

# Assignment 1

## Deriving the implicit formula for the cylinder

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To get the implicit equation, the following one needs to be solved:

$$\|x - c'\| - r = 0$$

where  $x$  is the intersection of the ray and the cylinder,  $c'$  is the projection of this point onto the axis of the cylinder and  $r$  its radius.

To compute  $c'$ , let us define  $\vec{a}$  as its normalized axis,  $c$  as its center and  $\vec{b}$  as the vector pointing from  $c$  to  $x$  - i.e.  $\vec{b} = x - c$ . By computing the dot product between  $\vec{b} \cdot \vec{a}$ , we get the distance between  $c$  and  $c'$ . We get  $c'$  by using the following:

$$c + (\vec{b} \cdot \vec{a}) \vec{a}$$

The fact that  $\vec{a}$  is already normalized is to be kept in mind. We finally get:

$$\|x - c - (\vec{b} \cdot \vec{a}) \vec{a}\| - r = 0$$

$$\begin{aligned} 0 &= \|x - c - (\vec{b} \cdot \vec{a}) \vec{a}\| - r \\ &= \left(x - c - (\vec{b} \cdot \vec{a}) \vec{a}\right)^2 - r^2 \end{aligned}$$

Now, let us replace  $x$  with the explicit ray equation  $o + t\vec{d}$ , where  $o$  is its origin point and  $\vec{d}$  its vector, and  $\vec{b}$  with its definition.

$$\begin{aligned} 0 &= \left(o + t\vec{d} - c - ((o + t\vec{d} - c) \cdot \vec{a}) \vec{a}\right)^2 - r^2 \\ &= \left(t\vec{d} + o - c - ((o + t\vec{d} - c) \cdot \vec{a}) \vec{a}\right)^2 - r^2 \\ &= \left(t\vec{d} + o - c - (t\vec{d} \cdot \vec{a}) \vec{a} - ((o - c) \cdot \vec{a}) \vec{a}\right)^2 - r^2 \\ &= \left(t(\vec{d} - (\vec{d} \cdot \vec{a}) \vec{a}) + o - c - ((o - c) \cdot \vec{a}) \vec{a}\right)^2 - r^2 \end{aligned}$$

Now, let's define  $x = \vec{d} - (\vec{d} \cdot \vec{a}) \vec{a}$  and  $y = o - c - ((o - c) \cdot \vec{a}) \vec{a}$ . We thus get the following:

$$\begin{aligned} 0 &= (tx + y)^2 - r^2 \\ &= t^2 x^2 + t 2xy + y^2 - r^2 \end{aligned}$$

Let's remember that a vector squared is the result of applying the scalar product on itself. We finally get as values for our quadratic function:

$$\begin{aligned} A = x^2 &= \left( \vec{d} - (\vec{d} \cdot \vec{a}) \vec{a} \right) \cdot \left( \vec{d} - (\vec{d} \cdot \vec{a}) \vec{a} \right) \\ B = 2xy &= 2 \left( \vec{d} - (\vec{d} \cdot \vec{a}) \vec{a} \right) \cdot (o - c - ((o - c) \cdot \vec{a}) \vec{a}) \\ C = y^2 &= (o - c - ((o - c) \cdot \vec{a}) \vec{a}) \cdot (o - c - ((o - c) \cdot \vec{a}) \vec{a}) \end{aligned}$$