Assignment 1

Deriving the implicit formula for the cylinder

Ding Markus Mamie Robin Montial Charline

March 5, 2019

To get the implicit equation, the following one needs to be solved:

$$\| \, x - c' \, \| - r = 0$$

where x is the intersection of the ray and the cylinder, c' is the projection of this point onto the axis of the cylinder and r its radius.

To compute c', let us define $\vec{a}$ as its normalized axis, c as its center and $\vec{b}$ as the vector pointing from c to x – i.e. $\vec{b} = x - c$. By computing the dot product between $\vec{b} \cdot \vec{a}$, we get the distance between c and c'. We get c' by using the following:

$$c + (\vec{b} \cdot \vec{a}) \, \vec{a}$$

The fact that $\vec{a}$ is already normalized is to be kept in mind. We finally get:

$$\left\| \, x - c - (\vec{b} \cdot \vec{a}) \, \vec{a} \, \right\| - r = 0$$

$$\begin{aligned} 0 &= \left\| \, x - c - (\vec{b} \cdot \vec{a}) \, \vec{a} \, \right\| - r \\ &= \left(x - c - (\vec{b} \cdot \vec{a}) \, \vec{a} \right)^2 - r^2 \end{aligned}$$

Now, let us replace x with the explicit ray equation $o + t \, \vec{d}$, where o is its origin point and $\vec{d}$ its vector, and $\vec{b}$ with its definition.

$$\begin{aligned} 0 &= \left(o + t \, \vec{d} - c - ((o + t \, \vec{d} - c) \cdot \vec{a}) \, \vec{a} \right)^2 - r^2 \\ &= \left(t \, \vec{d} + o - c - ((o + t \, \vec{d} - c) \cdot \vec{a}) \, \vec{a} \right)^2 - r^2 \\ &= \left(t \, \vec{d} + o - c - (t \, \vec{d} \cdot \vec{a}) \, \vec{a} - ((o - c) \cdot \vec{a}) \, \vec{a} \right)^2 - r^2 \\ &= \left(t \, (\vec{d} - (\vec{d} \cdot \vec{a}) \, \vec{a}) + o - c - ((o - c) \cdot \vec{a}) \, \vec{a} \right)^2 - r^2 \end{aligned}$$

Now, let's define $x = \vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}$ and $y = o - c - ((o - c) \cdot \vec{a})\,\vec{a}$. We thus get the following:

$$\begin{aligned} 0 &= (t\,x + y)^2 - r^2 \\ &= t^2\,x^2 + t\,2xy + y^2 - r^2 \end{aligned}$$

Let's remember that a vector squared is the result of applying the scalar product on itself. We finally get as values for our quadratic function:

$$\begin{aligned} A = x^2 &= \left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \cdot \left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \\ B = 2xy &= 2\,\left(\vec{d} - (\vec{d} \cdot \vec{a})\,\vec{a}\right) \cdot (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) \\ C = y^2 - r^2 &= (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) \cdot (o - c - ((o - c) \cdot \vec{a})\,\vec{a}) - r^2 \end{aligned}$$