Part-1.1

$$z = f(x,y) = ax + by + c$$

$$\frac{\partial z}{\partial x} = a + 0 + 0 = a$$

$$\frac{\partial z}{\partial y} = 0 + b + 0 = b$$

$$\nabla f(x,y) = \begin{bmatrix} a \\ b \end{bmatrix}$$

Part-1.2

$$z = f(X) = f(x_1, x_2, \cdots, x_N)$$

$$= \sum_{i=1}^{N} a_i (x_i - b_i) + S$$

$$= a_1 x_1 + a_2 x_2 + \cdots + a_N x_N + d$$

$$\frac{\partial z}{\partial x_1} = a_1 + 0 + \cdots + 0 = a_1$$

$$\frac{\partial z}{\partial x_2} = 0 + a_2 + \cdots + 0 = a_2$$

$$\vdots$$

$$\frac{\partial z}{\partial x_N} = 0 + 0 + \cdots + a_N = a_N$$

$$\nabla f(x,y) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

Part-1.3

$$Z = f(x, y) = A(x-x_0)^2 + B(y-y_0)^2 + C$$

$$Z = A(x^2 - 2x \cdot x_0 + x_0^2) + B(y-y_0)^2 + C$$

$$= A(x-x_0)^2 + B(y^2 - 2y \cdot y_0 + y_0^2) + C$$

$$\frac{\partial Z}{\partial x} = 2Ax - 2A \cdot x_0$$

$$= 2A(x-x_0)$$

$$\frac{\partial Z}{\partial y} = 2By - 2B \cdot y_0$$

$$= 2B(y-y_0)$$

$$\nabla f(x, y) = \begin{bmatrix} 2A(x-x_0) \\ 2B(y-y_0) \end{bmatrix}$$

Part-1 . 4

$$X^T = (3 \quad 1 \quad 4) \quad [1\times3]$$

$$y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \quad [3\times1]$$

$$B^T = \begin{pmatrix} 5 & 2 & 4 \\ 3 & 5 & 1 \end{pmatrix} \quad [2\times3]$$

$$X \cdot X = (9 + 1 + 16) = 26$$

$$X \cdot y^T = (6 + 5 + 4) = 15$$

$$X \times y = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix} \quad [3\times3]$$

$$y \times X = (2\times3 + 1\times5 + 4\times1)$$
$$= (15) \quad [1\times1]$$

$$A \times X = \begin{pmatrix} 12+5+8 \\ 9+1+20 \\ 18+4+12 \end{pmatrix} = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix} \quad [3\times1]$$

$$A \times B = \begin{pmatrix} 39 & , & 38 \\ 19 & , & 37 \\ 41 & , & 50 \end{pmatrix} \quad [3\times2]$$

B.reshape $(1,6) = (3 \quad 5 \quad 5 \quad 2 \quad 1 \quad 4) \quad [1\times6]$

Part-1 : LLS - Single Variable.

$$L(m, b) = \sum_{i=1}^{N} (y_i - M(\hat{x_i}, m, b))^2$$

$$= \sum_{i=1}^{N} (y_i - m\hat{x_i} - b)^2$$

$$\frac{\partial L}{\partial m} = 2 \cdot \sum_{i=1}^{N} (y_i - m\hat{x_i} - b) \cdot (-\hat{x_i})$$

$$= -2 \sum_{i=1}^{N} (y_i - m\hat{x_i} - b) \cdot \hat{x_i}$$

$$= -2 \sum_{i=1}^{N} (y_i \hat{x_i} - m\hat{x_i}^2 - b\hat{x_i})$$

$$= -2 \sum_{i=1}^{N} y_i \hat{x_i} + 2 \sum_{i=1}^{N} m\hat{x_i}^2 + 2 \sum_{i=1}^{N} b\hat{x_i}$$

$$\frac{\partial L}{\partial b} = 2 \sum_{i=1}^{N} (y_i - m\hat{x_i} - b) \cdot (-1)$$

let $0 = -2 \sum_{i=1}^{N} (y_i - m\hat{x_i} - b) \cdot$

$$\Rightarrow \sum_{i=1}^{N} (y_i) = \sum_{i=1}^{N} m\hat{x_i} + N \cdot b$$

$$N \cdot \bar{y_i} = m \cdot N \cdot \bar{x_i} + N \cdot b$$

$$b = \bar{y_i} - m \cdot \bar{x_i}$$

Let $\dfrac{\partial L}{\partial m} = 0$, we have:

$$-\sum_{i=1}^{N} x_i y_i + m \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i = 0$$

$$\begin{cases} \sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i \\ b = \bar{y}_i - m \bar{x}_i \end{cases}$$

$$\Rightarrow \sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + (\bar{y} - m\bar{x}) \sum_{i=1}^{N} x_i$$

$$\sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + \bar{y} \sum_{i=1}^{N} x_i - m \sum_{i=1}^{N} \bar{x} \cdot x_i$$

$$m \left(\sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} \bar{x} \cdot x_i \right) = \sum_{i=1}^{N} x_i y_i - \bar{y} \sum_{i=1}^{N} x_i$$

$$\begin{cases} m = \dfrac{\sum_{i=1}^{N} (x_i y_i - x_i \bar{y})}{\sum_{i=1}^{N} (x_i^2 - x_i \bar{x})} \\ \\ \bar{x} = \dfrac{1}{N} \sum_{i=1}^{N} x_i \ ; \ \bar{y} = \dfrac{1}{N} \sum_{i=1}^{N} y_i \end{cases}$$

$$\Rightarrow m = \dfrac{\sum_{i=1}^{N} x_i y_i - \dfrac{1}{N} \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \dfrac{1}{N} \cdot \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} x_i}$$

$$= \dfrac{\sum_{i=1}^{N} x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\sum_{i=1}^{N} x_i^2 - n \cdot \bar{x}^2}$$

$$= \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{N} X_i^2 - \bar{X} \sum_{i=1}^{N} X_i}$$

$$= \frac{COV(XY)}{\sum_{i=1}^{N} X_i^2 - 2\bar{X}\sum_{i=1}^{N} X_i + \bar{X}\sum_{i=1}^{N} X_i}$$

$$= \frac{COV(X,Y)}{\sum_{i=1}^{N} X_i^2 - 2\bar{X}\sum_{i=1}^{N} X_i + n \cdot \bar{X}^2}$$

$$= \frac{COV(X,Y)}{\sum_{i=1}^{N}(X_i - \bar{X})^2}$$

$$= \frac{COV(X,Y)}{Var(X)}$$

$$\Rightarrow b = \bar{Y} - \frac{COV(X,Y)}{Var(X)} \cdot \bar{X}$$

Multi-variable

$$\begin{cases} X = \{(x_1, y_1) \cdots (x_n, y_n)\} \\ Y = ax + b \end{cases}$$

$$Oss = SSE = S(a) = \sum_{i}^{N} (y_i - ax - b)^2$$

$$\frac{\partial S}{\partial a} = \sum 2(y_i - ax_i - b)(-x_i)$$

$$= 2\sum (x_iy_i + ax_i^2 + bx_i)$$

$$\frac{\partial S}{\partial b} = \sum 2(y_i - ax_i - b)(-1)$$
$$= \sum -2(y_i - ax_i - b)$$
$$= 2\sum(-y_i + ax_i + b)$$