ENG-6001B

Electricity Generation and Distribution

001 – Lab Report 1
Tuned filter and power factor correction

by

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1. Introduction

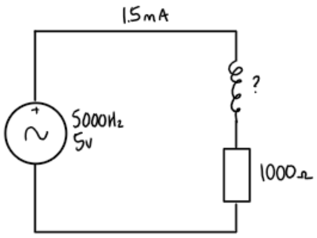
This exercise explores the design and engineering of two commonly found and used electronic circuits, along with circuit analysis. The two circuits are; a band pass filter, and a power factor correction circuit. A band pass filter circuit is a circuit that only allows a certain range of frequencies through it, attenuating frequencies it is designed to not permit. An example application where these would be used would be tuning a radio to a particular radio station's frequency. Power factor correcting a circuit is done to make the circuit as efficient as possible - the total power consumed by the circuit is by active power or resistive components, like LEDs, heating elements, etc., and reactive/wattless power consumed by inductors or capacitors is minimised. This is important as it can significantly improve a circuit's efficiency by reducing current consumption.

2. Description of the assignment

This assignment required completing multiple set tasks. Section 2.1 involves calculating, finding and validating a value for an appropriate inductor for the two aforementioned circuits, 2.2 involves designing, assembling and verifying a tuned band pass filter, and 2.3 involves designing and constructing a power factor correction circuit. Finally, section 2.4 involves analysing AC and DC circuits.

2.1. Selection of the value of an inductor

a. The first task involved finding the value of an inductor to be used in the two circuits. The circuit to do this is an alternating current AC voltage source connected in series with a 1000 Ohm resistor and an inductor. The circuit's input frequency (5KHz) and voltage amplitude (5V) were provided, along with the circuit's current amplitude (1.5mA). As we are provided both the voltage and current of the circuit, we can use Ohm's law to find the total resistance or impedance of the circuit. With this we can simply then subtract the 1000 Ohms the resistor provides leaving us with the value of the inductor. This gave a value of 0.1012160929 Henrys. A real inductor of this value was then measured with an RCL Bridge, giving a value of 0.1009 Henrys.



$V_s = 5\sin(2\pi \times 5000)$

$I = 0.0015A$

$Z_T = 1000 + j\omega L = 1000 + j\,2\pi \times 5000 \times L$

2 i. $|V| = 5v \quad |i| = 1.5mA$

$|z| = \frac{5}{1.5mA} = \frac{10000}{3}$

$|z|^2 = R^2 + (\omega L)^2$

$\left(\frac{10000}{3}\right)^2 = 1000^2 + (\omega L)^2$

$11111111.11 = 1000000 + \omega^2 L^2 \rightarrow 10111111.11 = \omega^2 L^2$

$10111111.11 = (2\pi f)^2 L^2 = (10000\pi)^2 L^2$

$3179.797338 = 10000\pi L \quad L = 0.1012160929\,H$

Actual measured: $0.1009\,H$

Figure 1 A diagram of the circuit, along with calculating the value of the inductor.

b. & c. With the real measured value of the inductor and combination of resistors used to obtain ~1000 Ohms, the current amplitude was calculated and predicted for the circuit. This gave a predicted exact amplitude of 1.504437mA. The circuit was then constructed and measured. Due to the equipment not cooperating the current had to be calculated from voltage and resistance measurements (4.75V and 998.8 Ohms). This gave the value of 1.452mA.

b. $0.0015\,A$ theory $R = j\omega L$ $j \times 10000\pi \times 0.1009$

$0.1009\,H$ $R_T = 998.8 + 3169.867j$

$998.8\,\Omega$ $|R_T| = \sqrt{998.8^2 + 3169.867^2}$

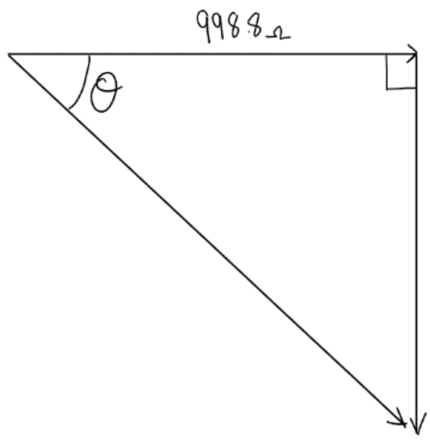
$5 = |I| \times |R_T|$

$5 = |I| \times 3323.5$

$|I| = 0.001504437\,A = 1.5\,mA$

Figure 2 Calculating the exact current using the real measured value of the inductor.

d. Finally the phase between current and input voltage was calculated. This can be done with trigonometry and complex numbers, using the resistor as the real component (998.8 Ohms) and the inductor's impedance as the complex component ($(2\pi \times 5 \times 0.1009j) = 3169.867$ Ohms). This gives an angle or phase of -72.51077296 degrees. The angle is negative due to the inductor making the current lag behind the input voltage. The actual and measured phase was -71 degrees.



3169.9... Ω

$\theta = \tan^{-1}\left(\frac{3169.866987}{998.8}\right) = -72.51077296°$ (negative since inductor)

Measured value : $-71°$

Figure 3 Calculating phase difference between the current and voltage of the circuit.

2.2. Design, assembly, and verification of a tuned filter

a. The next task was to design the band pass filter, with the requirements of a 5000 Hz resonant frequency with a Q factor of 20. The inductor of measured value 0.1009 Henrys from the previous task was to be used, and the capacitor and resistor values were to be calculated. Out of the two unknowns the capacitor was calculated first. This should be done as capacitors come in a smaller variety of values compared to resistors, so the resistor value should cater for the capacitor value rather than the other way around. This was done with the formula $w_0 = \frac{1}{\sqrt{LC}}$ where w_0 is the resonant frequency 5000Hz, and L being the inductor of value 0.1009H. This gives us the capacitor value of 0.1004174268 nano-Farads.

$$w = \frac{1}{\sqrt{LC}} \rightarrow 2\pi \times 5000 = \frac{1}{\sqrt{LC}} \rightarrow 10000\pi = \frac{1}{\sqrt{0.1009 \times C}} \rightarrow \sqrt{0.1009 \times C} = \frac{1}{10000\pi} \rightarrow C = 1.004174268 \times 10^{-8} F$$

$$Q = \frac{1}{wCR} \rightarrow 20 = \frac{1}{2\pi \times 5000 \times C \times R} \rightarrow \frac{1}{20} = 10000\pi \times R \times 1.004174268 \times 10^{-8} \rightarrow R = 158.4933494 \Omega$$

Figure 4 Calculating the value of the capacitor and resistors for the filter with the given parameter and component values.

b. The circuit was then constructed and measured to validate the theoretical values. This was done by measuring the resistor voltage over the voltage supply, over a range of frequencies (100 – 50,000Hz) in regular intervals of 1000Hz. As we find the peak resonance of the circuit smaller intervals were used for accuracy. This was done with a log scale for frequency. The measured resonant frequency and Q factor of the circuit differed with the theoretical results. Resonant frequency was quite close, being 4710Hz instead of 5000. This is expected as real-world values are not exact and could be affected by internal resistances. However, Q Factor was very far off, being 6.7286 instead of the desired 20. This could be from the combination of internal resistances of the real-world circuit, as well as a lack of accuracy from reading the graph. This could be contacted by reducing the value of the resistor or capacitor, which would increase the Q factor for this circuit.

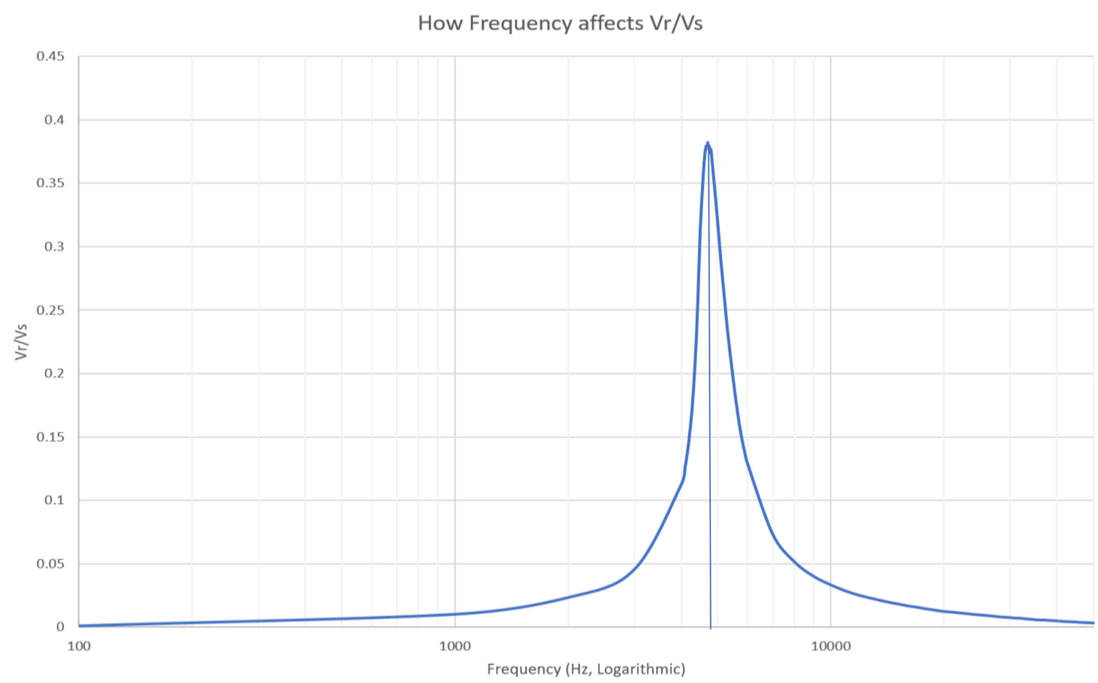


Figure 5 A graph showing the ratio of Vr/Vs and how it is affected by frequency.

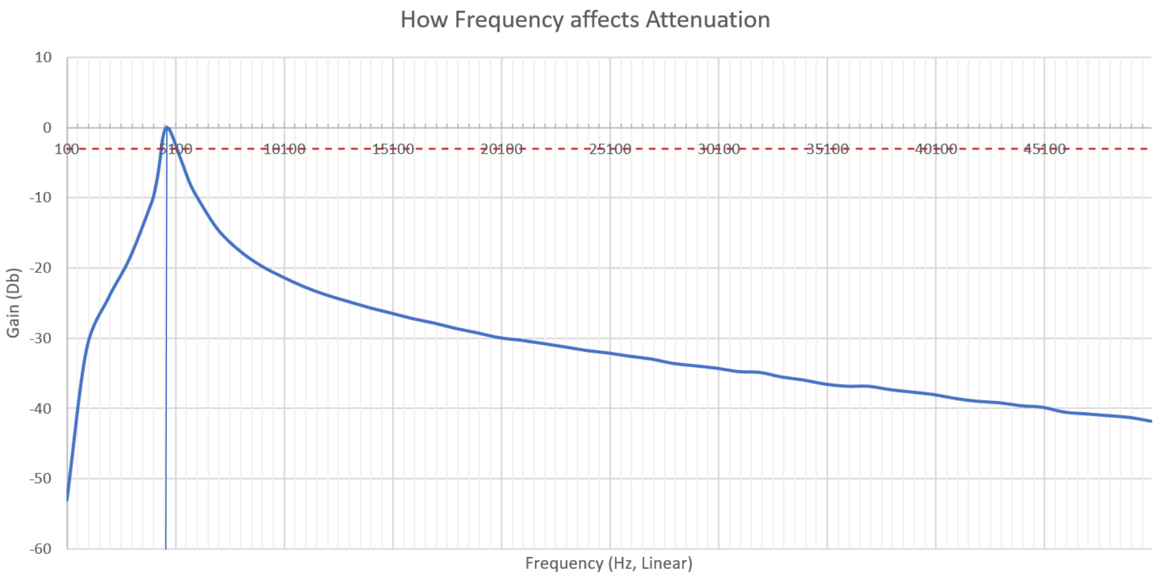


Figure 6 A similar graph showing negative Gain or attenuation affected by frequency. This was used to measure an estimate for the Q factor.

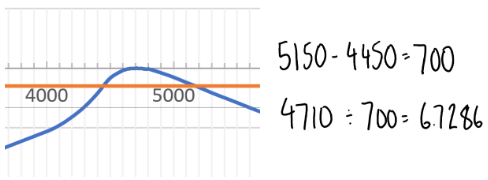


Figure 7 Calculating Q Factor from the previous graph.

c. Around the resonant frequency the phase between the voltage source v_S and the resistor v_R is 0 degrees or is in phase. The phase decreases to -90 degrees as we decrease the frequency to 1000Hz and increases to 90 degrees as frequency is increased to 50,000Hz. Graphing this shows a Sigmoidal shaped line.

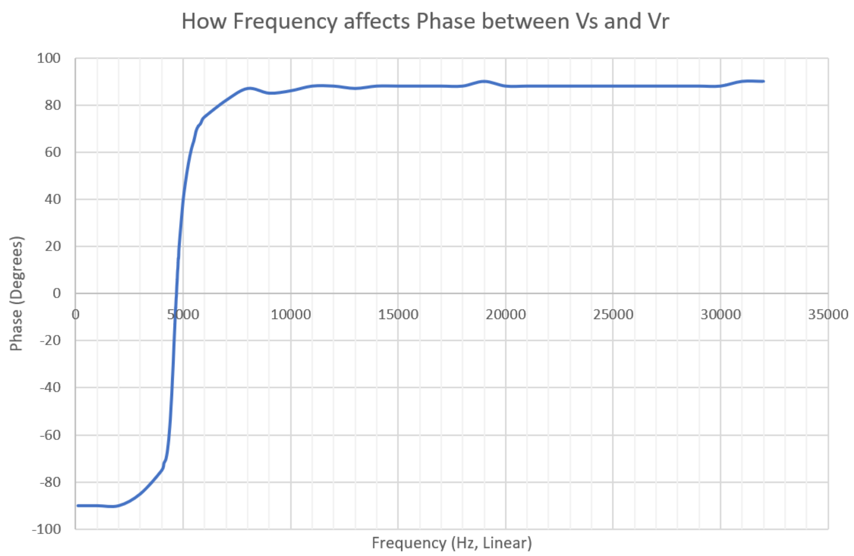


Figure 8 A graph showing how phase is affected by frequency. It is sigmoidal in shape.

d. The final part to this task required passing a square wave at 1000 Hz through the circuit and viewing and discussing the output.

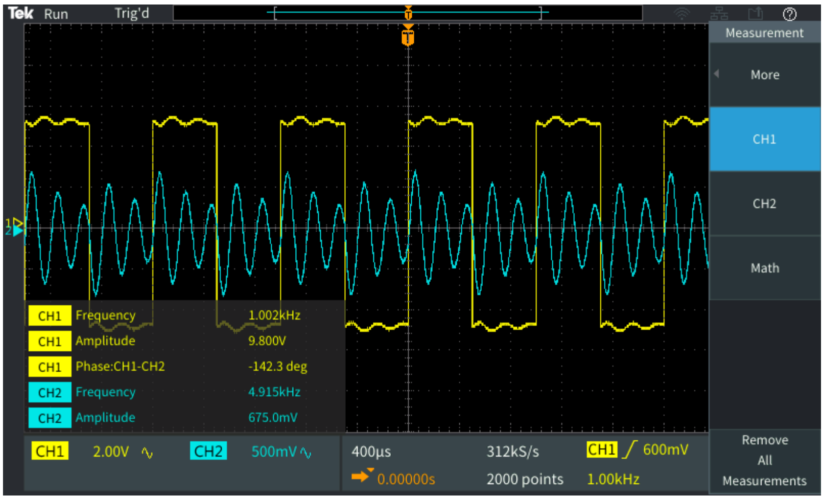


Figure 9 Readings showing the input voltage supply of a square wave and the output from the filter constructed.

The filter circuit outputs a sinusoidal wave of 5000 Hz from the 1000 Hz Square wave input. This is due to how square waves can be represented as a combination of many sinusoidal waves. Any regular wave of any shape can be made by summing several sinusoidal waves together – a process called additive synthesis. We can see this with a Fourier transform/spectrogram graph of an example 5Hz square wave pictured in figure 10. The output we see with our 1000Hz square wave then is to be expected, as the filter has filtered all but one of these sinusoidal waves out, which is the 5000 Hz sine wave we see on channel 2 in figure 9. The amplitude is also as expected, as it is just one of several sinusoidal waves that form the total 10V 1000Hz wave. The waves are also aligned - every one square wave has five sinusoidal waves.

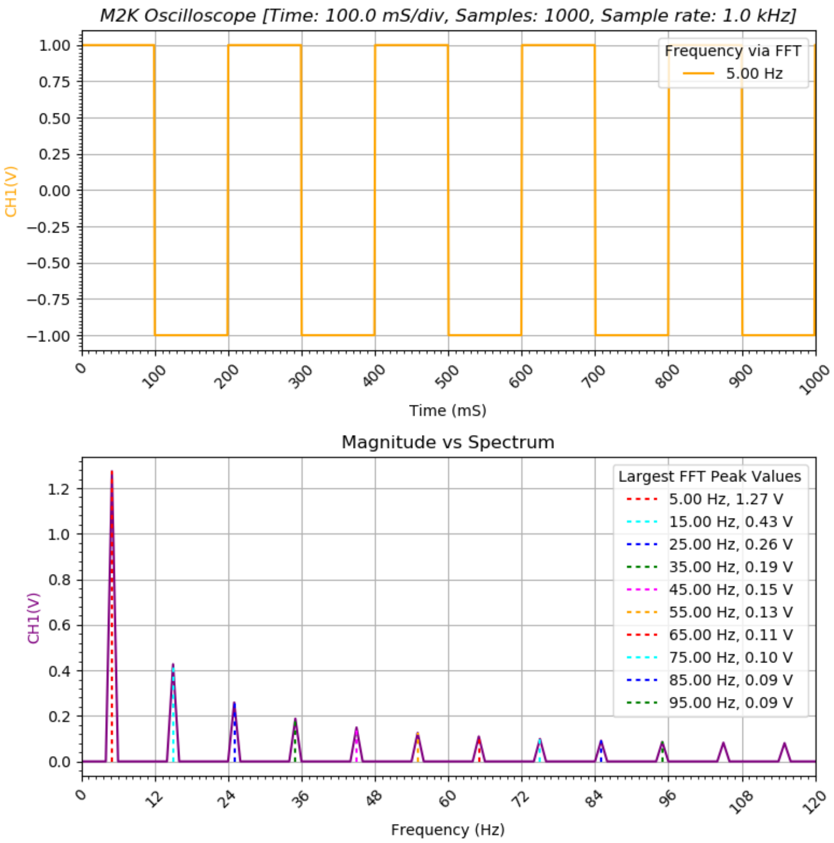


Figure 10 A Spectrogram graph of an example square wave showing that the square wave is constructed of multiple sinusoidal waves of different frequencies.

2.3. Design, implementation, and verification of a power-factor correction circuit

This task involves correcting the power factor of a circuit, with the aim of making the circuit use less current for the same power output, and be therefore more efficient. This is done by reducing or eliminating wattless power consumption, making all total/apparent power consumption used by resistive, real or useful components such as LEDs in a real-world application.

Power factor is essentially the ratio between useful, real power consumption over total power consumption for a given circuit. A power factor of 1, or where all of a circuit's power consumption is from real components, would be desirable.

We were to use the same 0.1009 Henry inductor from prior tasks, with a voltage supply running at 1000 Hz. We chose a voltage of 10 Volts peak to peak. To power factor correct a circuit, one must have an inefficient circuit in the first place, so the task required finding a resistor of a value where the resistive and reactive components have the same resistance/impedance – where the apparent power is at an angle of 45 degrees. If we $cos(45)$, this shows us the circuit would have a power factor of 0.707. To achieve this angle, the resistor's resistance should then be the same as the inductor's impedance – 633.9733975 Ohms.

$X_L = j\omega L$ $X_L = j \times 2\pi \times 1000 \times 0.1009 = 633.9733975j\Omega$

R is = to L if angle is $45°$ so $R = 633.9733975\Omega$

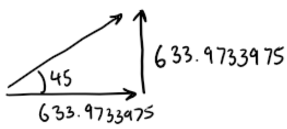


Figure 11 Calculating the resistor value if the phase difference was 45 degrees.

b. Next, the circuit was constructed and the voltage of the source and inductor were measured. Results are expected – the inductor is roughly the calculated 45 degrees out of phase from the supply (40.86 degrees), and the voltage is roughly half the supply (6.600V), due to being in series with a resistor of the same inductance/resistance. The values are slightly off, and this can be due to internal resistances and non-ideal real-world component values.

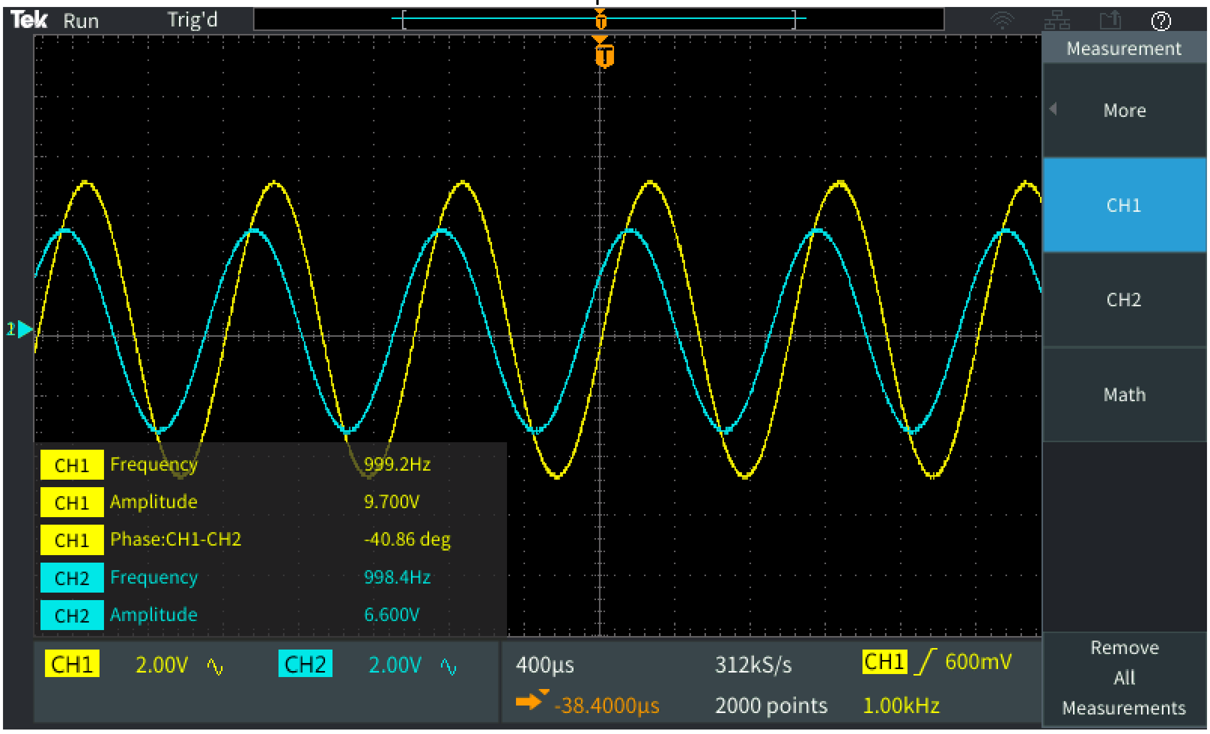


Figure 12 Readings of the voltage supply input on channel 1 with the inductor reading on channel 2. As you can see it is out of phase and its amplitude diminished.

c. The next task was to power correct the circuit, using a capacitor where its value was to be calculated. The pre-corrected power factor can be found by performing $cos(45)$ (or $cos(40.85)$ if using actually measured values), the angle being the phase of the inductor from the supply. This gives us a power factor of 0.707, or 0.756, depending on ideal or actual values. The value of the capacitor can then be found with $wC = \frac{B}{A^2+B^2}$, where A and B are the values of the resistor and inductor. Solving for C gives us the value 0.1255217835 micro-Farads for the capacitor.

$$wC = \frac{633.97}{633.97^2+633.97^2} = \frac{1}{633.97+633.97} = \frac{1}{1267.946795} = wC = 2\pi \times f \times C \rightarrow$$

$$\rightarrow 2000\pi \times C = \frac{1}{1267.946795} = 1.255217835\times10^{-7}\,F$$

Figure 13 Calculating the capacitor's value from the circuit's frequency, inductor and resistor values.

d. The capacitor along with an additional resistor was added to the circuit to check the correction. As shown phase (and therefore power factor) has been corrected.

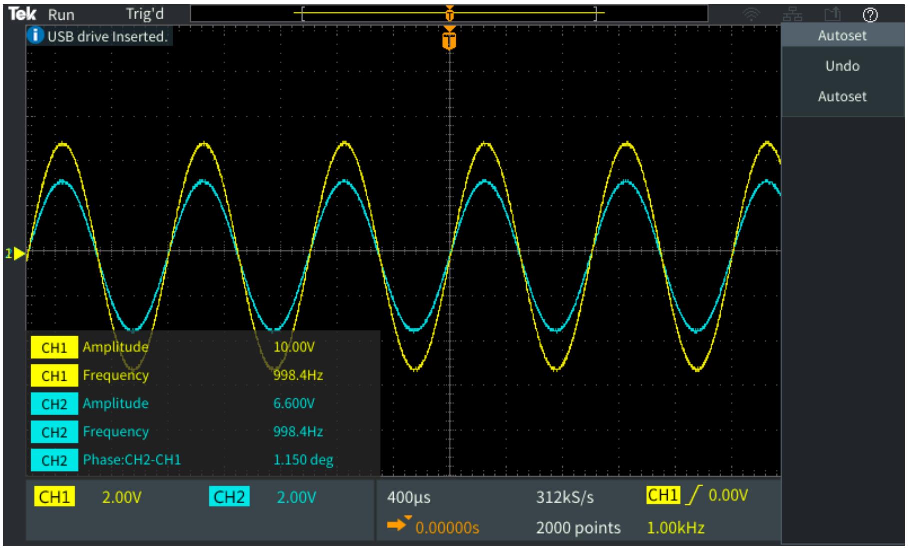


Figure 14 Readings of the power factor corrected circuit. The output and input are now in phase.

Performing $cos(0)$ or $cos(1.15)$ - the theoretical value or the actually obtained value, gives us a power factor value of 1 or 0.999 in ideal or actual values respectively. Voltage also shows a roughly ideal drop, as the capacitor has a resistance/inductance equal to the original resistor and inductor it is connected in parallel to. This creates a voltage divider where R1 and R2 are equal, meaning voltage over Rs and the capacitor should be equal, and 5 volts each. The real value of 6.600V is similar to the voltage measured in the previous task, and both can be explained by internal and not exact resistances in the circuit.

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times \frac{1}{1267.946795}} = \frac{1267.946795}{j} = -1267.946795j\,\Omega$$

Figure 15 Calculating the impedance of the capacitor from previously calculated wC. It is double that of the resistor and inductor.

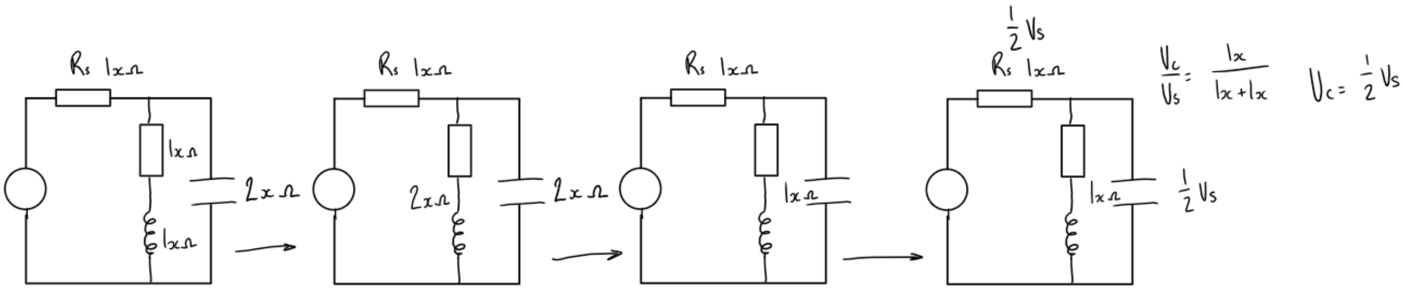
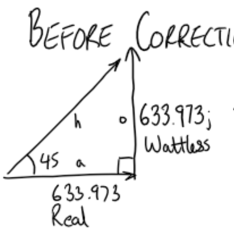


Figure 16 Diagrams showing how voltage is halved between Rs and the rest of the components, x being the resistance/impedance value of the components.

Calculating current required to provide the same amounts of power before and after correction shows the justification for this task – the circuit now consumes ~29% less current than before. This is essentially free energy savings, as the functionality of the circuit has not been affected in any way.

BEFORE CORRECTION



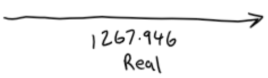
$$Z = h = \sqrt{633.973^2 + 633.973^2} = 896.5737769\,\Omega$$

$$V_{RMS} = \frac{5}{\sqrt{2}} = 3.535533906\,V$$

$$I_{RMS} = \frac{3.535533906}{896.5737769} = 3.943383129 \times 10^{-3}\,A$$

AFTER CORRECTION

$$Z = (633.973 + 633.973j) \| (-1267.946j) = \frac{(633.973 + 633.973j)(-1267.946j)}{633.973 + 633.973j - 1267.946j} = 1267.946\,\Omega$$



$$I_{RMS} = \frac{3.535533906}{1267.946} = 2.788392951 \times 10^{-3}\,A$$

~ 29% Less current used.

Figure 17 Calculating the current consumed for the current before and after power factor correction. An improvement of ~29%.

2.4. Questions on design and analysis of DC and AC circuits

a.

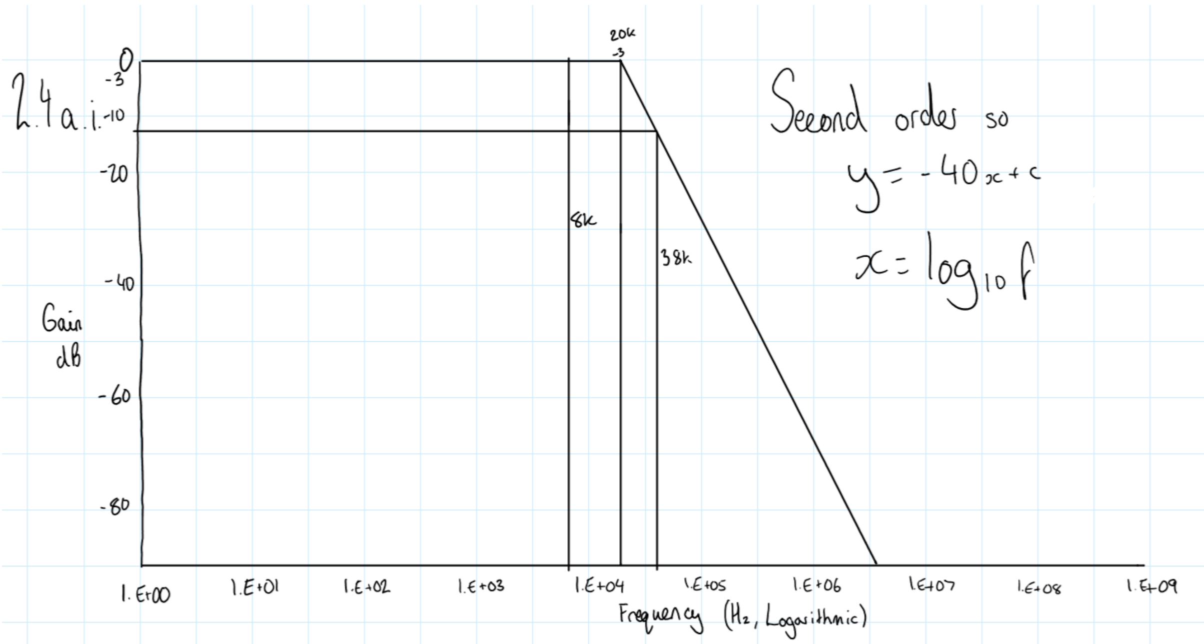


Figure 18 y = mx + c can be used. This is a low pass filter so the gradient is negative. It is second order so the gradient is 40.

At $f = f_c$ 20000 Hz = 0dB

$$0\,dB = -40 \log_{10} 20000 + c$$

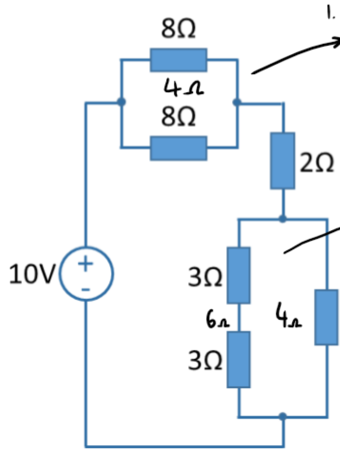
$$c = 172.0411998\,dB$$

ii. So at $f = 8k$ $-40 \log_{10} 8000 + 172.0411998\,dB = 15.91760032 \rightarrow$ Not, so 0dB

iii. at $f = 38k$ $-40 \log_{10} 38000 + 172.0411998\,dB = -11.15014406\,dB$

Figure 19 Intercepts at the values 8000 and 38000 can then be calculated.

b.



1. $R=\left(\frac{1}{8}+\frac{1}{8}\right)^{-1}=4\Omega$

$I_T=\frac{V_T}{R}=\frac{10}{4}=\frac{5}{2}A$

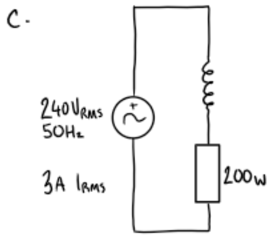
2. $R=\left(\frac{1}{6}+\frac{1}{12}\right)^{-1}=4\Omega$

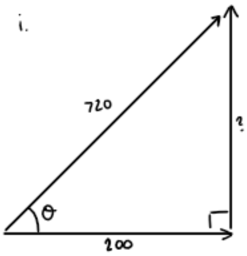
3. $\frac{V_{OUT}}{V_S}=\frac{R_1}{R_1+R_2+R_3}\rightarrow\frac{V_{OUT}}{10}=\frac{4}{4+2+4}$

$V_{OUT}=10\left(\frac{4}{10}\right)=4v$

Figure 18 Calculating V Out can be done with parallel resistor and voltage divider theory.

c.





$|S|=240\times3=720\,VA$ → Apparent power (total)

200w dissapated → Active power (real)

$720^2=200^2+x^2$ $x=691.6646586\,VAR$ → Reactive power (imag)

Figure 20 Trigonometry can be done to find out the value for reactive power, angle, and other values.

ii. $\cos\theta=\frac{200}{720}$ $\theta=73.87237979$

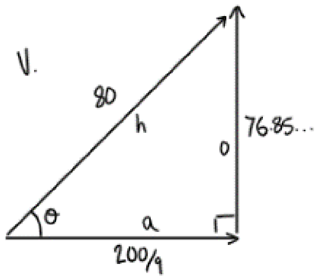
iii. $\cos(73.87237979)=0.277777$

iv. $P(W)=I_{RMS}^2R$

$200=3^2\times R\rightarrow R=\frac{200}{9}\Omega$

$Q(VAR)=I_{RMS}^2\times X\rightarrow 691.6646586=9\times X=76.85162873\Omega j\rightarrow L=\frac{76.851}{50\times2\pi}=0.2446263319H$

v.



$c=\sqrt{76.85^2+\left(\frac{200}{9}\right)^2}=80$ $Z=\frac{200}{9}+76.85162873j$

$\theta=\tan^{-1}\left(\frac{76.85}{200/9}\right)=73.87237979^\circ$ $\omega C=\frac{76.85162873}{\left(\frac{200}{9}\right)^2+76.85162873^2}$ →

Previous Power factor = $\cos(73.872)=0.27$

$\rightarrow 2\pi f\times C=\frac{76.85162873}{\left(\frac{200}{9}\right)^2+76.85162873^2}$

$2\pi\times50\times C=\frac{76.85162873}{\left(\frac{200}{9}\right)^2+76.85162873^2}$ $C=3.772445504F$

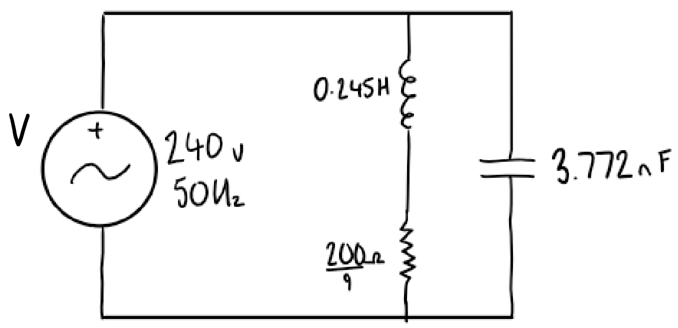
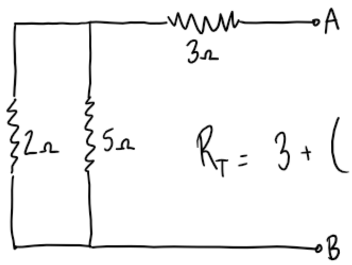


Figure 21 The final power factor corrected circuit.

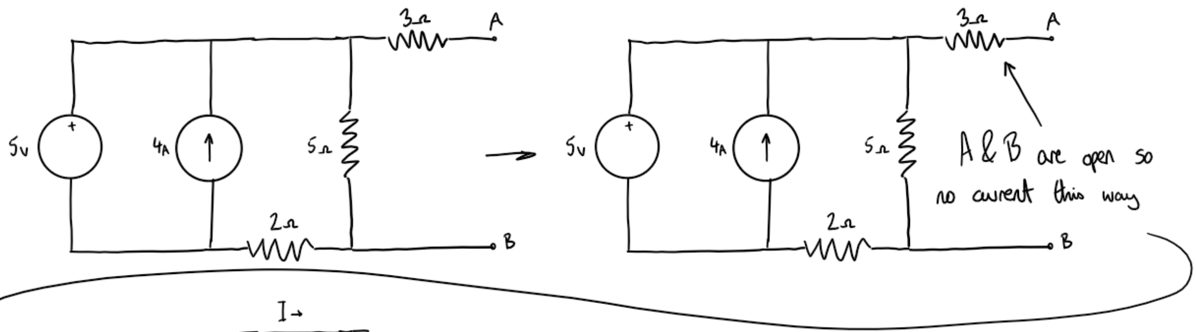
d.

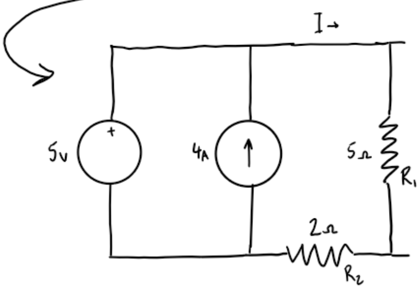
d. Finding R_T - Voltage shorted, Current opened



$$R_T = 3 + \left(\frac{1}{2} + \frac{1}{5}\right)^{-1} = \frac{31}{7}\Omega \text{ OR } 4.429\Omega$$

Figure 22 RTh can be found by shorting voltage sources and opening current sources.





$$\frac{V_1}{V_s} = \frac{R_1}{R_1+R_2}$$

$$V_1 = 5\left(\frac{5}{7}\right) = \frac{25}{7}V$$

$$V_2 = \frac{10}{7}V$$

$$V = I \times R$$

$$I = \frac{V}{R} = \frac{25/7}{5} = \frac{10/7}{2} = \frac{5}{7}A$$

$$V_{Th} = V_1 = \frac{25}{7}V \text{ OR } 3.571V$$

$$I_{Th} = \frac{V_{Th}}{R_{Th}} = \frac{25/7}{31/7} = \frac{25}{37}A \text{ OR } 0.\dot{6}7\dot{5}\ A$$

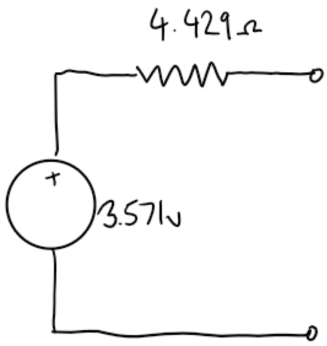


Figure 2319 The final Thevenin circuit

3. General discussion

This assignment served to teach the basics of designing circuits and the considerations behind doing so practically in the real world. The two main circuits demonstrated what they do and how they work and the steps taken in designing and building them. It also points out the problems faced outside of theory. For example, actual results and values can differ from what is expected and calculated. This can be due to components not being exact in the real world with tolerances to take into consideration, as well as considerations such as internal resistances of components such as power supplies. Circuits may also have to be designed around other real-world constraints such as availability. Components such as resistors, capacitors and inductors aren't available in unlimited variations, so sensible and available values should be chosen when doing theory. For example, capacitors and inductors are available with less variation compared to resistors, and resistors can also be combined to form even more variations, so when designing a circuit capacitor and inductor values should be chosen first, with resistors being chosen around them afterwards, as was done in this assignment.

The assignment also served to demonstrate the theory behind circuit design and analysis, including calculating component values, predicting results and outputs, as well as designing the circuits with desired characteristics in the first place.

Bibliography

2.1

- Lecture week 2: Introduction to AC Circuits, Slides 32-41

- Lecture week 3: Phasors and complex impedances, Slides 24,25

2.2

- Lecture week 3: Phasors and complex impedances, Slides 31, 34

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- signal analysis - Square wave FFT results show frequency that seems too low? - Signal Processing Stack Exchange

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2.3

- Lecture week 4: Power in AC Circuits, Slides 18, 24, 27, 28

2.4

- Lecture week 5: Filters, Slides 15,

- Seminar week 7: Filters, Question 2

- Lecture week 1: Introduction to DC electrical circuits, Slides 25, 30,31

- Lecture week 2: Introduction to AC Circuits, Slides 5-12

- Thevenin's Theorem - Circuit Analysis - YouTube

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- Thevenin's Theorem - Explanation, Solved Examples, Limitations (byjus.com)

- https://byjus.com/physics/thevenin-theorem/#:~:text=Example%3A%201%20For%20the%20analysis%20of%20the%20above,Find%20the%20equivalent%20voltage.%20...%20More%20items...%20