Suitably impressive thesis title

Robin Timmis

Your College
University of Oxford

A thesis submitted for the degree of
Doctor of Philosophy

Michaelmas 2014

Abstract

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Acknowledgements

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List of Figures

x

A List of Symbols and Abbreviations

e . . . . Absolute charge of an electron $= 1.602 \times 10^{-19}\,\text{C}$

ϵ_0 . . . . Permittivity of free space $= 8.854 \times 10^{-12}\,\text{F}\,\text{m}^{-1}$

λ_D . . . . Debye length $\equiv \sqrt{\frac{\epsilon_0 K T_e}{n_e e^2}}$

n_e . . . . Plasma electron number density as a function of position

n_i . . . . Plasma ion number density as a function of position

T_e . . . . Plasma electron temperature

Z . . . . Ion charge state in units of e

1D, 2D, 3D . . One-, two- or three-dimension(al)

Otter . . . . . One of the finest of water mammals.

Hedgehog . . . Quite a nice prickly friend.

*Neque porro quisquam est qui dolorem ipsum quia
dolor sit amet, consectetur, adipisci velit...*

*There is no one who loves pain itself, who seeks after
it and wants to have it, simply because it is pain...*

— Cicero's *de Finibus Bonorum et Malorum*

1

Introduction

Contents

1.1   A plan

Actually the very first thing to say will be the stuff about light i think, about diagnostic tools essentially always being about controlling the various properties of light (ie electromagnetic waves)

sections to include start with the laser and our entrance into the multi-petawatt regime with no signs of stopping - ref 40 in alex savin thesis - it looks like that is saying huge growth in laser power check that and maybe write in thesis but don't include the figure. Also include description of CPA and OPCPA what is a plasma modelling plasma with PIC codes intense lasers and absorption mechanisms?? yes for sure but dont do all just those just below ZVP simulation units and similarity parameter Frames of reference - lab, sim, ablating front sruface of plasma

what is the story? Based on first section by Alex after the abstract Plasma is ubiquitous in our known universe and plasma provides us huge opportunities as a tool to improve our lives (From chen) what we can see in the sky is a result of that stuff being in the plasma state. Lasers can do so much now and are only getting more powerful all the time thanks to CPA and since developments (discuss) Simulataneously our ability to understand the physics has been aided by an explosion in computing power (peter HEDP paper) In this thesis we discuss some of the opportunities that relativistic laser plasma physics offers us with solid density targets - note that note about solids v gases at this point. Perhaps even before the debye length, define what we mean by the temperature of the plasma??

An unused statement about ion immobility Assume for now that the ion-electron mass ratio is infinite, that is to say the ions are approximately immobile for the timescales under consideration, generally true for a fair few relativistic laser pulse cycles (In later sections the mobility of plasma ions will prove very important but for now this is ignored.).

1.2   The definition of a plasma

As outlined in F. Chen's definitive textbook 'Introduction to Plasma Physics and Controlled Fusion' [**chen20116**], a plasma must fulfil three criteria, namely,

1. Ionisation: a plasma must consist of both charged and neutral particles, of course this alone cannot define a plasma, any gas will contain some degree of ionisation;

2. Quasineutrality: while locally there can be (often extreme) electromagnetic forces and charge concentrations at work, over the length scales of the plasma, such forces are screened out and the plasma bulk remains net neutral in charge;

3. Collective behaviour: unlike in a gas where collisions dominate, the particles in a plasma generate electromagnetic fields that interact at a distance and

thus a particle's motion depends not only on its immediate viscinity but on the surrounding plasma conditions, indeed often it is the so-called 'collisionless' plasmas where collisions can be safely neglected that are of most interest, as is the focus of this thesis.

1.2.1   The Debye length

The Debye length describes the extent to which a plasma can shield electromagnetic fields within and so remain quasineutral. Consider an infinitely extending plasma with a test charge placed at some point, then what would be the potential $\phi(\mathbf{x})$? If the plasma had no kinetic energy, the charged particles would arrange themself immediately ajacent to the test charge and once this equilibrium state was reached there would be no electromagnetic fields present. Realistically the plasma will have some temperature, likely a very large temperature and so some particles will be able to escape the potential of the test charge and thus leak electromagnetic fields into the plasma bulk. Poisson's equation reads

$$\epsilon_0 \nabla^2 \phi = -e(Zn_i - n_e), \tag{1.1}$$

where $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ is the permittivity of free space, $e = 1.602 \times 10^{-19}\,\mathrm{C}$ is the charge of an electron, Z is the plasma ion charge in units of e and n_i and n_e are the number densities of plasma ions and electrons.

Since the electrons are significantly more mobile than the ions due to their lower mass, it is in general the electrons and not the ions that respond to the test charge and the ions can be assumed to provide a constant background of positive charge density. If the number density of electrons follows a Boltzmann temperature distribution in the presence of a potential energy $-e\phi$, then

$$n_e = n_{e,0} e^{e\phi/KT_e}, \tag{1.2}$$

where $n_{e,0}$ is the electron number density far from the test charge, $n_i = n_{e,0}/Z$ and KT_e is the electron temperature. Note that in plasmas it is very common for different species to have differing temperatures depending on the mechanism for energy absorption and the timescales for collisions compared to the timescale of the study.

Substituting equation 1.2 into equation 1.1 and Taylor expanding the exponential term in the limit that the plasma is weakly coupled ($e\phi << KT_e$), obtains

$$\nabla^2 \phi = \frac{\phi}{\lambda_D^2}, \tag{1.3}$$

where

$$\lambda_D \equiv \sqrt{\frac{\epsilon_0 KT_e}{n_e e^2}}, \tag{1.4}$$

is the *Debye length* and describes the thickness of the charge sheath surrounding the test charge. For quasineutrality to hold for the plasma bulk, its spatial dimensions must extend beyond a few Debye lengths.

1.2.2 The plasma parameter

In order for the above description to be statistically valid, there must be a large number of charged particles within the shielding sheath. The number of particles within the Debye sphere can be computed as

$$N_D = \frac{4}{3}\pi \lambda_D^3 n. \tag{1.5}$$

Note that, as discussed above, in most cases it is most suitable to choose the number density n to be the number density of electrons. To ensure the plasma is suitably ionised (criterion 1) and that the plasma engages in collective behaviour (criterion 3),

$$N_D >>> 1. \tag{1.6}$$

1.2.3 Collisionality and the plasma frequency

Collective behaviour not only depends on the ability for large numbers of particles to interact via electromagnetic forces but that these forces dominate over collisions in describing particle trajectories. Taking ω as the typical frequency of plasma oscillations and τ as the average time between collisions, for a plasma (as opposed to a gas) must satisfy
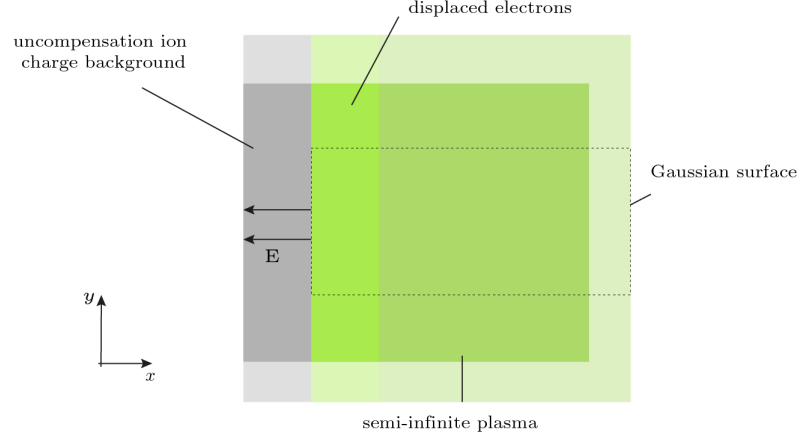
$$\omega\tau > 1. \tag{1.7}$$

Figure 1.1: Diagram to illustrate the plasma frequency derivation

It now remains to determine what is the typical frequency of collisions in a given plasma. While the types of plasma waves and their associated frequencies of oscillation are multitudinous, the characteristic frequency, the *plasma frequency*, ω_p, is the most straightforward. It describes the response of electrons to charge imbalances within an infinite uniform plasma at rest in the absence of magnetic fields or temperature fluctuations. As noted in section 1.2.1, the ions provide a constant backgorund of positive charge.

Consider an semi-infinite plasma existing for $x > 0$, with electron density n_e and ion density n_e/Z with charge state Z^1. Suppose the electron fluid is displaced into the plasma bulk a distance Δr as in figure 1.1. The displaced electrons will experience a restoring force, $-eE$, perpendicular to the surface due to the electron-ion charge imbalance. Applying the Lorentz force equation, the equation of motion for electrons on that surface is therefore

$$m_e \frac{\mathrm{d}^2 \Delta r}{\mathrm{d}t^2} = -eE. \tag{1.8}$$

Applying Gauss' law as described in figure blah, the uncompensated charge leads to

$$E = \frac{en_e \Delta r}{\epsilon_0}, \tag{1.9}$$

therefore

$$\frac{d^2 \Delta r}{dt^2} = -\frac{e^2 n_e}{m_e \epsilon_0} \Delta r. \tag{1.10}$$

---

[1]This description has direct relevance to the Zero Vector Potential mechanism which will be made clear later.

Equation 1.10 clearly describes a simple harmonic oscillator with a characteristic frequency given by the plasma frequency,

$$\omega_p = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}}.$$ (1.11)

2

The Zero Vector Potential Absorption Mechanism

Contents

2.1   Introduction

Now is presented the Zero Vector Potential mechanism of attosecond absorption, proposed by *Baeva et al* [**baeva__2006__TheoryHighorderHarmonic**] and later developed by *Savin et al* [**savin2017AttosecondscaleAbsorptionExtreme**, **savin2019EnergyAbsorptionLaserQED**]. Laser energy absorption in dense plasmas was first proposed by Wilks and Kruer [**wilks1997**], a ponderomotive mechanism where plasma electrons are heated directly by the laser pulse via the so-called $\mathbf{J} \times \mathbf{B}$ force.

This thesis focuses on the so-called 'post-ponderomotive' regime where the frequency of the plasma oscillations ($\omega_p \sim \sqrt{n_e}$) are greater than the $\mathbf{J} \times \mathbf{B}$ induced plasma electron oscillations at $2\omega_L$. The plasma electrons are then fast enough to compensate the ponderomotive pressure of the laser pulse with the formation of electrostatic fields between electrons and ions and so respond adiabatically to the

$\mathbf{J} \times \mathbf{B}$ force. Hence plasma electrons cannot be heated directly by the laser pulse. Note that this requires a sufficiently steep density gradient around the relativistic critical density surface (where $S = 1$) to shift the main interaction to a region where this condition on the overdensity is satisfied. In this case the ponderomotive pressure of the laser compresses the electrons at the front surface of the plasma and so shifts the laser-plasma surface interaction to plasma densities well beyond the relativistic critical density, leaving behind a positive space charge. This electron-ion charge separation leads to the formation of a *pseudo-capacitor* electrostatic field.

Interestingly working through the condition between ω_p and ω_L in normalised units suggests the criterion for this regime is $S > 4$, slightly more constraining than $S > 1$ as is typically stated [**savin2019thesis**].

[Come back to this and write up stuff about actually it being the relativistc frequncy and critical density surfaces that matter and how this does indeed put a stricter condition on the bulk S value]

So we have entered a regime of adiabaticity where the plasma skin layer is confined within a potential well consisting of the ponderomotive pressure and the Coulomb potential. Consider a relativistic linearly polarised laser pulse obliquely incident, with an angle of incidence of θ, on a semi-infinite plasma, existing for $x > 0$. The Hamiltonian of a single electron confined within the potential well is

$$\mathcal{H} = c\sqrt{m_e^2 c^2 + |\mathbf{p}|^2} + e\Phi. \tag{2.1}$$

Here the first term is the electron energy, U, extracted from the invariant of the relativistic 4-momentum of the electron, $\mathbf{P}^\mu = (U/c, \mathbf{p})$,

$$\mathbf{P}^\mu \cdot \mathbf{P}_\mu = \frac{U^2}{c^2} - |\mathbf{p}|^2 = m^2 c^2. \tag{2.2}$$

Note that while there has been growing interest in the curvature of spacetime by relativistic lasers [cite edward here], for modern high power lasers this effect is small and not relevant for this thesis. Throughout the inner product of 4-vectors is defined with the Minkowski Metric. [find alex citation on pg 89 of thesis]

The second term of equation 2.1 describes the contribution to the electron's energy from the electrostatic potential of the pseudo-capacitor. Decomposing the electron's 3-momentum into orthogonal components: p_{prop}, along the laser propagation direction, p_{pol}, along the polarisation axis of the laser pulse and $p_\perp$, perpendicular to both, two simplifications can be made. Firstly, by canonical conservation of transverse momentum, $p_{\text{pol}} = eA$, where A is the laser vector potential. Secondly, in the case of a *p*-polarised laser pulse (the known optimum for ZVP electron bunch generation), the forces at play confine the electron trajectory to the p_{prop}-p_{pol} plane and the interaction geometry is in essence two-dimensions (2D).

[include a diagram alluding to this?-it is basically since B is out of the plane and all other E fields are in the plane, also perhaps provide a foot not here to explain how incidentally this all provides a succinct explanation of why p is better?]

Explicitly, the Hamiltonian is now

$$\mathcal{H} = c\sqrt{m_e^2 c^2 + p_{\text{prop}}^2 + e^2 A^2} + e\Phi. \tag{2.3}$$

From equation 2.3 it is clear that should the vector potential pass through zero, one of the walls of the potential well is suppressed, allowing electrons in the in the skin layer to escape the plasma, breaking adiabaticity. The necessity of vector potential zeros for this violent reconstruction of the plasma surface led Baeva et al [**baeva2011ZeroVectorPotential**] to coin the term 'Zero Vector Potential' mechanism to describe this process. Indeed, while in standard calculations a laser pulse will exponentially decay within a skin layer without passing through zero, Baeva et al [**baeva2011ZeroVectorPotential**] were able to demonstrate in **PIC!** (**PIC!**) simulations that for this regime, zeros are able to propagate through the skin layer of the plasma. The explanation for this difference in mechanics relies on a Doppler shift in the laser field due to the relativistic motion of the ablating plasma surface, and the mathematical formalism of this process proceeds as follows.

[I think before this point it would be good to enter in the language of electron bunches or sheaths - I will continue the dicussion assuming this concept has been introduced].

As the Zero Vector Potential (ZVP) mechanism is a relativistic phenomenon, it is essential to consider the laser pulse propagating through a relativistically ablating electron bunch (i.e. with some component of its velocity anti-parallel to the laser pulse propagation direction). Transforming to the rest frame of the ablating front, beyond the relativistic critical density surface, the vector potential of the laser pulse will be an evanescent wave, at the spatial centre of the laser pulse, it can be described simply by

$$\mathbf{A}'_{\mathrm{L}}(t', r') = A'_0 \cos(\omega'_{\mathrm{L}} t') \exp(-r'/\delta') \hat{\mathbf{r}}'_{\mathrm{pol}} = A'_{\mathrm{L}} \hat{\mathbf{r}}'_{\mathrm{pol}}, \qquad (2.4)$$

where the primed symbols indicate that these quantities are measured in the rest frame of the expanding front, A'_0 is the vector potential amplitude and ω'_{L} is the frequency of the laser pulse, r' is the propagation distance of the laser into the plasma, δ' is the skin depth and $\hat{\mathbf{r}}'_{\mathrm{pol}}$ a unit vector defining the polarisation direction of the laser pulse. Un-primed coordinates will indicate the lab frame measurements.

[For sure include a diagram of this]

While in previous demonstrations of the vector potential zeros, it was assumed that the ablation occurs normal to plasma surface, it is now known that this ablation occurs in the specular reflection direction and it is necessary to confirm that zeros are still predicted. Consider a p-polarised laser pulse confined to the x-y plane incident with an angle of incidence θ on an ablating overdense plasma expanding with velocity $-v_f \hat{\mathbf{x}}$ in the lab frame, as in figure 2.1. The direction of polarisation is

$$\hat{\mathbf{r}}_{\mathrm{pol}} = \hat{\mathbf{x}} \sin 2\theta - \hat{\mathbf{y}} \cos 2\theta \qquad (2.5)$$

and the velocity of the rest frame of the ablating front relative to the lab frame is $-v_f \hat{\mathbf{x}}$.

Applying the Lorentz transformation to the electromagnetic 4-potential,

$$\mathbf{A}_\mu = (\phi/c, \mathbf{A}), \qquad (2.6)$$

explicitly,

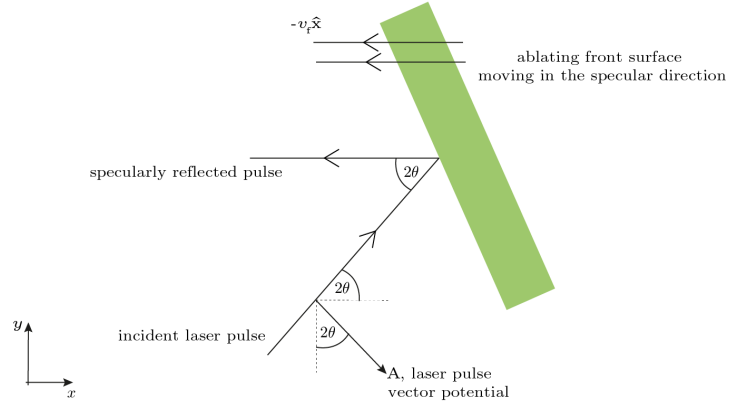$$\mathbf{A}'_\mu = \Lambda^\mu_\nu \mathbf{A}'_\nu, \qquad (2.7)$$

Figure 2.1: Diagram of a *p*-polarised laser pulse incident on an ablating overdense plasma. The laser is incident obliquely at an angle of θ and is reflected specularly. The plasma ablates specularly also. The interaction geometry is confined to a 2D plane.

where Λ^μ_ν, the Lorentz transform in this geometry is

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.8}$$

and here $\beta = -v_{\rm f}/c$, $\gamma = 1/\sqrt{1-\beta^2}$. Immediately from the y-coordinate transformation,

$$A'_{\rm L} \cos 2\theta' = A_{\rm L} \cos 2\theta. \tag{2.9}$$

Applying the headlight effect for a source moving at an angle 2θ to the boosted frame,

[I still need to include the headlight effect derivation]

$$\cos(2\theta') = \frac{\cos(2\theta) - \beta}{1 - \beta \cos(2\theta)} \tag{2.10}$$

and rearranging equation 2.9, the vector potential in the lab frame is

$$A_{\rm L} = \frac{1 - \beta \sec(2\theta)}{1 - \beta \cos(2\theta)} A'_0 \cos(\omega'_L t') \exp(-r'/\delta'). \tag{2.11}$$

Writing the boosted frame space-time coordinates in terms of the lab frame coordinates,

$$ct' = \gamma(ct - \beta x), \tag{2.12}$$

$$x' = \gamma(x - \beta ct), \tag{2.13}$$

yields

$$A_{\mathrm{L}} = A_0 \cos\left(\omega_L t - kx\right) \exp\left(-\frac{\sqrt{(x-\beta c t)^2 + (y/\gamma)^2}}{\delta}\right), \qquad (2.14)$$

where

$$A_0 = \frac{1 - \beta \sec\left(2\theta\right)}{1 - \beta \cos\left(2\theta\right)} A'_0, \qquad (2.15)$$

$$\omega_L = \gamma \omega'_L, \qquad (2.16)$$

$$k = \frac{\beta \gamma \omega'_L}{c}, \qquad (2.17)$$

$$\delta = \frac{\delta'}{\gamma}. \qquad (2.18)$$

The oscillatory term in equation 2.14 demonstrates the propagation of vector potential zeros within the plasma target. From the structure of this term it would appear that these zeros are expelled from the plasma along the specular direction at a speed (recall beta is negative - change this earlier in the theory so that that negative sign is more explicit in the result.)

$$v_\phi = \frac{\omega_L}{k} = \frac{c}{\beta} = -\frac{c^2}{v_{\mathrm{f}}}. \qquad (2.19)$$

Could also discuss here about how relativistic similarity theory derives that zeros move at speed c but how that cannot be valid since then we would always have infinitely thin radiation pulses, unless there is an extended range of zero? I suppose there is some radiation happening around the peak? Good questions.. One remaining consideration is we require that the zero gets through the whole electron bunch which is generally at very high density but is also very thin, in a way is this skin depth not what precisely determines the bunch width? The bunch will be compressed until the skin depth goes to zero across it perhaps? Things to think about.

Appendices

Cor animalium, fundamentum eft vitæ, princeps omnium, Microcofmi Sol, a quo omnis vegetatio dependet, vigor omnis & robur emanat.

The heart of animals is the foundation of their life, the sovereign of everything within them, the sun of their microcosm, that upon which all growth depends, from which all power proceeds.

— William Harvey [**harvey_exercitatio_1628**]

A

Review of Cardiac Physiology and Electrophysiology

Contents

Appendices are just like chapters. Their sections and subsections get numbered and included in the table of contents; figures and equations and tables added up, etc. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Sed et dui sem. Aliquam dictum et ante ut semper. Donec sollicitudin sed quam at aliquet. Sed maximus diam elementum justo auctor, eget volutpat elit eleifend. Curabitur hendrerit ligula in erat feugiat, at rutrum risus suscipit. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Integer risus nulla, facilisis eget lacinia a, pretium mattis metus. Vestibulum aliquam varius ligula nec consectetur. Maecenas ac ipsum odio. Cras ac elit consequat, eleifend ipsum sodales, euismod nunc. Nam vitae tempor enim, sit amet eleifend nisi. Etiam at erat vel neque consequat.

A.1   Anatomy

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A.2   Mechanical Cycle

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A.3   Electrical Cycle

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A.4 Cellular Electromechanical Coupling

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