Matrices

Outline

- What is a matrix?
- Notation of a Matrices
- Order of a Matrix
- Adding two Matrices
- Subtracting two Matrices
- Scalar Multiplication
- Multiplying two Matrices

What are matrices?

Matrices is a plural form of a matrix, which is a rectangular array or a table where numbers or elements are arranged in **rows** and **columns**.

What are matrices?

They can have any number of columns and rows.

Different operations can be performed on matrices such as addition, scalar multiplication, multiplication, transposition, etc.

Notation of Matrices

If a matrix has m rows and n columns, then it will have $m \times n$ elements

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot\cdot & a_{1n} \\ a_{21} & a_{22} & \cdot\cdot & a_{2n} \\ \cdot & \cdot & & \\ & & \cdot\cdot & a_{3n} \\ \cdot & \cdot & & \\ a_{m1} & a_{m2} & \cdot\cdot & a_{mn} \end{bmatrix}$$

Notation of Matrices

A matrix is represented by the uppercase letter, in this case, 'A'

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & a_{3n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Notation of Matrices

and the elements in the matrix are represented by the lower case letter and two subscripts representing the position of the element in the number of row and column in the same order,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & a_{3n} \\ \cdot & \cdot & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Notation of Matrices

For example,

$$A_{11} = 1$$

$$A_{21} = 5$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Order of a Matrix

The size of a matrix (which is known as the order of the matrix) is determined by the number of rows and columns in the matrix.

$$\text{Rows}\left\{\underbrace{\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}}_{\text{Columns}}\right.$$

Order of a Matrix

The order of a matrix with 2 rows and 2 columns is represented as 2×2 and is read as 2 by 2

$$\text{Rows}\left\{\underbrace{\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}}_{\text{Columns}}\right.$$

Order of a Matrix

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\boxed{2 \times 1}$$

$$B = \begin{bmatrix} 1 \end{bmatrix}$$

$$\boxed{1 \times 1}$$

Order of a Matrix

$$C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$\boxed{1 \times 3}$$

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\boxed{3 \times 2}$$

Adding Matrices

Adding matrices can only be possible if the number of rows and columns of both matrices are the same.

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

Adding Matrices

While adding two matrices, we add the corresponding elements

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

Adding Matrices

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+7 & 3+4 \\ 5+(-3) & -4+5 \end{bmatrix}$$

$$\boxed{A + B = \begin{bmatrix} 9 & 7 \\ 2 & 1 \end{bmatrix}}$$

Subtracting Matrices

Subtracting matrices can only be possible if the number of rows and columns of both matrices are the same.

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

Subtracting Matrices

While subtracting two matrices, we add the corresponding elements

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

Subtracting Matrices

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2-7 & 3-4 \\ 5-(-3) & -4-5 \end{bmatrix}$$

$$\boxed{A - B = \begin{bmatrix} -5 & -1 \\ 8 & -9 \end{bmatrix}}$$

Scalar Multiplication

The product of a matrix A with any number 'c' is obtained by multiplying every entry of the matrix A by c.

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$$

Scalar Multiplication

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$$

$$4A = \begin{bmatrix} 4 \times 2 & 4 \times 3 \\ 4 \times 5 & 4 \times (-4) \end{bmatrix}$$

$$\boxed{4A = \begin{bmatrix} 8 & 12 \\ 20 & -16 \end{bmatrix}}$$

Multiplying Matrices

Multiplying matrices is possible only **if the number of columns in the first matrix and rows in the second matrix are equal.**

$$A = \begin{bmatrix} 2 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

Multiplying Matrices

$$A = \begin{bmatrix} 2 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

$$\boxed{1 \times 3}$$

$$\boxed{3 \times 1}$$

Multiplying Matrices

$$A = \begin{bmatrix} 2 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

$$\boxed{1 \times 3}$$

$$\boxed{3 \times 1}$$

If we multiply $A \times B$, we would need to multiply each row element in A with the column elements in B

Multiplying Matrices

$$A = \begin{bmatrix} \boxed{2} & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} \boxed{3} \\ 4 \\ -5 \end{bmatrix}$$

$$A \times B = [(2 \times 3)]$$

Multiplying Matrices

$$A = \begin{bmatrix} 2 & \boxed{5} & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ \boxed{4} \\ -5 \end{bmatrix}$$

$$A \times B = [(2 \times 3) + (5 \times 4)]$$

Multiplying Matrices

$$A = \begin{bmatrix} 2 & 5 & \boxed{6} \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 4 \\ \boxed{-5} \end{bmatrix}$$

$$A \times B$$
$$= [(2 \times 3) + (5 \times 4) + (6 \times -5)]$$

Multiplying Matrices

$$A = \begin{bmatrix} 2 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

$$A \times B$$

$$= [(2 \times 3) + (5 \times 4) + (6 \times -5)]$$

$$A \times B$$

$$= [6 + 20 - 30]$$

$$A \times B$$

Multiplying Matrices

$$B = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 5 & 6 \end{bmatrix}$$

$$\boxed{3 \times 1} \qquad \boxed{1 \times 3}$$

If we multiply $B \times A$, we would need to multiply each row element in B with the column elements in A

Multiplying Matrices

$$B = \begin{bmatrix} \boxed{3} \\ 4 \\ -5 \end{bmatrix} \qquad A = \begin{bmatrix} \boxed{2 \quad 5 \quad 6} \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 6 & 15 & 18 \\ & & \\ & & \end{bmatrix}$$

Multiplying Matrices

$$B = \begin{bmatrix} 3 \\ \boxed{4} \\ -5 \end{bmatrix} \qquad A = \boxed{\begin{bmatrix} 2 & 5 & 6 \end{bmatrix}}$$

$$B \times A = \begin{bmatrix} 6 & 15 & 18 \\ 8 & 20 & 24 \\ & & \end{bmatrix}$$

Multiplying Matrices

$$B = \begin{bmatrix} 3 \\ 4 \\ \boxed{-5} \end{bmatrix} \qquad A = \begin{bmatrix} \boxed{2 \quad 5 \quad 6} \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 6 & 15 & 18 \\ 8 & 20 & 24 \\ -10 & -25 & -30 \end{bmatrix}$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$\boxed{2 \times 3}$$

$$\boxed{3 \times 4}$$

Multiplying Matrices

$$A = \begin{bmatrix} \boxed{1 \quad 4 \quad -2} \\ 3 \quad 5 \quad -6 \end{bmatrix}$$

$$B = \begin{bmatrix} \boxed{\begin{matrix} 5 \\ 3 \\ -2 \end{matrix}} & \begin{matrix} 2 \\ 6 \\ 9 \end{matrix} & \begin{matrix} 8 \\ 4 \\ 7 \end{matrix} & \begin{matrix} -1 \\ 5 \\ -3 \end{matrix} \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 21 & & & \\ & & & \end{bmatrix}$$

$$(1 \times 5) + (4 \times 3) + (-2 \times -2) = 21$$

Multiplying Matrices

$$A = \begin{bmatrix} \boxed{1 \quad 4 \quad -2} \\ 3 \quad 5 \quad -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 21 & 8 & & \\ & & & \end{bmatrix}$$

$$(1 \times 2) + (4 \times 6) + (-2 \times 9) = 8$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 5 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 21 & 8 & 10 & \\ & & & \end{bmatrix}$$

$$(1 \times 8) + (4 \times 4) + (-2 \times 7) = 10$$

Multiplying Matrices

$$A = \begin{bmatrix} \boxed{1 \quad 4 \quad -2} \\ 3 \quad 5 \quad -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 21 & 8 & 10 & 25 \\ & & & \end{bmatrix}$$

$$(1 \times -1) + (4 \times 5) + (-2 \times -3) = 25$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ \boxed{3 \quad 5 \quad -6} \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & & & \end{bmatrix}$$

$$(3 \times 5) + (5 \times 3) + (-6 \times -2) = 42$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ \boxed{3 \quad 5 \quad -6} \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B$$

$$= \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & & \end{bmatrix}$$

$$(3 \times 2) + (5 \times 6) + (-6 \times 9) = -18$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ \boxed{3 \quad 5 \quad -6} \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B$$

$$= \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & 2 & \end{bmatrix}$$

$$(3 \times 8) + (5 \times 4) + (-6 \times 7) = 2$$

Multiplying Matrices

$$A = \begin{bmatrix} 1 & 4 & -2 \\ \boxed{3 \quad 5 \quad -6} \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 2 & 8 & -1 \\ 3 & 6 & 4 & 5 \\ -2 & 9 & 7 & -3 \end{bmatrix}$$

$$A \times B$$

$$= \begin{bmatrix} 21 & 8 & 10 & 25 \\ 42 & -18 & 2 & 40 \end{bmatrix}$$

$$(3 \times -1) + (5 \times 5) + (-6 \times -3) = 40$$

Transpose of a Matrix

The transpose of a matrix is done when we replace the rows of a matrix to the columns and columns to the rows.

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 7 & 5 \end{bmatrix}$$

Transpose of a Matrix

The transpose of a matrix A is denoted by A^T

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 7 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 & 7 \\ -2 & 0 & 5 \end{bmatrix}$$

Transpose of a Matrix

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 7 & 5 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 3 & 7 \\ -2 & 0 & 5 \end{bmatrix}$$

$$\boxed{3 \times 2} \qquad \boxed{2 \times 3}$$

Determinant of a Matrix

The determinant is a special number that can be calculated from a square matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = (a \times d) - (b \times c)$$

Determinant of a Matrix

The determinant helps us find the inverse of a matrix, tells us things about the matrix that are useful in systems of linear equations, calculus and more.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = (a \times d) - (b \times c)$$

Determinant of a Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = (a \times d) - (b \times c)$$

Determinant of a Matrix

$$A = \begin{bmatrix} 3 & 5 \\ -4 & 7 \end{bmatrix}$$

$$|A| = (3 \times 7) - (5 \times -4)$$

$$|A| = 41$$

Determinant of a Matrix

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$|A| = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

Determinant of a Matrix

$$A = \begin{bmatrix} \boxed{a_1} & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$|A| = \boxed{a_1} \boxed{\begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix}} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

Determinant of a Matrix

$$A = \begin{bmatrix} a_1 & \boxed{b_1} & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$|A| = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - \boxed{b_1} \boxed{\begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix}} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

Determinant of a Matrix

$$A = \begin{bmatrix} a_1 & b_1 & \boxed{c_1} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$|A| = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + \boxed{c_1} \boxed{\begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}}$$

Determinant of a Matrix

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 5 & 7 & 6 \\ -8 & 1 & 9 \end{bmatrix}$$

$$|A| = 2\begin{bmatrix} 7 & 6 \\ 1 & 9 \end{bmatrix} - 4\begin{bmatrix} 5 & 6 \\ -8 & 9 \end{bmatrix} + (-3)\begin{bmatrix} 5 & 7 \\ -8 & 1 \end{bmatrix}$$

Determinant of a Matrix

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 5 & 7 & 6 \\ -8 & 1 & 9 \end{bmatrix}$$

$$|A| = 2[(7 \times 9) - (1 \times 6)] - 4[(5 \times 9) - (6 \times -8)] + (-3)[(5 \times 1) - (7 \times -8)]$$

$$|A| = 2[(63) - (6)] - 4[(45) - (-48)] + (-3)[(5) - (-56)]$$

$$|A| = 2(57) - 4(93) + (-3)(61)$$

$$\boxed{|A| = -441}$$