Power assignment in a wireless communication system

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Abstract

In wireless networks, power allocation is an effective technique for prolonging the lifetime of the network terminals. Generally, optimum power allocation improves the efficiency of wireless systems. When power allocation is properly done, source information can reach the destination efficiently. In this project, we have tried to maximise the min SINR for a MIMO (Multiple Input Multiple Output) system.

1 Problem

We consider n transmitters with powers $p_1, p_2, \ldots, p_n \geq 0$, transmitting to n receivers. These powers are the optimisation variables in the problem. We let $G \in \mathbf{R}^{n \times n}$ denote the matrix of path gains from the transmitters and receivers; $G_{ij} \geq 0$ is the path gain from transmitter j to receiver i. The signal power at receiver i is then $S_i = G_{ii} p_i$, and the interference power at receiver i is $I_i = \sum_{G_{k \neq i}} G_{ik} p_k$. The signal to interference noise ratio, denoted SINR, at the receiver i, is given by $\frac{S_i}{I_i + \sigma_i}$, where $\sigma_i > 0$ is the (self-) noise power in receiver i. The objective in the problem is to maximize the minimum SINR ratio, over all the receivers, that is :

$$\text{maximize} \min_{i=1,\ldots,n} \frac{S_i}{I_i + \sigma_i}$$

There are a number of constraints on the powers that must be satisfied, in addition to the obvious one $p_i \geq 0$. The first is a maximum allowable power for each transmitter, i.e., $p_i \leq P_i^{max}$, where $P_i^{max} > 0$ is given. In addition, the transmitters are partitioned into groups, with each group sharing the same power supply, so there is a total power constraint for each group of transmitter powers. More precisely, we have subsets $K_1, \ldots, K_m$ of $\{1, \ldots, n\}$ with $K_1 \cup \cdots \cup K_m = \{1, \ldots, n\}$ and $K_j \cup K_l = 0$ if $j \neq l$. For each group K_l, the total associated transmitter power cannot exceed $P_l^{\text{gp}} > 0$:

$$\sum_{k \in K_l} p_k \leq P_l^{\text{gp}}, l = 1, \ldots, m.$$

Finally, we have a limit $P_k^{\text{rc}} > 0$ on the total received power at each receiver:

$$\sum_{k=1}^{n} G_{ik} p_k \leq P_i^{\text{rc}}, i = 1, \ldots, n.$$

Formulate the SINR maximisation problem as a generalised linear-functional program.

2 Mathematical Formulation

The objective is to maximise the minimum SINR of the system under certain power constraints.
The objective function is :

$$\text{minimize} \max_{i=1,\ldots,n} \frac{I_i + \sigma_i}{S_i}$$

The constraints are given as follows:

1. Each transmitter power $0 \leq p_i \leq P_i^{\max}$

2. If the transmitters are partitioned into m non overlapping groups, $K_1, \dots, K_m$, which share a common power supply with total power $P_l^{\text{gp}} : \sum_{k \in K_l} p_k \leq P_l^{\text{gp}}$
3. There is a maximum power that each receiver can receive P_i^{rc} and is given by the following expression :

$$\sum_{k=1}^{n} G_{ik} p_k \leq P_i^{\text{rc}}$$

The objective function with constraints can be rewritten as :

$$\begin{gathered} \text{minimize} \max_{i=1,\dots,n} \frac{I_i + \sigma_i}{S_i} \\ 0 \leq p_i \leq P_i^{\max} \\ \sum_{k \in K_l} p_k \leq P_l^{\text{gp}} \\ \sum_{k=1}^{n} G_{ik} p_k \leq P_i^{\text{rc}} \end{gathered}$$

3 Implementation

The objective function we formulated is quasiconvex. This cannot be solved directly using CVXPY (CVXPY is an open source Python-embedded modeling language for convex optimization problems). Therefore, to solve this we have used bisection method.
Rewriting the objective function as :

$$I_i + \sigma_i \leq S_i \alpha$$

where $\alpha = \gamma^{-1} \geq 0$ is a constraint.
Now we define initial Lower bound "L_0" and Upper bound "U_0" on α, such that $L < \alpha^* < U$ where α^* the optimal value of α.
Using an arbitrary objective function, feasibility is checked for α_0 and we take the initial value of α_0 which is given as :

$$\alpha_0 = \frac{1}{2}(L_0 + U_0)$$

From feasibility, the new lower bound and upper bound is determined:
If α_0 is feasible then,

$$L_1 = L_0, \quad U_1 = \alpha_0 \quad \text{and} \quad \alpha_1 = \frac{1}{2}(L_1 + U_1).$$

If α_0 is infeasible then,

$$L_1 = \alpha_1, \quad U_1 = U_0 \quad \text{and} \quad \alpha_1 = \frac{1}{2}(L_1 + U_1).$$

This bisection process is repeated until $U_N - L_N < \epsilon$, where ϵ is desired tolerance.
Example. Consider there are 5 transmitters i.e. n = 5 and the path gain from transmitter j to receiver i is 0.5 and 0.1 otherwise i.e the path gain form the matrix $G \in \mathbf{R}^{n \times n}$ is defined as

$$G = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.8 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

$P_i^{\max} = 1$ for all transmitters and the transmitters are split into groups, each with $P_l^{\text{gp}} = 1.8$. In first group, we group 1 and 2 while in second, we group 3,4 and 5.Let the max power that each receiver can receive is 4, i.e., $P_i^{\text{rc}} = 4$ and $\sigma_i = 0.1$.
The output obtained is :

Minimum SINR = 1.454
Power = [0.666 0.666 0.666 0.666 0.666]

4 Conclusion

The power allocation problem we studied in our project for MIMO system is very useful for future wireless technologies like massive MIMO in which base stations are equipped with a very large number of antenna elements to improve spectral and energy efficiency.