Example: Use Kuhn-Tucker to solve, where $p_x, p_y, m > 0$:

$$\max_{x,y} xy \quad s.t. \quad m \ge p_x x + p_y y \quad x \ge 0 \quad y \ge 0$$

Lagrangian: =
$$X y + \lambda [M - Px \cdot x - Pyy] + \mu_x(x-0)$$

+ My (y-0)

(i) FOCs:
$$Z_x = y - \lambda p_x + \mu_x = 0$$

 $Z_y = x - \lambda p_y + \mu_y = 0$

(ii) Constraints:

$$m \ge p_x \times + p_y \cdot y$$

 $\times 50$

(iii) Complementary slackness conditions:

$$\lambda [m-p_{x}:x-p_{y}:y]=0$$
 $\mu_{x}(x-0)=0$ $\mu_{y}(y-0)=0$

$$\mu_{x}(x-0)=0$$

(iv) Non-negative multipliers:

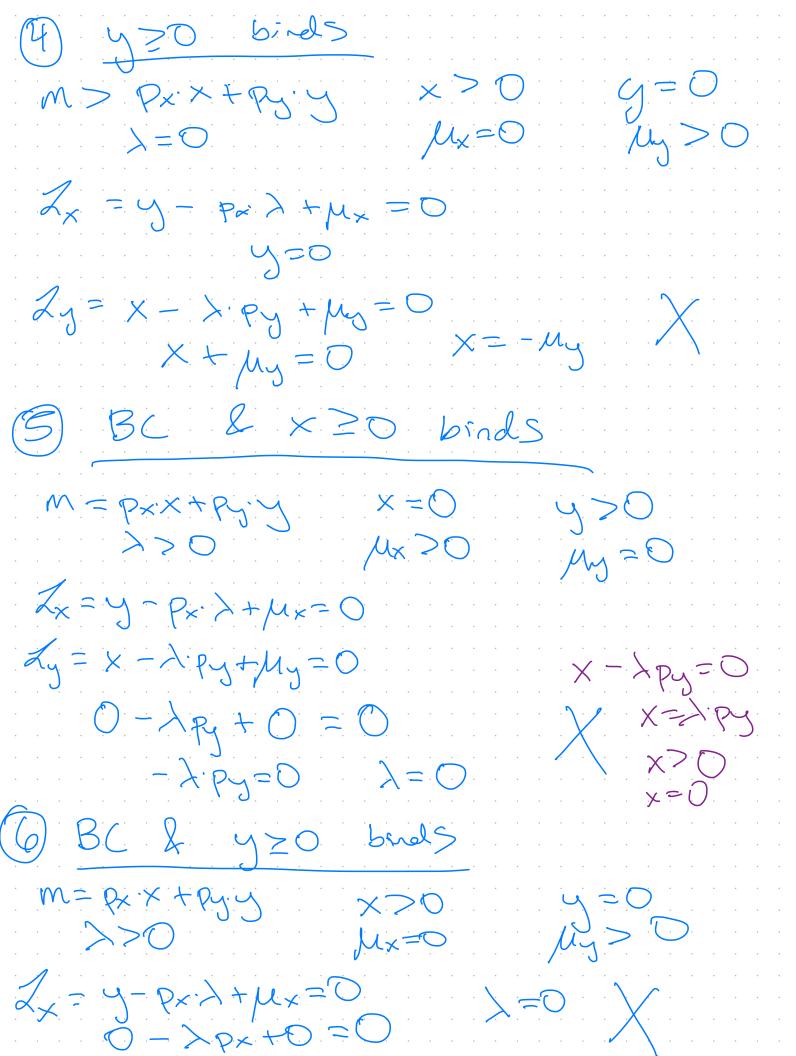
$$\lambda \geq 0$$

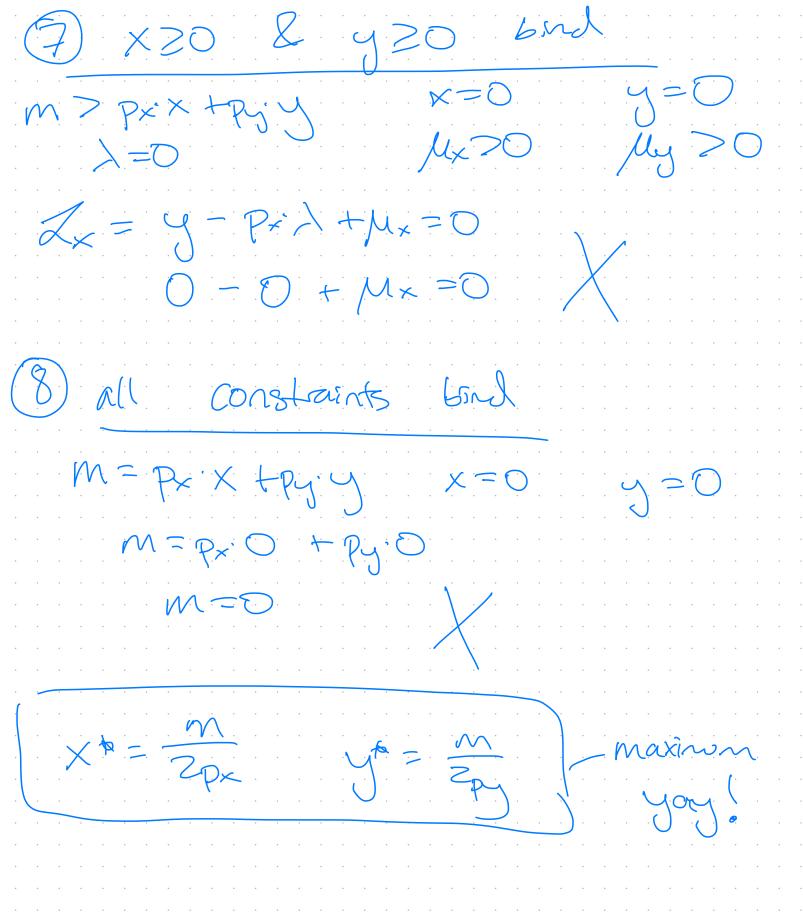
Mx 20

1. No constraints bind

- 2. The budget constraint binds
- 3. $x \ge 0$ binds
- 4. $y \ge 0$ binds
- 5. Budget constraint and $x \ge 0$ bind
- 6. Budget constraint and $y \ge 0$ bind
- 7. $x \ge 0$ and $y \ge 0$ bind
- 8. All constraints bind

1 no constraints bind M> Pxx+Pyy 450 ×20 /w = 0 Mx=0 × contradiction? Z = 1 4 - 1 Px 2 + 1/4 = 0 y = 0 + 0 = 02 BC binds $M = Px \times Py \cdot y$ X > C My=0 1 / 2 = 0 $Z_{x} = y - \lambda p_{x} + \mu_{x} = 0$ $y = \frac{P \times X}{P y}$ $y - \lambda p_{\times} = 0$ 2y= x-2.py +My=0 $\lambda = \frac{\times}{Py}$ X-ZPy = D y= ~ > m=Pxxx+py= Pxxx $\chi^{\star} = \frac{m}{2px}$ $(3) \times 20 \text{ bals}$ $m > p_x \times + p_y \cdot y$ $\frac{1}{2} \frac{1}{2} > \frac{1}{2} \frac{1}{2}$ X=0 $\mu_{x}>0$ Mig = 0 $Z_{x} = y - \lambda \cdot p_{x} + M_{x} = 0$ y = - Mx y + Mx = 0





Example: Use Kuhn-Tucker to solve, where $p_x, p_y, m > 0$:

$$\max_{x,y} x + y \quad s.t. \quad m \ge p_x x + p_y y \quad x \ge 0 \quad y \ge 0$$

$$A = x + y + \lambda [m - p_x \times - p_y y] + \mu_x \times + \mu_y \cdot y$$

$$A_x = 1 - p_x \cdot \lambda + \mu_x = 0$$

$$A_y = 1 - p_y \cdot \lambda + \mu_y = 0$$

$$M > P_{x} \times + P_{y} \cdot y$$
 $X > 0$
 $\lambda = 0$ $\mu_{x} = 0$
 $\lambda = 1 - 0 + 0 = 0$

$$Z_{x} = (-p_{x}, \lambda + \mu_{x} = 0)$$

Pxxtpjy=m

W= bx.X +bh.d

Zx = 1 = 1 = 1 = 0 2y=11=10 $(3) \times 20$ binds $M > P_X \times + P_{yy}$ X = 0 $\lim_{n \to \infty} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n}$ L= 1-0+ Hx=0 (H) 420 65 rds $M > Px \times + Py \cdot y \times > 0$ $\lambda = 0$ $\mu_{x} = 0$ My 20 $\mathcal{L}_{x} = (1 - 10) + 10 = 10$

$$Z_{x} = 1 - p_{x} \lambda + \mu_{x} = 0$$

$$Z_{y} = 1 - p_{y} \lambda + \mu_{y} = 0$$

$$S = P_{x} \times p_{y} \times p_{y} \times p_{y} \times p_{y} = 0$$

$$A_{y} = P_{y} \times p_{y} \times p_{y} \times p_{y} = 0$$

$$A_{y} = 1 - p_{y} \lambda = 0 \quad p_{y} \lambda = 1 \quad \lambda = p_{y}$$

$$A_{x} = 1 - p_{x} \lambda + \mu_{x} = 0$$

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Zx = 1 - Px 2 + Mx = 0 Ly= 1- Py + Hy = 0 BC L y 20 bind M-PxX tpy.y $\mathcal{Z}_{x} = 1 - p_{x} \cdot \lambda = 0$ Zy = 11- Py, 2 + My =0 $-\frac{PS}{Px} + \mu_y = 0$ $My = P_{x}$ $P_{x} - 1$ $Y \Rightarrow P_{x}$ $Y \Rightarrow P_{x}$ $Y \Rightarrow P_{x}$

 $\mathcal{J}_{x} = 1 - p_{x} \lambda + \mu_{x} = 0$ 2 = 1 - Py 2 + My = 0 (7) x20 l y20 binel x = 0 Mx > 0 My > 0 m > Px x + Py y $\mathcal{I}_{\times} = (-0) + \mathcal{M}_{\times}$ $\mathcal{M}_{\times} = -1$ (8) all constaints Brid M=PxX+Pyy $\times = 0$