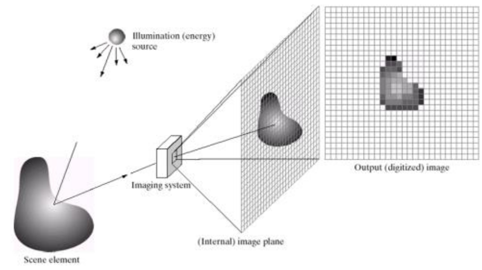
Digital Image Fundamentals

Xiaojun Qi

1

Digital Image Acquisition Process



a b c d e

An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

2

Image Digitization

Physical Image → Sampling → Quantization → Digital Image

Sampling: Digitizing the coordinate values and partitioning the x-y plane into a grid (integer). That is, amplitude value (e.g., f(x, y)) is sampled at discrete points.

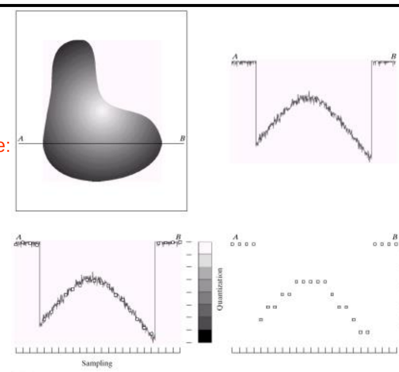
Quantization: Digitizing the amplitude values in gray levels (integer). [Lmin, Lmax] → [(Lmin, A1), (A1, A2), … (An, Lmax)] → [0, 255]

where n is the number of quantization levels (also referred to as gray level)

3

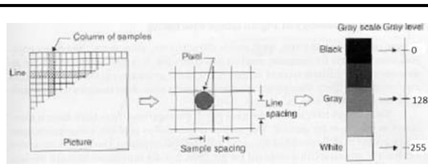
1D signal case:

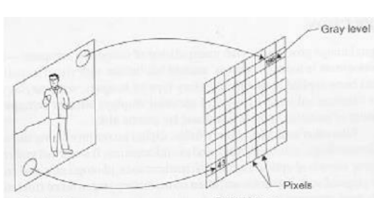
Digitizing the continuous image along the line segment AB



a b
c d

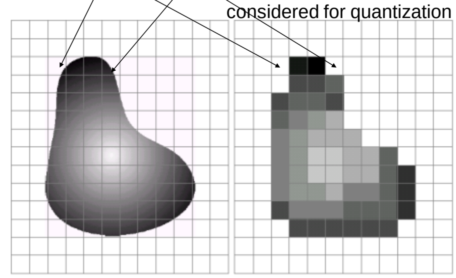
FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.





5

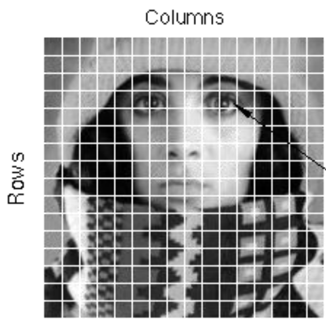
Image areas occupying less than half of the pixel area will not be considered for quantization



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

6



Questions:

Suppose that the image is sampled as shown on the left side:

1. How many rows are there in the image?
2. How many columns are there in the image?
3. What is the size of the image?

7

Digital Images

- An image is a mapping of spatial coordinates (x, y) into values identified with a set of gray levels.
- Pixel or picture element is a triple
  (x, y, g(x, y)) = (x, y, L), where L is a gray level. The number of gray levels typically is **an integer power of 2** due to processing, storage, and sampling hardware considerations.
- The gray level 256 is fairly common because the data can be stored in one byte.

8

DIP Concepts

- Image:
  An image may be defined as a 2-D function, *f(x,y)*, where *x* and *y* are spatial coordinates, and the amplitude of *f* at any pair of coordinates *(x, y)* is called the **intensity** or **gray level** of the image at that point.

- Digital Image:
  x, *y*, and the amplitude values of *f* are all **finite, discrete quantities.**

9

(a) (b)

130	146	133	95	71	71	62	78
130	146	133	92	62	71	62	71
139	146	146	120	62	55	55	55
139	139	139	116	117	112	117	110
139	139	139	139	139	139	139	139
116	112	139	139	139	143	124	139
156	159	159	159	159	146	159	159
168	159	156	159	159	159	139	159

(c) A 8*8 array

(a) Image of a face (b) subimage from the right eye region
(c) Matrix notation of the subimage

10

DIP Concepts (Cont.)

- Pixels (Picture Elements, Image Elements, Pels):
  The elements in a digital image with a particular location and value.

- Digital Image Processing:
  Manipulation and analysis of digital images (pictorial information) by computer.

11

An *M*×*N* digital image can be represented in the following compact matrix form:

$$f(x,y) = \begin{bmatrix} f(1,1) & f(1,2) & \ldots\ldots & f(1,N) \\ f(2,1) & f(2,2) & \ldots\ldots & f(2,N) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ f(M,1) & f(M,2) & \ldots\ldots & f(M,N) \end{bmatrix}$$

- Common choices for *M*, *N* and *L*
  $M = 2^m$, $N = 2^n$, and $L = 2^k$
  examples for *M*, *N*: 256 x 256 or 512 x 512
  examples for *L*: 256 (*k*=8 bits/pixel), 4096 (*k*=12 bits/pixel)
- Storage requirements
  $M \times N \times k$ bits (M=N=1024, k=8, 1MB image)

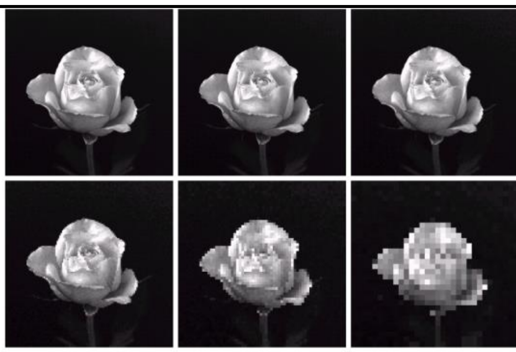
12

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

The number of bits required to store square images with various values of N and k.

- Pixel is the smallest unit in an image. It depends on the sampling.
- Resolution is the smallest number of discernible line pairs per unit distance.
  - Dense sampling produces a high resolution image
  - Coarse sampling produces a low resolution image
- Spatial resolution of an image is the physical size of a pixel in that image. That is, spatial resolution is the smallest discernible detail in an image.
  - A digital image of size M by N has a spatial resolution of $M \times N$ pixels.
- Gray-level resolution of an image is the number of gray-levels in that image. It depends on the number of bits used for quantization. It also determines the gray-level (intensity) range.
  - An L-level digital image has a gray-level resolution of L level.



Ex1: Effects of varying the number of samples in a digital image while keeping the number of gray levels constant

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

One Possible Implementation

Step 1: Subsampling: Delete every other row and column from the original image.

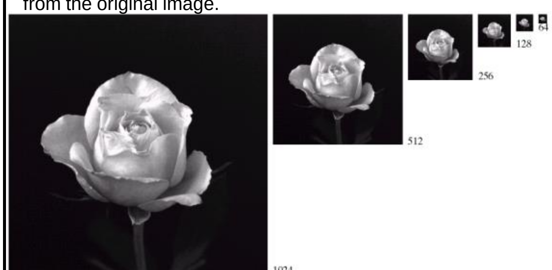
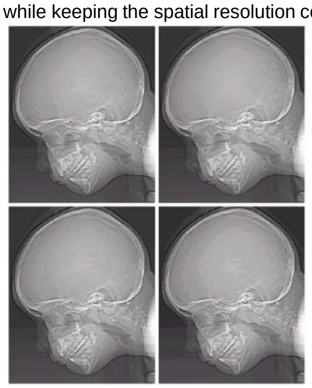


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

Step 2: Scale up: Bring all the subsampled images up to the size of the original image by **row and column pixel replication**.

Ex2: Effects of varying the number of gray levels in a digital image while keeping the spatial resolution constant

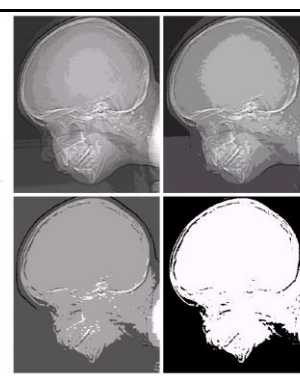


a b
c d

FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

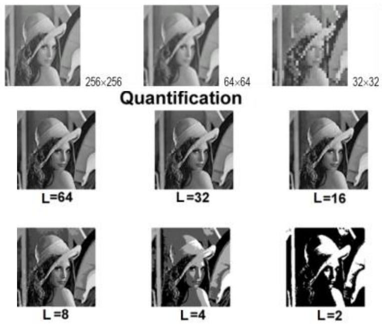


It is accomplished by using: Original Intensity * (2^k-1) / 255 where k is the decreased bits for the gray level.

More Illustration Examples

Sampling

Quantification



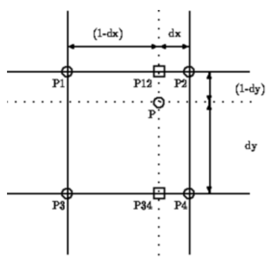
19

Zooming and Shrinking Digital Images

- Zooming Digital Images (Oversampling)
  1. Nearest Neighbor Interpolation
     – Pixel replication: It is applicable when we want to increase the size of an image an integer number of times.
  2. Bilinear Interpolation

- Shrinking Digital Images (Undersampling)
  1. Row-Column deletion

20



Bilinear Interpolation

P12 = dx P1 +(1-dx) P2

P34 = dx P3 + (1-dx) P4

P = dyP12 +(1-dy) P34

=dxdyP1 + (1-dx)dyP2+dx(1-dy)P3+(1-dx)(1-dy)P4

21

Three Coordinate Systems

1. Cartesian Coordinate: The first component *x* increases to the right, while the second component *y* increases upward. Origin is at lower left corner and usually starts with (0, 0).
   – All of the Matlab interpolation functions and the graphics routines, use this coordinate system.

22

Three Coordinate Systems

2. Array (Matrix) Coordinate: The first component i (row) increases downward, while the second component j (column) increases to the right. Origin is at upper left corner and usually starts with (1, 1). Also refers to row-column (i.e., x-y) coordinate systems.
   – It is the Matlab's coordinate system for matrices. Matlab uses this system for matrix subscript notation.

23

Three Coordinate Systems

3. Pixel Coordinate: The first component *x* increases to the right, while the second component *y* increases downward. Origin is at upper left corner and usually starts with (1, 1).
   – In Image Processing Toolbox of Matlab, it uses the matrix coordinate system for direct image matrix subscripting (Image Matrix Manipulation), but the pixel coordinate system for everything else. When the syntax for a function uses *x* and *y*, it refers to the pixel coordinate system.

24

Digital Image Notations

- (x, y): spatial coordinates [or p=(x, y): a point]
- g(x, y), or g(p): image gray-level at (x, y)
- (x, y, g(x,y)): Pixel
- G: Image Grid
- Nx: Number of x's on the image grid (Height)
- Ny: Number of y's on the image grid (Width)
- Nt=Nx × Ny: Total number of pixels on the image grid.
- L: The number of gray-levels. The gray-levels range from 0 to L-1.

25

Neighbors and Neighborhood

- Np: The neighbors of a pixel p
  - N_4(p): 4 horizontal and vertical neighbors of p
  - N_D(p): 4 diagonal neighbors of p
  - N_8(p): 4 horizontal and vertical neighbors of p plus 4 diagonal neighbors of p; that is N_4(p) and N_D(p)
- Wp: The window of a pixel p

  It is the neighborhood of a pixel p with certain gray-level values
  - Wp = {Np, g(w)} where g(w) = g(p') for all p' in Np

26

Adjacency: [V is a set of gray-level values]

- 4-Adjacency: p and q with values from V are 4-adjacent if q is in the set N_4(p).
- 8-Adjacency: p and q with values from V are 8-adjacent if q is in the set N_8(p).
- m-Adjacency (mixed adjacency): p and q with values from V are m-adjacent if
  - q is in N_4(p) or
  - q is in N_D(p) and the set N4(p) ^ N4(q) has no pixels whose values are from V

0	1	1
0	1	0
0	0	1

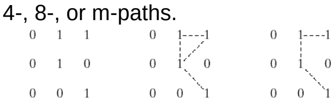
Question:
Which pixels are the 4-adjaceny, 8-adjacency and m-adjacency of the pixel p where V = {1}

27

Path

A digital path (or curve) from a pixel p with coordinates (x, y) to a pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates (x_0, y_0), (x_1, y_1), …, (x_n, y_n) where (x_0, y_0) = (x, y), (x_n, y_n) = (s, t), and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for 1<=i<=n.

Based on type of adjacency specified, the path can be defined as 4-, 8-, or m-paths.



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

There is no ambiguity in the m-path.

28

Let S represent a subset of pixels in an image.

- Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called connected component of S.
- If it only has one connected component, then set S is called a connected set.
- Each connected component in an image is called a region of the image.
- Boundary (border or contour) of a region R: It is the set of pixels in the region that have at least one neighbors that are not in R.

29

1	1	1	0	0	0	0	0
1	1	1	0	1	1	0	0
1	1	1	0	1	1	0	0
1	1	1	0	0	0	1	0
1	1	1	0	0	0	1	0
1	1	1	0	0	0	1	0
1	1	1	0	0	1	1	0
1	1	1	0	0	0	0	0

Suppose S={1} and a path is based on 4-adjacency
- How many connected components are there in this 8 by 8 binary image?
- How many regions are there in this binary image?
- What is the boundary for each region?

30

Sample Matlab Codes

Calling Matlab Function bwlabel(a, 4) returns:

$$\begin{matrix} 1&1&1&0&0&0&0&0\\ 1&1&1&0&1&1&0&0\\ 1&1&1&0&1&1&0&0\\ 1&1&1&0&0&0&1&0\\ 1&1&1&0&0&0&1&0\\ 1&1&1&0&0&0&1&0\\ 1&1&1&0&0&1&1&0\\ 1&1&1&0&0&0&0&0 \end{matrix} \longrightarrow \begin{matrix} 1&1&1&0&0&0&0&0\\ 1&1&1&0&2&2&0&0\\ 1&1&1&0&2&2&0&0\\ 1&1&1&0&0&0&3&0\\ 1&1&1&0&0&0&3&0\\ 1&1&1&0&0&0&3&0\\ 1&1&1&0&0&3&3&0\\ 1&1&1&0&0&0&0&0 \end{matrix}$$

31

Distance and Distance Measures

- D_e Distance (Euclidean Distance)
  $D_e(p, q) = [(x - s)^2 + (y - t)^2]^{1/2}$
- D_4 Distance (City-Block Distance)
  $D_4(p, q) = |x - s| + |y - t|$.
- D_8 Distance (Chessboard Distance)
  $D_8(p, q) = \max(|x - s|, |y - t|)$.
- D_m Distance: The shortest m-path between the two points. All the pixels along the path must be m-adjacent to each other.

Distance Measures

1. D(p, q) >= 0 ;
2. D(p, q) = D(q, p) ;
3. D(p, z) <= D(p, q) + D(q, z).

$$\begin{matrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}$$

What is the distance between these two points?

Image Format

- There are many different Raster image formats such as TIFF, JPEG, GIF, and PNG. They all can be organized as follows:
  - Image Header (in ASCII):
    - magic number designed to identify the file as an image with that specific format
    - image size (i.e., height and width)
    - Depth (i.e., gray-level)
    - date, creator, etc
  - Image Data (in binary or ASCII):
    They are arranged in a sequential order

33

- PPM (Portable Pixel Map)

Image header consists of:

1. P6 [Two-character magic number]
2. Number of Columns (Width) Number of Rows (Height)
3. Max intensity

Image data in Raw Bytes (binary):

A raster of Height rows, in order from top to bottom. Each row consists of Width pixels, in order from left to right. Each pixel is a triplet of red, green, and blue samples, in that order. Each sample is represented in pure binary by either 1 or 2 bytes. If the Maxval is less than 256, it is 1 byte. Otherwise, it is 2 bytes.

34

- PPM (Portable Pixel Map)

Image header consists of:

1. P3 [Two-character magic number]
2. Number of Columns (Width) Number of Rows (Height)
3. Max intensity

Image data in ASCII:

A raster of Height rows, in order from top to bottom. Each row consists of Width pixels, in order from left to right. Each pixel is a triplet of red, green, and blue samples, in that order.

35

An Example of PPM (Portable Pixel Map)

```
P3
# Example.ppm
4 4
15
0 0 0     0 0 0     0 0 0     15 0 15
0 0 0     0 15 7    0 0 0     0 0 0
0 0 0     0 0 0     0 15 7    0 0 0
15 0 15   0 0 0     0 0 0     0 0 0
```

36

- PGM (Portable Gray Map): It is a popular format for gray-scale images

Header consists of:

1. P5 [Two-character magic number]
2. Number of Columns (Width) Number of Rows (Height)
3. Max intensity

Image data in raw bytes (binary)

37

- PGM (Portable Gray Map): It is a popular format for gray-scale images

Header consists of:

1. P2 [Two-character magic number]
2. Number of Columns (Width) Number of Rows (Height)
3. Max intensity

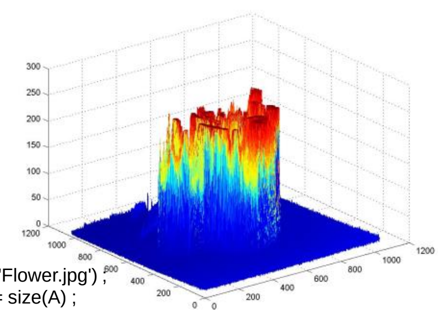
Image data in ASCII

38

Images as Surfaces

- Image Data may also be viewed as a surface.
- View the function g(x, y) as forming a surface as x and y vary.
- Surface plot of the intensity information

39




```
A = imread('Flower.jpg') ;
[Row, Col] = size(A) ;
X = 1 : Row ;
Y = 1 : Col ;
[X, Y] = meshgrid(X, Y) ;
mesh(X, Y, A) ;  %% Note: Flower.jpg is a grayscale picture
```

40

Multi-channel Images and Color

- N-channel or N-band Image:
  - On a common referenced coordinate systems, there is a sequence of gray-level images [g1(x,y), g2(x,y), …, gn(x, y)]
- Color Model: Represent colors and their relationship to each other.

41

Color Models

- The purpose of a color model (also called color space or color system) is to facilitate the specification of colors in some standard, generally accepted way.
- In essence, a color model is a specification of a coordinate system and a subspace within that system where each color is represented by a single point.
- Most color models in use today are oriented either toward hardware (such as for color monitor and printers) or toward applications where color manipulation is a goal (such as in the creation of color graphics for animation).

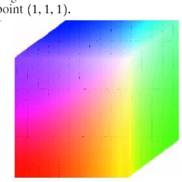
42

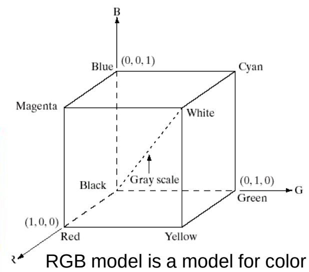
- RGB (red-green-blue): It is an additive model.
  - F(x,y) = (r, g, b)
- CMY (cyan-magenta-yellow): It is a subtractive model since all three primaries are subtracted from white light to produce the required color.
- YIQ: Recoding of RGB for transmission efficiency and for maintaining compatibility with monochrome television standards.
- HIS (hue-intensity-saturation): Decouple the intensity component from the color information. H is the dominant color, S is the degree of non-dilution in white, and I is the relative brightness.

43

Color Models
-- RGB Color Model

FIGURE 6.7 Schematic of the RGB color cube. Points along the main diagonal have gray values, from black at the origin to white at point (1, 1, 1).



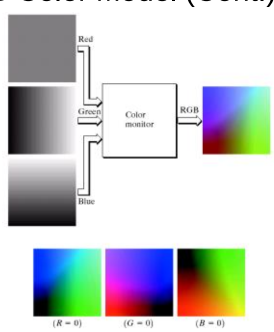


RGB model is a model for color monitors and a broad class of color video cameras.

44

Color Models
-- RGB Color Model (Cont.)

a
b
FIGURE 6.9 (a) Generating the RGB image of the cross-sectional color plane (127, *G*, *B*). (b) The three hidden surface planes in the color cube of Fig. 6.8.



45

Color Models
-- CMY Color Model

- Most devices that deposit colored pigments on paper, such as color printers and copiers, require CMY data input or perform an RGB to CMY conversion internally.
- This conversion is performed using the simple operation (Note: All color values have been normalized to the range [0, 1])

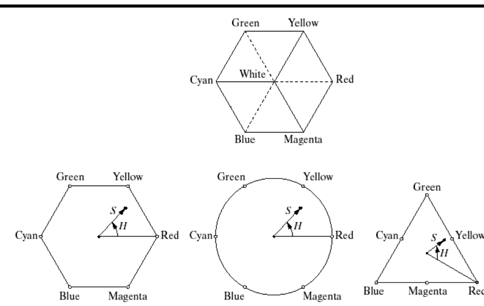
$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

46

Color Models
-- HSI Color Model

- HSI (Hue, Saturation, and Intensity) color model decouples the intensity component from the color-carrying information (hue and saturation) in a color image.
  - Hue is a color attribute that describes a pure color (pure yellow, orange, or red).
  - Saturation gives a measure of the degree to which a pure color is diluted by white light.
  - Intensity is one of the key factors in describing color sensation.
- As a result, the HSI model is an ideal tool for developing image processing algorithms based on color descriptions that are natural and intuitive to humans.

47



a
b c d
FIGURE 6.13 Hue and saturation in the HSI color model. The dot is an arbitrary color point. The angle from the red axis gives the hue, and the length of the vector is the saturation. The intensity of all colors in any of these planes is given by the position of the plane on the vertical intensity axis.

48

Color Models
-- HSI Color Model (Cont.)

- Given an image in RGB color format, the H component of each RGB pixel is obtained by using the equation:

$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases}$$

with

$$\theta = \cos^{-1}\left\{ \frac{\frac{1}{2}[(R-G)+(R-B)]}{\left[(R-G)^2+(R-B)(G-B)\right]^{1/2}} \right\}$$

How about the denominator is zero?

49

Color Models
-- HSI Color Model (Cont.)

- The saturation component is given by

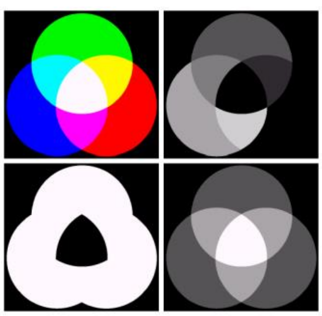
$$S = 1 - \frac{3}{(R+G+B)}[\min(R,G,B)]$$

- The intensity component is given by

$$I = \frac{1}{3}(R+G+B)$$

- Here the RGB values have been normalized to the range [0, 1] and that the angle theta is measured with respect to the red axis of the HSI space.

50



a b
c d

FIGURE 6.16 (a) RGB image and the components of its corresponding HSI image: (b) hue, (c) saturation, and (d) intensity.

51

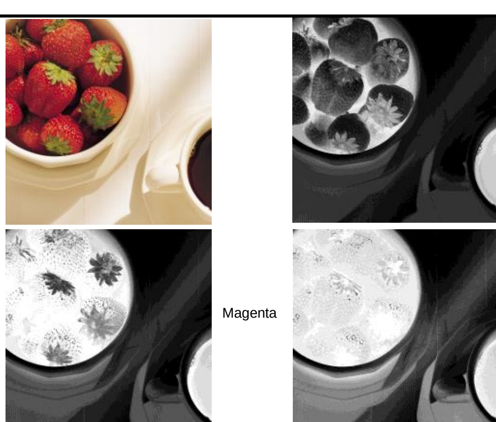
Other Color Models

TABLE 3.6 Transformations from NTSC Receiver Primary to Different Coordinate Systems. Input Vector is $[R_N\ G_N\ B_N]^T$.

Output vector	Transformation matrix	Comments
$\begin{bmatrix} R \\ G \\ B \end{bmatrix}$	$\begin{pmatrix} 1.167 & -0.146 & -0.151 \\ 0.114 & 0.753 & 0.159 \\ -0.001 & 0.059 & 1.128 \end{pmatrix}$	CIE spectral primary system
$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$	$\begin{pmatrix} 0.607 & 0.174 & 0.201 \\ 0.299 & 0.587 & 0.114 \\ 0.000 & 0.066 & 1.117 \end{pmatrix}$	CIE X, Y, Z system
$\begin{bmatrix} U \\ V \\ W \end{bmatrix}$	$\begin{pmatrix} 0.405 & 0.116 & 0.133 \\ 0.299 & 0.587 & 0.114 \\ 0.145 & 0.827 & 0.627 \end{pmatrix}$	CIE UCS tristimulus system
$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$	$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix}$	NTSC transmission system

Y: Luminance; I: Hue; Q: Saturation

2



Cyan

Magenta

Yellow

53

Red

Green

Blue

54

55

Matlab

- Arguments to functions are passed by reference (i.e., pointers) in most cases to conserve memory. If an argument is modified by the function, then the argument is passed by value, which means a copy of the argument is made.
- For example the file Change.m contains the following:

```
function B = Change(A)
A = A + 10;
B=A;
```

56