Basic (Low-Level) DIP -- Part I

Image Enhancement in the Spatial Domain

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Introduction

· Objective:

To process an image so that the result is more suitable than the original image for a specific application. → Problem Oriented

- To suppress undesired distortions
- Emphasize and sharpen image features for display and analysis
- There is no general theory of image enhancement. When an image is processed for visual interpretation, the viewer is the ultimate judge of how well a particular method works. > Subjective Process

Two broad categories of image enhancement approaches

- Spatial Domain Methods: Direct manipulate pixels in an image.
- Frequency Domain Methods: Modify the Fourier or Wavelet transform of an image.
- Normally, enhancement techniques use various combinations of methods from these two categories.

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 Image processing in spatial domain may be represented as

$$g(x, y) = T(f(x, y))$$

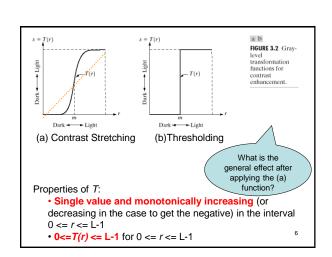
where f(x, y) is the input image, g(x, y) is the processed image, T is the operator on f(x, y).

- In general, the operator *T* can operate on:
 - A single pixel $(x, y) \rightarrow point processing$
 - A group of pixels → mask processing

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Introduction to Point Processing

- Point processing modifies each pixel intensity independently based on a function *T*. Let the intensity of pixel before modification be *r* and after modification be s, then s = T(r).
- T is often referred to as:
 Gray-level (Intensity or Mapping)
 transformation function.



Introduction to Mask Processing (Filtering)

- T operates on a neighborhood of pixels.
- Neighborhood about a point (x,y) normally is a square or rectangular subimage area centered at (x, y).
- The general approach is to use a function of the values of f in a predefined neighborhood of (x, y) to determine the value of g at (x, y).

FIGURE 3.1 A
3 × 3
neighborhood
about a point
(x, y) in an image.

Normally, square and
rectangular masks are the
most predominant ones
used in the enhancement.

Ex2:

Mask (filter, kernel, template, or window)
Processing Function: Use a function of the
values of f in a 3×3 neighborhood of (x, y) to
determine the values of g at (x, y)

Point Processing Methods -- Basic Gray Level Transformations Basic Gray-level Transformation Function Used for Image Enhancement Regular 33 Some basic pay-level Transformation Function Used for Image Enhancement Regular 34 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Used for Image Enhancement Regular 35 Some basic pay-level Transformation Function Image Enhanc

The values of pixels, before and after processing, will be denoted by r and s, respectively.

1) Image Negative: s = L - 1 - r;

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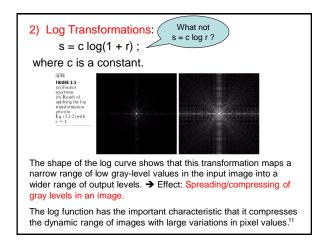
| Image Negative: s = L - 1 - r;

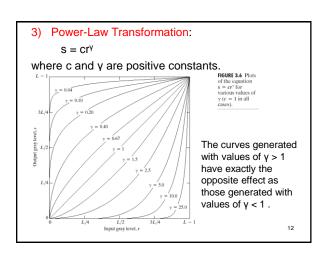
| Image Negative: s = L - 1 - r;

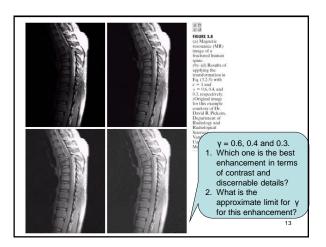
| Image Negative: s = L - 1 - r;

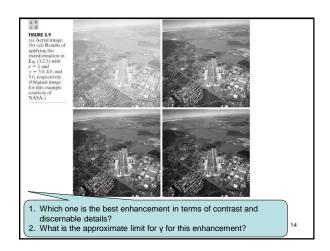
| Image Negative: s = L - 1 - r;

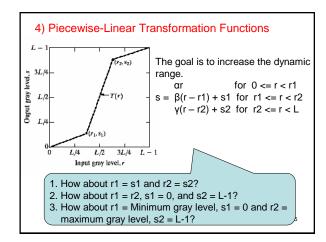
|

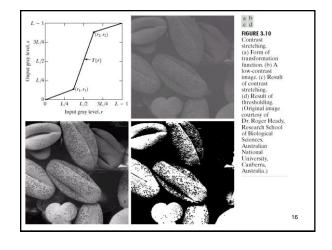


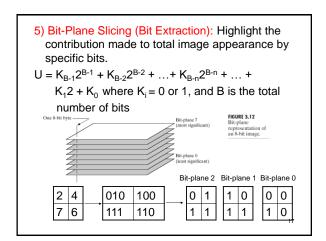


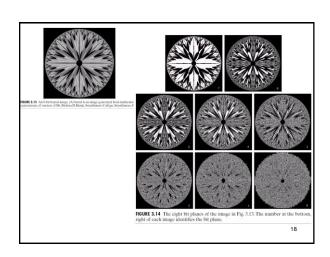












Point Processing Methods -- Histogram Processing

- The **histogram** of a digital image with gray levels in the range [0, L-1] is a discrete function $h(r_k) = n_k$, where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k .
- A normalized histogram is given by p(r_k) = n_k/n for k = 0, 1, ..., L-1 and n is the total number of pixels in the image. That is, p(r_k) gives an estimate of the probability of occurrence of gray level r_k.

Dark image

Bright image

Less contrast image

It is reasonable to conclude that an image whose pixels tend to occupy the entire range of possible gray levels and, in addition, tend to be distributed uniformly, will have an appearance of high contrast and will exhibit a large variety of gray tones.

The net effect will be an image that shows a great deal of gray-level detail and has high dynamic range.

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Point Processing Methods

- -- Properties of the Image Histogram
- Histogram clustered at the low end: Dark Image
- Histogram clustered at the high end: Bright Image
- Histogram with a small spread: Low contrast Image
- Histogram with a wide spread: High contrast Image

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Point Processing Methods -- Histogram Equalization

- Histogram equalization is to increase the dynamic range of an image by an nonlinear intensity transformation function such that the histogram of the transformed image covers the whole dynamic range with equal probability.
 - → Stretch the contrast by redistributing the gray-level values uniformly.
 - It is fully automatic compared to other contrast stretching techniques.

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Derivation of the Histogram Equalization

- Let r and s be the intensity before and after histogram equalization and s = T(r).
- Let the normalized histogram (probability density) before and after histogram equalization be f(r) and g(s).
- Let Fs and Fr be the cumulative density functions (CDF) for s and r respectively.
- The goal is to identify a function T such that g(s) = 1 for 0 < s < 1 [This derivation is given in the textbook in section 3.3.1]. In other word, s is uniformly distributed within its range (i.e., histogram is as smooth as possible).

Random Its value Derivation of the Variable Histogram Equalization

$$F_{r}^{\downarrow}(r') = P(r \le r) = P(T^{-1}(s) \le r) = P(s \le T(r))$$

= $F_{s}(T(r))$

since
$$F_s(s) = P(s \le s) = \int_{t=0}^{s} g(t)dt = s$$
. Thus,

 $F_{\rm r}(r) = F_{\rm s}(T(r)) = T(r)$

This means the transfer function T is the CDF of r.

However, the equalized histogram cannot be flat due to discrete approximation of the continuous density function by histogram.

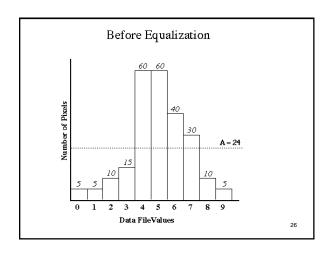
Point Processing Methods -- Discrete Histogram Equalization

Given an image of M x N with the maximum intensity being Imax:

- 1. Obtain the histogram H(k), k = 0, 1, ..., Imax
- 2. Compute the cumulative normalized histogram T(k): $T(k) = \sum_{i=0}^{k} H(i) / MN$
- Compute the transformed intensity by: g_k = (L-1) * T(k), where L is the maximum gray level of the new processed image

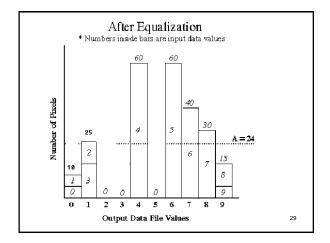
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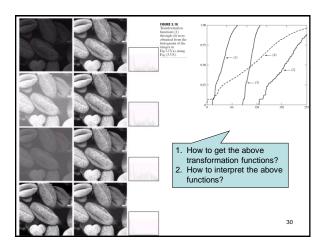
4. Scan the image and set the pixel with the intensity k to g_k.

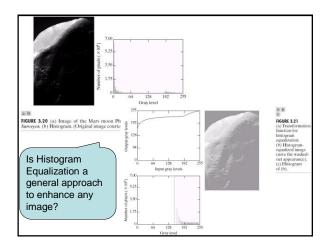


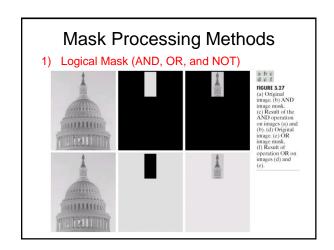
Total Number of Pixels: 5+5+10+15+60+60+40+30+10+5=240. Step 1: H(0)=5; H(1)=5; H(2)=10; H(3)=15; H(4)=60; H(5)=60; H(6)=40; H(7)=30; H(8)=10; H(9)=5; Step 2: T(0)=5/240; T(1)=10/240; T(2)=20/240; T(3)=35/240; T(4)=95/240; T(5)=155/240; T(6)=195/240; T(7)=225/240; T(8)=235/240; T(9)=240/240;

• Step 3: $G0 = 0.1875 \rightarrow 0$; $G1 = 0.3750 \rightarrow 0$; $G2 = 0.7500 \rightarrow 1$; $G3 = 1.3125 \rightarrow 1$; $G4 = 3.5625 \rightarrow 4$; $G5 = 5.8125 \rightarrow 6$; $G6 = 7.3125 \rightarrow 7$; $G7 = 8.4375 \rightarrow 8$; $G8 = 8.8125 \rightarrow 9$; $G9 = 9.0000 \rightarrow 9$; • Step 4: Original Intensity \rightarrow New Intensity \rightarrow Number $0 \rightarrow 0 \rightarrow 5$; $1 \rightarrow 0 \rightarrow 5$; $2 \rightarrow 1 \rightarrow 10$; $3 \rightarrow 1 \rightarrow 15$; $4 \rightarrow 4 \rightarrow 60$; $5 \rightarrow 6 \rightarrow 60$; $6 \rightarrow 7 \rightarrow 40$; $7 \rightarrow 8 \rightarrow 30$; $8 \rightarrow 9 \rightarrow 10$; $9 \rightarrow 9 \rightarrow 5$;









Mask Processing Methods

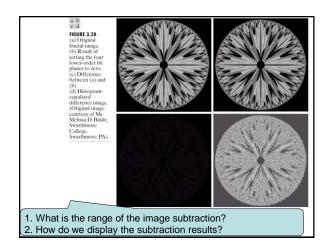
2) Image Subtraction

 The difference between two images f(x, y) and h(x, y), expressed as:

$$g(x, y) = f(x, y) - h(x, y)$$

is obtained by computing the difference between all pairs of corresponding pixels from *f* and *h*.

- Image subtraction is primarily used to detect objects (surveillance) or remove an object (background removal)
- It involves two images, one is called mask image (i.e., h) and the other is current image (i.e., f).

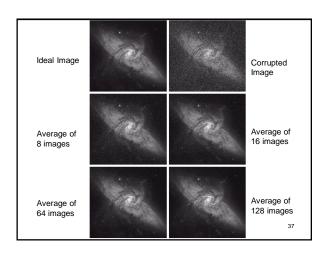


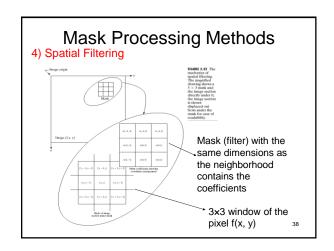
Mask Processing Methods

3) Image Averaging

- The purpose is to reduce image noise by averaging a sequence of images of the same scene but obtained at different times with small intervals.
- Assume the same noise characteristics (i.e., the noise is uncorrelated and has zero average value) for all images, averaging can therefore reduce noise.

Gaussian Noise (Random Noise) follows the Gaussian Distribution $y = \frac{1}{\sigma^{2} \sqrt{2\pi}} e^{-\frac{x^{2}}{2\sigma^{2}}}$ The Gaussian distribution shows the probability y of finding a deviation x from the mean (x = 0), according to the equation stated, where e is the base of natural logarithms, and σ is the standard deviation. The probability of larger deviations can be seen to decrease rapidly.



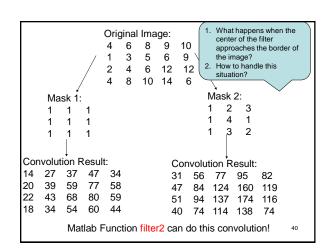


- Spatial filtering alters image intensity of a pixel based on its intensity, the intensities of the neighboring pixels, and the coefficients of the corresponding mask. That is:
 - For a linear spatial filtering, the response is given by a sum of products of the filter coefficients with the corresponding image pixels in the area spanned by the filter mask.
 - In general, linear filtering of an image f of size MxN with a filter mask of size mxn is given by the expression:

$$g(x, y) = \sum_{s=-at=-b}^{a} w(s, t) f(x+s, y+t)$$

where a = (m-1)/2 and b = (n-1)/2

 It is accomplished by convolving the mask with the original image.



Three types of Spatial Filters:

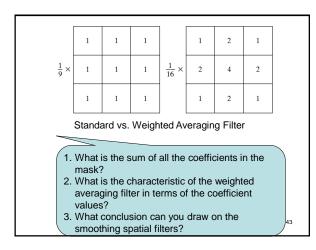
- Low-pass filter: Enhance low frequency components of the image while eliminating or reducing high frequency components.
- High-pass filter: Enhance high frequency components of the image while eliminating or reducing low frequency components.
- Band-pass filter: Enhance certain range of frequency components of the image while eliminating or reducing frequency components beyond the range.

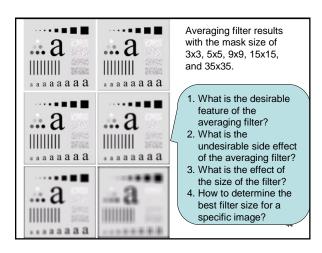
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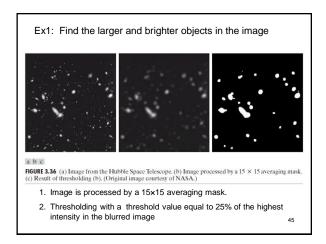
Mask Processing Methods

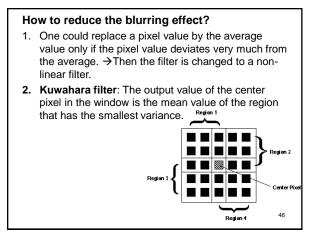
4.1) Smoothing Spatial Filters

- It is a linear low-pass filter
- A standard averaging filter replaces the value of every pixel in an image by the average of the gray levels in the window defined by the filter mask. This process results in an image with reduced "sharp" transitions in gray levels.
- A weighted averaging filter uses different coefficients at different spatial locations.





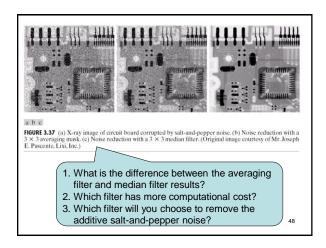




Mask Processing Methods

4.2) Order-Statistics Filters

- It is a nonlinear low-pass filter
- Its filtering result is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
 - Median filter: Replace center pixel with median of the gray levels in the window of that pixel (the original pixel value is included in the computation of the median).



The filter can be shown as a convolution mask, for example, for an average 3x3 filter What does this filter look like? 999000 .9900. 999000 .8530. 909000 .8541. .9900. .8541. .9900. 999090 999000 .9641. .9900. 999000 average filter result median filter result image 49

· Determine the size of the mask

- The size of the mask must be larger than the scale of the noise but smaller than the dimensions of any structure in the image that is important to subsequent analysis. That is, features (e.g. lines or spots) that are smaller than half the mask can be selectively eliminated as noise (or at least not features of interest).
- 2. The larger the mask, the longer the ranking process (for the median filter) or the computational process (for the averaging filter) takes
- Comparison between averaging and median filters:

The median filter is superior to the smoothing filter in that it does not smooth or blur the boundaries of regions or features in the image.

Mask Processing Methods

4.3) Sharpening Spatial Filters

- The goal of the sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.
 - Sharpening could be accomplished by spatial differentiation.
 - Image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying gray-level values.

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The first - order derivative of a 1D function f(x) is:

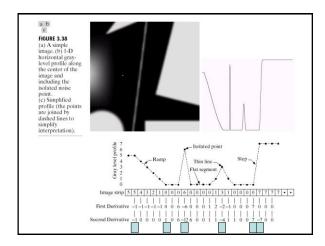
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

The second - order derivative of a 1D function f(x) is:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

- · First derivative:
 - 1. Zero in flat segment
 - 2. Nonzero at the onset of a gray-level step or ramp
 - 3. Nonzero along the ramp.
 - Second derivative:
 - 1. Zero in flag area.
 - 2. Nonzero at the onset and end of a gray-level step or ramp.
 - 3. Zero along ramps of constant slope.

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- Compare the response between 1st and 2nd derivatives:
 - 1st derivatives generally produce thicker edges in an image.
 - 2nd derivatives have a stronger response to fine detail, such as thin lines and isolated points.
 - 1st derivatives generally have a stronger response to a gray-level step.
 - 2nd derivatives produce a double response at step changes in gray-level values in an image.
 - For similar changes in gray-level values in an image, the response from the 2nd derivative is stronger to a line than to a step, and to a point than to a line.

The Laplacian Filter (2nd Derivatives)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

= [f(x+1, y) + f(x-1, y) - 2f(x, y)]

+[f(x, y+1)+f(x, y-1)-2f(x, y)]

= [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)

f(x+1,y)+f(x-1,y)+f(x,y+1)							
	0	1	0	1	1	1	
	1	-4	1	1	-8	1	
	0	1	0	1	1	1	
	0	-1	0	-1	-1	-1	
	-1	4	-1	-1	8	-1	
					٠.		

a b c d
FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).
(b) Mask used to implement an extension of this equation that diagonal headings of the diago

nt an n of this that the

1. What is the sum of the

entries in the mask?
2. What is the difference

spatial filters?

between the sum of the

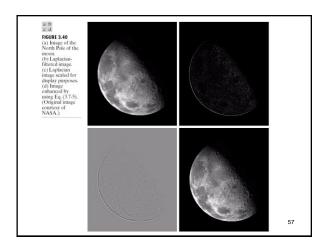
smoothing and sharpening

ther tations

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- Laplacian Filter is an isotropic filter. That is, its response is independent of the direction of the discontinuities in the image to which the filter is applied.
- Laplacian Filter is a linear operator since the derivatives of any order are linear operations.
- Laplacian Filter highlights gray-level discontinuities in an image and deemphasizes regions with slowly varying gray levels since it is a derivative operator.
- The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if the center coefficien t of} \\ & \text{Laplacian mask is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{if the center coefficien t of} \\ & \text{Laplacian mask is positive} \end{cases}$$



Simplify the Laplacian Enhancement Filter:

$$g(x, y) = f(x, y) - [f(x+1, y) + f(x-1, y)]$$

$$+ f(x, y+1) + f(x, y-1)] + 4f(x, y)$$

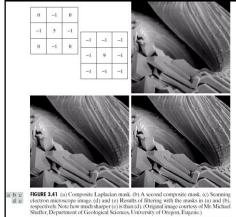
$$= 5f(x, y) - [f(x+1, y) + f(x-1, y)$$

$$+ f(x, y+1) + f(x, y-1)$$

Based on the above formulation, a composite Laplacian filter can be constructed. What does this composite filter look like?

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The results obtained with the mask containing the diagonal terms usually are a little sharper than those obtained with the basic mask.

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Unsharp Masking

Unsharp Masking: Subtract a blurred version of an image from the image itself to sharpen the image.

$$f_s(x, y) = f(x, y) - f_{blur}(x, y)$$

· High-Boost Filtering

High Boost Filtering is a slight further generalization of unsharp masking.

$$f_{hb}(x, y) = Af(x, y) - f_{blur}(x, y)$$

$$= (A-1)f(x, y) + f(x, y) - f_{blur}(x, y)$$

$$= (A-1)f(x, y) + f_s(x, y)$$

where $A \ge 1$

The Gradient Enhancement – 1st Derivatives

The gradient of f at coordinates (x, y) is defined:

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of this vector is given by:

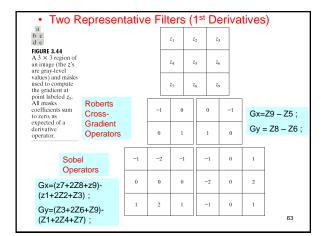
$$mag(\nabla f) = \left[G_x^2 + G_y^2\right]^{1/2} = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

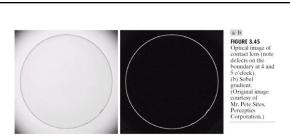
 $mag(\nabla f)$ can be approximated by using absolute values:

$$mag(\nabla f) \approx |G_x| + |G_y|$$

- The gradient of f is a 2D column vector. The components of the gradient vector itself are linear operators, but the magnitude of this vector is not because of the squaring and square root operations.
- Since the magnitude of the gradient vector is isotropic, it often is referred to as the gradient.
- For computational simplicity, an absolute value approximation of the magnitude is used since it still preserves relative changes in gray levels.
- First derivative enhancement filters are also referred to as gradient edge detectors.

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The edge defects are also quite visible, but with the added advantage that constant or slowly varying shades of gray have been eliminated.

The ability to enhance small discontinuities in an otherwise flat gray field is another important feature of the gradient

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Mask Processing Methods -- Local Histogram Equalization

- Instead of performing a histogram for the whole image, histogram equalization is performed locally on a small region of the image adaptively.
- This can be achieved using a window and moving the center of this window across the image from pixel to pixel. At each location, histogram equalization is performed on the portion of the image covered by the window. The pixel value at the center of the window is mapped to a new gray-level based on histogram equalization function. This process repeats until the last pixel of the image, with window moving one pixel at a time.

Mask Processing Methods -- Local Enhancement by Statistics Several Important Statistical Concepts

Mean (Average): $m = \sum_{i=0}^{D-1} r_i p(r_i);$

mean:

The nth moment of r about its $\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i);$

Variance

 $\mu_2(r) = \sigma^2(r) = \sum_{i=1}^{L-1} (r_i - m)^2 p(r_i).$

In a digital image, the mean is a measure of average gray level; the variance (or standard deviation, which is a square root of the variance) is a measure of average contrast

Mask Processing Methods

-- Local Enhancement by Statistics The Interpretation of Some Important Statistical

Concepts

- Variance (The 2nd Moment)
 - A small variance indicates a tight grouping.
 - A large variance indicates the opposite.
- · The 3rd Moment
 - Indicate how many points on each side of the average
 - =0: symmetric about the mean
 - <0: A low bias with respect to the average
 - >0: A high bias with respect to the average

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Local enhancement by contrast manipulation is an ideal approach to try on problems where part of the image is acceptable, but other parts may contain hidden features of interest.

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Let f(x, y) represent the value of an image pixel at any image coordinates (x, y),

Let g(x, y) represent the corresponding enhanced pixel at those coordinates.

Ther

$$g(x, y) = \begin{cases} E \times f(x, y) & \text{if } m_{S_{xy}} \le k_0 M_G \text{ and } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x, y) & \text{otherwise} \end{cases}$$

where, E, k_0, k_1 , and k_2 are specified parameters;

 M_G is the global mean of the input image;

 D_G is its global standard deviation

 S_{xy} denotes a neighborhood of specified size, centered

at (x, y).

