Image Segmentation

Xiaojun Qi

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Outlines

- Discontinuity-Based Approach+PostProcessing
  - Point Detection
  - Line Detection
  - Edge Detection
- Similarity-Based Approach
  - Thresholding Technique
  - Region Splitting: Quad Picture Tree + Merging
  - Region Growing: Seeded Region Growing
  - Watershed Algorithm

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Importance of Segmentation

- Segmentation is generally the first stage in any attempt to analyze or interpret an image automatically.
- Segmentation bridges the gap between low-level image processing and high-level image processing.
- Some kinds of segmentation technique will be found in any application involving the detection, recognition, and measurement of objects in images.

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Importance of Segmentation (Cont.)

- The role of segmentation is crucial in most tasks requiring image analysis. The success or failure of the task is often a direct consequence of the success or failure of segmentation.
- However, a reliable and accurate segmentation of an image is, in general, very difficult to achieve by purely automatic means.

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Application of Segmentation

- Industrial inspection
- Optical character recognition (OCR)
- Tracking of objects in a sequence of images
- Classification of terrains visible in satellite images.
- Detection and measurement of bone, tissue, etc., in medical images.

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Image Segmentation Example

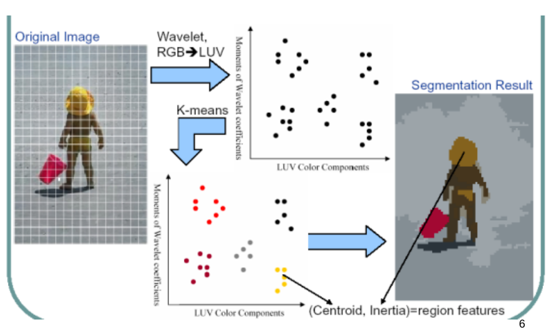

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Image Segmentation
-- Descriptive Definition

- Segmentation subdivides an image into its constituent regions or objects. That is, it partitions an image into distinct regions that are meant to correlate strongly with objects or features of interest in the image.
- Segmentation can also be regarded as a process of grouping together pixels that have similar attributes.
- The level to which the subdivision is carried depends on the problem being solved. That is, segmentation should stop when the objects of interest in an application have been isolated.→ There is no point in carrying segmentation past the level of detail required to identify those elements.

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Image Segmentation
-- A Math Oriented Descriptive Definition

- It is the process that partitions the image pixels into **non-overlapping regions** such that:

1. Each region is homogeneous (i.e., uniform in terms of the pixel attributes such as intensity, color, range, or texture, and etc.) and connected.
2. The union of adjacent regions is not homogeneous.

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Image Segmentation
-- Pure Mathematical Definition

- $\{R_i\}$ is a segmentation of an entire image R if:
1. $R = \bigcup_{i=1}^{n} R_i$, the union of all regions covers entire R
2. $R_i \cap R_j = \phi$, for all i and j, $i \neq j$, there is no overlap of the regions
3. $P(R_i) = True$ for i = 1, 2, …, n, P is the logical uniformity predicate defined over the points in set R_i
4. $P(R_i \cup R_j) = False$, for $i \neq j$ and Ri and Rj are neighboring regions.
5. Ri is a connected region, i = 1, 2, …, n.

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Image Segmentation
-- Explanation

1. All pixels must be assigned to regions.
2. Each pixel must belong to a single region only.
3. Each region must be uniform.
4. Any merged pair of adjacent regions must be non-uniform.
5. Each region must be a connected set of pixels.

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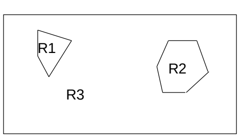
Several Predicate Examples

1. P(R)=True, if $|g(x_1, y_1) - g(x_2, y_2)| <= \varepsilon$ for all (x_1, y_1), (x_2, y_2) in R
2. P(R)=True, if $T_1 <= g(x, y) <= T_2$ for all (x, y) in R where T_1 and T_2 are thresholds that define the region.



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Several Predicate Examples

3.
$$P(R) = \begin{cases} \text{True} & \text{if } |f(j,k) - f(m,n)| \leq \Delta, \\ \text{False} & \text{otherwise} \end{cases}$$

Where (j, k) and (m, n) are the coordinates of neighboring pixels in region R.

This predicate states that a region R is uniform if (and only if) any two neighboring pixels differ in gray-level by no more than Δ.

4.
$$P(R) = \begin{cases} \text{True} & \text{if } |f(j,k) - \mu_R| \leq \Delta, \\ \text{False} & \text{otherwise} \end{cases}$$

Where f(j, k) is the gray-level of a pixel with coordinates (j, k) and μ_R is the mean grey level of all pixels in R

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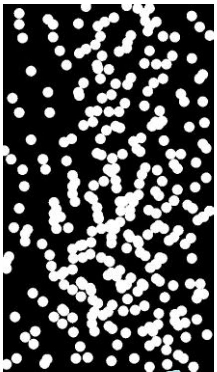

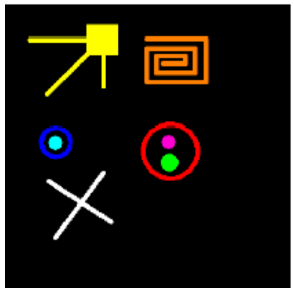

How many regions does each of these two pictures have?

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How many regions does each of these two pictures have?

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Image Segmentation Strategies

- Image segmentation algorithms generally are based on one of two basic properties of intensity values: discontinuity and similarity.
- Discontinuity based approach: Partition an image based on abrupt changes in intensity.
- Similarity based approach: Partition an image based on regions that are similar according to a set of predefined criteria.
  - Thresholding
  - Region growing
  - Region splitting and merging

Do you already know discontinuity in the past?

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Discontinuity vs. Similarity

- Techniques based on discontinuity attempt to partition the image by detecting abrupt changes in gray level. → Point, line, and edge detectors.
- Techniques based on similarity attempt to create the uniform regions by grouping together connected pixels that satisfy predefined similarity criteria. Therefore, the results of segmentation may depend critically on these criteria and on the definition of connectivity.
- The approaches based on discontinuity and similarity mirror one another in the sense that completion of a boundary is equivalent to breaking one region into two.

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Discontinuity-Based Approach

Discontinuity based approach should yield pixels lying only on edges. In practice, this set of pixels seldom characterizes an edge completely because of noise, breaks in the edge from non-uniform illumination, and other effects that introduce spurious intensity discontinuities.

→Thus edge detection algorithms typically are followed by linking procedures to assemble edge pixels into meaningful edges. Several basic approaches are:

- Local Processing
- Global Processing Via the Hough Transform
- Global Processing via Graph-Theoretic Technique

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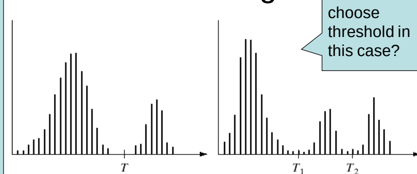
Similarity-Based Approach -- Thresholding

This histogram-based approach assumes that different features in an image give rise to distinct peaks in its histogram.

How to choose threshold in this case?



a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Case A: The image is composed of one light object on a dark background, in such a way that object and background pixels have gray levels grouped into two dominant modes.

Case B: Two types of light objects on a dark background.

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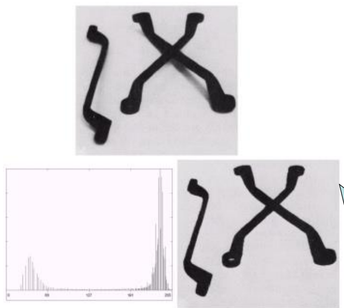
Thresholding Ex 1

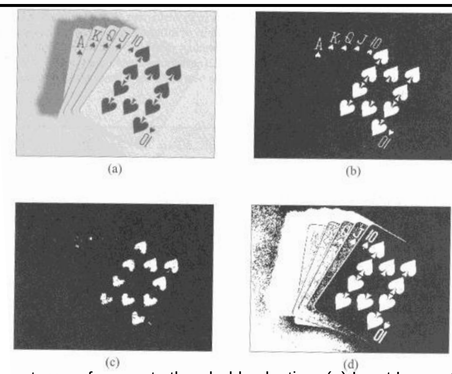


a
b c

FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with T midway between the maximum and minimum gray levels.

In thresholded images, we usually regard the non-zero values as interesting and a value of 0 as having no significance. However, this case is opposite to this convention.

Ex 2



Importance of accurate threshold selection. (a) Input Image (b) Correct choice of threshold (T = 90) (c) Threshold too low (T = 40) (d) Threshold too high (T = 215)

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Determination of Threshold T -- An Iterative Approach

1. Select an initial estimate for T.
2. Segment the image using T. This will produce two groups of pixels: G1 consisting of all pixels with gray level values > T and G2 consisting of pixels with gray level values <=T.
3. Compute the average gray levels µ1 and µ 2 for the pixels in regions G1 and G2.
4. Compute a new threshold value: T = (µ1+ µ2)/2.
5. Repeat step 2 through 4 until the difference in Ts in successive iterations is smaller than a predefined parameter T_0.

How do you select an initial estimate of T?

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Determination of Threshold T -- A Matlab Approach

- Matlab function “graythresh”
- It uses Otsu’s method to choose threshold value k that maximizes the between class variances.
- The performance is similar to the one achieved for the approach in the previous slide.

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Similarity-Based Approach -- Summary of Thresholding Method

- Thresholding groups together pixels according to some global attribute, such as grey level.
- Two pixels at opposite corners of an image will both be detected if they both have grey levels above the threshold, even though they are probably not related in any meaningful way. It is possible to distinguish between these two pixels if we additionally take into account the fact that pixels belonging to a single object are close to one another. → Connectivity Consideration

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Similarity-Based Approach -- Region Splitting

Basic Idea: Divide an image into smaller and smaller regions until all the pixels in the different regions satisfy the predefined uniformity predicate or uniformity measure for that region.

Steps:

- It begins with the entire image in one region.
- The region are then split to form sub-regions which satisfy the basic segmentation criterion using a suitable uniformity predicate or uniformity measure.

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Some Uniformity (Similarity) Measures

Determine whether the region should be split

- Consider the case of region R split into the subregions R_1, R_2, R_3, and R_4. Let μ be the mean of R and μ_i be the mean of R_i.

R_1	R_2
R_3	R_4

- The simplest Measure:
  if $|\mu - \mu_i| < \varepsilon$, then R_i is uniform. If all the regions were uniform then R would not be split into the sub-regions.

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Uniformity (Similarity) Measure 1

$$U_1(R) = \frac{\sum_{i=1}^{4} \#(R_i)\sigma_i^2}{\#(R)}$$

What is the formula to calculate the variance?

$$\mu_2(r) = \sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i).$$

- Where #(R) is the number of pixels in the region R.
- The variance σ_i^2 is a measure of the uniformity of each sub-region R_i
- A large variance indicates the region is less uniform. Therefore, a smaller U1(R) corresponds to a more uniform region R. If the region is non-uniform, then split the region.

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Uniformity (Similarity) Measure 2

$$U_2(R) = \frac{\sum_{i=1}^{4} (\mu_i - \mu)^2}{\sum_{i=1}^{4} \sigma_i^2}$$

- If U2(R) is large, then R is not uniform and should be split into the four sub-regions.
- It indicates that the sub-regions R_i differ from R but are themselves uniform.

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Uniformity (Similarity) Measure 3

$$U_3(R) = 1 - \sum_{R_j} \frac{w_j \sigma_j^2}{\sigma_{\max}^2}$$

$$Here \quad \sigma_{\max}^2 = \frac{(g_{\max} - g_{\min})^2}{2}$$

- Where g_{max} and g_{min} are maximum and minimum gray-level values in the region R and w_j is a weight associated with the sub-region R_j.

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Uniformity (Similarity) Measure 4

$$U_4(R) = \frac{(n_1 + n_2)(\mu_1 - \mu_2)^2}{n_1\sigma_1^2 + n_2\sigma_2^2} = \frac{n\sigma^2}{n_1\sigma_1^2 + n_2\sigma_2^2} - 1$$

- If the U4(R) is low, the region should be merged.
- The term n is the number of points in both regions while σ^2 is the variance of the two combined regions.

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Uniformity (Similarity) Measure Example

2	2	4	4
2	2	2	2
1	1	5	5
3	3	5	5

R1	R2
R3	R4

μ = 3, μ1 = 2, μ2 = 3, μ3 = 2, μ4 = 5.

$$\sigma_1^2 = \frac{\sum_{p \in R1} (g(p) - 2)^2}{4} = 0;$$

$$\sigma_2^2 = \frac{\sum_{p \in R2} (g(p) - 3)^2}{4} = \frac{1+1+1+1}{4} = 1;$$

$$\sigma_3^2 = \frac{\sum_{p \in R3} (g(p) - 2)^2}{4} = \frac{1+1+1+1}{4} = 1;$$

$$\sigma_4^2 = \frac{\sum_{p \in R4} (g(p) - 5)^2}{4} = 0.$$

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Uniformity (Similarity) Measure Example

Simplest Measure: | μ- μi| <= 2 .

$$U1(R) = \frac{4\times 0 + 4\times 1 + 4\times 1 + 4\times 0}{16} = 0.5;$$

$$U2(R) = \frac{(2-3)^2 + (3-3)^2 + (2-3)^2 + (5-3)^2}{0+1+1+0} = \frac{6}{2} = 3;$$

$$U3(R) = 1 - \frac{1\times 0 + 1\times 1 + 1\times 1 + 1\times 0}{(5-1)^2 / 2} = 1 - 1/4 = 0.75 .$$

$$U4(R) = \frac{8\times(2-3)^2}{4\times 0 + 4\times 1} = \frac{8}{4} = 2 \text{ for sub-regions } R1 \text{ and } R2 .$$

Where U1, U2, U3, and U4 represent the uniform measure 1, 2, 3, and 4

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Special Data Structures
-- Picture Tree in Region Splitting

- A picture tree is a graph structure in the form of a tree that has an arc between two nodes (i.e., R1 and R2, where they respectively represent two regions) of the graph, if R2 is contained in R1 (i.e., R1 is the parent node of R2).
- It indicates that R2 is a sub-region of the region R1.

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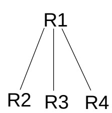
Picture Tree Example

- In the following example, R1 includes R2, R3, and R4, and these sets form a partition of R1.



- Picture tree structure is useful in implementing and describing region splitting and region merging segmentation methods.

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Special Data Structures
-- Quad Picture Tree (QPT) in Region Splitting

- Quad picture tree (QPT): It is a picture tree with the original image as the start region and progressively divides each region into four (square) sub-regions with an equal number of pixels.
- The QPT can be used to guide the search for regions with uniform gray-levels for gray-level images by developing a measure of uniformity (homogeneity) to test the regions as candidates to be split.

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QPT Example

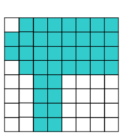
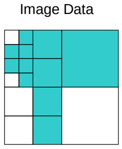
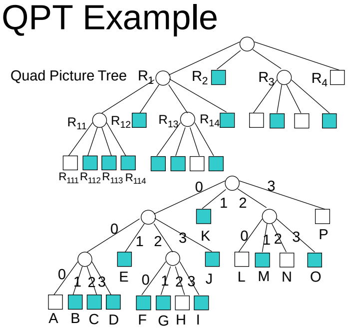


Image Data



Partition into uniform regions



001(B), 002(C), 003(D), 01x(E), 020(F), 021(G), 023(I), 03x(J), 1XX(K), 21X(M), 23X(O)

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QPT Example (Cont.)

- Cutset: It is a subset S of the nodes such that:
  - No two nodes in S belong to a path from the root node to a leaf node
  - The addition of any node destroys property.
- A segmentation of an image corresponds to a cutset of the QPT.
- The linear quadtree stores only the black nodes. A coding scheme is used to code the four descendents of a node. Each node is coded into the digits 0, 1, 2, 3.

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Similarity-Based Approach -- Region Merging

1. Define a splitting method to segment the image into small atomic regions satisfying the basic segmentation criterion using a suitable uniformity predicate.
2. Define a method for merging adjacent regions. That is, merge two adjacent regions which satisfy the merging conditions and the basic segmentation criterion (i.e., the uniform predicate for the union of these two adjacent regions is true).
3. Repeat the merging procedure. If no regions can be merged, then stop.

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Similarity-Based Approach -- Other Approaches

1. Region splitting and merging: Combine two approaches, i.e., region splitting and region merging.
2. Hybrid approach: Combine edge detection method with the region based approach.

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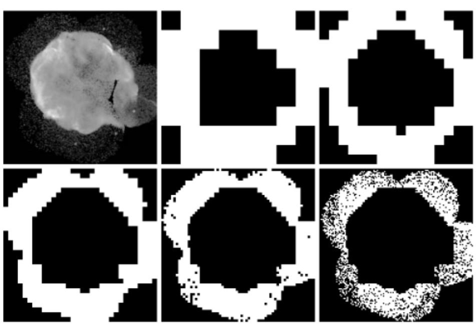

a b c
d e f

FIGURE 10.17 Image segmentation by a split-and-merge procedure. (a) Original image. (b) through (f) results of segmentation using function `splitmerge` with values of `mindim` equal to 32, 16, 8, 4, and 2, respectively. (Original image courtesy of NASA.)

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Similarity-Based Approach -- Region Growing

- Region growing is a bottom-up procedure that starts with a set of seed pixels. The aim is to grow a uniform, connected region from each seed. A pixel is added to a region if and only if
  - It has not been assigned to any other region
  - It is a neighbor of that region
  - The new region created by addition of the pixel is still uniform.
- In general, it starts with a single pixel (seed) and add new pixels slowly.

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Seeded Region Growing -- Non-Math Perspective

Basic Ideas:

1. Choose the seed pixels.
2. Check the neighboring pixels and add them to the region if they are similar to the seed by using a certain predicate.
3. Repeat step 2 for each of the newly added pixels; stop if no more pixels can be added.

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Seeded Region Growing Notations

- Let T be the set of pixels that are neighbors to some atomic region. That is:

$$T = \{ p \mid p \notin \bigcup_i A_i \ \ and \ \ (N(p) \bigcap (\bigcup_i A_i)) \neq \phi \}$$

where $N(p)$ is the 8- or 4 or m-neighbor region of pixel p. The set of A_i are initially the atomic regions.

- If pixel p is in T, then for every atomic region A_i such that

$$A_i \cap N(p) \neq \Phi \text{, then let } \delta_i(p) = | g(p) - \text{mean } \{ g(A_i) \} |$$

This is the difference between p and A_i, which is computed as the difference between the gray-level of p and the average gray-level in A_i.

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Seeded Region Growing Notations

- The minimum of these differences over A_i is defined as $\delta(p) = \min\{\ \delta_i(p) \mid N(p) \cap A_i \neq \phi\}$
- The point we are interested in is:

$$p' = \{\ p \in T \mid \delta(p) \text{ is a minimum } \}$$

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Seeded Region Growing Algorithm -- Math Perspective

1. Label the pixels in the atomic regions with their initial labeling.
2. Find the pixels in T. Assign each pixel in T to a temporary label based on its neighboring region. Put these pixels in a linear list, which is sorted in an ascending order according to $\delta_i(p)$.
3. Remove the first pixel p from the linear list while the list is not empty.

   3.1 Set p to the same label as its neighboring region.
   3.2 Update the mean of the affected (i.e., expanded) region.
   3.3 If the new neighbors of the expanded region already have temporary labels, label these neighbors as boundary pixels. These boundary pixels will not be carried on to the further process.
   3.4 Otherwise, assign the new neighbors a temporary label based on their neighboring region.
   3.5 Recalculate the $\delta_i(p)$ for each temporarily labeled neighboring pixel of the expanded region. Order the linear list in ascending order according to the new $\delta_i(p)$.

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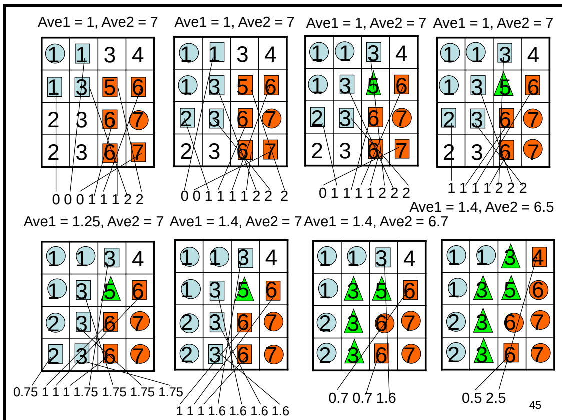

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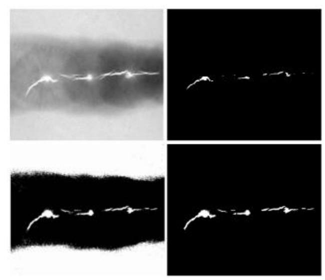

a b
c d

FIGURE 10.14 (a) Image showing defective welds. (b) Seed points. (c) Binary image showing all the pixels (in white) that passed the threshold test. (d) Result after all the pixels in (c) were analyzed for 8-connectivity to the seed points. (Original image courtesy of X-TEK Systems, Ltd.)

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Similarity-Based Approach Watershed Algorithm

- Suppose that a hole is punched in each regional minimum and that the entire topography is flooded from below by letting water rise through the holes at a uniform rate. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging. The flooding will eventually reach a stage when only the tops of the dams are visible above the water line. These dam boundaries correspond to the divide lines of the watersheds. They are the (continuous) boundaries extracted by a watershed segmentation algorithm.

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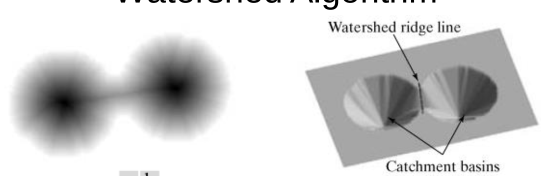
Watershed Algorithm



a b

FIGURE 10.18 (a) Gray-scale image of dark blobs. (b) Image viewed as a surface, with labeled watershed ridge line and catchment basins.

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Watershed Algorithm
-- Threshold Set

- Let g be a gray level function. The set

$$T_k = \{p \mid g(p) \le k\}$$

is called the threshold set of g at level k, where p is a pixel.

4	8	3	7
1	8	7	7
2	6	2	1
8	8	2	1

What are T_ks for k=0, 1, 2, …, 7 for this mini-image?

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Watershed Algorithm
-- Minimum

- M is a minimum if M is a plateau (i.e., connected region) of pixels with value k. All the pixels in M have gray-level k. And one can not reach a lower altitude without climbing higher in gray-level values.

$\forall p \in M, \forall q \notin M$, where $g(q) \le g(p)$ and for every path

$pa = (p_0, p_1, \ldots, p_l)$, where $p_0 = p, p_l = q$,

there is an i such that

$g(p_i) > g(p_0) = g(p)$.

The path here indicates that the start point has a higher or equal pixel intensity than the end point.

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Minimum Example

9	9	9	9	9	9	9	9
9	8	8	8	8	8	9	9
9	7	4	8	3	7	8	9
9	7	1	8	7	7	1	9
9	7	2	6	2	1	1	9
9	8	8	8	2	1	2	9
9	8	9	9	8	9	8	9
9	9	9	9	9	9	9	9

The image has the following minimums:

Minimum A with intensity 1:

Minimum B with intensity 1:

Minimum C with intensity 3:

Is there any other minimum in this mini-image?

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Watershed Algorithm
-- Geodesic Distance

- The geodesic distance between pixel p and q in a set A is the minimum of the lengths of all paths between pixel p and q.

$$d_A(p,q) = \min\{length(pa)\}$$

where pa is a path between p and q in a set A }

- A path in a set A between pixel p and q is a sequence of pixels pa = $\{p_0, p_1, \ldots, p_k\}$, where $p_0 = p$, $p_k = q$ and each p_i is in A. In addition, each p_i and p_{i+1} are adjacent (i.e., connected).

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Watershed Algorithm
-- Geodesic Influence Zone

- The geodesic influence zone of Bi in set A is defined as

$$iz_A(B_i) = \{p \in A \mid \forall j, d_A(p, B_i) < d_A(p, B_j), i \ne j\}$$

where each Bi is a connected component that partitions A. That is, $B = \bigcup_i B_i$ and B is a subset of A.

- $d_A(p, B)$ represents the geodesic distance between a point p in A and a set B where B is a subset of A. It is the minimum of the lengths of all paths from p to any point in B.

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Watershed Algorithm
-- Skeleton by Influence Zone

- The term $skiz_A(B) = A \setminus iz_A(B) = A \setminus \bigcup_i iz_A(B_i)$ represents elements in A which are not in geodesic influence zone and is called the skeleton by influence zones of A.
- They are points equidistant from two or more connected components.

Would I use the following formula to represent the skeleton by influence zones?

$$skiz_A(B) = \bigcup_i \{p \in A \mid \forall j, d_A(p, B_i) = d_A(p, B_j), i \ne j\}$$

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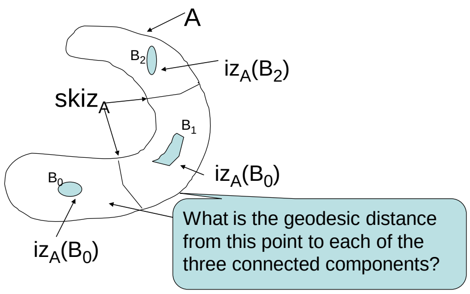
Example



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Watershed Algorithm -- Catchment Basin or Watershed

- Let M be a minimum region. Then a catchment basin C(M) consists of the pixels from which there is a downhill path to M.
- The pixels at gray-level k or less in C(M) are defined as:

$$C_K(M) = \{ p \in C(M) \mid g(p) \leq K \} = C(M) \cap T_k$$

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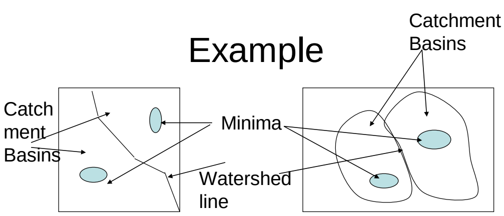
Example



Three types of points are considered:

1) Points belonging to a regional minimum.

2) Points at which a drop of water, if placed at the location of any of those points, would fall with certainty to a single minimum; → Catchment Basin

3) Points at which water would be equally likely to fall to more than one minimum. → Watershed Line

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Application and Use of Watershed Algorithm

- One of the principal applications of watershed segmentation is in the extraction of nearly uniform (bloblike) objects from the background.
- In practice, we often see watershed segmentation applied to the gradient of an image, rather than to the image itself. In this formulation, the regional minima of catchment basins correlate nicely with the small value of the gradient corresponding to the objects of interest.

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Watershed Segmentation Using the Distance Transform

1	1	0	0	0
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0
0	1	1	1	0

0.00	0.00	1.00	2.00	3.00
0.00	0.00	1.00	2.00	3.00
1.00	1.00	1.41	2.00	2.24
1.41	1.00	1.00	1.00	1.41
1.00	0.00	0.00	0.00	1.00

a b

FIGURE 10.19
(a) Small binary image.
(b) Distance transform.

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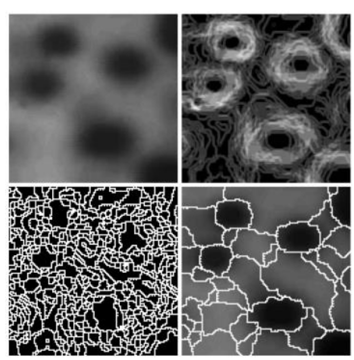

a b
c d

FIGURE 10.21
(a) Gray-scale image of small blobs. (b) Gradient magnitude image. (c) Watershed transform of (b), showing severe oversegmentation. (d) Watershed transform of the smoothed gradient image; some oversegmentation is still evident. (Original image courtesy of Dr. S. Beucher, CMM/Ecole de Mines de Paris.)

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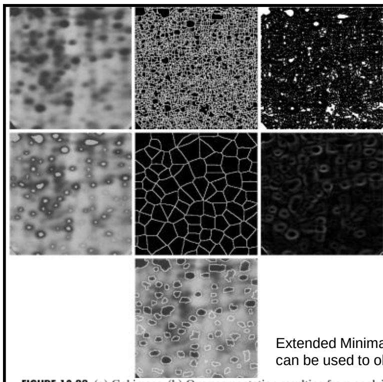

Marker-Controlled Watershed Segmentation

A marker is a connected component belonging to an image. It can be internal inside each of the objects of interest and external markers within the background.

Extended Minima Transform Technique can be used to obtain internal markers.

FIGURE 10.22 (a) Gel image. (b) Oversegmentation resulting from applying the watershed transform to the gradient magnitude image. (c) Regional minima of gradient magnitude. (d) Internal markers. (e) External markers. (f) Modified gradient magnitude. (g) Segmentation result. (Original image courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Watershed Algorithm -- Non-Math Perspective

- The objective of watersheds algorithm is to find the watershed lines.
- The basic procedure to calculate the catchment basin is to use immersion procedure. That is:

  Start with the **lowest** minimum regions and then determine the catchment basins by adding the pixels at the next gray-level that is in the catchment basin of the minimum. At each pixel where the catchment basins would merge then build a boundary between the basins.

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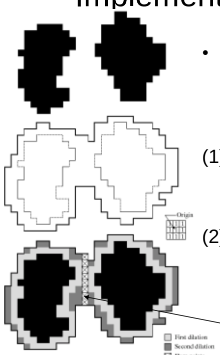
Watershed Algorithm -- Implementation Perspective



- Each of the connected components is dilated by the 3x3 square SE. The dilation is subject to two conditions:

(1)The dilation has to be constrained to the black regions and later dilated resultant regions.

(2)The dilation can not be performed on points that would cause the sets being dilated to merge.

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Watershed Algorithm -- Walk-through

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

Minima

Watershed Line

Step 1:

Find the lowest minimum regions

What is the lowest minimum in this example?

Note: Here a 3x3 cross SE is used for illustration.

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At Intensity 3

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

At Intensity 4

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

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At Intensity 5

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

At Intensity 6

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

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What is the geodesic distance from these two points to each of the components?

At Intensity 7

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

At Intensity 8

8	8	8	8	8	8	8	8	8
7	6	6	7	7	7	7	7	8
7	5	6	7	6	5	4	3	8
6	4	6	7	6	5	2	2	8
6	4	3	5	7	6	2	2	8
6	3	3	5	6	7	5	3	8
6	3	3	6	6	7	6	4	8
7	6	6	6	7	8	8	8	8

What is the geodesic distance from these two points to each of the components?

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(a) Original Image (b) Watershed lines

1. How many regional minimums does this image have?
2. What do the watershed lines look like?

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Watershed Algorithm -- Math Perspective

Define the following recurrence for $h \in [h_{min}, h_{max}]$:

$$X_{h_{min}} = T_{h_{min}} = \{p \mid g(p) = h_{min}\}$$

$$X_{h+1} = X_h \cup \min_{h+1} \cup (IZ_{T_{h+1}}(X_h) \setminus T_h)$$

The watershed of the image is the complement of $X_{h_{max}}$:

$$Watershed(g) = A \setminus X_{h_{max}}$$

What do these three union components represent?

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Other Approaches -- Neighborhood Region Growing

- All the pixels are considered as the nodes forming a graph.
- If p and p' are neighboring pixels, then connect them with an arc if |g(p) – g(p')| < ε where ε is a parameter.

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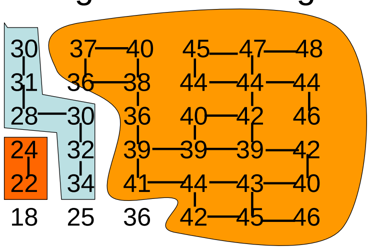
Neighborhood Region Growing



$|g(p) – g(p')| < 4$

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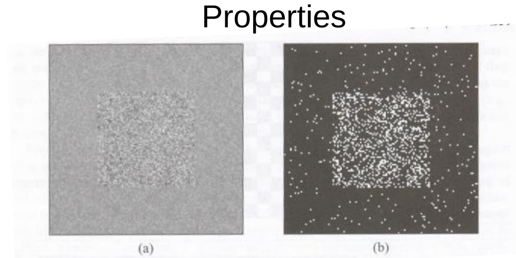
Segmentation Using Other Image Properties



Performance of gray level thresholding on textured images

(a)Image of a square (b) Result of thresholding the image in (a)

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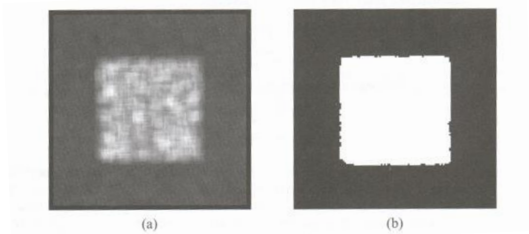

- Texture segmentation using gray level variance

  (a)Variance image (b) Result of thresholding the image in (a)

  Here the variance image is obtained by gray level variance in 7x7 regions of the textured image.

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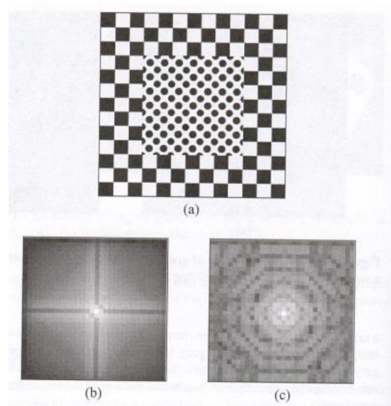

(a)An image consisting of two repeating patterns.

(b)Spectrum of a 32x32 sample of the background pattern.

(c)Spectrum of a 32x32 sample of the smaller patterned region.

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