

# Dragons as Rational Economic Agents in a Non-Competitive Environment

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## Abstract

The following paper describes a universe in which dragons are perfectly mobile, perfectly rational, and in possession of full information of their respective system. These dragons are human-fearing and fundamentally antisocial in nature, leading them to avoid interactions with humans and other dragons while still maximizing their consumption of prey. Dragons with full information and perfect mobility migrate frequently to maintain a steady caloric intake without perturbing any one region of humans to a degree to which the humans will feel compelled to hunt the dragons to extinction. The dragons will likely have a deleterious but not necessarily destructive impact on the larger ecological sub-systems. We believe dragons will primarily transition through four phases: Static Disequilibrium, Dynamic Disequilibrium, Dynamic Equilibrium, and Static Equilibrium. The early dragons will possess significant advantages with regards to resource availability, but the overall population of dragons will ultimately be capped at a small, finite size as the global threat of humans will likely force dragons to only exist in true harmony within regions unoccupied by humans at present—namely, the polar regions. The paper finishes with a letter suggesting changes to the "Games of Thrones" universe to author George R. Martin.

## 1 Introduction

This paper describes a mathematical solution to the question: what if dragons existed on Earth? The paper approaches the topic as a hyperbolic counter-argument of neoclassical theories regarding perfect competition. What if economic agents, rather than competing away all profits as information asymmetries become ever slimmer, chose to be fundamentally averse to risk and competition? What if agents instead sought to constantly exploit novel niches so that the system never settled on a singular equilibrium point, but rather existed in a state of non-linear dynamics which create a constant dynamic tension between the present known and the unknowable future? And what if these economic agents were not humans, but dragons: beings capable of understanding the pitfalls of mob mentality and choosing instead to live as antisocial nomads who

constantly exploit new, previously unforeseen niches within a closed system? This paper suggests a model which will seek to understand such a world; in which, rather than attempting to seek equilibrium or stasis points, the model acknowledges that the dragons will never truly complete their quest to avoid competition and thus will consequently produce a series of indeterminate emergent economic models.

## 2 Problem Statement

In the fictional television series Game of Thrones, three dragons are raised by Daenerys Targaryen, the “Mother of Dragons.” When hatched, the dragons are small, roughly 10 kg, and after a year grow to roughly 30-40 kg. They continue to grow throughout their life depending on the conditions and amount of food available to them. However, what if these dragons did not exist merely in the realm of imagination; but rather were alive on Earth today? How would dragon characteristics, behavior, habits, and diet affect their respective interaction with the environments? What needs and requirements—including area, caloric requirements, and climate conditions—would the dragons need to satisfy in order to survive? What would be the ecological impact of these dragons be?

## 3 Model

### 3.1 Dragon Characteristics

1. **Dragon Weight:** Dragons are born at 10 kg. Within the first year, these dragons grow to a weight ranging between 30-40 kg. The dragon’s weight proceeds over time and eventually reaches a maximum weight; this maximum weight is determined largely by the proportion of food available to the dragon over time.
2. **Dragon Abilities:** All dragons are able to breathe fire; they have extremely strong, scaly surfaces which make them resistant to trauma; they are able to travel great distances with little proportional energy expenditure; they have very keen senses; and they reproduce asexually as the benefit of reproducing with a mate is outweighed by cost of competition for limited resources.
3. **Climate Invulnerability:** Dragons are ambivalent to climate and temperature due to tough exteriors and a high ability to maintain homeostasis
4. **Apex Predator:** Dragons are the apex predators in conjunctions with humans. Only humans and other dragons can kill dragons.

### 3.2 Dragon Behavior

1. **Human-fearing:** The key factor of concern for dragons is human hunting, since humans demonstrate an ability and tendency to hunt dragons to extinction
2. **Rational, Profit-Maximizing Agents:** Dragons are rational profit-maximizing economic agents driven by incentives. Given the mythological history of humans hunting dragons, dragons are inherently risk averse. Therefore, dragons seek to optimize the quantity of prey available to them in a given region while minimizing the risk of interacting with humans or other dragons. In this situation we are assuming perfect information—because dragons are able to fly great distances and have highly keen senses, they are able to gather the sufficient quantity of information needed to evaluate the optimal habitat by a single flight over the globe.
3. **Antisocial, Non-Competitive Dragons:** Dragons are inherently antisocial, because the costs of being nearby humans and other dragons far outweigh the benefits of forming communities. However, dragons do exhibit a high degree of collaboration as a result of this antisocial behavior. All dragons possess the same knowledge regarding regions of the world and the risk and reward for each region—but because dragons are antisocial, they prefer occupying their own unique niche to competing with a larger or equivalently-sized dragon for a marginally more advantageous niche. Therefore, dragons display an aversion to competition—which is intrinsically built upon information asymmetries which do not exist within this particular group. Instead, they independently sort themselves by mapping each dragon—ranked by size—to its correspondingly ranked geographic regions—determined by the ratio of perceived rewards versus perceived risks.
4. **Perfect Mobility:** Because dragons have perfect information, as the ratio of rewards to risk becomes disadvantageous, dragons are able to migrate to an unoccupied niche. They migrate frequently enough so that they rarely hunt a niche to extinction and do not experience large short term declines in their rate of growth.
5. **Sleep Patterns:** Dragons are primarily nocturnal creatures—as they co-adapted alongside humans, they recognized the optimal time for hunting occurs when humans are asleep.
6. **Hunting Patterns:** Dragons hunt everyday to satisfy their caloric needs.
7. **Flight Patterns:** Dragons fly over the surface of the Earth every morning to observe each ecoregion. If the marginal perceived net reward extracted from a particular region is exceeded by the perceived net reward of an unoccupied region, the dragon will migrate. Else, the dragon will remain put.

8. **Migratory Patterns:** If all ecoregions are occupied, the displaced dragon will enter the weakest dragon's ecoregion. Each dragon has a probability of victory proportional to their relative weights to each other.

### 3.3 Dragon Diet

1. **Caloric Requirements:** Everyday dragons require a quantity of calories proportional to their current weight as well as the weight of their growth.
2. **Diet Type:** Dragons are primarily carnivorous.

### 3.4 Interactions with Environment

1. **Ecological Impact:** Dragons are not inherently destructive beings because, due to their inherent self-interest, the cost of destroying the surrounding environment poses no intrinsic benefit to the dragon and ultimately wastes energy. Therefore, the dragon destroys land only inadvertently due to its need to hunt prey consistently. Dragons primarily hunt by breathing fire on their prey (in order to cook their prey, thus making for a far more satisfactory dining experience). Therefore, the ecological damage is directly proportional to the amount of prey hunted—which is itself simply a function of the dragon's caloric requirements.
2. **Caloric Intake:** Everyday dragons must consume a quantity of calories. Because dragons do not ever hunt a niche to extinction, unless the entire population of Earth was eradicated, dragons would find a food source. If a dragon's caloric intake cannot be met in a given niche, the dragon will migrate to a different niche.
3. **Area Requirements:** The Earth is divided into 40 subregions. Each region is approximately of equal area—this acknowledges that, regardless of weight or size, dragons prefer to leave ample room between other dragons so as to avoid confrontation or competition.
4. **Community Requirements:** In order for the larger global community to support dragons without damaging the world's ecosystems, it would make sense for humans to provide high-calorie subsidies to offer dragons to offset their voracious appetites. Thus, the air dropping of care packages full of synthetic meat would enable human communities to minimize dragon migration and ensure minimal danger to human communities.

## 4 Simulation Setup

1. Divide the Earth into 40 distinct ecoregions. Each ecoregion will have 2 key factors:
  - (a) Dragon probability of being hunted: varies proportionally with regard to human population density

- (b) Estimated relative prey population per land area.
- 2. Initialize three objects of type dragon. Each dragon will have a weight and age determined by a normal distribution between age [0,4] years and weight[30,100] kg.
- 3. Create 40 objects of type eco-region.
- 4. Rank each eco-region using probability of being hunted  $\times \frac{\text{available calories}}{\text{livable land mass}}$ .
- 5. Assign the heaviest dragon the best land object, the second heaviest the second best, and so on.
- 6. Each day, the following tasks are carried out:
  - (a) Hunted - if the dragon is hunted, it dies; else, it lives.
  - (b) Hunt- the dragon gathers food fit for its caloric needs.
  - (c) Reproduce - every dragon is asexual. As such, every turn there is a  $\frac{1}{1000}$  \* weight chance of it producing an offspring on any given day. Only one offspring can be produced with each reproductive event. Once an offspring is produced, the child travels to the next optimum uninhabited eco-region.
  - (d) Move - Implement risk-reward function to assess dragon's migration choice.
- 7. Each day, we can assess ecological impact, prey population in the world and in each ecoregion, dragon weight, and so on.

## 5 Equations and Variables Governing the Model

### 5.1 Main Equations and Variables

- 1. **t**: Time, with a timescale of one day
- 2.  **$\omega(t)$** , Dragon weight growth over time: The rate of growth,  $\frac{d\omega}{dt}$ , will vary as a function of available prey within the population according to Lotka-Volterra Differential Models [2].

$$\frac{d\omega}{dt} = r(1 - \frac{\omega(t)}{\omega_{\max}})\omega(t)$$

- 3.  **$h(t)$** , Human population growth over time: Humans experience a small, but non-zero growth over the course of the simulation. Population density and growth rate are inversely related.

$$\frac{dh}{dt} = r(1 - \frac{h(t)}{h_{\max}})$$

4.  $e(\omega(t))$ . Ecological impact: Shows ecological damage as a function of dragon population growth. Frequent dragon migration reduces the chances of irreversible short term damage.

Impact varies proportionally with dragon weight (heavier dragons need more calories and hence cause more damage) until a limiting point of total destruction,  $E_{\max}$

$$e \propto (\omega(t)) \times \beta$$

5.  $P(\omega(t))$ , Prey population: Describes how prey population varies as a function of dragon population growth over time. Like ecological impact, frequent dragon migration reduces the chances of irreversible short term damage.

$$\begin{aligned}\frac{d\omega}{dt} &= \alpha\omega - \beta\omega p \\ \frac{dp}{dt} &= \delta\omega p - \lambda p\end{aligned}$$

Note\* We will add a detailed description of the coefficients  $\alpha, \beta, \delta, \lambda$  in the lengthened version of this paper

6.  $P(K)$ , Risk/probability of being hunted: The sum total probabilities of three key factors: human population size, ecological damage, and dragon size. Regions with higher human populations, greater ecological damage, and larger dragons cause dragons to have a higher chance of being hunted. Applying the Law of Total Probability, we have

$$P(K) = P(K|\frac{w(t)}{W_{\max}}) + P(K|\frac{H(t)}{H_{\max}}) + P(K|\frac{E(\omega(t))}{E_{\max}})$$

When  $\omega = W_{\max}$ ,  $H = H_{\max}$ , and  $E = E_{\max}$ , the dragon has a probability  $P(K)=1$  of being hunted.

7.  $R$ , the return function: Signifies the potential profit a dragon can attain by occupying a specific geographic region.

$$R = \frac{P(\omega(t))}{P(K)^2}$$

High values of  $P(\omega(t))$  are offset by high values of  $P(K)$  by a power of 2 because of dragon's intrinsic risk-aversion. Dragons must find a balance between the reward and threat that maximizes reward given the minimum upper bound of an allowable threat in the region.

## 5.2 Constants

Values for constants were taken with consideration from real-world predator prey systems.

1.  $\omega$ , density of predators. Weight of dragons per unit of land area.
2.  $\mathbf{p}$ , density of prey. Established in eco-region definition, correlated to real-world prey density of the corresponding area of the world.
3.  $\alpha$ , intrinsic rate of prey population increase. Random variable (0.3, 0.7)
4.  $\beta$ , predation rate coefficient. Random variable (0,1)
5.  $\delta$ , reproduction rate of predators per 1 prey eaten, Random variable (0.001,0.1)
6.  $\lambda$ , predator mortality rate, Random variable (0.3, 0.7)

## 5.3 Threat-Reward Modeling for Dragon Behavior

Dragons perceive the threat of human presence and the reward of increased prey presence. Dragons are naturally risk-averse as the threat of human hunting is more dangerous than living in low-prey-density regions. For the threat and reward functions, we use a profit model. Using a Mean Variance Optimization / Optimum Portfolio model, we optimize the position on the optimal portfolio.

$$\text{Net Gain} = \frac{\text{Reward}}{\text{Risk}^2}$$

$$\text{Net Gain} = \frac{P(\omega(t))}{\frac{1}{9} \left( \frac{e(\omega(t))}{e_{\max}} + \frac{h(t)}{H_{\text{total}}} + \frac{\omega(t)}{\omega_{\max}} \right)^2}$$

Dragons only switch from region 1 to region 2 if

$$\frac{\text{Reward}_1}{\text{Risk}_1^2} > \frac{\text{Reward}_2}{\text{Risk}_2^2}$$

## 6 Analysis

\*\*Note: Unfortunately, due to a last-minute loss of one of our team members, we did not have the time to adjust time schedules effectively and implement the model. Therefore, the following analysis will focus primarily on fundamental analysis of the governing equations without actual programmatic implementation. We recognize this is a significant weakness of this paper, but due to the unfortunate timing we believe that fundamental analysis will offer a necessary and rigorous (but clearly not sufficiently comprehensive) understanding.

We may examine four fundamental cases which would arise from the emergence of dragons on Earth:

1. **Static Disequilibrium**
2. **Dynamic Disequilibrium**
3. **Dynamic Equilibrium**
4. **Static Equilibrium**

Each of the following will be described below:

### 6.1 Static Disequilibrium

In the scenario in which dragons lack perfect information, perfect rationality, perfect mobility, and also possess a voracious appetite, this will represent the case of static disequilibrium. This would occur most often among young dragons who lack the ability to fly great distances, are still relatively immature, and have a higher caloric need to satisfy their early-stage growth. Under this situation, young dragons would settle into a singular region and consume all in their sight so that the rate of predation would far exceed the rate of prey growth:

$$\beta > \alpha \tag{1}$$

Under these conditions, there would be three probable outcomes (which would depend in equal parts on randomness and specific initial conditions):

1. All prey and wildlife is destroyed; as the dragon cannot satisfy its hunger, it will die out.
2. Due to the massive levels of wildlife destruction, humans will hunt down and kill the dragon.
3. The dragon will mature fast enough to develop rationality and the ability to fly and thus switch strategies to dynamic disequilibrium

This least-optimal case suggests that it will be extremely difficult and unlikely for young dragons to ever reach maturity (the antisocial nature of dragons suggests parents do not actually foster their young). Therefore, the actual maximum population potential of dragons will be significantly curbed by the high rate of death among young dragons. This of course will preserve the stability of the overall system and ensure enough food is in place; which is in part why the rational parents deliberately do not raise their young.

### 6.2 Dynamic Disequilibrium

Dynamic disequilibrium is characterized by younger dragons who have successfully evolved perfect mobility but still possess voracious appetites due to their high rate of growth early on and are only semi-rational. These dragons will have a beta level far exceeding alpha for any given region:

$$\beta \gg \alpha \tag{2}$$



These dragons will enter a region and cause tremendous damage so that the return gained is rapidly outpaced by the risk incurred and the dragon will be forced to migrate within only a few time steps. The dragon will migrate so frequently that geographic regions it decimated earlier in the simulation will not have time to revitalize (based on the intrinsic rates of prey growth and ecological restoration) before the dragon returns to this region, thus suggesting that the dragon's logical course of action will be to continue until Earth is decimated. These dragons will consequently have three possible fates:

1. It will invade another dragon's domain, and one of these two dragons will die with a probability proportional to the two dragons relative normalized weights.
2. It will cause enough destruction in every region with high human density that it will be classified an international threat and consequently will have a very high probability of being hunted down and killed.
3. It will mature sufficiently to become fully rational and switch strategies to Dynamic Equilibrium.

### 6.3 Dynamic Equilibrium

Dynamic Equilibrium is defined as the optimal strategy for the perfectly mobile and perfectly rational dragon in a world devoid of information asymmetries. As opposed to the prior positions of disequilibrium (in which dragons over-consume finite resources and are consequently hunted down at extreme rates), the dragon in dynamic equilibrium recognize two fundamental factors:

1. The caloric needs of a given mature dragon can never be satisfied by a single region.
2. If a dragon over-consumes any given region, it will be hunted down.

Therefore, the dragon must straddle the line between satisfying caloric requirements (as it is not yet finished growing) and over-consuming within a fixed region or within the system as a whole. Therefore, the dragon will have a beta value roughly equivalent to or slightly greater than the alpha value of a given region:

$$\beta \geq \alpha \tag{3}$$

The dragons semi-stable consumption rate will cause damage to prey populations and larger ecological systems, but not to the same disruptive degree of the prior two phase strategies. It will migrate when the return to risk ratio of a given region is outweighed by the next best option; but this will occur far less frequently, and overall the net rate of prey populations and ecological growth of the dragon's first occupied region will return to its state by the time the dragon cycles through every other region. Thus, the overall risk to return ratio will be balanced throughout the region and dragons will live in a dynamic harmony with the larger global ecosystem by taking advantage of their significant mobility.

## 6.4 Static Equilibrium

The static equilibrium will occur among the oldest dragons who have begun to lose their perfect mobility but who replace this mobility with optimized levels of wisdom (characterized by possessing higher levels of risk aversion and the recognition that the lowest probability of being hunted will occur in a stable equilibrium with few to no humans present). In this scenario, the dragons will curb their appetites so that the predation rate is always less than or at most equal to the prey growth rate:

$$\beta \leq \alpha \tag{4}$$

This is a reasonable assumption because more energy can be conserved by stable equilibrium in a fixed location versus a dynamic equilibrium by means of constant migration, so calories lost by restricting diet can be compensated for by minimizing other expenditures of energy. In addition, because these dragons are no longer growing, they will have far smaller appetites. Although the original ecosystem may experience a temporary disruption as a result of the dragon's emergence, over time the ecosystem will stabilize as the dragon establishes its particular niche within the environment. Under this model, dragons would almost never need to migrate. Dragons would likely settle into geographic zones with a high prey population, a sustainable level of prey growth, and a very small human presence. Under these conditions, the dragons would most likely exist in the polar regions. For the purpose of this paper, we will define this strategy as the end-game strategy of the wisest dragons who have survived thus far.

## 6.5 Optimal Solution

As three dragons are introduced into Earth's ecosystem, they will enter the phase of Static Disequilibrium. However, due to the high degree of open land unoccupied by humans within the polar regions, it is probable at least two of the initial three dragons will progress to Dynamic Disequilibrium. Once again, due to such a wide area of untapped land, the principles of "Increasing Returns" (first articulated by economist Brian Arthur of Stanford University and the Santa Fe Institute, not to be confused with neoclassical "Increasing Returns") suggest that the first-movers in a system have significant advantages [1]. Therefore, these early dragons will likely not be hunted down in the early phases (especially once mobility is gained). As these dragons develop perfect rationality, they will transition to the optimal solution, Dynamic Equilibrium, and co-exist in harmony with humans and the larger ecological landscape. As the dragons grow older and reproduce, more dragons will enter the system. As scarce resources become depleted by their parents, however, it will become far more difficult for young dragons to avoid detection than it was for their parents—again in line with Arthur's "Increasing Returns" hypothesis. As older dragons switch to Static Equilibrium strategies, younger dragons will have a better opportunity to transition into the Dynamic strategy phases. However, the nature of the increasing human threat will suggest that there is a maximum amount of dragons that can exist within the system until new dragons are hunted

down at a probability high enough that dragons can only maintain sustainable populations in regions with low human densities.

## 7 Shortcomings of Model

We recognize there are inherent shortcomings to the model, including but certainly not limited to the following arguments:

1. **Perfect Rationality** In this model, we have assumed perfect rationality among fully-mature dragons. We realize the assumption that any model which suggests non-human agents are more rational economic agents than humans themselves may feel silly or short-sighted; however, the intention of this paper is to question the ramifications of such a system, not to understand whether or not it aligns with reality (indeed, dragons do not align with reality—so such a perfect rational model working in practice is similarly as fantastical as fire-breathing monsters possessing intuitive mathematical brilliance).
2. **Perfect Information** We assumed perfect information as a necessary corollary to perfect rationality. We acknowledge that almost no example of purely perfect information exists; but again, our model is concerned more with the ramifications of "what if" then the necessary supposition that this necessarily is so.
3. **Perfect Mobility** We assumed dragons were perfectly mobile and could travel tremendous distances with little to not energy expenditure. We built this assumption off of the question's initial condition that dragons can travel great distances. We believe this makes sense because dragons are incredibly massive creatures with huge wingspans; so a combination of utilizing principles of aerodynamics in conjunction with tremendous amounts of muscle mass contributing to wing power suggests flying great distances wouldn't pose a significant problem. However, a disagreement of this principle assumption could naturally form a counter-paper. However, the core principle is that mobility is a key factor for niche specialization.
4. **Lack of Human Coordination** The fact that humans do not coordinate against dragons (ie. humans in region A would not be aware of the damage caused by dragons in region B) in this model is the greatest weakness of the model—especially with the inter-connectivity offered by the Internet and a plethora of social media outlets. However, this was a necessary simplification to make for this particular model; although deliberation on this question could serve as a fascinating extension to this paper.
5. **Dragon Incentives** We assumed in this model that dragons are purely profit-maximizing agents who are anti-social in nature. We recognize that this assumption essentially eliminates all components of anthropology from the dragon's psychology. Rather than striving for companionship or

intrinsic purpose in life, the dragons seek only to eat, reproduce, and not be killed. This too was a necessary assumption, although the ramifications of this assumption could lead towards fascinating insights into the flaws of more general neoclassical theories of economics built upon similar assumptions.

## 8 Conclusion/Letter to George R.R. Martin

Dear George R.R. Martin,

In the above paper we have described a scenario we believe may be helpful for an extension to the "Game of Thrones" universe. Our team has analyzed the likely outcome in a world where three dragons were randomly dropped onto Earth. Our dragons are similar to those found in *A Song of Ice and Fire*, with more conscious thought to acknowledge the long history of free-thinking hyperintelligent dragons in mythology. We have assumed that the dragons would behave as intelligent, antisocial, competition-averse beings capable of understanding the risk and reward of each geographic location on Earth. We present you with some of our findings in the hope that they might enhance the realism of your next novel.

We devised an agent-based model that places three dragons with randomly assigned physical characteristics into three random locations on Earth, which we represented as a series of discrete regions. Given the strength of dragons, we assumed that dragons could travel from one region to any other in a single day, and that climate would have no effect on their geographic preferences compared to the risk/reward of prey availability vs chances of being hunted by humans.

Though we did not have an implemented model to test all our hypotheses, fundamental analysis of the equations we developed revealed the nature of dragon behavior on Earth. We identified four states of dragon existence, each of which corresponds to a different group of dragons. Young dragons, who did not have experience to interpret risk/reward of different regions and had a rapidly increasing appetite, represented static disequilibrium, where they would hunt niches to extinction, destroy massive amounts of wildlife, and either be killed by humans or grow up and transition to a different stage of life. Young dragons' high likelihood of being hunted by humans indicates that humans could manage the dragon population with a strategy targeting them, a great potential plot point for a future novel.

Adolescent dragons, who may behave more rationally than their younger counterparts, are still voraciously hungry and lack perfect logic. These dragons cause tremendous damage due to their high appetite, so they have to migrate extremely frequently, so frequently that they may hunt every region before any region can recuperate. An abundance of adolescent dragons leads to the complete destruction of all land areas on Earth. Alternately, if an adolescent encroaches on another dragon's territory, it may be killed—the stronger dragon will win. This dragon may also be killed by humans, as it is not strong or wise enough to evade hunting. If it manages to survive, it may become an adult dragon. We

anticipated that dragon territorial death-matches may be of interest to you, and could provide more information on the topic if necessary.

Adult dragons have perfect mobility and logical reasoning, with an ability to recognize the risks and rewards for each geographic region and more stable caloric requirements. They do not need to migrate as frequently, and can cultivate a dynamic harmony with the larger global ecosystem by taking advantage of their mobility. The eldest, wisest dragons begin to lose their perfect mobility, but instead have higher levels of risk aversion and avoid humans at rates much higher than their younger counterparts. They seek unoccupied regions of the Earth and settle there, reducing their calorie needs by limiting their migration needs. In an optimal solution, dragons enter and progress through the four stages of equilibrium, cycling through until only a few older dragons survive in regions of extreme weather unoccupied by humans, such as the poles or deserts.

Though the adjustment period would be undoubtedly turbulent for humans, dragons, and the ecosystems of Earth, intelligent dragons could one day settle into natural harmony with the rest of Earth. The situation need not involve calamity, and the period of turbulence before the settling could undoubtedly provide you with rich plot details for your work. We hope you are able to glean some inspiration from our work, and we would be eager to provide any additional information needed.

Regards,

Association for Economic Dragon Ecology

## References

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