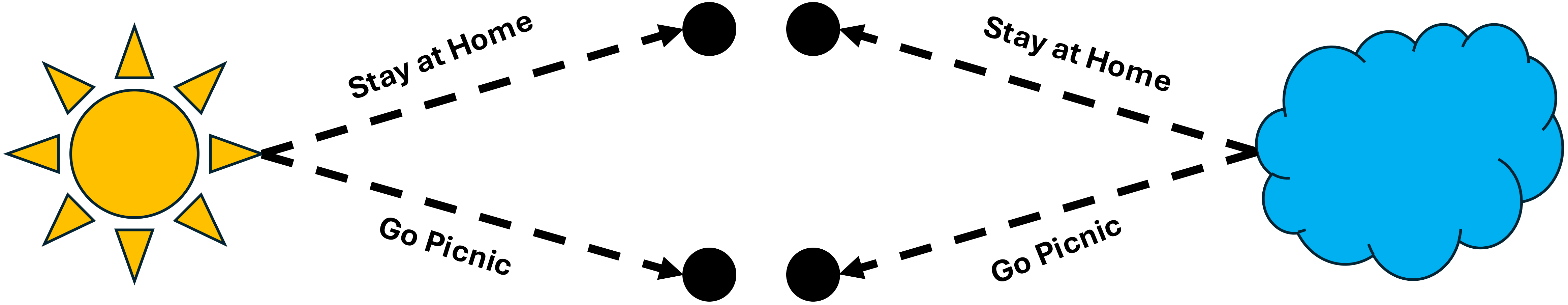
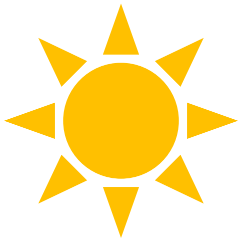
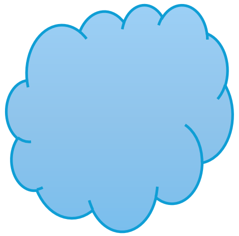
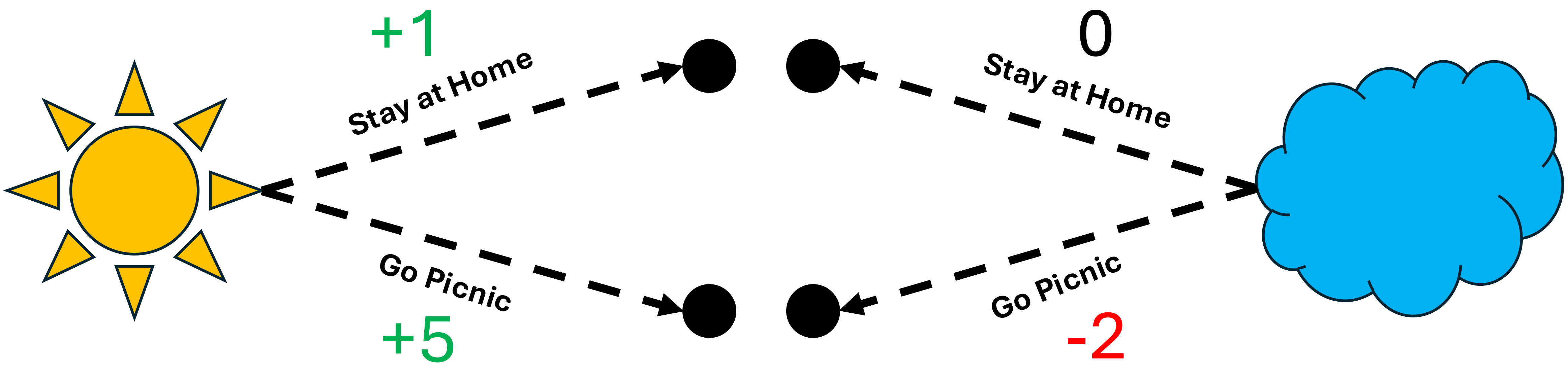


States s: {Sunny, Cloudy}

Actions a: {Go to Picnic, Stay at Home}

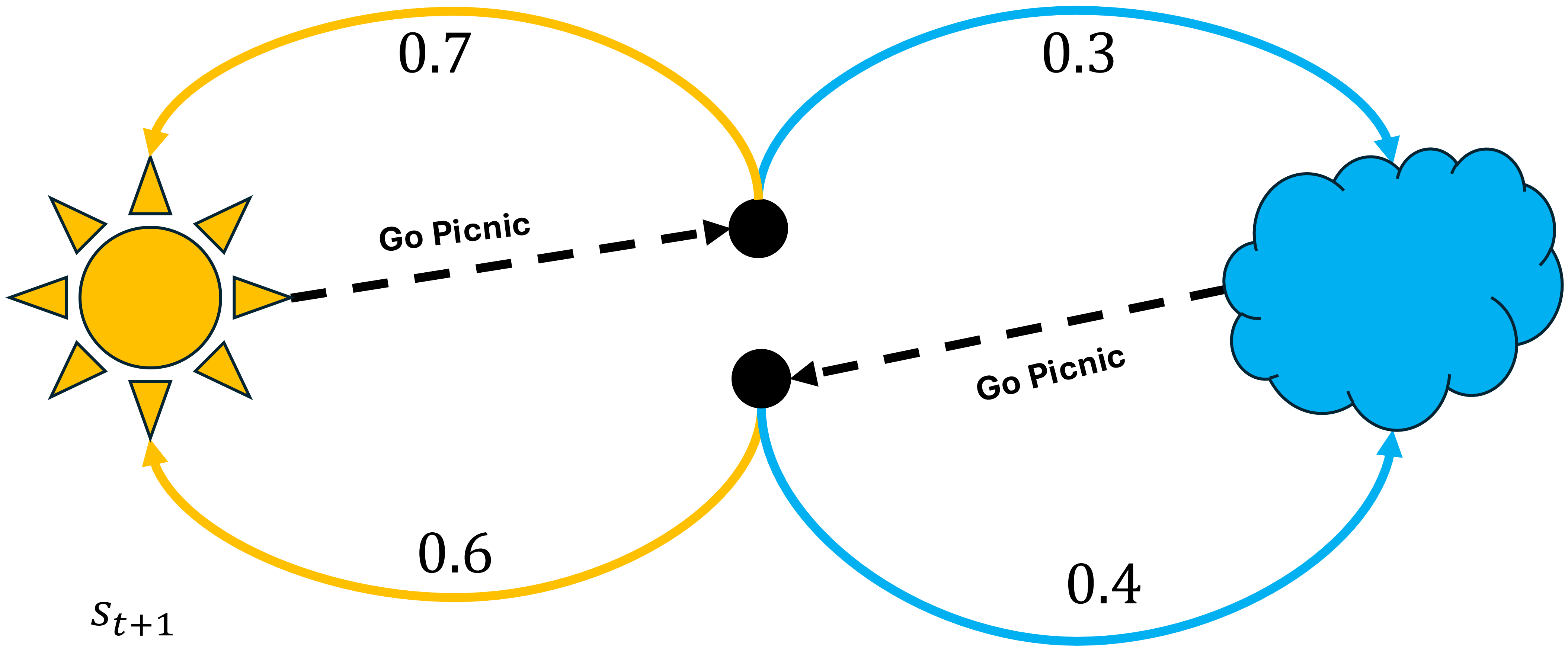
Discount $\gamma = 0.9$

	Go to Picnic	Stay at Home
	+5	+1
	-2	0



	Go to Picnic	Stay at Home
☀	+5	+1
☁	-2	0

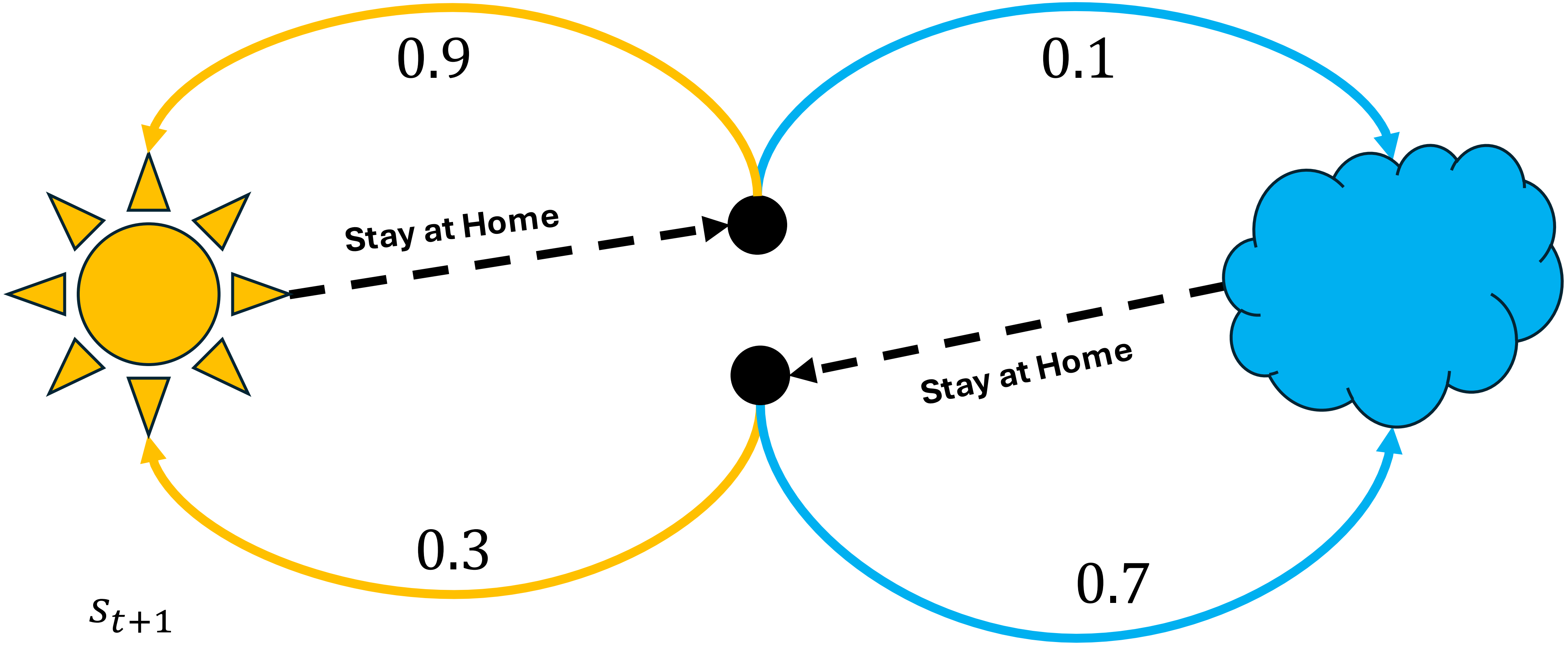
$$R_{picnic} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \quad R_{stay} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



		s_{t+1}	
s_t		0.7	0.3
		0.6	0.4

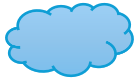
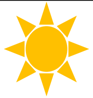
State Transition Matrix

$$P_{picnic} = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$



s_{t+1}

s_t

		
	0.9	0.1
	0.3	0.7

State Transition Matrix

$$P_{stay} = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

Step 1: Compute state-wise average reward under the policy π

For **Sunny**:

$$r_\pi = 0.5\ \times\ (5) + 0.5\ \times\ (1) = 2.5 + 0.5 = 3$$

For **Cloudy**:

$$r_\pi = 0.5\ \times\ (-2) + 0.5\ \times\ (0) = -1 + 0 = -1$$

$$\boxed{r_\pi = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$$

That's "what you get right now" on average if you follow the policy

Step 2: Compute the policy transition matrix

Row 1 (Sunny):

- $P\pi(1,1) = 0.5 \times 0.7 + 0.5 \times 0.9 = 0.35 + 0.45 = 0.80$
- $P\pi(1,2) = 0.5 \times 0.3 + 0.5 \times 0.1 = 0.15 + 0.05 = 0.20$

Row 2 (Cloudy):

- $P\pi(2,1) = 0.5 \times 0.6 + 0.5 \times 0.3 = 0.30 + 0.15 = 0.45$
- $P\pi(2,2) = 0.5 \times 0.4 + 0.5 \times 0.7 = 0.20 + 0.35 = 0.55$

$$\boxed{P_\pi = \begin{bmatrix} 0.80 & 0.20 \\ 0.45 & 0.55 \end{bmatrix}}$$

This is how we move between states on average under the policy π

Step 3: Write the Bellman expectation equations $v_\pi(sunny)$

General Form:

$$v_\pi(s) = r_\pi(s) + \gamma \sum P_\pi(s, s')\, v_\pi(s')$$

$$v_1 = 3 + 0.9(0.8\, v_1 + 0.2\, v_2)$$

$$v_1 = 3 + 0.72\, v_1 + 0.18\, v_1$$

$$v_1 - 0.72\, v_1 - 0.18\, v_2 = 3$$

$$\boxed{0.28 v_1 - 0.18 v_2 = 3}$$

Step 3: Write the Bellman expectation equations $v_\pi(cloudy)$

General Form:

$$v_\pi(s) = r_\pi(s) + \gamma \sum P_\pi(s, s')\, v_\pi(s')$$

$$v_2 = -1 + 0.9(0.45\, v_1 + 0.55\, v_2)$$

$$v_2 = -1 + 0.405\, v_1 + 0.495\, v_2$$

$$v_2 - 0.405\, v_1 - 0.495\, v_2 = -1$$

$$\boxed{-0.405\, v_1 + 0.505\, v_2 = -1}$$

Step 4: Write the Bellman expectation equations $v_\pi(cloudy)$

$$\boxed{0.28v_1 - 0.18v_2 = 3}$$

$$0.28v_1 = 3 + 0.18v_2$$

$$\frac{0.28v_1}{0.28} = \frac{3 + 0.18\,v_2}{0.28}$$

$$\boxed{v_1 = \frac{3 + 0.18\,v_2}{0.28}}$$

Step 4: Solve for $v_\pi(cloudy)$

$$\boxed{-0.405\, v_1 + 0.505\, v_2 = -1}$$

$$-0.405(\frac{3 + 0.18\, v_2}{0.28}) + 0.505 v_2 = -1$$

$$(-0.405\ \times\ \frac{3}{.28})(-0.405\ \times\ \frac{.18}{.28} v_2) + 0.505 v_2 = -1$$

$$(-0.405\ \times\ 10.714)(-0.405\ \times\ 0.642857 v_2) + 0.505 v_2 = -1$$

$$-4.339 - 0.2607 v_2 + 0.505 v_2 = -1$$

$$-4.339 + (0.505 - 0.2607) v_2 = -1$$

$$-4.339 + 0.2443 v_2 = -1$$

$$0.2443 v_2 = -1 + 4.339$$

$$\boxed{v_\pi(cloudy) = \frac{-1 + 4.339}{0.2443} = \frac{3.339}{0.2443} = 13.65}$$

Step 4: Solve for $v_\pi(sunny)$

$$\boxed{v_1 = \frac{3 + 0.18\, v_2}{0.28}}$$

$$v_1 = \frac{3+0.18\,(13.65)}{0.28} = \frac{3+2.457}{0.28} = \frac{5.457}{0.28}$$

$$\boxed{v_\pi(sunny) = 19.489}$$

Step 5: Write the Bellman optimality equations

General Form:

$$v_*(s) = \max_a\{R(s,a) + \gamma s' \sum P(s' \mid s,a) v*(s')\}$$

For Sunny (v_1) using Picnic:

$$v_*(sunny) \ = 5 + 0.9(0.7\, v_1 + 0.3\, v_2)$$

For Cloudy (v_2) using Picnic:

$$v_*(cloudy) = -2 + 0.9(0.6\, v_1 + 0.4\, v_2)$$

Step 5: Write the Bellman optimality equations

Sunny:

$$v_1 = 5 + 0.63\, v1 + 0.27\, v2$$

$$v_1 - 0.63v_1 - 0.27v_2 = 5$$

$$\boxed{0.37v_1 - 0.27v_2 = 5}$$

Cloudy:

$$v2 = -2 + 0.54\, v_1 + 0.36\, v_2$$

$$\boxed{-0.54\, v_1 + 0.64\, v_2 = -2}$$

Step 6: Solve for v_*

From the Sunny equation:

$$0.37v_1 = 5 + 0.27\ v_2$$

$$v_1 = \frac{5 + 0.27\ v_2}{0.37}$$

$$\mathit{Using\ the\ equation\ for\ cloudy,} \qquad \boxed{-0.54\, v_1 + 0.64\, v_2 = -2}$$

$$-0.54\left(\frac{5 + 0.27\ v_2}{0.37}\right) + 0.64v_2 = -2$$

Step 6: Solve for v_*(cloudy)

$$-0.54 \times \frac{5}{0.37} = -4.339$$

$$-0.54 \times \frac{0.27}{0.37} = -0.394\, v_2$$

$$-4.339 - 0.394\, v_2 + 0.64 v_2 = -2$$

$$-4.339 + (0.64 - 0.394)\, v_2 = -2$$

$$-4.339 + 0.245\, v_2 = -2$$

$$0.245\, v_2 = -2 + 4.339 = 5.297$$

$$0.245\, v_2 = 5.297$$

$$\boxed{v_*(\text{cloudy}) = \frac{5.297}{0.245} = 21.538}$$

Step 6: Solve for v_*(sunny)

$$v_1 = \frac{5 + \ 0.27 v_2}{0.37}$$

$$v_1 = \frac{5 + 0.27 \ \times \ 21.538}{0.37}$$

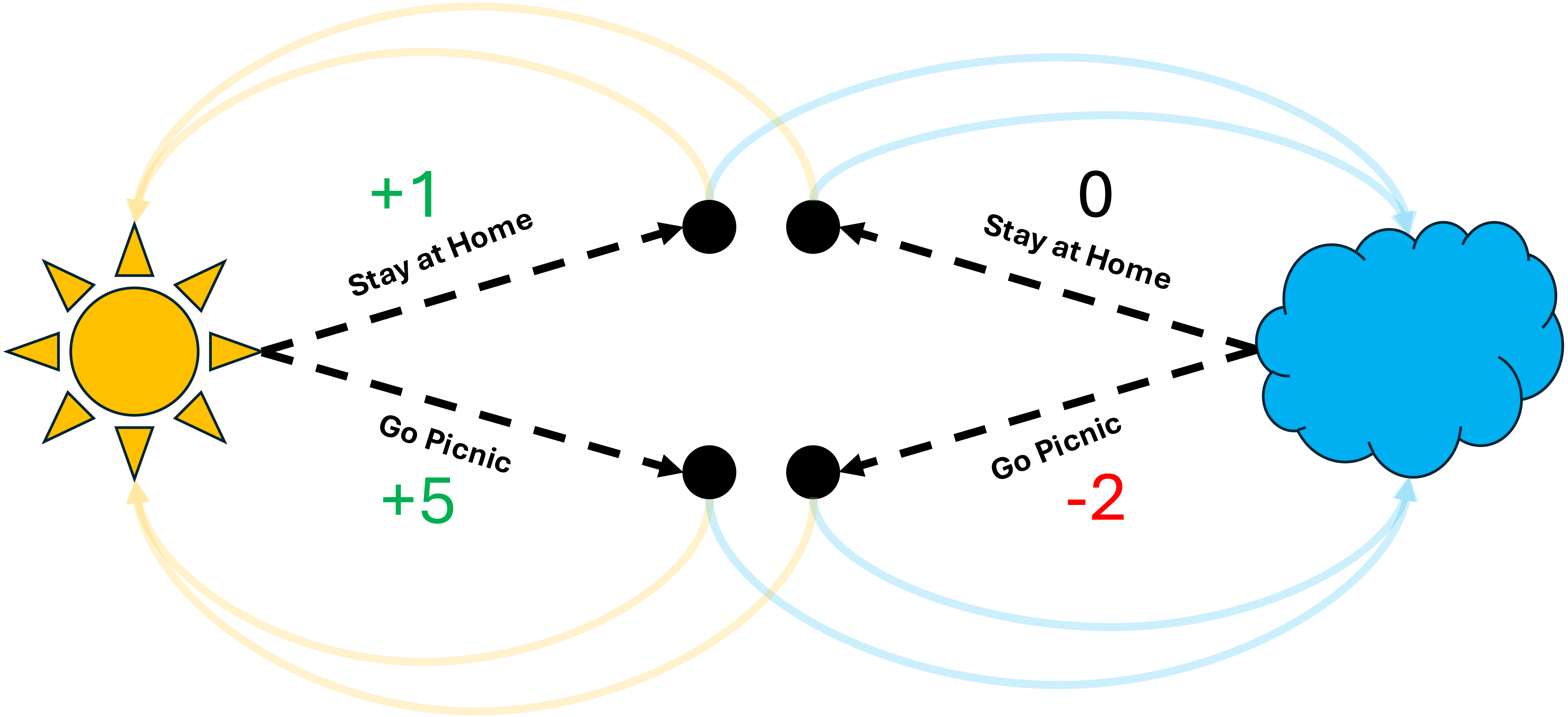
$$v_1 = \frac{5 + 5.81}{0.37} = \frac{10.81}{0.37}$$

$$\boxed{v_*(\text{sunny}) = 29.23}$$

$$\boxed{v_*(\text{cloudy}) = 21.538}$$

Step 7: Solve for q_*

- $\mathrm{q}(1, \mathrm{Picnic}) = 5 + 0.9(0.7v_1 + 0.3v_2) = 29.23$
- $\mathrm{q}(1, \mathrm{Stay}) = 1 + 0.9(0.9v_1 + 0.1v_2) = 26.61$
- $\mathrm{q}(2, \mathrm{Picnic}) = -2 + 0.9(0.6v_1 + 0.4v_2) = 21.53$
- $\mathrm{q}(2, \mathrm{Stay}) = 0 + 0.9(0.3v_1 + 0.7v_2) = 21.46$


+1
Stay at Home
Go Picnic
+5
0
Stay at Home
Go Picnic
-2