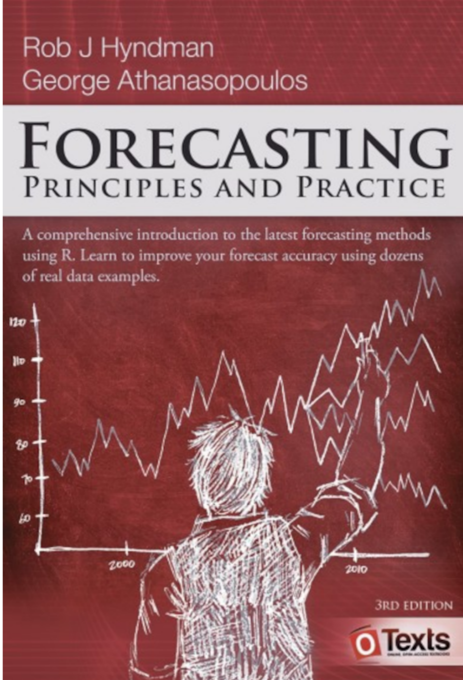
2. Time series graphics

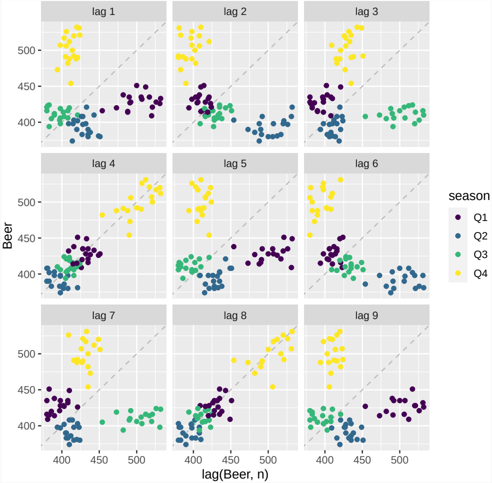
2.8 Autocorrelation

OTexts.org/fpp3/



Example: Beer production

```
new_production |> gg_lag(Beer, geom = "point")
```



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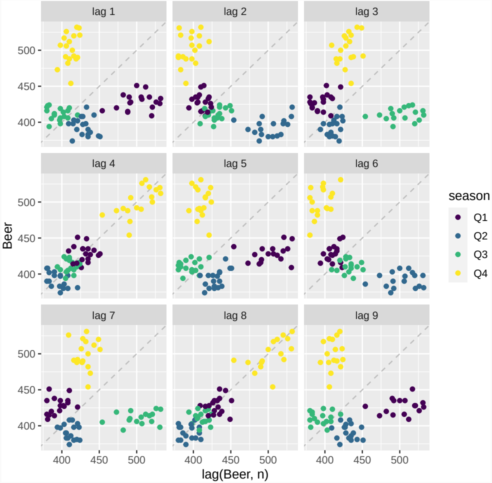

- Each graph shows y_t plotted against y_{t-k} for different values of k.
- The autocorrelations are the correlations associated with these scatterplots.
- r_1 = Correlation(y_t, y_{t-1})
- r_2 = Correlation(y_t, y_{t-2})
- r_3 = Correlation(y_t, y_{t-3})

$\vdots$

Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T} \sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})$$

$$\text{and} \qquad r_k = c_k / c_0$$

Autocorrelation

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$$c_k = \frac{1}{T} \sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k / c_0$$

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

Autocorrelation

Results for first 9 lags for beer data:

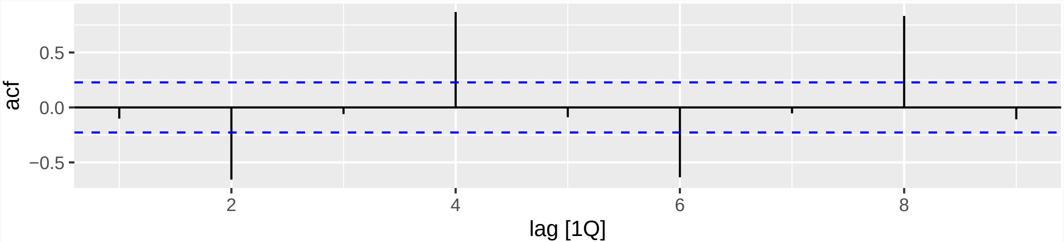
```
new_production |> ACF(Beer, lag_max = 9)
```

```
## # A tsibble: 9 x 2 [1Q]
##        lag     acf
##   <cf_lag>   <dbl>
## 1       1Q -0.102
## 2       2Q -0.657
## 3       3Q -0.0603
## 4       4Q  0.869
## 5       5Q -0.0892
## 6       6Q -0.635
## 7       7Q -0.0542
```

Autocorrelation

Results for first 9 lags for beer data:

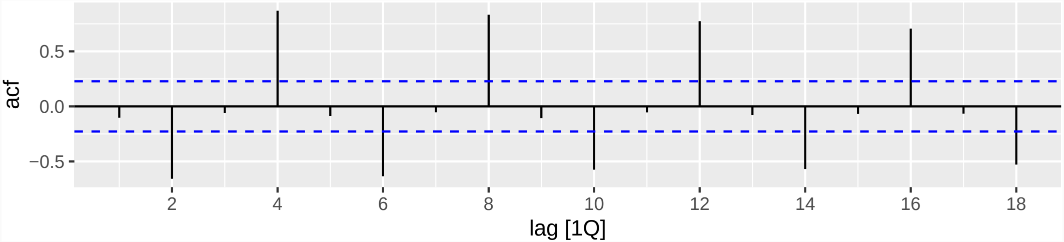
```
new_production |> ACF(Beer, lag_max = 9) |> autoplot()
```



- Together, the autocorrelations at lags 1, 2, . . . , make up the *autocorrelation* or ACF.
- The plot is known as a **correlogram**

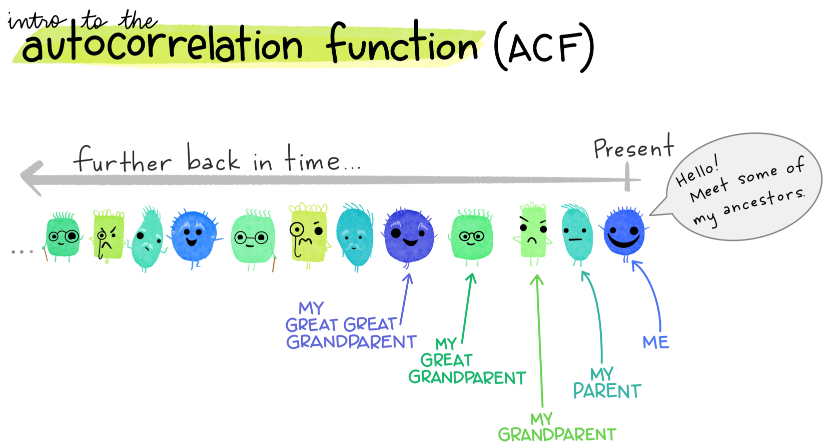
Autocorrelation

```
new_production |> ACF(Beer) |> autoplot()
```



- r_4 higher than for the other lags due to **the seasonal pattern in the data**: peaks tend to be **4 quarters** apart and troughs tend to be **4 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.

Autocorrelation functions

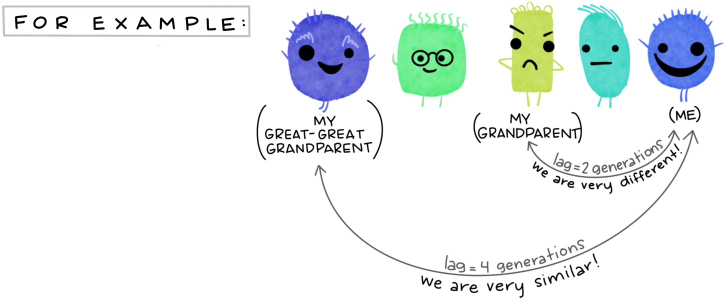


Artwork by @allison_horst

Autocorrelation functions

in our family MONSTERS tend to be...

- A little similar to their parent and great-grandparent
- Very different from their grandparent
- Very similar to their great-great grandparent



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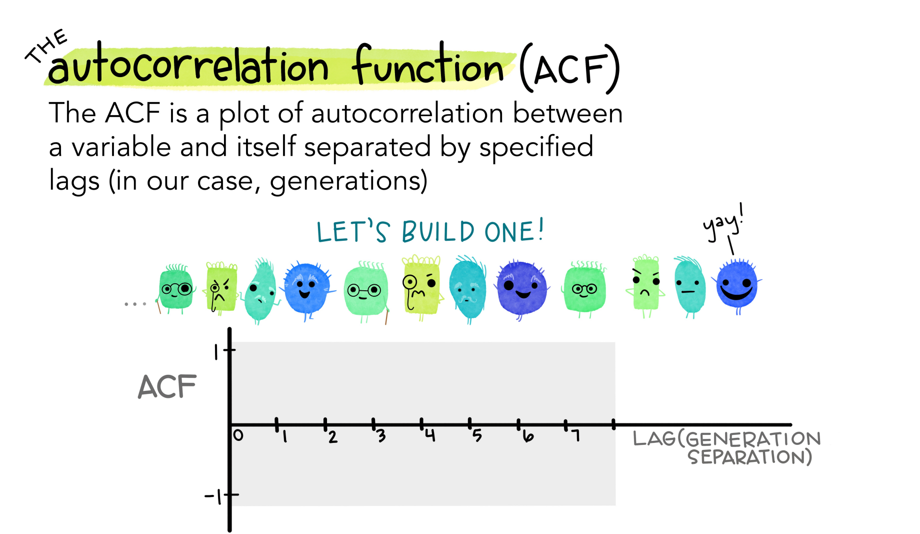
Autocorrelation functions



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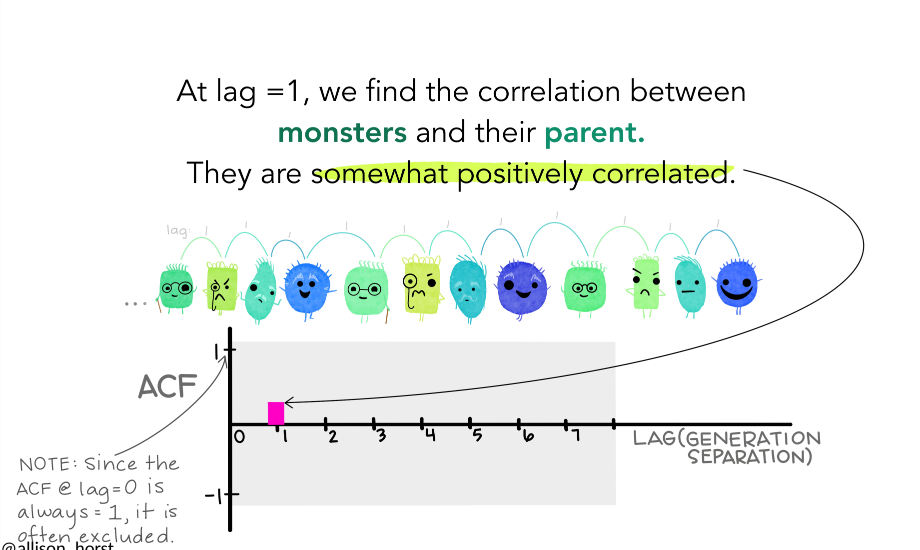
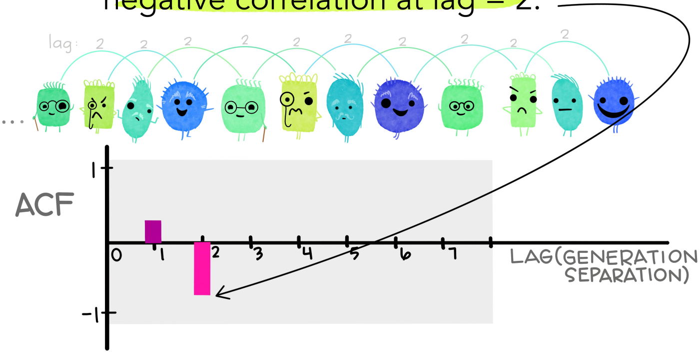
Autocorrelation functions



Artwork by @allison_horst

Autocorrelation functions

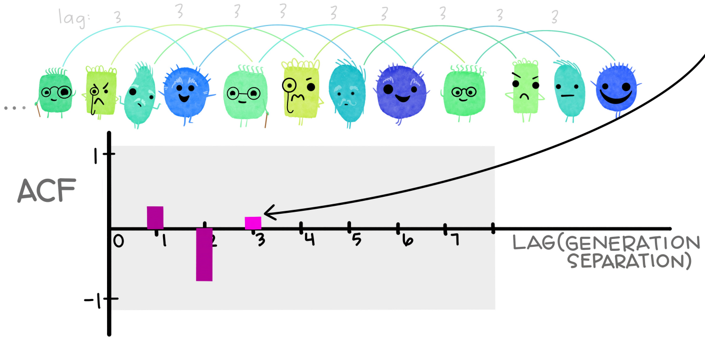
At lag = 2, we find the correlation between **monsters** and their **grandparent**. Since they tend to be very different, we find a negative correlation at lag = 2.



Artwork by @allison_horst

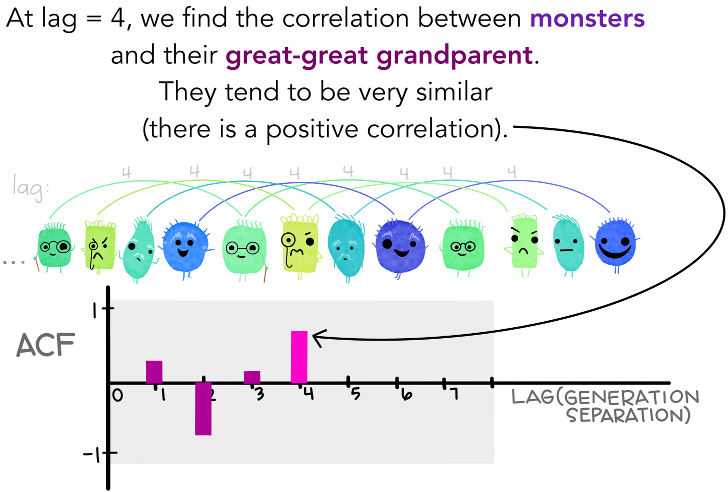
Autocorrelation functions

At lag = 3, we find the correlation between **monsters** and their **great-grandparent.** They are slightly positively correlated.



Artwork by @allison_horst

Autocorrelation functions



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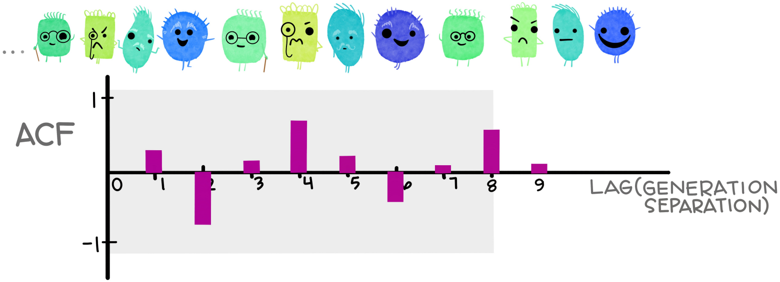
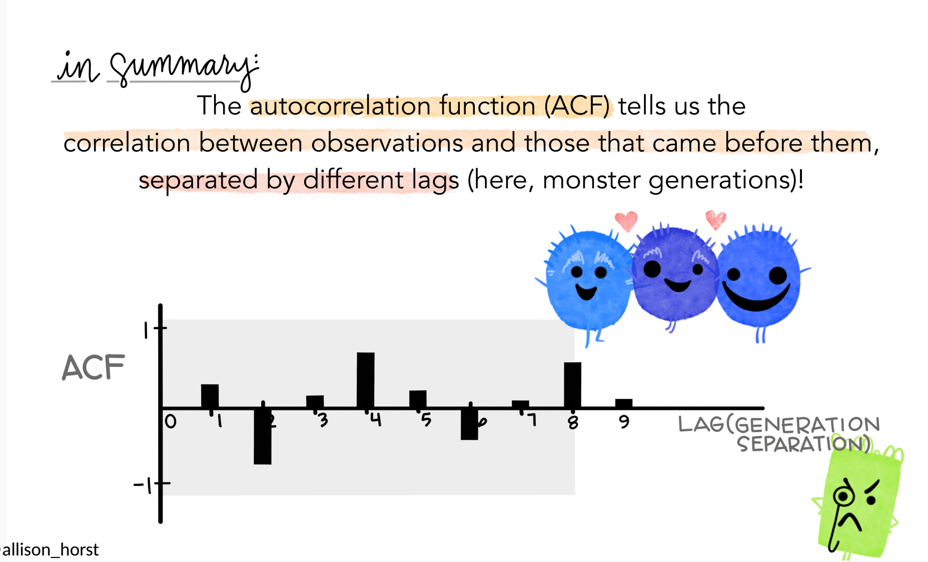
Autocorrelation functions

...and we continue finding the correlations as we increase the lag (generations) between the monsters...



Artwork by @allison_horst

Autocorrelation functions



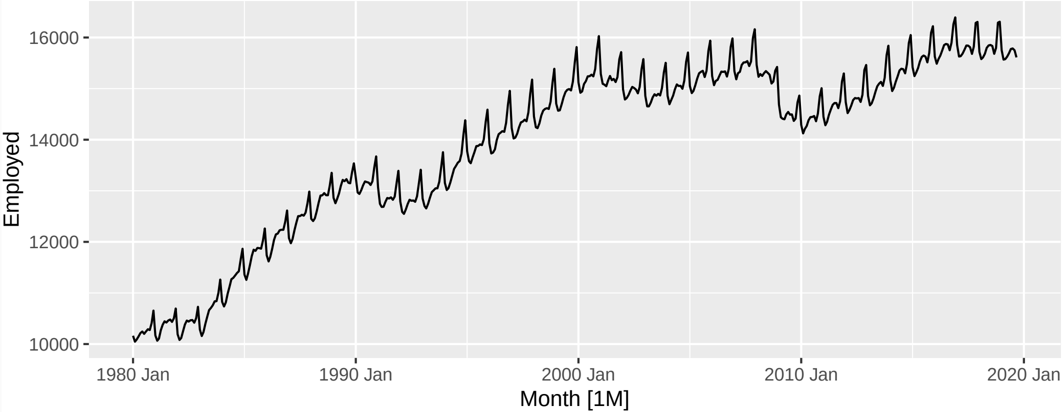
Artwork by @allison_horst

Trend and seasonality in ACF plots

- When data have a trend, the autocorrelations for small lags tend to be large and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects.

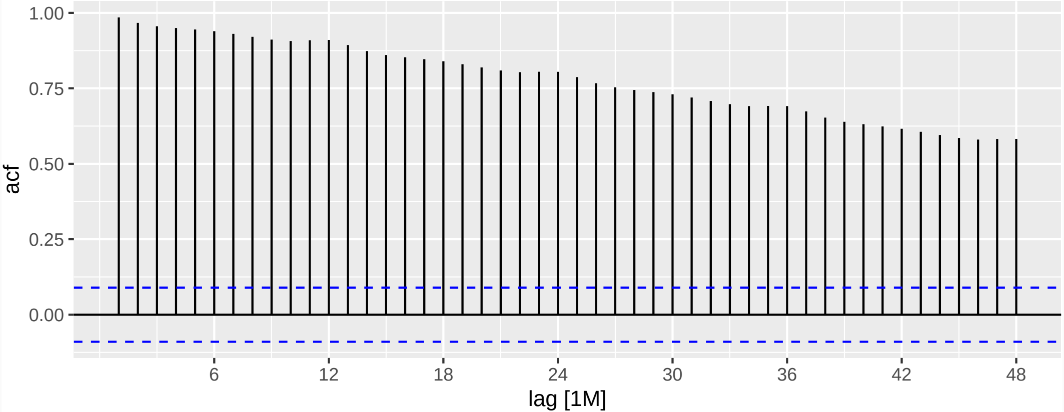
US retail trade employment

```
retail <- us_employment |>
  filter(Title == "Retail Trade", year(Month) >= 1980)
retail |> autoplot(Employed)
```



US retail trade employment

```
retail |>
  ACF(Employed, lag_max = 48) |>
  autoplot()
```



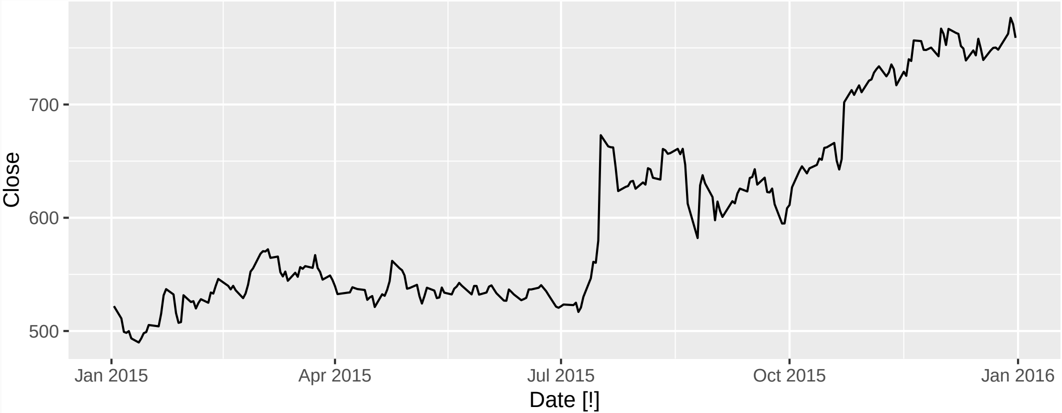
Google stock price

```
google_2015 <- gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2015) |>
  select(Date, Close)
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##    Date       Close
##    <date>     <dbl>
##  1 2015-01-02  522.
##  2 2015-01-05  511.
##  3 2015-01-06  499.
##  4 2015-01-07  498.
##  5 2015-01-08  500.
##  6 2015-01-09  493.
##  7 2015-01-12  490.
##  8 2015-01-13  493.
##  9 2015-01-14  498.
## 10 2015-01-15  499.
```

Google stock price

```
google_2015 |> autoplot(Close)
```



Google stock price

```
google_2015 |>
  ACF(Close, lag_max = 100)
```

```
## # A tsibble: 100 x 2 [1]
##          lag   acf
##     <cf_lag> <dbl>
##  1         1 0.982
##  2         2 0.959
##  3         3 0.937
##  4         4 0.918
##  5         5 0.901
##  6         6 0.883
##  7         7 0.865
##  8         8 0.849
##  9         9 0.834
## 10        10 0.818
## # ... with 90 more rows
```

Google stock price

```r
google_2015 |>
  ACF(Close, lag_max = 100) |>
  autoplot()
```