Cosmos College of Management and Technology

Department of Science and Humanities

Algebra and Geometry

Assignment II

Submission Date: **2080-05-03**

Infinite Series

1. Find the center and radius of convergence and interval of convergence of the power series
   (a) $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n5^n}$
   (b) $\sum_{n=1}^{\infty} \frac{2^n(x+4)^n}{n}$
   (c) $\sum_{n=0}^{\infty} \frac{(x)^{2n+1}}{n!}$
   (d) $\sum_{n=1}^{\infty} \frac{(-1)^n(2x-1)^n}{n6^n}$
   (e) $\sum_{n=0}^{\infty} \frac{(10)^{n+1}}{3^{2n}} x^n$
   (f) $(x-2)+\frac{(x-2)^2}{4}+\frac{(x-2)^3}{9}+\frac{(x-2)^4}{16}+\ldots\ldots\ldots\ldots$
2. Test the convergence or divergence of the following series
   (a) $\frac{2}{3^2}+\frac{3}{4^2}+\frac{4}{5^2}+\ldots$
   (b) $\sum \frac{n+1}{2n+3}$
   (c) $\sum_{n=0}^{\infty}[\sqrt{n^2+1}-n]$
   (d) $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots$
   (e) $\frac{2}{9}+\frac{3}{16}+\frac{4}{25}+\cdots$
   (f) $1+\frac{2}{9}+\frac{3}{16}+\frac{4}{25}+\cdots$
   (g) $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\cdots$
   (h) $\sum[\sqrt{n^3+1}-\sqrt{n^3-1}]$

3. Prove the necessary condition for the convergence of an infinite series $\sum u_n$ is $\lim_{n\to\infty} u_n$ = 0. Also show that this is not sufficient condition.

Vector Space

1. Define eigen values and vectors of a square matrix with its characteristic equation. Find Eigen value and Eigen vector of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
2. State Cayley Hamilton Theorem and use it to find the inverse of $\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.
3. Define eigen value and eigen vectors of the square matrix. Find eigen value and vectors of the square matrix *A* = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$.
4. Define basis of a vector space over the field *F*. Check the vectors (1,1,1), (1,3,2), (-1,0,1) form a basis of R^3 or not.
5. Define Linear transformation. Check the following transformation is linear or not?

   (a) $T: \boldsymbol{R^2} \to \boldsymbol{R^2}$ defined by $T(x, y) = (x + 2, y)$.

   (b) $T: \boldsymbol{R^2} \to \boldsymbol{R}$ defined by $T(x, y) = |x + y|$.
6. Check the linearly dependence and independence of the vectors {(1,1,1), (1,2,1), (2,3,3)}.
7. Define subspace of a vector space *V*. Show that *W={(x,y): 2x+y=0}* is a subspace of R^2.

Matrix and System of Linear Equations

1. Maximize the following linear programming problem by using simplex method. $Z = 300x_1 + 500x_2$ subject to $x_1 + 4x_2 \le 30, x_1 + x_2 \le 5, 2x_1 + x_2 \le 30$
2. Minimize *Z* = $8x_1$+ $9x_2$ subject to $x_1 + x_2 \ge 5, 3x_1 + x_2 \ge 21, x_1 \ge 0, x_2 \ge 0$ and solve it by dual method.

3. Apply simplex method to solve, maximize: Z = $15x_1 + 10x_2$ subjected to $2x_1 + 2x_2 \le 10; 2x_1 + 3x_2 \le 10;\ x_1, x_2 \ge 0$
4. Maximize $Z = -3x_1 + 7x_2$ subject to the constraints $2x_1 + 3x_2 \le 5$, $5x_1 + 2x_2 \ge 3, x_2 \le 1, x_1, x_2 \ge 0$ by Big M method.

Vector Algebra

1. Define vector triple product. Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$.
2. Show that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.
3. Define scalar triple product. Prove that the scalar triple product of three vectors represent a volume of a parallelepiped. What conclusion can be drawn if $\vec{a} \times (\vec{b} \times \vec{c}) = 0$?
4. Show that the position of dot and cross can be interchanged in scalar triple product, without changing its value. Also derive the formula for scalar triple product in determinant form.
5. Define scalar triple product. Show that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ if and only if the vectors $\vec{a}$ and $\vec{c}$ are collinear.
6. Define reciprocal system of vectors. Find the reciprocal vectors of **a** = 2**i** + 3**j** – **k**, **b** = **i** – **j** -2**k** and **c** = - **i** + 2**j** + 2**k**.