

Suitably impressive thesis title

Robin Timmis

Your College

University of Oxford

*A thesis submitted for the degree of
Doctor of Philosophy*

Michaelmas 2014

Abstract

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Acknowledgements

Personal

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A List of Symbols and Abbreviations

e	Absolute charge of an electron = 1.602×10^{-19} C
ϵ_0	Permittivity of free space = 8.854×10^{-12} F m ⁻¹
λ_D	Debye length $\equiv \sqrt{\frac{\epsilon_0 K T_e}{n_e e^2}}$
n_e	Plasma electron number density as a function of position
n_i	Plasma ion number density as a function of position
T_e	Plasma electron temperature
Z	Ion charge state in units of e
1D, 2D, 3D . .	One-, two- or three-dimension(al)
Otter	One of the finest of water mammals.
Hedgehog . . .	Quite a nice prickly friend.

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There is no one who loves pain itself, who seeks after it and wants to have it, simply because it is pain...

— Cicero's *de Finibus Bonorum et Malorum*

1

Introduction

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1.1 A plan

Actually the very first thing to say will be the stuff about light i think, about diagnostic tools essentially always being about controlling the various properties of light (ie electromagnetic waves)

sections to include start with the laser and our entrance into the multi-petawatt regime with no signs of stopping - ref 40 in alex savin thesis - it looks like that is saying huge growth in laser power check that and maybe write in thesis but don't include the figure. Also include description of CPA and OPCPA what is a plasma modelling plasma with PIC codes intense lasers and absorption mechanisms?? yes for sure but dont do all just those just below ZVP simulation units and similarity parameter Frames of reference - lab, sim, ablating front sruface of plasma

what is the story? Based on first section by Alex after the abstract Plasma is ubiquitous in our known universe and plasma provides us huge opportunities as a tool to improve our lives (From chen) what we can see in the sky is a result of that stuff being in the plasma state. Lasers can do so much now and are only getting more powerful all the time thanks to CPA and since developments (discuss) Simultaneously our ability to understand the physics has been aided by an explosion in computing power (peter HEDP paper) In this thesis we discuss some of the opportunities that relativistic laser plasma physics offers us with solid density targets - note that note about solids v gases at this point. Perhaps even before the debye length, define what we mean by the temperature of the plasma??

An unused statement about ion immobility Assume for now that the ion-electron mass ratio is infinite, that is to say the ions are approximately immobile for the timescales under consideration, generally true for a fair few relativistic laser pulse cycles (In later sections the mobility of plasma ions will prove very important but for now this is ignored.).

1.2 The definition of a plasma

As outlined in F. Chen's definitive textbook 'Introduction to Plasma Physics and Controlled Fusion' [chen20116], a plasma must fulfil three criteria, namely,

1. Ionisation: a plasma must consist of both charged and neutral particles, of course this alone cannot define a plasma, any gas will contain some degree of ionisation;
2. Quasineutrality: while locally there can be (often extreme) electromagnetic forces and charge concentrations at work, over the length scales of the plasma, such forces are screened out and the plasma bulk remains net neutral in charge;
3. Collective behaviour: unlike in a gas where collisions dominate, the particles in a plasma generate electromagnetic fields that interact at a distance and

thus a particle's motion depends not only on its immediate vicinity but on the surrounding plasma conditions, indeed often it is the so-called 'collisionless' plasmas where collisions can be safely neglected that are of most interest, as is the focus of this thesis.

1.2.1 The Debye length

The Debye length describes the extent to which a plasma can shield electromagnetic fields within and so remain quasineutral. Consider an infinitely extending plasma with a test charge placed at some point, then what would be the potential $\phi(\mathbf{x})$? If the plasma had no kinetic energy, the charged particles would arrange themselves immediately adjacent to the test charge and once this equilibrium state was reached there would be no electromagnetic fields present. Realistically the plasma will have some temperature, likely a very large temperature and so some particles will be able to escape the potential of the test charge and thus leak electromagnetic fields into the plasma bulk. Poisson's equation reads

$$\epsilon_0 \nabla^2 \phi = -e(Zn_i - n_e), \quad (1.1)$$

where $\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$ is the permittivity of free space, $e = 1.602 \times 10^{-19} \text{ C}$ is the charge of an electron, Z is the plasma ion charge in units of e and n_i and n_e are the number densities of plasma ions and electrons.

Since the electrons are significantly more mobile than the ions due to their lower mass, it is in general the electrons and not the ions that respond to the test charge and the ions can be assumed to provide a constant background of positive charge density. If the number density of electrons follows a Boltzmann temperature distribution in the presence of a potential energy $-e\phi$, then

$$n_e = n_{e,0} e^{e\phi/KT_e}, \quad (1.2)$$

where $n_{e,0}$ is the electron number density far from the test charge, $n_i = n_{e,0}/Z$ and KT_e is the electron temperature. Note that in plasmas it is very common for different species to have differing temperatures depending on the mechanism for energy absorption and the timescales for collisions compared to the timescale of the study.

Substituting equation 1.2 into equation 1.1 and Taylor expanding the exponential term in the limit that the plasma is weakly coupled ($e\phi \ll KT_e$), obtains

$$\nabla^2 \phi = \frac{\phi}{\lambda_D^2}, \quad (1.3)$$

where

$$\lambda_D \equiv \sqrt{\frac{\epsilon_0 KT_e}{n_e e^2}}, \quad (1.4)$$

is the *Debye length* and describes the thickness of the charge sheath surrounding the test charge. For quasineutrality to hold for the plasma bulk, its spatial dimensions must extend beyond a few Debye lengths.

1.2.2 The plasma parameter

In order for the above description to be statistically valid, there must be a large number of charged particles within the shielding sheath. The number of particles within the Debye sphere can be computed as

$$N_D = \frac{4}{3}\pi\lambda_D^3 n. \quad (1.5)$$

Note that, as discussed above, in most cases it is most suitable to choose the number density n to be the number density of electrons. To ensure the plasma is suitably ionised (criterion 1) and that the plasma engages in collective behaviour (criterion 3),

$$N_D \gg \gg 1. \quad (1.6)$$

1.2.3 Collisionality and the plasma frequency

Collective behaviour not only depends on the ability for large numbers of particles to interact via electromagnetic forces but that these forces dominate over collisions in describing particle trajectories. Taking ω as the typical frequency of plasma oscillations and τ as the average time between collisions, for a plasma (as opposed to a gas) must satisfy

$$\omega\tau > 1. \quad (1.7)$$

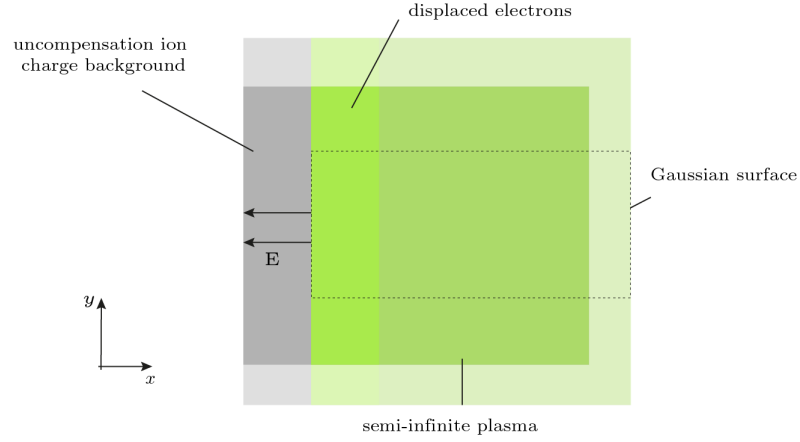


Figure 1.1: Diagram to illustrate the plasma frequency derivation. THIS FIGURE NEEDS E POINTING THE OTHER WAY

It now remains to determine what is the typical frequency of collisions in a given plasma. While the types of plasma waves and their associated frequencies of oscillation are multitudinous, the characteristic frequency, the *plasma frequency*, ω_p , is the most straightforward. It describes the response of electrons to charge imbalances within an infinite uniform plasma at rest in the absence of magnetic fields or temperature fluctuations. As noted in section 1.2.1, the ions provide a constant background of positive charge.

Consider an semi-infinite plasma existing for $x > 0$, with electron density n_e and ion density n_e/Z of charge state Z ¹. Suppose the electron fluid is displaced by some perfectly isotropic force into the plasma bulk a distance $(\Delta x)\hat{\mathbf{x}}$ as in figure 1.1. The total charge of displaced electrons within a surface area of σ is

$$Q = -en_e\sigma\Delta x. \quad (1.8)$$

Applying Gauss' law to the surface detailed in figure 1.1, the uncompensated charge leads to

$$-\sigma E\hat{\mathbf{x}} = \frac{Q}{\epsilon_0}\hat{\mathbf{x}} = -\frac{en_e\sigma\Delta x}{\epsilon_0}\hat{\mathbf{x}} \quad (1.9)$$

at the electron surface. By the Lorentz force, the displaced electrons will experience a restoring force, $-eE\hat{\mathbf{x}}$, perpendicular to the surface due to the electron-ion charge

¹This description has direct relevance to the Zero Vector Potential mechanism which will be made clear later.

imbalance. The equation of motion for electrons on that surface is therefore

$$m_e \frac{d^2 \Delta x}{dt^2} = -eE = -\frac{e^2 n_e}{\epsilon_0} \Delta x. \quad (1.10)$$

Equation 1.10 clearly describes a simple harmonic oscillator with a characteristic frequency given by the plasma frequency,

$$\omega_p = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}. \quad (1.11)$$

2

The Zero Vector Potential Absorption Mechanism

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2.1 Introduction

Of primary interest in this thesis is the interaction of a relativistically intense short pulse laser interacting with a solid density plasma target with a sharp density gradient. Now is presented the Zero Vector Potential mechanism of attosecond absorption of laser pulse energy, proposed by *Baeva et al* [**baeva_2006_TheoryHighorderHarmonic**] and later developed by *Savin et al* [**savin2017AttosecondscaleAbsorptionExtreme**, **savin2019EnergyAbsorptionLaserQED**]. Laser energy absorption in dense plasmas was first proposed by Wilks and Kruer [**wilks1997**], a ponderomotive mechanism where plasma electrons are heated directly by the laser pulse via the so-called $\mathbf{J} \times \mathbf{B}$ force.

This thesis focuses on the so-called ‘post-ponderomotive’ regime where the frequency of the plasma oscillations ($\omega_p \sim \sqrt{n_e}$) are greater than the $\mathbf{J} \times \mathbf{B}$ induced plasma electron oscillations at $2\omega_L$. The plasma electrons are then fast enough to compensate the ponderomotive pressure of the laser pulse with the formation of electrostatic fields between electrons and ions and so respond adiabatically to the $\mathbf{J} \times \mathbf{B}$ force. Hence plasma electrons cannot be heated directly by the laser pulse. Note that this requires a sufficiently steep density gradient around the relativistic critical density surface (where $S = 1$) to shift the main interaction to a region where this condition on the overdensity is satisfied. In this case the ponderomotive pressure of the laser compresses the electrons at the front surface of the plasma and so shifts the laser-plasma surface interaction to plasma densities well beyond the relativistic critical density, leaving behind a positive space charge. This electron-ion charge separation leads to the formation of a *pseudo-capacitor* electrostatic field.

Interestingly working through the condition between ω_p and ω_L in normalised units suggests the criterion for this regime is $S > 4$, slightly more constraining than $S > 1$ as is typically stated [savin2019thesis].

[Come back to this and write up stuff about actually it being the relativistic frequency and critical density surfaces that matter and how this does indeed put a stricter condition on the bulk S value]

So we have entered a regime of adiabaticity where the plasma skin layer is confined within a potential well consisting of the ponderomotive pressure and the Coulomb potential. Consider a relativistic linearly polarised laser pulse obliquely incident, with an angle of incidence of θ , on a semi-infinite plasma, existing for $x > 0$. The Hamiltonian of a single electron confined within the potential well [herbert_classical_mechanics] is

$$\mathcal{H} = c\sqrt{m_e^2 c^2 + |\mathbf{p}|^2} - e\Phi. \quad (2.1)$$

Here the first term is the electron energy, U , extracted from the invariant of the relativistic 4-momentum of the electron, $\mathbf{P}^\mu = (U/c, \mathbf{p})$,

$$\mathbf{P}^\mu \cdot \mathbf{P}_\mu = \frac{U^2}{c^2} - |\mathbf{p}|^2 = m^2 c^2. \quad (2.2)$$

Note that while there has been growing interest in the curvature of spacetime by relativistic lasers [cite edward here], for modern high power lasers this effect is small and not relevant for this thesis. Throughout the inner product of 4-vectors is defined with the Minkowski Metric. [find alex citation on pg 89 of thesis]

The second term of equation 2.1 describes the contribution to the electron's energy from the electrostatic potential of the pseudo-capacitor. Decomposing the electron's 3-momentum into orthogonal components: p_{prop} , along the laser propagation direction, p_{pol} , along the polarisation axis of the laser pulse and p_{\perp} , perpendicular to both, two simplifications can be made. Firstly, by canonical conservation of transverse momentum, $p_{\text{pol}} = eA$, where A is the laser vector potential. Secondly, in the case of a p -polarised laser pulse (the known optimum for ZVP electron bunch generation), the forces at play confine the electron trajectory to the p_{prop} - p_{pol} plane and the interaction geometry is in essence two-dimensions (2D).

[include a diagram alluding to this?-it is basically since B is out of the plane and all other E fields are in the plane, also perhaps provide a foot not here to explain how incidentally this all provides a succinct explanation of why p is better?]

Explicitly, the Hamiltonian is now

$$\mathcal{H} = c\sqrt{m_e^2c^2 + p_{\text{prop}}^2 + e^2A^2} - e\Phi. \quad (2.3)$$

From equation 2.3 it is clear that should the vector potential pass through zero, one of the walls of the potential well is suppressed, allowing electrons in the in the skin layer to escape the plasma, breaking adiabaticity. The necessity of vector potential zeros for this violent reconstruction of the plasma surface led Baeva et al [**baeva2011ZeroVectorPotential**] to coin the term ‘Zero Vector Potential’ mechanism to describe this process. Indeed, while in standard calculations a laser pulse will exponentially decay within a skin layer without passing through zero, Baeva et al [**baeva2011ZeroVectorPotential**] were able to demonstrate in **PIC!** (**PIC!**) simulations that for this regime, zeros are able to propagate through the skin layer of the plasma. The explanation for this difference in mechanics relies

on a Doppler shift in the laser field due to the relativistic motion of the ablating plasma surface, and the mathematical formalism of this process proceeds as follows.

[I think before this point it would be good to enter in the language of electron bunches or sheaths - I will continue the dicussion assuming this concept has been introduced].

As the Zero Vector Potential (ZVP) mechanism is a relativistic phenomenon, it is essential to consider the laser pulse propagating through a relativistically ablating electron bunch (i.e. with some component of its velocity anti-parallel to the laser pulse propagation direction). Transforming to the rest frame of the ablating front, beyond the relativistic critical density surface, the vector potential of the laser pulse will be an evanescent wave, at the spatial centre of the laser pulse, it can be described simply by

$$\mathbf{A}'_L(t', r') = A'_0 \cos(\omega'_L t') \exp(-r'/\delta') \hat{\mathbf{r}}'_{\text{pol}} = A'_L \hat{\mathbf{r}}'_{\text{pol}}, \quad (2.4)$$

where the primed symbols indicate that these quantities are measured in the rest frame of the expanding front, A'_0 is the vector potential amplitude and ω'_L is the frequency of the laser pulse, r' is the propagation distance of the laser into the plasma, δ' is the skin depth and $\hat{\mathbf{r}}'_{\text{pol}}$ a unit vector defining the polarisation direction of the laser pulse. Un-primed coordinates will indicate the lab frame measurements.

[For sure include a diagram of this]

While in previous demonstrations of the vector potential zeros, it was assumed that the ablation occurs normal to plasma surface, it is now known that this ablation occurs in the specular reflection direction and it is necessary to confirm that zeros are still predicted. Consider a p-polarised laser pulse confined to the x - y plane incident with an angle of incidence θ on an ablating overdense plasma expanding with velocity $-v_f \hat{\mathbf{x}}$ in the lab frame, as in figure 2.1. The direction of polarisation is

$$\hat{\mathbf{r}}_{\text{pol}} = \hat{\mathbf{x}} \sin 2\theta - \hat{\mathbf{y}} \cos 2\theta \quad (2.5)$$

and the velocity of the rest frame of the ablating front relative to the lab frame is $-v_f \hat{\mathbf{x}}$.

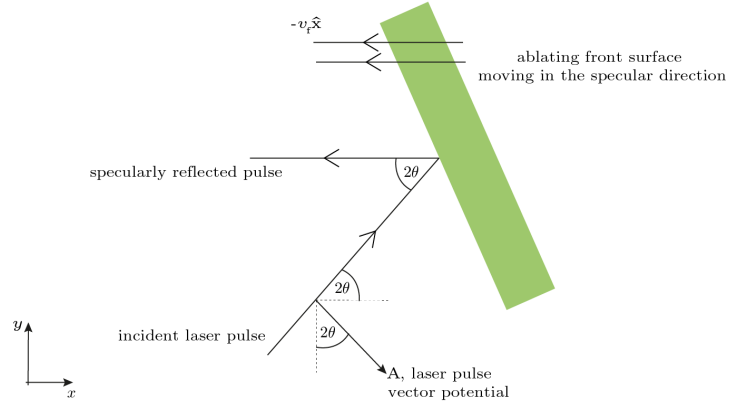


Figure 2.1: Diagram of a p -polarised laser pulse incident on an ablating overdense plasma. The laser is incident obliquely at an angle of θ and is reflected specularly. The plasma ablates specularly also. The interaction geometry is confined to a 2D plane.

Applying the Lorentz transformation to the electromagnetic 4-potential,

$$\mathbf{A}_\mu = (\phi/c, \mathbf{A}), \quad (2.6)$$

explicitly,

$$\mathbf{A}'_\mu = \Lambda^\mu_\nu \mathbf{A}_\nu, \quad (2.7)$$

where Λ^μ_ν , the Lorentz transform in this geometry is

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.8)$$

and here $\beta = -v_f/c$, $\gamma = 1/\sqrt{1-\beta^2}$. Immediately from the y -coordinate transformation,

$$A'_L \cos 2\theta' = A_L \cos 2\theta. \quad (2.9)$$

Applying the headlight effect for a source moving at an angle 2θ to the boosted frame,

$$\cos(2\theta') = \frac{\cos(2\theta) - \beta}{1 - \beta \cos(2\theta)} \quad (2.10)$$

and rearranging equation 2.9, the vector potential in the lab frame is

$$A_L = \frac{1 - \beta \sec(2\theta)}{1 - \beta \cos(2\theta)} A'_0 \cos(\omega'_L t') \exp(-r'/\delta'). \quad (2.11)$$

Writing the boosted frame space-time coordinates in terms of the lab frame coordinates,

$$ct' = \gamma(ct - \beta x), \quad (2.12)$$

$$x' = \gamma(x - \beta ct), \quad (2.13)$$

yields

$$A_L = A_0 \cos(\omega_L t - kx) \exp\left(-\frac{\sqrt{(x - \beta ct)^2 + (y/\gamma)^2}}{\delta}\right), \quad (2.14)$$

where

$$A_0 = \frac{1 - \beta \sec(2\theta)}{1 - \beta \cos(2\theta)} A'_0, \quad (2.15)$$

$$\omega_L = \gamma \omega'_L, \quad (2.16)$$

$$k = \frac{\beta \gamma \omega'_L}{c}, \quad (2.17)$$

$$\delta = \frac{\delta'}{\gamma}. \quad (2.18)$$

The oscillatory term in equation 2.14 demonstrates the propagation of vector potential zeros within the plasma target. From the structure of this term it would appear that these zeros are expelled from the plasma along the specular direction at a speed (recall beta is negative - change this earlier in the theory so that that negative sign is more explicit in the result.)

$$v_\phi = \frac{\omega_L}{k} = \frac{c}{\beta} = -\frac{c^2}{v_f}. \quad (2.19)$$

Could also discuss here about how relativistic similarity theory derives that zeros move at speed c but how that cannot be valid since then we would always have infinitely thin radiation pulses, unless there is an extended range of zero? I suppose there is some radiation happening around the peak? Good questions.. One remaining consideration is we require that the zero gets through the whole electron bunch which is generally at very high density but is also very thin, in a way is this skin depth not what precisely determines the bunch width? The bunch will be compressed until the skin depth goes to zero across it perhaps? Things to think about.

Qualitatively, and in summary, for sufficiently intense laser pulses, electrons on the radiated surface of a solid target are accelerated by the laser to relativistic

velocities at a fraction of a laser pulse cycle and therefore electrons both follow similar trajectories and are able to respond adiabatically to the $\mathbf{J} \times \mathbf{B}$ force of the laser pulse. They therefore form a high charge density thin coherent electron sheath on the front surface of the plasma but displaced inwards from the immobile ions (ions are approximately immobile on the timescale of a laser pulse cycle) via the ponderomotive pressure of the laser. This charge separation generates a longitudinal electrostatic pseudocapacitor field that confines electrons to a potential well on the front surface of the plasma, preventing further propagation of the electron bunch into the plasma bulk. When the zero of the vector potential passes through the electron bunch, the ponderomotive pressure instantaneously vanishes and electrons are ejected specularly from the target, copropagating with the zeroes and gaining energy as they discharge the pseudocapacitor field. Coherent synchrotron emission occurs concurrently. The electron bunch is then rotated by the laser pulse and launched into the bulk at high energy.

SOMething to think about: which comes first? electron bunch acceleration across pseudo capacitor or zeroes? Laser accelerates electrons in laser propagation direction, however cannot propagate further into the plasma so only get motion parallel to surface, once potential well disrupted, acceleration is perp to surface so combined, electrons travel in specular direction. So actually what we are saying is the zeroes will go in whatever direction the surface ablates in, but the surface will move in a direction dependent on the components of electron

The headlight effect

This most likely goes in the appendix. But storing here for now.

The headlight effect describes the beaming of an isotropically emitting source travelling at some velocity relative to an observer. Consider the geometry of figure 2.1 with the source (the laser pulse) travelling at an angle 2θ to the observer (in this case, the ablating front). A photon with energy E emitted from the rest frame of the source (in this case the lab frame) has a 4-momentum

$$\mathbf{P}_\mu = \left(\frac{E}{c}, \frac{E}{c} \cos 2\theta, \frac{E}{c} \sin 2\theta \right). \quad (2.20)$$

As the interaction geometry is confined to a 2D plane, the z -component can be safely neglected. Applying the lorentz boost of equation 2.8,

$$\begin{aligned}\frac{E'}{c} &= \gamma \left(\frac{E}{c} - \beta \frac{E}{c} \cos 2\theta \right) \\ \frac{E'}{c} \cos 2\theta' &= \gamma \left(\frac{E}{c} \cos 2\theta - \beta \frac{E}{c} \right).\end{aligned}\tag{2.21}$$

Solving these equations for the angle in the boosted frame,

$$\cos 2\theta' = \frac{\cos 2\theta - \beta}{1 - \beta \cos 2\theta}.\tag{2.22}$$

Conservation of generalised transverse momentum

This should most likely go in the appendix/ before ZVP Hamiltonian discussion.

Whilst it is commonly stated within the field of laser-solid interactions, it would appear that some nuance/detail is missing from the discussion which in turn shrouds the ZVP mechanism in confusion.

Consider a holonomic system of N relativistic particles under the influence of electromagnetic forces. A particle j with charge e_j and mass m_j experiences a scalar potential,

$$U_j = e_j(\Phi - \mathbf{A} \cdot \mathbf{v}_j)\tag{2.23}$$

and hence the system is described by the Lagrangian

$$L = \sum_{j=1}^N \left(-m_j c^2 \sqrt{1 - \beta_j^2} - e_j(\Phi - \mathbf{A} \cdot \mathbf{v}_j) \right),\tag{2.24}$$

where $\beta^2 = \mathbf{v}_j \cdot \mathbf{v}_j / c^2$ [goldstein_classical_mechanics]. The generalised momentum corresponding to coordinate x_j is

$$p_{j,x} = \frac{\partial L}{\partial \dot{x}_j} = m_j \dot{x}_j + e_j A_x,\tag{2.25}$$

explicitly, the generalised momentum describes both the linear mechanical momentum and the momentum of the electromagnetic field. Via Noether's theorem, if L is independent of x_j , *i.e.* spatially homogeneous along x for particle j , then

$$\dot{p}_{j,x} = 0.\tag{2.26}$$

Considering a p-polarised Gaussian laser pulse, axis of polarisation along x , A_x will be approximately constant. Integrating and noting that initially there is no linear or electromagnetic momentum, the generalised transverse momentum conservation equation for an electron at the plasma-vacuum boundary is obtained, namely,

$$p_T = eA, \quad (2.27)$$

where p_T is the electron momentum along the polarisation axis of the laser pulse. and A its vector potential.

Note that this is only valid provided the radiating electron does not radiate along the direction of \mathbf{A} as discussed by Sokolov *et al* [sokolov_2009_DynamicsEmittingElectrons]. But we do know this radiation is specular so this is true for normal incidence but not specular?? Really need to consider what the EM fields are in the surface from combined Incident and reflected. The implications of this should maybe be considered?? Also note that this is only true for gaussian pulses with spatial profiles \gg than electron trajectories (*i.e.* twice the relativistic larmor radius).

2.1.1 ZVP electron bunch energies

In [baeva_2011_ZeroVectorPotential], Baeva *et al* propose energy scalings for electron bunches produced in the ZVP regime as a function of the laser intensity and plasma density, finding that one of the key statements of similarity theory ($p \sim a_0 S^x$, where x is some integer value, THIS NEEDS A CITE I THINK IT APPEARS IN BAEVAS ORIGINAL HHG PAPER) holds for the ZVP mechanism. Later this was then extended to three-dimensions (3D) by Savin *et al* [savin_2017_AttosecondscaleAbsorptionExtreme]. What follows is that discussion with a more close consideration of the both consequences and constants of proportionality.

(Pherhaps I should redo this discussion condering infinitesimal areas of the plasma surface to show how variation can exist across the surface) Defo do this! And say that provided the variation is small, rel to what though? the surface remains approximately flat -> that is totally not true

Consider again the semi-infinite block of plasma proposed in figure 2.1, normally irradiated by a laser pulse with wavelength λ_L and peak electric field, E_L . It is now the ponderomotive pressure of the laser that displaces the electron fluid. Consider just one laser cycle. The electron surface moves inwards until the pressure exerted by the peak instantaneous ponderomotive pressure of the laser pulse cycle,

$$\mathbf{P}_L = \epsilon_0 E_L^2 \hat{\mathbf{x}} = \epsilon_0 \left(\frac{a_0 \omega_L m_e c}{e} \right)^2 \hat{\mathbf{x}} \quad (2.28)$$

is equal and opposite to the pressure exerted by the pseudo-capacitor field,

$$\mathbf{P}_C = \frac{QE}{\sigma} \hat{\mathbf{x}} = -\frac{(en_e \Delta x)^2}{\epsilon_0} \hat{\mathbf{x}} \quad (2.29)$$

using equations 1.8 and 1.9. Equating the magnitudes of \mathbf{P}_L and \mathbf{P}_C , the maximum displacement inwards of electrons is

$$\Delta x \hat{\mathbf{x}} = \frac{c}{\omega_L} \frac{a_0}{\bar{n}_e} \hat{\mathbf{x}} = \frac{1}{kS} \hat{\mathbf{x}}, \quad (2.30)$$

where k is the wave-vector of the laser pulse. Correspondingly,

$$E = \frac{en_e}{\epsilon_0} \Delta x = \frac{\omega_L c m_e a_0}{e} = E_L. \quad (2.31)$$

Applying the results of equations 2.30 and 2.31, when the ponderomotive pressure vanishes and the electron bunch is launched across the pseudo-capacitor, the relativistic kinetic energy gained by a single electron is

$$T = \int \mathbf{F} \cdot d\mathbf{s} = \int_{\Delta x}^0 -eE dx = \int_{\Delta x}^0 -\frac{en_e x}{\epsilon_0} dx = \frac{1}{2} m_e c^2 \frac{a_0^2}{\bar{n}_e} \quad (2.32)$$

or an electron gamma factor,

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = 1 + \frac{a_0^2}{2\bar{n}_e}. \quad (2.33)$$

Assuming all displaced electrons are captured by the pseudo-capacitor field and launched as a coherent bunch, the total kinetic energy of the electron bunch is

$$U_{ZVP} = n_e \sigma \Delta x T = \frac{\sigma n_c}{k} \times m_e c^2 \frac{a_0^3}{\bar{n}_e}. \quad (2.34)$$

It is now interesting to compare equation 2.34 to the laser energy deposited upon the plasma surface and therefore consider what fraction of the laser energy can be absorbed via the ZVP mechanism. Using $E = E_L$, 2.34 can be rewritten as

$$U_{\text{ZVP}} = \frac{1}{2\omega_L S} \sigma c \epsilon_0 E_L^2. \quad (2.35)$$

For the case of normal incidence, bunches are produced at a frequency of $2\omega_L$, naturally following the frequency of the $\mathbf{J} \times \mathbf{B}$ force. Assuming a sinusoidal plane wave incident with surface area σ , the energy absorbed in half a laser cycle is

$$U_{L,1/2} = \sigma \frac{T}{2} \langle I_L \rangle = \frac{2\pi}{4\omega_L} \sigma c \epsilon_0 E_L^2. \quad (2.36)$$

Hence,

$$\eta_{\text{ZVP}} = \frac{U_{\text{ZVP}}}{U_{L,1/2}} = \frac{1}{\pi S}. \quad (2.37)$$

Interestingly, this new analytical result predicts the trend observed by A. Savin [savin_2019_ModellingLaserPlasmaInteractions] in **PIC!** simulations both in magnitude and in scaling. Indeed, A. Savin demonstrated

$$\eta_{\text{ZVP}} \sim S^{-1.000(3)}, \quad (2.38)$$

however, this result led A. Savin to conclude increasing S increases the energy in the reflected **HHG!** (**HHG!**) beam thus increasing high harmonic efficiency, seemingly in tension with the vast majority of the work on this regime [CITE CITE CITE]. The resolution arises from the following: there are two distinct conversion efficiencies which describe the reflected harmonic spectrum: the conversion efficiency into the whole reflected beam and the conversion efficiency into individual harmonics. While the total conversion into the reflected beam decreases for decreasing S , the slope of the harmonic spectrum also decreases and therefore while A. Savin is absolutely correct, the important parameter (the slope of the harmonic spectra) follows the opposite trend.

Indeed high X-ray harmonic efficiency necessitates high inefficiencies in the production of ZVP electron bunches as higher energy bunches produce more

coherent reflected radiation, a caveat not often considered in the quest for higher-order harmonics.

[Side note for my reference: bunch is made and at peak compression at peak of laser pulse. then laser field oscillates, inside the bunch they experience an $a_0 = 0$ (? maybe) outside there is a minimum in the field, then the bunch is rotated back when it hits the next peak of the laser pulse (ie $a_0 = 0$? outside plasma?) the upshot of this is I can then say there is a distance of $\lambda/2$ between bunch peak displacement and radiation point and since peak displacement $= 1/kS$, for all S , the pseudocapacitor has been completely discharged and therefore there has been the maximum energy gain before HHG production - important for electron bunch gamma factor discussion.]

Next up: the unification of RES and ZVP.

2.1.2 ZVP bunches oblique incidence scaling and internal bunch structure

This section is inspired by [gonoskov_2011_UltrarelativisticNanoplasmonicsRoute] and [vincenti_2014_OpticalPropertiesRelativistic] supplementary material but different where I believe they have gone wrong to generalise the ZVP mechanism energy scalings.

The below section needs cleaning up of minus signs etc, take the convention laser propagates in the positive direction therefore drift of electrons and ions in frame with normal incidence is in negative direction.

As has previously been stated, if the plasma-vacuum boundary is sufficiently steep, the plasma electrons will respond adiabatically to the laser pulse and arrange themselves to form a pseudocapacitor longitudinal electric field E_C at the plasma surface. At all points in this adiabatic phase, the surface electrons will be in a quasi-static equilibrium *i.e.* there will be a balance between the electromagnetic forces on them. Consider again a laser pulse incident on a solid density plasma existing for $x > 0$ at angle θ . Transforming to the reference frame where the laser is incident normally (quantities in this frame are indicated by the primed

symbol), the electron and ion bulk plasma species are now streaming with a velocity $\mathbf{v}_d = -c \sin \theta \hat{\mathbf{y}}$. From the Lorentz force law along the longitudinal direction ($\hat{\mathbf{x}}$), for a displacement of the electron fluid x'_e (one assumes that the expression for a single electron at the surface describes the surface as via relativity all electrons follow similar trajectories), travelling at speed \mathbf{v}'

$$-e(\mathbf{v}'(x'_e) \times (\mathbf{B}'_L(x'_e) + \mathbf{B}'_i(x'_e)) \cdot \hat{\mathbf{x}} + E'_C(x'_e)) = 0, \quad (2.39)$$

where

$$B'_L = \frac{m_e \omega'_L a_0 \sin(\omega'_L t' - k' x'_e)}{e} \hat{\mathbf{z}} \quad (2.40)$$

and B_i originates from the uncompensated ion current, $\mathbf{J}_i = Z e n'_i(x'_e) \mathbf{v}_d$, where the electron fluid has been displaced. As before, from equation 1.9,

$$E'_C = \frac{e n'_e x'_e}{\epsilon_0}. \quad (2.41)$$

Note that there is no contribution to the laser magnetic field here from the reflected laser pulse since the assumption is that during this pushing phase all laser pulse energy is converted into electrostatic potential energy, this is supported by the attosecond duration of the reflected harmonic beam (*i.e.* it is not produced during this phase). Maxwell-Ampère's Law states

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \quad (2.42)$$

Noting that by symmetry there can be no variation in the magnetic field with y' or z' it becomes clear that

$$-\frac{d(\mathbf{B}'_i)_{z'}}{dx'} = \mu_0 (\mathbf{J}_i)_{y'}. \quad (2.43)$$

Integrating equation 2.43 from $-\infty$ to x'_e , noting that $\mathbf{B}_i = 0$ at infinity and assuming a constant density profile n'_i for $x > 0$,

$$\mathbf{B}'_i(x'_e) = \mu_0 e n'_e x'_e c \sin(\theta) \hat{\mathbf{z}}. \quad (2.44)$$

Using equations 2.40, 2.41 and 2.44 and making the very reasonable approximation that the relativistic electrons on the surface move at speed $v'_y \approx \pm c$ at peak displacement ($x'_e = x'_p$), 2.39 can be written as

$$-e \left(\pm c \left(\pm \frac{m_e \omega'_L a_0}{e} + \mu_0 e n'_e x'_p c \sin \theta \right) + \frac{e n'_e x'_p}{\epsilon_0} \right) = 0. \quad (2.45)$$

Note that to be in the laser pushing phase the first term must be negative, corresponding to \mathbf{v}' and \mathbf{B}'_L having the opposite sign, hence,

$$c \left(-\frac{m_e \omega'_L a_0}{e} \pm \mu_0 e n'_e x'_p c \sin \theta \right) + \frac{e n'_e x'_p}{\epsilon_0} = 0, \quad (2.46)$$

where here the \pm tracks the sign of \mathbf{v}' . After some manipulation, one arrives at

$$x'_p = \frac{1}{k' S' (1 \pm \sin \theta)}. \quad (2.47)$$

Transforming back to the lab frame, naturally,

$$x_p = \frac{1}{k S (1 \pm \sin \theta)}. \quad (2.48)$$

Already this is quite a result, reducing to equation 2.30 for $\theta = 0$ and predicting the suppression and enhancement of the two surface oscillations per laser pulse cycle. Explicitly, for a laser pulse propagating at $y = x \tan \theta$, the peak displacement of the electron surface is enhanced for \mathbf{B}_L in the $+\hat{\mathbf{z}}$ -direction and suppressed for \mathbf{B}_L in the $-\hat{\mathbf{z}}$ -direction.

Consider now acceleration of the electron bunch across the pseudocapacitor field in the boosted frame,

$$T' = \int \mathbf{F}' \cdot d\mathbf{s}' = \int_{x'_p}^0 -e E'_C(x'_e) dx'_e = \frac{e n'_e (x'_p)^2}{2 \epsilon_0} = \frac{1}{2} m_e c^2 \frac{a_0^2}{\bar{n}'_e (1 \pm \sin \theta)^2}. \quad (2.49)$$

Again, transforming back to the lab frame, noting the gain in energy from crossing the pseudo-capacitor

The linearity of four-vectors ensures

$$\Delta \mathbf{P}^\mu = \left(\frac{\Delta E}{c}, \Delta \mathbf{p} \right) \quad (2.50)$$

is also a four-vector. The Lorentz transform for change in energy is thus

$$\Delta E = \gamma \left(\Delta E' - \frac{\mathbf{v}_d}{c} \cdot \Delta \mathbf{p}' \right), \quad (2.51)$$

where

$$\gamma = \frac{1}{\sqrt{1 + \sin^2 \theta}} = \frac{1}{\cos \theta}. \quad (2.52)$$

Hence for energy gain $\Delta E' = T'$ in the boosted frame (where $\Delta p_y = 0$),

$$T = \gamma T'. \quad (2.53)$$

Using equations 2.49 and 2.52 and recalling $\bar{n}_e = \bar{n}'_e/\gamma$,

$$T = m_e c^2 \frac{a_0 S}{2(1 \pm \sin \theta)^2} \quad (2.54)$$

Nb this could have simply been established in the lab frame and considering F.ds which is just the same [The electron bunch then accelerates across the pseudocapacitor in the specular direction, the force] maybe put this derivation in appendix.

Therefore, as with peak displacement, the energy gained by the electron bunch via the ZVP mechanism is suppressed in one half cycle and enhanced in the second.

Currently this model favours an angle of incidence of $\pi/2$, however this does not match with observations. The resolution comes by considering the maximum energy gained crossing the pseudocapacitor before interaction with the next peak of the laser pulse (and therefore launching of the electron bunch back towards the plasma bulk). If both electron bunch and laser pulse move at speed c , the electron bunch will reach the next peak of the laser pulse after propagating a distance $\lambda_L/4$ along the x -axis. Adjusting the limits of equation 2.49,

$$T = \begin{cases} m_e c^2 \frac{a_0 S}{2(1 \pm \sin \theta)^2} & x_p \geq \lambda_L/4 \\ m_e c^2 \frac{a_0 S}{2(1 \pm \sin \theta)^2} - & x_p < \lambda_L/4 \end{cases} \quad (2.55)$$

Big note: The highest electron gamma factors comes from those electrons at the very front surface who are accelerated before high density is reached and quasistatic equilibrium reached (there are therefore very few of them), these few electrons do not satisfy ZVP relation for energy, instead ponderomotive + ZVP which at most would be twice the predicted gamma which is quite some difference

HOWEVER for all experiments so far, ZVP energy gain is small since S large so ponderomotive is more sensible.

Also mention alex's point about using 2D simulations.

Earlier write something along the lines of explaining while ZVP absorption does represent laser energy absorption, bulk heating occurs rather indirectly.

Perhaps derive equivalence between res and ZVP here?

Then section of a typical bunch properties

Then Energy scalings

Then QED? Then experiment?

Appendices

Cor animalium, fundamentum est vitæ, princeps omnium, Microcofmi Sol, a quo omnis vegetatio dependet, vigor omnis & robur emanat.

The heart of animals is the foundation of their life, the sovereign of everything within them, the sun of their microcosm, that upon which all growth depends, from which all power proceeds.

— William Harvey [harvey__exercitatio__1628]



Review of Cardiac Physiology and Electrophysiology

Appendices are just like chapters. Their sections and subsections get numbered and included in the table of contents; figures and equations and tables added up, etc. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Sed et dui sem. Aliquam dictum et ante ut semper. Donec sollicitudin sed quam at aliquet. Sed maximus diam elementum justo auctor, eget volutpat elit eleifend. Curabitur hendrerit ligula in erat feugiat, at rutrum risus suscipit. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Integer risus nulla, facilisis eget lacinia a, pretium mattis metus. Vestibulum aliquam varius ligula nec consectetur. Maecenas ac ipsum odio. Cras ac elit consequat, eleifend ipsum sodales, euismod nunc. Nam vitae tempor enim, sit amet eleifend nisi. Etiam at erat vel neque consequat.