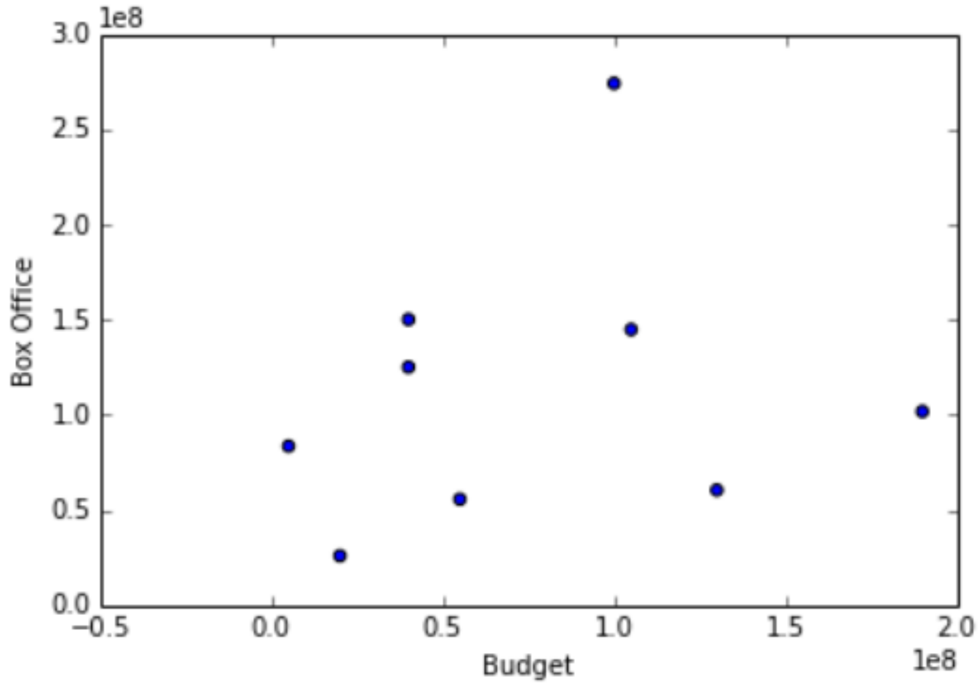
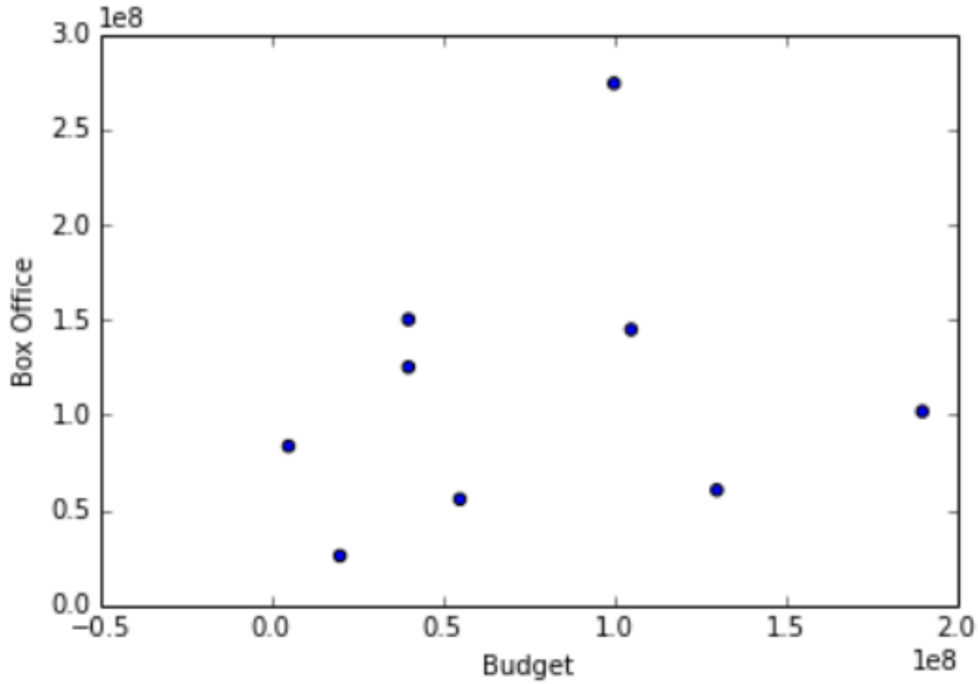
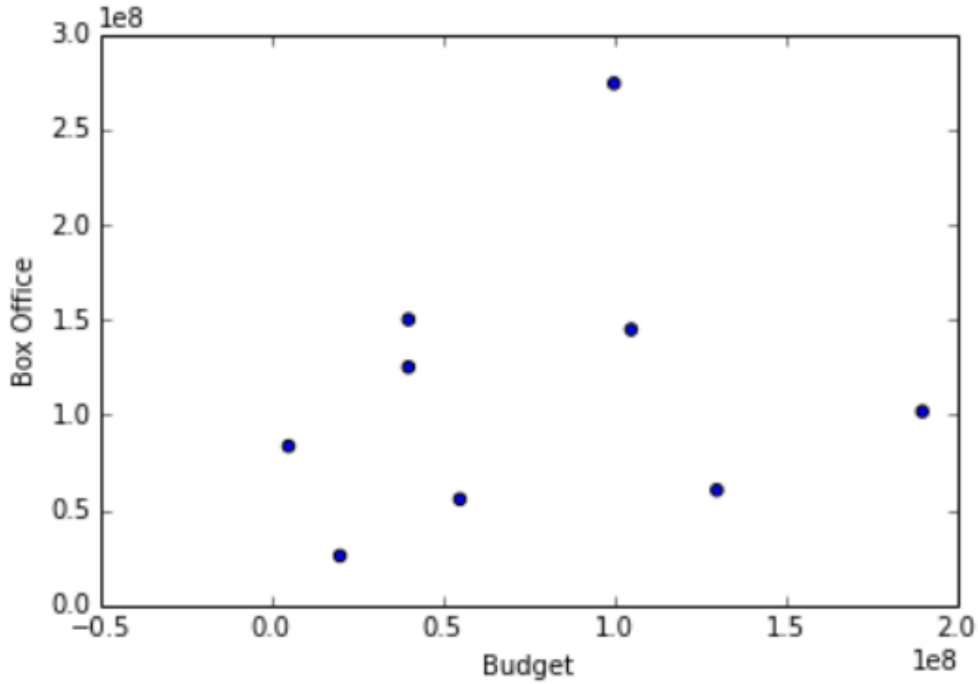
# Linear Regression


1e8
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Box Office
−0.5
0.0
0.5
1.0
1.5
2.0
Budget
1e8




$$y_\beta(x) = \beta_0 + \beta_1 x$$

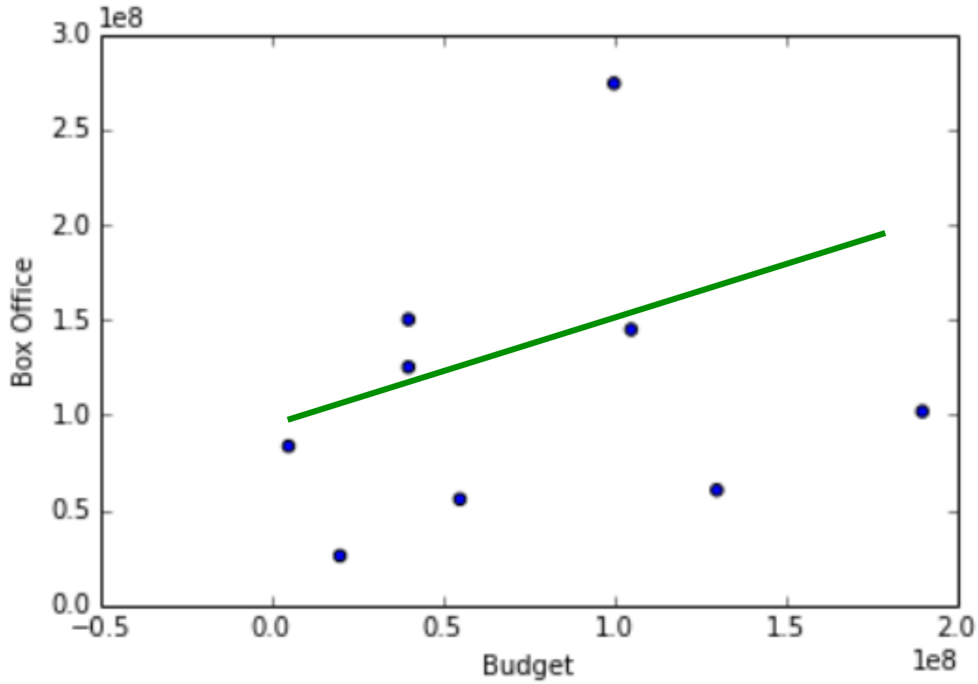



coef 0 coef 1

$$y_{\beta}(x) = \beta_0 + \beta_1 x$$

Gross of movie

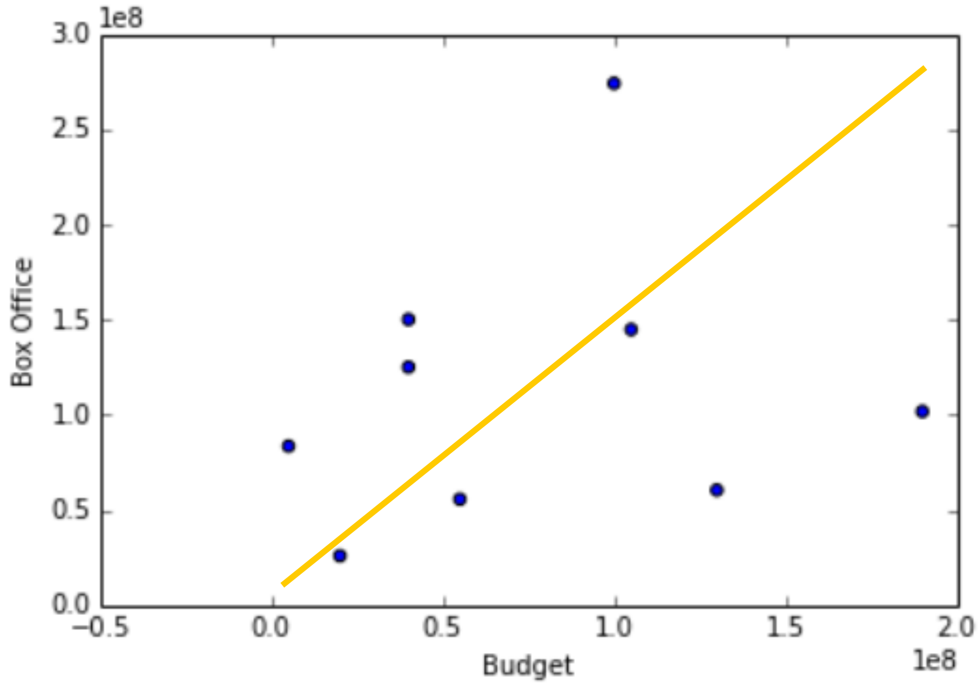
Budget of movie




$$y_\beta(x) = \beta_0 + \beta_1 x$$

$\beta_0 = 80 million$

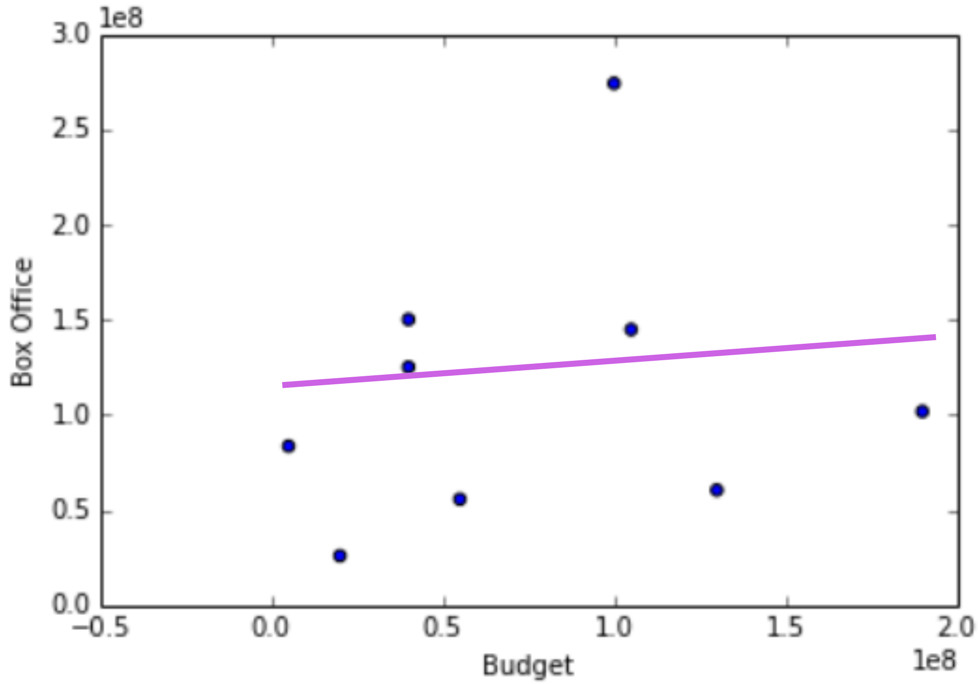
$\beta_1 = 0.5$




$$y_\beta(x) = \beta_0 + \beta_1 x$$

$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

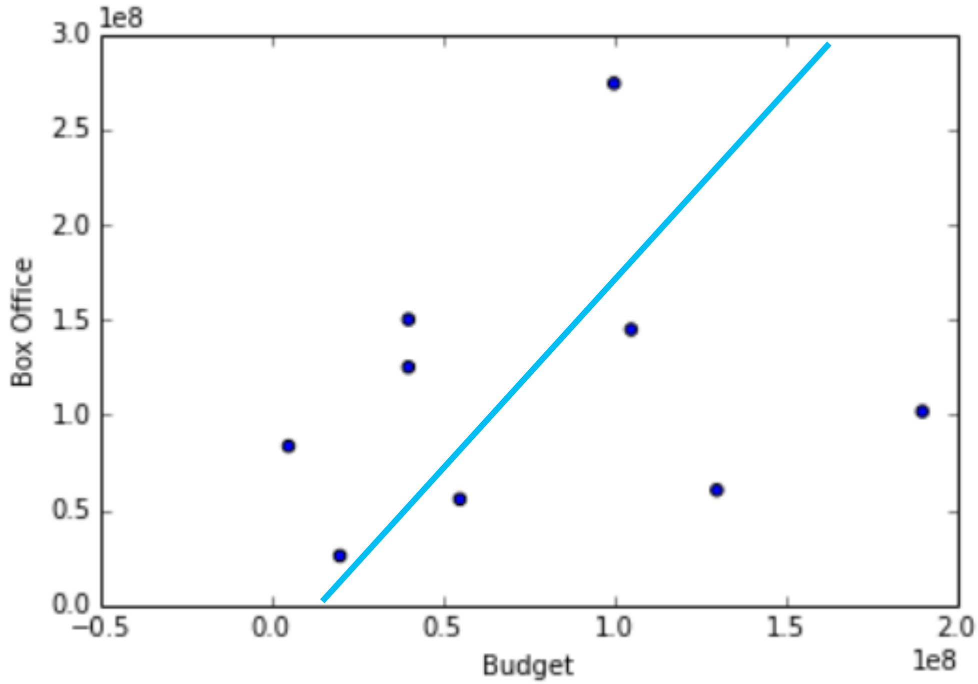



$$y_\beta(x) = \beta_0 + \beta_1 x$$

$\beta_0 = 80million$
$\beta_1 = 0.5$

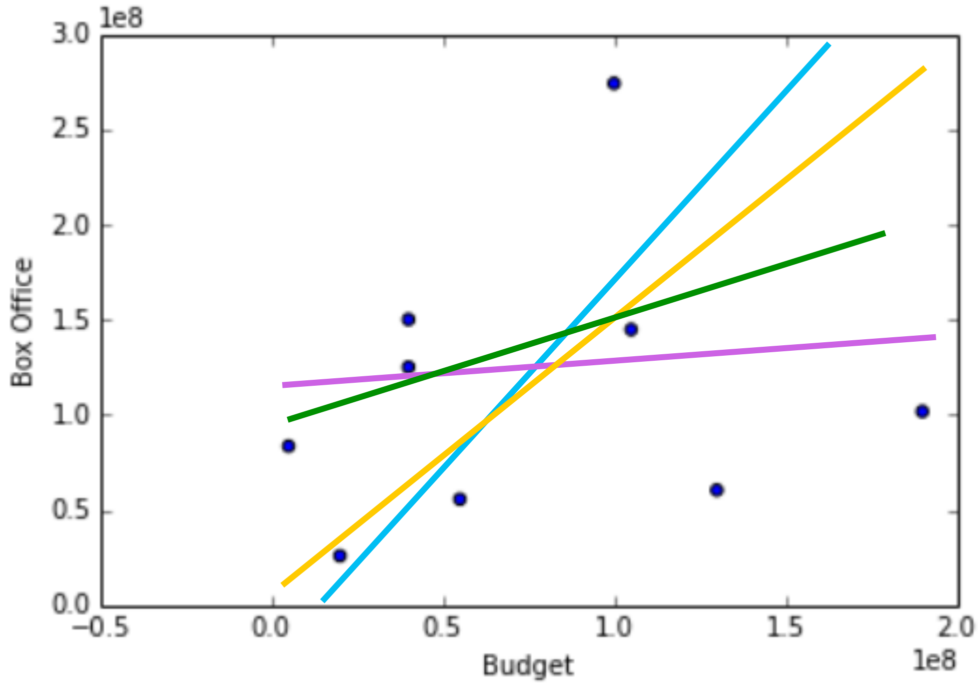
$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120million$
$\beta_1 = 0.1$




$$y_\beta(x) = \beta_0 + \beta_1 x$$

| | | | |
|---|---|---|---|
| $\beta_0 = 80 million$ | $\beta_0 = 0$ | $\beta_0 = 120 million$ | $\beta_0 = 30 million$ |
| $\beta_1 = 0.5$ | $\beta_1 = 1.5$ | $\beta_1 = 0.1$ | $\beta_1 = 2$ |




$$y_\beta(x) = \beta_0 + \beta_1 x$$

$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
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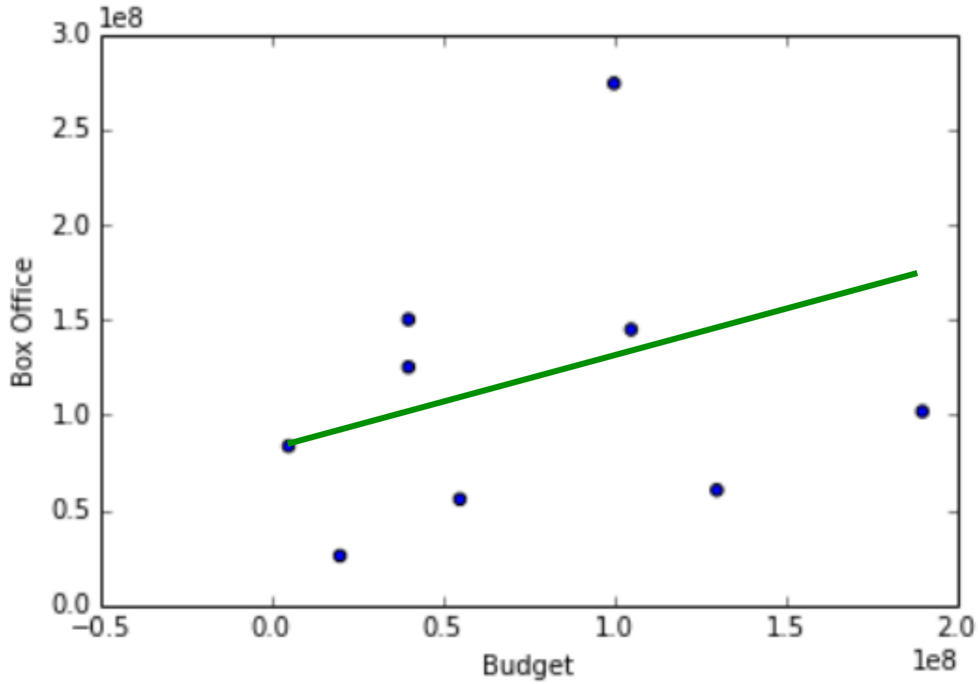
$\beta_0 = 120 million$
$\beta_1 = 0.1$

$\beta_0 = 30 million$
$\beta_1 = 2$



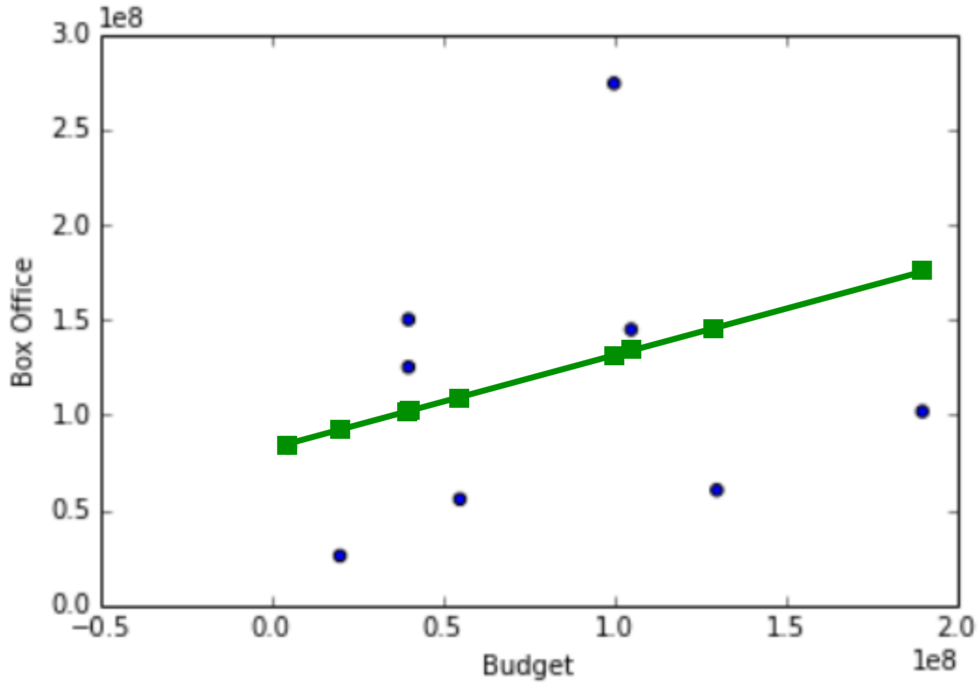

$$y_\beta(x) = \beta_0 + \beta_1 x$$

$\beta_0 = 80 million$ $\beta_1 = 0.5$

$\beta_0 = 0$ $\beta_1 = 1.5$

$\beta_0 = 120 million$ $\beta_1 = 0.1$

$\beta_0 = 30 million$ $\beta_1 = 2$




$$y_\beta(x) = \beta_0 + \beta_1 x$$

| | | | |
|---|---|---|---|
| $\beta_0 = 80 million$ | $\beta_0 = 0$ | $\beta_0 = 120 million$ | $\beta_0 = 30 million$ |
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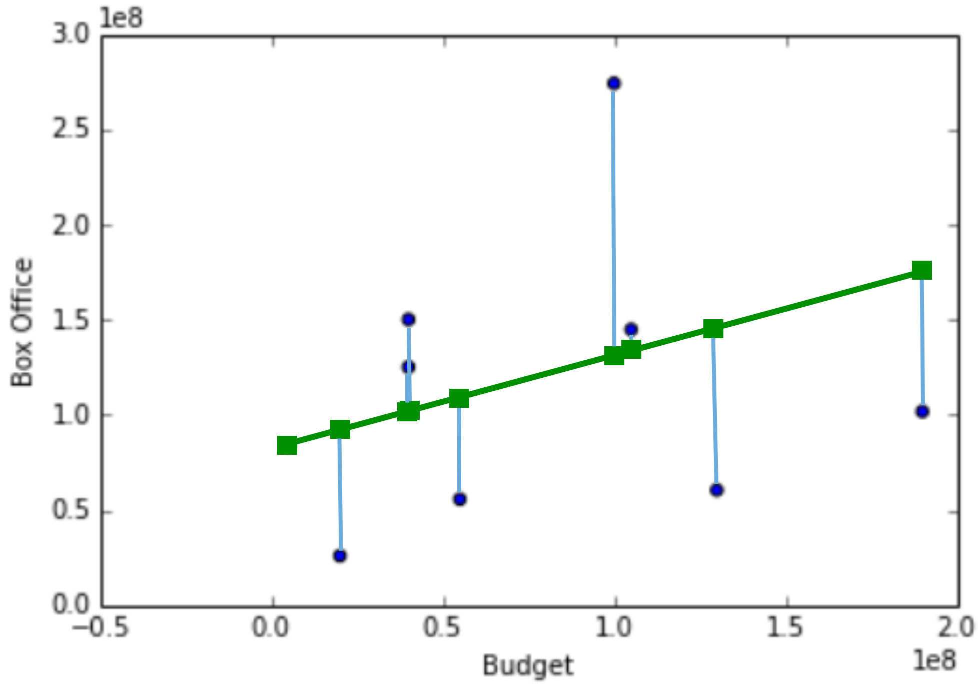



$$y_\beta(x_{obs}^{(0)}) - y_{obs}^{(0)}$$

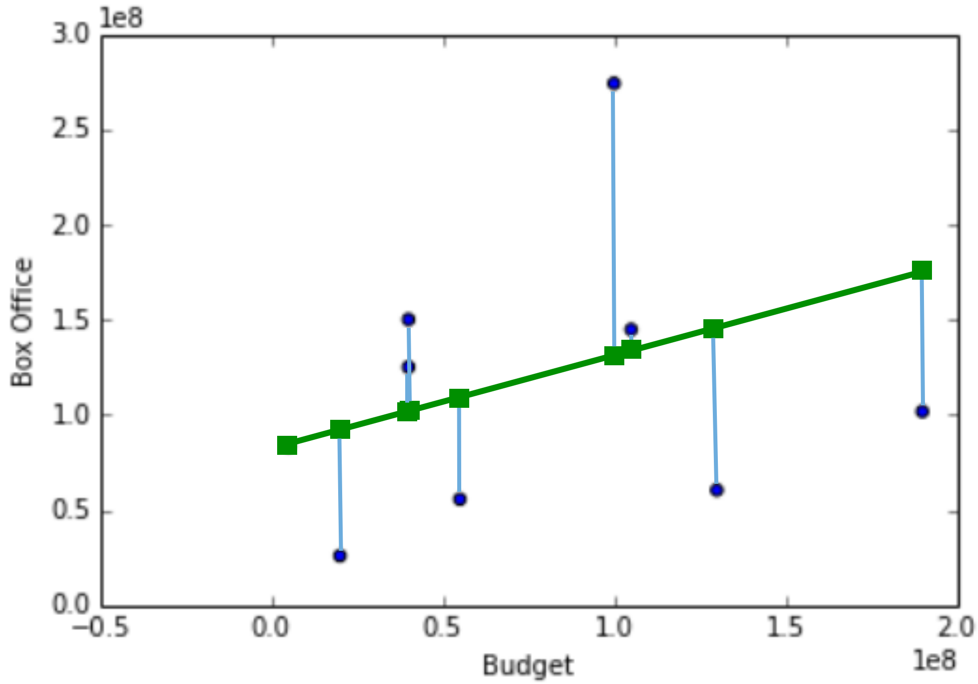
$$y_\beta(x_{obs}^{(1)}) - y_{obs}^{(1)}$$

$$y_\beta(x_{obs}^{(2)}) - y_{obs}^{(2)}$$

$$y_\beta(x_{obs}^{(3)}) - y_{obs}^{(3)}$$

Predicted value by model – Observed value

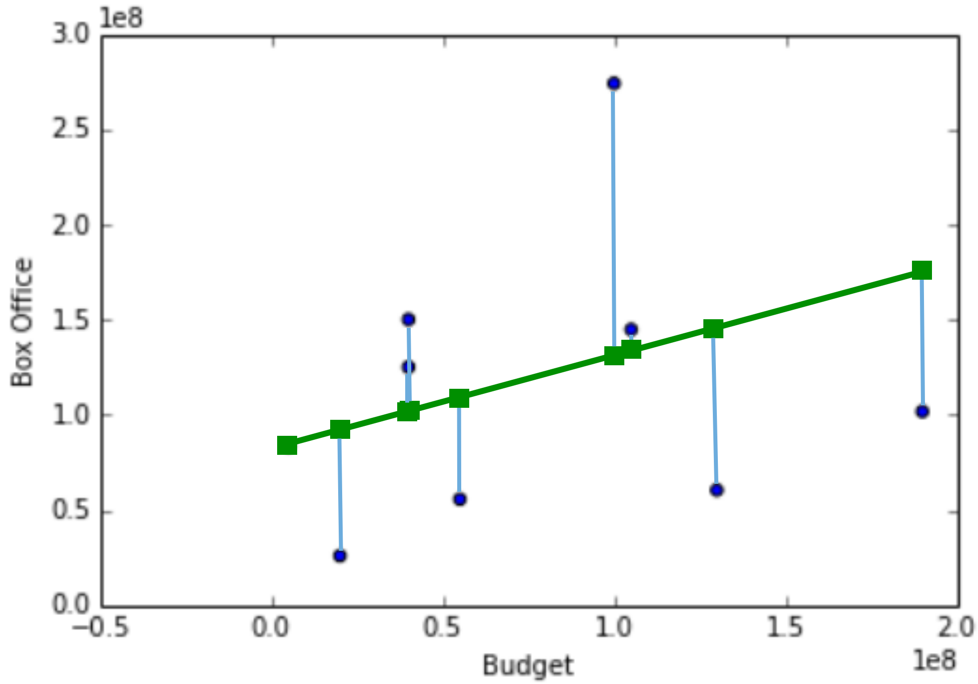
β0 =80M, β1=0.5




Predicted value by model – Observed value

β0 =80M, β1=0.5

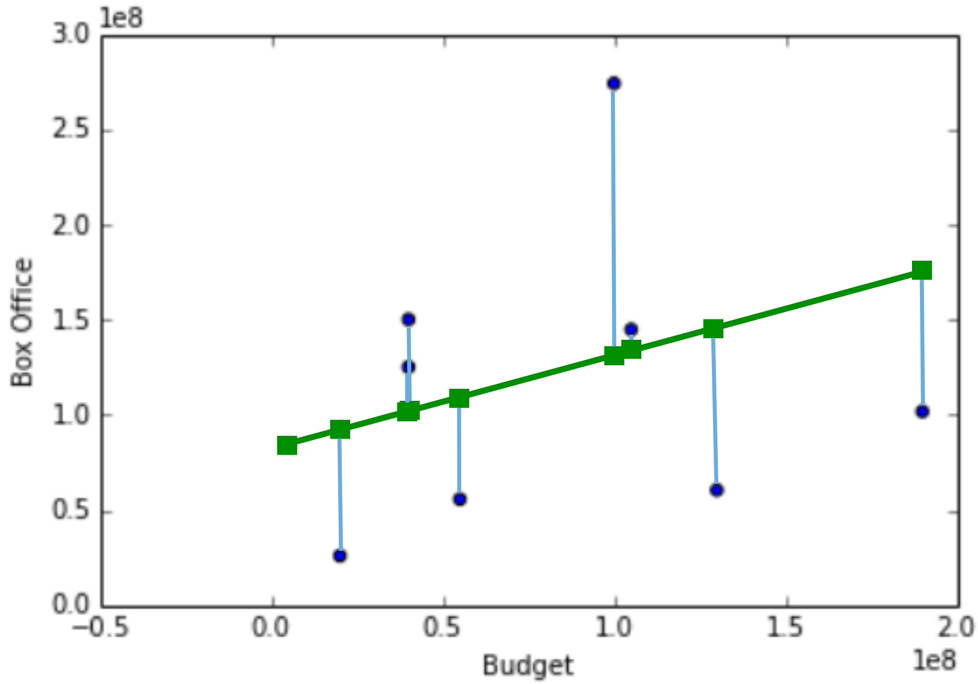
$$y_{\beta}(x_{obs}^{(i)}) - y_{obs}^{(i)}$$




Predicted value by model – Observed value

β0 =80M, β1=0.5

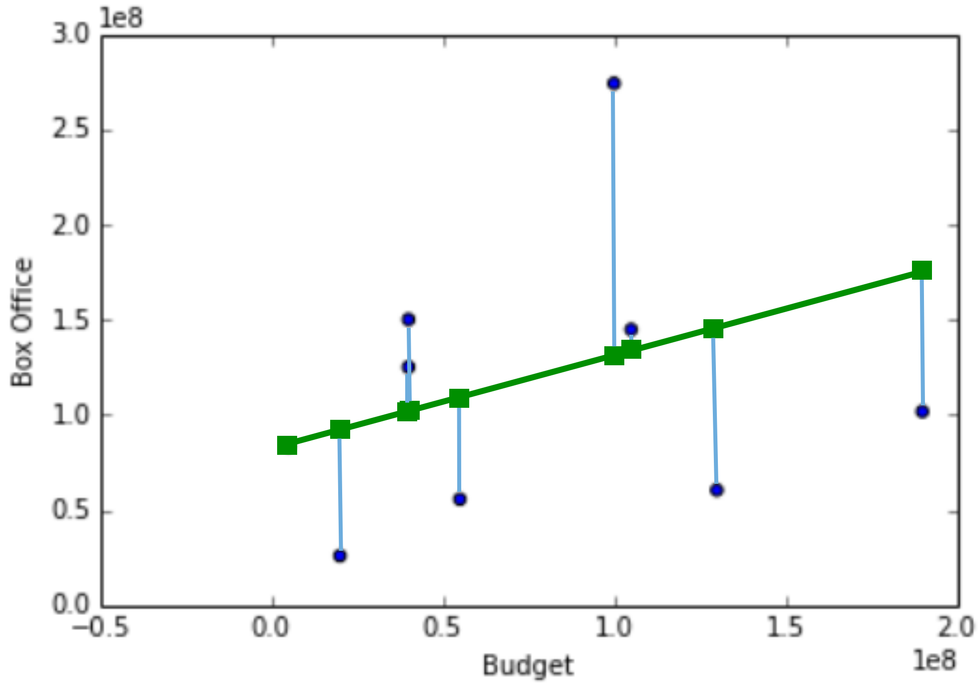
$$(\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)}$$




Predicted value by model – Observed value

β0 =80M, β1=0.5

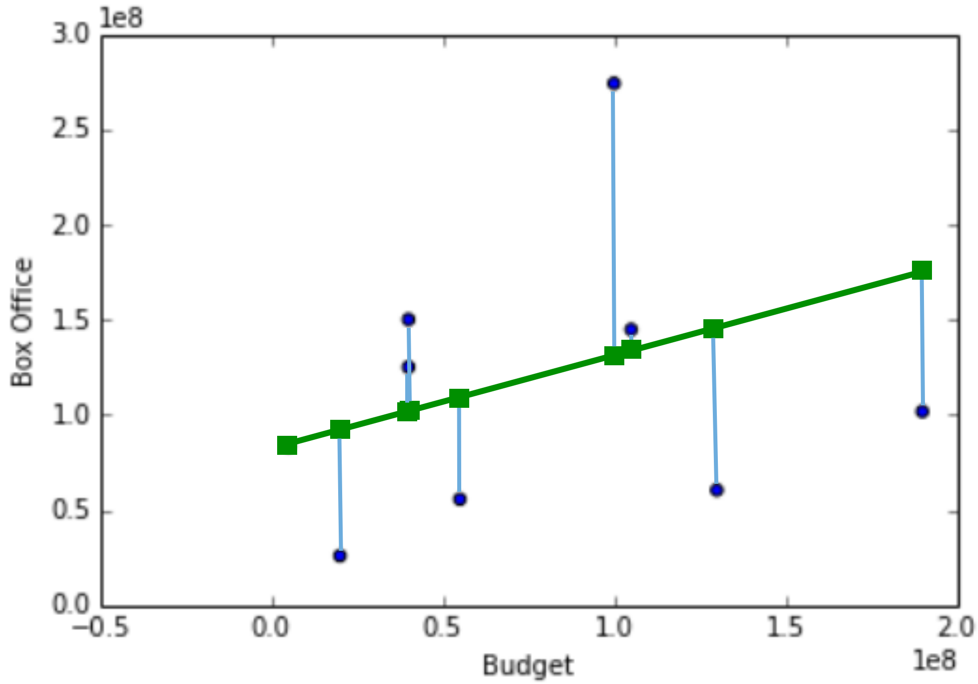
$$\sum_{i=1}^{m}\left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2$$




Predicted value by model – Observed value

β0 =80M, β1=0.5

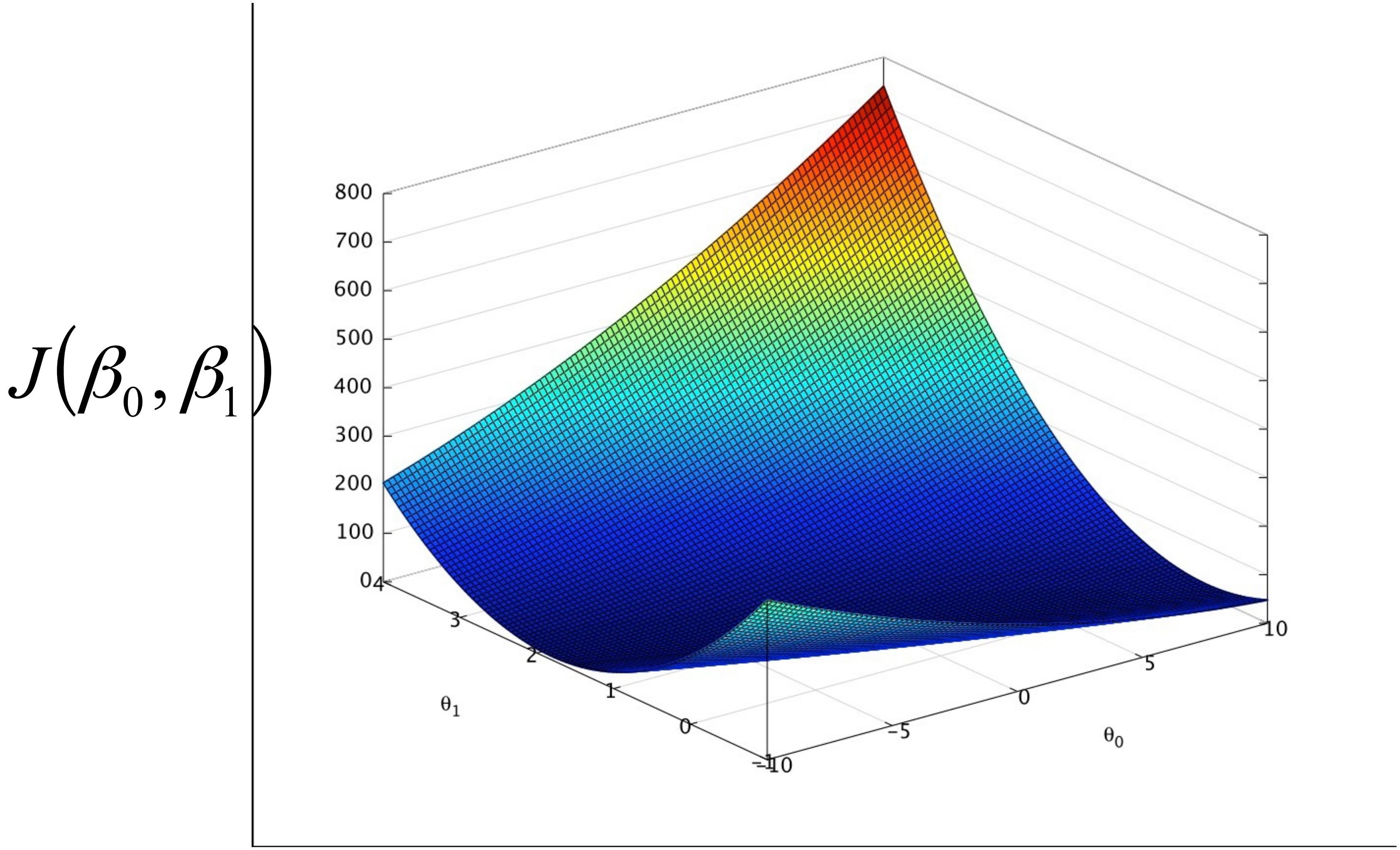
$$\min_{\beta_0,\beta_1} \sum_{i=1}^{m} \left( (\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$



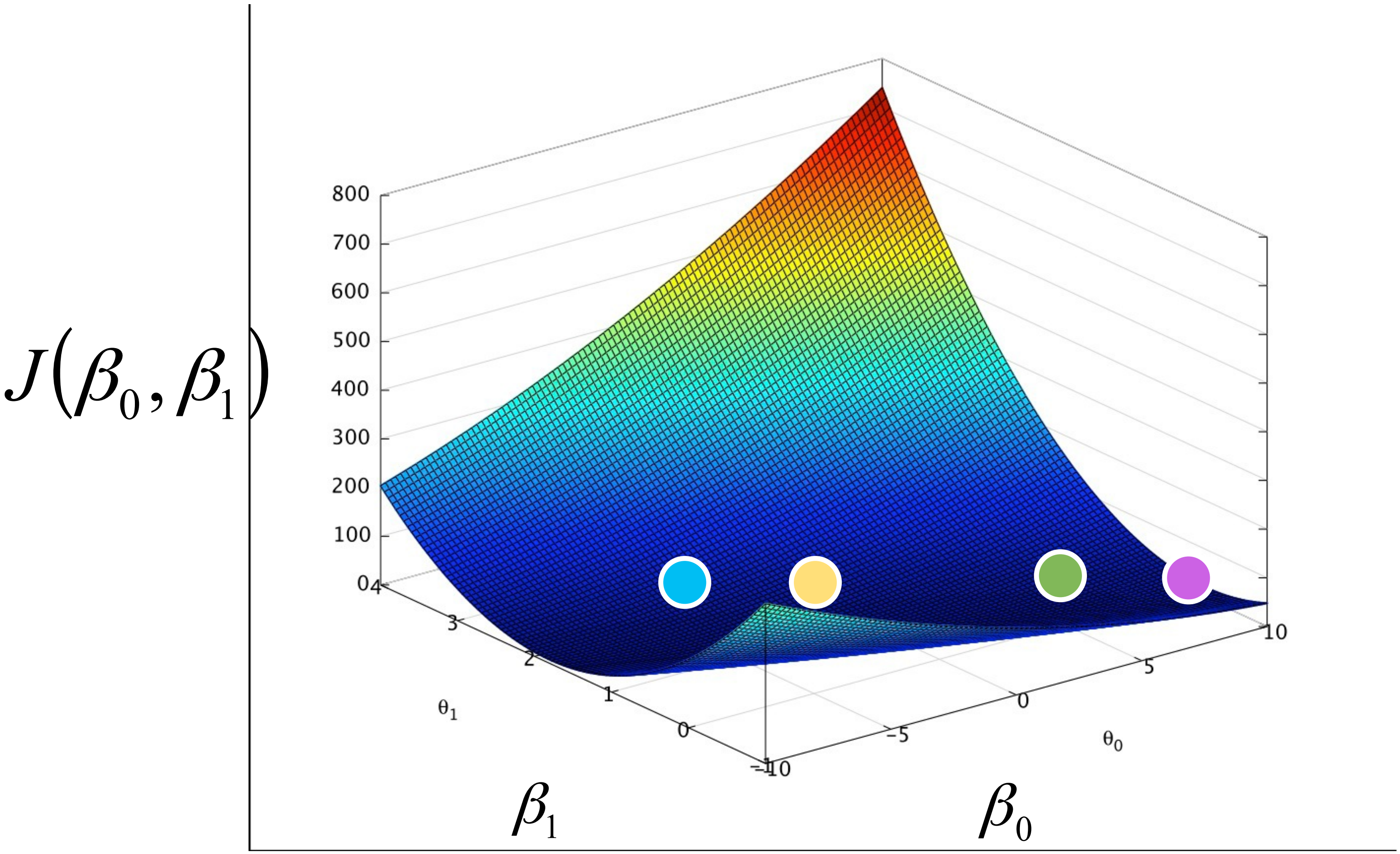

Cost function

Takes a model (specific parameter values), returns score

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left( (\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$




$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left( (\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

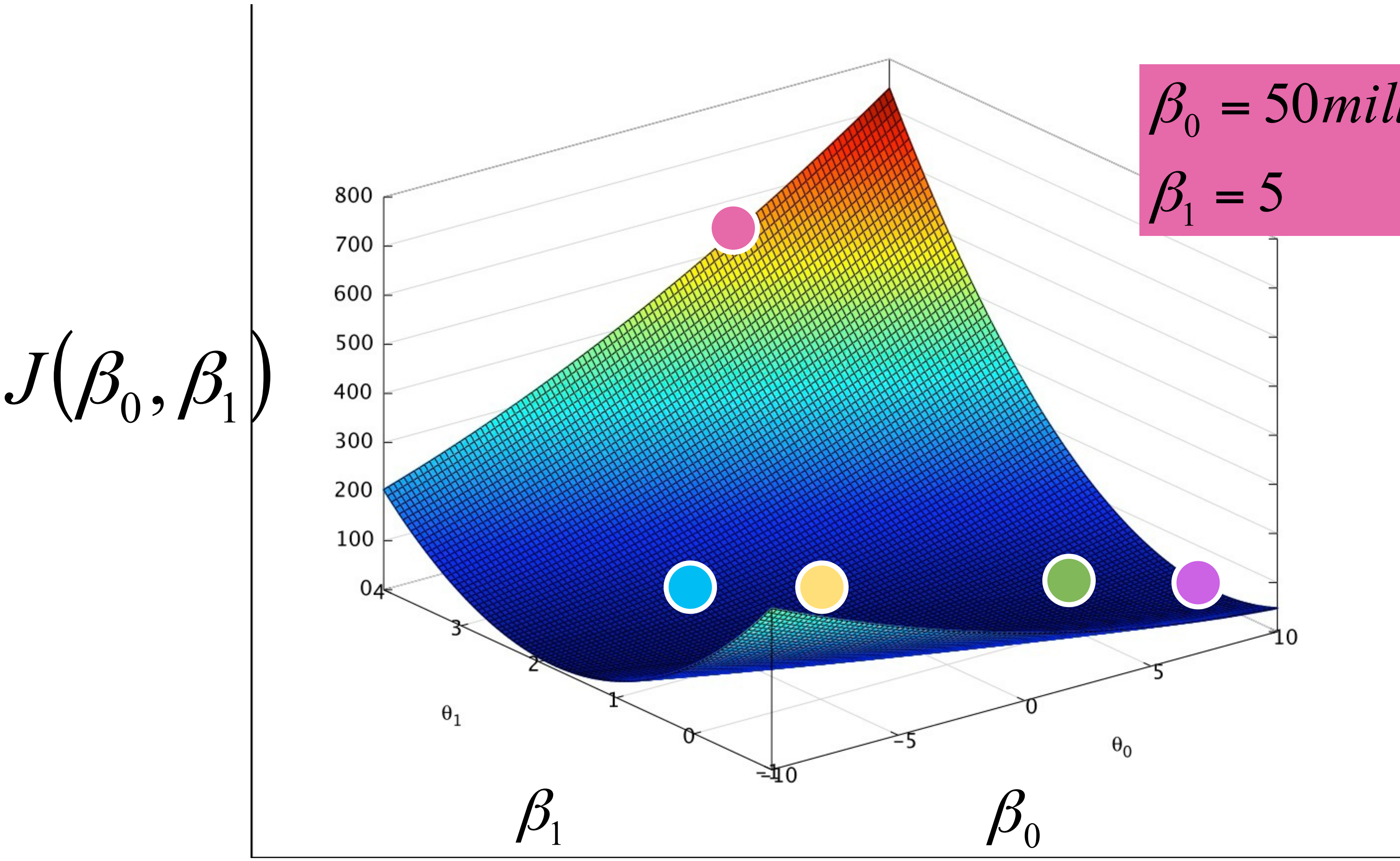



$\beta_0 = 80million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120million$
$\beta_1 = 0.1$

$\beta_0 = 30million$
$\beta_1 = 2$



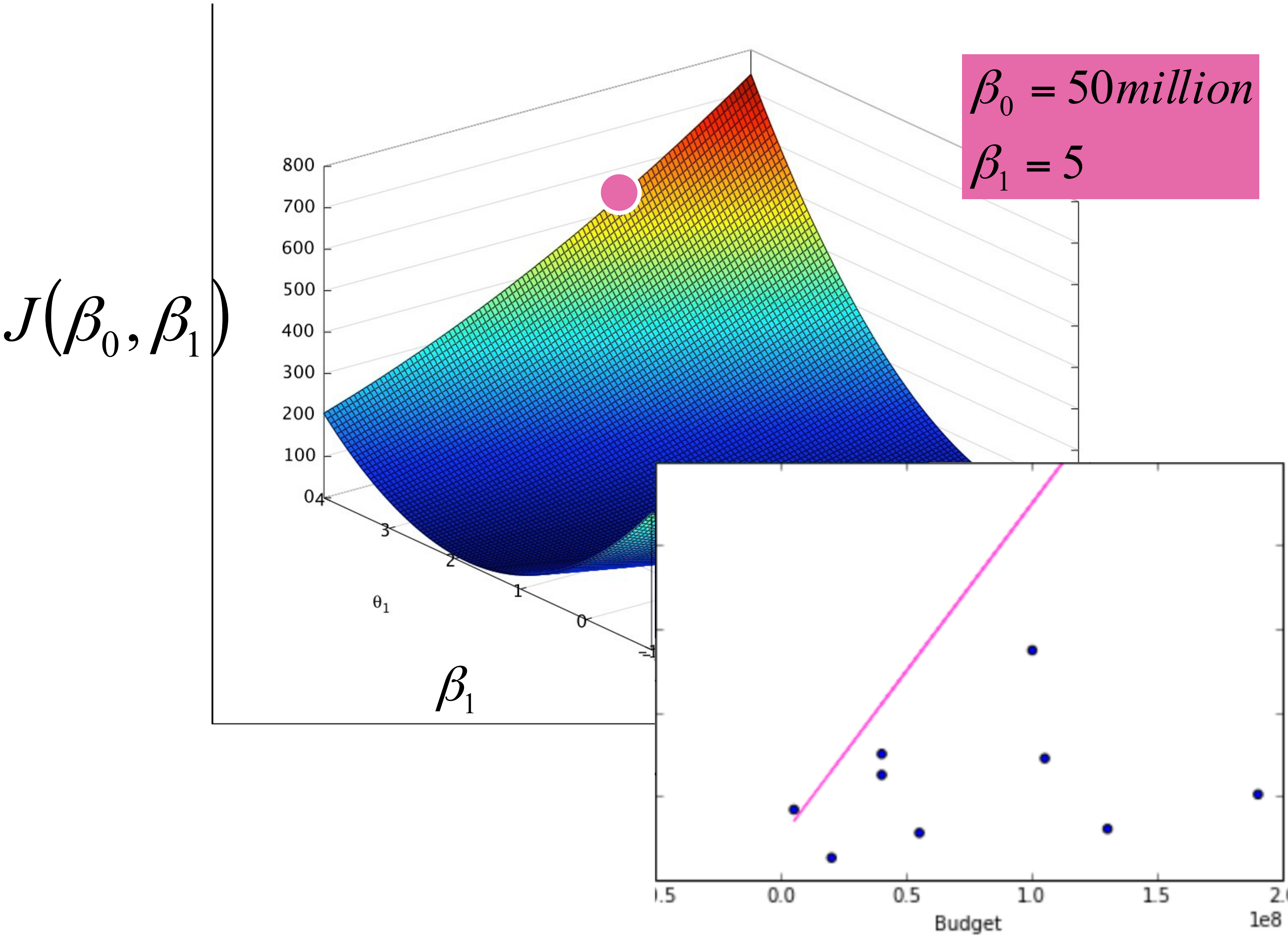

$\beta_0 = 50 million$
$\beta_1 = 5$

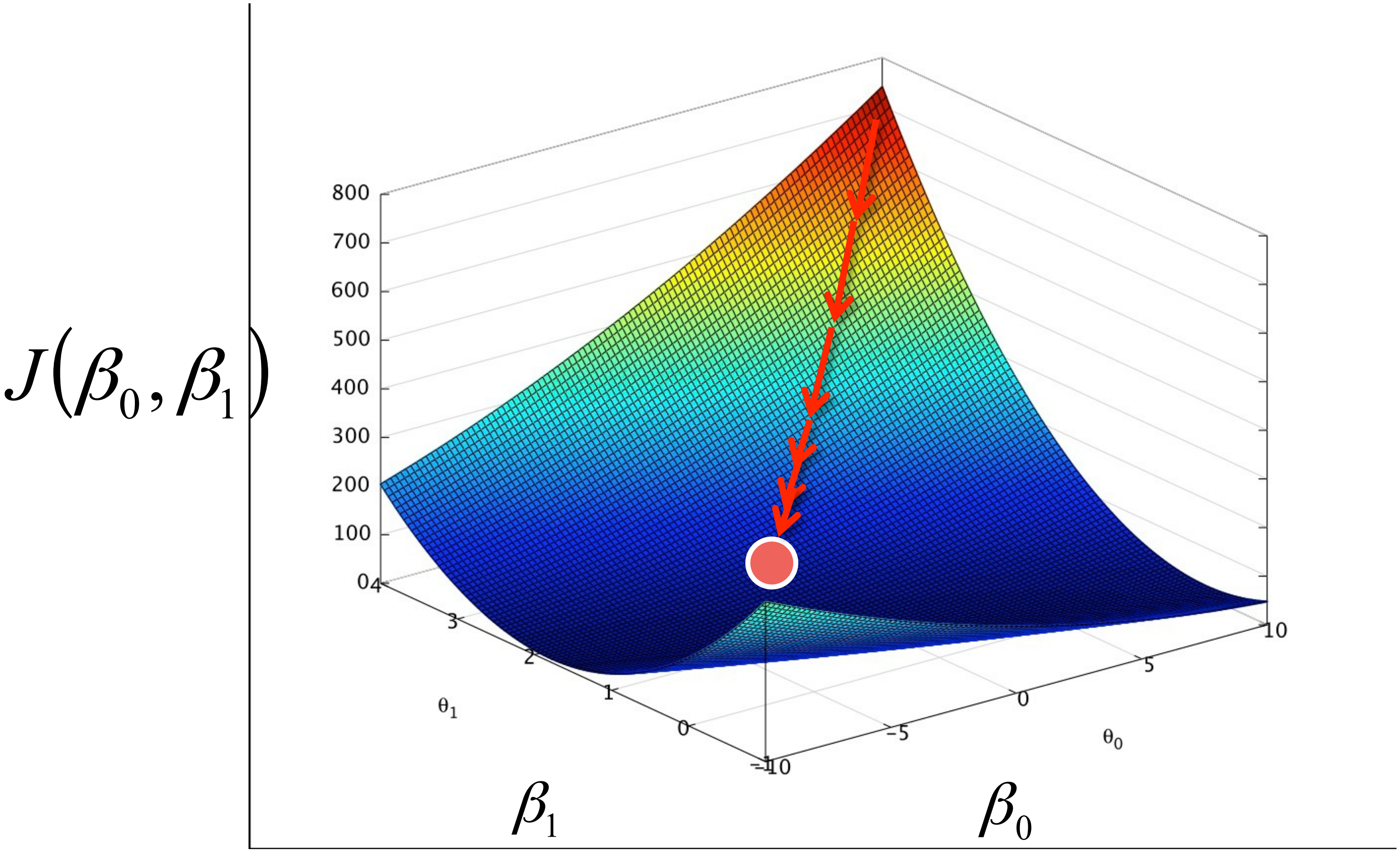
$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

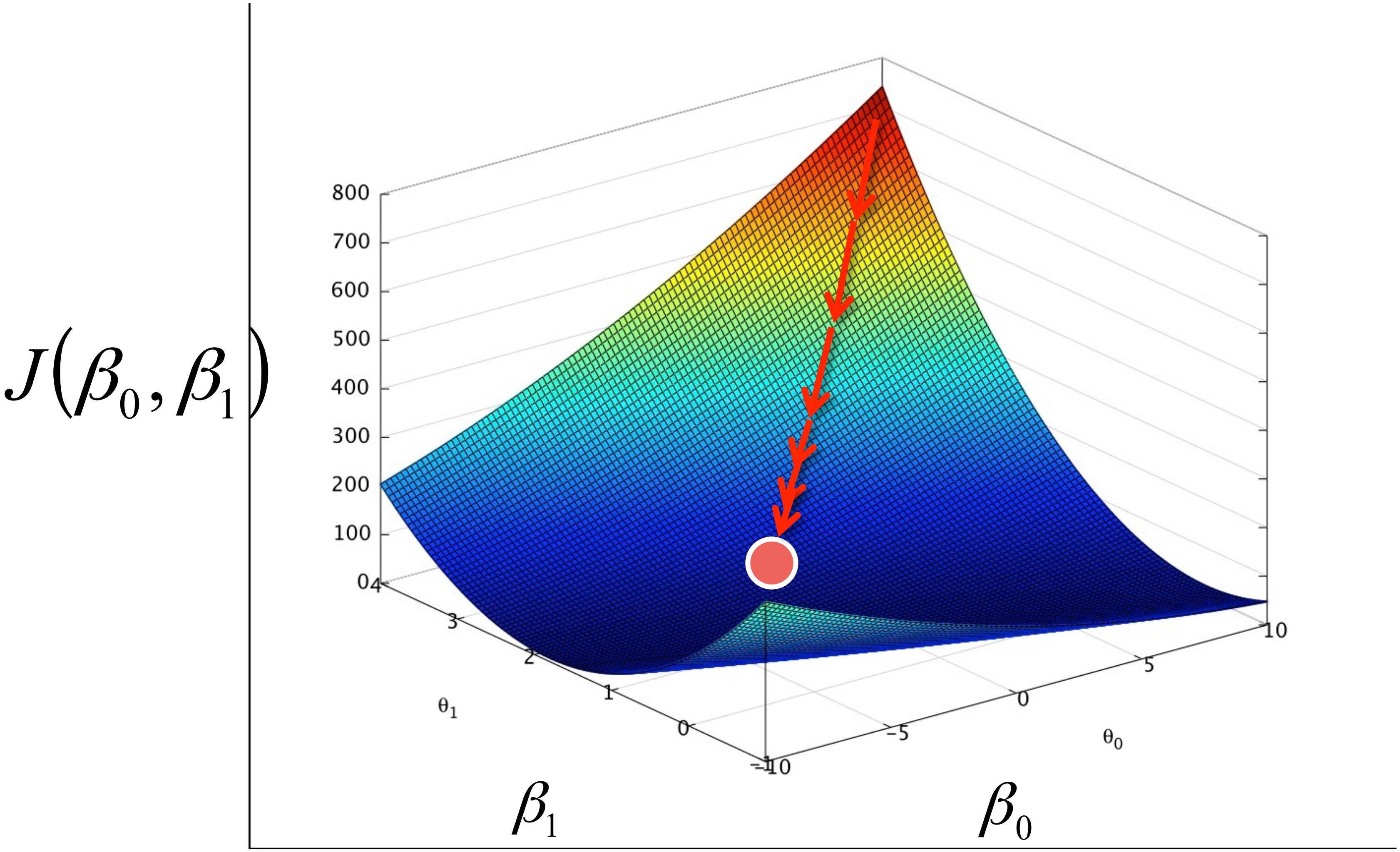
$\beta_0 = 120 million$
$\beta_1 = 0.1$

$\beta_0 = 30 million$
$\beta_1 = 2$


$\beta_0 = 50 million$
$\beta_1 = 5$
$J(\beta_0, \beta_1)$
800
700
600
500
400
300
200
100
0
4
3
2
1
0
$\theta_1$
$\beta_1$
0.0
0.5
1.0
1.5
2.0
Budget
1e8

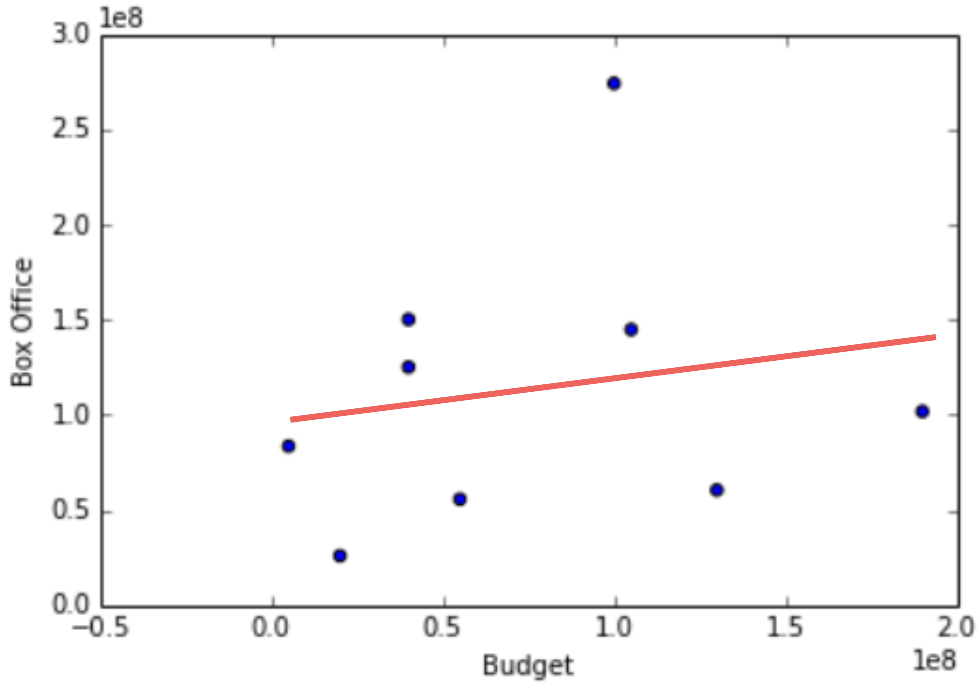



**import s**tatsmodels.formula.api **as** sm
linmodel = sm.OLS(Y, X).fit()




$$\beta_0 = 94.68 million$$

$$\beta_1 = 0.1$$




$$y_\beta(x) = \beta_0 + \beta_1 x$$

$$\beta_0 = 94.68 million$$

$$\beta_1 = 0.1$$

# Models and Randomness

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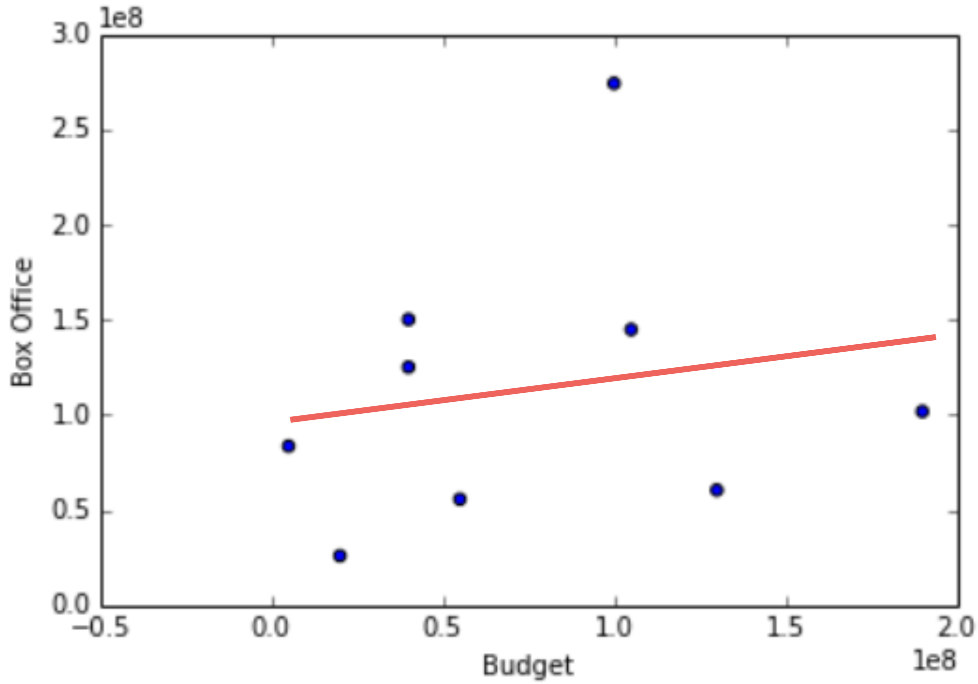
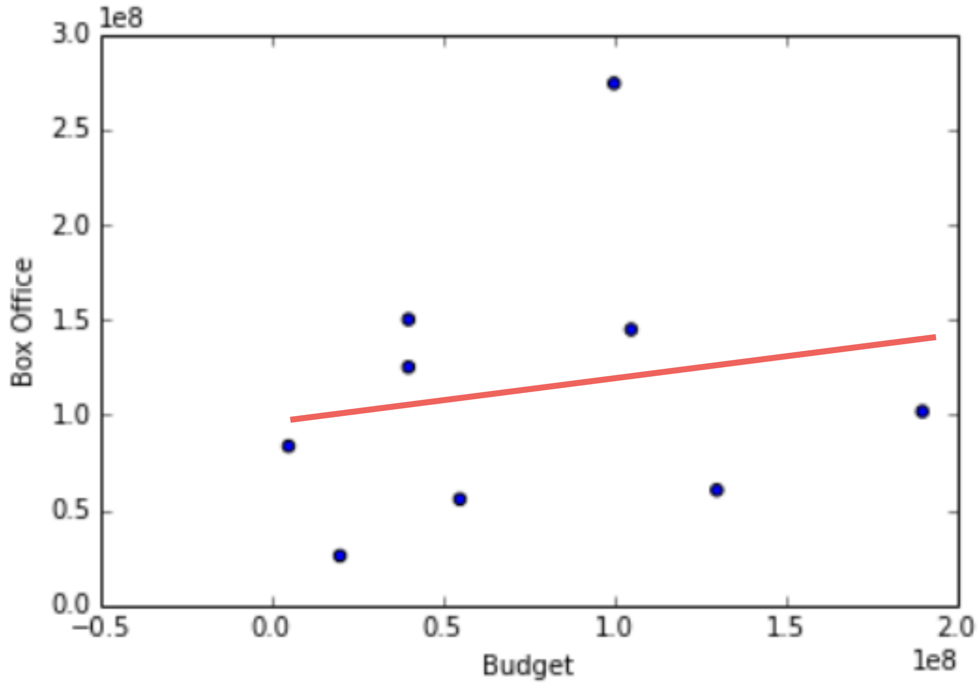



$$y = \beta_0 + \beta_1 x + \varepsilon$$

Random for each movie

$$\beta_0 = 94.68 million$$

$$\beta_1 = 0.1$$




$$y = \beta_0 + \beta_1 x + \varepsilon$$

$$\beta_0 = 94.68 million$$

$$\beta_1 = 0.1$$

**Random**
Normal
distribution
Mean=0
Stdev=
$67,762,000

$$\beta_0 = 94.68 million$$
$$\beta_1 = 0.1$$

$$y = \beta_0 + \beta_1 x + \varepsilon$$

**Random**
Normal distribution
Mean=0
Stdev= $67,762,000

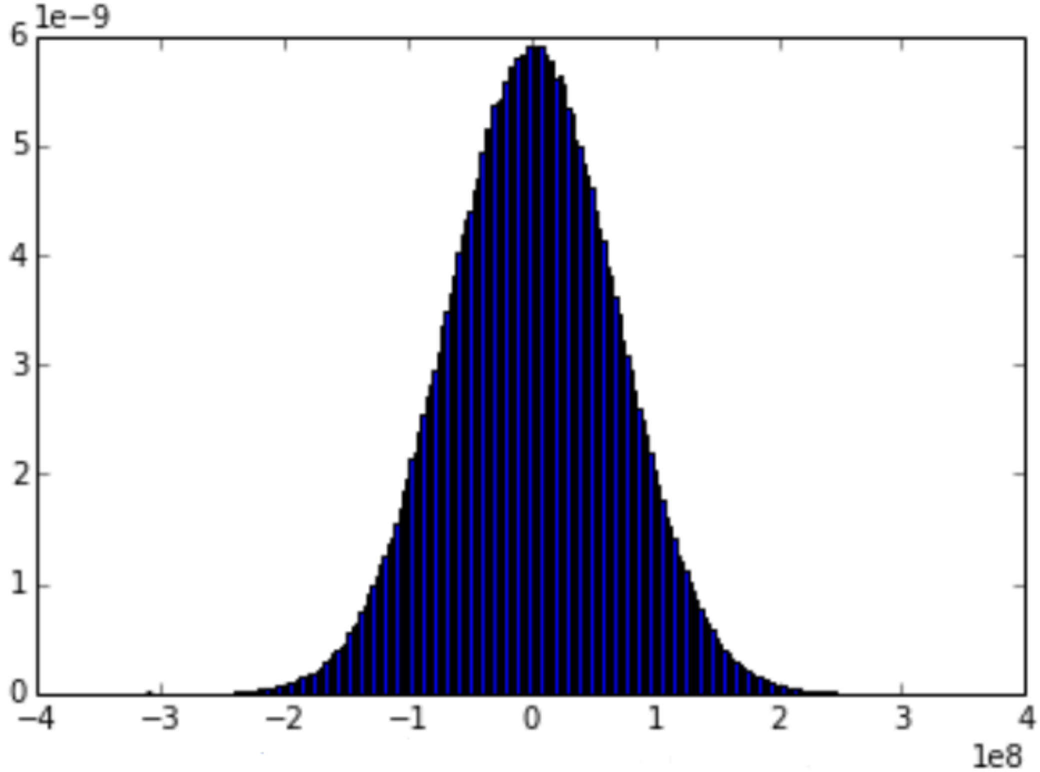
$\beta_0 = 94.68 million$

$\beta_1 = 0.1$

$$y = \beta_0 + \beta_1 x + \varepsilon$$

**Random**
Normal distribution
Mean=0
Stdev= $67,762,000

pdf(ε)



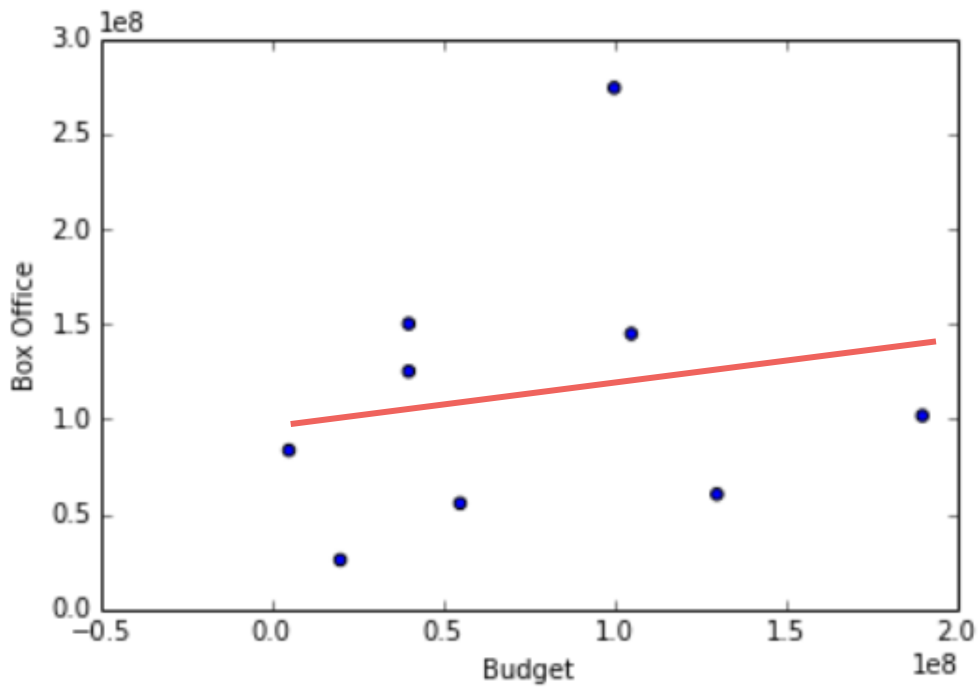

ε

(Note axes!)

$$\beta_0 = 94.68 million$$
$$\beta_1 = 0.1$$

$$y = \beta_0 + \beta_1 x + \varepsilon$$

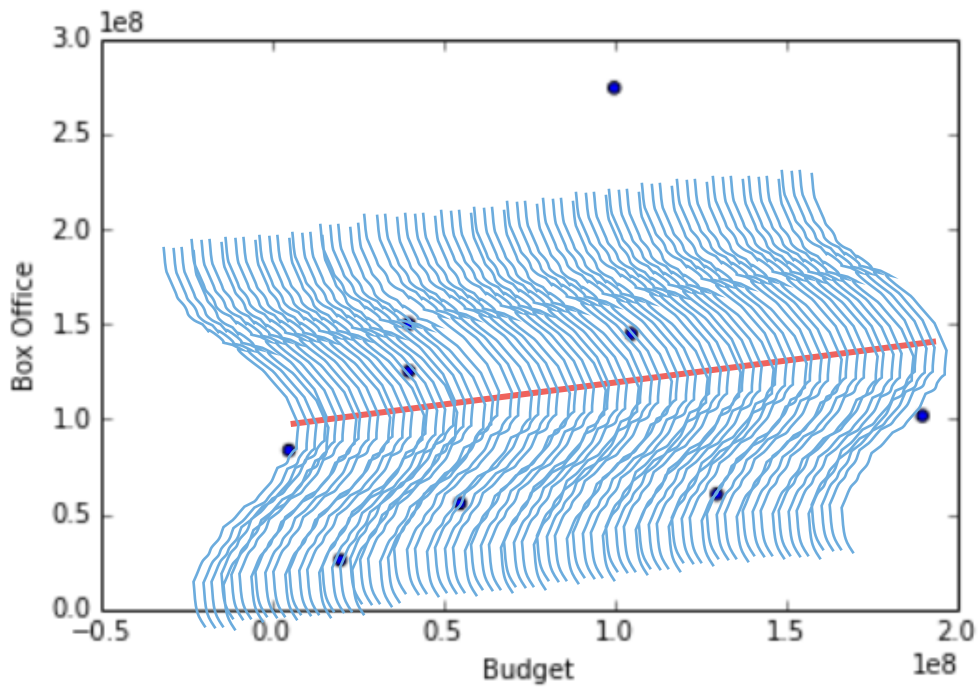
**Random**
Normal distribution
Mean=0
Stdev= \$67,762,000



$$\beta_0 = 94.68 million$$
$$\beta_1 = 0.1$$

$$y = \beta_0 + \beta_1 x + \varepsilon$$

**Random**
Normal distribution
Mean=0
Stdev= $67,762,000

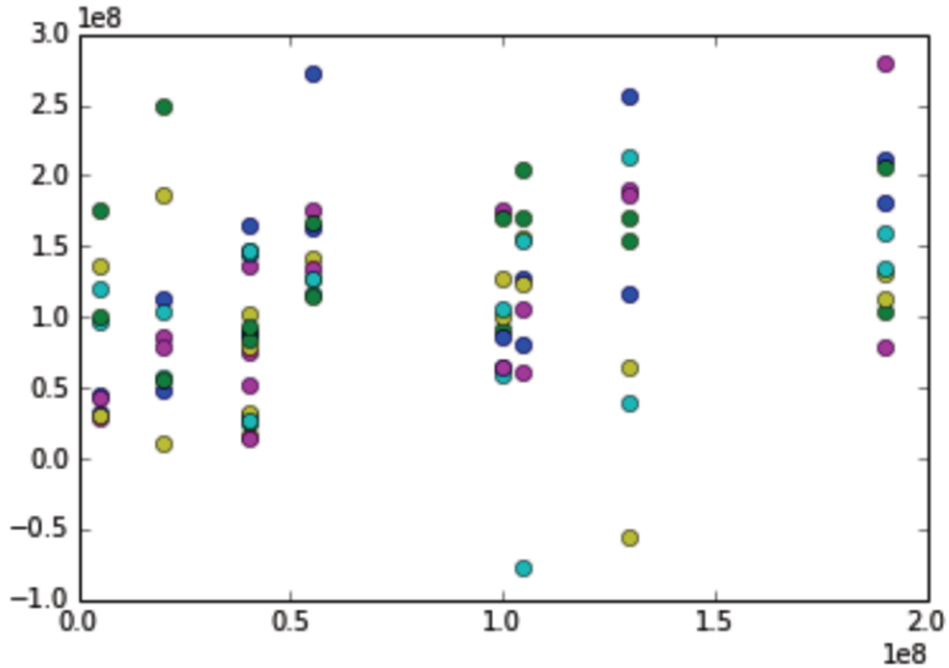


$$y = \beta_0 + \beta_1 x + \varepsilon$$

```
def underlying_gross_model(budget):
  return 94.68e6 + 0.248*budget +random.gauss(0,67762000)
```

$$y = \beta_0 + \beta_1 x + \varepsilon$$

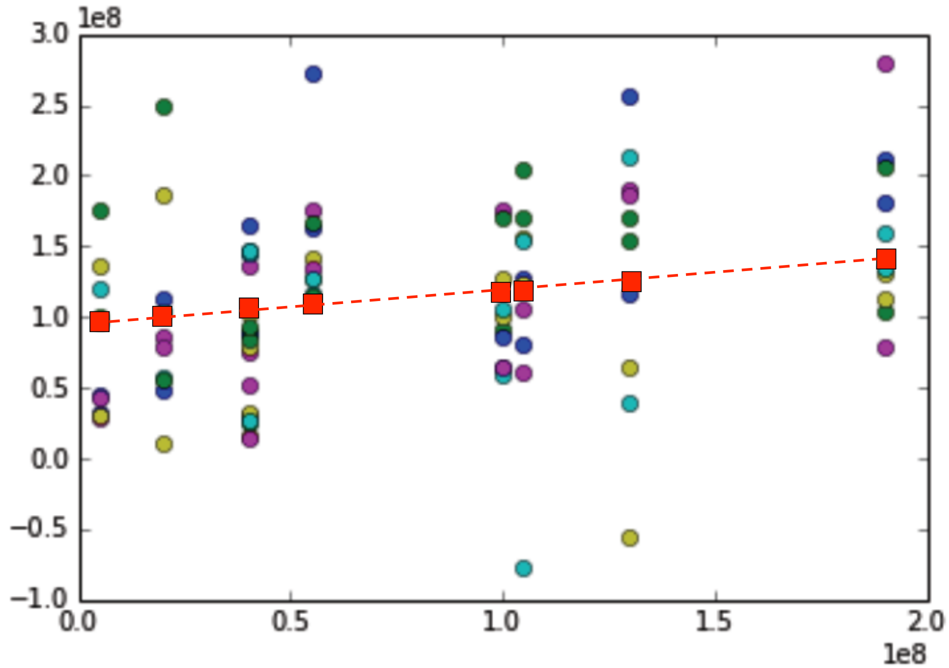
```python
def underlying_gross_model(budget):
  return 94.68e6 + 0.248*budget +random.gauss(0,67762000)
```




Possible Values in alternative universes

$$y = \beta_0 + \beta_1 x + \varepsilon$$

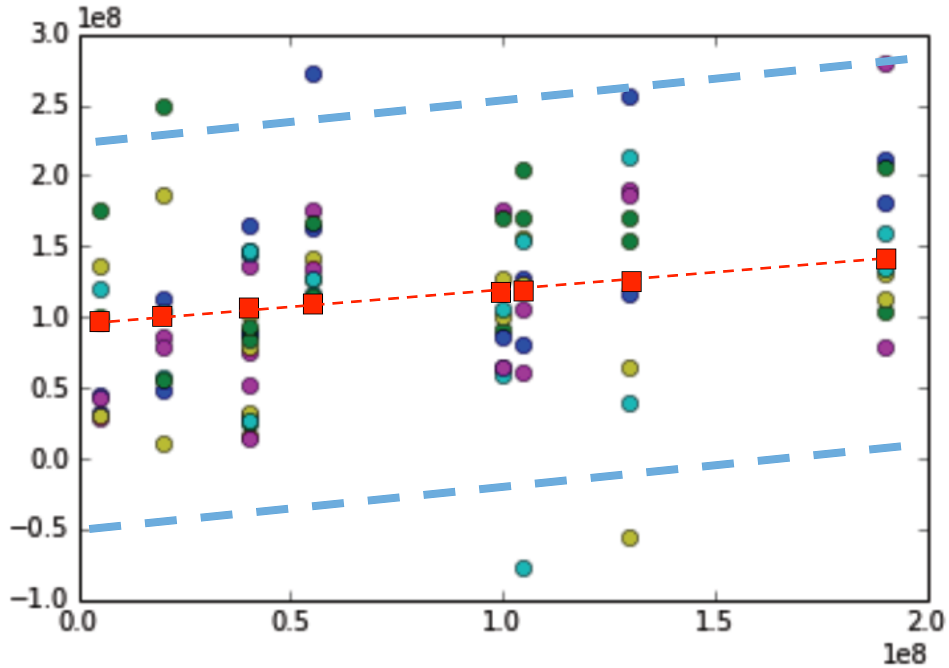
```
def underlying_gross_model(budget):
  return 94.68e6 + 0.248*budget +random.gauss(0,67762000)
```




Expected value is β0+β1x (without ε)

$$y = \beta_0 + \beta_1 x + \varepsilon$$

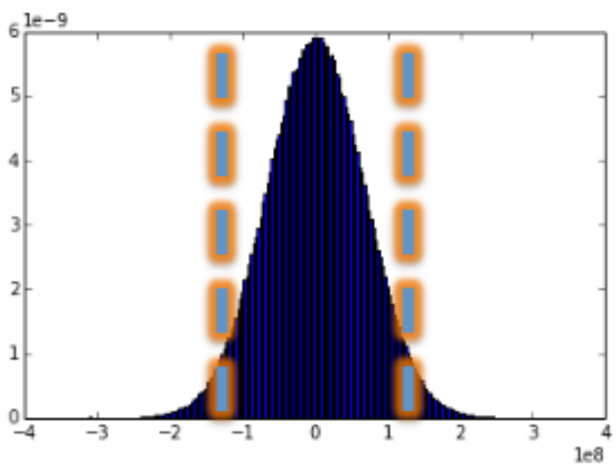
```
def underlying_gross_model(budget):
  return 94.68e6 + 0.248*budget +random.gauss(0,67762000)
```

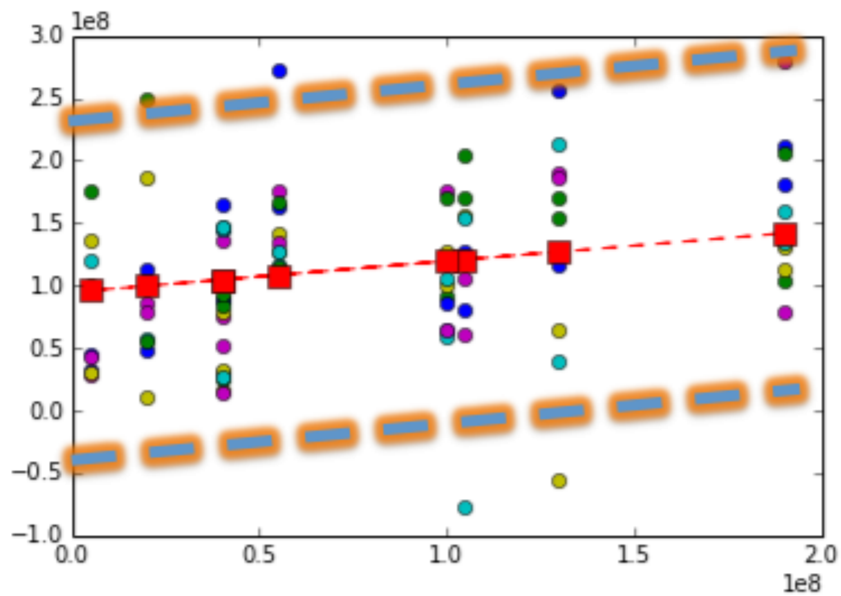



95% prediction interval

$$y = \beta_0 + \beta_1 x + \varepsilon$$

```
def underlying_gross_model(budget):
  return 94.68e6 + 0.248*budget +random.gauss(0,67762000)
```

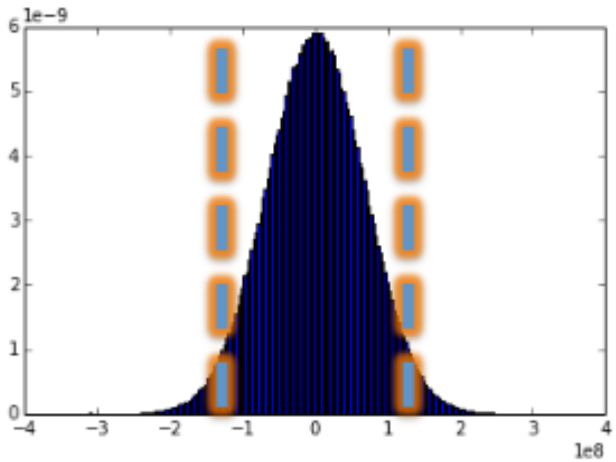


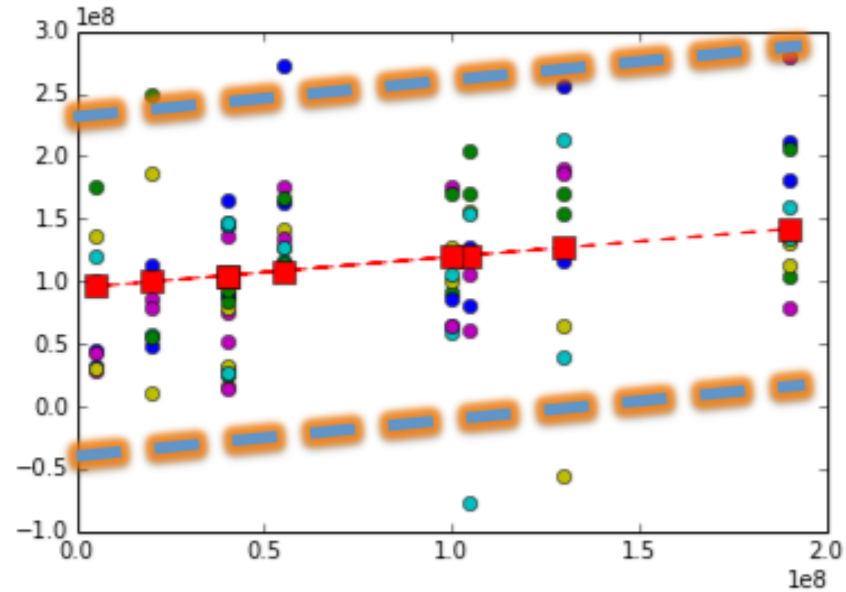


95% prediction interval

$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

```
def underlying_gross_model(budget):
  return random.gauss(94.68e6 + 0.248*budget, 67762000)
```




95% prediction interval

# Multiple Linear Regression

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$$y_{\beta}(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$$

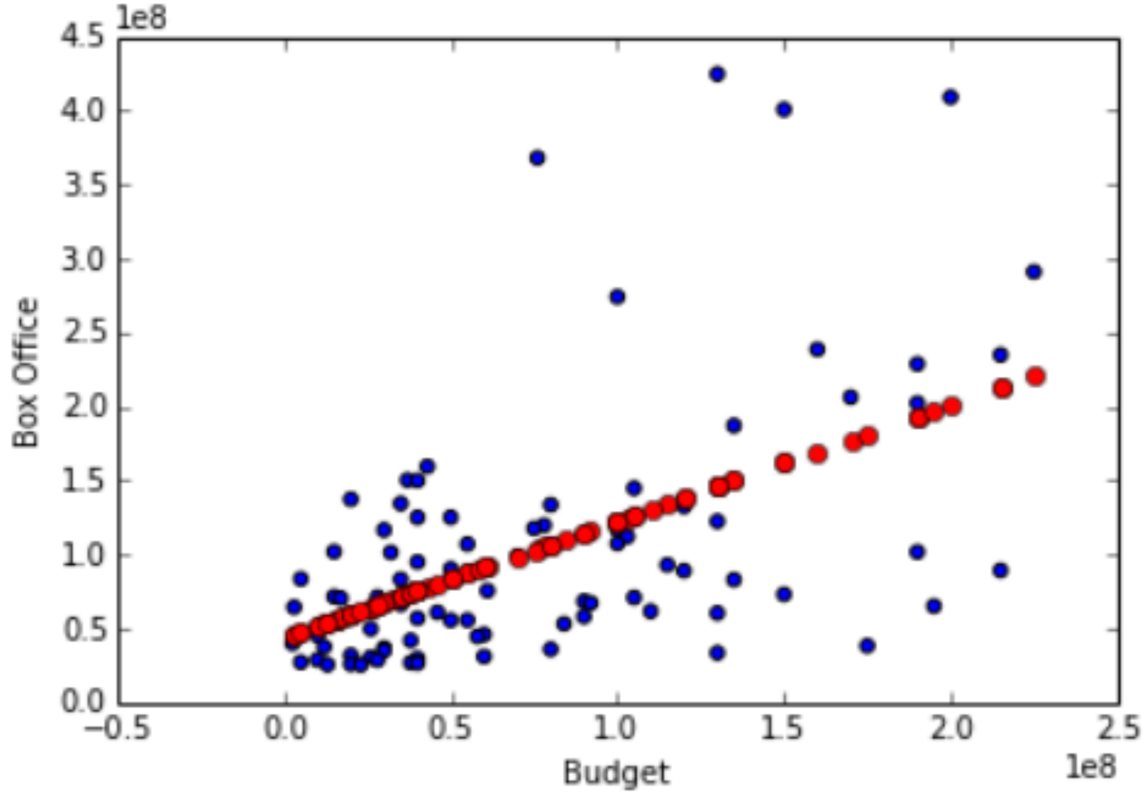
$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$$

$$\min J(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$$

to find the best fitting model

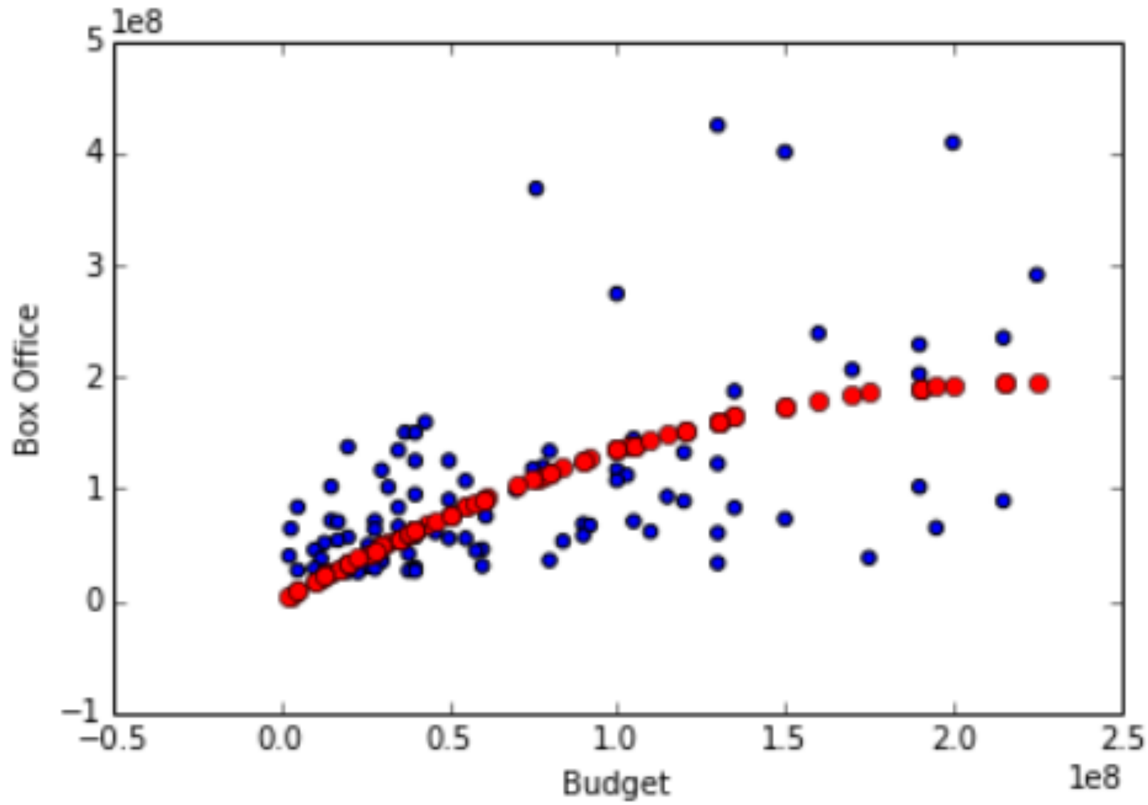
# Polynomial regression

$$y_\beta(x) = \beta_0 + \beta_1 x$$



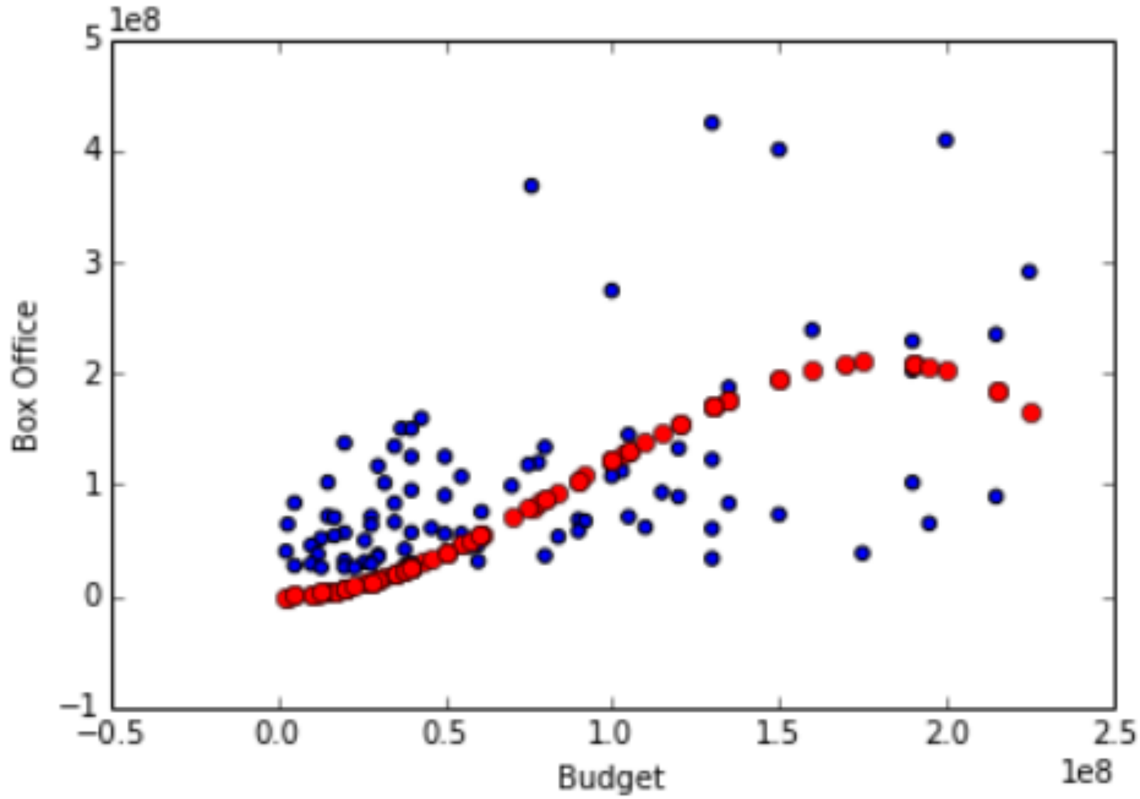
# Polynomial regression

$$y_\beta(x) = \beta_0 + \beta_1 x + \beta_2 x^2$$



# Polynomial regression

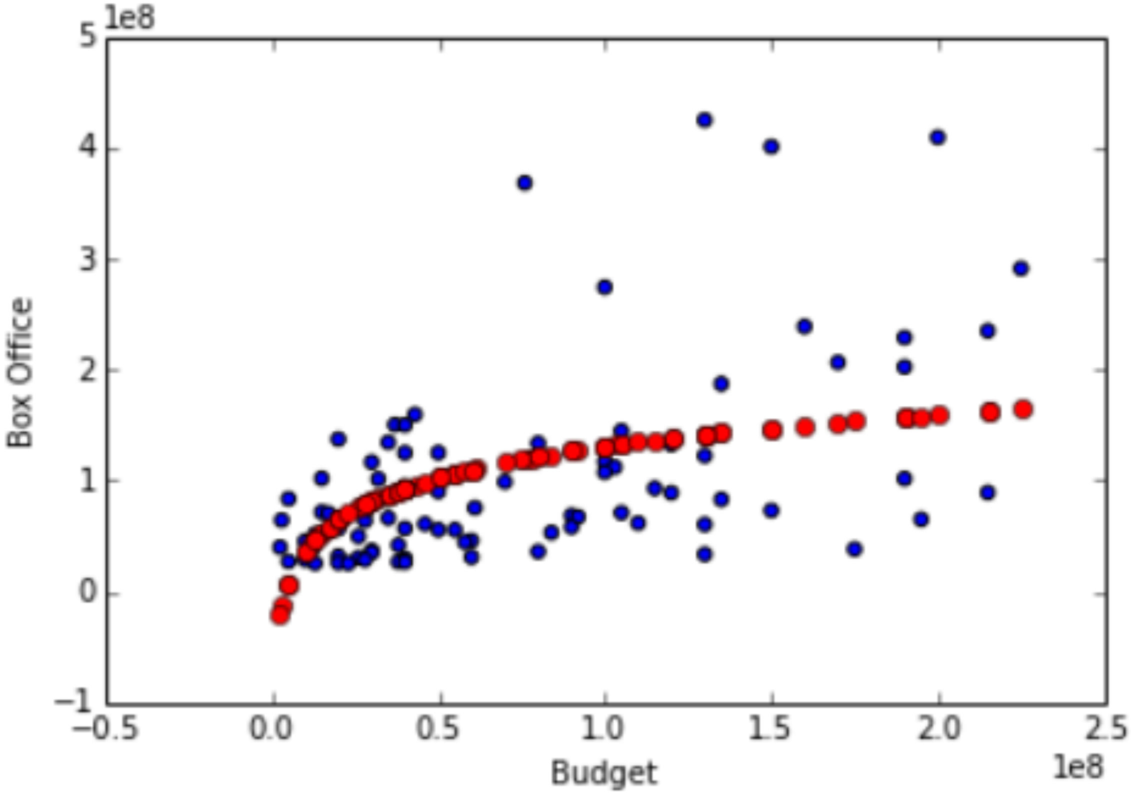
$$y_\beta(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$



# Other functional forms

## log

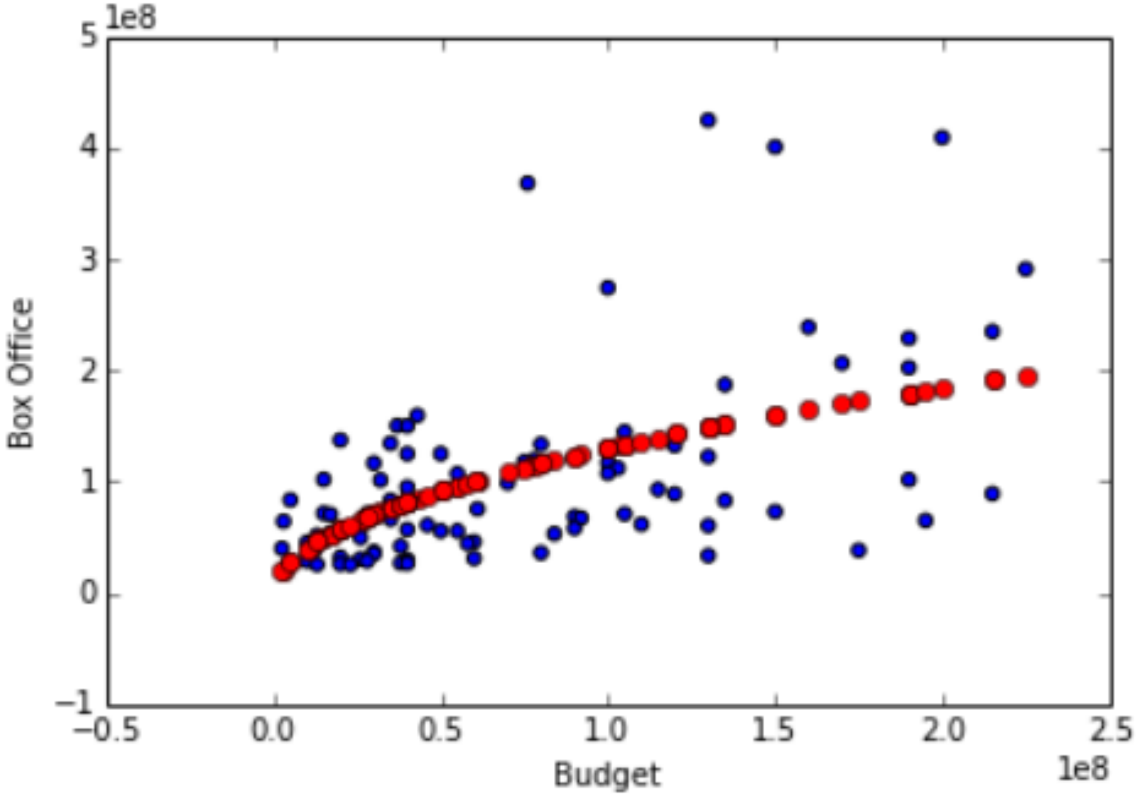
$$y_\beta(x) = \beta_0 + \beta_1 \log(x)$$



# Other functional forms

## square root

$$y_\beta(x) = \beta_0 + \beta_1 \sqrt{x}$$



## Possible to combine variables

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3)$$

## Possible to combine variables

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3)$$

## Interactions

(example: existence of both genres has an each extra effect, different than the sum of each)

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3)$$

Linear Regression is not "linear"
because we're fitting "a line."
We also fit many other forms.
It's "linear" because the features are combined in a linear fashion ( $\Sigma \beta_i f(x_i)$ ).

Linear

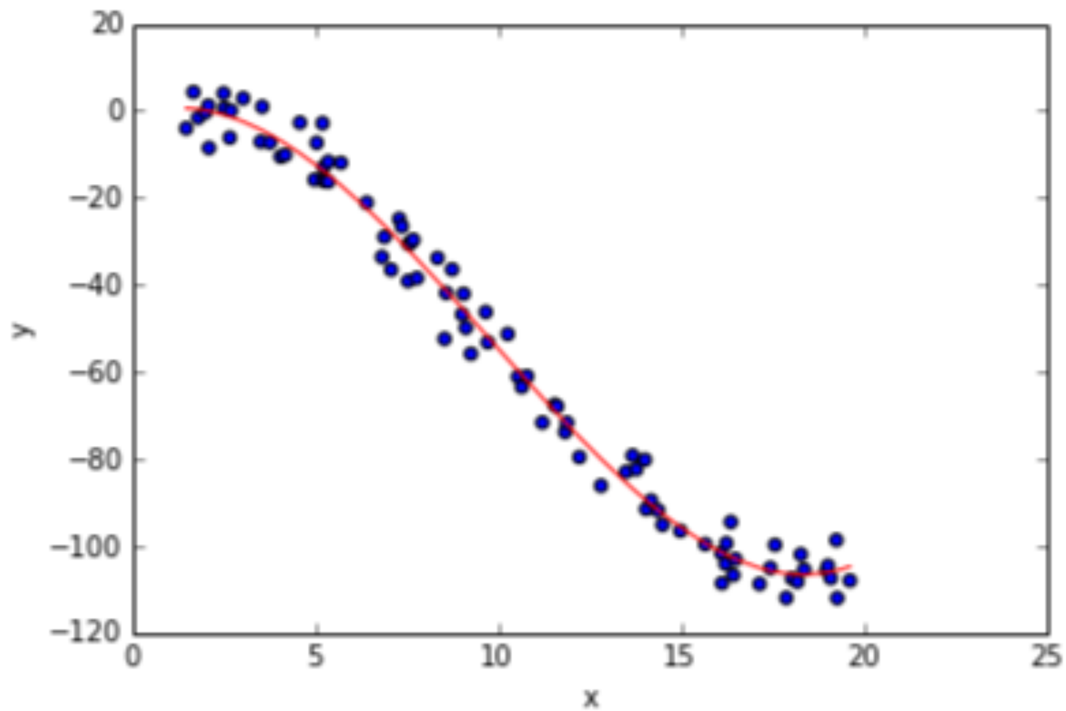
$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2^{-1}$$

Nonlinear

$$y_\beta(x) = \beta_0 + \beta_1 e^{\beta_2 x_1} + \frac{\beta_3 x_2}{(1 + \beta_4 x_2)}$$

# How to choose functional forms to try?

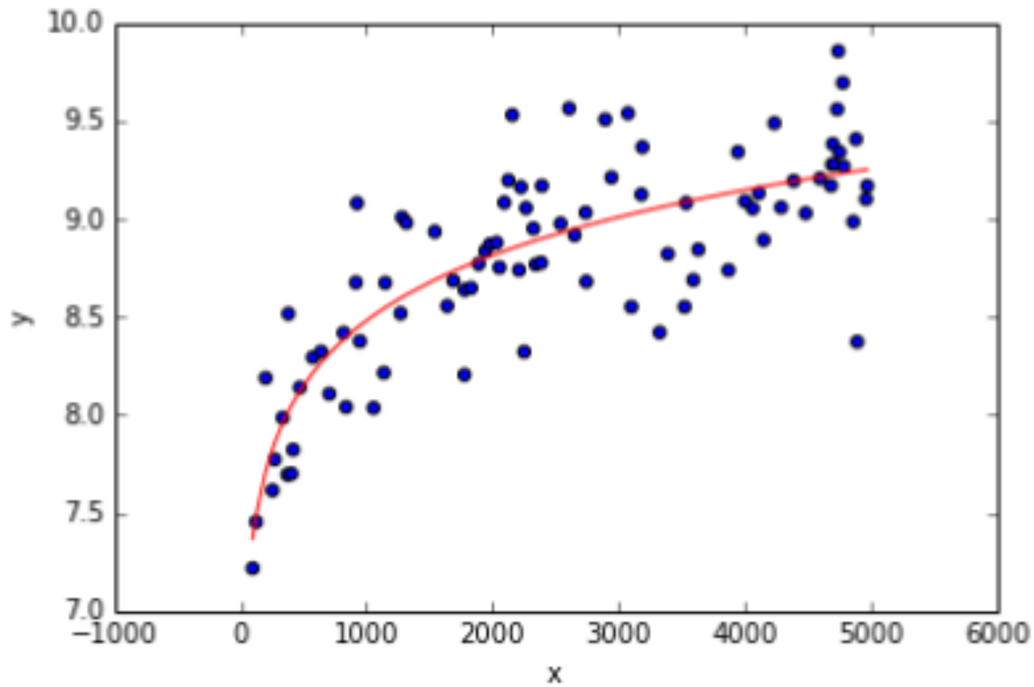
## Check one on one relationship of variable with outcome




$$y_{\beta}(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

# How to choose functional forms to try?

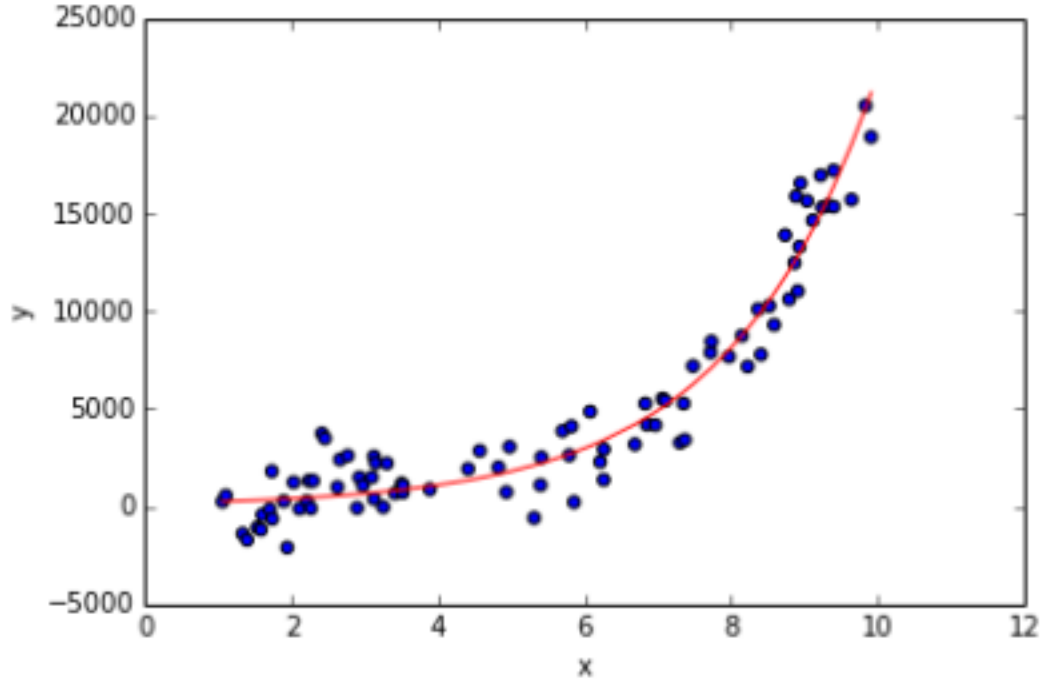
## Check one on one relationship of variable with outcome




$$y_\beta(x) = \beta_0 + \beta_1 \log(x)$$

# How to choose functional forms to try?

## Check one on one relationship of variable with outcome




$$\log(y_\beta(x)) = \beta_0 + \beta_1 x$$