

High Frequency parameters

Formulation of S parameters

Introduction

At **high frequencies** (RF & microwave), signals do not behave like simple voltages and currents.

Instead, they travel as **electromagnetic waves** through transmission lines.

This introduces challenges such as:

- Reflection due to impedance mismatch
- Standing waves
- Losses in connectors and cables
- Phase delays
- Electromagnetic interference (EMI)

Traditional circuit parameters like **Z, Y, and H** fail at GHz frequencies because it is difficult to measure current/voltage directly without disturbing the signal.

To solve this, engineers use **Scattering Parameters (S-parameters)** — the universal language of high-frequency systems like:

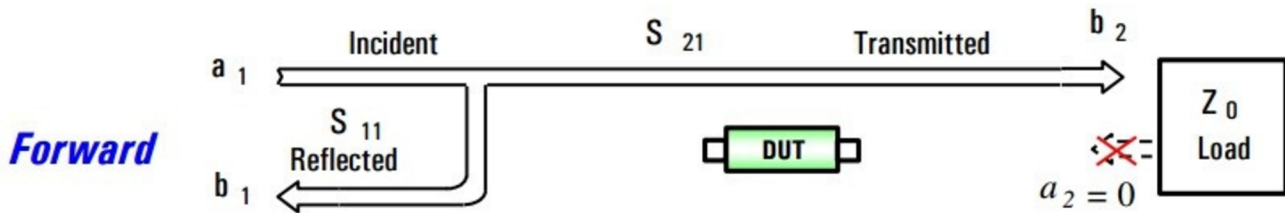
- 5G antennas
- Satellite receivers
- Radar modules
- Wi-Fi routers
- Microwave circuits

S-parameters describe how much signal is **reflected**, **transmitted**, or **lost** when waves travel through a device.

S-parameter Measurement (2 port)

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

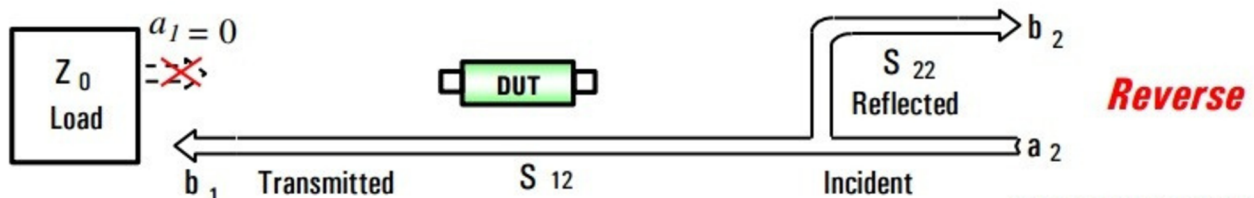


$$S_{11} = \frac{\text{Reflected}}{\text{Incident}} = \frac{b_1}{a_1} \Big|_{a_2 = 0}$$

$$S_{21} = \frac{\text{Transmitted}}{\text{Incident}} = \frac{b_2}{a_1} \Big|_{a_2 = 0}$$

$$S_{22} = \frac{\text{Reflected}}{\text{Incident}} = \frac{b_2}{a_2} \Big|_{a_1 = 0}$$

$$S_{12} = \frac{\text{Transmitted}}{\text{Incident}} = \frac{b_1}{a_2} \Big|_{a_1 = 0}$$



High-Frequency Network Overview

A typical RF/microwave system consists of:

- Antennas
- Power amplifiers
- Low-noise amplifiers (LNAs)
- Filters & duplexers
- Mixers
- Transmission lines (coax/microstrip/waveguide)

All these components are modeled using **S-parameters**, because they allow engineers to predict:

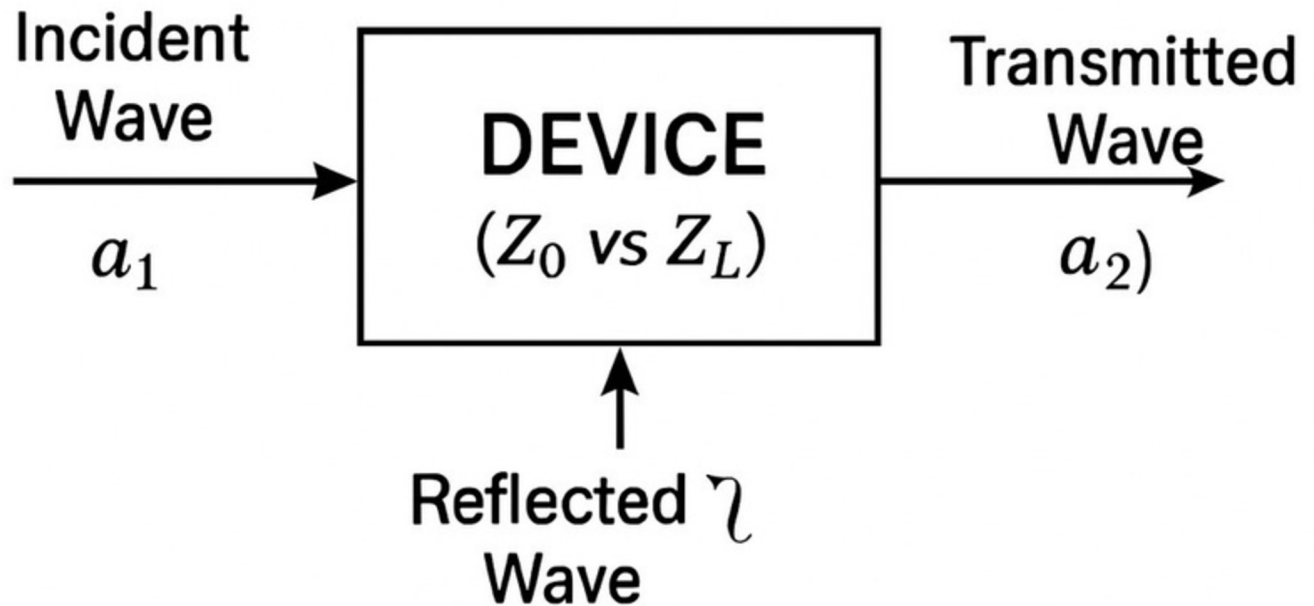
- Gain
- Loss
- Isolation
- Return loss
- Stability
- Bandwidth
- Noise performance

Even a small mismatch in these components can cause major performance drops at high frequencies.

Reflection Coefficient (Γ)

The **reflection coefficient** (Γ) measures how much of the wave reflects back due to impedance mismatch.

Reflection Coefficient (Γ)



Reflected Wave

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Real-Time Use:

In RF circuits, Γ determines:

- ✓ How well the antenna matches to $50\ \Omega$
- ✓ Whether power returns back to the amplifier
- ✓ How stable communication remains at high frequencies

A large $\Gamma \rightarrow$ high reflection \rightarrow signal distortion.

VSWR (Voltage Standing Wave Ratio)

VSWR expresses how severe the standing waves are on the transmission line.

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Good Match: VSWR $\approx 1.1 : 1$

Bad Match : VSWR $> 3 : 1$

Why VSWR Matters

At high frequencies, a poor VSWR causes:

- Signal drop
- Amplifier overheating
- Reduced communication range
- Data errors

Low VSWR ensures **smooth wave propagation**.

Return Loss & Insertion Loss

Return Loss (RL)

Shows how much signal is reflected back.

$$RL = -20 \log |\Gamma| \quad RL = -20 \log |\Gamma|$$

High RL \rightarrow low reflection \rightarrow good matching.

Insertion Loss (IL)

Shows how much signal is lost while passing through a device.

$$IL = -20 \log \left| \frac{V_{out}}{V_{in}} \right| \quad IL = -20 \log \left| \frac{V_{out}}{V_{in}} \right|$$

At GHz frequencies:

- Long microstrip lines
- SMA connectors
- Filters and duplexers

must all have **low IL** to maintain signal strength.

Formulation of the S-Parameter Matrix

In high-frequency circuits, signals are treated as **waves**, not currents/voltages.

Let:

- a_1, a_2 : incident waves
- b_1, b_2 : reflected waves

S-parameter matrix for a 2-port network:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Meaning of Each S-Parameter

Parameter	Meaning	Application
S₁₁	Input reflection	Matching of input port
S₂₁	Forward transmission	Gain of amplifier / filter response
S₁₂	Reverse transmission	Isolation, feedback stability
S₂₂	Output reflection	Output matching

Power Delivered to the Load

The power delivered to a load is governed by incident power and reflection:

$$P_{\text{delivered}} = |a_1|^2 (1 - |S_{11}|^2)$$

More reflection → less power delivered.

Application Example:

In a high-frequency amplifier:

- If S_{11} is high (bad match) → most power reflects → amplifier saturates or becomes unstable
- If S_{21} is high (good gain) → strong output signal

That's why S-parameters determine amplifier quality.

Real-Time Engineering Example

Imagine you are designing a **5G wireless module**:
The chain:

Antenna → LNA → Bandpass Filter → Mixer → Amplifier → PCB traces

Transmission line applications:

Component	High-Frequency Parameter Use
Antenna	Match S11 for better radiation
LNA	Use S21 to maximize gain with low noise
Filter	IL, S21 for band selection
Mixer	S12 to minimize leakage
PCB microstrip	VSWR, RL to minimize signal loss

If any stage mismatches:

- Internet speed drops
- Signal becomes noisy
- System overheats
- Communication range reduces
-

S-parameters prevent these issues.

Conclusion

High-frequency systems depend heavily on **transmission line theory** and **S-parameters** to ensure smooth, stable, and efficient signal flow.

Parameters like **Γ** , **VSWR**, **Return Loss**, **Insertion Loss**, and **S-matrix values** determine:

- Signal quality
- Power transfer
- Device matching
- System stability
- From antennas to high-speed amplifiers and filters, every RF component is designed using S-parameters. They are the backbone of all modern wireless and microwave technologies.

Every time you use 5G, Wi-Fi, or radar — S-parameters are silently working behind the scenes, ensuring your signal stays strong and clean.

