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Chapter 1: Introduction.

→ Set

→ A set is a well defined collection of distinct objects. For example: $\{1, 2, 3\}$ is the set of three numbers 1, 2 and 3. "{" indicates beginning of set and "}" indicates its end. Every object betn them is seperated by comma. is a member of the set.

Set of vowels : $\{a, e, i, o, u\}$

set of integer : $\{-2, -1, 0, 1, 2\}$

Real No; any number except $\{-\infty, \infty\}$

① Roster Form: In this form a set is described by listing elements seperated by commas with braces $\{\}$

② Set Builder form OR listing Method: Here, a set is described by a characterizing property $g(x)$ of its elements x. eg: $\{x : g(x) \text{ holds}\}$ or, $\{x \mid g(x) \text{ holds}\}$

eg: set of 20 Natural numbers:

$S = \{x \in N ; x < 20\}$

eg: $\{x \mid 1 \le x \le 2$ and x is a real number$\}$ represents the set of real numbers between 1 and 2.

eg: $\{x \mid x$ is the square of an integer and $x \le 100\}$ represents the set $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

③ Third way is to give a procedure, to generate the members of the set.

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First, basic elements of the set are presented.

i.e, $0 \in N$ — (1)

Then a method is given to generate elements of the ~~same~~ set.

i.e for any number x if $x \in N$, then $x+1 \in N$ — (2)

Thirdly, a statement is given that excludes undesirable elements:

i.e. Nothing is in N unless it is obtained from (1) and (2)

Hence by defn,

by (1) 0 is put into N

by (2), since 0 is in N, $0+1 (=1)$ is in N

Again by (2), $1+1 (=2)$ is in N

If we don't have (3), 0.5, 1.5, 2.5 can be included, which are not natural numbers.

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* Type of sets

1) Null set; (Empty set) or Void set; The set which contains no element is called the null set.
Denoted by ϕ, $\{\}$. $\{\phi\}$ is not null set.

2) Finite set; A set is called a finite set if it is either empty set or its elements can be listed by natural numbers.
eg: set of first five natural numbers $S = \{1, 2, 3, 4, 5\}$

3) Infinite set; A set whose elements cannot be listed by the natural numbers,
i.e $N = \{1, 2, 3, 4 \ldots\}$

* Cardinal number or Cardinality of a set.

→ If a set has n distinct elements for some natural numbers n, n is the cardinality (size) of S and S is a finite set.
for example the cardinality of the set $\{3, 1, 2\}$ is 3.

* Equivalent set;

Two sets are said to be equivalent set if their cardinal number are same,
i.e $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$
$n(A) = 4$ $n(B) = 4$
$\therefore n(A) \sim n(B)$

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*** Equal set :** Two sets are said to be equal sets if their cardinal number and elements are same

i.e $A = \{1, 2, 3, 4\}$ & $B = \{2, 4, 1, 3\}$

$n(A) = 4$ & $n(B) = 4$

*** Subset :** Let A and B be two sets, if every elements of A is an element of B then A is called a subset of B. It is denoted by $A \subseteq B$.

eg: $A = \{1, 2, 3\}$
& $B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \subset B$ → proper subset.

→ $A = \{a, b\}$ subset no : 2^n

∴ $2^2 = 4$ elements

Hence subsets are $\{\phi, \{a\}, \{b\}, \{a, b\}\}$

Q. $A \{a, b, c\}$: $2^3 = 8$

Note (1) Every set is a subset of itself.

(2) The total number of subset of a finite set containing n element is 2^n

(3) The empty set is a subset of every set.

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*** Power Set:** Let A be a set. Then the collection of all the subsets of A is called the power set of A and is denoted by by P(A)

$$P(A) = \{ S : S \subset A \}$$

eg: If $A = \{a, b, c\}$

its subsets is given by $2^3 = 8$

Hence, $\{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

Now;

$P(A) = \{\phi, \{a\} \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

*** Universal set:** A set that contains all sets in a given context is called universal set.

eg: $U = \{a, e, i, o, u\} \rightarrow A = \{a, e, i\}$

$B = \{i, o, u\}$

Set operation:

→ sets can be combined in a number of different ways to produce another set.

(I) Union of sets

(II) Intersection of sets

(III) Difference of sets

(IV) Complement of set.

(V) Product of set.

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(I) Union of set : The union of sets A and B, denoted by $A \cup B$.

if $A = \{1,2,3\}$ and $B = \{4,5\}$ then $A \cup B = \{1,2,3,4,5\}$

(II) Intersection of sets : The intersection of set A and B is denoted by $A \cap B$.

If $A = \{1,2,3\}$ and $B = \{1,2,4,5\}$ then

$A \cap B = \{1,2\}$

if $A = \{1,2,3\}$ and $B = \{4,5\}$ then $A \cap B = \phi$

(III) Difference of sets : The difference of sets A and B, denoted by $A - B$.

For eg: i) $A = \{1,2,3\}$ and $B = \{1,2,4,5\}$ then

$A - B = \{3\}$

if $A = \{1,2,3\}$ and $B = \{4,5\}$ then

$A - B = \{1,2,3\}$

Note: $A - B \neq B - A$.

(IV) Complement of sets : The complement of set A is the set of all elements in the universal set U but not in A. Denoted by $\overline{A}$

eg: Let $U = \{1,2,3,4,5,6\}$ and $A = \{1,2,3\}$ then

$\overline{A} = \{4,5,6\}$

eg: $U = \{d,a,n,g,e,r\}$ and $A = \{a,n,g,e,r\}$

$\therefore \overline{A} = \{d\}$

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(M) Cartesian Product: A cartesian product of two sets A and B is a set containing ordered pair from A and B. Denoted by $A \times B$

eg: Let $A = \{1, 2, 3\}$ and $B = \{c, d\}$

Find $A \times B$:

$A \times B = \{(1,c), (1,d), (2,c), (2,d), (3,c), (3,d)\}$

Properties of sets:

(1) Commutative law; $A \cup B = B \cup A$

$A \cap B = B \cap A$

(2) Associative law; $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

(3) Distributive law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(2) Associative law eg:

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7\}$ ... LHS.

Again; $B \cup C = \{3, 4, 5, 6, 7\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$ ... RHS

(3) Distributive; $A = \{3, 5, 7, 9\}$, $B = \{3, 8, 9\}$ $C = \{1, 2, 3, 9\}$

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Q. If $U = \{1,2,3,4,5,6,7,8,9\}$,
$A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$

Verify that:

(i) $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

(ii) $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

→ Soln: (i) $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

Now $\overline{A} = \{1,3,5,7,9\}$ and $\overline{B} = \{1,4,6,8,9\}$

For LHS $\overline{(A \cup B)}$, first we perform $(A \cup B)$

$\therefore (A \cup B) = \{2,4,6,8\} \cup \{2,3,5,7\}$

$= \{2,3,4,5,6,7,8\}$

Now $\overline{(A \cup B)} = \{1,9\}$ → comparing with universal

RHS $\overline{A} \cap \overline{B} = \{1,3,5,7,9\} \cap \{1,4,6,8,9\}$ set.

$= \{1,9\}$

$\therefore$ LHS = RHS. Hence verified.

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

LHS : $A \cap B = \{2\}$

$\overline{(A \cap B)} = \{1,3,4,5,6,7,8,9\}$ — (i)

RHS : $\overline{A} \cup \overline{B} = \{1,3,5,7,9\} \cup \{1,4,6,8,9\}$

$= \{1,3,4,5,6,7,8,9\}$ — (ii)

LHS = RHS.

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De Morgan's law proof Algebrically.

(I) $(A \cup B)' = A' \cap B'$

(II) $(A \cap B)' = A' \cup B'$

{ If $x \in A$, if we prove x also belongs to B i.e. $x \in B$ then $A \subset B$ (A is the subset of B) }

(I) To prove $(A \cup B)' = A' \cap B'$

→ Let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in (A' \cap B')$

$\therefore (A \cup B)' \subset A' \cap B'$ — ①

Also :

- $x \in (A \cup B)$ $\Rightarrow x \in A$ or $x \in B$
- If $x \notin (A \cap B)$ $\Rightarrow x \notin A$ or $x \notin B$

Also, { $x \in A$, and $x \notin A'$ vice versa. }

Again from RHS.

$y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$.

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\therefore (A' \cap B') \subset (A \cup B)'$ — ②

From ① and ② $A' \cap B' = (A \cup B)'$

Rules:

① $x \in (A \cup B)$
→ $x \in A$ or $x \in B$

② $x \notin (A \cap B)$
→ $x \notin A$ or $x \notin B$

③ $x \in (A \cap B)$
→ $x \in A$ and $x \in B$

④ $x \notin (A \cup B)$
→ $x \notin A$ and $x \notin B$

⑤ $x \in A$
→ $x \notin A'$ and vice versa.

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(ii) $(A \cap B)' = A' \cup B'$

Let $x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in (A' \cup B')$

Now, $(A \cap B)' \subset A' \cup B'$

Again from RHS. $y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\therefore (A' \cup B') \subset (A \cap B)'$

* Show that $A-(B \cup C) = (A-B) \cap (A-C)$

→ soln: Let us take the LHS as $x \in A-(B \cup C)$

$\Rightarrow x \in A$ & $x \notin (B \cup C)$

$\Rightarrow x \in A$ & $(x \notin B$ & $x \notin C)$

$\Rightarrow x \in A$ & $x \notin B$ and $x \in A$ & $x \notin C$

$\Rightarrow x \in A-B$ and $x \in A-C$

$\Rightarrow x \in ((A-B) \cap (A-C))$

$\therefore A-(B \cup C) \subseteq (A-B) \cap (A-C)$

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Conversly,

Let $x \in ((A-B) \cap (A-C))$

$\Rightarrow x \in (A-B)$ and $x \in (A-C)$

$\Rightarrow x \in A$ & $x \notin B$ and $x \in A$ & $x \notin C$

$\Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in (A-(B \cup C))$

Hence, proved LHS = RHS.

Show that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ → DeMorgan's law.

Show that $(A \cup B) \cup C = A \cup (B \cup C)$

→ Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

Therefore $A \cup (B \cup C) \subseteq (A \cup B) \cup C$

Conversly,

$x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

Here proved. //