$\ell\,(\ell+1)$-weighted extrapolation equations

Let $D_\ell = \ell\,(\ell+1)\,C_\ell$, then we require for the linear extrapolation that

$$D_\ell = D_L + (\ell - L)\,\frac{dD_\ell}{d\ell}, \tag{1}$$

where L is the ℓ of the last known C_ℓ, and

$$\frac{dD_\ell}{d\ell} = \frac{D_L - D_{L-1}}{L-(L-1)} = \frac{D_L - D_{L-1}}{L-L+1} = D_L - D_{L-1}. \tag{2}$$

So

$$D_\ell = D_L + (\ell - L)\,(D_L - D_{L-1}) = (\ell - L + 1)\,D_L + (-\ell + L)\,D_{L-1}. \tag{3}$$

Inserting the definition of D_ℓ gives

$$\begin{aligned}\ell\,(\ell+1)\,C_\ell &= (\ell - L + 1)\,L\,(L+1)\,C_L + (-\ell + L)\,(L-1)\,(L-1+1)\,C_{L-1}\\ &= (\ell - L + 1)\,L\,(L+1)\,C_L + (-\ell + L)\,L\,(L-1)\,C_{L-1},\end{aligned} \tag{4}$$

so

$$C_\ell = \frac{(\ell - L + 1)\,L\,(L+1)}{\ell\,(\ell+1)}C_L + \frac{(-\ell + L)\,L\,(L-1)}{\ell\,(\ell+1)}C_{L-1}. \tag{5}$$

So as a matrix, if we have a matrix $\mathbf{M}$ that takes us from a vector of C_ℓs $\mathbf{C}'$ to an extrapolated vector $\mathbf{C}$ as

$$\mathbf{C} = \mathbf{M}\mathbf{C}', \tag{6}$$

then its elements $M_{\ell\ell'}$ will be given by

$$M_{\ell\ell'} = \begin{cases} \delta_{\ell\ell'} & \text{if } \ell <= L;\\ \dfrac{(-\ell + L)\,L\,(L-1)}{\ell\,(\ell+1)} & \text{if } \ell > L \text{ and } l' = L-1;\\ \dfrac{(\ell - L + 1)\,L\,(L+1)}{\ell\,(\ell+1)} & \text{if } \ell > L \text{ and } l' = L;\\ 0 & \text{otherwise.}\end{cases} \tag{7}$$