Multi-User Clustering and Power Allocation for RIS-assisted NOMA Systems with Imperfect Phase compensation

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***Abstract*—In this letter, we analyze the performance of the intelligent reflecting surface (IRS) assisted downlink nonorthogonal multiple access (NOMA) systems in the presence of imperfect phase compensation. We derive an upper bound on the imperfect phase compensation to achieve minimum required data rate for each user. Using this bound, we propose an user pairing algorithm to maximize the network throughput. We then derive bounds on the power allocation factors. Through numerical evaluation and simulations, we show that the proposed algorithms significantly outperform the state-of-the-art algorithms in the presence of phase imperfections.**

***Index Terms*—Intelligent-reflecting surfaces, non-orthogonal multiple access, power allocation, and user pairing.**

I. Introduction

NON-ORTHOGONAL multiple access (NOMA) is considered a key radio access technique for fifth-generation (5G) and beyond 5G networks [1]. In NOMA, the users are allocated the same time and frequency resources but are multiplexed across the power domain to achieve multi-fold improvement in the network capacity. At the receiver side of NOMA systems, the successive interference cancellation is employed to decode the transmitted data. Similar to NOMA, intelligent reflecting surface (IRS) is another key technology to improve coverage for the beyond-5G networks [2]. An IRS consists of a large number of passive antenna elements where the reflection from each antenna is controlled to direct the signal towards a particular user. Note that, unlike the spatial multiplexing scenario, NOMA is preferred in situations where the channel vectors of users are in the same direction [3]. However, this is not always possible in conventional wireless systems, whereas, in the case of IRS-assisted systems, the network operator can control direction of the user channel vectors by tuning the IRS [3]. Motivated by this, IRS has been analyzed along with NOMA to achieve better network capacity and enhanced coverage [2], [3], [4], [5]. The practical IRS systems have imperfections in the phase control because of hardware limitations and channel estimations errors [6], [7]. These imperfections in the phase compensation have a significant impact on the data rates observed by the users. However, limited works in the literature consider these imperfections while analyzing the network performance [3], [6]. In [3], the authors have proposed a novel design for IRS-assisted NOMA transmissions and have analyzed the impact of hardware impairments. In [7], the authors, have presented a joint optimal training sequence and reflection pattern for IRS systems to minimize the mean squared error of the channel estimation. In [6], the authors have evaluated the performance of orthogonal multiple access (OMA) systems in the presence of imperfect phase compensation. In [8], [9], [10], [11], [12], the authors have shown that network performance of the NOMA systems is heavily dependent on the user pairing. Hence, IRS-assisted NOMA systems have to consider the imperfections in the phase compensation while pairing the users. Otherwise, the enhanced network throughputs will not be realized in practice. To the best of our knowledge, none of the existing works in the literature have proposed user pairing and power allocation for IRS-assisted NOMA systems with imperfect phase compensation.

In view of the aforementioned details, this letter presents the first work that discusses the following contributions.

1) We derive bounds on the power allocation factors and the imperfect phase compensation to achieve minimum required data rates in IRS-assisted NOMA systems.
2) Using the derived bounds, we propose user pairing algorithm for IRS-assisted downlink NOMA systems.
3) Through numerical evaluation and simulation, we validate the derived bounds and show that the proposed user pairing algorithm and power allocation factors significantly outperform the state-of-the-art algorithms.

II. SYSTEM MODEL

We consider a base station (BS) with M antennae and an IRS with N antennae, where IRS is activated by a controller connected to the BS as shown in Fig. 1. The channels coefficients between the BS to IRS and ith user to IRS are denoted by h_R and h_i, respectively, and are defined as follows [6]

$$\begin{aligned}\mathbf{h}_R &= \beta_I \mathbf{a}_N(\phi_I^a, \phi_I^e)\mathbf{a}_M^H(\psi_B^a, \psi_B^a)\\ \mathbf{h}_i &= \beta_i \mathbf{a}_N(\psi_I^a, \psi_I^a)\end{aligned} \tag{1}$$

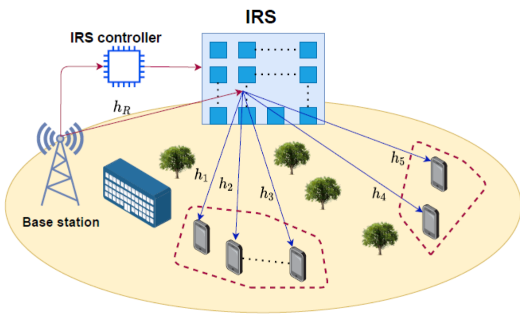

Figure 1: System Model

Fig. 1. System Model

where $\{\}^H$ is the Hermitian of the matrix, β_I and β_i are the distance dependent losses of BS to IRS link and IRS to i^{th} user link, respectively, ϕ_I^a and ϕ_I^e are the angle of arrival (AoA) in azimuth and elevation at the IRS, respectively, ψ_B^a and ψ_B^e are the angle of departure (AoD) in azimuth and elevation at the BS, respectively, ψ_I^a and ψ_I^e are the AoD in the azimuth and elevation at the IRS, respectively, and $\mathbf{a}_X(v^a, v^e)$ is the array factor that captures the beamforming gain. For a planar array with X antenna elements, we assume $\sqrt{X}$ elements in the horizontal and vertical direction of the planar array, and thus, define the array factor as follows [6]:

$$\mathbf{a}_N(v^a, v^e) = [1, \ldots, e^{j\frac{2\pi d}{\lambda}}(x\text{sin}v^a\text{sin}v^e + y\text{cos}v^e), \\ \ldots e^{j\frac{2\pi d}{\lambda}}((\sqrt{X}-1)\text{sin}v^a\text{sin}v^e + (\sqrt{X}-1)\text{cos}v^e)]^T \quad (2)$$

where $0 \leq x, y \leq (\sqrt{X}-1)$ are the indices of antenna elements in the planar array, d is the spacing between antenna elements, λ is the wavelength, v^a and v^e are the desired directions in azimuth and elevation, respectively. We denote the diagonal matrix that captures the reflection of the IRS as $\mathbf{\Theta}$ define each diagonal element of $\mathbf{\Theta}$ as $e^{j\theta_k}$ [3], where $k \in [1, N]$ is the antenna index and $\theta_k \in [0, 2\pi]$ is the phase reflection coefficient. Ideal phase control is difficult to achieve in real-time because of various factors like imperfect channel estimation [7], finite resolution while applying phase shifters, etc. All these factors result in imperfect phase compensation, and hence, we consider the actual reflection matrix to be $\tilde{\mathbf{\Theta}}$ with each diagonal element defined as $e^{\tilde{\theta_k}}$, where $\tilde{\theta_k} = \Theta_k + \hat{\theta_k}, \hat{\theta_k}$ being the phase noise. We consider $\hat{\theta_k}$ to be uniformly distributed over $[-\delta, \delta]$ with $\delta \in [0, \pi]$. With all this information, the signal received by the i^{th}user in an OMA system is formulated as [3]

$$y_i^{\text{OMA}} = \mathbf{h}_i^H \tilde{\mathbf{\Theta}} \mathbf{h}_R \sqrt{P_t} s_i + n,$$

where s_i is the data transmitted to the i^{th} user. Without loss of generality, we assume $s_i' s$ to be independent and identically distributed with zero mean and unit variance, n denotes the additive white Gaussian noise with zero mean and variance of σ^2, and P_t is the available transmit power at the BS. The signal-to-interference-plus-noise ratio (SINR) of the i_{th} user in the OMA system is formulated as

$$\gamma_i^{\text{OMA}} = \frac{P_t ||\mathbf{h}_i^H \tilde{\mathbf{\Theta}} \mathbf{h}_R||^2}{I + \sigma^2} \quad (3)$$

where I is the interference power received at the user. In case of an IRS-assisted downlink NOMA system, we consider that the BS transmits $\sqrt{P_t}(\sum i = 1U\sqrt{\alpha_i}s_i)$, where U represents the number of users paired together, and s_i and α_i denotes the transmitted data and the fraction of power allocated to the i^{th} user, respectively. We assume the channel gains of the users satisfy $||h_i \mathbf{\Theta} h_R||^2 \geq ||h_i \mathbf{\Theta} h_R||^2, \forall j > i$. We consider that user i correctly decodes and removes the interference from the user j , where $j > i$. Further, $0 < \alpha_i < 1$ and $\sum_{i=1}^{U} \alpha_i = 1$. The signal received by the i^{th} user in NOMA is given by [3]

$$y_i^{\text{OMA}} = \mathbf{h}_i^H \tilde{\mathbf{\Theta}} \mathbf{h}_R \sqrt{P_t} \left(\sum_{i=1}^{U} \sqrt{\alpha} s_i \right) + n$$

Thus, we define the SINR of ith user in a NOMA as follows:

$$\gamma_i^{\text{NOMA}} = \frac{\alpha_i P_t ||\mathbf{h}_i^H \tilde{\mathbf{\Theta}} \mathbf{h}_R||^2}{\sum_{j=1}^{i-1} \alpha_j P_t ||\mathbf{h}_j^H \tilde{\mathbf{\Theta}} \mathbf{h}_R||^2 + I + \sigma^2} \quad (4)$$

We define channel state information (CSI) of i^th user (γ_i^{CSI}) as

$$\gamma_i^{\text{CSI}} = \frac{P_t ||\mathbf{h}_i^H \tilde{\mathbf{\Theta}} \mathbf{h}_R||^2}{I + \sigma^2} = \frac{P_t |\beta_i \beta_I|^2 N^2 M}{I + \sigma^2}$$

As $N \to \infty$, the normalized achievable data rates in an IRS-assisted OMA and NOMA systems are as follows:

$$R_i^{\text{OMA}} = \frac{1}{U} \log_2(1 + \gamma_i^{\text{CSI}} sinc^2(\delta)), \quad (5)$$

$$R_i^{\text{NOMA}} = \log_2(1 + \frac{\alpha_i \gamma_i^{\text{CSI}} sinc^2(\delta)}{\sum_{j=1}^{i-1} \alpha_j \gamma_i sinc^2(\delta) + 1}) \quad (6)$$

$$(7)$$

Also, the SINR of i^{th} user in a OMA and NOMA are as follows:

$$\gamma_i^{\text{OMA}} = \gamma_i^{\text{CSI}} sinc^2(\delta) \quad (8)$$

$$\gamma_i^{\text{NOMA}} = \frac{\alpha_i \gamma_i^{\text{CSI}} sinc^2(\delta)}{\sum_{j=1}^{i-1} \alpha_j \gamma_i sinc^2(\delta) + 1} \quad (9)$$

III. COMPUTATION OF BOUNDS

In this section, we derive a lower bound on the power allocation factor and an upper bound on the imperfections in phase compensation for a NOMA cluster under consideration in terms of channel coefficients.

Further, based on NOMA principle [10], the following conditions hold.

$$\alpha_G > \alpha_{G-1} > \cdots > \alpha_2 > \alpha_1 \tag{10}$$

$$\sum_{i=1}^{G} \alpha_i = 1 \tag{11}$$

From (10), we conclude the following result

$$\sum_{j=i+1}^{G} \alpha_j > (G-i)\alpha_i \tag{12}$$

A. Lower Bound on α_i

We define $\bar{R_1}$ and $\bar{R_G}$ as the minimum rates required by strongest and weakest user, respectively. For the lower bound on α_i, we assume that rate of i^{th} user in NOMA (R_i^{NOMA}) should be greater than or equal to the minimum rate required by i^{th} user ($\bar{R}_i$). Thus, by considering $R_i^{\text{NOMA}} > \bar{R}_i$, we get

$$\log_2\left(1+\gamma_i^{\text{NOMA}}\right) > \bar{R}_i \tag{13}$$

$$\gamma_i^{\text{NOMA}} > 2^{\bar{R}_i} - 1$$

$$\frac{\alpha_i \gamma_i^{\text{OMA}}}{1+\sum_{j=1, j\neq i}^{i-1} \gamma_i^{\text{OMA}}} > 2^{\bar{R}_i} - 1$$

substituting in the above equation $\sum_{j=1,j\neq i}^{i-1} \alpha_j = 1 - \alpha_i - \sum_{j=j+1}^{\text{G}} \alpha_j$

$$\frac{\alpha_i}{1+(1-\alpha_i-\sum_{j=i+1}^{\text{G}}\alpha_j)\gamma_i^{\text{OMA}}} > \frac{2^{\bar{R}_i}-1}{\gamma_i^{\text{OMA}}} \tag{14}$$

$$\alpha_i > \left(\frac{2^{\bar{R}_i}-1}{\gamma_i^{\text{OMA}}}\right)\left(1+(1-\alpha_i-\sum_{j=i+1}^{\text{G}}\alpha_j)\gamma_i^{\text{OMA}}\right) \tag{15}$$

$$\alpha_i > \left(\frac{2^{\bar{R}_i}-1}{\gamma_i^{\text{OMA}}}\right)\left(1+(1-\alpha_i-(G-i)\alpha_i)\gamma_i^{\text{OMA}}\right) \tag{16}$$

After rearranging all terms, we got :

$$\alpha_i > \frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{OMA}})}{\gamma_i^{\text{OMA}}(1+(2^{\bar{R}_i}-1)(1+G-i))}$$

Substituting the value of $\gamma_i^{\text{OMA}} from (8)$, we get

$$\alpha_i > \frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{CSI}}\text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}}\text{sinc}^2(\delta)(1+(2^{\bar{R}_i}-1)(1+G-i))} \tag{17}$$

Note that $\alpha_{i\text{LB}}$ in (17) specifies the minimum power required for the i^{th} user to achieve the minimum required data rates.

B. Upper Bound on Imperfect Phase Compensation

Using the bound in (17)

$$\alpha_i > \frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{CSI}}\text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}}\text{sinc}^2(\delta)(1+(2^{\bar{R}_i}-1)(1+G-i))}$$

$$\alpha_{i-1} > \frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{CSI}}\text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}}\text{sinc}^2(\delta)(1+(2^{\bar{R}_i}-1)(2+G-i))}$$

Since, $\alpha_i > \alpha_{i-1}$, we can write,

$$\frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{CSI}}\text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}}\text{sinc}^2(\delta)(1+(2^{\bar{R}_i}-1)(1+G-i))} > \frac{(2^{\bar{R}_i}-1)(1+\gamma_i^{\text{CSI}}\text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}}\text{sinc}^2(\delta)(1+(2^{\bar{R}_i}-1)(2+G-i))}$$

After simplification we get,

$$\text{sinc}^2(\delta) > \frac{A-B}{C-D} \triangleq \text{sinc}^2(\delta_{\text{UB}}) \tag{18}$$

where,

$$A = \gamma_i^{\text{CSI}}\left[1+(2^{\bar{R}_i}-1)(G-i+1)\right](2^{\bar{R}_{i-1}}-1)$$
$$B = \gamma_{i-1}^{\text{CSI}}\left[1+(2^{\bar{R}_{i-1}}-1)(G-i+2)\right](2^{\bar{R}_i}-1)$$
$$C = \gamma_{i-1}^{\text{CSI}}\gamma_i^{\text{CSI}}\left[\left[1+(2^{\bar{R}_{i-1}}-1)(G-i+2)\right](2^{\bar{R}_i}-1)\right]$$
$$D = \gamma_{i-1}^{\text{CSI}}\gamma_i^{\text{CSI}}(2^{\bar{R}_{i-1}}-1)\left[(1+(2^{\bar{R}_i}-1)(G-i+1)\right]$$

The constraint in (18) specifies that, there exists a practically feasible α_i which achieves minimum data rate requirements for any $\delta \leq \delta_{\text{UB}}$. Note that when we consider $\bar{R}_i = R_i^{\text{OMA}}$, δ_{UB} is computable at the base station as it is only dependent on γ_i^{CSI}. From (18), we conclude that it is beneficial to pair the users in IRS-assisted NOMA systems only when δ_{UB} with that user pair is greater than or equal to δ. Otherwise, the data rates achieved by the users in NOMA will not be higher than their OMA counterparts.

C. MSD between succesive users

In case of G users clustered in NOMA system, we define the MSD between two users for achieving higher NOMA rates. For simplicity consider,

$$\mathcal{X}_G(\gamma_i^{\text{OMA}}) = (2^{\bar{R}_i}-1), \quad \mathcal{X}_G(\gamma_{i-1}^{\text{OMA}}) = (2^{\bar{R}_{i-1}}-1)$$
$$D_i = \frac{[1+\gamma_{i-1}^{\text{OMA}}]\mathcal{X}_G(\gamma_{i-1}^{\text{OMA}})}{[1+\gamma_i^{\text{OMA}}]\mathcal{X}_G(\gamma_i^{\text{OMA}})}, E_i = (G-i+1)\gamma_i^{\text{OMA}}\mathcal{X}_G(\gamma_i^{\text{OMA}})$$
$$E_{i-1} = (G-i+2)\gamma_{i-1}^{\text{OMA}}\mathcal{X}_G(\gamma_{i-1}^{\text{OMA}})$$

Rewritting (18);

$$\frac{\gamma_{i-1}^{\text{OMA}}+E_{i-1}}{\gamma_i^{\text{OMA}}+E_i} > D_i$$
$$\gamma_{i-1}^{\text{OMA}}+E_{i-1} > D_i\gamma_i^{\text{OMA}}+D_iE_i$$

Substituting the value of E_{i-1} in above equation;

$$\gamma_{i-1}^{\text{OMA}} > \frac{D_i \gamma_i^{\text{OMA}} + D_i E_i}{1 + (G-i+2)\mathcal{X}_G(\gamma_{i-1}^{\text{OMA}}}$$

$$\gamma_{i-1}^{\text{OMA}} - \gamma_i^{\text{OMA}} > \frac{D_i \gamma_i^{\text{OMA}} + D_i E_i}{1 + (G-i+2)\mathcal{X}_G(\gamma_{i-1}^{\text{OMA}}} - \gamma_i^{\text{OMA}}$$

Thus, we define the MSD between users $i-1$ and i as follows

$$\Delta_{i-1,i}^{\text{MSD}} = \frac{D_i \gamma_i^{\text{OMA}} + D_i E_i}{1 + (G-i+2)\mathcal{X}_G(\gamma_{i-1}^{\text{OMA}}} - \gamma_i^{\text{OMA}} \tag{19}$$

Note that for a cluster of G users, we have $G-1$ combinations of $\Delta_{i-1,i}^{\text{MSD}}$ and δ_{UB} values. In case (18) and (19) are satisfied for all these combinations, then all the G users can be clustered together to achieve NOMA rates higher than their OMA counterparts. Further, we next use the MSD formulated in (19) and the lower bound on power allocation formulated in (10) to propose MUC, AMUC and power allocation algorithms for NOMA systems, respectively.

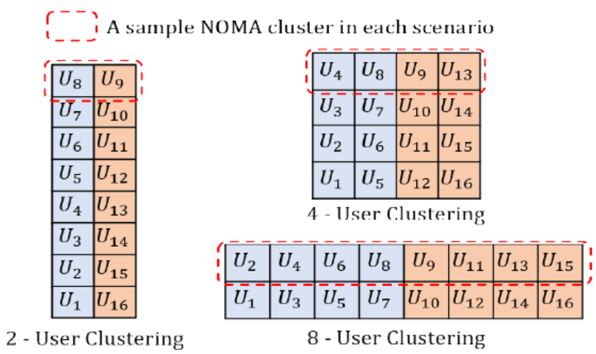

Fig. 2. various user clustering scenarios for multi-user NOMA

IV. PROPOSED ALGORITHM

In this section, we explain the procedure for the clustering of users. We propose MUC and AMUC algorithms to achieve higher NOMA user rates and compare their performance with a conventional near-far (NF) [14] user pairing algorithm. Given a set of users in a cluster, we then present power allocation for each user.

A. Muti-User Clustering (MUC) Algorithm

We use the upper bound on δ_{UB} (18) and the MSD criterion (18) to propose an MUC algorithm for a generalized number of users as follows. With G users in a cluster, we first evaluate δ_{UB} as in (18) and $\Delta_{i-1,i}^{\text{MSD}}$ as in (19) for all the $G-1$ combinations. We consider clustering these G users in NOMA only when (18) and (19) are satisfied for all the $G-1$ combinations. Else, all the G users are designated as OMA users. This way, the individual rates of each user will never be less than that of the corresponding OMA rates. Since, we check the criteria in (18) and (19) for $G-1$ times in each of N/G clusters, the complexity of the proposed MUC is $\mathcal{O}(2N/G \times (G-1))$. Further, the probability of users not meeting MSD criterion increases with an increase in the number of users in a cluster and imperfections in phase compensation. Hence, with large sized user clustering and higher values of δ, the MUC algorithm will designate most of the users as OMA. To address this issue, next, we propose an AMUC algorithm.

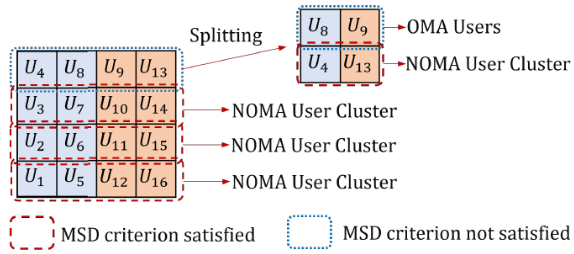

Fig. 3. Splitting of 4 users cluster into 2 users cluster

B. Adaptive Multi-User Clustering (AMUC) Algorithm

In the AMUC algorithm, whenever the G users in a cluster fail to meet δ_{UB} and $\Delta_{i-1,i}^{\text{MSD}}$, we split the cluster into two halves. For this new cluster of users, we again perform the criterion check as formulated in (18) and (19). If the criterion is met for all the combinations, we continue clustering those $G/2$ users in NOMA. Otherwise, we continue splitting this cluster again into two new halves. We continue this procedure until we end up with a single user. If the MSD criterion is not met for any user clustering, then that single user will be designated as OMA user. Further, while splitting a cluster into two halves, we follow the NF [14] user pairing procedure. Otherwise, the throughput gains will not be achieved. In Fig. 3, we have presented an example of splitting a 4-user cluster into two 2-user clusters. Note that the 2 users clustered together after the splitting process in Fig. 3 are exactly the same as they would have been in the case of 2-user clustering in Fig. 2.

The complexity of the proposed algorithm is calculated as follows. For each of N/G clusters, the proposed algorithm initially checks (18) and (19) criterion for $(G-1)$ combinations. If the conditions are not satisfied, then it splits the cluster into two halves and checks the criterions again. This results into additionally $4 \times ((G/2) - 1)$ computations. This procedure is continued till only one user is left in a cluster. Thus, in a worst case scenario, the complexity of the proposed AMUC algorithm is $\mathcal{O}((2N/G)(G\log_2 G - ((1-G)/(1-\log_2 G))))$. Next, we propose the power allocation algorithm for a given set of clustered users.

Algorithm 1 Proposed Algorithm

Require: $U, \gamma_i^{\text{CSI}}, N, G, \delta$

Variables: r and i are variables representing cluster index and user index, respectively. $Flag$ is a variable used to keep track of MSD criterion.

Function Clustering (U,G,N):
U' = sort(U, 'descend');
$U'' \leftarrow$ Reshape U' into G columns;
$U_{Clustered} \leftarrow$ Flip users of U'' in columns $G/2$ to G;
for $r = 1 \rightarrow N/G$ **do**
  $U_r \leftarrow$ Pick G users from r^{th} row of $U_{Clustered}$
  for $i = 2 \leftarrow G$ **do**
    if $\delta < \delta_{\text{UB}}$ and $\gamma_{i-1}^{\text{OMA}} - \gamma_{i-1}^{\text{OMA}} > \Delta_{i-1}^{\text{MSD}}$ **then**
      $Flag = 0$;
      for $i = G \rightarrow 1$ **do**
        $\alpha_i = \alpha_{i\text{LB}}$
      end for
      if $\sum_{k=1}^{G} \alpha_k \leq 1$ **then**
        $Flag = 0$;
      else
        $Flag = 1$;
        break;
      end if
    else
      $Flag = 1$
      break;
    end if
  end for
  if $Flag = 0$ **then**
    Consider all the users for NOMA;
  else
    if AMUC **then**
      Clustering(U_r,$G/2$,G);
    else
      MUC;
      Consider all users for OMA;
    end if
  end if
end for

C. Power Allocation

Based on the lower bound formulated in (10), we allocate the minimum power required for each user as follows

$$\alpha_i > \frac{(2^{\bar{R}_i} - 1)(1 + \gamma_i^{\text{CSI}} \text{sinc}^2(\delta))}{\gamma_i^{\text{CSI}} \text{sinc}^2(\delta)(1 + (2^{\bar{R}_i} - 1)(1 + G - i))}$$

We begin allocation with user G, as α_G is dependent only on γ_G^{CSI} . We then use this allocated power α_G and $\gamma_{G-1}^{\text{OMA}}$ to recursively compute the power allocation factor α_{G-1}. Likewise, we continue allocation till α_1. Further, when we allocate the minimum power required for each user based on (9), $\sum_{i=1}^{G} \alpha_i < 1$. Hence, to maximize the achievable sum rates, we allocate the remaining power $(1 - (\sum_{i=1}^{G} \alpha_i < 1))P_t$ to the strong user. We have presented a pseudo-code to implement the proposed MUC, AMUC, and power allocation in Algorithm 1.

V. RESULTS AND DISCUSSION

For evaluation, we assumed cluster of two users. In Fig. 4 and 5, we present the comparison of data rates with varying α_1. We consider $[\gamma_1^{\text{CSI}}, \gamma_2^{\text{CSI}} = 8, 2]$dB with $\delta = 0°$ and $\delta = 11°$ in Fig. 3 and 5, respectively.

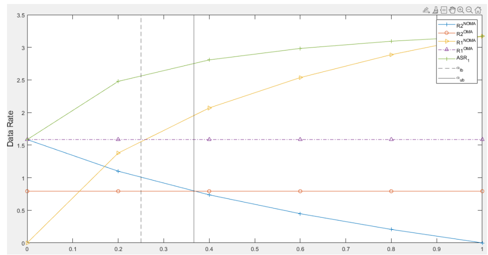

Fig. 4. Data rate vs strong user power allocation factor

Since the SINRs of the users in NOMA pair are same, the minimum required rates by the users $(\bar{R}_1 \text{and} \bar{R}_2)$ are same in both the cases.

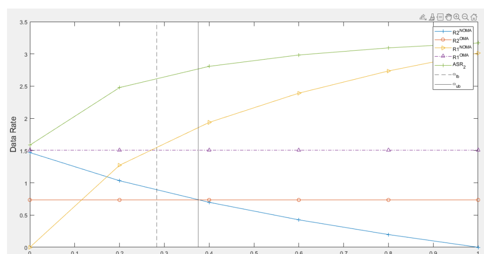

Fig. 5. Data rate vs strong user power allocation factor

However, the individual NOMA rates vary with δ and are better in case of $\delta = 0°$. Comparison of achievable data rates with varying δ (in degrees) is shown in Fig. 6.
Finally, we have verified our results for two user cluster with the base paper and obtained the exact plot.

VI. CONCLUSION

We have derived bounds on power allocation and imperfect phase compensation, for multi-user clustering in a downlink NOMA system. We have proposed adaptive multi-user clustering and power allocation algorithms for a generalized number of users in a cluster. We have also shown that an increase in the number of users in a cluster does not always achieve higher NOMA rates.

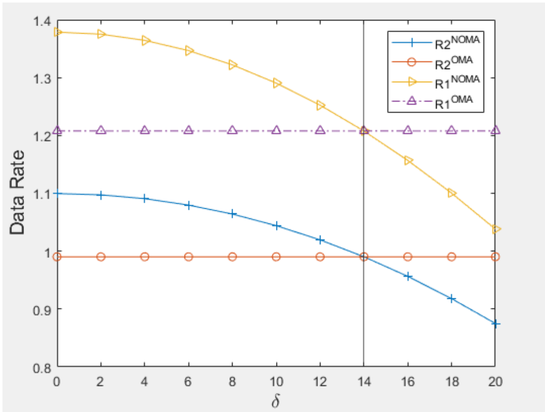

Fig. 6. Data rate vs imperfect phase compensation

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