

Part-1.1

$$Z = f(x, y) = ax + by + c$$

$$\frac{\partial Z}{\partial x} = a + 0 + 0 = a$$

$$\frac{\partial Z}{\partial y} = 0 + b + 0 = b$$

$$\nabla f(x, y) = \begin{bmatrix} a \\ b \end{bmatrix}$$

Part-1.2

$$Z = f(X) = f(x_1, x_2, \dots, x_N)$$

$$= \sum_{i=1}^N a_i (x_i - b_i) + c$$

$$= a_1 x_1 + a_2 x_2 + \dots + a_N x_N + c$$

$$\frac{\partial Z}{\partial x_1} = a_1 + 0 + \dots + 0 = a_1$$

$$\frac{\partial Z}{\partial x_2} = 0 + a_2 + \dots + 0 = a_2$$

$$\frac{\partial Z}{\partial x_N} = 0 + 0 + \dots + a_N = a_N$$

$$\nabla f(x, y) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

Part-1.3

$$Z = f(x, y) = A(x - x_0)^2 + B(y - y_0)^2 + C$$

$$Z = A(x^2 - 2x \cdot x_0 + x_0^2) + B(y - y_0)^2 + C$$

$$= A(x - x_0)^2 + B(y^2 - 2y \cdot y_0 + y_0^2) + C$$

$$\frac{\partial Z}{\partial x} = 2Ax - 2A \cdot x_0$$

$$= 2A(x - x_0)$$

$$\frac{\partial Z}{\partial y} = 2By - 2B \cdot y_0$$

$$= 2B(y - y_0)$$

$$\nabla f(x, y) = \begin{pmatrix} 2A(x - x_0) \\ 2B(y - y_0) \end{pmatrix}$$

Part 1.4

$$X^T = (3 \quad 1 \quad 4)$$

$$Y^T = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 5 & 2 & 4 \\ 3 & 5 & 1 \end{pmatrix}$$

$$X \cdot X : (9 + 1 + 16) = 26$$

$$X \cdot Y^T : (6 + 5 + 4) = 15$$

$$X \cdot Y = \begin{pmatrix} 6 & 15 & 3 \\ 2 & 5 & 1 \\ 8 & 20 & 4 \end{pmatrix}$$

$$Y \cdot X = (2 \times 3 + 1 \times 5 + 4 \times 1) \\ = (15)$$

$$A \times X = \begin{pmatrix} 12 + 5 + 8 \\ 9 + 1 + 20 \\ 18 + 4 + 12 \end{pmatrix} = \begin{pmatrix} 25 \\ 30 \\ 34 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 39, & 38 \\ 19, & 37 \\ 41, & 50 \end{pmatrix}$$

$$B.\text{reshape}(1,6) = (3 \quad 5 \quad 5 \quad 2 \quad 1 \quad 4)$$

Part-1 : LLS - Single Variable.

$$L(m, b) = \sum_{i=1}^N (y_i - \hat{m}(x_i^*), m, b)^2$$
$$= \sum_{i=1}^N (y_i - mx_i^* - b)^2$$

$$\frac{\partial L}{\partial m} = 2 \cdot \sum_{i=1}^N (y_i - mx_i^* - b) \cdot (-x_i^*)$$
$$= -2 \sum_{i=1}^N (y_i - mx_i^* - b) \cdot x_i^*$$
$$= -2 \sum_{i=1}^N (y_i x_i^* - mx_i^{*2} - bx_i^*)$$
$$= -2 \sum_{i=1}^N y_i x_i^* + 2 \sum_{i=1}^N mx_i^{*2} + 2 \sum_{i=1}^N bx_i^*$$

$$\frac{\partial L}{\partial b} = 2 \sum_{i=1}^N (y_i - mx_i^* - b) \cdot (-1)$$

let 0 = $-2 \sum_{i=1}^N (y_i - mx_i^* - b)$

$$\Rightarrow \sum_{i=1}^N (y_i) = \sum_{i=1}^N mx_i^* + n \cdot b$$

$$n \cdot \bar{y}_i = m \cdot n \cdot \bar{x}_i + n \cdot b$$

$$b = \bar{y}_i - m \cdot \bar{x}_i$$

Let $\frac{\partial L}{\partial m} = 0$, we have:

$$-\sum_{i=1}^N x_i y_i + m \sum_{i=1}^N x_i^2 + b \sum_{i=1}^N x_i = 0$$

$$\begin{cases} \sum_{i=1}^N x_i y_i = m \sum_{i=1}^N x_i^2 + b \sum_{i=1}^N x_i \\ b = \bar{y} - m \bar{x} \end{cases}$$

$$\Rightarrow \sum_{i=1}^N x_i y_i = m \sum_{i=1}^N x_i^2 + (\bar{y} - m \bar{x}) \sum_{i=1}^N x_i$$

$$\sum_{i=1}^N x_i y_i = m \sum_{i=1}^N x_i^2 + \bar{y} \sum_{i=1}^N x_i - m \sum_{i=1}^N \bar{x} \cdot x_i$$

$$m \left(\sum_{i=1}^N x_i^2 - \sum_{i=1}^N \bar{x} \cdot x_i \right) = \sum_{i=1}^N x_i y_i - \bar{y} \sum_{i=1}^N x_i$$

$$m = \frac{\sum_{i=1}^N (x_i y_i - x_i \bar{y})}{\sum_{i=1}^N (x_i^2 - x_i \bar{x})}$$

$$\left\{ \begin{array}{l} \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad ; \quad \bar{y} = \frac{1}{N} \sum_{i=1}^N y_i \end{array} \right.$$

$$\Rightarrow m = \frac{\sum_{i=1}^N x_i y_i - \frac{1}{N} \sum_{i=1}^N x_i \cdot \sum_{i=1}^N y_i}{\sum_{i=1}^N x_i^2 - \frac{1}{N} \cdot \sum_{i=1}^N x_i \cdot \sum_{i=1}^N x_i}$$

$$= \frac{\sum_{i=1}^N x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\sum_{i=1}^N x_i^2 - n \cdot \bar{x}^2}$$

$$= \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^N x_i^2 - \bar{x} \sum_{i=1}^N x_i}$$

$$= \frac{\text{cov}(X, Y)}{\sum_{i=1}^N x_i^2 - 2\bar{x} \sum_{i=1}^N x_i + \bar{x} \sum_{i=1}^N x_i}$$

$$= \frac{\text{cov}(X, Y)}{\sum_{i=1}^N x_i^2 - 2\bar{x} \sum_{i=1}^N x_i + n \cdot \bar{x}^2}$$

$$= \frac{\text{cov}(X, Y)}{\sum_{i=1}^N (x_i - \bar{x})^2}$$

$$= \frac{\text{cov}(X, Y)}{\text{Var}(X)}$$

$$\Rightarrow b = \bar{y} - \frac{\text{cov}(X, Y)}{\text{Var}(X)} \cdot \bar{x}$$

Multi-variable

$$\begin{cases} X = \{(x_1, y_1) \dots (x_n, y_n)\} \\ Y = ax + b \end{cases}$$

$$\text{loss} = \text{SSR} = S(a) = \sum_{i=1}^n (y_i - ax - b)^2$$

$$\begin{aligned} \frac{\partial S}{\partial a} &= \sum_{i=1}^n 2(y_i - ax_i - b)(-x_i) \\ &= -2 \sum_{i=1}^n (x_i y_i + ax_i^2 + bx_i) \end{aligned}$$

$$\begin{aligned}
 \frac{\partial S}{\partial b} &= \leq 2(y_i - ax_i - b)(-1) \\
 &= \leq -2(y_i - ax_i - b) \\
 &= 2 \leq (-y_i + ax_i + b)
 \end{aligned}$$