---(i)}$$

Now, putting $x = \frac{y-3}{2}$ in $f(x)$,

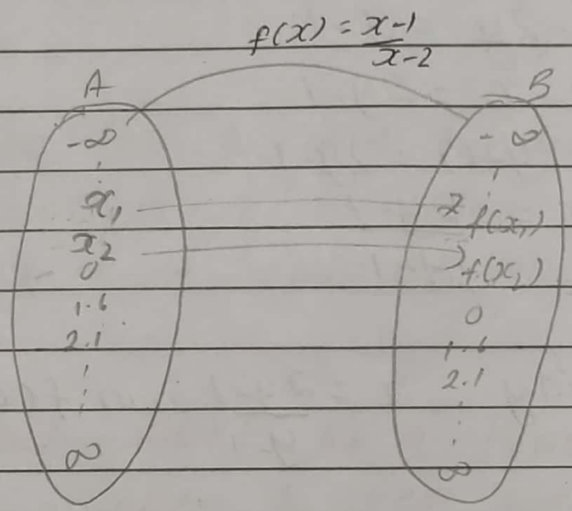
$$f\left(\frac{y-3}{2}\right) = 2\left(\frac{y-3}{2}\right) + 3$$

$f\left(\frac{y-3}{2}\right) = y-3+3$
$= y$

$\therefore$ The given function is bijective.

Qn 2. $f(x) = \dfrac{x-1}{x-2}$ $\quad A = R - \{2\}$, $\quad B = R - \{1\}$

Sol^n



where,
$A = R - \{2\}$
$B = R - \{1\}$.

1. One to one function.

Let x_1 and $x_2 \in A$

$$f(x_1) = \frac{x_1 - 1}{x_1 - 2} \quad \text{--- (i)}$$

$$f(x_2) = \frac{x_2 - 1}{x_2 - 2} \quad \text{--- (ii)}$$

Now, Equating (i) & (ii)

$$\frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_2 - 1)(x_1 - 2)$

$\Rightarrow x_1 x_2 - 2x_1 - x_2 + 2 = x_1 x_2 - 2x_2 - x_1 + 2$

$\Rightarrow 2x_1 + x_2 = 2x_2 + x_1$

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$\Rightarrow 2x_1 - x_1 = 2x_2 - x_2$

$\Rightarrow x_1 = x_2$

2. Onto function.

let $y \in B$

$$y = \frac{x-1}{x-2}$$

$\Rightarrow xy - 2y = x - 1$

$\Rightarrow xy - x = 2y - 1$

$\Rightarrow x(y-1) = 2y-1$

$\Rightarrow x = \frac{2y-1}{y-1}$

Now, putting $x = \frac{2y-1}{y-1}$ in $f(x)$

$$f\left(\frac{2y-1}{y-1}\right) = \frac{\frac{2y-1}{y-1} - 1}{\frac{2y-1}{y-1} - 2}$$

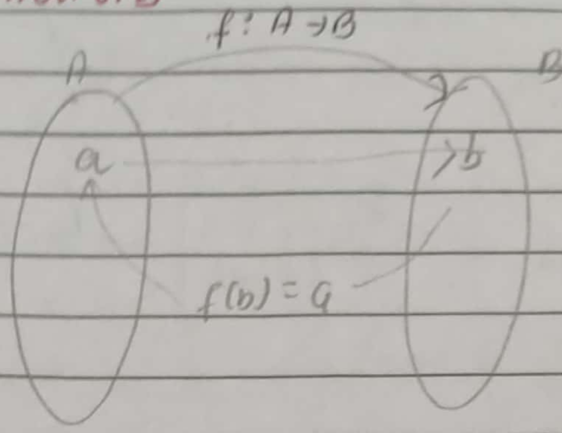
$$= \frac{\frac{2y-1-y+1}{y-1}}{\frac{2y-1-2y+2}{y-1}}$$

$$= \frac{y}{1}$$

$$= y$$

Inverse and Composition functions

a. Inverse functions

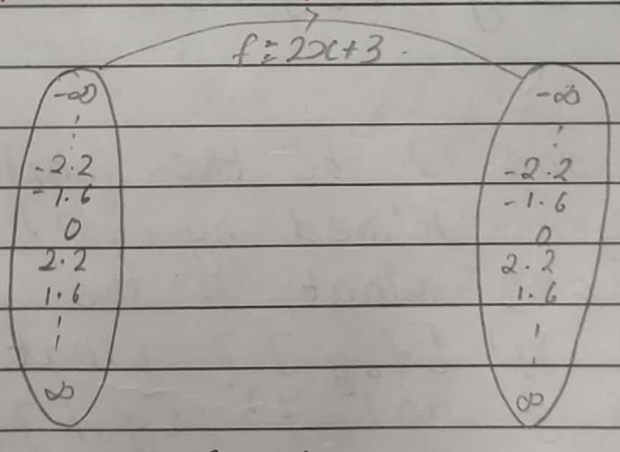


Hint:

- Replace $f(x)$ with y
- Solve for 'x' in terms of y
- Replace x with $f^{-1}(y)$

Let f be the bijective function from set A to set B the a function f^{-1} is called inverse function if and only if $f^{-1}(b) = a$ whenever $f(a) = b$

Q.1 Let $f(x) = 2x+3$ be a function from set ~~A to B~~ of real number to real number. find its inverse.



first check (Bijective)

$$y = 2x+3$$

$$\frac{y-3}{2} = x \quad \text{—(i)}$$

$$f^{-1}(y) = \frac{y-3}{2}$$

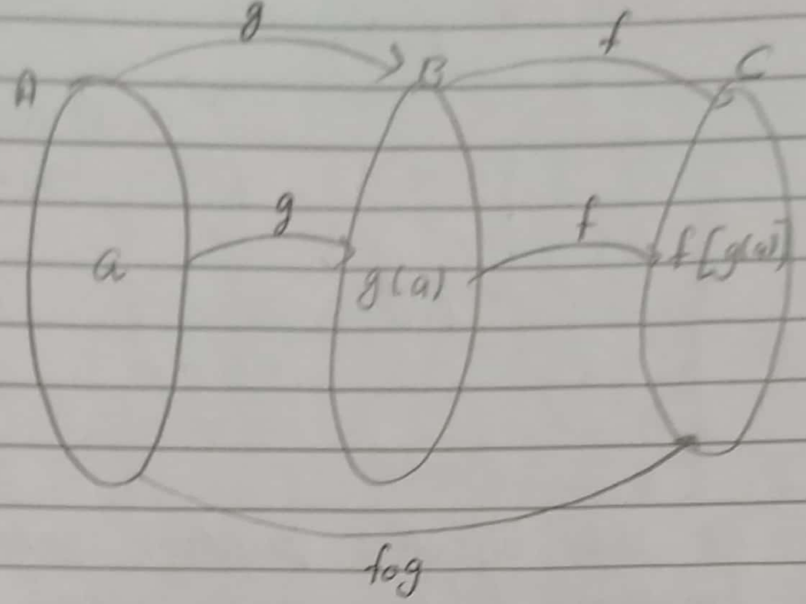
Check:

$x = 2, y = 7$

$f(2) = 2 \cdot 2 + 3 = 7$

$$f^{-1}(7) = \frac{7-3}{2} = \frac{4}{2} = 2$$

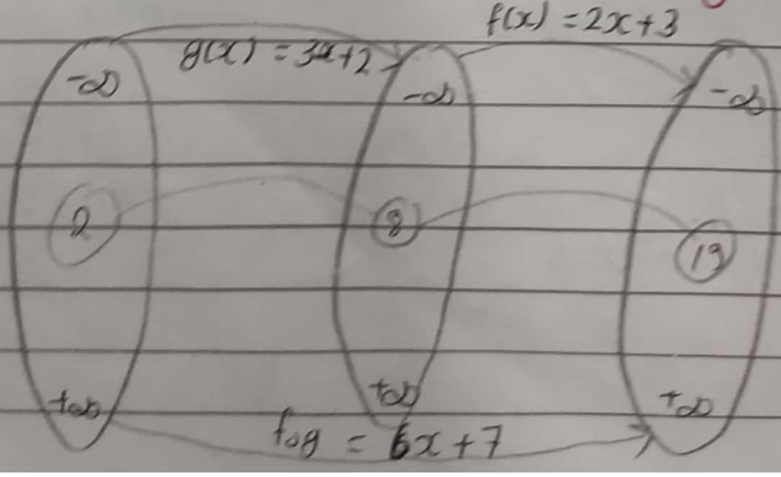
b. Composite Function [$f \circ g$]



Let g be the function from A to B and f be the function from B to C then the composite function is denoted by $f \circ g$ which is given by,

$$f \circ g = f[g(a)]$$

Q.1. Let 'f' and 'g' be the function from set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$. What is the composition of 'f' and 'g' [$f \circ g$]? What is the composition of 'g' and 'f' [$g \circ f$]?

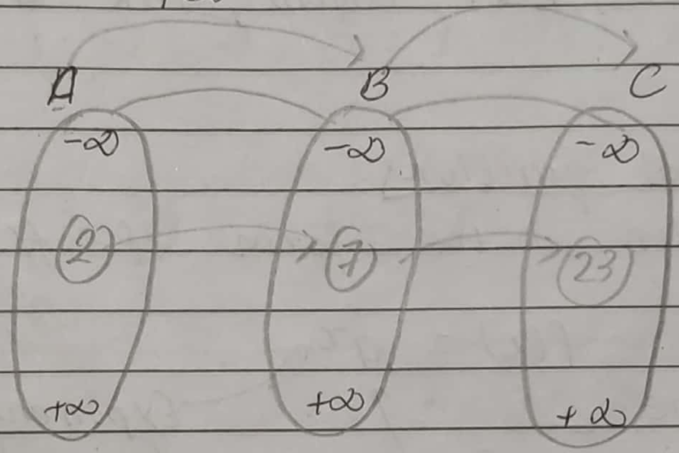


(a) $f \circ g$

$$
\begin{aligned}
f \circ g &= f[g(x)] \\
&= f[3x+2] \\
&= 2(3x+2)+3 \\
&= 6x+4+3 \\
&= 6x+7
\end{aligned}
$$

$\therefore f \circ g = 6x+7$ |

(b) $g \circ f$ $\quad f(x) = 2x+3 \quad g(x) = 3x+2$



$$
\begin{aligned}
g \circ f &= g[f(x)] \\
&= g[2x+3] \\
&= 3(2x+3)+2 \\
&= 6x+11
\end{aligned}
$$

Boolean functions

A Boolean function is mathematically defined as

$$f: B^n \to B;$$

where,

$$B = \{0, 1\}$$

n = +ve integer represention total inputs.

Boolean function are implemented by using logic gates:

OR, AND, NOT, NAND, NOR, XOR, XNOR.

Exponential functions

A function in the form of $f(x) = a^x$ exponential function

$$f(x) = a^x$$

where, a = Base (constant), x = Exponent (variable)

Floor and ceiling functions

a. Floor functions

The floor function assigns to the real number x, the largest integer that is less than or equal to x. It is denoted by

$$\boxed{\lfloor x \rfloor}$$

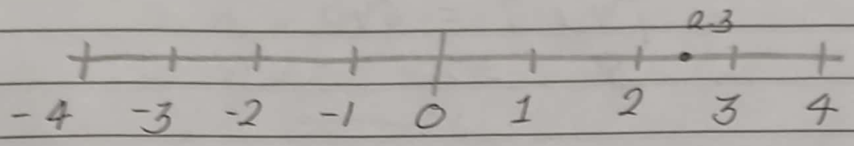
b. Ceiling function.

The ceiling function to the real number x, the smallest integer that is greater than or equal to x:

$$\boxed{\lceil x \rceil}$$

Example:

(a) $x = 2.3$

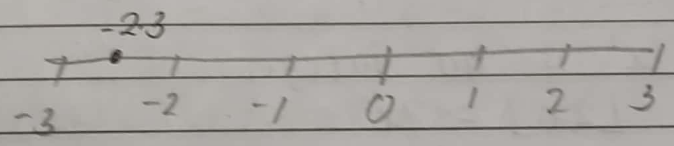


So:

$\lfloor x \rfloor = 2$

$\lceil x \rceil = 3$

(b) $x = -2.3$



$\lceil -2.3 \rceil = -2$

$\lfloor -2.3 \rfloor = -3$

(c) $x = 4$ → Integer

$\lceil x \rceil = \lfloor x \rfloor = 4$

(d) -0.5

$\lceil 0.5 \rceil = 0$ $\qquad$ $\lfloor -0.5 \rfloor = -1$

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Formulae:

n = integer
x = real number

ⓐ $\lfloor x+n \rfloor = \lfloor x \rfloor + n$
ⓑ $\lceil x+n \rceil = \lceil x \rceil + n$

$\lceil 6 + 1.1 \rceil = 6 + \lceil 1.1 \rceil$
$\Rightarrow \lceil 7.1 \rceil = 6+2$
$\Rightarrow 8 = 8$

Q1)- Prove that if 'x' is real number:
$$\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$$

case I: 'x' is an integer
$\lfloor 2x \rfloor = 2x$
$\lfloor x \rfloor = x$

$\lfloor x + \frac{1}{2} \rfloor = x + \lfloor 0.5 \rfloor$
$= x + 0$
$= x$

$2x = x + x$
$\Rightarrow 2x = 2x$

(True)

case II: 'x' is not integer
we can write,
$x = n + r$ where
r = fractional part of x $(0<r<1)$

n = largest integer less than x.

Now,

$$\lfloor 2x \rfloor = \lfloor 2(n+r) \rfloor$$
$$= \lfloor 2n + 2r \rfloor$$
$$= \lfloor 2n \rfloor + \lfloor 2r \rfloor$$
$$= 2n + \lfloor 2r \rfloor \text{ ——— (a)}$$

$$\lfloor x \rfloor = \lfloor n + r \rfloor$$
$$= n + \lfloor r \rfloor$$
$$= n + 0$$
$$= n \text{ ——— (b)}$$

$$\lfloor x + 0.5 \rfloor = \lfloor (n+r) + 0.5 \rfloor$$
$$= \lfloor n + r + 0.5 \rfloor$$
$$= n + \lfloor r + 0.5 \rfloor \text{ ——— (c)}$$

$$2n + \lfloor 2r \rfloor = n + n + \lfloor r + 0.5 \rfloor$$
$$\Rightarrow 2n + \lfloor 2r \rfloor = 2n + \lfloor r + 0.5 \rfloor$$
$$\Rightarrow \lfloor 2r \rfloor = \lfloor r + 0.5 \rfloor$$

- If $0.5 \le r < 1$,
  $1 = 1$
  (True)

- If $0 < r < 0.5$
  0.4
  $0 = 0$
  (True)

Qn2. Prove that if 'n' is odd number then

$$\left\lceil \frac{n^2}{4} \right\rceil = (n^2+3)/4$$

(odd number, $n = 2\overset{\curvearrowright \text{ integer.}}{k} + 1$)

Soln

$$\left\lceil \frac{n^2}{4} \right\rceil = \left\lceil \frac{(2k+1)^2}{4} \right\rceil$$

$$= \left\lceil \frac{4k^2+4k+1}{4} \right\rceil$$

$$= \left\lceil k^2 + k + \frac{1}{4} \right\rceil$$

$$= k^2 + k + \left\lceil 1/4 \right\rceil$$

$$= k^2 + k + \lceil 0.25 \rceil$$

$$= k^2 + k + 1$$

multiplying by 4;

$$= \frac{4(k^2+k+1)}{4}$$

$$= \frac{4k^2 + 4k + 4}{4}$$

$$= \frac{(2k)^2 + 2 \cdot 2k + 1 + 1^2 + 3}{4}$$

$$= \frac{n^2+3}{4}.$$

Qn3 If 'n' is odd then

(a) $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$ (b) $\left\lceil \frac{n}{2} \right\rceil = \frac{n+1}{2}$

(a) soln

$$\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{2k+1}{2} \right\rfloor$$
$$= \left\lfloor k + \frac{1}{2} \right\rfloor$$
$$= k + \lfloor 0.5 \rfloor$$
$$= k$$
$$= \frac{2k}{2}$$
$$= \frac{2k+1-1}{2}$$
$$= \frac{n-1}{2}$$

proved

(b) soln

$$\left\lceil \frac{n}{2} \right\rceil = \left\lceil \frac{2k+1}{2} \right\rceil$$
$$= \left\lceil k + \frac{1}{2} \right\rceil$$
$$= k + \lceil 0.5 \rceil$$
$$= k + 1$$
$$= \frac{2k+1}{2}$$
$$= \frac{2k+2}{2} = \frac{2k+1+1}{2} = \frac{n+1}{2}$$

proved

2 questions

Chapter-1

Basic Discrete Structure

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Set Theory

Computer representation of sets and set operation

Let, $U = \{1, 2, 3, 5, 8, 9, 10\}$
$A = \{1, 8, 10\}$
$B = \{1, 5, 9\}$

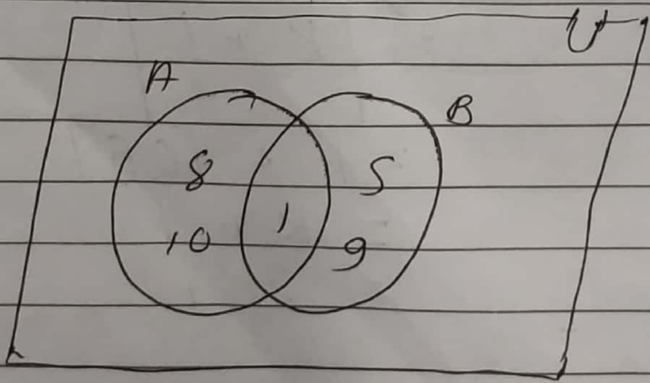
We use bit string to represent sets

	1	2	3	4	5	6	7	8	9	10
U =	1	1	1	0	1	0	0	1	1	1

	1	2	3	4	5	6	7	8	9	10
A =	1	0	0	0	0	0	0	1	0	1

	1	2	3	4	5	6	7	8	9	10
B =	1	0	0	0	1	0	0	0	1	0

a. Union Operation
$A \cup B = \{1, 5, 8, 9, 10\}$

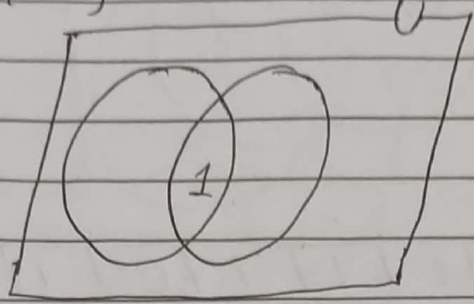


{ We perform bitwise OR betn A & B.

	1	2	3	4	5	6	7	8	9	10
AB =	1	0	0	0	1	0	0	1	1	1

(b) Intersection Operation

$A \cap B = \{1\}$

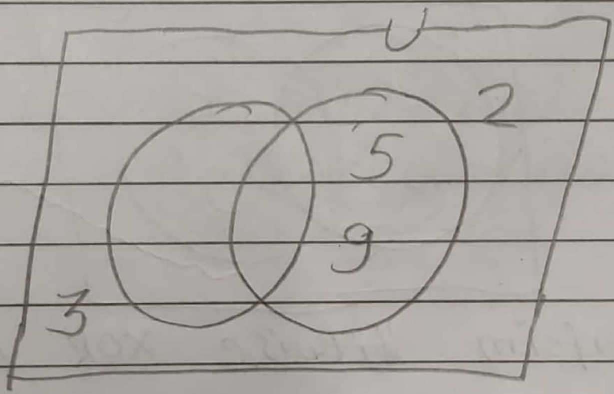


& We perform bitwise between A & B.

	1	2	3	4	5	6	7	8	9	10
A & B =	1	0	0	0	0	0	0	0	0	0

(C) Complement Operation

$\bar{A} = \{2, 3, 5, 9\}$

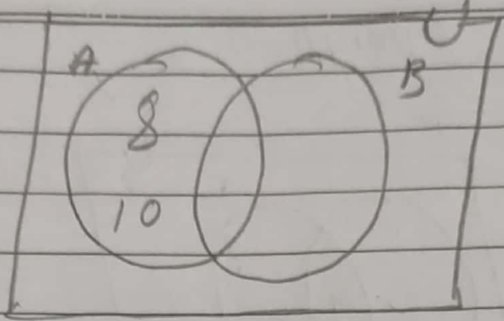


We person bitwise NOT

	1	2	3	4	5	6	7	8	9	10
A & B =	0	1	1	*	1	*	*	0	1	0

(d) ~~Symmetric~~ Set Difference:

$A - B = \{8, 10\}$



We perform bitwise AND betⁿ A & $(\overline{B})$

$!B =$

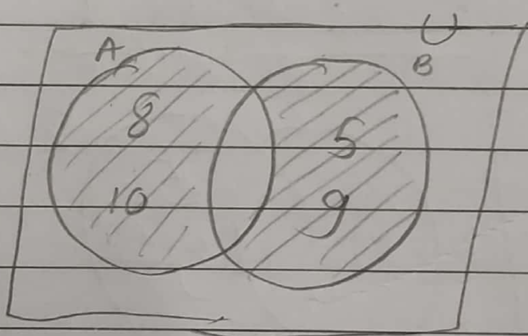
1	2	3	4	5	6	7	8	9	10
0	1	1	1	0	1	1	1	0	1

$A \& (!B) =$

1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	1	0	1

(e) Symmetric Difference.

$A \oplus B = (A-B) \cup (B-A)$

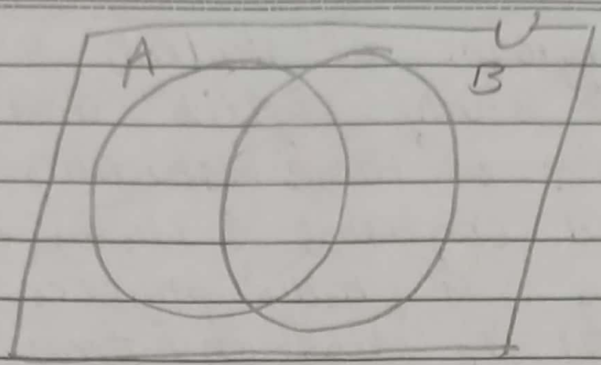


We perform bitwise XOR betⁿ A & B

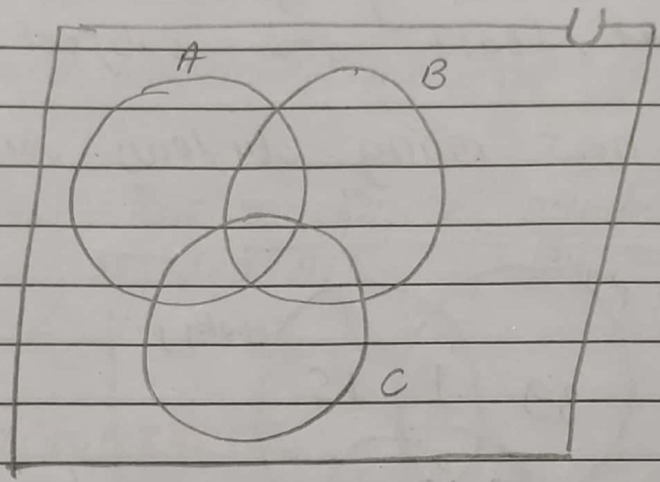
$A \oplus B =$

1	2	3	4	5	6	7	8	9	10
0	0	0	0	1	0	0	1	1	1

Inclusion / Exclusion



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

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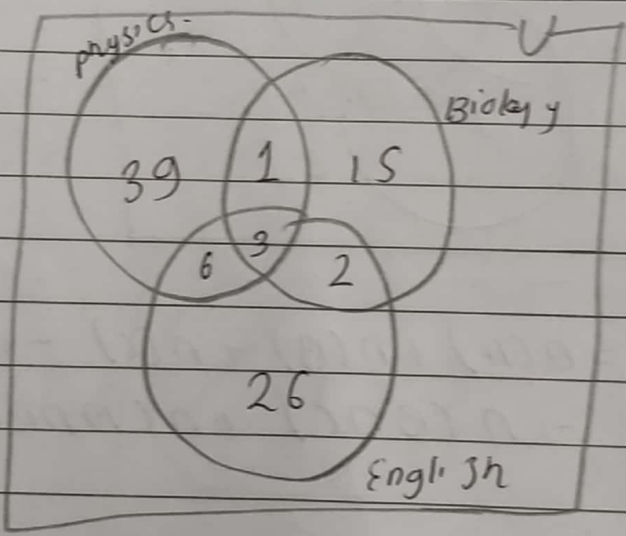
Qn. Among a group of students, 49 study physics, 37 study English and 21 study biology. If 9 of these students study physics and English, 5 study english and Biology, 4 study physics and biology 3 study all the subjects. Find the no. of students in the group if all students study at least one subject?

@ Find how many students study only physics



$n(P \cup E \cup B) = 92$

Number Theory

(a) Prime Number

The number that is only divisible by "1 and itself"

1-50 = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

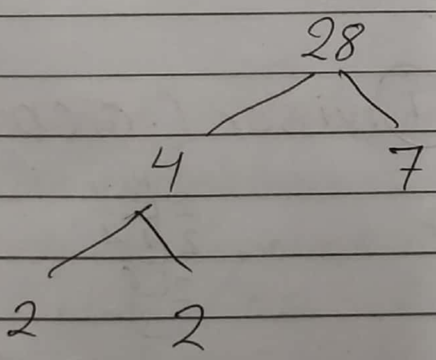
The only even prime no. is 2.

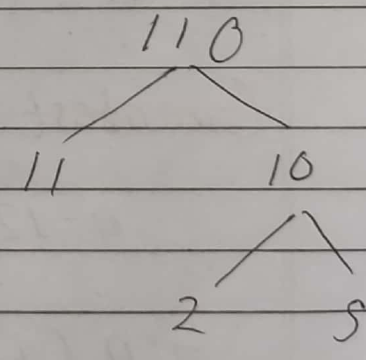
The largest known prime number till date:

$$2^{82,589,933} - 1$$

Prime factorization

"Every number can be written as a product of prime numbers"





$28 = 2^2 \cdot 7$

$110 = 2 \cdot 5 \cdot 11$

The Division Algorithm

Let 'a' be an integer and 'd' be a positive integer. Then there are unique integers 'q' and 'r' with $0 \le r < d$ such that.

$$\boxed{a = dq + r} \qquad \text{—— (i)}$$

divident division quotient remainder

Example: * $\frac{11}{3}$

Here $a = 11$, & $d = 3$

$11 = 3 \cdot 3 + 2$

* $\left\{ \frac{-11}{3} \right\}$

$\Rightarrow -11 = (-4)(3) + 1$

Greatest Common Divisor (GCD)

HCF

$a = 12$ $b = 24$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

GCD (12, 24)

Factors of $a = 2 \times 2 \times 3$

factors of $b = 2 \times 2 \times 2 \times 3$

$HCF = 2 \times 2 \times 3$

$GCD(12, 24) = 12.$

CamScanner

Let a and b be the two integers. The largest integer 'd' such that $d | a$ and $d | b$ is called greatest common Divisor (GCD).

GCD (36, 42)

Factor of a = 1, 2, 3, 4, 6, 9, 12, 36

Factor of b = 1, 2, 3, 6, 7, 14, 21, 42

Relatively Prime

Two integers a & b are called relatively prime if GCD is 1.

Factors of 17 = 1, 17

Factors of 22 = 1, 2, 11, 22

GCD = 1

Pairwise Relatively Prime:

The integers $a_1, a_2, \ldots a_n$ are pairwise relatively prime if GCD $(a_i, a_j) = 1$ whenever $1 \leq i < j \leq n$.

Qn. 1 Determine whether the integers "10, 17, 21" are pairwise relatively prime

Soln

GCD (10, 17) = 1

GCD = (17, 21) = 1

GCD = (10, 21) = 1

10, 19, 24

Soln

GCD (10, 19) = 1

GCD (19, 24) = 1

GCD (10, 24) = 2

GCD using Prime factorization

Qn1 GCD = (120, 500)

Prime factors of 120 = $2 \cdot 2 \cdot 2 \cdot 5 \cdot 3 = 2^3 \cdot 3^1 \cdot 5^1$

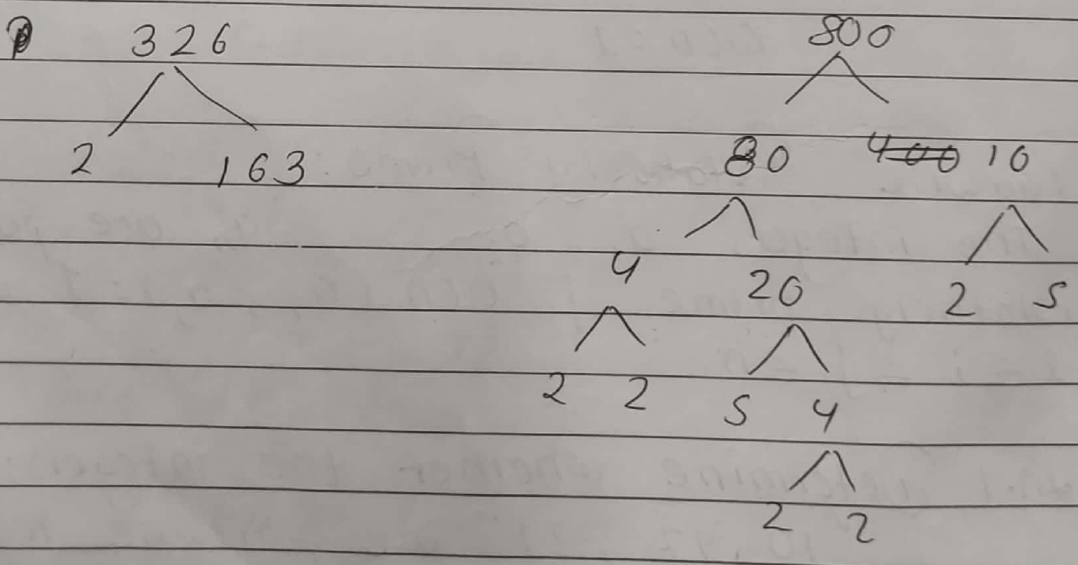
Prime factors of 500 = $2 \cdot 5 \cdot 5 \cdot 5 \cdot 2 = 2^2 \cdot 3^0 \cdot 5^3$

GCD = $2^2 \cdot 3^0 \cdot 5^1$

= 20

Qn.2. GCD (326, 800)

Soln



Prime factors of 326 = $2 \cdot 163$

Prime factors of 500 = $2^5 \cdot 5^2$

GCD = 2.

Least common Multiple (LCM)

$a = 3$, $b = 15$

multiple of 3 = 3, 6, 9, 12, 15, 18, 21 ...

multiple of 15 = 15, 30, 45, 60, 75, 90 ....

LCM (3, 15) = 15.

Prime factor

Prime factor $3 = 3 \cdot 5^0$

Prime factor $15 = 3 \cdot 5$

$LCM = 3 \cdot 5$

$= 15$

Qn. 2. $a = 120$, $b = 500$

Prime factor :

Prime factor of $120 = 2^2 \cdot 3 \cdot 2 \cdot 5 = 2^3 \cdot 3^1 \cdot 5^1$

Prime factor of $500 = 5^2 \cdot 2 \cdot 2 \cdot 5 = 2^2 \cdot 3^0 \cdot 5^3$

$LCM (120, 500) = 2^3 \cdot 3^1 \cdot 5^3$

$= 3000$

The Eucledian Algorithm (: Finding GCD)

Step I: Given two integers 'a' and 'b' where $a \geq b$. Divide 'a' and 'b' to get remainder 'r'.

Step II: If $r = 0$, then GCD of 'a' and 'b' is equal to 'b'.

Step III: If $r \neq 0$, then replace 'a' with 'b' with 'r' and repeat step I and II.

Qn1 Find the GCD of 414 and 662 using Eucledian Algorithm.

Soln

$a = 662$

$b = 414$

S.N	a	b	$r = a \% b$
1.	662	414	248
2.	414	248	166
3.	248	166	82
4.	166	82	2
5.	82	2	0

GCD = 2

GCD (662, 414) = 2

Qn 2. Find the GCD of 56 and 15 using Eucledian Algorithm

$a = 56$

$b = 15.$

S.N	a	b	r = a % b
1.	56	15	11
2.	15	11	~~3~~ 4
3.	11	4	3
4.	4	3	1
5.	3	1	0

$GCD(56, 15) = \underline{1}$

The Extended Eucledian Algorithm (: Finding GCD & linear combinator)

$GCD(a, b) = Sa + tb.$

1. Initialize.

$(r_0, S_0, t_0) = (a, 1, 0)$

$(r_1, S_1, t_1) = (b, 0, 1)$

2. compute:

$q = \frac{r_1}{r_0}$

3 Comute:

$r_2 = r_0 - q * r_1$

$S_2 = S_0 - q \times S_1$

$t_2 = t_0 - q * t_1$

4. Set

$(r_0, s_0, t_0) = (r_1, s_1, t_1)$

$(r_1, s_1, t_1) = (r_2, s_2, t_2)$

5. Repeat step 2 to 4 until $r_2 = 0$.

6. $GCD(a, b) = r_1$

$s = s_1$

$t = t_1$

The GCD of two number 'a' and 'b' can be expressed as.

linear combinator.

$$\boxed{GCD(a, b) = sa + tb} \quad \text{——} (i)$$

where,

's' and 't' are known as Bizout's coefficient

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Qn) Express GCD of (83, 19) as linear combinaler of 83 and 19.

~~Soln~~
Here,

$$GCD(83, 19) = 83 * s + 19 * t$$

$q = \frac{r_0}{r_1}$	r_0	s_0	t_0	r_1	s_1	t_1	$r_2 = r_0 - q \times r_1$	$s_2 = s_0 - q \times s_1$	$t_2 = t_0 - q \times t_1$
4	83	1	0	19	0	1	7	1	-4
2	19	0	1	7	1	-4	5	-2	9
1	7	1	-4	5	-2	9	2	3	-13
2	5	-2	9	2	3	-13	1	-8	35.
2	2	3	-13	1	-8	35	0	19	-83.

~~GCD(~~ $1 = -8 * 83 + 19 \times 35$

xxx

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Qn2. Express GCD of (56, 15) as linear combinator of 56 and 15.

Soln

$GCD(56, 15) = 56 \times S + 15 \times t$

$q = \frac{r_0}{r_1}$	r_0	s_0	t_0	r_1	s_1	t_1	$r_2 = r_0 - q \times r_1$	$s_2 = s_0 - q \times s_1$	$t_2 = t_0 - q$
3	56	1	0	15	0	1	11	1	-3
1	15	0	1	11	1	-3	4	-1	4
2	11	1	-3	4	-1	4	3	3	-11
1	4	-1	4	3	3	-11	1	-4	15
3	3	3	-11	1	-4	15	0	–	–

$$1 = 56 \times (-4) + 15 \times (15)$$

Congurent Modulo

Two integers a and b are said to be congurent modulo ~~and~~ (m) if and only if they have same remainder when divided by m.

$$\boxed{a \equiv b \bmod m}$$

(a is congurent modulo to b mod m)

Eg:

(a)		(b)
16		11
1	5	1
	(m)	

$$16 \equiv 11 \bmod 5$$

a is congurent modulo to b mod m if m divides a - b

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Prove:

Let 'm' be a positive integer. If $a \equiv b \mod m$ and $c \equiv d \mod m$ then:

(i) $(a+c) \equiv (b+d) \pmod m$

(ii) $ac \equiv bd \pmod m$

Soln

Here, We have,

$$a \equiv b \mod m$$

i.e $\frac{a-b}{m} = k$ (integer)

$$a = b + mk \quad \text{---(i)}$$

we have

$$c \equiv d \mod m$$

ie $\frac{c-d}{m} = l$ (integer)

$$c = ml + d \quad \text{---(ii)}$$

Adding (i) and (ii).

$$(a+c) = (b+d) + ml + mk$$

$$(a+c) - (b+d) = ml + mk$$

$$\Rightarrow \frac{(a+c) - (b+d)}{m} = l + k \text{ (integer)}$$

$$\Rightarrow \therefore (a+c) \equiv (b+d) \mod m.$$

Now, multiplyng (i) & (ii)

$$ac = (b+mk)(ml+d)$$

$$\Rightarrow ac = bd + bml + m^2kl + mkd.$$

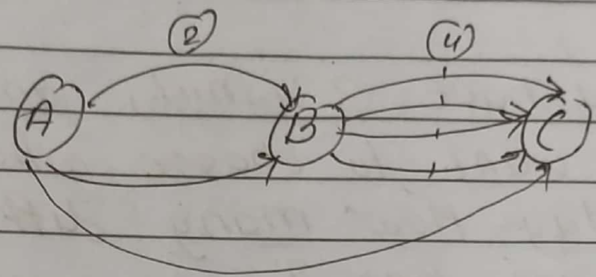
$$\Rightarrow \frac{ac - bd}{m} = bl + mkl + kd \Rightarrow \text{(Integer)}.$$

$$\therefore ac = bd \mod m$$

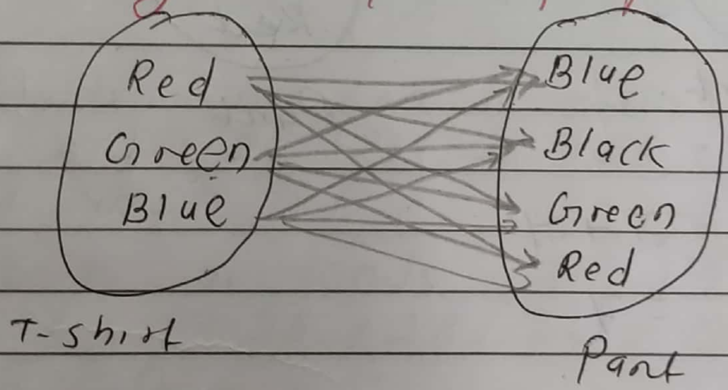
Basic of counting

① Product rule

If there are 'n' ways to do some things and 'm' ways to do another thing then the task can be done in $n \times m$ number of ways.



Qn 1 Suppose you have 3 t-shirts and 4 pair of pants. How many different outfits can you create by selecting one pair of pant and one t-shirt



3×4
$= 12$

2 Sum rule

If there are 'n' choices for one action and 'm' choices for another action and two actions cannot be done at same time, then there are 'n+m' ways to choose one of these actions

Qn1 suppose you have 3 tshirts and 4 pair of pants. You want to choose what outfit to wear today. How many different outfit choices do you have?

Red
Green
Blue

T shirt

Blue
Black
Green
Red

Pant

$3 + 4$
$= 7$

Qn2. Suppose you want to create a password using four uppercase letter and 2 digits. How many password you can create?

A
⋮
Z

26

0
⋮
9

10

$\underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10}$ $= 26^4 10^2$
$= 45697600$

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1. How many strings of eight uppercase English letter are there?

a. If letters can be repeated.

$\Rightarrow \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26}$

$= (26)^8$

b. If letters cannot be repeated.

$\underline{26} * \underline{25} * \underline{24} * \underline{23} * \underline{22} * \underline{21} * \underline{20} \times \underline{19}$

c. starts with 'A' and letters can be repeated.

$\underset{A}{\underline{1}} * \underline{26} * \underline{26} * \underline{26} * \underline{26} * \underline{26} * \underline{26} * \underline{26}$

$(26)^7$

d. starts and ends with 'BO'

$\underset{B}{\underline{1}} * \underset{O}{\underline{1}} \quad \underline{26} \quad \underline{26} \quad \underline{26} \quad \underline{26} \quad \underset{B}{\underline{1}} * \underset{O}{\underline{1}}$

e. starts or ends with 'BO'

$\underset{B}{\underline{1}} \times \underset{O}{\underline{1}} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} +$

$\underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \quad \underset{B}{\underline{1}} \quad \underset{O}{\underline{1}}$

$= (26)^6 + (26)^6$

Imp

Pigeonhole Principle

If there are more pigeons than pigeon hole then at least one pigeonhole must contain more than one pigionhole

If there are 'n' object and 'm' container $(n > m)$ then at least one container contains more than one object.

In a group of people, at least two people must have same birth month.

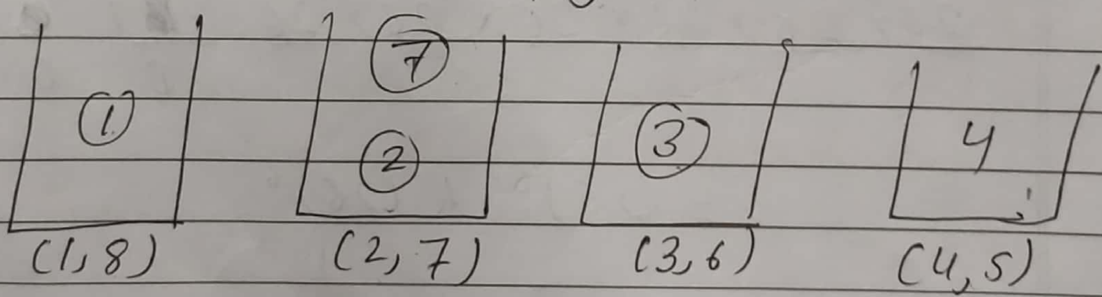
In a group of 366 people, at least two people must have same birth dat

Q.no. Show that if any five numbers are choosen from 1 to 8, then two of them will add to nine?

Soln

Here,

We define Pigeonhole as.



Each five choosen number must belong to one of these pigeonhole.

Since, there are only 4 pigeonholo, at least two of them will fall in same pigeonhole

Q.2 Show that if 8 positive integer are choosen than at least two number will have same remainder when divided by 7.

Soln

14	22	16	31	32	19	55 20
remainder 0	remainder 1	2	3	4	5	6

We divide the set of positive integer into 7 pigeonholes according to remainder when divided by 7.

V.imp (Generalized)

Extended Pigeonhole principle

If 'N' objects are placed into 'k' boxes, then there is at least one box containing $\left\lceil \frac{N}{k} \right\rceil$ objects.

Qn. Find the minimum no. of students in a class to be sure that 4 of them are born in same month.

Soln

$k = 12$

$N = ?$

$12 + 12 + 12 + 1$

$= 37$

$\left\lceil \frac{N}{12} \right\rceil \geq 4$

$\left\lceil \frac{37}{12} \right\rceil \geq 4$

$\lceil 3.1.. \rceil \geq 4$

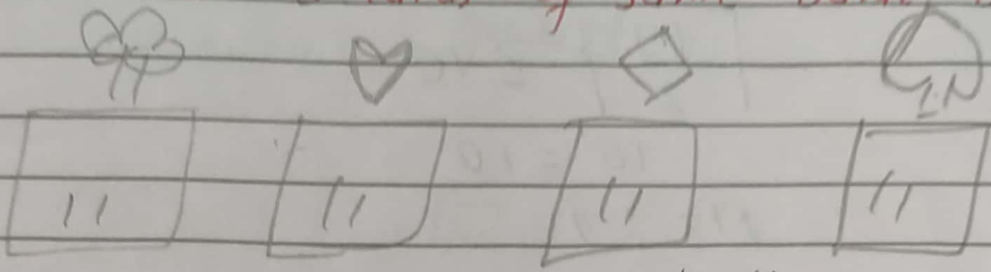
$\Rightarrow 4 = 4$

$N = 37$

Qn.2 How many cards must be selected from a standard deck of 52 cards to guarantee that:

(a) At least 3 cards of same suit are choos[en]

(b) At least 3 cards of same value are choosen.



$k = 4.$

$4 + 4 + 1 = 9$

(a)

$$\left\lceil \frac{N}{k} \right\rceil \geq 3$$

$$\left\lceil \frac{N}{4} \right\rceil \geq 3$$

$$N = 9$$

(b) $13 + 13 + 1$

$= 27$

$$\left\lceil \frac{N}{k} \right\rceil \geq 3$$

$$\left\lceil \frac{N}{13} \right\rceil \geq 3$$

$$N = 27.$$

(c) at least 10 cards are of same colour

$k=2$

$$\left\lceil \frac{N}{k} \right\rceil \geq 10$$

$$\left\lceil \frac{19}{2} \right\rceil \geq 10$$

$$10 = 10$$

$N = 19.$

Permutation

The arrangement of object in a specific order.

$$P(n,r) = \frac{n!}{(n-r)!}$$

Qn1. How many Permutation of letters "ABCDEFG" contain:

a. The string BCD
b. The string CFGA
c. The strings BA and GF
d. The string ABC and CDE

Soln

(a) (B, C, D) (A) (E) (F) (G)

$n=5, r=5$

$$P(n,r) = \frac{5!}{(5-5)!} = 120$$

(b) CFGA B D E

$n=4$
$r=4$

$$P(4,4) = \frac{4!}{(4-4)!} = 24$$

(c) BA GF C D E.

$n=5$, $r=5$

$$P(5,5) = \frac{5!}{(5-5)!} = 120$$

(d) ABC CDE K G

$n=3$, $r=3$.

$$P(3,3) = \frac{3!}{(3-3)!} = 6.$$

Combination

$$C(n,r) = \frac{n!}{(n-r)! \times r!}$$

"Combination is a way of selecting item from group in which order does not matter."

Qn.1. In how many different ways seven-person committes can be formed each containing 3 women from available set of 20 women & 4 men from available set of 30 men?

Soln
Here,

Total ways of selecting men = C(30,4)

Total ways of selecting women = C(20,3)

Total ways = C(30,4) * C(20,3)
= 31,241,700

Qn.2. How many bit string of length 10 contain
a. Exactly four 1's
b. At most four 1's
c. At least 1's
d. An equal number of 0's and 1's

Soln
Here,

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(a) Exactly four 1's

$n=10$

$r=4$

$C(10,4) = 210$

(b) At most four 1's

$n=10$

$r=0 \quad r=1 \quad r=2 \quad r=3$

$C(10,0) + C(10,1) + C(10,2$

$+ C(10,3)$

$= 176$

(c) At least four 1's

$n=10$

$r=4 \quad r=5 \quad r=6 \quad r=7 \quad r=8 \quad r=9 \quad r=10$

(d) Equal number of 0's and 1's

$n=10$

$r=5$

$C(10,5) = 252.$

Boolean Matrix Operations

"A matrix with entry with either '0' or '1' is called Boolean Matrix."

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{4 \times 3}$$

↑ row ↑ coloumn.

ⓐ Join (Addition) OR

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

ⓑ Meet (AND) ($\cap$)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} (1 \wedge 0) & (0 \wedge 0) & (0 \wedge 1) \\ & & \\ & & \end{bmatrix}$$

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Boolean Product (Multiplication)

→ Addition ⇒ OR

→ Multiply ⇒ AND

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

$$A \odot B = \begin{bmatrix} (0\wedge1)\vee(0\wedge1)\vee(1\wedge0) & . & . \\ (1\wedge1)\vee(1\wedge1)\vee(0\wedge0) & . & . \\ (0\wedge1)\vee(0\wedge1)\vee(0\wedge0) & . & . \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0+0+0 & 0+0+0 & 1 \\ 1+1+0 & 0+1+0 & 0+1+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$$

Chapter-6

Language, Automata and Grammar

Finite state machine
(FSM)

- FSM with output
  - Medey
  - Moorey
- FSM with output
  - DFA
  - NFA

Finite state Machine with output: (FSM)

A FSM is defined by,

$M = \{ I, O, S, f, g, \sigma$

where,

I = finite set of i/p symbols

O = ,, ,, ,, o/p ,,

S = ,, ,, ,, states

f = state transition function.

$f : S * I \rightsquigarrow S$

$g \Rightarrow$ output function

$g : S * I \rightsquigarrow O$

σ = an initial state.

Qn) FSM is defined as:

$I = \{a, b\}$, $O = \{0, 1\}$

$S = \{A, B, C\}$, $\sigma = A$

'f' and 'g' are defined by following Table.

S \ I	f: a	f: b	g: a	g: b
A	A	B	0	0
B	A	C	1	1
C	A	B	0	1

find the output corresponding to string "aabbaa"

soln

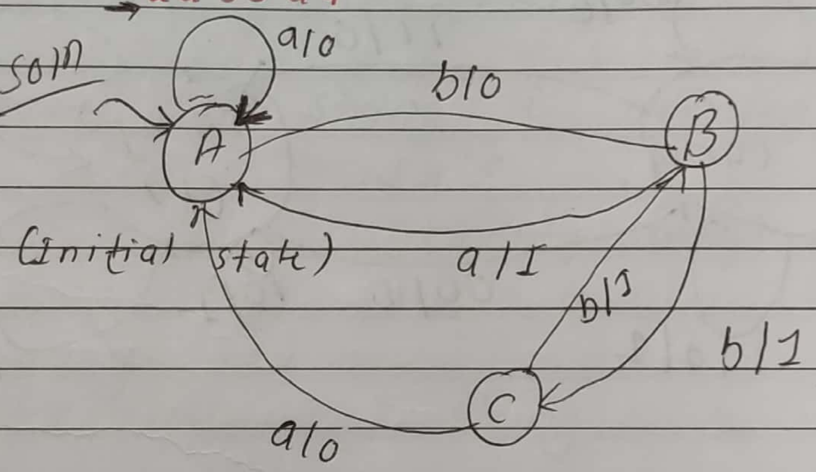


fig: Transition Diagram

current State	Input	Next State	output
A	a	A	0
A	a	A	0
A	b	B	0
B	b	C	1
C	a	A	0
A	a	A	0

Output 000100

Qn.2 Design FSM that performs serial addition. [8]

Sol

Sol^n

Here

$$\begin{array}{r} 0\ 1\ 1 \\ +\ 1\ 1\ 0 \\ \hline 1\ 0\ 0\ 1 \end{array}$$

ϕ 10 + 1

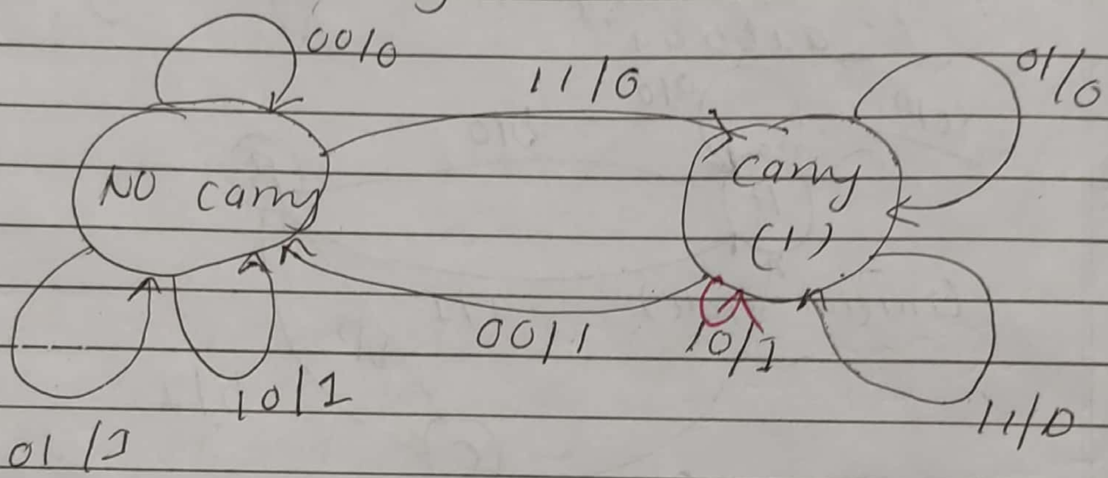
1

$I = \{00, 11, 10, 01\}$

$O = \{0, 1\}$

$S = \{$No carry, carry$\}$

σ = No carry.



S \ I	f 00	01	10	11	g 00	01	10	11
No carry	NC	NC	NC	C	0	1	1	0
carry (C)	NC	C	C	C	1	0	1	0

V.V.imp

Deterministic Finite Automata (DFA)

Few terminologies:

1. Alphabets (Σ)
   - ~~Collection~~ set of input symbols.

     $\Sigma = (a, b)$

     $\Sigma = (0, 1)$

2. String (W):
   - Collection of Alphabets.

     $W = abbaaa$, $|W| = 6$

     $W = 0011100$, $|W| = 7$

   Length zero string is denoted by Σ

   $W = \Sigma$

   (empty string)

3. Language (L):
   collection of string,

   $L_1 = \{ W : |W| = 2 \text{ over } \Sigma = (a, b) \}$

   $L_1 = \{ aa, bb, ab, ba \}$ (Finite)

   $L_2 = \{ W : \text{'W' starts with 'a' over } \Sigma (a, b) \}$

   $L_2 = \{ a, ab, aaa, \ldots \}$ (infinite)

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DFA is defined as

$$M = \{ \Sigma, S, f, \sigma, A \}$$

where,

Σ = input symbols.
S = set of states.

f = State transition function:

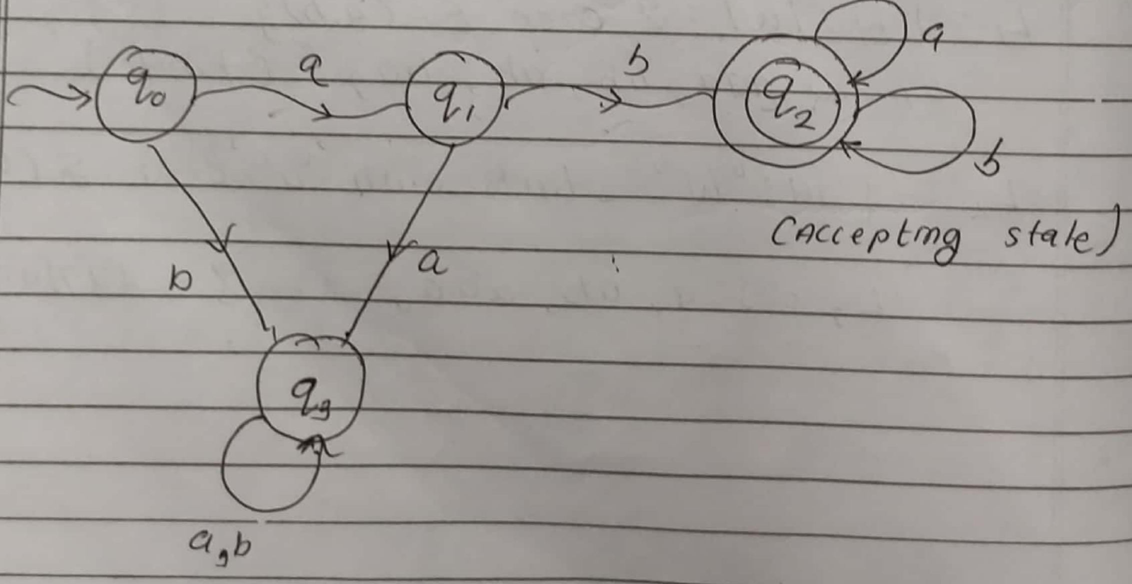
$$f : S \times \Sigma \to S$$

σ = an initial state.
A = set of accepting stable

Qn1 Design DFA for following Language:
$L = \{ W$: 'W' starts with 'ab' over $\Sigma = (a, b) \}$

Tips:

① Find the minimal string of Language
② Design DFA for that ~~language~~ string (start always with initial state).
③ Fill the gaps i.e complete all the transition.



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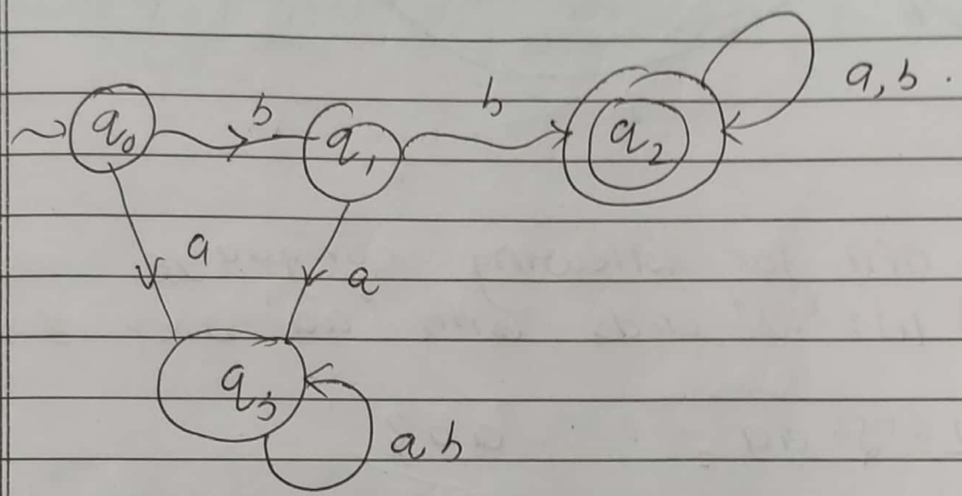
Example : (i) "abaaba"

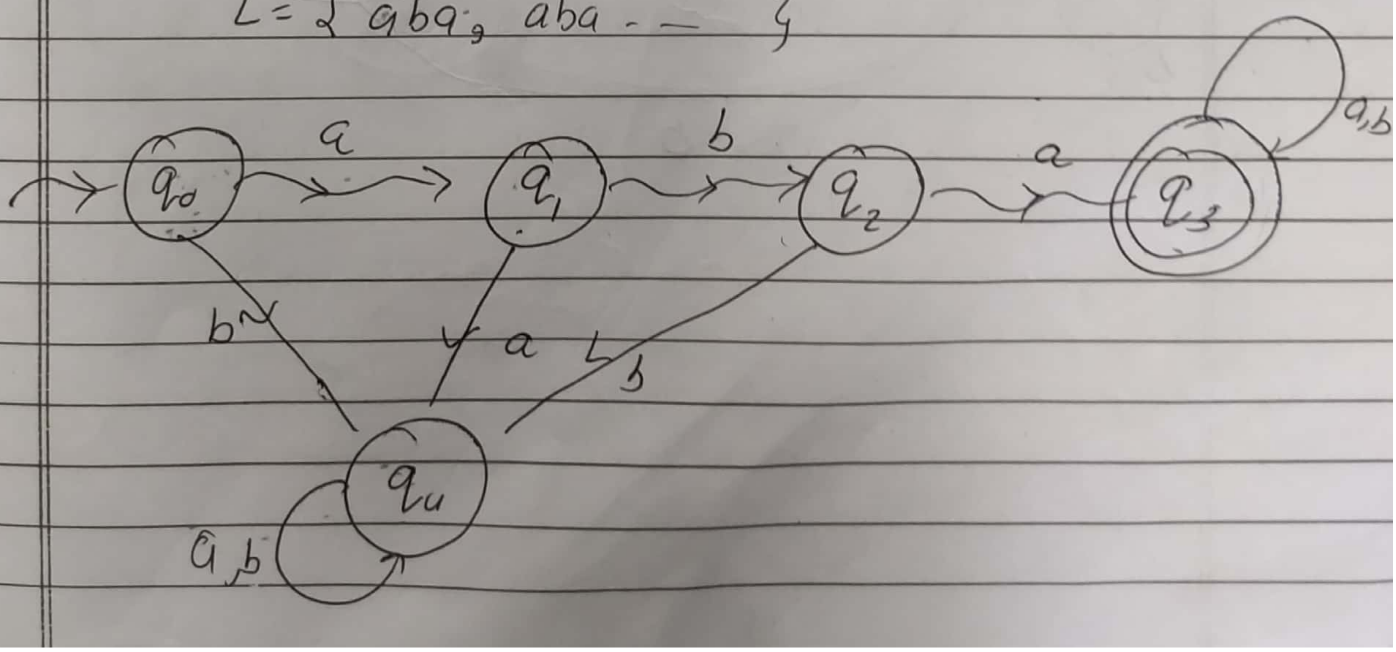
- Scan from left to right
- Start with initial state
- if the DFA stops at final state at last scan, the string is accepted else rejected.

Qn.2. Design DFA for following language
$L = \{ W : \text{"}W\text{" starts with "bb" over } \Sigma(a,b) \}$



Qn.3. Design DFA for following Language:
$L = \{ W : \text{"}W\text{" starts with "aba" over } \Sigma = (a,b) \}$

Soln
$L = \{ aba, aba \ldots \}$



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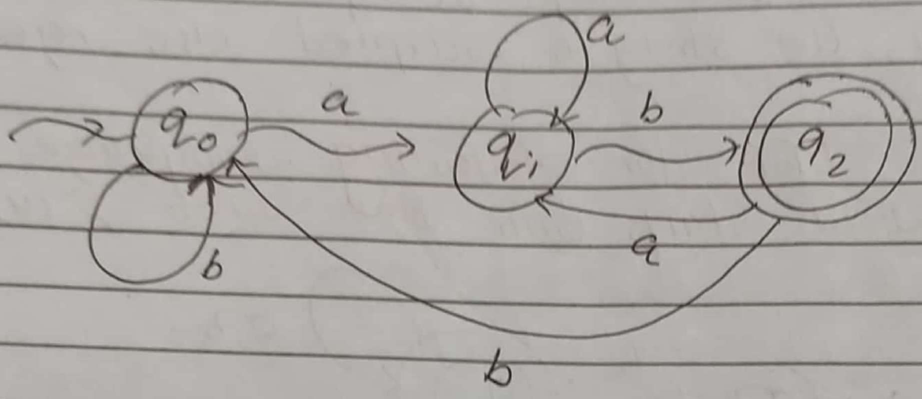
Qn 4. Design DFA for following Language.
$L = \{ W : `W' \text{ ends with "ab" over } \Sigma = (a,b) \}$

Sol^n

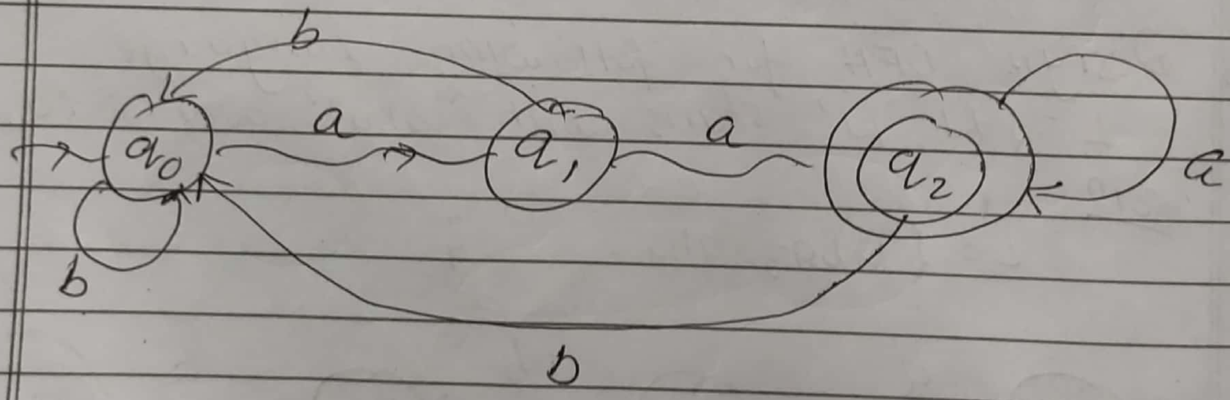
$L = \{ ab, \ldots\ldots ab \}$



Qn 5 Design DFA for following Language
$L = \{ W : `W' \text{ ends with `aa' over } \Sigma = (a,b) \}$

Sol^n

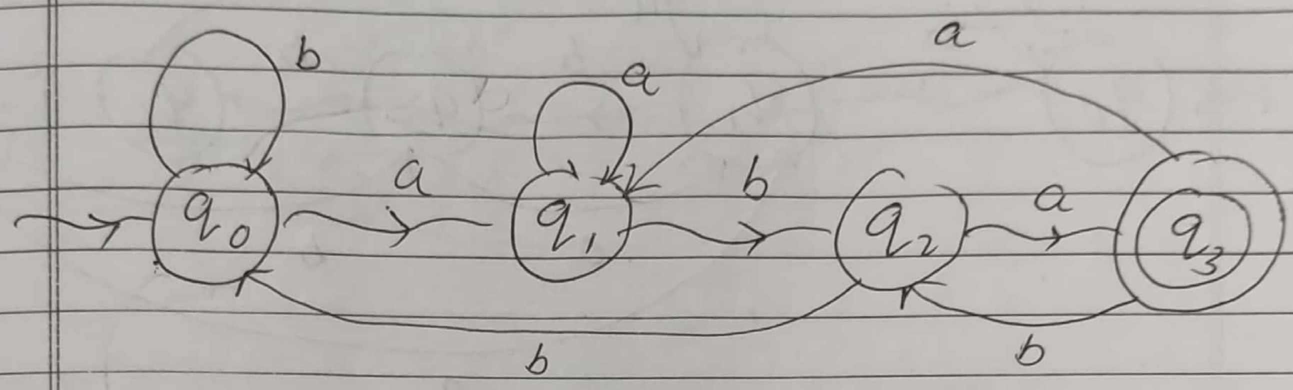
$L = \{ aa, \ldots\ldots aa \}$



Q.6. Design DFA for following Language.
$L = \{w : \text{'w' ends with 'aba' over } \Sigma = (a,b)\}$

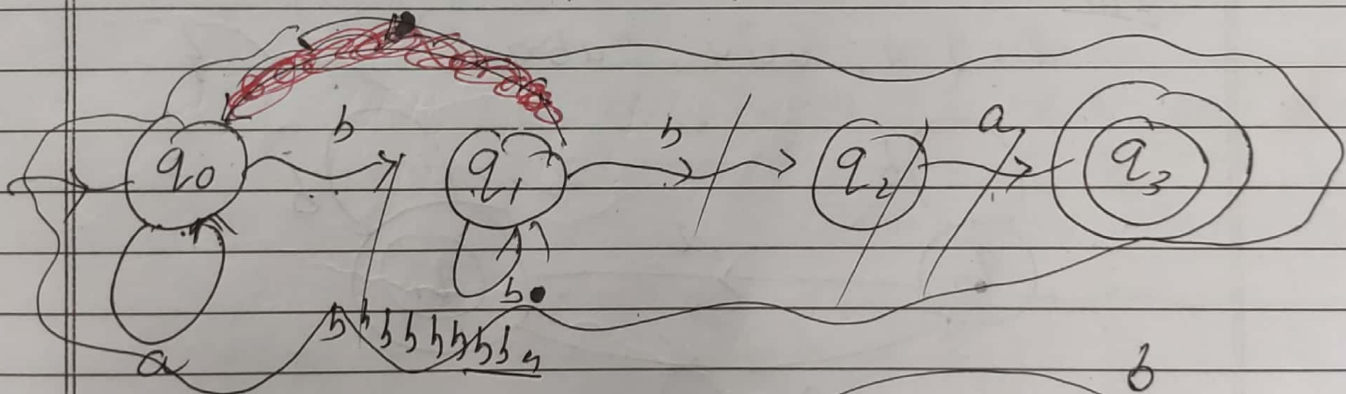
Soln

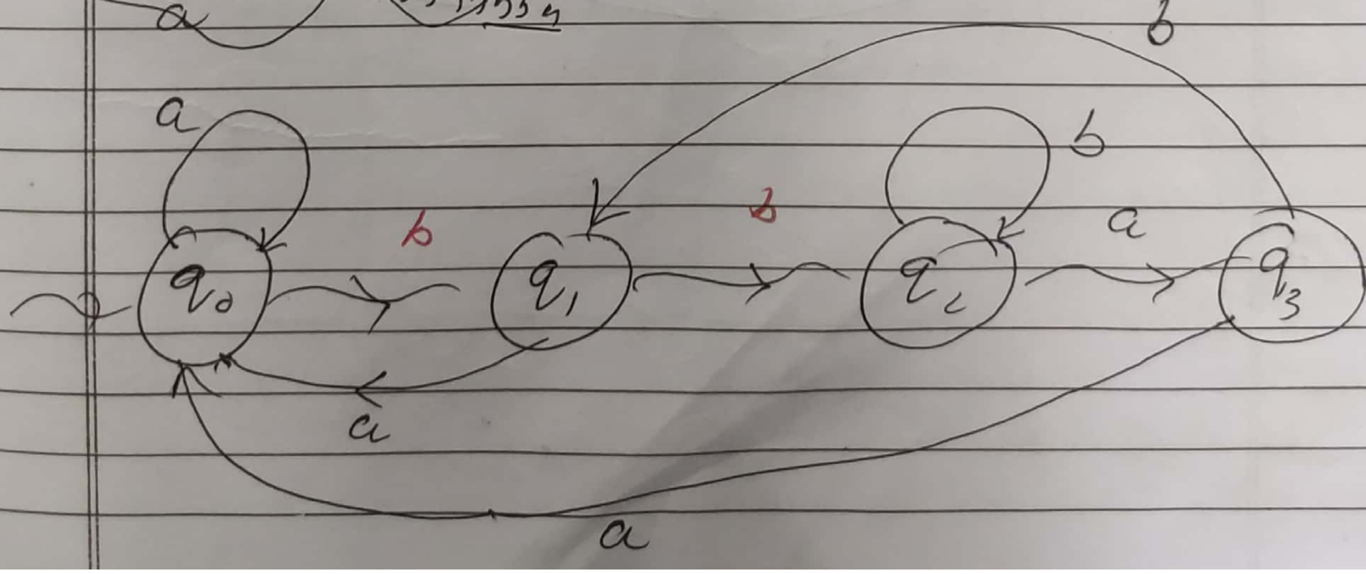
$L = \{ aba, \ldots aba \}$



7. $L = \{w : \text{'w' ends with "bba" over } \Sigma = (a,b)\}$

$L = \{ bba, \ldots bba \}$





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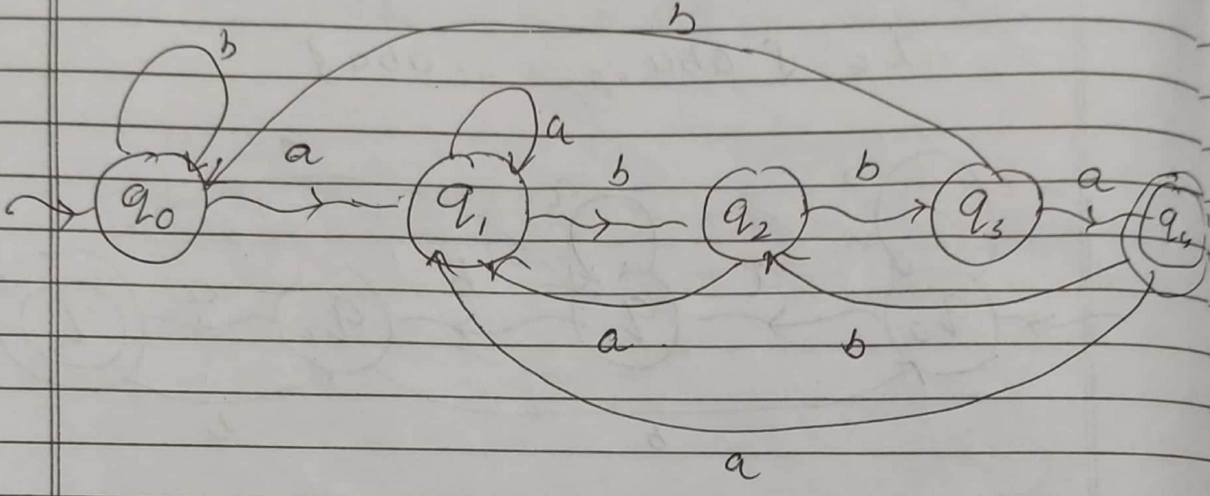
Qn7. L = { w : 'w' ends with "abba" over Σ = (a, b) }

Soln

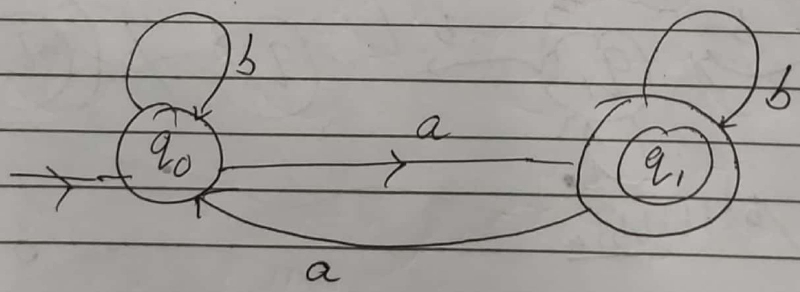
L = { abba, ... abba }



Qn1. L = { w : 'w' contains odd number of a's over Σ = (a, b) }

Soln

L = { a, aaa, abaa }

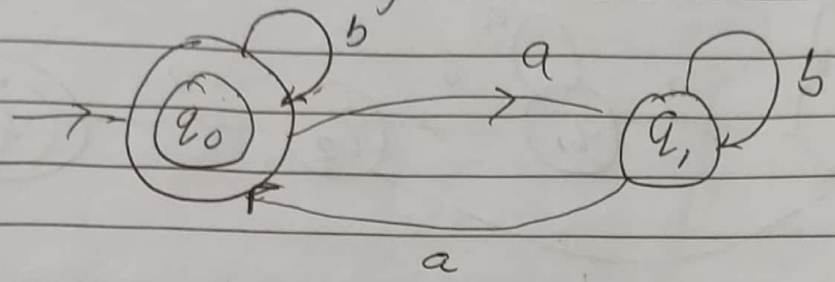


L = { w: 'w' contains even number of a's over Σ = {a,b} }

Sol^n

L = { ε, aa, }

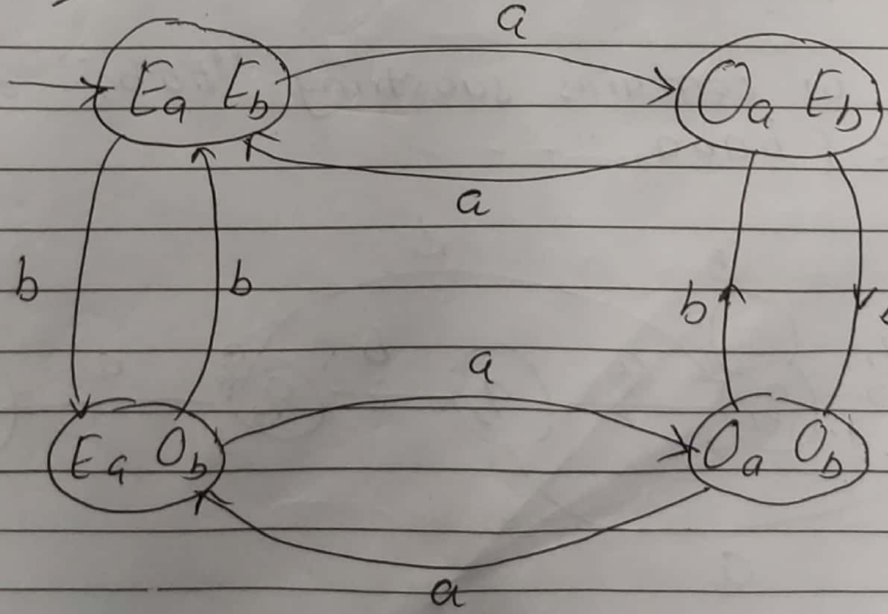
(If the language contains 'ε' then, initial state is the final state.)



3. L = {
→ w: contains odd no. of 'a' & odd number of 'b' ⟶ $O_a O_b$
→ w: contains even number of 'a' & even no. of 'b' ⟶ $E_a E_b$
→ w: contains odd number of 'a' & even no. of 'b' ⟶ $O_a E_b$
→ w: contains even no. of 'a' and odd no. of 'b' ⟶ $E_a O_b$

(Acc. to question make final state)

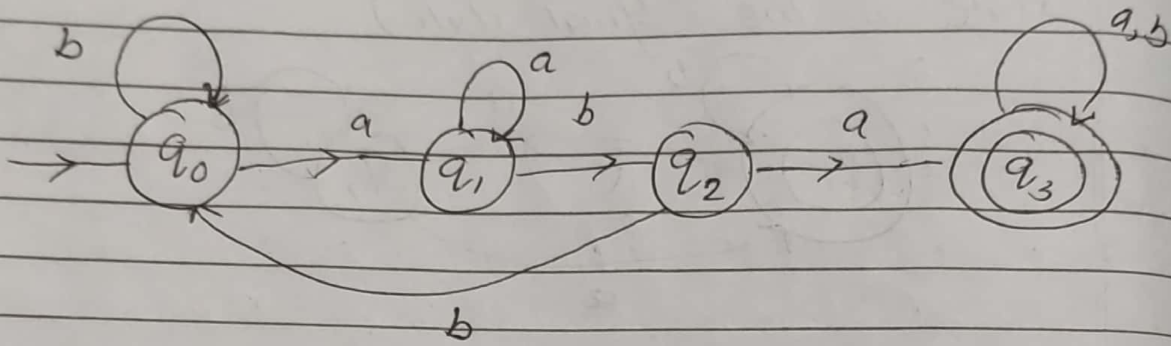
Sol^n



Qn4. $L = \{ w : w$ contains substring "aba" over $\Sigma = (a, b) \}$

soln

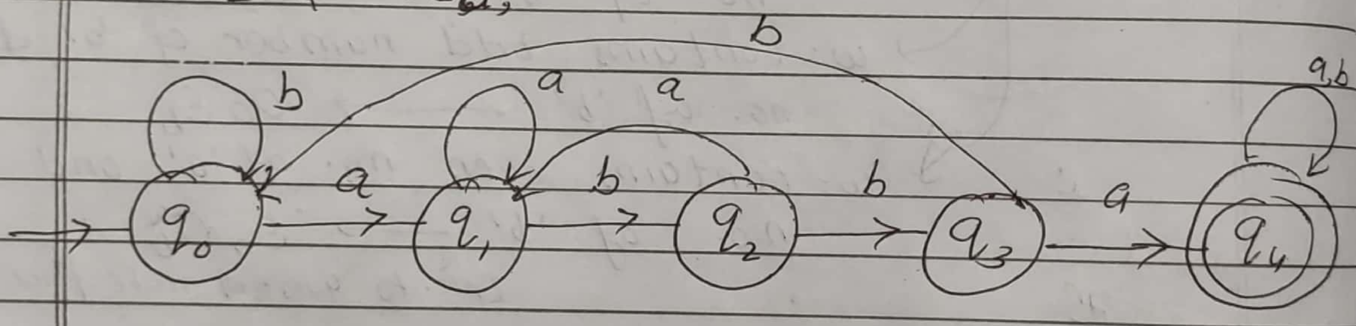
$L = \{ aba, \ldots$



Qn5. $L = \{ w : w$ contains substring "abba" over $\Sigma = (a, b)$

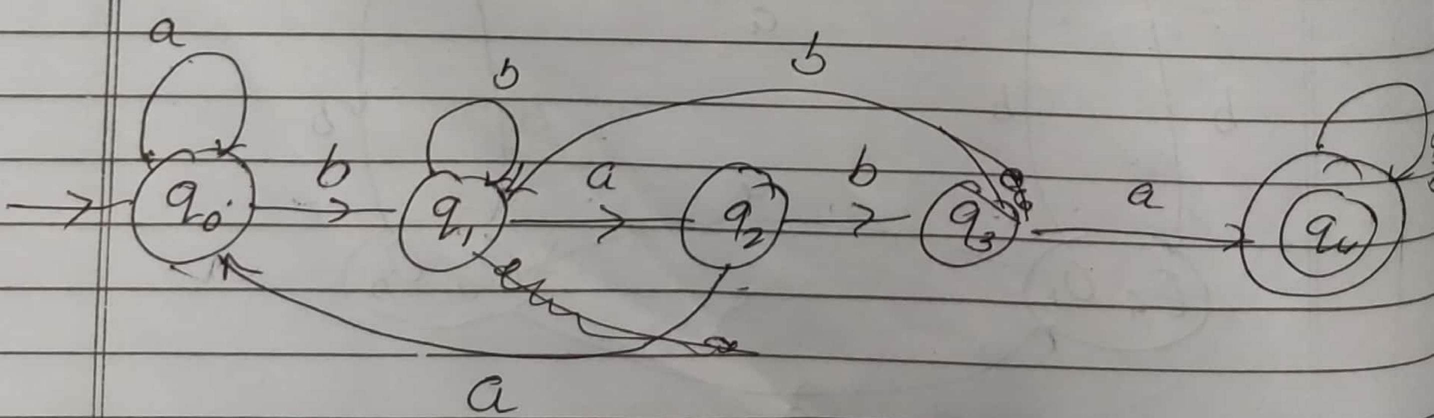
soln

$L = \{ abba, \ldots$



Qn6. $L = \{ w : w$ contains substring "baba" over $\Sigma = (a, b)$

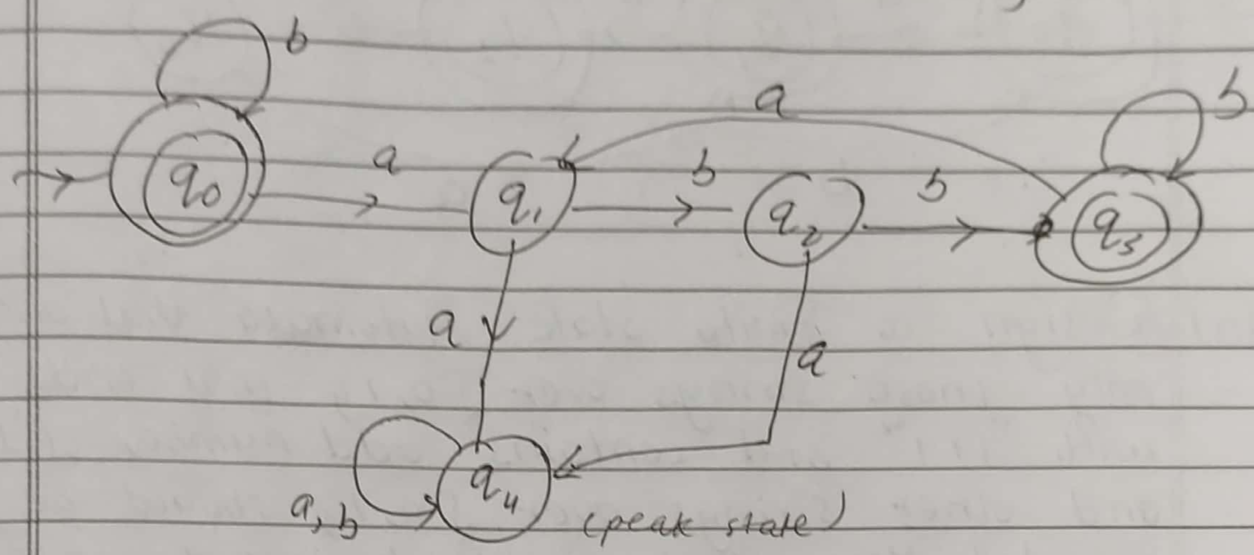
$L = \{ baba, \ldots$



Qn7 $L = \{ w :$ every 'a' is followed by "bb" over $\Sigma = (a,b) \}$

Soln

$L = \{ \epsilon,$ bbbbb, abb, $\}$



Qn8 $L = \{ w :$ 'w' doesn't ends with "aba" over $\Sigma = (a,b) \}$

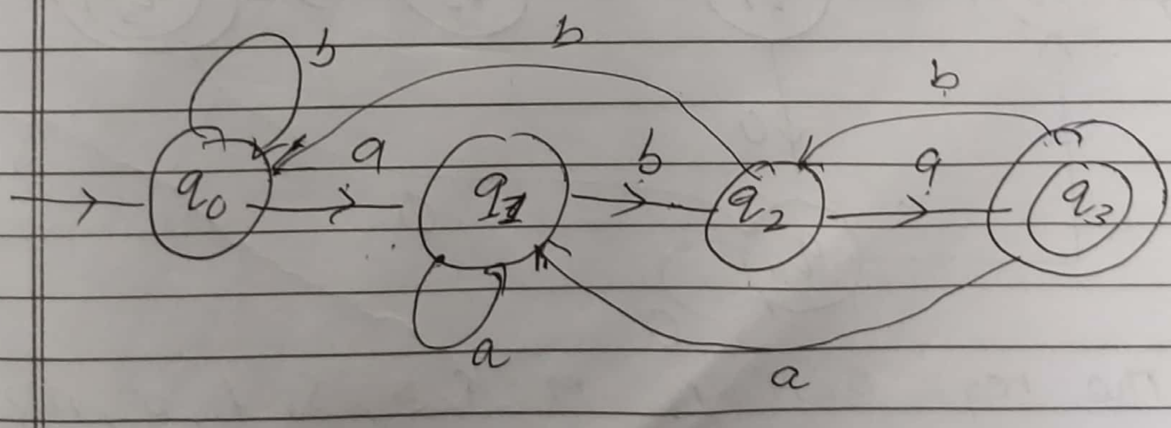
Note: For "doesnot" type question:

① construct DFA that "does" contain given propel

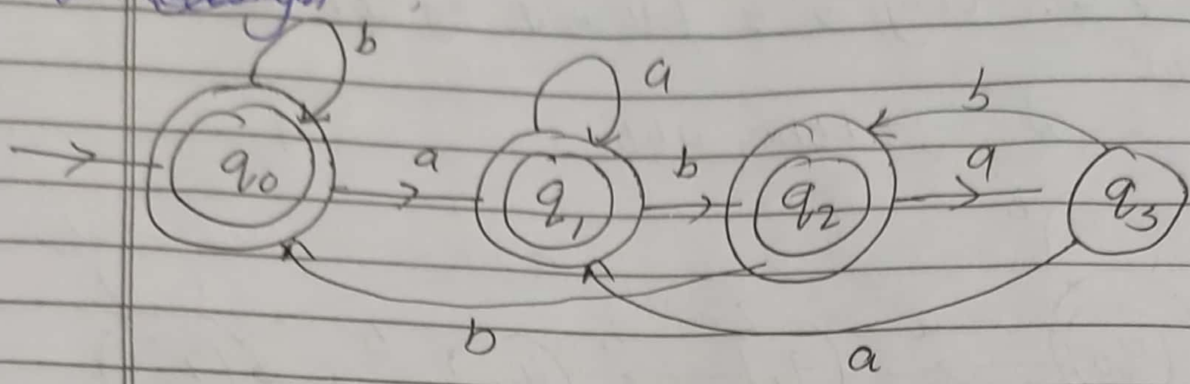
② Reverse final & non-final states.

① ends with "aba"

$L = \{$ aba, - - - -



Ans/ Design

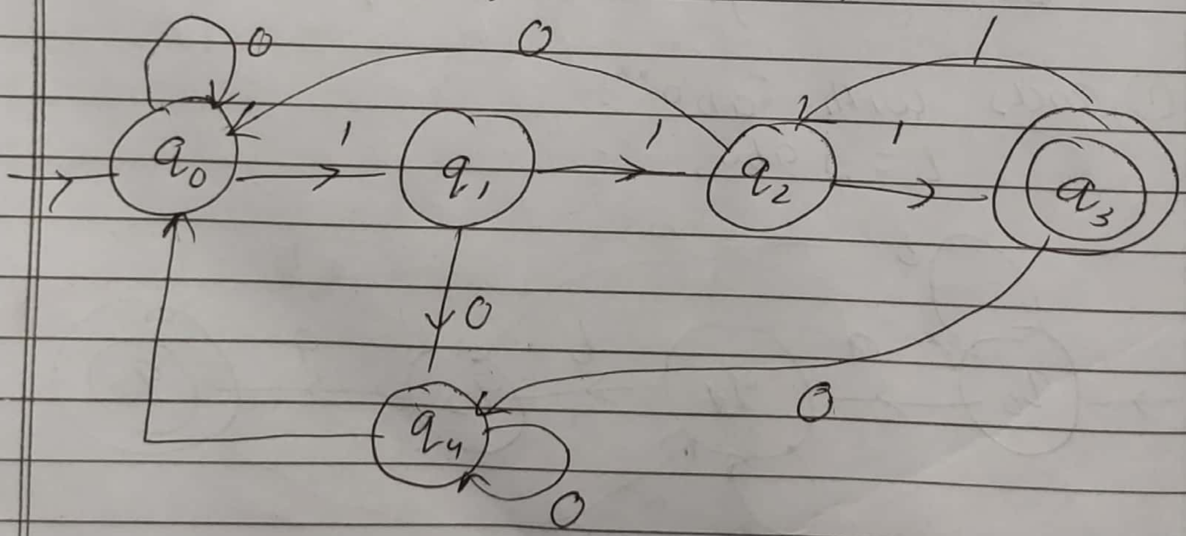


Qn 1. Design a Finite State Automata that accepts only those strings over {0, 1} that ends with "111" and contains odd number of '1' and other strings over {0, 1} should be rejected. Your design should include proper defination of finite-state automation, transition table and transition diagram.

Sol^n

Step I: transition diagram:

$L = \{111, \ldots$



The req. DFA is $M = \{\Sigma, S, f, \sigma, A\}$

where, $\Sigma = \{a, b\}$

$S = \{q_0, q_1, q_2, q_3, q_4\}$

$\sigma = q_0$

$A = \{ q_3 \}$

f is transition function define as:

δ \ I	0	1
$\rightarrow q_0$	q_0	q_1
q_1		
q_2		
q_3		