Assignment 1

Deriving the implicit formula for the cylinder

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To get the implicit equation, the following one needs to be solved:

$$\| x - c' \| - r = 0$$

where x is the intersection of the ray and the cylinder, c' is the projection of this point onto the axis of the cylinder and r its radius.

To compute c', let us define $\vec{a}$ as its normalized axis, c as its center and $\vec{b}$ as the vector pointing from c to x – i.e. $\vec{b} = x - c$. By computing the dot product between $\vec{b} \cdot \vec{a}$, we get the distance between c and c' since $\vec{a}$ is normal – it's an orthogonal projection. We get c' by using the following:

$$c + (\vec{b} \cdot \vec{a})\, \vec{a}$$

The fact that $\vec{a}$ is already normalized is to be kept in mind. We finally get:

$$\left\| x - c - (\vec{b} \cdot \vec{a})\, \vec{a} \right\| - r = 0$$

$$\begin{aligned} 0 &= \left\| x - c - (\vec{b} \cdot \vec{a})\, \vec{a} \right\| - r \\ &= \left(x - c - (\vec{b} \cdot \vec{a})\, \vec{a} \right)^2 - r^2 \end{aligned}$$

Now, let us replace x with the explicit ray equation $o + t\,\vec{d}$, where o is its origin point and $\vec{d}$ its vector, and $\vec{b}$ with its definition.

$$\begin{aligned} 0 &= \left(o + t\,\vec{d} - c - ((o + t\,\vec{d} - c) \cdot \vec{a})\, \vec{a} \right)^2 - r^2 \\ &= \left(t\,\vec{d} + o - c - ((o + t\,\vec{d} - c) \cdot \vec{a})\, \vec{a} \right)^2 - r^2 \\ &= \left(t\,\vec{d} + o - c - (t\,\vec{d} \cdot \vec{a})\, \vec{a} - ((o - c) \cdot \vec{a})\, \vec{a} \right)^2 - r^2 \\ &= \left(t\,(\vec{d} - (\vec{d} \cdot \vec{a})\, \vec{a}) + o - c - ((o - c) \cdot \vec{a})\, \vec{a} \right)^2 - r^2 \end{aligned}$$

Now, let's define $x = \vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}$ and $y = o - c - ((o - c) \cdot \vec{a})\,\vec{a}$. We thus get the following:

$$\begin{aligned} 0 &= (t\,x + y)^2 - r^2 \\ &= t^2\,x^2 + t\,2xy + y^2 - r^2 \end{aligned}$$

Let's remember that a vector squared is the result of applying the scalar product on itself. We finally get as values for our quadratic function:

$$\begin{aligned} A = x^2 &= \left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \cdot \left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \\ B = 2xy &= 2\,\left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \cdot (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) \\ C = y^2 - r^2 &= (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) \cdot (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) - r^2. \end{aligned}$$