SKALARNI PRODUKT $\quad \vec{x}\cdot\vec{y} \in \mathbb{R}$

Za $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ in $\vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ je $\vec{x}\cdot\vec{y} = \vec{x}\vec{y} = x_1y_1 + x_2y_2 + \ldots + x_ny_n$ skalarni produkt vektorjev $\vec{x}$ in $\vec{y}$.

Lastnosti skalarnega produkta

① $\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x}$ $\quad$ komutativnost / simetričnost

② $$(\alpha\vec{x})\cdot\vec{y} = \begin{bmatrix} \alpha x_1 \\ \vdots \\ \alpha x_n \end{bmatrix}\cdot\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \alpha x_1y_1 + \alpha x_2y_2 + \ldots + \alpha x_ny_n = \alpha(x_1y_1 + \ldots + x_ny_n) =$$
$$= \alpha(\vec{x}\cdot\vec{y})$$

$(\alpha\vec{x})\cdot\vec{y} = \alpha(\vec{x}\cdot\vec{y}) = \vec{x}\cdot(\alpha\vec{y})$ $\quad$ homogenost $\quad x, y \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$

③ $$\vec{x}\cdot(\vec{y}+\vec{z}) = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\cdot\begin{bmatrix} y_1+z_1 \\ \vdots \\ y_n+z_n \end{bmatrix} = x_1(y_1+z_1) + \ldots + x_n(y_n+z_n) \underset{\uparrow}{=}$$

distributivnost v $\mathbb{R}$

$$= x_1y_1 + x_1z_1 + \ldots + x_ny_n + x_nz_n = \vec{x}\cdot\vec{y} + \vec{x}\cdot\vec{z}$$ aditivnost

Lastnosti ② in ③ skupaj imenujemo **LINEARNOST**.

④ $$\vec{x}\cdot\vec{x} = x_1x_1 + \ldots + x_nx_n = \underset{\overset{\vee\!|}{0}}{x_1^2} + \ldots + \underset{\overset{\vee\!|}{0}}{x_n^2}$$ $\quad$ $\vec{x}\cdot\vec{x} \geq 0$

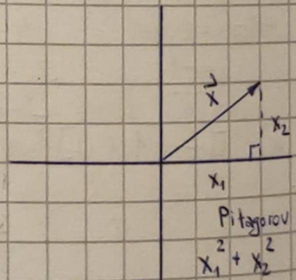
Kdaj je $\vec{x}\cdot\vec{x} = 0$?

$\vec{x}\cdot\vec{x} = 0 \iff x_1^2 + \ldots + x_n^2 = 0 \iff x_1 = \ldots = x_n = 0 \iff \vec{x} = \vec{0}$

$\vec{x}\cdot\vec{x} = 0 \iff \vec{x} = \vec{0}$ $\quad$ pozitivna definitnost

DOLŽINA VEKTORJA
(ali NORMA)

$\vec{x} \in \mathbb{R}^n$, dolžina $\|\vec{x}\| = \sqrt{\vec{x}\cdot\vec{x}}$



Pitagorov izrek
$x_1^2 + x_2^2$

$$\|\vec{x}\| = \sqrt{\vec{x}\cdot\vec{x}} = \sqrt{x_1^2 + \ldots + x_n^2}$$

Primer: $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\quad \vec{x}\cdot\vec{y} = 1\cdot2 + 2\cdot1 + 3\cdot(-1) = 1$

$\|\vec{x}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ $\qquad \|\vec{y}\| = \sqrt{2^2+1^2+(-1)^2} = \sqrt{6}$

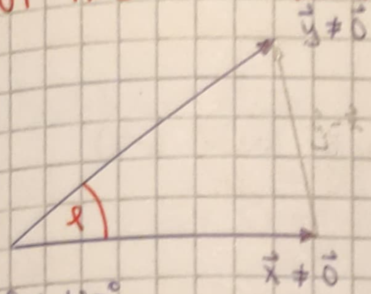
Vektor $\vec{x} \in \mathbb{R}^n$ je **ENOTSKI (NORMIRAN)**, če je $\|\vec{x}\| = 1$.

Primer: $\frac{1}{\sqrt{14}}\vec{x}$ je enotski vektor.

Vsak vektor $\frac{\vec{x}}{\|\vec{x}\|}$ je enotski vektor.

Edini vektor dolžine 0 je $\vec{0}$.

KOT MED VEKTORJEMA



$\varphi \leq 180°$

Če $\vec{x}, \vec{y} \neq 0$, potem:

$$\cos\varphi = \frac{\vec{x}\vec{y}}{\|\vec{x}\| \cdot \|\vec{y}\|}$$

Kosinusni izrek:

$$\|\vec{y}-\vec{x}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2 - 2 \cdot \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos\varphi$$

$$(\vec{y}-\vec{x})(\vec{y}-\vec{x}) = \vec{x}\vec{x} + \vec{y}\vec{y} - 2 \cdot \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos\varphi \quad \text{(def. dolžine)}$$

$$\cancel{\vec{y}\vec{y}} - \vec{x}\vec{y} - \vec{y}\vec{x} + \cancel{\vec{x}\vec{x}} = \cancel{\vec{x}\vec{x}} + \cancel{\vec{y}\vec{y}} - 2\|\vec{x}\| \cdot \|\vec{y}\| \cos\varphi \quad \text{(linearnost)}$$

$$\vec{x}\vec{y} + \vec{y}\vec{x} = 2\|\vec{x}\| \cdot \|\vec{y}\| \cos\varphi$$

$$2\vec{x}\vec{y} = 2\|\vec{x}\| \cdot \|\vec{y}\| \cos\varphi \quad /:2$$

$$\vec{x}\vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos\varphi$$

LASTNOSTI:

① Če $\varphi \in [0, \frac{\pi}{2})$, potem $\cos\varphi > 0$ in zato $\vec{x} \cdot \vec{y} > 0$.

Če $\varphi \in (\frac{\pi}{2}, \pi]$, potem $\cos\varphi < 0$ in zato $\vec{x} \cdot \vec{y} < 0$.

Če $\varphi = \frac{\pi}{2}$, potem $\cos\varphi = 0$ in zato $\vec{x} \cdot \vec{y} = 0$.

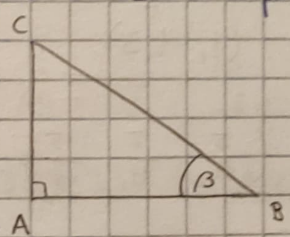
Vektorja $\vec{x}$ in $\vec{y}$ sta pravokotna (ortogonalna), če $\vec{x} \cdot \vec{y} = 0$.

$\Rightarrow$ Vektor $\vec{0}$ je pravokoten na vsak vektor.

$\boxed{P}$ $A(1,2,3)$, $B(2,2,1)$, $C(3,1,c)$

$\vec{AB} = (1,0,-2)$
$\vec{AC} = (2,-1,c-3)$
$\vec{BC} = (1,-1,c-1)$

1) $c = ?$ $\triangle ABC$ pravokotni



$$\|\vec{AB}\|^2 + \|\vec{AC}\|^2 = \|\vec{BC}\|^2$$

$$\cancel{1^2} + 2^2 + 2^2 + \cancel{1^2} + (c-3)^2 = \cancel{1} + \cancel{1} + (c-1)^2$$

$$\cancel{c^2} - 6c + 9 + 8 = \cancel{c^2} - 2c + 1$$

$$4c = 16$$

$$\underline{\underline{c = 4}} \implies C(3,1,4)$$

2) kot β

$\vec{BA} = (-1,0,2)$, $\|\vec{BA}\| = \sqrt{5}$

$\vec{BC} = (1,-1,3)$, $\|\vec{BC}\| = \sqrt{11}$

$$\cos\beta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \frac{(-1,0,2) \cdot (1,-1,3)}{\sqrt{5} \cdot \sqrt{11}} = \frac{5}{\sqrt{5} \cdot \sqrt{11}} = \frac{\sqrt{55}}{11}$$

$\beta \doteq 47{,}6°$

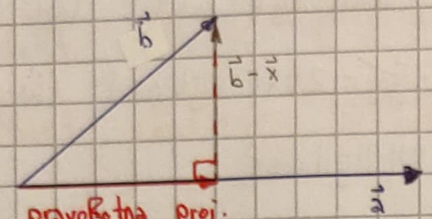
$\boxed{N1}$ $\vec{a}, \vec{b}$ $\|\vec{a}\| = 2$ $\|\vec{b}\| = 3$ $\varphi = \frac{\pi}{4}$

$$\vec{a}\vec{b} = 2 \cdot 3 \cdot \cos\frac{\pi}{4} = 3\sqrt{2}$$

$$(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = \vec{a}\vec{a} - \cancel{\vec{a}\vec{b}} + \cancel{\vec{b}\vec{a}} - \vec{b}\vec{b} = \|\vec{a}\|^2 - \|\vec{b}\|^2 = 2^2 - 3^2 = 4 - 9 = \underline{-5}$$

PRAVOKOTNA PROJEKCIJA (VEKTORJA NA VEKTOR)

Naj $\vec{a}, \vec{b} \neq \vec{0}$ vektorja v $\mathbb{R}^n$.



pravokotna proj. vektorja $\vec{b}$ na vektor $\vec{a}$

$\text{proj}_{\vec{a}}\vec{b} = \vec{x}$ ... proj $\vec{b}$ na $\vec{a}$ (vzdolž $\vec{b}-\vec{x}$)

LASTNOSTI:

① $\vec{x} = \alpha\vec{a}$ smer

② $\vec{a} \perp \vec{b}-\vec{x}$ pravokotna smer

oznaka: $\text{proj}_{\vec{a}}\vec{b}$

→ po ②: $\vec{a}\cdot(\vec{b}-\vec{x}) = 0$

po ①: $\vec{a}\cdot(\vec{b}-\alpha\vec{a}) = 0$

$$\vec{a}\vec{b} - \alpha\vec{a}\vec{a} = 0$$

$$\alpha\vec{a}\vec{a} = \vec{a}\vec{b} \quad |: \vec{a}\vec{a} \neq 0$$

$$\alpha = \frac{\vec{a}\vec{b}}{\vec{a}\vec{a}} \Longrightarrow \text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\cdot\vec{a} \;!$$

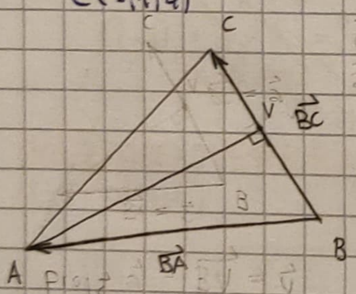
$$\|\text{proj}_{\vec{a}}\vec{b}\| = \|\vec{b}\|\cdot|\cos\varphi| = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$$

N2 $\triangle ABC$

A(1,2,3)

B(2,2,1)

C(3,1,4)



$\vec{BA} = (-1, 0, 2)$

$\vec{BC} = (1, -1, 3)$

$\vec{BV} = (v_1-2, v_2-2, v_3-1)$

$$\vec{BV} = \text{proj}_{\vec{BC}}\vec{BA} = \frac{\vec{BC}\cdot\vec{BA}}{\|\vec{BC}\|^2}\cdot\vec{BC}$$

$$\vec{BV} = \frac{(1,-1,3)\cdot(-1,0,2)}{11}\cdot(1,-1,3)$$

$$\vec{BV} = \frac{5}{11}\cdot(1,-1,3) = \left(\frac{5}{11}, -\frac{5}{11}, \frac{15}{11}\right)$$

$$\left(\frac{5}{11}, -\frac{5}{11}, \frac{15}{11}\right) = (v_1-2, v_2-2, v_3-1)$$

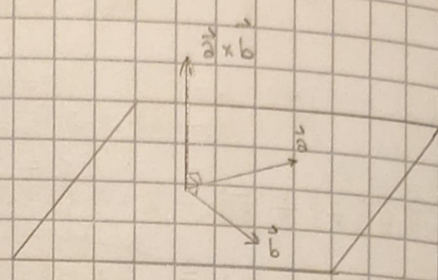
$$\left.\begin{aligned} v_1 - 2 &= \frac{5}{11} &\Rightarrow\quad v_1 &= \frac{27}{11} \\ v_2 - 2 &= -\frac{5}{11} &\Rightarrow\quad v_2 &= \frac{17}{11} \\ v_3 - 1 &= \frac{15}{11} &\Rightarrow\quad v_3 &= \frac{26}{11} \end{aligned}\right\} \quad V\left(\frac{27}{11}, \frac{17}{11}, \frac{26}{11}\right)$$

VEKTORSKI PRODUKT

def. le za vektorje v $\mathbb{R}^3$, $\vec{a} \times \vec{b} \in \mathbb{R}^3$

Za $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ in $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ je $\vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \in \mathbb{R}^3$ vektorski produkt vektorjev $\vec{a}$ in $\vec{b}$. Beremo $\vec{a}$ vektorsko $\vec{b}$.

Primer: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2\cdot 6 - 3\cdot 5 \\ 3\cdot 4 - 1\cdot 6 \\ 1\cdot 5 - 2\cdot 4 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$



Lastnosti vektorskega produkta

① $\vec{a} \times \vec{a} = \begin{bmatrix} a_2 a_3 - a_3 a_2 \\ a_3 a_1 - a_1 a_3 \\ a_1 a_2 - a_2 a_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$

② $\vec{b} \times \vec{a} = \begin{bmatrix} b_2 a_3 - b_3 a_2 \\ b_3 a_1 - b_1 a_3 \\ b_1 a_2 - b_2 a_1 \end{bmatrix} = -(\vec{a} \times \vec{b})$ ANTISIMETRIČNOST

③ Geometrijska lastnost: $\vec{a} \times \vec{b}$ je pravokoten na vsakega izmed njiju $(\vec{a}, \vec{b})$

$$(\vec{a} \times \vec{b}) \cdot \vec{a} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \cancel{a_1 a_2 b_3} - \cancel{a_1 a_3 b_2} + \cancel{a_2 a_3 b_1} - \cancel{a_1 a_2 b_3} + \cancel{a_1 a_3 b_2} - \cancel{a_2 a_3 b_1} = 0$$

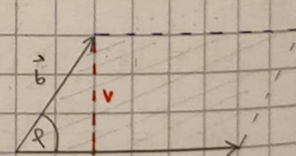
$\Rightarrow$ $(\vec{a} \times \vec{b}) \perp \vec{a}$ in podobno pokažemo $(\vec{a} \times \vec{b}) \perp \vec{b}$.

Za dva vektorja v ravnini lahko z lin. kombinacijami zapišemo vse vektorje v tej ravnini. Za vektor, ki kaže iz ravnine, potrebujemo vektorski produkt.

④ Geometrijska lastnost: $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \cdot \|\vec{b}\| \cdot |\sin \varphi|$

dolžina $\vec{a} \times \vec{b}$

ploščina paralelograma, napetega na $\vec{a}$ in $\vec{b}$



Ploščina paralelograma, napetega na $\vec{a}$ in $\vec{b}$ je dolžina vektorskega produkta $\vec{a} \times \vec{b}$.

Ploščina trikotnika, napetega na $\vec{a}$ in $\vec{b}$ je $\frac{1}{2}$ dolžine vek. produkta $\vec{a} \times \vec{b}$.

★ DOKAZ: $\|\vec{a} \times \vec{b}\|^2 = (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2 + (a_1 b_2 - a_2 b_1)^2 =$

$= a_2^2 b_3^2 + a_3^2 b_2^2 - 2 a_2 a_3 b_2 b_3 + a_3^2 b_1^2 + a_1^2 b_3^2 - 2 a_1 a_3 b_1 b_3 + a_1^2 b_2^2 + a_2^2 b_1^2 - 2 a_1 a_2 b_1 b_2$

$\|\vec{a}\|^2 \cdot \|\vec{b}\|^2 \sin^2 \varphi = \|\vec{a}\|^2 \|\vec{b}\|^2 (1 - \cos^2 \varphi) = \|\vec{a}\|^2 \|\vec{b}\|^2 - \|\vec{a}\|^2 \|\vec{b}\|^2 \cos^2 \varphi =$

$= (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1 b_1 + a_2 b_2 + a_3 b_3)^2 = \cancel{a_1^2 b_1^2} + a_1^2 b_2^2 + a_1^2 b_3^2 +$

$+ a_2^2 b_1^2 + \cancel{a_2^2 b_2^2} + a_2^2 b_3^2 + a_3^2 b_1^2 + a_3^2 b_2^2 + \cancel{a_3^2 b_3^2} - (\cancel{a_1^2 b_1^2} + \cancel{a_2^2 b_2^2} + \cancel{a_3^2 b_3^2} + 2 a_1 a_2 b_1 b_2 +$

$+ 2 a_1 a_3 b_1 b_3 + 2 a_2 a_3 b_2 b_3) = (\text{leva stran})^2$

Iz $\|\vec{a}\times\vec{b}\|^2 = \|\vec{a}\|^2\cdot\|\vec{b}\|^2\cdot\sin^2\varphi$ sledi: $\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\cdot\|\vec{b}\|\cdot|\sin\varphi|$. $\blacksquare$

Posebej: Kdaj je $\vec{a}\times\vec{b}=\vec{0}$?

$\vec{a}\times\vec{b}=0 \Leftrightarrow \|\vec{a}\times\vec{b}\|=0$

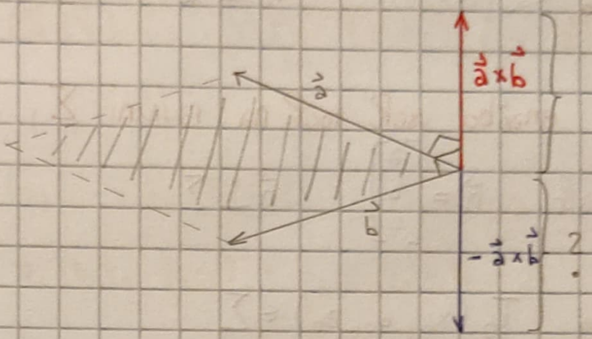
- $\vec{a}=0$
- $\vec{b}=0$
- $\varphi = k\pi,\ k\in\mathbb{Z} \Leftrightarrow \vec{a}$ in $\vec{b}$ kolinearna

$\vec{a}\times\vec{b}=\vec{0} \Leftrightarrow \vec{a}\parallel\vec{b}$

$\vec{a}\cdot\vec{b}=0 \Leftrightarrow \vec{a}\perp\vec{b}$

⑤ Geometrijska lastnost

$\Rightarrow$ pravilo desnosučnega vijaka



N3 $\vec{a},\vec{b},\vec{c}\in\mathbb{R}^3,\quad \alpha\in\mathbb{R}$ $\qquad \vec{a}=(a_1,a_2,a_3)\quad \vec{b}=(b_1,b_2,b_3)\quad \vec{c}=(c_1,c_2,c_3)$

DISTRIBUTIVNOST $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

LS: $$\vec{a}\times(\vec{b}+\vec{c}) = \begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\times\begin{bmatrix}b_1+c_1\\b_2+c_2\\b_3+c_3\end{bmatrix} = \begin{bmatrix}a_2(b_3+c_3)-a_3(b_2+c_2)\\a_3(b_1+c_1)-a_1(b_3+c_3)\\a_1(b_2+c_2)-a_2(b_1+c_1)\end{bmatrix}$$

DS: $$\vec{a}\times\vec{b}+\vec{a}\times\vec{c} = \begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\times\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}+\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\times\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{bmatrix}+\begin{bmatrix}a_2c_3-a_3c_2\\a_3c_1-a_1c_3\\a_1c_2-a_2c_1\end{bmatrix} =$$

$$= \begin{bmatrix}a_2(b_3+c_3)-a_3(b_2+c_2)\\a_3(b_1+c_1)-a_1(b_3+c_3)\\a_1(b_2+c_2)-a_2(b_1+c_1)\end{bmatrix} = LS \quad \blacksquare$$

HOMOGENOST $\vec{a}\times(\alpha\vec{b}) = \alpha\,\vec{a}\times\vec{b} = (\alpha\vec{a})\times\vec{b}$

L: $$\vec{a}\times(\alpha\vec{b}) = \begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\times\begin{bmatrix}\alpha b_1\\\alpha b_2\\\alpha b_3\end{bmatrix} = \begin{bmatrix}\alpha a_2b_3-\alpha a_3b_2\\\alpha a_3b_1-\alpha a_1b_3\\\alpha a_1b_2-\alpha a_2b_1\end{bmatrix}$$

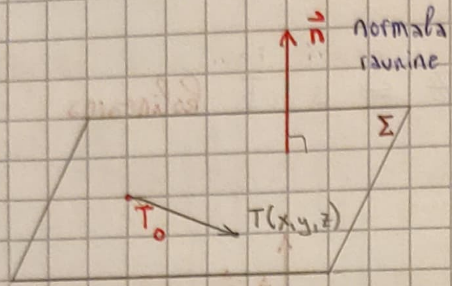
S: $$\alpha\,\vec{a}\times\vec{b} = \alpha\begin{bmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{bmatrix} = \begin{bmatrix}\alpha a_2b_3-\alpha a_3b_2\\\alpha a_3b_1-\alpha a_1b_3\\\alpha a_1b_2-\alpha a_2b_1\end{bmatrix} = L$$

D: $$(\alpha\vec{a})\times\vec{b} = \begin{bmatrix}\alpha a_1\\\alpha a_2\\\alpha a_3\end{bmatrix}\times\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} = \begin{bmatrix}\alpha a_2b_3-\alpha a_3b_2\\\alpha a_3b_1-\alpha a_1b_3\\\alpha a_1b_2-\alpha a_2b_1\end{bmatrix} = S = L \quad \blacksquare$$

ENAČBA RAVNINE

S čim lahko podamo ravnino:

- 3 točke
- 2 vektorja in začetna točka (fiksirana)
- s smerjo, na katero je ravnina pravokotna in točko na ravnini



$\vec{n} \perp \Sigma$

⇒ Določimo enačbo vseh točk na ravnini Σ, če poznamo:

- normalo $\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\vec{n} \perp \Sigma$
- točko $T_0(x_0, y_0, z_0) \in \Sigma$

Točka $T(x,y,z)$ leži na ravnini Σ, če $\overrightarrow{T_0T} \parallel \Sigma$, tj. $\overrightarrow{T_0T} \perp \vec{n}$.

$$\underbrace{\begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix}}_{\overrightarrow{T_0T}} \cdot \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\vec{n}} = 0 \Rightarrow \underline{a}(x-\underline{x_0}) + \underline{b}(y-\underline{y_0}) + \underline{c}(z-\underline{z_0}) = 0$$

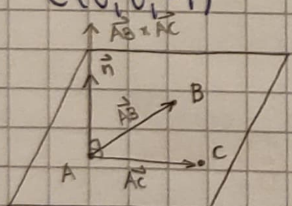
podčrtane člene imamo podane

$$\Rightarrow ax + by + cz = \underbrace{ax_0 + by_0 + cz_0}_{= d}$$

- a, b, c: koordinate $\vec{n}$
- x_0, y_0, z_0: koord. točke na Σ

Primer: $a=1,\ b=2,\ c=3$; $x_0=4,\ y_0=5,\ z_0=6$ $\Big\}$ $x + 2y + 3z = 32$

P $A(-1,2,-1)$, $B(2,-1,2)$, $C(0,0,-1)$ enačba ravnine skozi te točke



$\overrightarrow{AB} = (3, -3, 3)$

$\overrightarrow{AC} = (1, -2, 0)$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} \cancel{-3\cdot 0} - 3\cdot(-2) \\ 3\cdot 1 - \cancel{3\cdot 0} \\ 3\cdot(-2) - (-3)\cdot 1 \end{bmatrix} =$$

$-6 - (-3) = -6 + 3 = -3$

$$\vec{n} = \frac{1}{3}\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 6 \\ 3 \\ -3 \end{bmatrix} \cdot \frac{1}{3} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Enačba ravnine: $2x + y - z = 2\cdot 0 + 1\cdot 0 + (-1)\cdot(-1)$

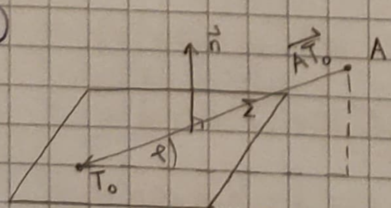
$$\underline{\underline{2x + y - z = 1}}$$

$d = 2\cdot 0 + 0 - (-1) = 1$ če vstavimo C

$d = 2\cdot 2 + (-1) - 2 = 1$ če vstavimo B (isto dobimo!)

N4 Σ ravnina, normala $\vec{n}$, $T_0 \in \Sigma$, $A \notin \Sigma$

①



② $\overrightarrow{AT_0}$ na $\vec{n}$

③ $\overrightarrow{T_0A} \cdot \vec{n} = \|\overrightarrow{T_0A}\| \cdot \|\vec{n}\| \overbrace{\cos(90° - \varphi)}^{= \sin\varphi}$

$$\varphi = \arcsin\left(\frac{\overrightarrow{T_0A} \cdot \vec{n}}{\|\overrightarrow{T_0A}\| \cdot \|\vec{n}\|}\right)$$

→ N4

④ $\vec{d} = \text{proj}_{\vec{n}} \overrightarrow{AT_0} = \frac{\vec{n}\cdot\overrightarrow{AT_0}}{\|\vec{n}\|^2}\cdot\vec{n}$

$\|\vec{d}\| = \frac{\vec{n}\cdot\overrightarrow{AT_0}}{\|\vec{n}\|}$

⑤ $\overrightarrow{T_0A}\cdot\vec{n}$ o legi A

$\overrightarrow{T_0A}\cdot\vec{n} = \|\overrightarrow{T_0A}\|\cdot\|\vec{n}\|\cdot\cos\varphi$

$\overrightarrow{T_0A}\cdot\vec{n} > 0$: leži desno od T_0 (nad/pod Σ)

$\overrightarrow{T_0A}\cdot\vec{n} < 0$: leži levo od T_0 (nad/pod Σ)

MEŠANI PRODUKT

$\vec{a},\vec{b},\vec{c}\in\mathbb{R}^3$ $\quad\vec{a}\times\vec{b}$ vektor, $\vec{a}\vec{b}$ skalar

Mešani produkt definiramo kot $(\vec{a}\times\vec{b})\cdot\vec{c}$. Velja $(\vec{a}\times\vec{b})\cdot\vec{c} = \vec{a}\cdot(\vec{b}\times\vec{c})$.

★ DOKAZ:

$$(\vec{a}\times\vec{b})\cdot\vec{c} = (a_2b_3 - a_3b_2)c_1 + (a_3b_1 - a_1b_3)c_2 + (a_1b_2 - a_2b_1)c_3$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1)$$

LS = DS ▨

Mešani produkt vektorjev $\vec{a},\vec{b},\vec{c}\in\mathbb{R}^3$ je enak:

$$(\vec{a},\vec{b},\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c} = \vec{a}\cdot(\vec{b}\times\vec{c})\in\mathbb{R}$$

N5 $\quad(\vec{a},\vec{b},\vec{c}) = (\vec{b},\vec{c},\vec{a}) = (\vec{c},\vec{a},\vec{b}) = -(\vec{a},\vec{c},\vec{b}) = -(\vec{b},\vec{a},\vec{c}) = -(\vec{c},\vec{b},\vec{a})$

① $(\vec{a},\vec{b},\vec{c}) = (\vec{a}\times\vec{b})\vec{c} = \vec{a}(\vec{b}\times\vec{c})$

② $(\vec{b},\vec{c},\vec{a}) = (\vec{b}\times\vec{c})\vec{a} = \vec{a}(\vec{b}\times\vec{c}) = ①$

③ $(\vec{c},\vec{a},\vec{b}) = (\vec{c}\times\vec{a})\vec{b} = \vec{c}(\vec{a}\times\vec{b}) = (\vec{a}\times\vec{b})\vec{c} = \vec{a}(\vec{b}\times\vec{c}) = ①$

④ $-(\vec{a},\vec{c},\vec{b}) = -(\vec{a}\times\vec{c})\vec{b} = -\vec{a}(\vec{c}\times\vec{b}) = -\vec{a}\cdot(-\vec{b}\times\vec{c}) = \vec{a}(\vec{b}\times\vec{c}) = ①$

⑤ $-(\vec{b},\vec{a},\vec{c}) = -(\vec{b}\times\vec{a})\vec{c} = -(-\vec{a}\times\vec{b})\vec{c} = (\vec{a}\times\vec{b})\vec{c} = \vec{a}(\vec{b}\times\vec{c}) = ①$

⑥ $-(\vec{c},\vec{b},\vec{a}) = -(\vec{c}\times\vec{b})\vec{a} = -(-\vec{b}\times\vec{c})\vec{a} = \vec{a}(\vec{b}\times\vec{c}) = ①$ ▨

GEOMETRIJSKI POMEN MEŠANEGA PRODUKTA

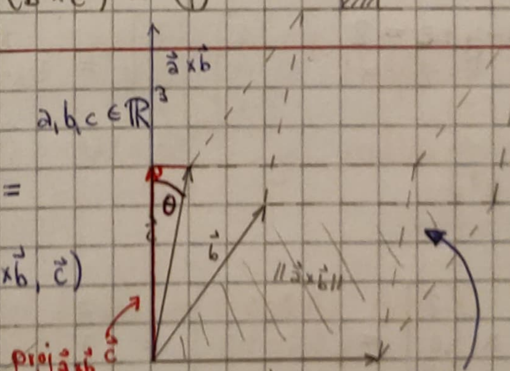
$a,b,c\in\mathbb{R}^3$

$$|(\vec{a},\vec{b},\vec{c})| = |(\vec{a}\times\vec{b})\cdot\vec{c}| = |\|\vec{a}\times\vec{b}\|\cdot\|\vec{c}\|\cdot\cos\theta| =$$

$\theta = \sphericalangle(\vec{a}\times\vec{b},\vec{c})$

$$= |\|\vec{a}\times\vec{b}\|\cdot\underbrace{\|\vec{c}\|\cdot\cos\theta}_{\|\text{proj}_{\vec{a}\times\vec{b}}\vec{c}\|}|$$

= višina paralelepipeda



paralelepiped (= 3D paralelogram)

⇒ dobimo prostornino paralelepipeda, napetega na $\vec{a},\vec{b}$ in $\vec{c}$

POSLEDICA: $(\vec{a},\vec{b},\vec{c}) = 0 \Leftrightarrow \vec{a},\vec{b},\vec{c}$ ležijo v isti ravnini (so koplanarni)