- Convex set:  $t\mathbf{x}_0 + (1-t)\mathbf{x}_1 \in A \quad \forall \ \mathbf{x}_0, \mathbf{x}_1 \in A, t \in [0,1]$
- Superior set for  $\mathbf{x}_0$ :  $\{\mathbf{x} \mid \mathbf{x} \in D \land f(\mathbf{x}) \geq f(\mathbf{x}_0)\}$
- Inferior set for  $\mathbf{x}_0$ :  $\{\mathbf{x} \mid \mathbf{x} \in D \land f(\mathbf{x}) \leq f(\mathbf{x}_0)\}$
- Quasiconcave function:  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \forall t \in [0,1]$
- Strictly quasiconcave function:  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$
- Quasiconvex function:  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\}\ \forall t \in [0, 1]$
- Strictly quasiconcave function:  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\}\ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$
- Concave shape: A line connecting two points is underneath
- Strictly concave shape: A line connecting two points is strictly underneath
- Convex shape: A line connecting two points is above
- Strictly convex shape: A line connecting two points is strictly above
- Homothetic function: level sets have the same slope along rays from the origin

## Cobb-Douglas Utility Function: $u(x,y) = x^{\alpha} y^{\beta}$

1. Superior set is always convex?

2. Inferior set is always convex?

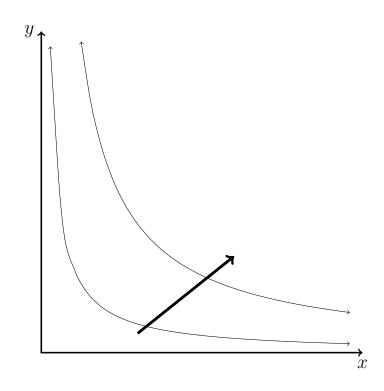
3. u is a quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0,1]$ 

4. u is a strictly quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$ 

- 5. u is a quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0, 1]$
- 6. u is a strictly quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0, 1)$

The level sets (indfference curves) have a ...

- 7. ... concave shape?
- 8. ... strictly concave shape?
- 9. ... convex shape?
- 10. ... strictly convex shape?
- 11. *u* is homothetic?

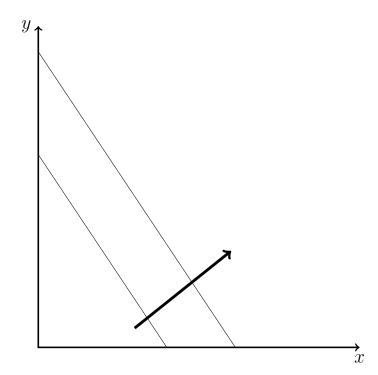


If both goods are "goods", then the indifference curves are downward sloping and increasing to the upper right.

### Perfect Substitutes Utility Function: u(x, y) = ax + by

- 1. Superior set is always convex?
- 2. Inferior set is always convex?
- 3. u is a quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0,1]$
- 4. u is a strictly quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$
- 5. u is a quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0, 1]$
- 6. u is a strictly quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0, 1)$

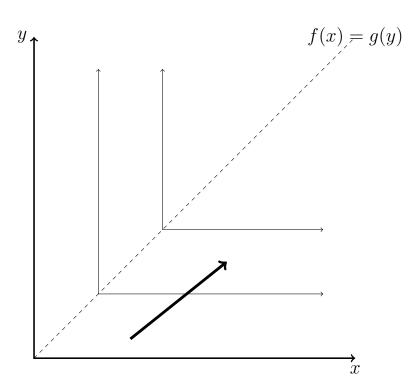
- 7. ... concave shape?
- 8. ... strictly concave shape?
- 9. ...convex shape?
- 10. ... strictly convex shape?
- 11. u is homothetic?



## **Perfect Complements Utility Function:** $u(x,y) = \min\{f(x), g(y)\}$

- 1. Superior set is always convex?
- 2. Inferior set is always convex?
- 3. u is a quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0,1]$
- 4. u is a strictly quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$
- 5. u is a quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0, 1]$
- 6. u is a strictly quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0, 1)$

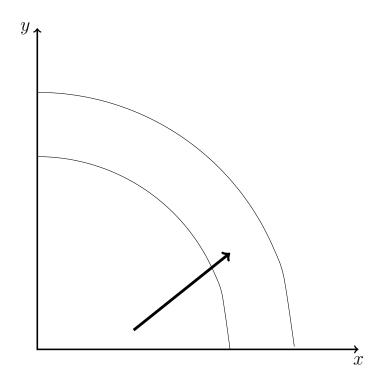
- 7. ... concave shape?
- 8. ... strictly concave shape?
- 9. ... convex shape?
- 10. ... strictly convex shape?
- 11. u is homothetic?



# Prefers Extremes Utility Function: $u(x,y) = x^2 + y^2$

- 1. Superior set is always convex?
- 2. Inferior set is always convex?
- 3. u is a quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0,1]$
- 4. u is a strictly quasiconcave function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min \{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$
- 5. u is a quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0, 1]$
- 6. u is a strictly quasiconvex function?  $f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0, 1)$

- 7. ... concave shape?
- 8. ... strictly concave shape?
- 9. ... convex shape?
- 10. ... strictly convex shape?
- 11. u is homothetic?



### Quasilinear Utility Function: $u(x, y) = x + \ln(y)$

- 1. Superior set is always convex?
- 2. Inferior set is always convex?
- 3. u is a quasiconcave function?

$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \ge \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \quad \forall t \in [0, 1]$$

4. *u* is a strictly quasiconcave function?

$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) > \min\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0,1)$$

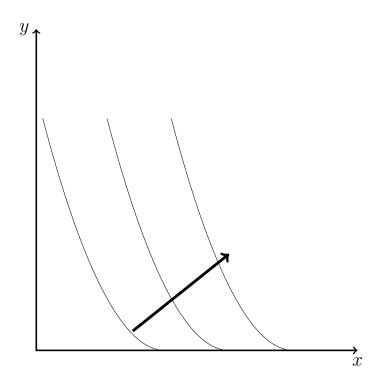
5. u is a quasiconvex function?

$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) \le \max\{f(\mathbf{x}_0), f(\mathbf{x}_1)\} \ \forall t \in [0, 1]$$

6. *u* is a strictly quasiconvex function?

$$f(t\mathbf{x}_0 + (1-t)\mathbf{x}_1) < \max\{f(\mathbf{x}_0), f(\mathbf{x}_0)\} \ \forall \mathbf{x}_0 \neq \mathbf{x}_1, \ t \in (0, 1)$$

- 7. ... concave shape?
- 8. ... strictly concave shape?
- 9. ... convex shape?
- 10. ... strictly convex shape?
- 11. u is homothetic?



### MRS Test

• 
$$u(x,y) = x^{\alpha} y^{\beta}$$

$$MRS(x,y) = \frac{\partial u/\partial x}{\partial u/\partial y}$$

$$\bullet \ u(x,y) = ax + by$$

What happens to the MRS as  $x \uparrow$ ?

$$\frac{\partial MRS}{\partial x} =$$

• 
$$u(x,y) = \min\{f(x), g(y)\}$$

What happens to the MRS as  $y \downarrow$ ?

$$\frac{-\partial MRS}{\partial y} =$$

$$\bullet \ u(x,y) = x^2 + y^2$$

• 
$$u(x,y) = x + \ln(y)$$