Regularization

OLS Regression Results

Dep. Variable:	DomesticTotalGross	**R-squared:**	0.286
Model:	OLS	**Adj. R-squared:**	0.278
Method:	Least Squares	**F-statistic:**	34.82
Date:	Sun, 14 Sep 2014	**Prob (F-statistic):**	6.80e-08
Time:	21:59:46	**Log-Likelihood:**	-1738.1
No. Observations:	89	**AIC:**	3480.
Df Residuals:	87	**BIC:**	3485.
Df Model:	1		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
Budget	0.7846	0.133	5.901	0.000	0.520 1.049
Ones	4.44e+07	1.27e+07	3.504	0.001	1.92e+07 6.96e+07

Omnibus:	39.749	**Durbin-Watson:**	0.674
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	99.441
Skew:	1.587	**Prob(JB):**	2.55e-22
Kurtosis:	7.091	**Cond. No.**	1.54e+08

$$AIC = 2k - 2\ln(L)$$

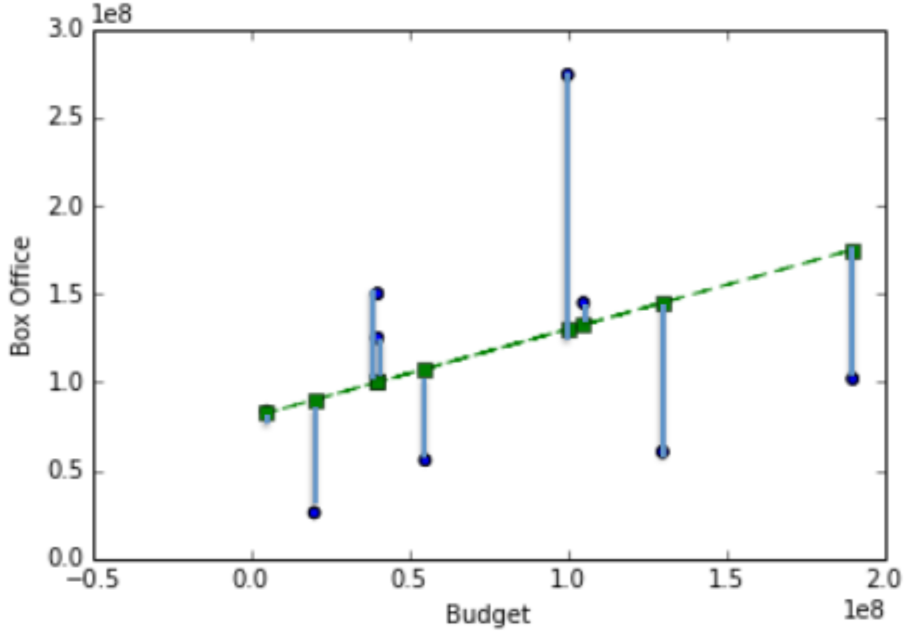
parameters

Log likelihood

While awarding goodness of fit, penalize model complexity

While awarding goodness of fit, penalize model complexity

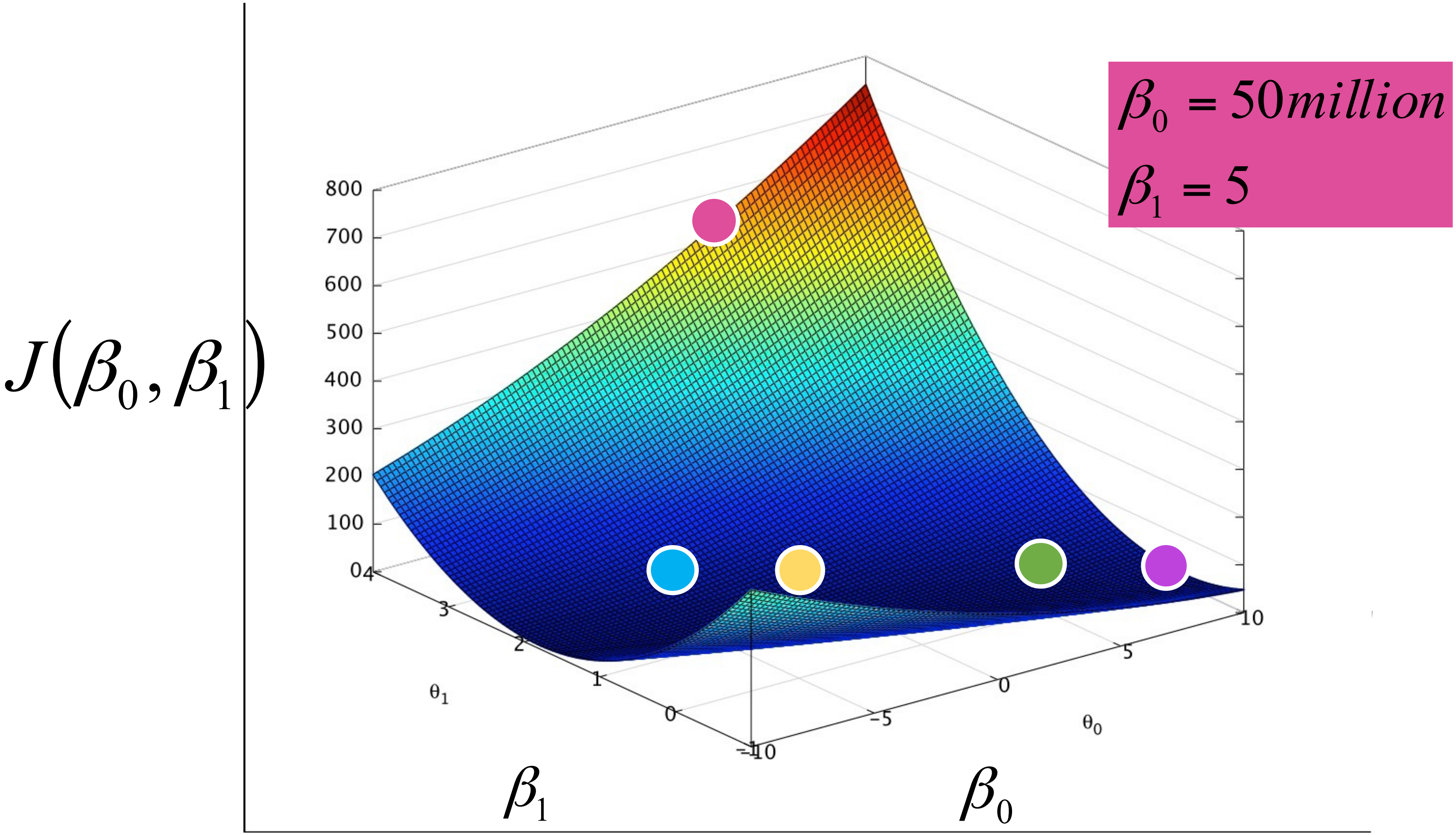
Why not do that while we are fitting?



Cost function

Takes a model (specific parameter values), returns a score

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$



$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120 million$
$\beta_1 = 0.1$

$\beta_0 = 30 million$
$\beta_1 = 2$

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Cost function

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(\left(\beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2$$

Cost function

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Cost function

Add a penalty for the size of each parameter!

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

L2 Regularization

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Ridge Regression

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Effect of λ?

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta (x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta (x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

zero

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

very small

$$J(\beta_0, \beta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda\sum_{j=1}^{k}\beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

“just right”

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

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VERY LARGE

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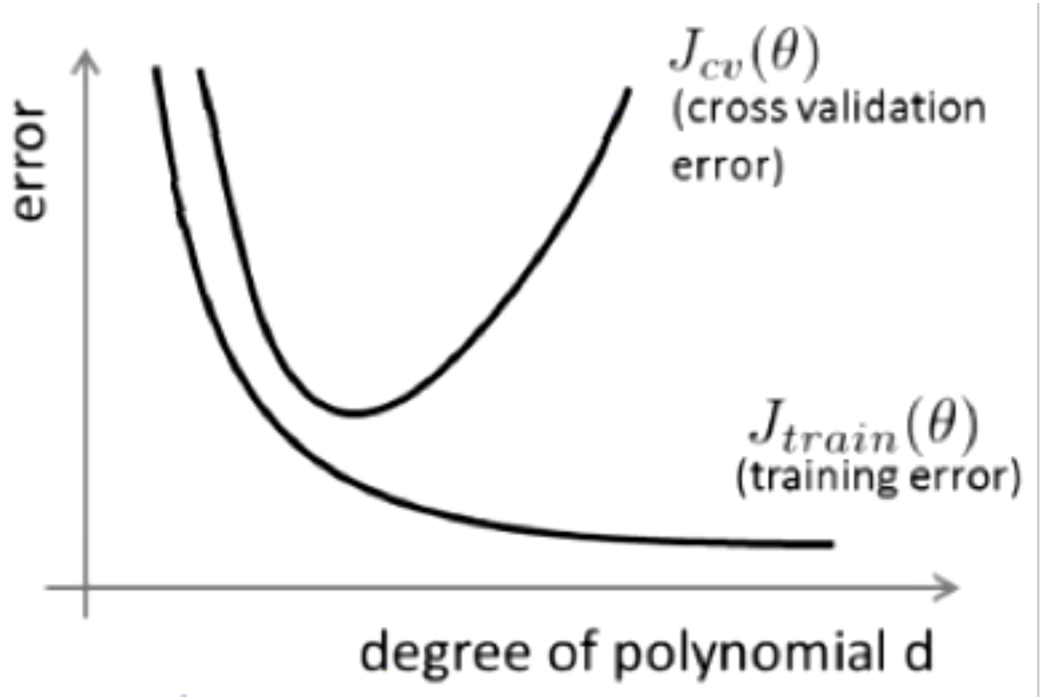
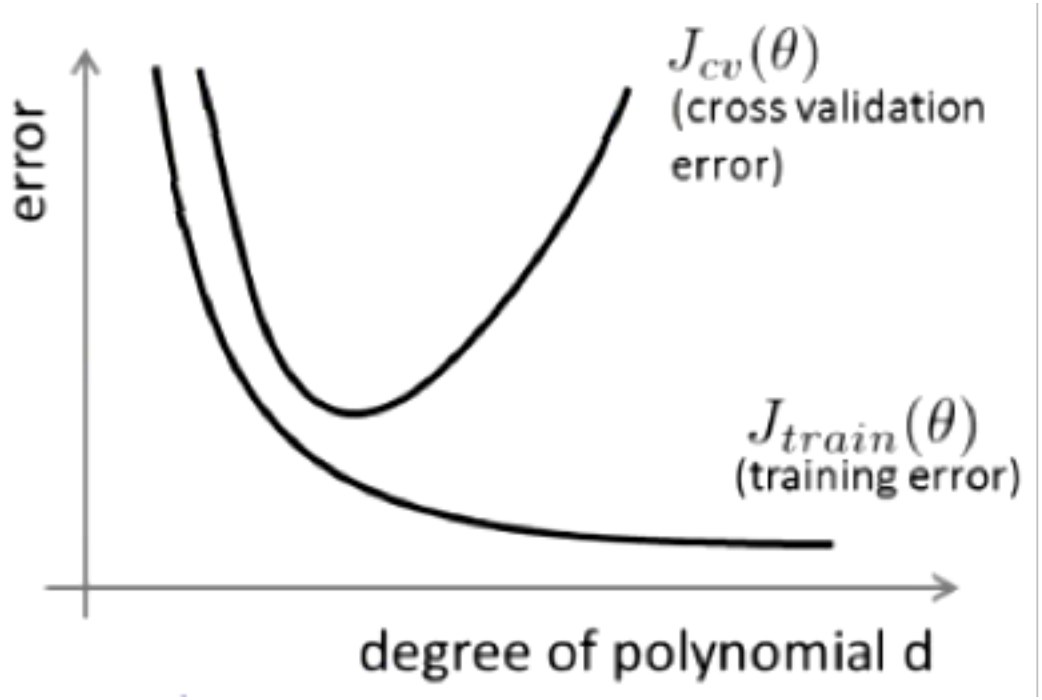
≈0 ≈0 ≈0 ≈0 ≈0

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

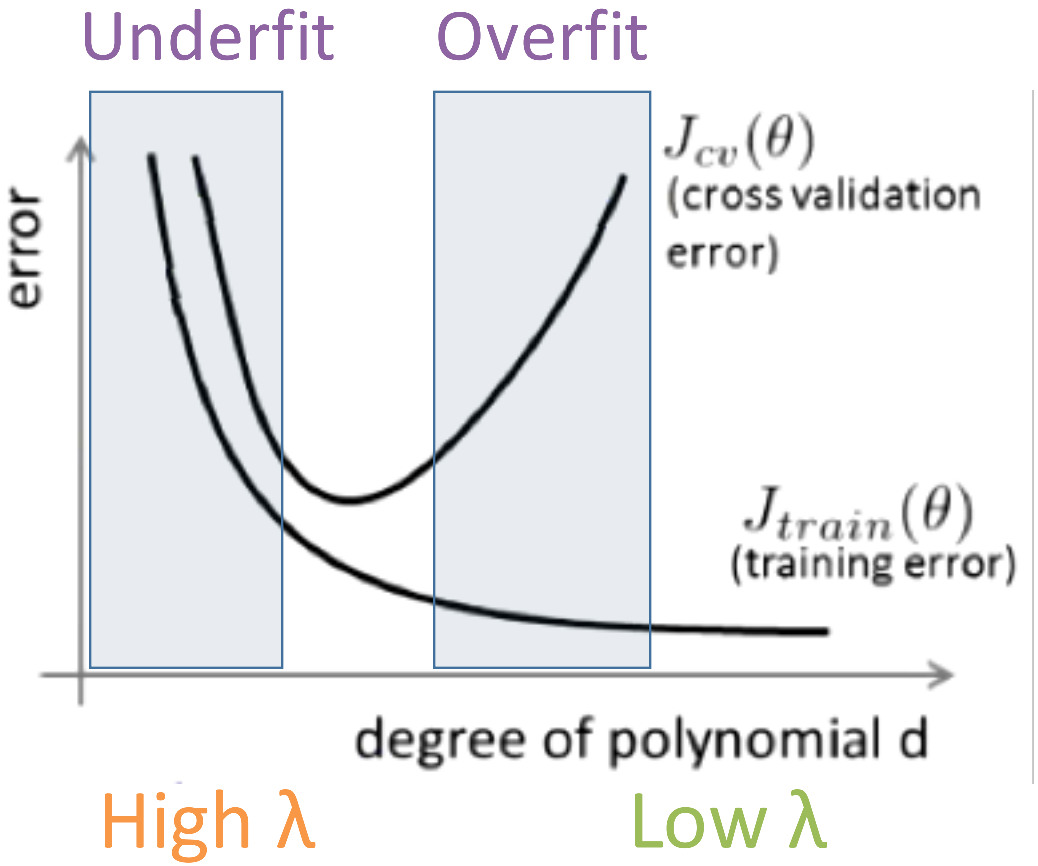
Error vs. regularization λ



Error vs. regularization λ



High λ Low λ



Underfit
Overfit
error
$J_{cv}(\theta)$
(cross validation error)
$J_{train}(\theta)$
(training error)
degree of polynomial d
High λ
Low λ

Ridge Regularization (L2)

$$J\left(\beta_0,\beta_1\right)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_{\beta}(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\beta_j^2$$

Ridge Regularization (L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Lasso Regularization (L1)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \left| \beta_j \right|$$

Ridge Regularization (L2)

$$J\left(\beta_0,\beta_1\right)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\beta_j^2$$

Lasso Regularization (L1)

$$J\left(\beta_0,\beta_1\right)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\left|\beta_j\right|$$

Elastic Net (L1 + L2)

$$J\left(\beta_0,\beta_1\right)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda_1\sum_{j=1}^{k}\left|\beta_j\right|+\lambda_2\sum_{j=1}^{k}\beta_j^2$$

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X,Y)
```

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Ridge Regularization:

```
from sklearn.linear_model import Ridge
model = Ridge(1.0)
model.fit(X, y)
```

λ (sklearn Calls It alpha)

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Lasso:

```
from sklearn.linear_model import Lasso
model = Lasso(1.0)
model.fit(X, y)
```

λ (sklearn Calls It alpha)

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Elastic Net:

```
from sklearn.linear_model import ElasticNet
model = ElasticNet(1.0, l1_ratio = 0.5)
model.fit(X, y)
```

total weight for the full penalty term

ratio of l1/l2 penalty

My model is not awesome enough.

What do I do?

Try these and check test error
(and AIC,BIC,etc.) again:

Use a smaller set of features

Regularization: Increase/decrease λ

Try adding polynomials

Check functional forms for each feature

Try including other features

Use more data (bigger training set)