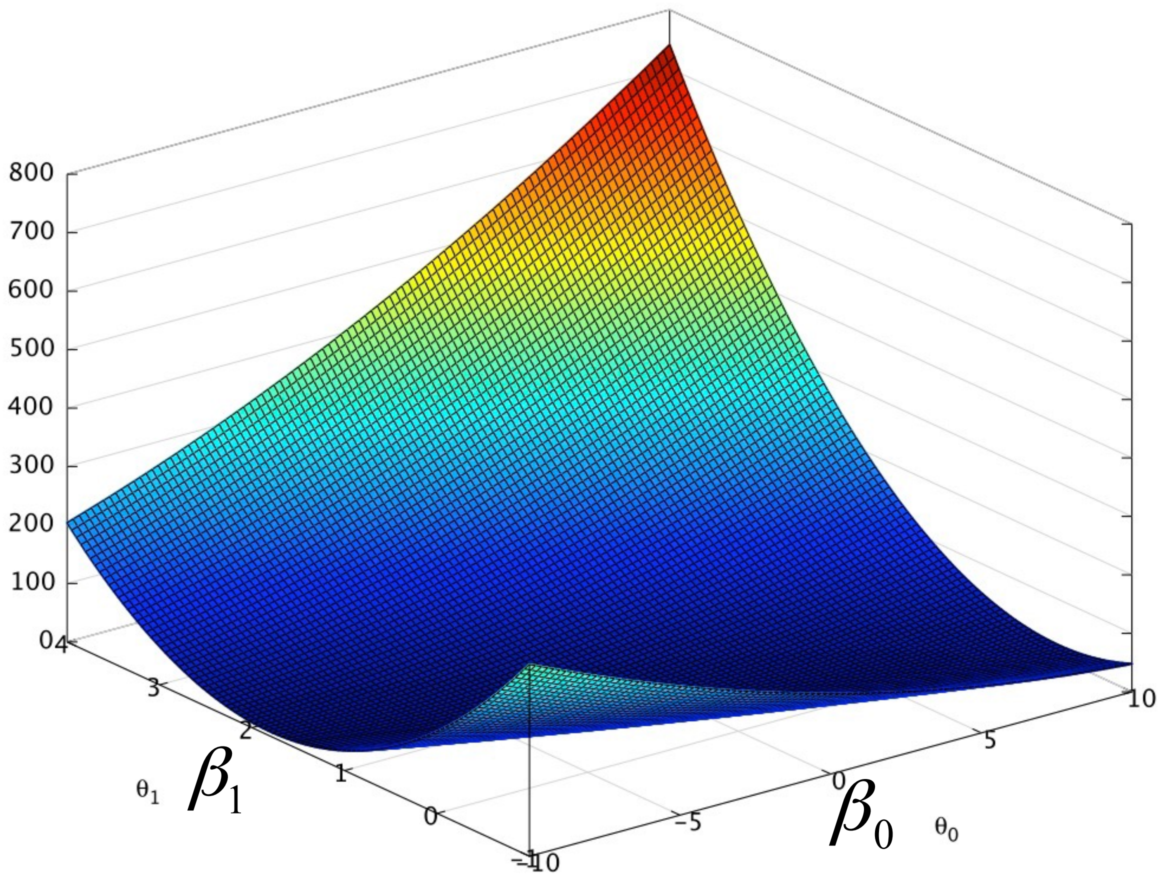
Stochastic Gradient Descent

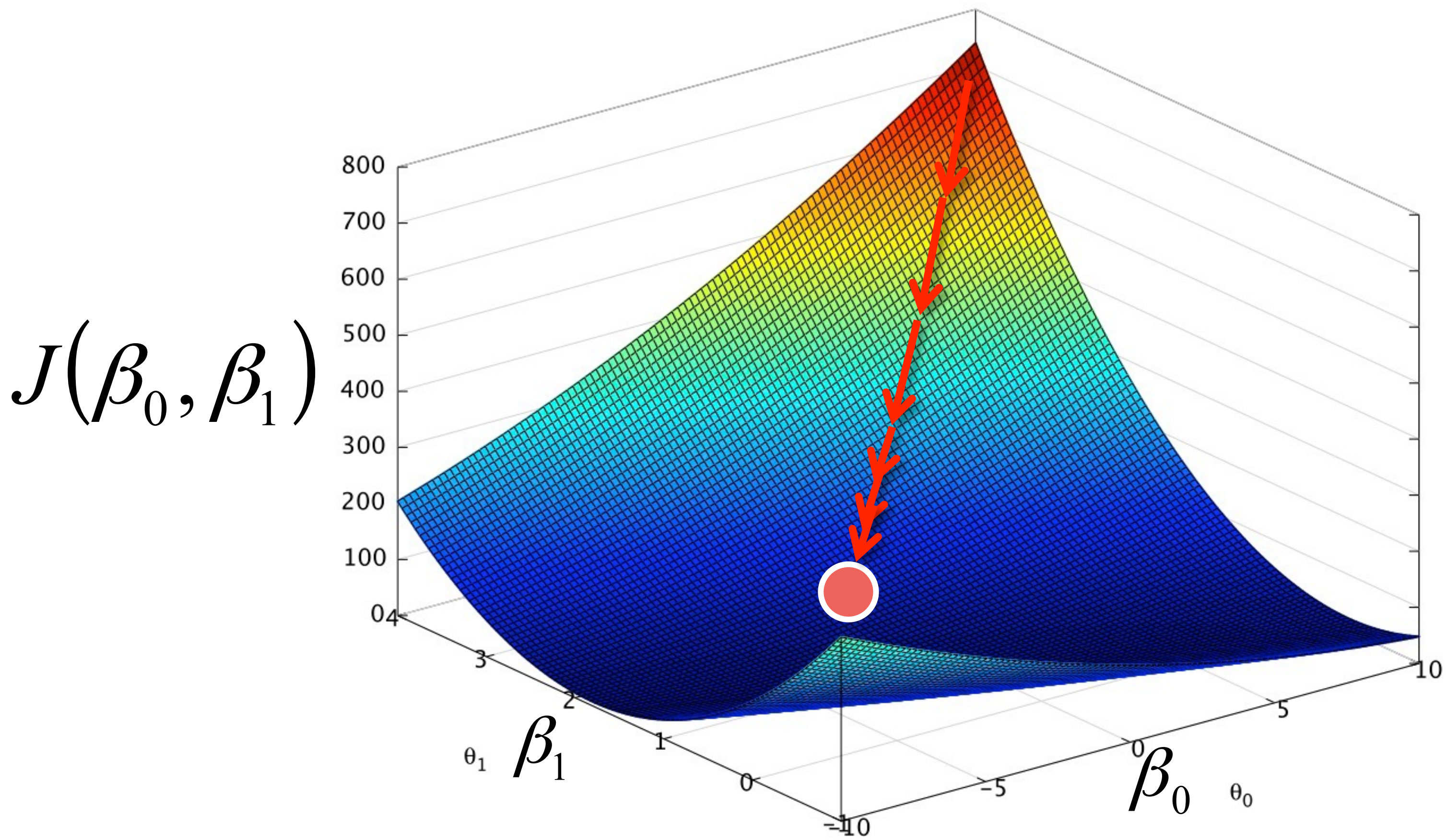
Gradient Descent

Start with a cost function J(β):

$$J(\beta_0, \beta_1)$$



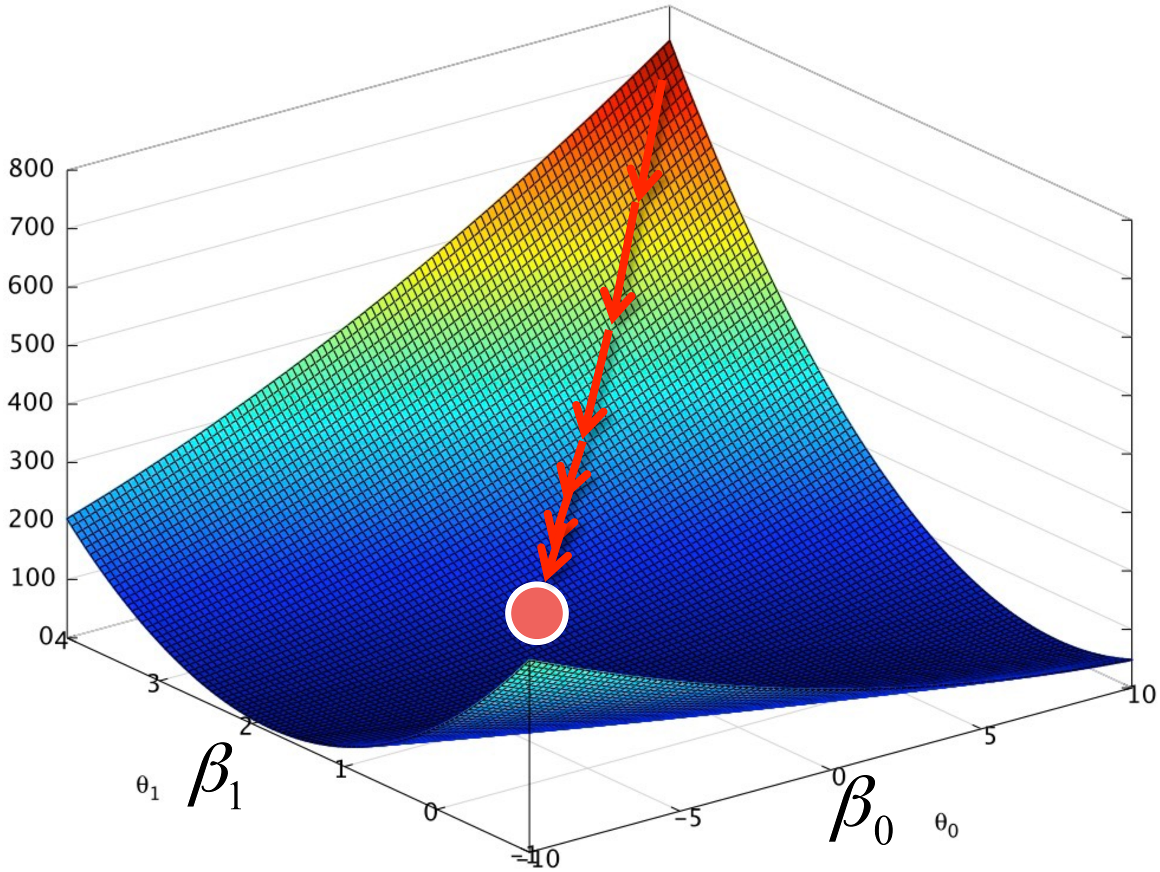
Gradient Descent



Gradient Descent

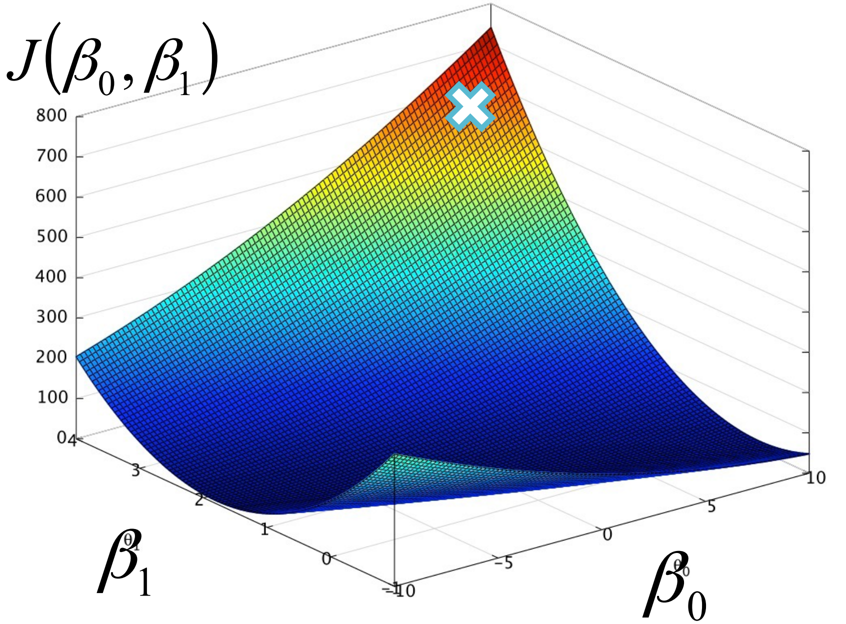
Then gradually move to the minimum.

$$J(\beta_0, \beta_1)$$



Gradient Descent with Linear Regression

How can we do this?

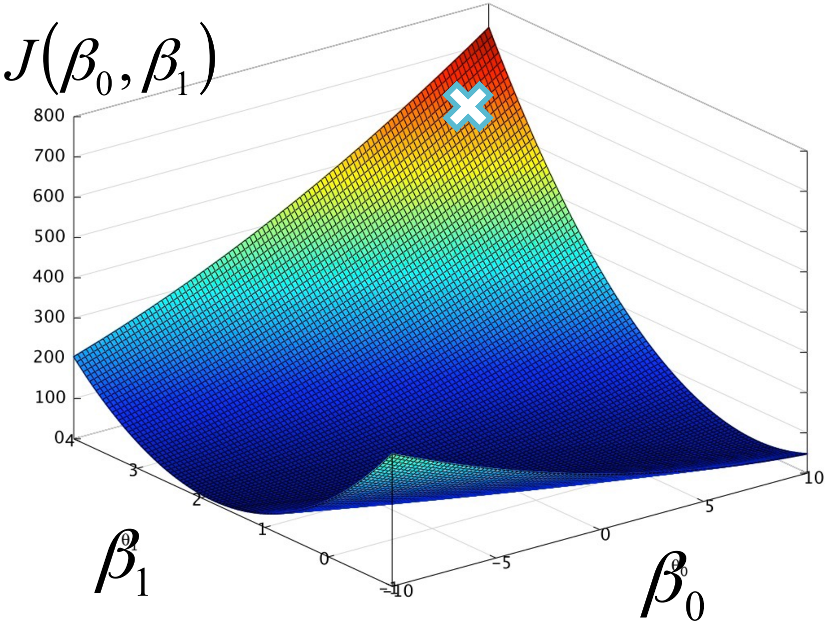


Gradient Descent with Linear Regression

How can we do this?

(without seeing the graph of J(β)!)

Start with the function J(β):



Gradient Descent with Linear Regression

How can we do this?

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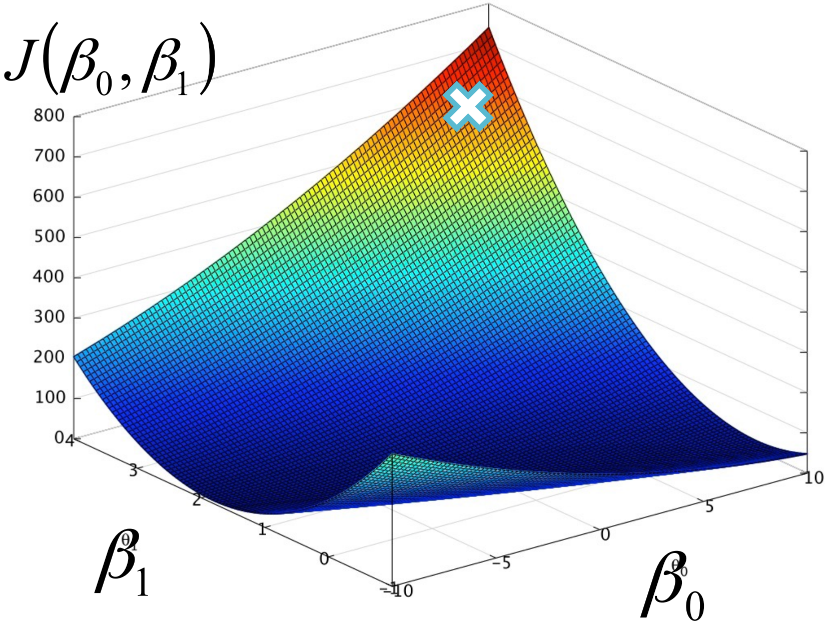
Start with the function J(β):

$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left((\beta_0+\beta_1 x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2$$



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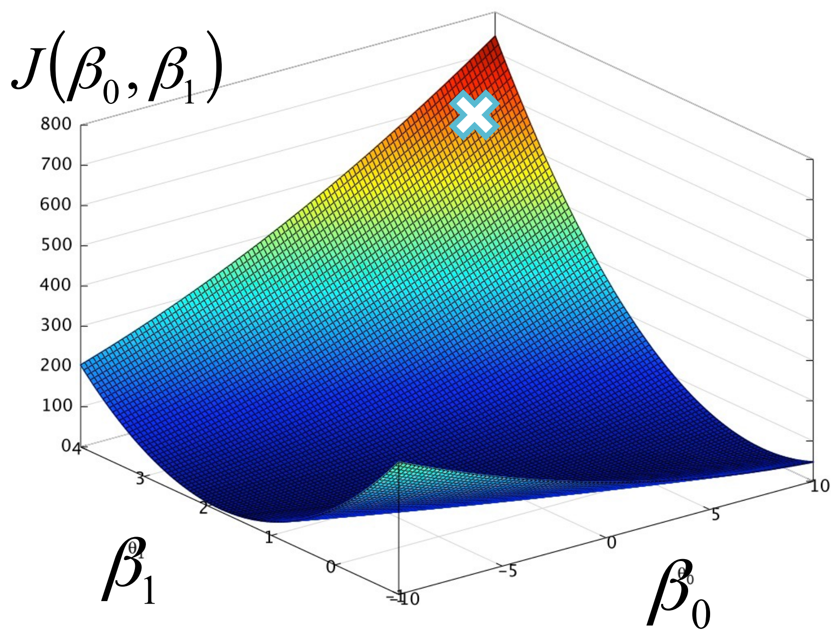
and compute its
<u>gradient vector</u> $\nabla J(\beta)$.



Gradient Descent with Linear Regression

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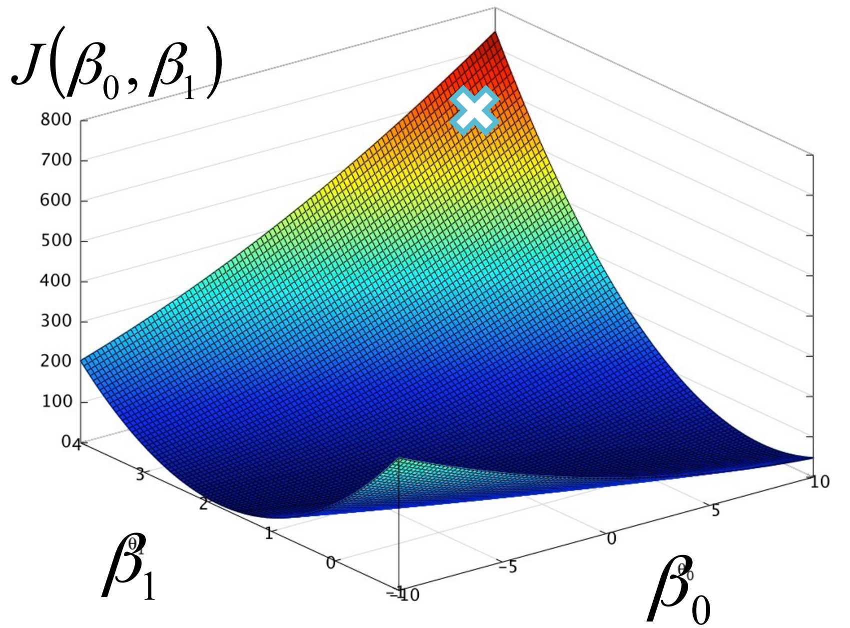
The gradient points
in the "direction of
maximum increase" of J.



Gradient Descent with Linear Regression

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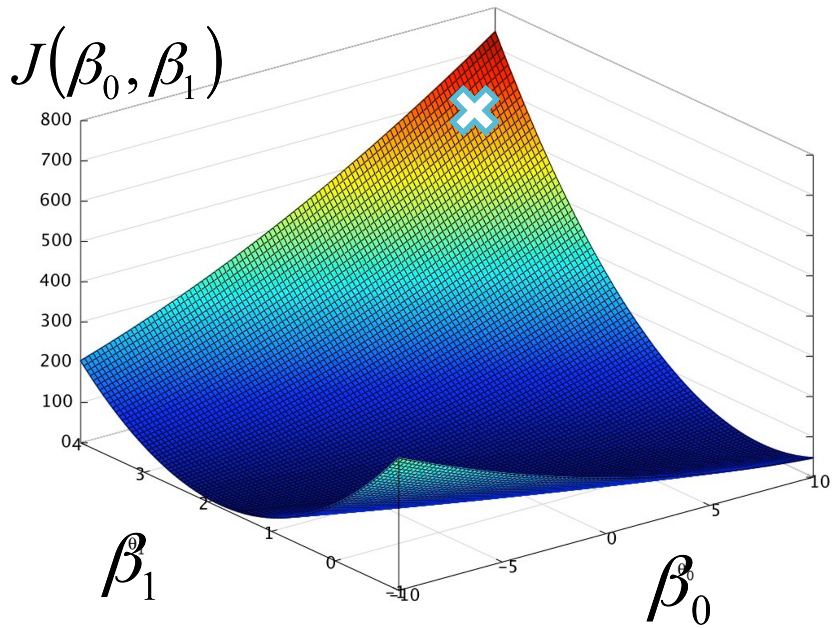
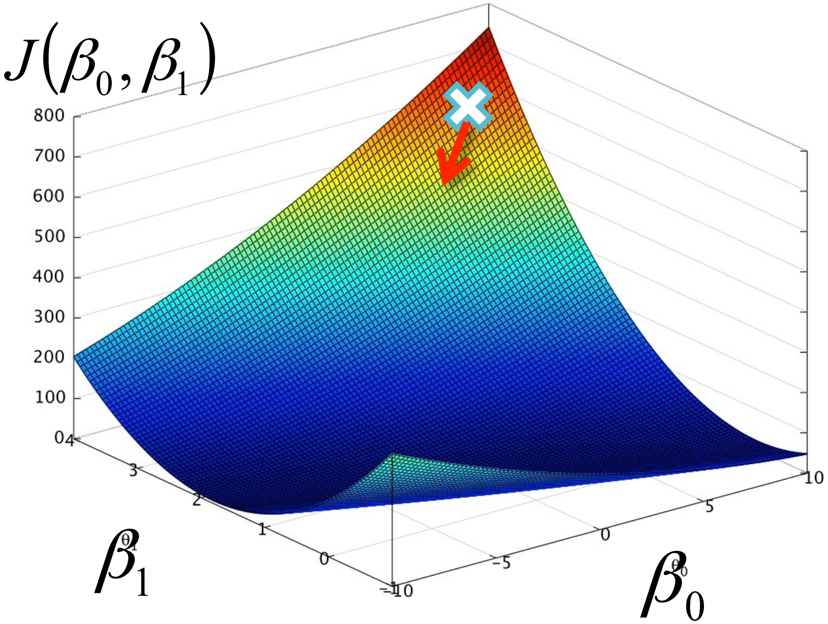


Gradient Descent with Linear Regression

$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left((\beta_0+\beta_1 x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2$$

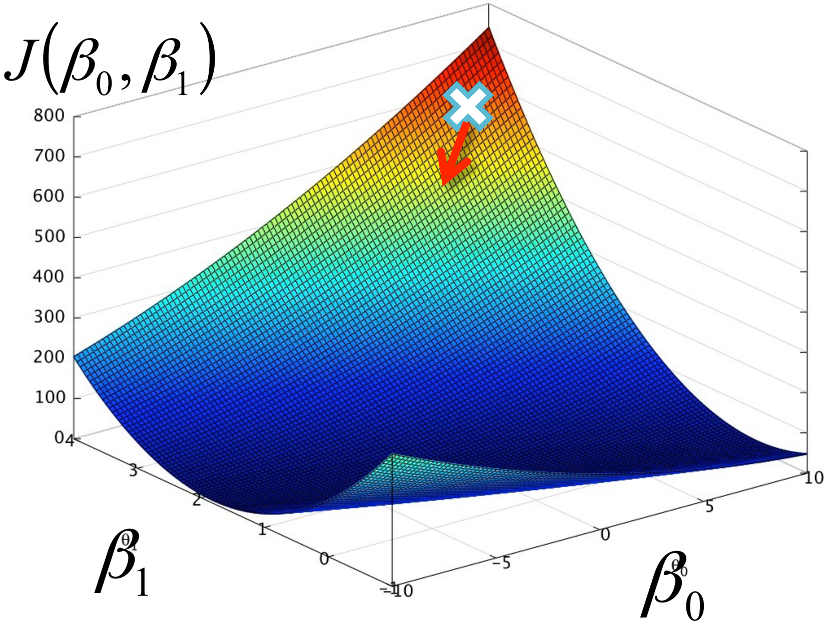
$$w_1 = w_0 - \alpha\nabla\sum_{i=1}^{m}\left((\beta_0+\beta_1 x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2$$



Gradient Descent with Linear Regression

$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left(\left(\beta_0+\beta_1 x_{obs}^{(i)}\right)-y_{obs}^{(i)}\right)^2$$

$$w_1=w_0-\alpha\left(\frac{\partial}{\partial\beta_0},\cdots,\frac{\partial}{\partial\beta_n}\right)\sum_{i=1}^{m}\left(\left(\beta_0+\beta_1 x_{obs}^{(i)}\right)-y_{obs}^{(i)}\right)^2$$



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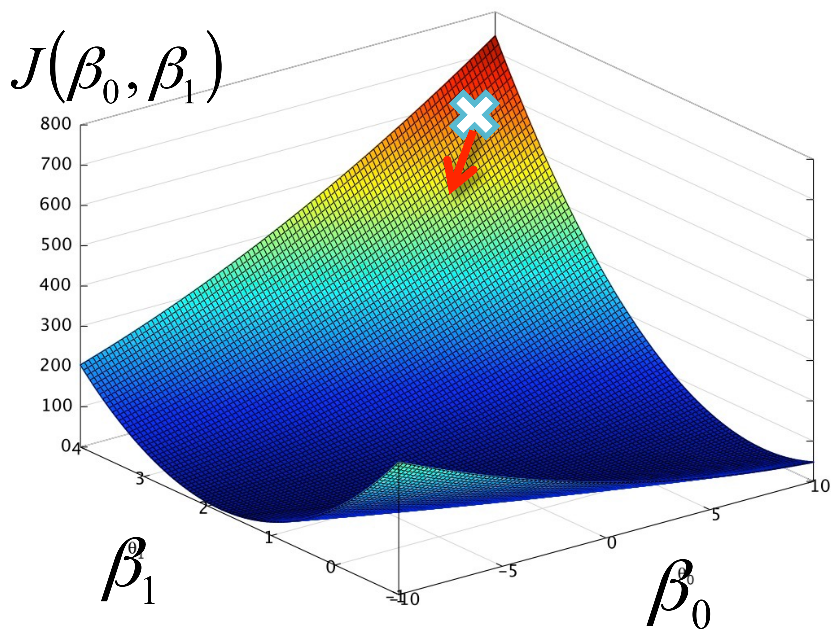
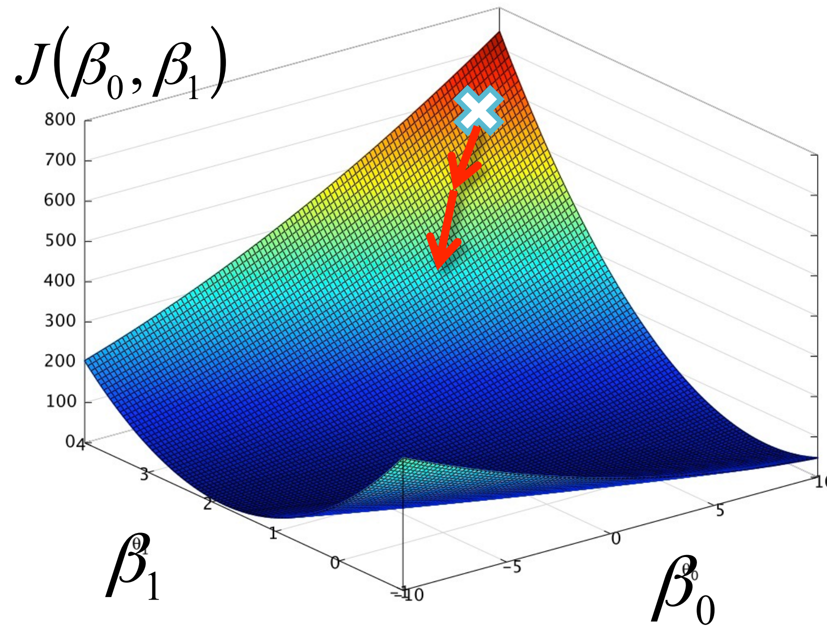


Gradient Descent with Linear Regression

$$J(\beta_0, \beta_1) = \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

$$w_2 = w_1 - \alpha \nabla \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$



Gradient Descent with Linear Regression

$$J\left(\beta_0, \beta_1\right) = \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

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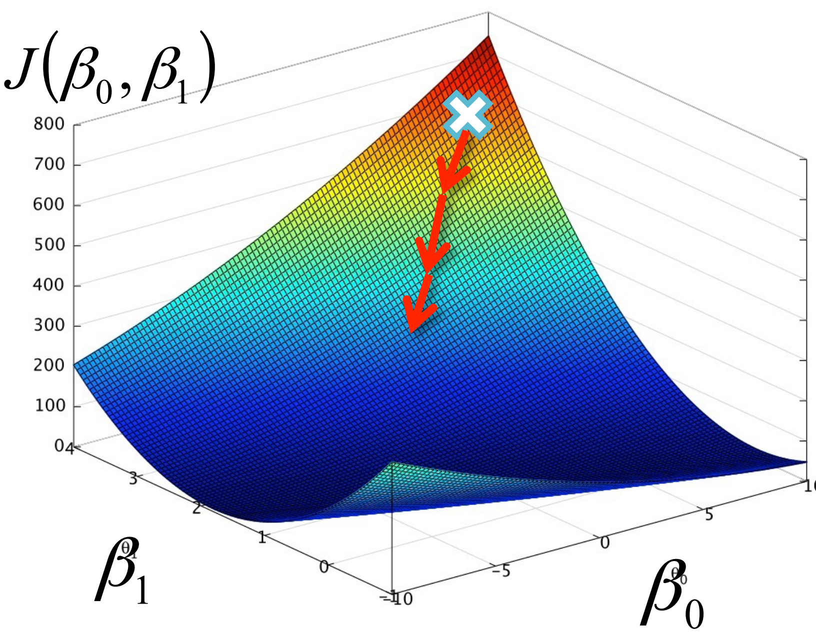

Gradient Descent with Linear Regression

$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left((\beta_0+\beta_1 x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2$$

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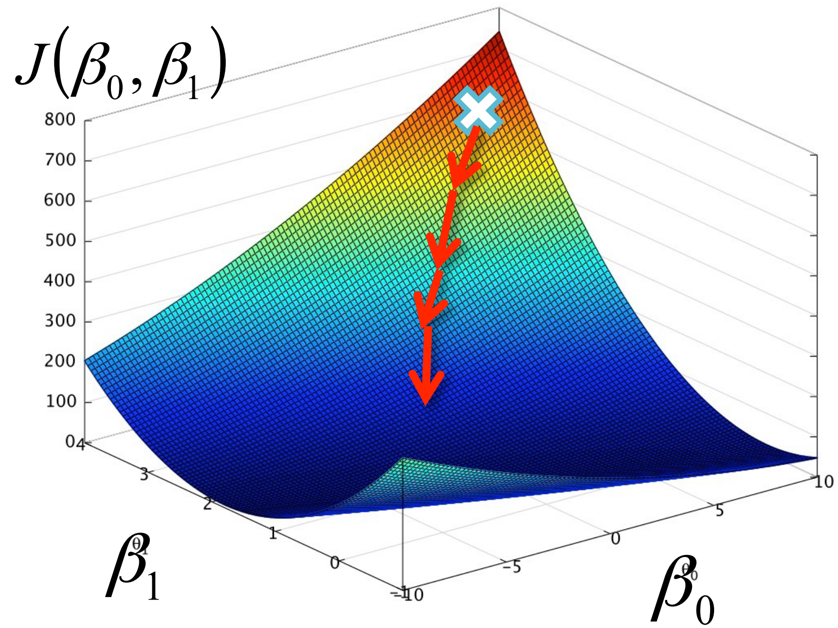
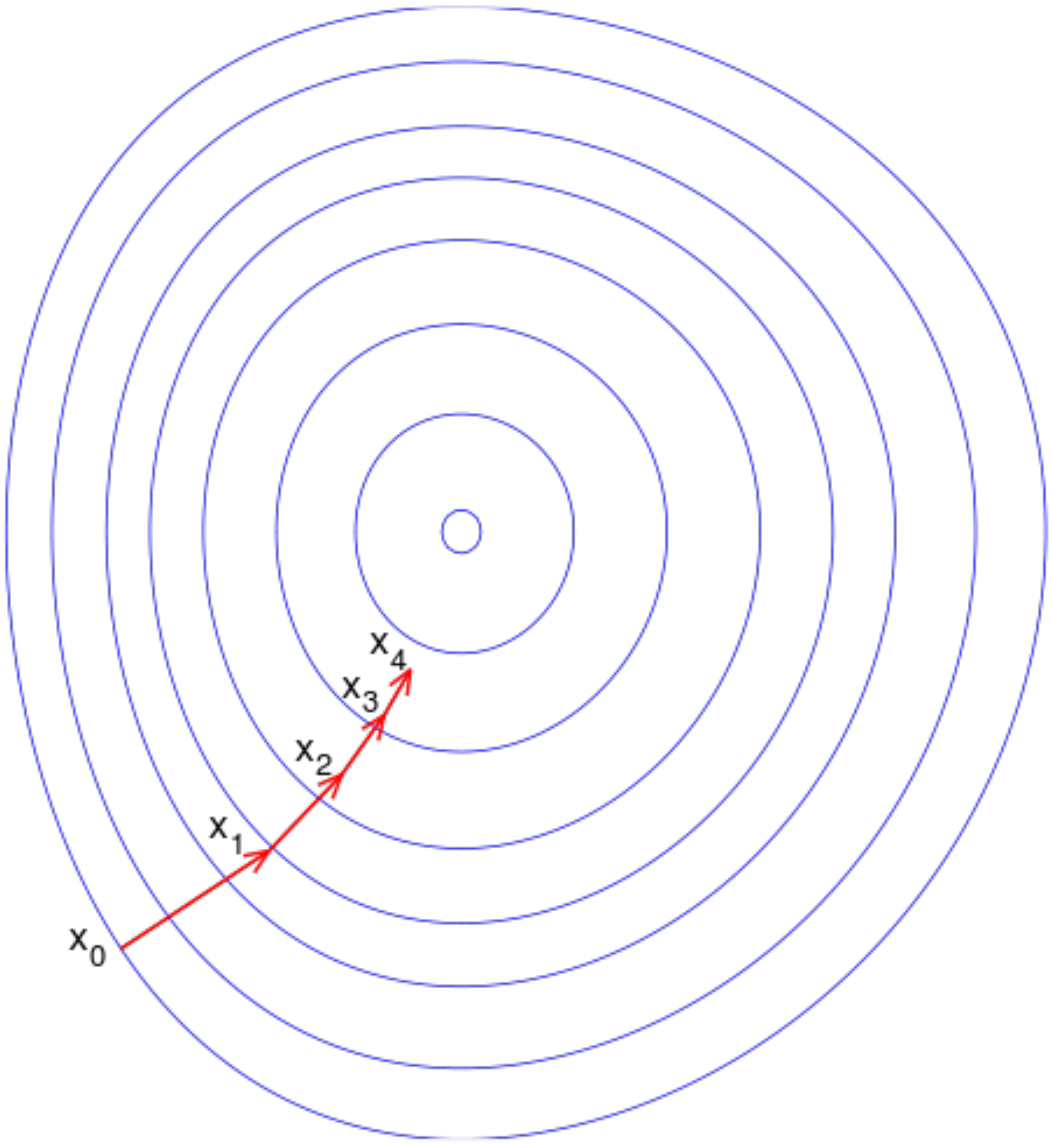
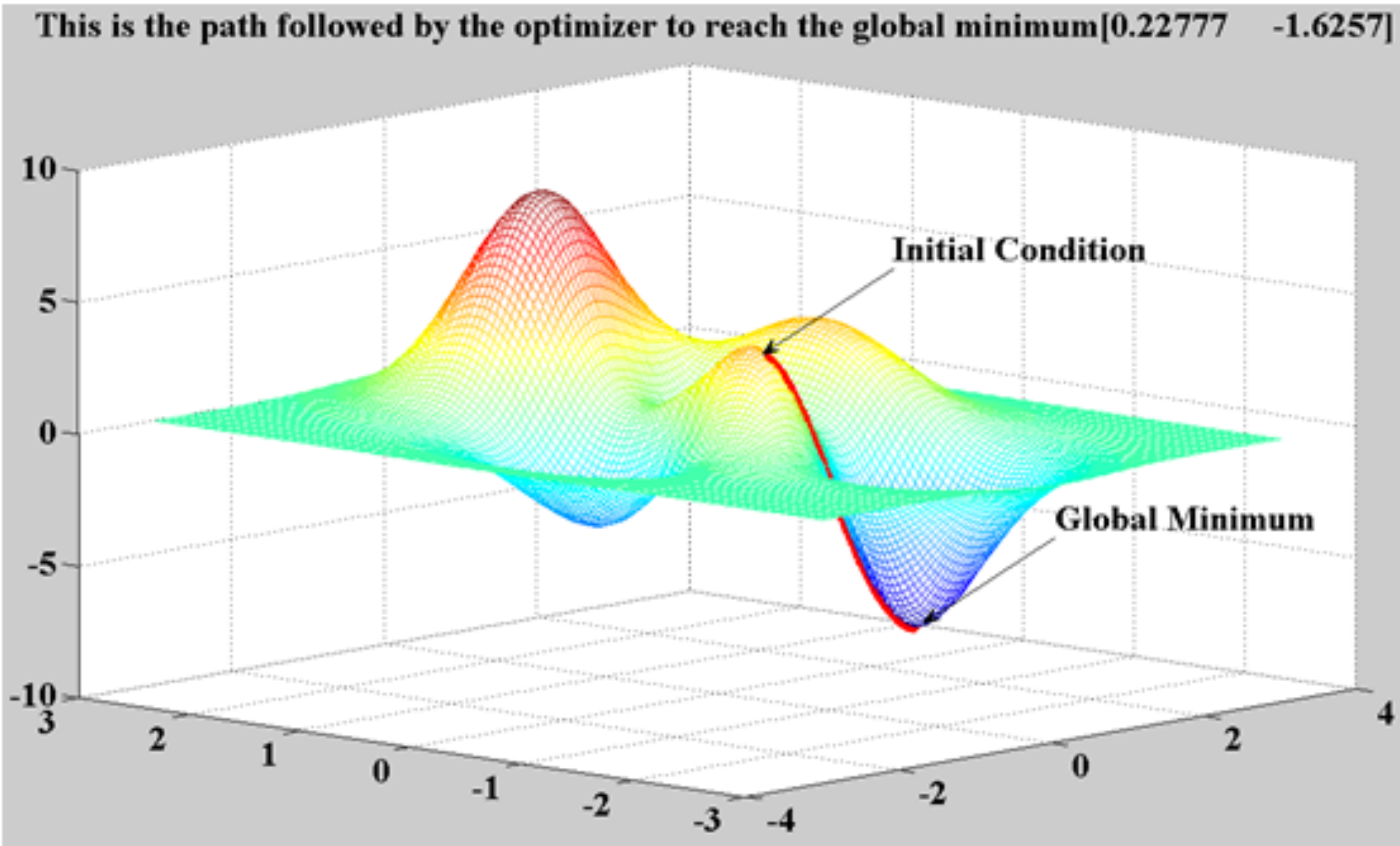


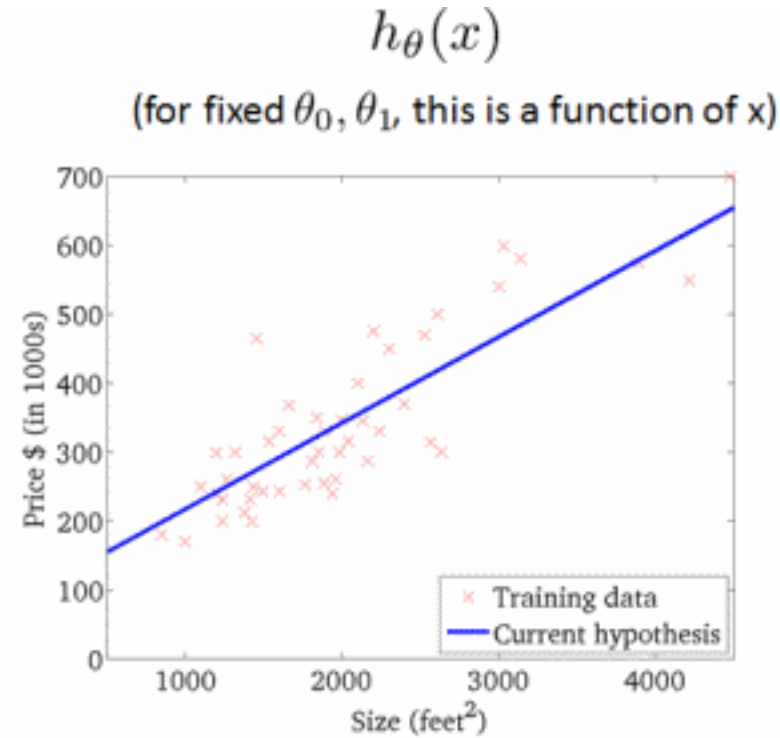
Gradient Descent

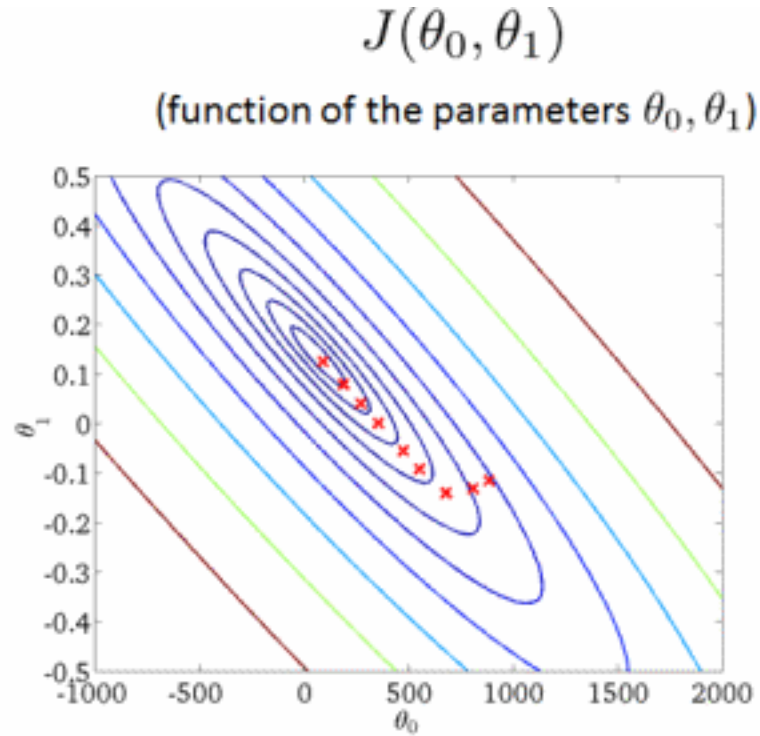


Gradient Descent



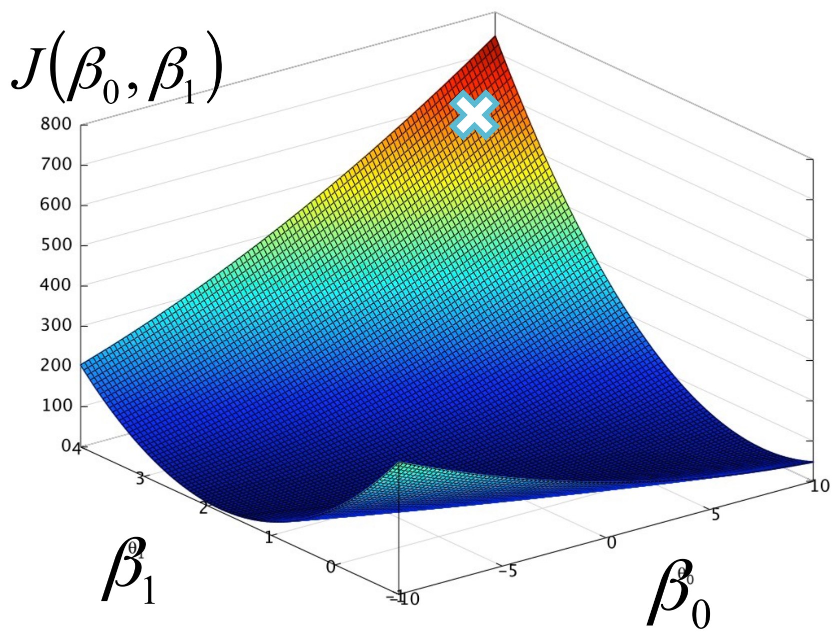
Gradient Descent





Stochastic Gradient Descent

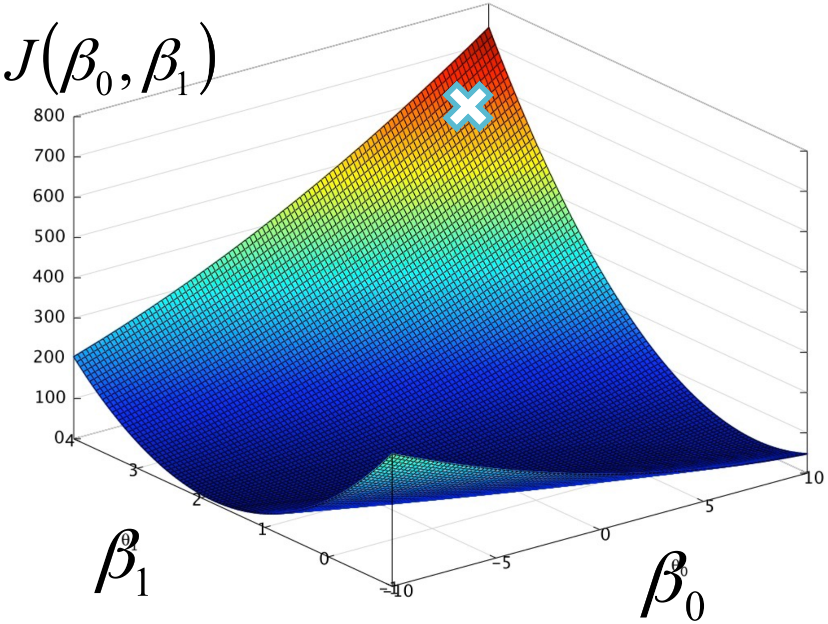
$$J\left(\beta_0, \beta_1\right) = \sum_{i=1}^{m}\left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2$$



Stochastic Gradient Descent

$$J(\beta_0, \beta_1) = \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Instead of using all points to find the gradient,
Only use a SINGLE point each time



Stochastic Gradient Descent

$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left(\left(\beta_0+\beta_1 x_{obs}^{(i)}\right)-y_{obs}^{(i)}\right)^2$$

Instead of using all points to find the gradient,
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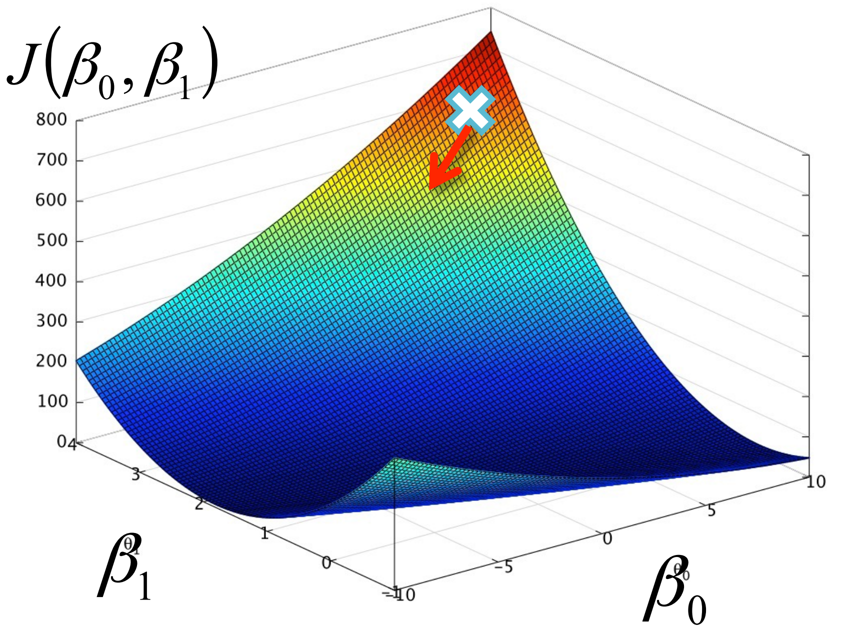
$$w_1=w_0-\alpha\nabla\left(\left(\beta_0+\beta_1 x_{obs}^{(0)}\right)-y_{obs}^{(0)}\right)^2$$



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$$J\left(\beta_0,\beta_1\right)=\sum_{i=1}^{m}\left(\left(\beta_0+\beta_1 x_{obs}^{(i)}\right)-y_{obs}^{(i)}\right)^2$$

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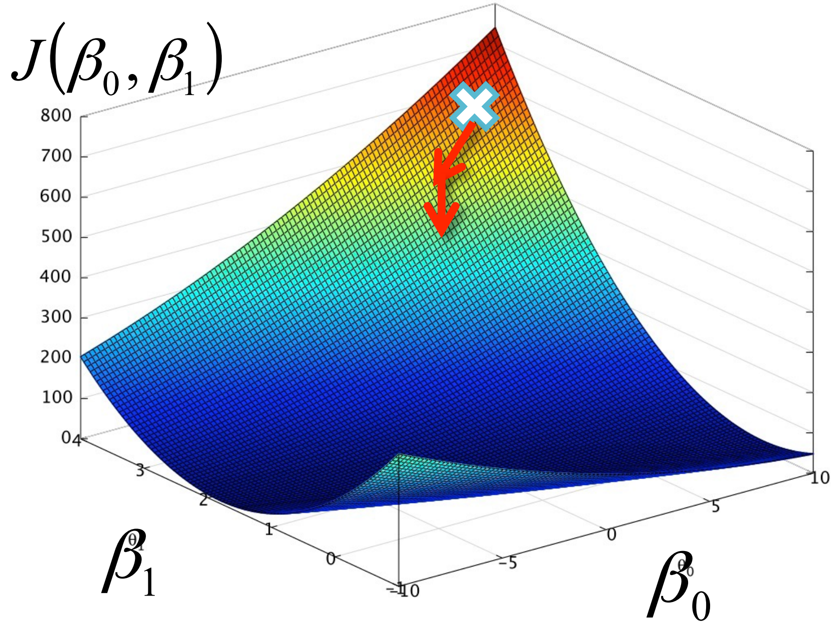
$$w_2=w_1-\alpha\nabla\left(\left(\beta_0+\beta_1 x_{obs}^{(1)}\right)-y_{obs}^{(1)}\right)^2$$



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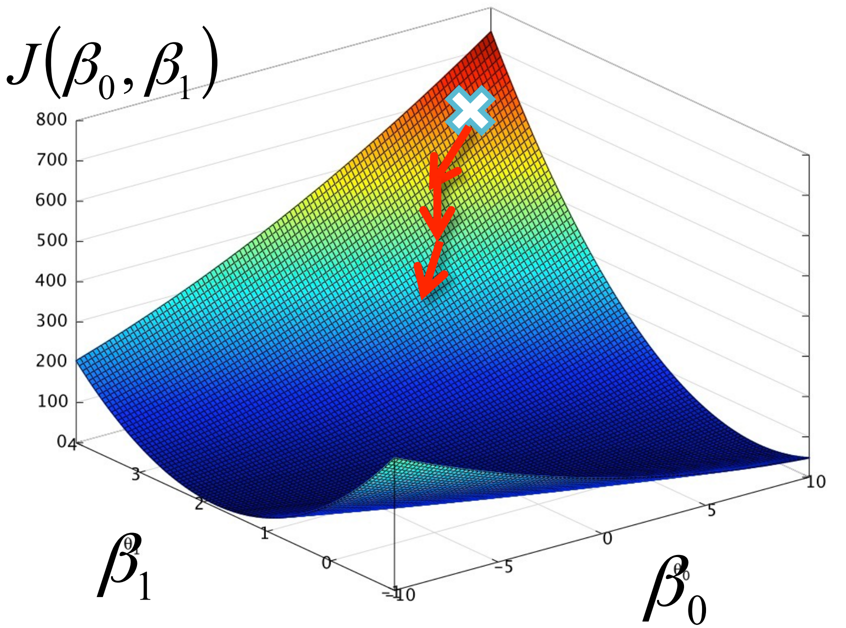
$$w_3 = w_2 - \alpha \nabla \left((\beta_0 + \beta_1 x_{obs}^{(2)}) - y_{obs}^{(2)} \right)^2$$



Stochastic Gradient Descent

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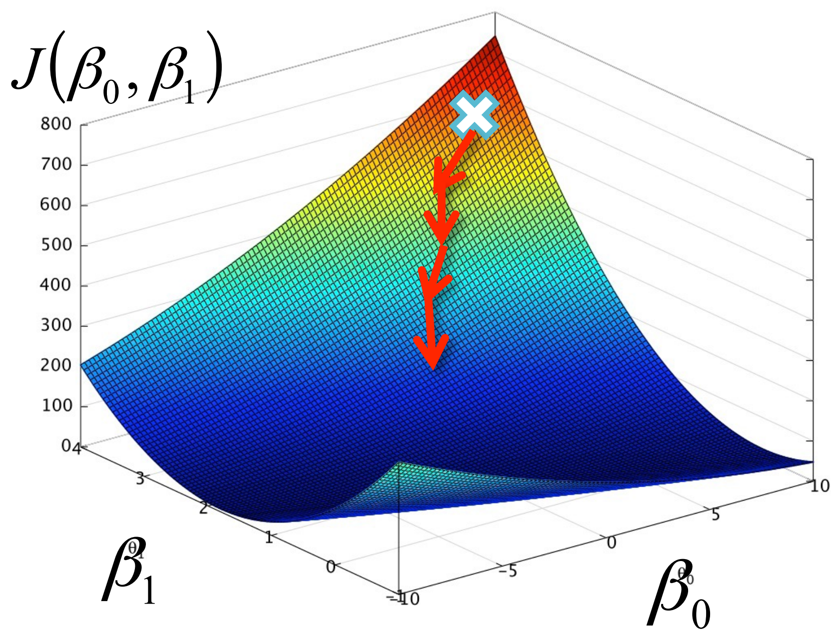
$$w_4=w_3-\alpha\nabla\left((\beta_0+\beta_1 x_{obs}^{(3)})-y_{obs}^{(3)}\right)^2$$



Faster

Derivative of single point at each step (instead of 100K)

Online Training

Only need to keep single point in memory

No need to store 100K rows, large data no problem

Covers Many Algorithms

Gradient Descent is the bottleneck for linear algorithms

Can do Linear Regression, Logistic Regression, SVMs

Some Implementations

Some Implementations

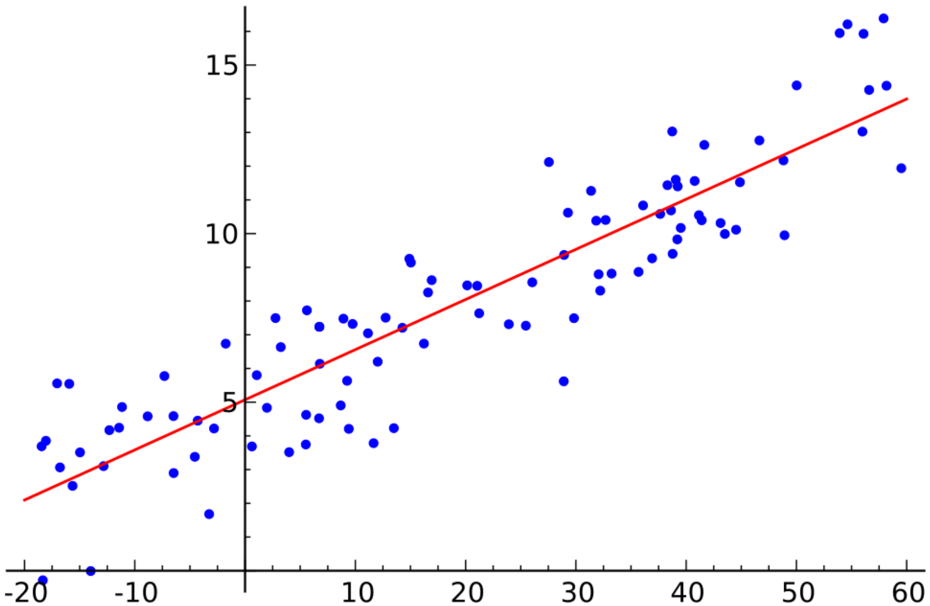
```
from sklearn.linear_model import SGDRegressor

from sklearn.linear_model import SGDClassifier
```

```python
from sklearn.linear_model import SGDRegressor
```

```
from sklearn.linear_model import SGDRegressor
```

```
SGDRegressor(loss='squared_loss')
```

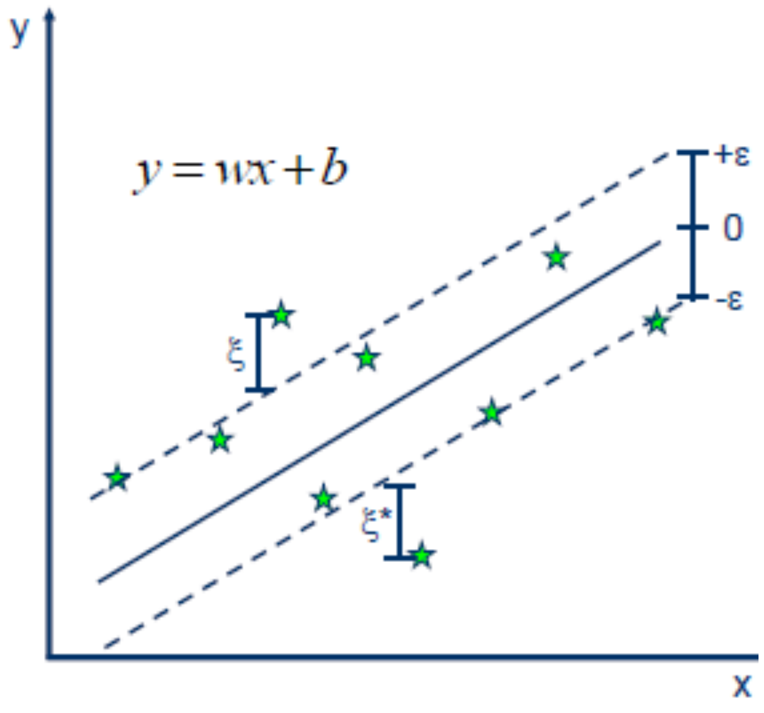


Sum of squared errors

squared loss ==
Linear Regression

```
from sklearn.linear_model import SGDRegressor
```

```
SGDRegressor(loss='epsilon_insensitive')
```



Best loss name ever

epsilon insensitive loss ==
SVM Regression

```
from sklearn.linear_model import SGDClassifier
```

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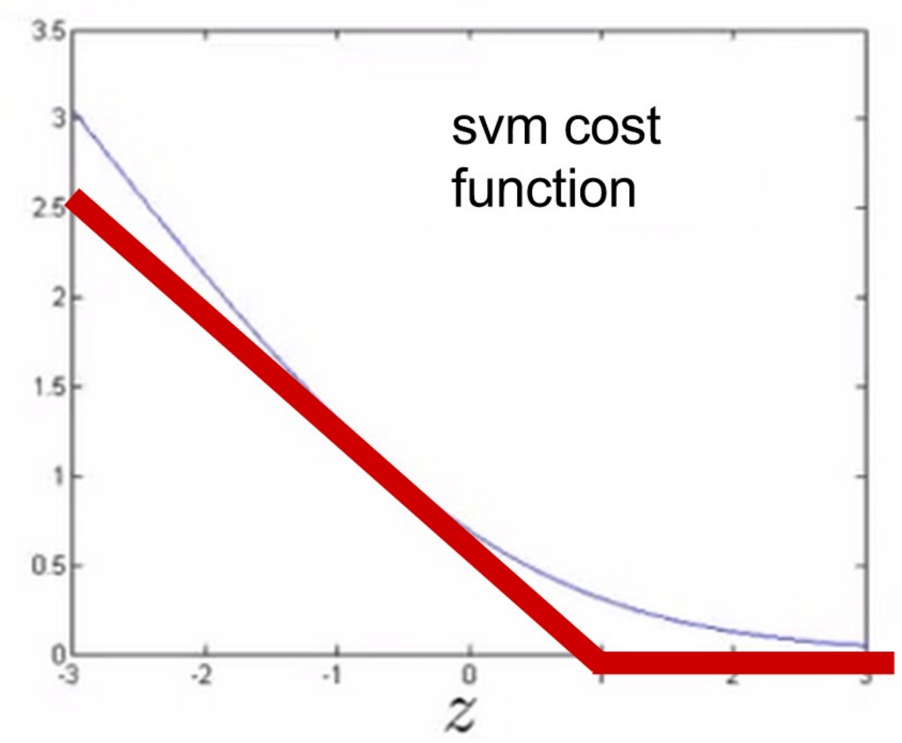
SGDClassifier(loss='hinge')
```

from sklearn.linear_model **import** SGDClassifier

SGDClassifier(loss='hinge')

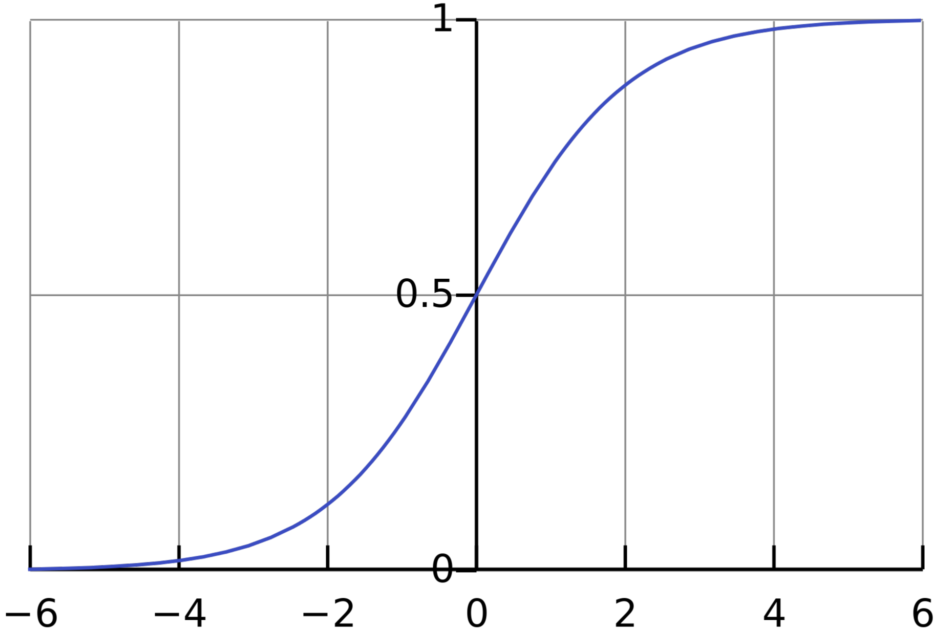


Looks like a hinge.

hinge loss == SVM

```
from sklearn.linear_model import SGDClassifier
```

```
SGDClassifier(loss='log')
```



This one's kind of clear

log loss == Logistic Regression

```
from sklearn.linear_model import SGDClassifier
```

```
SGDClassifier(alpha=0.0001,
              penalty='l2',
              l1_ratio=0.15)
```

Regularization parameters

Penalty values: 'l1', 'l2', 'elasticnet'