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Improved quantum-inspired evolutionary algorithm and its application to 3-SAT problems

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***Abstract*—An improved quantum-inspired evolutionary algorithm is presented in this paper. Quantum angle is adopted to present the quantum bit in the proposed algorithm. A novel quantum rotation gate strategy is adopted to adjust the direction of the quantum gate which is used to update the quantum population. The step size is adaptively adjusted rather than a fixed angle. Furthermore, the particle swarm optimization is added into the improved algorithm to accelerate the convergent speed and develop the local searching ability. To demonstrate the effectiveness and applicability of the proposed approach, several experiments are performed on the 3-SAT problems. The results show that it is feasible and effective to solve the 3-SAT problem using the proposed algorithm.**

quantum computation, quantum-inspired evolutionary algorithm, 3-SAT problem)

I. INTRODUCTION

Quantum computation was proposed by Benioff and Feynman in the early 1980s. Due to its unique computational performance, the quantum computation has attracted extensive attention of researchers [1-3]. However, quantum machine is not available now. So, many researches focus on the effect of quantum mechanics on traditional algorithm. Quantum-inspired evolutionary algorithm (QEA) was proposed in Ref.[4]. QEA used a Q-bit, which is defined as the smallest unit of information and a Q-bit individual as a string of Q-bits. Besides, a Q-gate is introduced as a variation operator to promote the optimization of the individuals. The QEA has many advantages such as small population size, capability of exploration and exploitation, speedy convergence and powerful global optimal ability. It has been applied in many fields for its powerful performance [5-8].

A Boolean satisfiability problem (SAT) involves a Boolean formula F consisting of a set of Boolean variables $x_1,x_2,\ldots x_n$. The formula F is in conjunctive normal form and it is a conjunction of m clauses $c_1,c_2,\ldots c_m$. Each clause c_i is a disjunction of one or more literals, where a literal is a variable x_j or its negation. A formula F is satisfiable if there is a truth assignment to its variables satisfying every clause of the formula, otherwise the formula is unsatisfiable. The goal is to determine a variable assignment satisfying all clauses. If the number of literals in each clause is equal to 3 or less than 3, the satisfiability problem is called 3-SAT problem. In this paper, we assume that each clause contains 3 variables.

The satisfiability problem (SAT) is the basic NP-hard problem in the field of computer science. As one of the six basic core NP-complete problems, it is the basic for many problems, for example, symbolic logic, reasoning, machine learning, VLSI test and verification, design of asynchronous circuits, sports planning and so on [9,10].

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II. IMPROVED QUANTUM-INSPIRED EVOLUTIONARY ALGORITHM

A. Quantum Angle

First, Qubit is present as $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$. We can look the qubit as the phase θ in a two-dimensional space. Then, in QEA, the transition of the qubit state can be realized by rotation of this phase θ. The definition of quantum angle is given as follows.

Definition: A quantum angle is defined as an arbitrary angle θ and a Q-bit is presented as [θ].

Then [θ] is equivalent to the original Q-bit as $\begin{bmatrix}\sin(\theta)\\ \cos(\theta)\end{bmatrix}$. It satisfies that $|\sin(\theta)|^2 + |\sin(\theta)|^2 = 1$ spontaneously. The modular $|\sin(\theta)|^2$ and $|\cos(\theta)|^2$ are the probabilities that the Q-bit exists in state "0" and state ''1'', respectively. Then an m-Q-bit could be replaced by $\left[\theta_1\middle|\theta_2\middle|\ldots\middle|\theta_m\right]$. The common rotation gate $\begin{bmatrix}\alpha_i'\\ \beta_i'\end{bmatrix} = \begin{vmatrix}\cos(\xi(\Delta\theta_i)) & -\sin(\xi(\Delta\theta_i))\\ \sin(\xi(\Delta\theta_i)) & \cos(\xi(\Delta\theta_i))\end{vmatrix}\begin{bmatrix}\alpha_i\\ \beta_i\end{bmatrix}$ is replaced by $[\theta_i'] = [\theta_i + \xi(\Delta\theta_i)]$. It can simplify the operation of the quantum gate.

B. Quantum Rotation Gate Strategy

Original QEA gets the adjustment direction by looking up a table. However, when quantum angle is adopted, the bit of a chromosome corresponds to a phase angle. The distance and

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direction can be known between each bit of an individual and the best individual. If the direction of current individual bits is different from the best individuals', the direction of current individual to the best individual is adjusted directly. The adjust step is $\Delta\theta$. The magnitude of it has an effect on the speed of convergence. If it is too big, the solutions may diverge or converge prematurely to a local optimum. Otherwise, if it is too small, the convergent speed may be very slow or in stagnation. In this paper, we defined $\Delta\theta$ as a variable:

$$\Delta\theta = 0.01\pi * \exp(-Iter / MaxIter) \quad (1)$$

where Iter is the current iteration, MaxIter is the maximal iteration. At the initial iterations, adopting a larger angel can accelerate the convergent speed. In the late iterations, a small angle is adopted to develop the local searching ability. The step size is adaptively adjusted in the algorithm.

III. SOLVING 3-SAT PROBLEM BASED ON IQEA

A. Representation of Solution

The most obvious way to represent a solution candidate for 3-SAT problem is to use a bit string of length *n*, where each variable is associated to one bit [12]. We adopt this method to represent the possible solutions. In QEA, we can get an n-bit binary string after observation period. The binary string corresponds to a truth assignment, which is considered as a possible solution.

B. Fitness Function

In this paper, the fitness function is defined as the number of clauses which are satisfied by a truth assignment. Therefore, the 3-SAT problem is transformed into an optimization problem, and the goal is how to get the maximum of the fitness function. If the maximum equal to n, the 3-SAT problem is satisfiable.

C. The Procedure of IQEA

The Procedure of IQEA is described in Fig. 1. The definitions of P(*t*), Q(*t*), *f*(X), PP(*t*) and B^t are defined as same as in the procedure of QEA [4]. PP(*t*) is defined as P(*t*). It presents the population of BPSO. In the procedure of IQEA, the PSO is combined with IQEA. The optimization of IQEA is divided to two phrases. At the first phrase, the particle swarm optimization (PSO) is adopted to optimize the PSO population. The population of PSO is twice of quantum population. Half of the PSO population is derived from n best individuals in the last iteration and others from the quantum population. The PSO will be run for some iteration and *n* best solutions and the best solution will be stored. Then, a guided quantum chromosome will be represented based on the best individual. Some quantum chromosomes will be produced by random distribution near this guided chromosome. These chromosomes will be put into quantum population as a quarter of individuals. At the second phrase, IQEA is adopted to optimize the quantum population. *n* solutions of quantum population obtained after IQEA are put into PSO population. It ensures that the population size of PSO is 2*n*.

```
Procedure of IQEA
Begin
   t=0, Random initialize Q(0) using quantum angle;
   Make P(0) by observing the state of Q(0);
   Initialize PP(0) using Q(0);
   While (not termination condition)do
   Begin
  t=t+1;
  Optimize PP(t) by BPSO;
  Store n best solutions among PP(t);
  Using the best solution to generate n/4 individuals
  Replace n/4 individuals in Q(t) and get a new Q(t);
  While (not termination condition)do
  Begin
     Make P(t) by observing the state of Q(t);
     Evaluate f( x_j^t );
     Adopt quantum rotation gate strategy to update
Q(t);
     Store the best solutions among P(t) into B^t and
         f(B^t);
  End
  Add the P(t) into PP(t);
End
End
```

Figure 1. The procedure of IQEA

IV. RESULTS

Two groups of benchmark are presented to examine the validity of the proposed algorithm. One group is come from SATLIB database. Another group is used by Gottlieb. They are random 3-SAT instances with a ratio clauses-to-variables of 4.3. Ref.[13] showed that these problems are hard to solve.

A. Benchmark1

The problems of SATLIB database have been widely applied to test the performance of different algorithms [14]. According to the number of the variables, we choose 100 problems form the dataset and divide them 10 groups, denoted by sat1, sat2, …, sat10. The results in Ref. [15] are considered and our results are added with respect to the same comparison criteria.

The population size of the IQEA is 30. Each instance is tested 50 times and the number of iteration is limited to 10^6. If it can get the solution in the limited iteration, the run is considered to be successful. The success rate (SR) is the number of successful runs divided by the total number of runs. The results are showed in Tab.1.

In Tab.1, the quantum genetic algorithm and WalkSAT are chosen to compare with the IQEA. To the ten datasets, the success rates of IQEA almost beyond 90%. Its performance is superior to QGA. For the characteristics of quantum computation, the IQEA can get a satisfied solution with a small population.

TABLE I. COMPARISON WITH QGA AND WALKSAT

datasets	SR		
	IQEA	QGA	WalkSat
Sat1	1.00	1.00	1.00
Sat2	1.00	1.00	1.00
Sat3	1.00	1.00	1.00
Sat4	0.95	0.92	1.00
Sat5	0.93	0.90	1.00
Sat6	0.92	0.82	0.98
Sat7	0.90	0.87	0.86
Sat8	0.90	0.73	0.88
Sat9	0.88	0.61	0.93
Sat10	0.83	0.45	0.87

B. Benchmark2

Gottlieb et al. proposed several evolutionary algorithms for SAT [16]. Results presented in that paper are considered and our results are added. The same criteria and parameters are adopted as benchmark1.

The data provided in Tab.2 are: the number of the instances chose in the experiments (n.b.), the number of variables in each instance (var.), the success rate which is the number of successful runs divided by the total number of runs (SR). From Tab.2, we can conclude that the improved QEA can solve the 3-SAT problem as well as RFEA2+ do. With the increasing of the problem scale, the proposed algorithm also can get a good result. However, the proposed algorithm is in essence an evolutionary algorithm. From the two benchmarks, we found that the convergent speed is slow in the anaphase when the problem scale is very large. How to develop the convergent speed in the anaphase is also a challenge. Meanwhile, as an incomplete algorithm for SAT problem, the proposed algorithm can be used to solve MAXSAT problems.

TABLE II. RESULTS FOR BENCHMARK2

Instances			IQEA	RFEA2	RFEA2+	FlipGA	ASAP
suit	n.b.	var.	SR	SR	SR	SR	SR
A	3	30	1.00	1.00	1.00	1.00	1.00
A	3	40	1.00	1.00	1.00	1.00	1.00
A	3	50	1.00	1.00	1.00	1.00	1.00
A	3	100	0.96	0.99	0.97	0.87	1.00
B	20	50	1.00	1.00	1.00	1.00	1.00
B	20	75	0.95	0.95	0.96	0.82	0.87
B	20	100	0.88	0.77	0.81	0.57	0.59
C	10	20	1.00	1.00	1.00	1.00	1.00
C	10	40	1.00	1.00	1.00	1.00	1.00
C	10	60	1.00	0.99	0.99	1.00	1.00
C	10	80	0.93	0.92	0.95	0.73	0.72
C	10	100	0.91	0.72	0.79	0.62	0.61

V. CONCLUSIONS

Quantum computation is a new computing model which is different from the classical computation. Due to its unique computational performance, the quantum computing has attracted extensive attention of researchers. The QEA is the combination of quantum computation and evolutionary algorithms. It has many advantages such as small population size, capability of exploration and exploitation, speedy convergence and powerful global optimal ability. It has been applied in many fields for its good performance. In this paper, we propose an improved quantum-inspired evolutionary algorithm and extend the application field of the QEA by using it to solve the 3-SAT problem. The quantum angle is adopted in the improved algorithm. And a novel quantum rotation gate strategy is adopted to update the population. Furthermore, to accelerate the convergent speed, the particle swarm optimization is combined with the IQEA. Experimental results show that the improved algorithm it is feasible and effective to solve 3-SAT problems. It can be as a new incomplete method to solve satisfiability problems.

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