1 Question

I would like to visualize the equivalence between copula density and marginal density: Based on a trivariate distribution, I would like to visualize the bivariate conditional distribution on the marginal and the copula level. I am uncertain if my theoretical understanding is fully correct because I fail to correctly visualize the copula distribution.

1.1 Marginal Conditional Distribution

The easy part is the conditional marginal distribution. It is simply given by:

$$f(x_1, x_2|x_3) = c(u_1, u_2, u_3)f(x_1)f(x_2).$$

1.2 Copula Conditional Distribution

Here is where I am a bit confused. First, I establish the following:

- Conditional copula

$$C_{12|3}(u_1|u_3, u_2|u_3) = \frac{d}{du_3}C_{123}(u_1, u_2, u_3).$$

- Conditional copula density

$$\frac{d^2C_{12|3}}{du_1du_2} = c_{12|3}(u_1|u_3, u_2|u_3) = c_{123}(u_1, u_2, u_3).$$

(See source p. 21 and table p.23)
Mathematically, I understand the equivalence between joint copula density and conditional copula density: The conditional copula given the conditional probabilities is equivalent to the joint copula given unconditional probabilities, right?

How does this fit into the deconstruction used by Vine copulas?

$$c(u_1, u_2, u_3) = c_{13}(u_1, u_2)c_{23}(u_2, u_3)c_{12|3}(u_1|u_3, u_3|u_3).$$

Using the fact that $c_{12|3} = c_{123}$, we would have $c_{12} \cdot c_{23} = 1$ which seems wrong.
Also, regarding my initial goal: Using contour plots, I expected the joint and the conditional copula density to look the same due to their equality. And I also expected the maximizer of the marginal conditional bivariate density to be the maximizer of the copula densities. Neither is the case.

That is, I expected $x_1^*, x_2^* = argmax_{x_1,x_2}f(x_1, x_2|x_3)$ to also maximize the conditional copula density after the following transformation:

Maximizer of joint density in x_1 dimension

$$u_1^* = F(x_1^*).$$

Maximizer of conditional density in x_1 dimension

$$u_1^*|u_3 = h_{1|3}^{-1}(u_1^*|u_3).$$

Where the h-function is given by:

$$h_{1|3}(u_1, u_3) = \frac{d}{du_3} C_{13}(u_1, u_3).$$

Vice versa for u_2. Also, if the above would hold, and joint copula density is equal to conditional copula density, would the maximizers not be equal, too? i.e. $u_1^* = u_1^*|u_3$?

So my questions are:

- Is my understanding of the mathematical equivalence correct? (The conditional copula given the conditional probabilities is equivalent to the joint copula given unconditional probabili- ties)
- Why are their contours not identical if they are identical in value?
- How does this fit into the vine deconstruction? $(c(u_1, u_2, u_3) = c_{13}(u_1, u_2)c_{23}(u_2, u_3)c_{12|3}(u_1|u_3, u$
- Are my thoughts regarding the maximizers wrong?