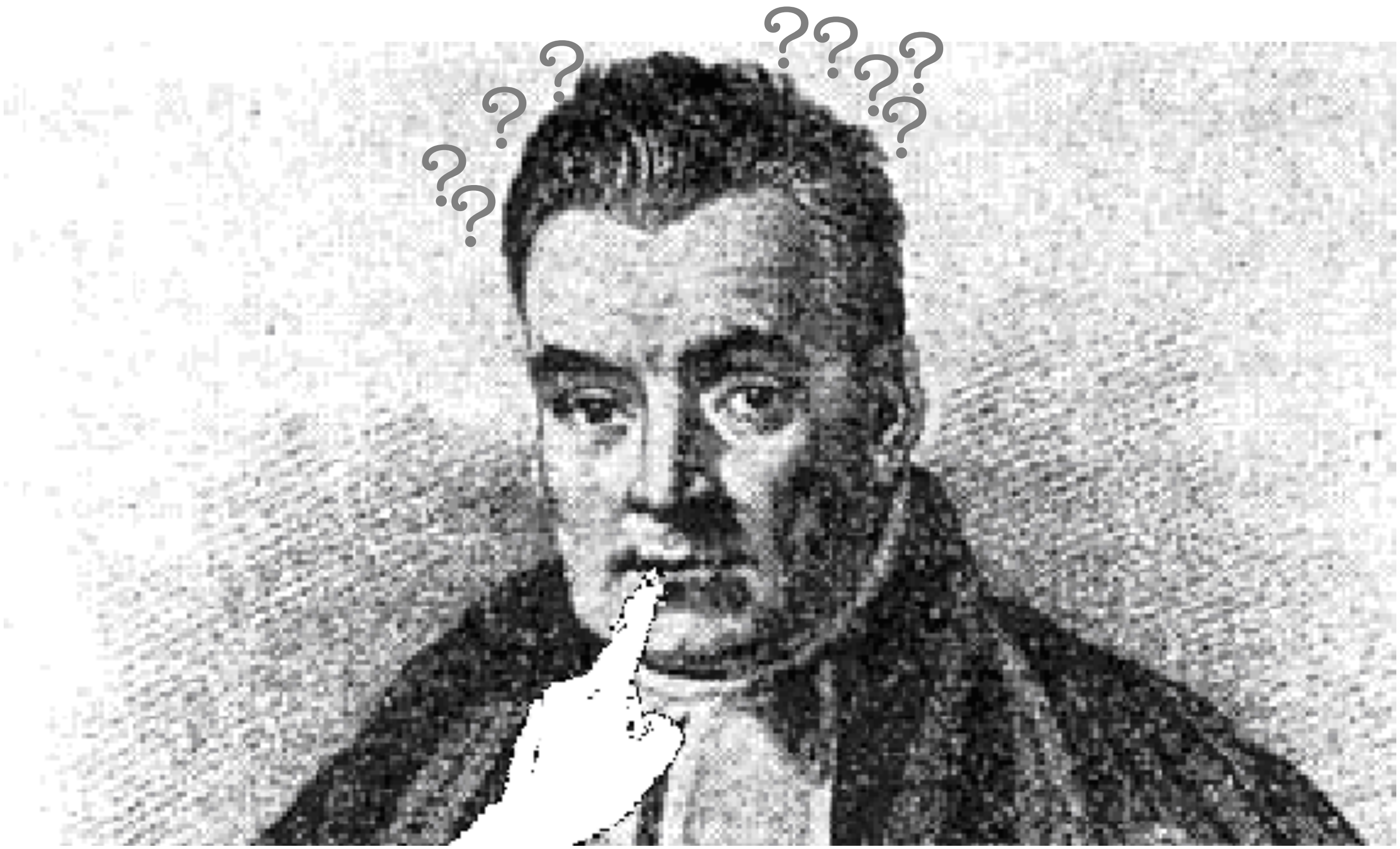
Naive Bayes

Naïve



Updating the state of
knowledge

step by step

with new information

What is classification?

Deciding among hypotheses (labels),
using information we have (features)
for each example

3 Features: Votes on 3 Bills

2 Labels: Democrat / Republican

Prediction:

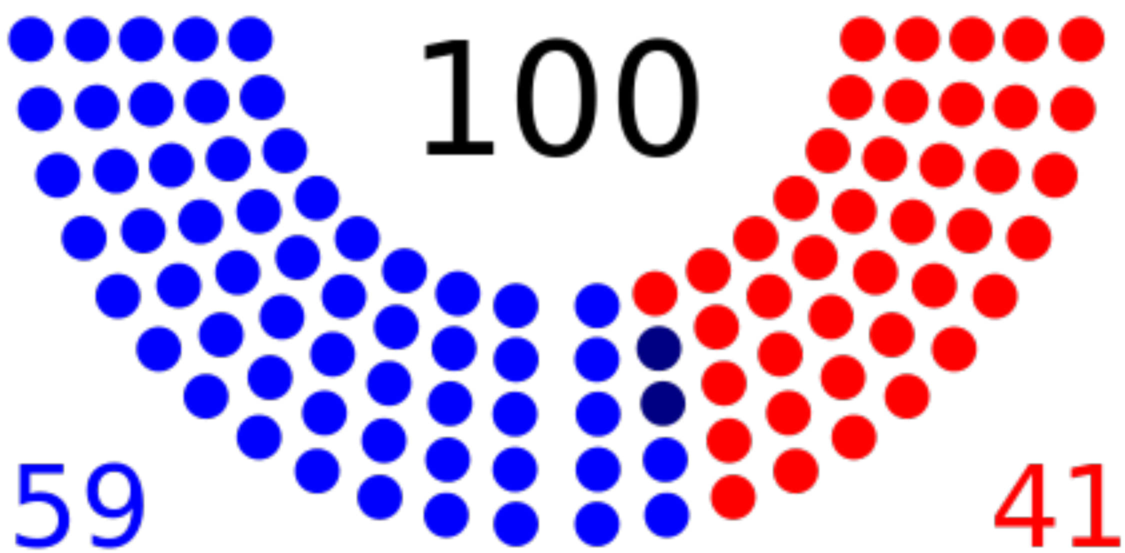
I know your votes, I’m trying to guess your party

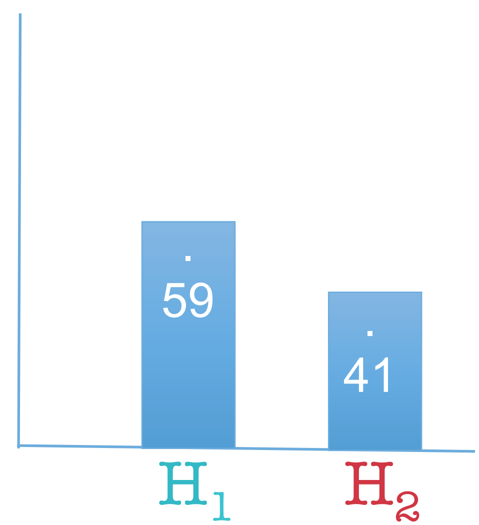
Prior: Initial belief

What’s my best guess without any vote info?

I’d guess democrat since there are more of them.

P(Democrat) = 0.59





Prior: Initial belief

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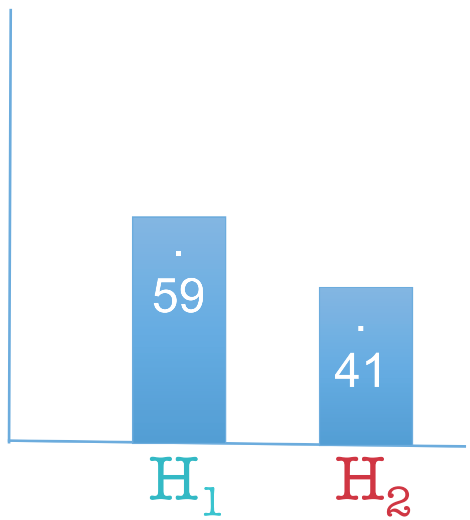
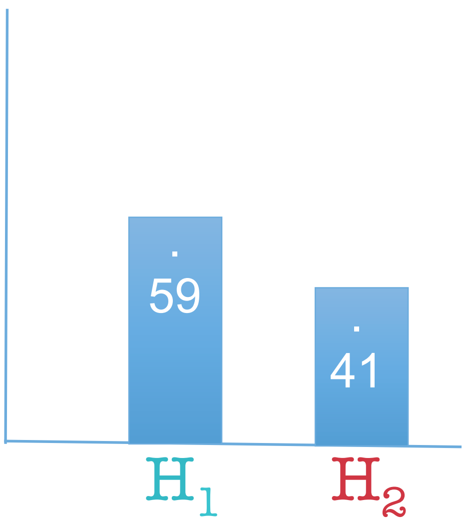


Prior: Initial belief
P(Democrat) = 0.59

New information (feature 1):
Voted YES on Net Neutrality



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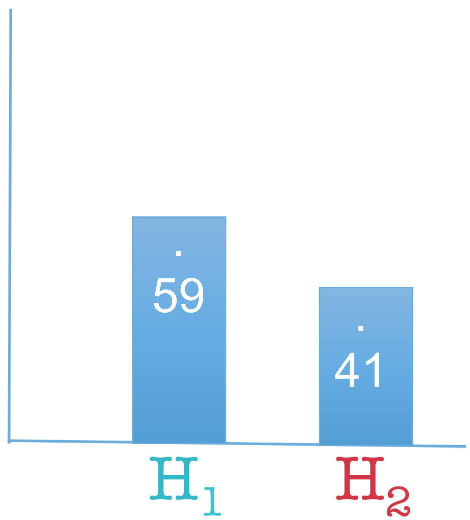
$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\, P(Dem)}{P(Y_{NN})}$$



Prior: Initial belief

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New information (feature 1):

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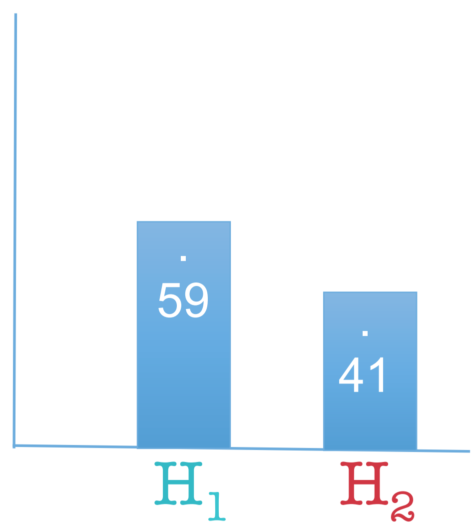
likelihood prior

posterior

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

evidence
(normalization factor)



Prior: Initial belief

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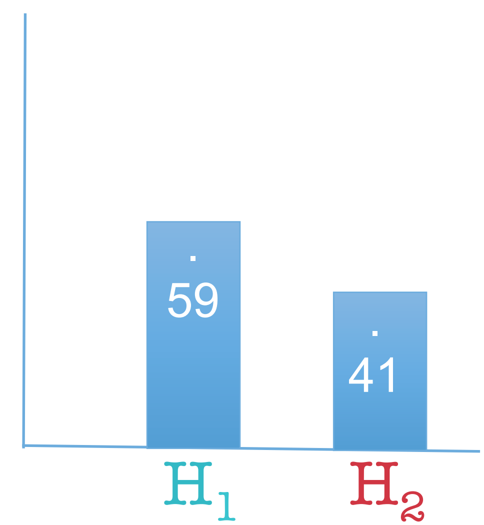
New information (feature 1):

Voted YES on Net Neutrality

$P(Y_{NN}|Dem)$

Prob. of voting yes on net neutrality if you're democrat

$$\underbrace{P(Dem|Y_{NN})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN}|Dem)}^{\text{likelihood}}\ \overbrace{P(Dem)}^{\text{prior}}}{\underbrace{P(Y_{NN})}_{\text{evidence (normalization factor)}}}$$



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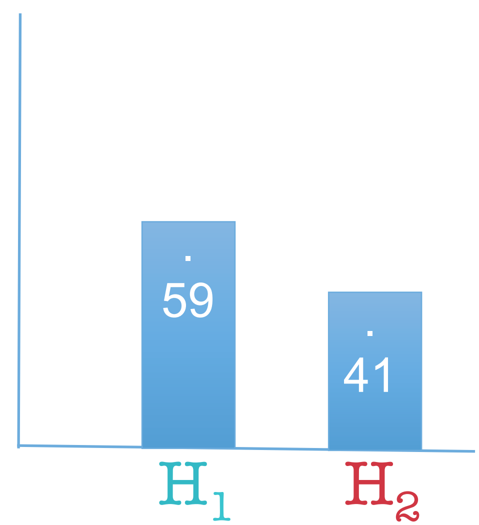
$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\ P(Rep)}{P(Y_{NN})}$$



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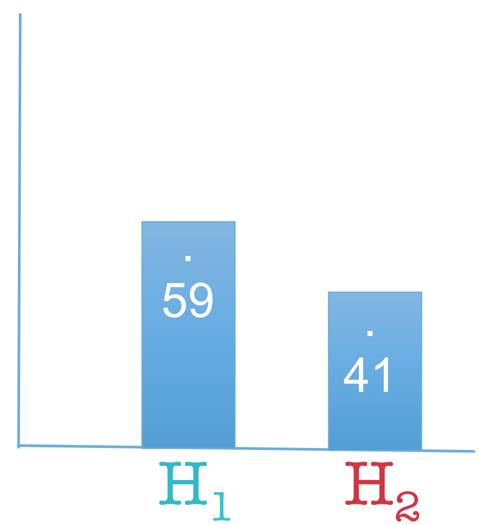
$P(Y_{NN}|Rep)$

$$P(Rep|Y_{NN})=\frac{P(Y_{NN}|Rep)\ P(Rep)}{P(Y_{NN})}$$

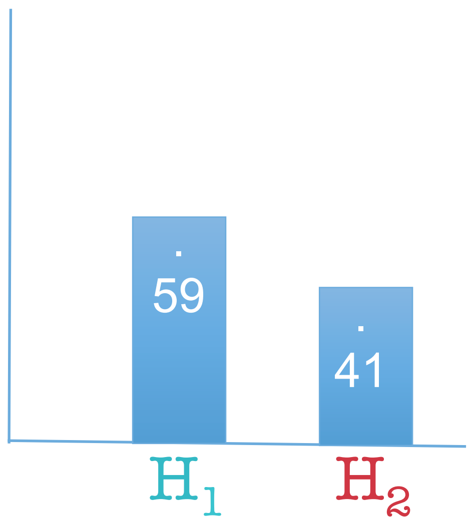


$P(Y_{NN}|Dem)$

Prob. of voting yes
on net neutrality
if you're democrat

$P(Y_{NN}|Rep)$

Prob. of voting
yes
on net neutrality
if you're
republican



Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$P(Y_{NN}|Dem)$
Prob. of voting yes
on net neutrality
if you're democrat

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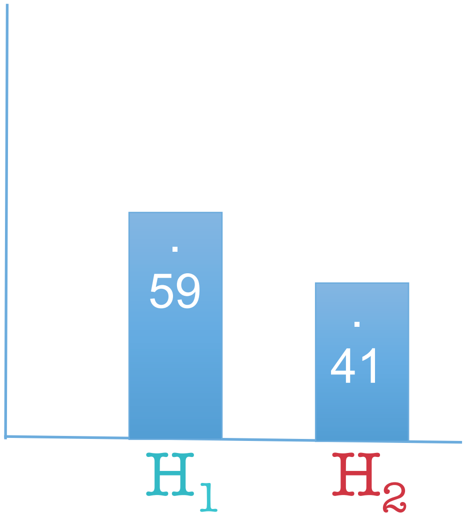
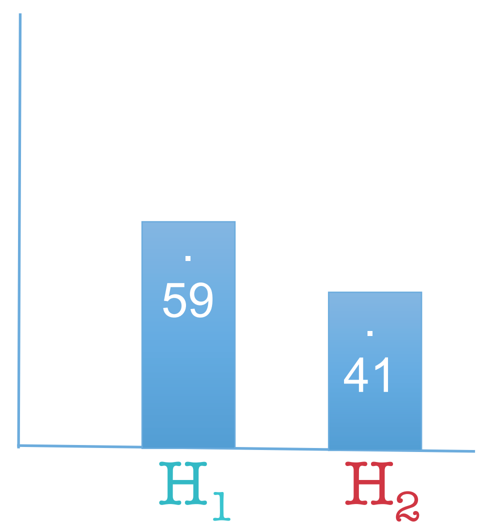


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|Dem) \approx \frac{\#\text{ democrats that } Y_{NN}}{\#\text{ all democrats}}$$

$$P(Y_{NN}|Rep) \approx \frac{\#\text{ republicans that } Y_{NN}}{\#\text{ all republicans}}$$

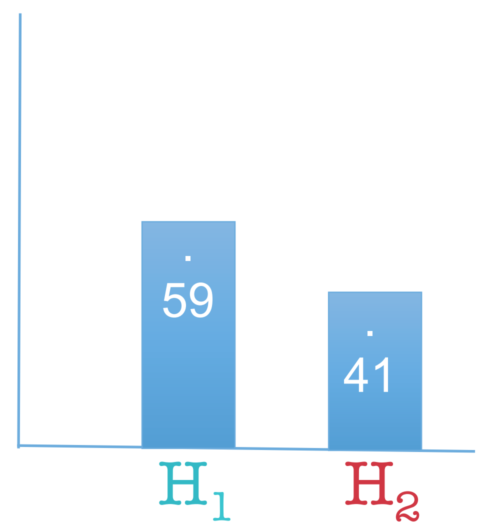


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|\text{Dem}) \approx \frac{\#\text{ democrats that } Y_{NN}}{\#\text{ all democrats}}$$

$$P(Y_{NN}|\text{Rep}) \approx \frac{\#\text{ republicans that } Y_{NN}}{\#\text{ all republicans}}$$

For likelihoods of discrete data,
training/fitting means counting!
(and estimating likelihoods by dividing counts)

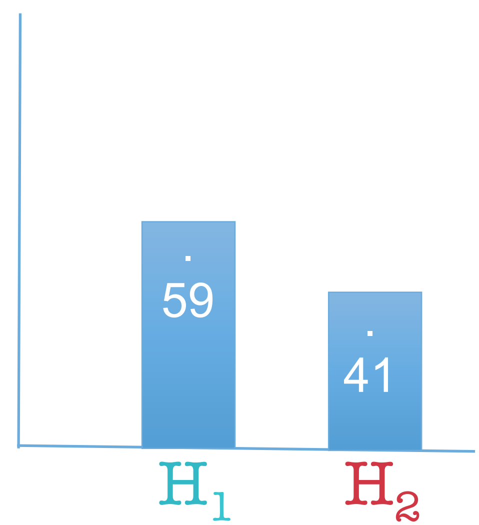


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|\text{Dem}) \approx \frac{56}{59} = 0.949$$

$$P(Y_{NN}|\text{Rep}) \approx \frac{34}{41} = 0.829$$

For likelihoods of discrete data,
training/fitting means counting!
(and estimating likelihoods by dividing counts)



Prior: Initial belief

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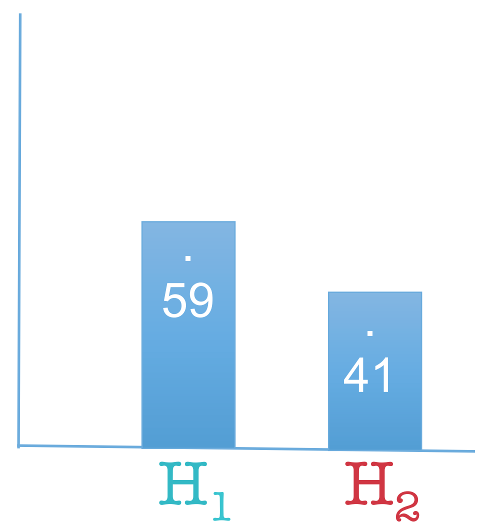
$$\underset{\text{posterior}}{P(Dem|Y_{NN})} = \frac{\overset{\text{likelihood}}{P(Y_{NN}|Dem)}\ \overset{\text{prior}}{P(Dem)}}{\underset{\text{evidence (normalization factor)}}{P(Y_{NN})}}$$

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\ P(Rep)}{P(Y_{NN})}$$



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New information (feature 1):

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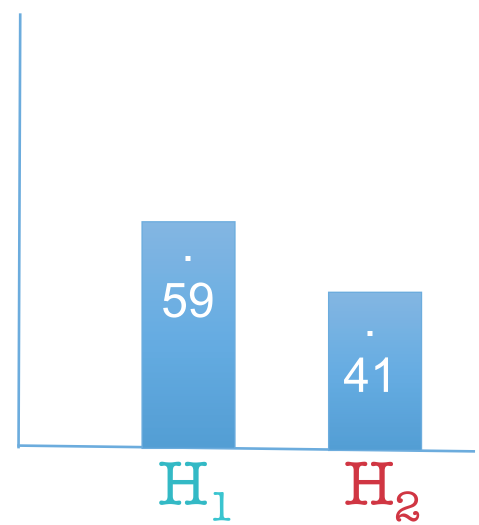
$$\underset{\text{posterior}}{P(\text{Dem}|Y_{NN})} = \frac{\overset{\text{likelihood}}{0.949} \;*\; \overset{\text{prior}}{0.59}}{\underset{\text{evidence (normalization factor)}}{P(Y_{NN})}}$$

$$P(\text{Rep}|Y_{NN}) = \frac{0.829 \;*\; 0.41}{P(Y_{NN})}$$



Current belief

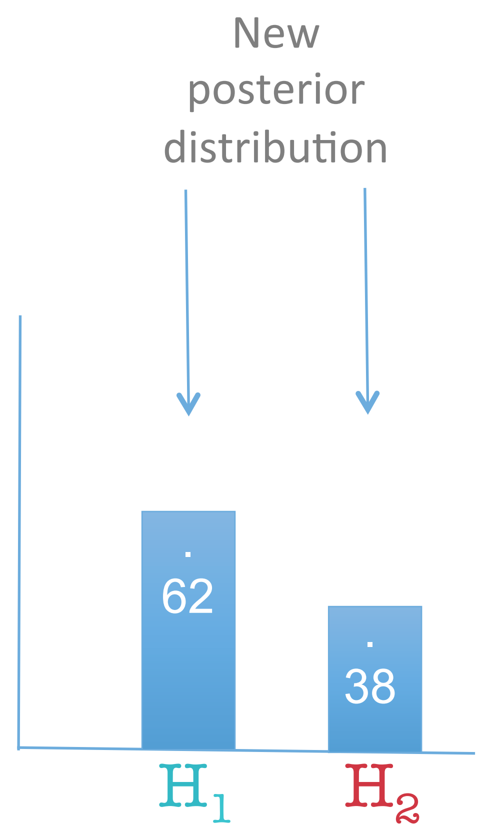
$P(\text{Democrat} | Y_{NN}) = 0.62$

New information (feature 1):

Voted YES on Net Neutrality

$$\underset{\text{posterior}}{P(\text{Dem}|Y_{NN})} = \frac{\overset{\text{likelihood}}{0.949} * \overset{\text{prior}}{0.59}}{\underset{\text{evidence (normalization factor)}}{P(Y_{NN})}}$$

$$P(\text{Rep}|Y_{NN}) = \frac{0.829 * 0.41}{P(Y_{NN})}$$

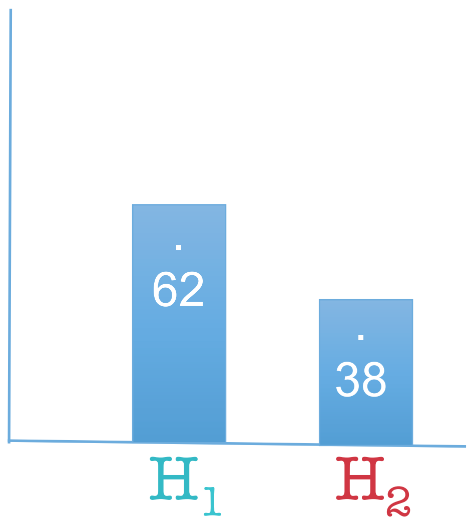


Current belief

$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts



Current belief

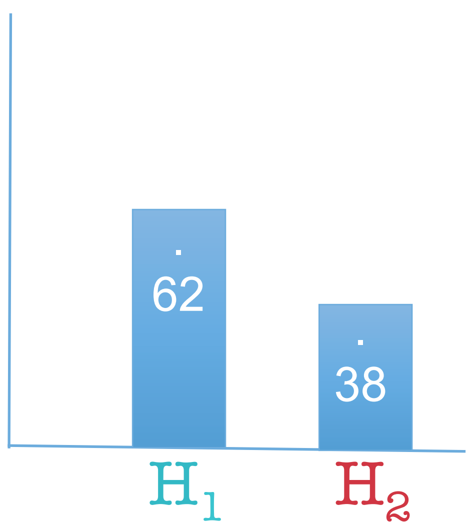
$P(\text{Democrat} | Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem}|Y_{NN}, Y_{TC}) = \frac{P(Y_{TC}|\text{Dem})\, P(\text{Dem}|Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep}|Y_{NN}, Y_{TC}) = \frac{P(Y_{TC}|\text{Rep})\, P(\text{Rep}|Y_{NN})}{P(Y_{TC})}$$



Current belief

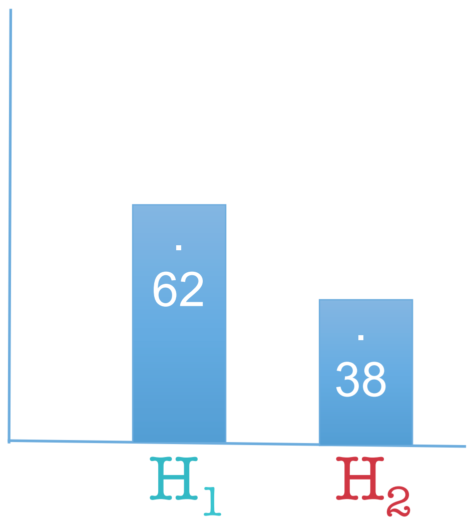
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

$P(Y_{TC} \mid \text{Dem})$

$$\approx \frac{10}{59} = 0.169$$

$P(Y_{TC} \mid \text{Rep})$

$$\approx \frac{35}{41} = 0.854$$



Current belief

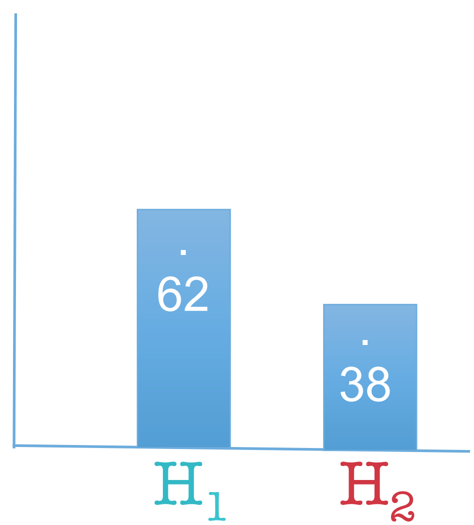
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN})}{P(Y_{TC})}$$



Current belief

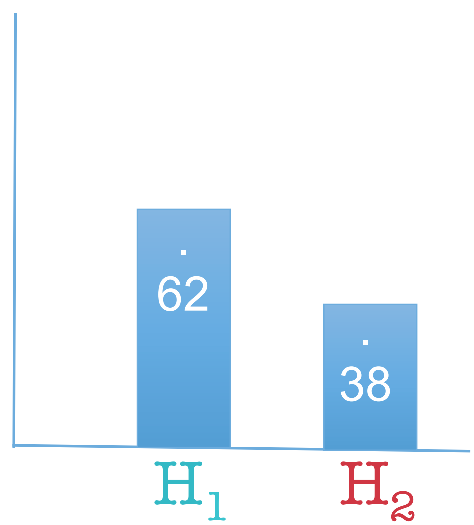
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$



Current belief

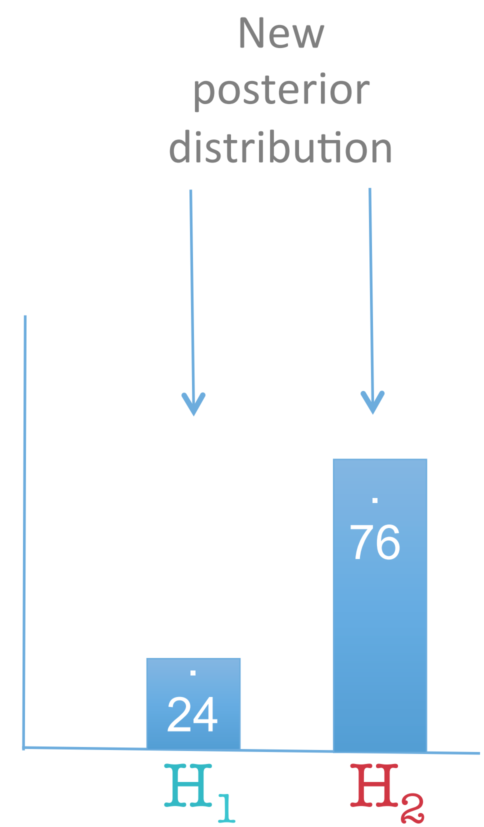
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$

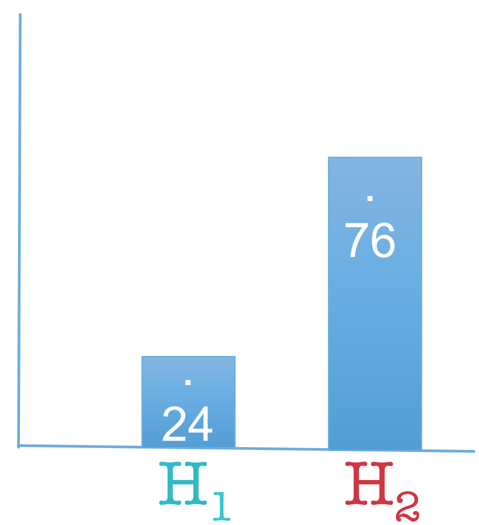


Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns



Current belief

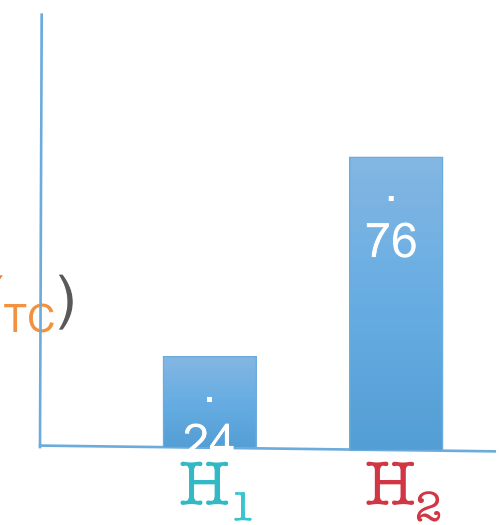
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(Dem \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid Dem)\, P(Dem \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(Rep \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid Rep)\, P(Rep \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

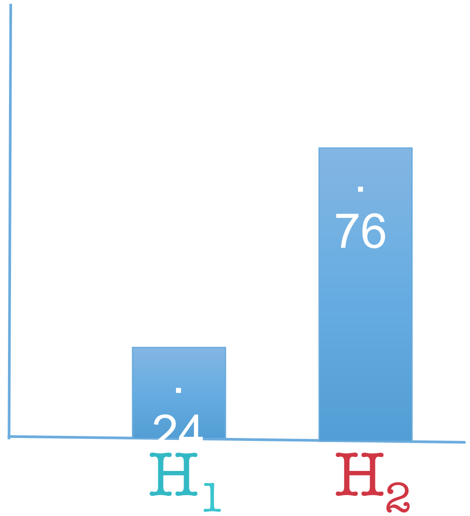
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

$P(N_{LG} \mid \text{Dem})$

$\approx \dfrac{53}{59} = 0.898$

$P(N_{LG} \mid \text{Rep})$

$\approx \dfrac{23}{41} = 0.561$



Current belief

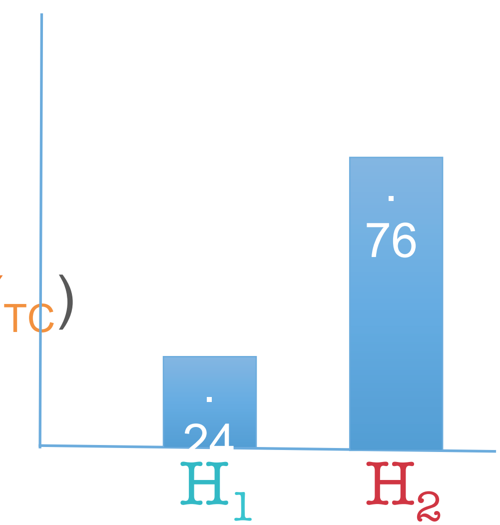
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

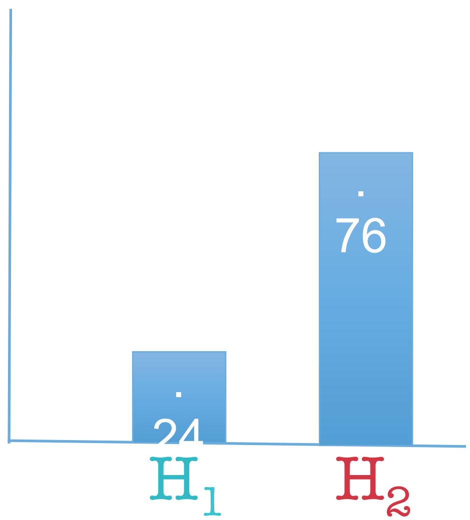
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.898 \quad * \quad 0.24}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.561 \quad * \quad 0.76}{P(N_{LG})}$$



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$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

New information (feature 3):

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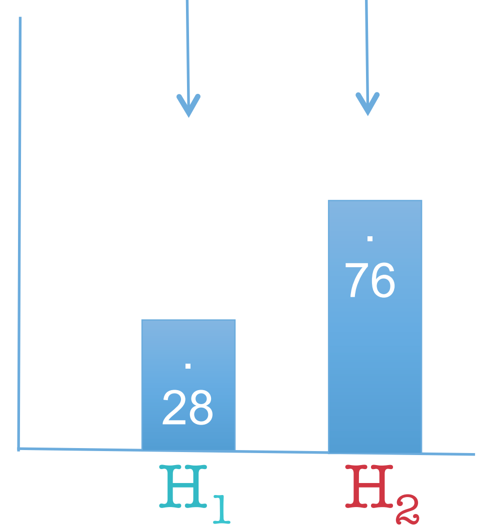
New posterior distribution



Current belief

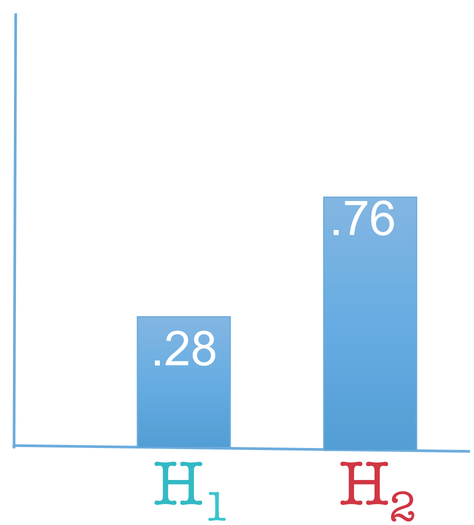
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.28$

Classify this person that voted

Yes on Net Neutrality (Y_{NN}),

Yes on Tax Cuts (Y_{TC}),

No on License-free Guns (N_{LG})



Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

Classify this person that voted

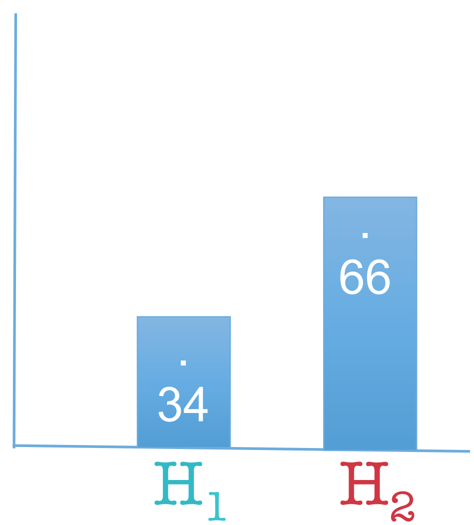
Yes on Net Neutrality (Y_{NN}),

Yes on Tax Cuts (Y_{TC}),

No on License-free Guns (N_{LG})

My strongest belief is in H_2,

I classify this person with the label Republican.



Naïve Bayes

Training:

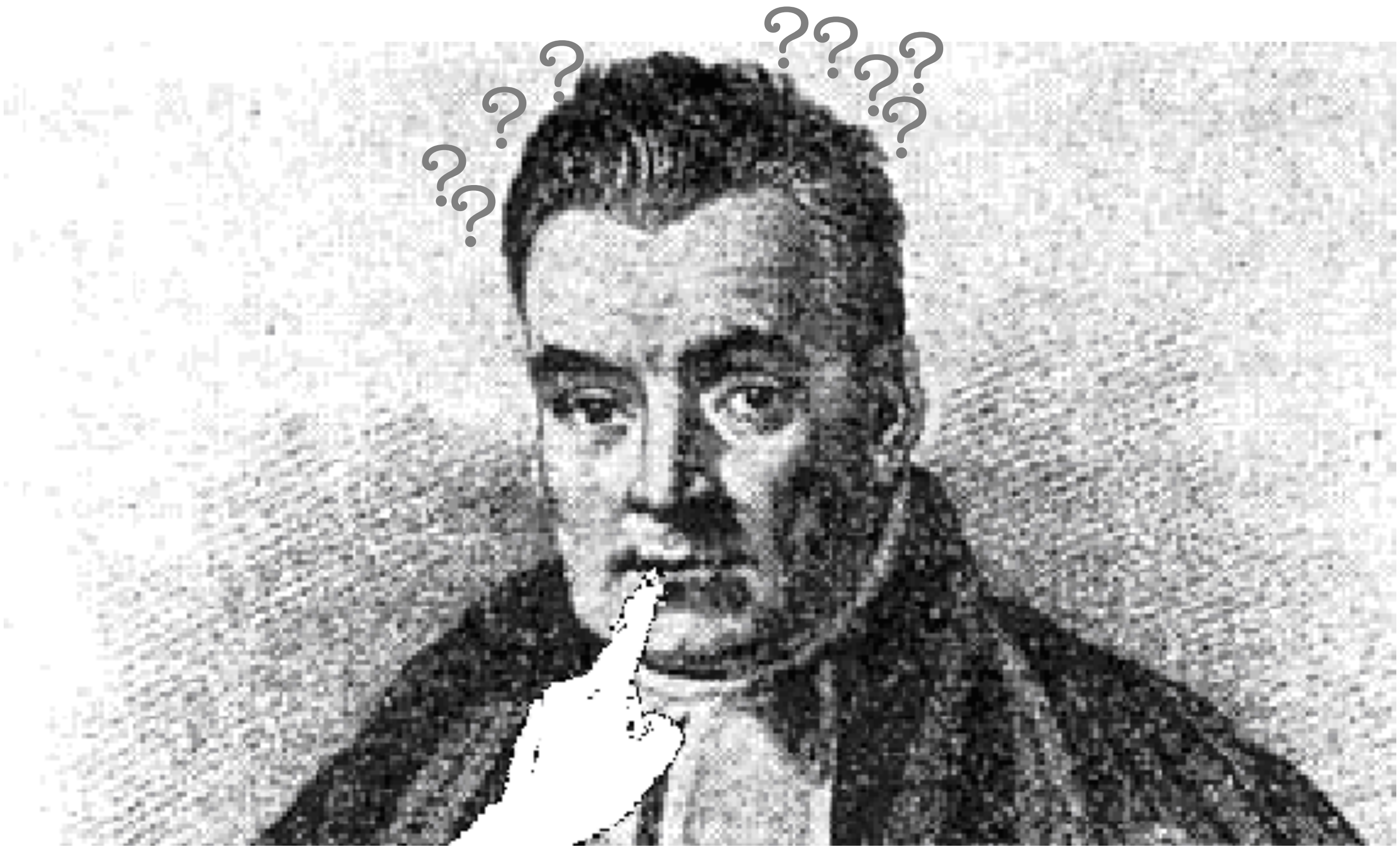
Count and calculate the likelihood of each feature value for each class:

$P(Y_{NN}|Dem)$
$P(Y_{NN}|Rep)$
$P(Y_{TC}|Dem)$
$P(Y_{TC}|Rep)$
$P(Y_{LG}|Dem)$
$P(Y_{LG}|Rep)$

Prediction:

Use Bayes to update priors with the likelihoods,
Pick label with the highest posterior probability.

What was the naïve part?



We assumed each feature is independent

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Easier to see in a single update rather than sequential

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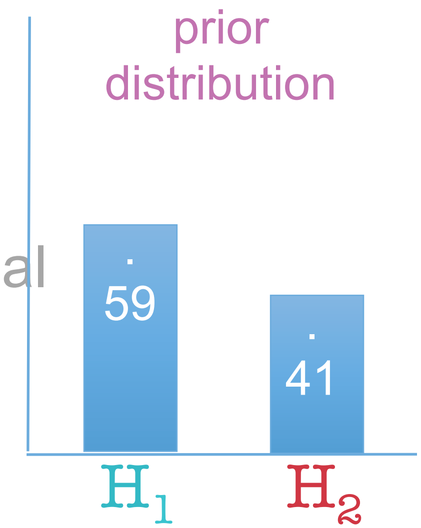
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{P(Y_{NN},Y_{TC},N_{LG}|Dem) \quad P(Dem)}{P(Y_{NN},Y_{TC},N_{LG})}$$



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Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\overset{\text{posterior}}{P(Dem|Y_{NN},Y_{TC},N_{LG})} = \frac{\overset{\text{likelihood}}{P(Y_{NN},Y_{TC},N_{LG}|Dem)}\ \overset{\text{prior}}{P(Dem)}}{P(Y_{NN},Y_{TC},N_{LG})}$$

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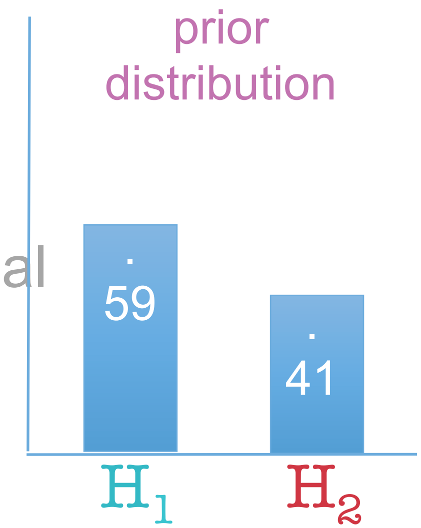


Easier to see in a single update rather than sequential

Prob. of this example having label Democrat, given the values Yes, Yes and No on the features NN, TC and LG

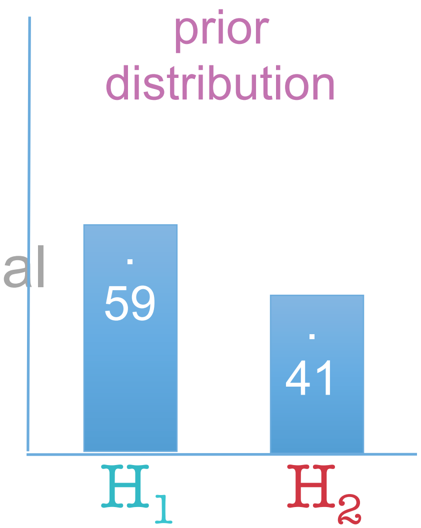
posterior — likelihood — prior

$$P(Dem \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(Y_{NN}, Y_{TC}, N_{LG} \mid Dem)\; P(Dem)}{P(Y_{NN}, Y_{TC}, N_{LG})}$$

Independence Assumption:

$$P(Y_{NN}, Y_{TC}, N_{LG} \mid Dem) = P(Y_{NN} \mid Dem)\, P(Y_{TC} \mid Dem)\, P(N_{LG} \mid Dem)$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

likelihood prior

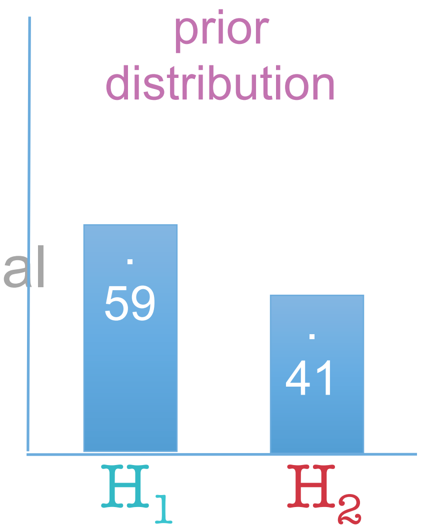
posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{P(Y_{NN},Y_{TC},N_{LG}|Dem)\ P(Dem)}{P(Y_{NN},Y_{TC},N_{LG})}$$

Independence Assumption:
$P(Y_{NN},Y_{TC},N_{LG}|Dem) = P(Y_{NN}|Dem)\ P(Y_{TC}|Dem)\ P(N_{LG}|Dem)$

Not even close in most cases!
Naïve Bayes still works well.

We assumed each feature is independent



Easier to see in a single update rather than sequential

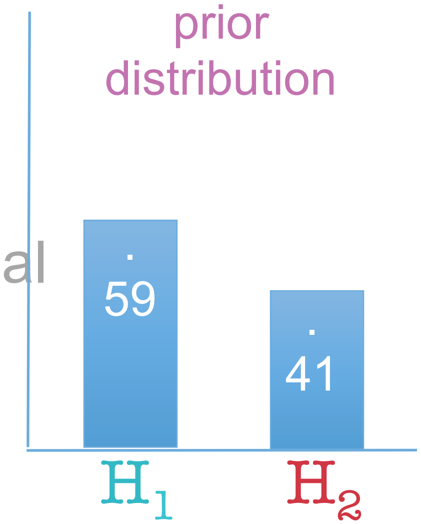
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

likelihood prior

posterior

$$\overset{\text{posterior}}{P(Dem|Y_{NN},Y_{TC},N_{LG})} = \frac{\overset{\text{likelihood}}{P(Y_{NN}|Dem)\,P(Y_{TC}|Dem)\,P(N_{LG}|Dem)}\;\overset{\text{prior}}{P(Dem)}}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat, given the values Yes, Yes and No on the features NN, TC and LG

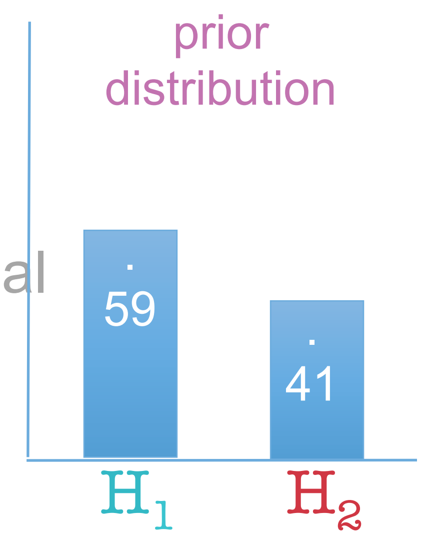
posterior

likelihood

prior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{0.949 * 0.169 * 0.898 \quad * \quad 0.59}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

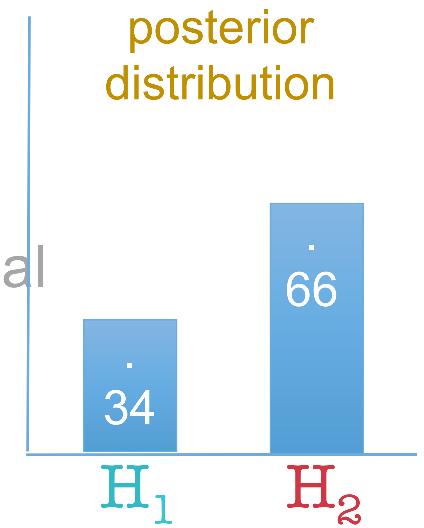
likelihood prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{0.949 * 0.169 * 0.898 \quad * \quad 0.59}{P(Y_{NN},Y_{TC},N_{LG})}$$

$$P(Rep|Y_{NN},Y_{TC},N_{LG}) = \frac{0.829 * 0.854 * 0.561 \quad * \quad 0.41}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$P(Dem|Y_{NN},Y_{TC},N_{LG}) = 0.28$

$P(Rep|Y_{NN},Y_{TC},N_{LG}) = 0.66$

predict!

What about multiple classes?












Straightforward!

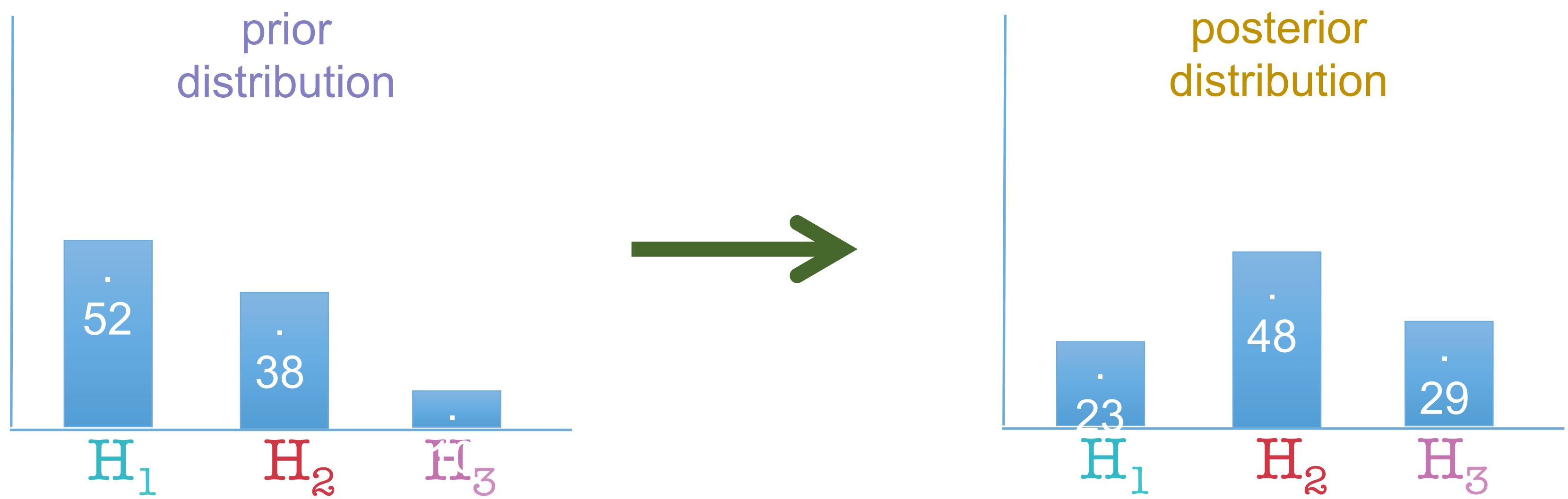
Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG})$$
$$P(Rep|Y_{NN},Y_{TC},N_{LG})$$
$$P(Indep|Y_{NN},Y_{TC},N_{LG})$$

Straightforward!

Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem|Y_{NN},Y_{TC},N_{LG})$$
$$P(Rep|Y_{NN},Y_{TC},N_{LG})$$
$$P(Indep|Y_{NN},Y_{TC},N_{LG})$$



Flavors of Bayes in `sklearn`:

Numeric Features:
Gaussian Naïve Bayes

Features that are 0 or 1 (and both matter):
Bernoulli Naïve Bayes

Features that are count-like (and only non-zero matters):
Multinomial Naïve Bayes

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Which did we do?