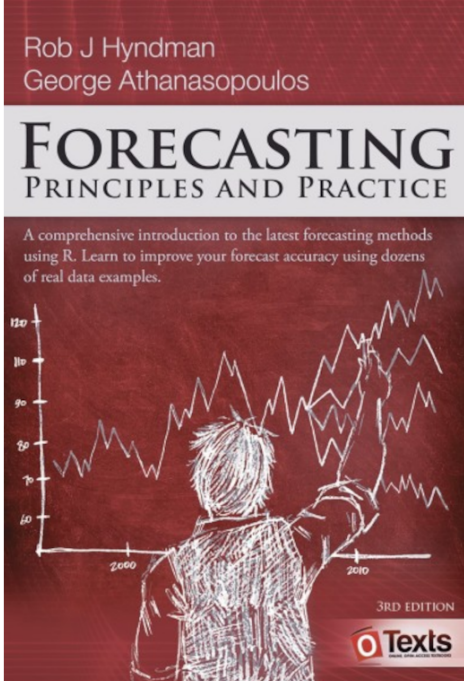
ETC3550/ETC5550
Applied forecasting

Ch2. Time series graphics

OTexts.org/fpp3/



Outline

1. Time series in R
2. Example: Australian prison population
3. Example: Australian pharmaceutical sales
4. Time plots
5. Seasonal and subseries plots
6. Lag plots and autocorrelation
7. White noise

Outline

`tsibble` objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country              GDP Imports Exports Population
##    <dbl> <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

`tsibble` objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

tsibble objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index Key                <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

tsibble objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index Key                  Measured variables
##   1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##   2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##   3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##   4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##   5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##   6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##   7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##   8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##   9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

`tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      <qtr> <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

`tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index Keys                    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

`tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index    Keys                Measure
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

`tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index  Keys                  Measure
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

`tsibble` objects

- A `tsibble` allows storage and manipulation of multiple time series in R.
- It contains:
  - An index: time information about the observation
  - Measured variable(s): numbers of interest
  - Key variable(s): optional unique identifiers for each series
- It works with tidyverse functions.

The `tsibble` index

Example

```
mydata <- tsibble(
  year = 2012:2016,
  y = c(123, 39, 78, 52, 110),
  index = year
)
mydata
```

```
## # A tsibble: 5 x 2 [1Y]
##    year     y
##   <int> <dbl>
## 1  2012   123
## 2  2013    39
## 3  2014    78
## 4  2015    52
## 5  2016   110
```

The `tsibble` index

Example

```
mydata <- tibble(
  year = 2012:2016,
  y = c(123, 39, 78, 52, 110)
) |>
  as_tsibble(index = year)
mydata
```

```
## # A tsibble: 5 x 2 [1Y]
##    year     y
##   <int> <dbl>
## 1  2012   123
## 2  2013    39
## 3  2014    78
## 4  2015    52
## 5  2016   110
```

The `tsibble` index

For observations more frequent than once per year, we need to use a time class function on the index.

```
z
```

```
## # A tibble: 5 x 2
##   Month    Observation
##   <chr>          <dbl>
## 1 2019 Jan          50
## 2 2019 Feb          23
## 3 2019 Mar          34
## 4 2019 Apr          30
## 5 2019 May          25
```

The `tsibble` index

For observations more frequent than once per year, we need to use a time class function on the index.

```
z |>
  mutate(Month = yearmonth(Month)) |>
  as_tsibble(index = Month)
```

```
## # A tsibble: 5 x 2 [1M]
##      Month Observation
##      <mth>       <dbl>
## 1 2019 Jan          50
## 2 2019 Feb          23
## 3 2019 Mar          34
## 4 2019 Apr          30
```

The `tsibble` index

Common time index variables can be created with these functions:

Frequency	Function
Annual	`start:end`
Quarterly	`yearquarter()`
Monthly	`yearmonth()`
Weekly	`yearweek()`
Daily	`as_date(), ymd()`
Sub-daily	`as_datetime()`

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Australian prison population

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv")
```

```
## # A tibble: 3,072 x 6
##    date       state gender legal     indigenous count
##    <date>     <chr> <chr>  <chr>     <chr>      <dbl>
##  1 2005-03-01 ACT   Female Remanded  ATSI           0
##  2 2005-03-01 ACT   Female Remanded  Other          2
##  3 2005-03-01 ACT   Female Sentenced ATSI           0
##  4 2005-03-01 ACT   Female Sentenced Other          0
##  5 2005-03-01 ACT   Male   Remanded  ATSI           7
##  6 2005-03-01 ACT   Male   Remanded  Other         58
##  7 2005-03-01 ACT   Male   Sentenced ATSI           0
##  8 2005-03-01 ACT   Male   Sentenced Other          0
##  9 2005-03-01 NSW   Female Remanded  ATSI          51
## 10 2005-03-01 NSW   Female Remanded  Other        131
## # ... with 3,062 more rows
```

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv") |>
  mutate(Quarter = yearquarter(date))
```

```
## # A tibble: 3,072 x 7
##    date       state gender legal     indigenous count Quarter
##    <date>     <chr> <chr>  <chr>     <chr>       <dbl>   <qtr>
##  1 2005-03-01 ACT   Female Remanded  ATSI            0 2005 Q1
##  2 2005-03-01 ACT   Female Remanded  Other           2 2005 Q1
##  3 2005-03-01 ACT   Female Sentenced ATSI            0 2005 Q1
##  4 2005-03-01 ACT   Female Sentenced Other           0 2005 Q1
##  5 2005-03-01 ACT   Male   Remanded  ATSI            7 2005 Q1
##  6 2005-03-01 ACT   Male   Remanded  Other          58 2005 Q1
##  7 2005-03-01 ACT   Male   Sentenced ATSI            0 2005 Q1
##  8 2005-03-01 ACT   Male   Sentenced Other           0 2005 Q1
##  9 2005-03-01 NSW   Female Remanded  ATSI           51 2005 Q1
## 10 2005-03-01 NSW   Female Remanded  Other         131 2005 Q1
```

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv") |>
  mutate(Quarter = yearquarter(date)) |>
  select(-date)
```

```
## # A tibble: 3,072 x 6
##    state gender legal     indigenous count Quarter
##    <chr> <chr>  <chr>     <chr>      <dbl>   <qtr>
##  1 ACT   Female Remanded  ATSI           0 2005 Q1
##  2 ACT   Female Remanded  Other          2 2005 Q1
##  3 ACT   Female Sentenced ATSI           0 2005 Q1
##  4 ACT   Female Sentenced Other          0 2005 Q1
##  5 ACT   Male   Remanded  ATSI           7 2005 Q1
##  6 ACT   Male   Remanded  Other         58 2005 Q1
##  7 ACT   Male   Sentenced ATSI           0 2005 Q1
##  8 ACT   Male   Sentenced Other          0 2005 Q1
##  9 NSW   Female Remanded  ATSI          51 2005 Q1
```

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv") |>
  mutate(Quarter = yearquarter(date)) |>
  select(-date) |>
  as_tsibble(
    index = Quarter,
    key = c(state, gender, legal, indigenous)
  )
```

```
## # A tsibble: 3,072 x 6 [1Q]
## # Key:       state, gender, legal, indigenous [64]
##    state gender legal    indigenous count Quarter
##    <chr> <chr>  <chr>    <chr>      <dbl>   <qtr>
##  1 ACT   Female Remanded ATSI           0 2005 Q1
##  2 ACT   Female Remanded ATSI           1 2005 Q2
##  3 ACT   Female Remanded ATSI           0 2005 Q3
##  4 ACT   Female Remanded ATSI           0 2005 Q4
##  5 ACT   Female Remanded ATSI           1 2006 Q1
```

Outline

Australian Pharmaceutical Benefits Scheme

Australian Pharmaceutical Benefits Scheme

The **Pharmaceutical Benefits Scheme** (PBS) is the Australian government drugs subsidy scheme.

Australian Pharmaceutical Benefits Scheme

The **Pharmaceutical Benefits Scheme** (PBS) is the Australian government drugs subsidy scheme.

- Many drugs bought from pharmacies are subsidised to allow more equitable access to modern drugs.
- The cost to government is determined by the number and types of drugs purchased. Currently nearly 1% of GDP.
- The total cost is budgeted based on forecasts of drug usage.
- Costs are disaggregated by drug type (ATC1 x15 / ATC2 84), concession category (x2) and patient type (x2), giving $84 \times 2 \times 2 = 336$ time series.

Working with `tsibble` objects

PBS

```
## # A tsibble: 67,596 x 9 [1M]
## # Key:       Concession, Type, ATC1, ATC2 [336]
##       Month Concession   Type    ATC1  ATC1_desc ATC2  ATC2_desc Scripts  Cost
##       <mth> <chr>        <chr>   <chr> <chr>     <chr> <chr>       <dbl> <dbl>
##  1 1991 Jul Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   18228 67877
##  2 1991 Aug Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   15327 57011
##  3 1991 Sep Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   14775 55020
##  4 1991 Oct Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   15380 57222
##  5 1991 Nov Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   14371 52120
##  6 1991 Dec Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   15028 54299
##  7 1992 Jan Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   11040 39753
##  8 1992 Feb Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   15165 54405
##  9 1992 Mar Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   16898 61108
## 10 1992 Apr Concessional Co-pay~ A     Alimenta~ A01   STOMATOL~   18141 65356
## # ... with 67,586 more rows
```

Working with `tsibble` objects

We can use the `filter()` function to select rows.

```
PBS |>
  filter(ATC2 == "A10")
```

```
## # A tsibble: 816 x 9 [1M]
## # Key:       Concession, Type, ATC1, ATC2 [4]
##       Month Concession   Type    ATC1  ATC1_~1 ATC2  ATC2_~2 Scripts   Cost
##       <mth> <chr>        <chr>   <chr> <chr>   <chr> <chr>     <dbl>  <dbl>
##  1 1991 Jul Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   89733 2.09e6
##  2 1991 Aug Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   77101 1.80e6
##  3 1991 Sep Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   76255 1.78e6
##  4 1991 Oct Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   78681 1.85e6
##  5 1991 Nov Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   70554 1.69e6
##  6 1991 Dec Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   75814 1.84e6
##  7 1992 Jan Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   64186 1.56e6
##  8 1992 Feb Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   75899 1.73e6
##  9 1992 Mar Concessional Co-pay~ A     Alimen~ A10   ANTIDI~   89445 2.05e6
```

Working with `tsibble` objects

We can use the `select()` function to select columns.

```
PBS |>
  filter(ATC2 == "A10") |>
  select(Month, Concession, Type, Cost)
```

```
## # A tsibble: 816 x 4 [1M]
## # Key:       Concession, Type [4]
##       Month Concession   Type           Cost
##       <mth> <chr>        <chr>         <dbl>
##  1 1991 Jul Concessional Co-payments 2092878
##  2 1991 Aug Concessional Co-payments 1795733
##  3 1991 Sep Concessional Co-payments 1777231
##  4 1991 Oct Concessional Co-payments 1848507
##  5 1991 Nov Concessional Co-payments 1686458
##  6 1991 Dec Concessional Co-payments 1843079
##  7 1992 Jan Concessional Co-payments 1564702
##  8 1992 Feb Concessional Co-payments 1732508
```

Working with `tsibble` objects

We can use the `summarise()` function to summarise over keys.

```
PBS |>
  filter(ATC2 == "A10") |>
  select(Month, Concession, Type, Cost) |>
  summarise(total_cost = sum(Cost))
```

```
## # A tsibble: 204 x 2 [1M]
##       Month total_cost
##       <mth>      <dbl>
##  1 1991 Jul    3526591
##  2 1991 Aug    3180891
##  3 1991 Sep    3252221
##  4 1991 Oct    3611003
##  5 1991 Nov    3565869
##  6 1991 Dec    4306371
##  7 1992 Jan    5088335
##  8 1992 Feb    2814520
```

Working with `tsibble` objects

We can use the `mutate()` function to create new variables.

```
PBS |>
  filter(ATC2 == "A10") |>
  select(Month, Concession, Type, Cost) |>
  summarise(total_cost = sum(Cost)) |>
  mutate(total_cost = total_cost / 1e6)
```

```
## # A tsibble: 204 x 2 [1M]
##       Month total_cost
##       <mth>      <dbl>
##  1 1991 Jul       3.53
##  2 1991 Aug       3.18
##  3 1991 Sep       3.25
##  4 1991 Oct       3.61
##  5 1991 Nov       3.57
##  6 1991 Dec       4.31
##  7 1992 Jan       5.09
```

Working with `tsibble` objects

We can use the `mutate()` function to create new variables.

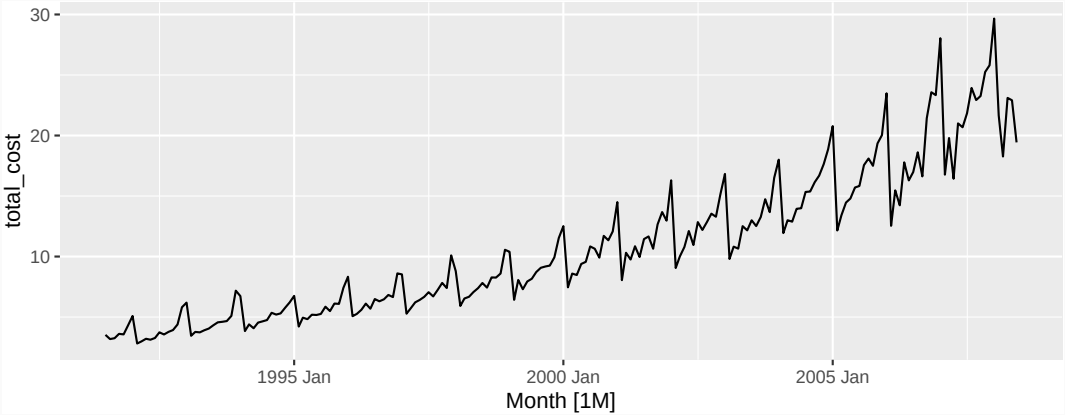
```
PBS |>
  filter(ATC2 == "A10") |>
  select(Month, Concession, Type, Cost) |>
  summarise(total_cost = sum(Cost)) |>
  mutate(total_cost = total_cost / 1e6) -> a10
```

```
## # A tsibble: 204 x 2 [1M]
##       Month total_cost
##       <mth>      <dbl>
##  1 1991 Jul       3.53
##  2 1991 Aug       3.18
##  3 1991 Sep       3.25
##  4 1991 Oct       3.61
##  5 1991 Nov       3.57
##  6 1991 Dec       4.31
##  7 1992 Jan       5.09
```

Outline

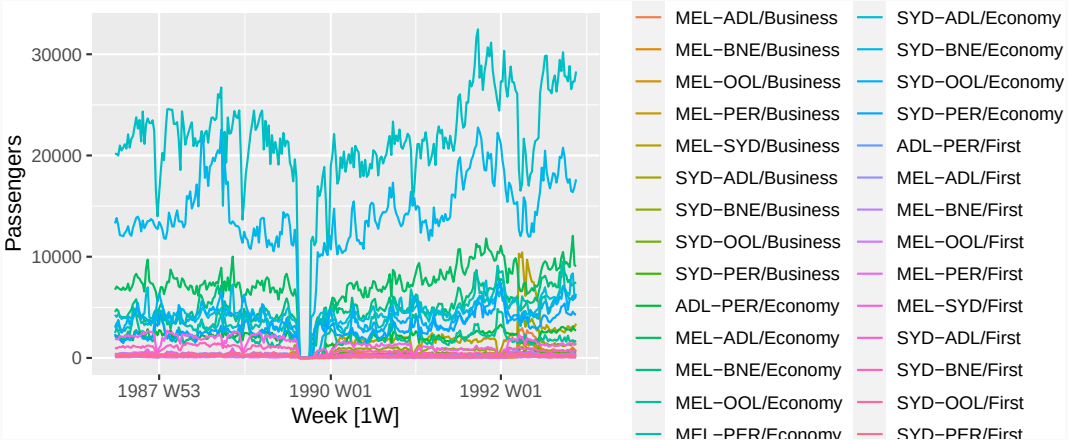
Time plots

```
a10 |>
  autoplot(total_cost)
```



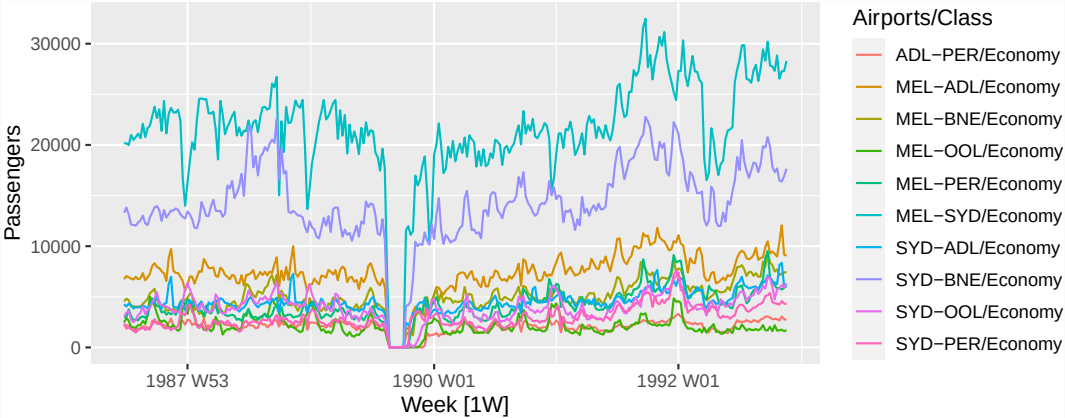
Ansett airlines

```
ansett |>
  autoplot(Passengers)
```



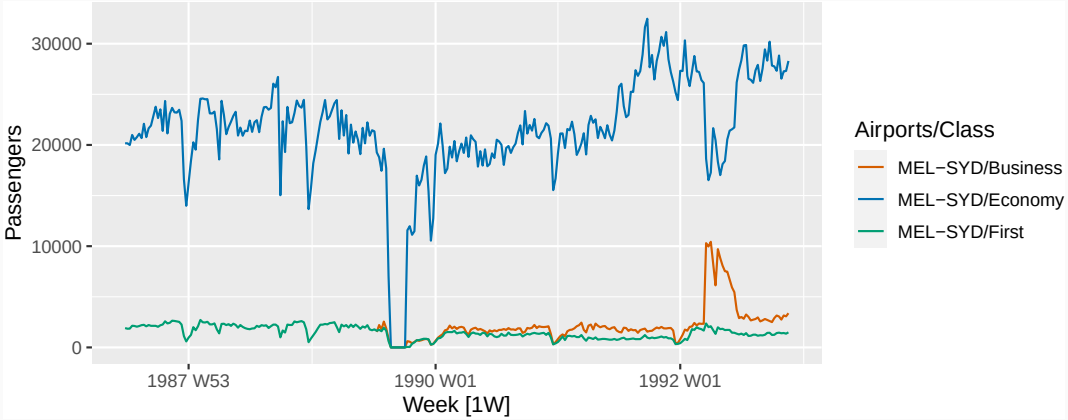
Ansett airlines

```
ansett |>
  filter(Class == "Economy") |>
  autoplot(Passengers)
```



Ansett airlines

```
ansett |>
  filter(Airports == "MEL-SYD") |>
  autoplot(Passengers)
```



Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

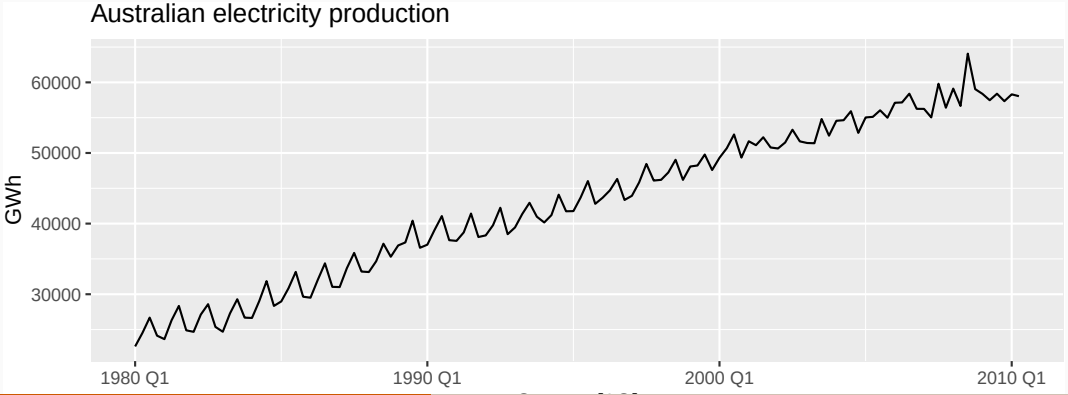
Time series components

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

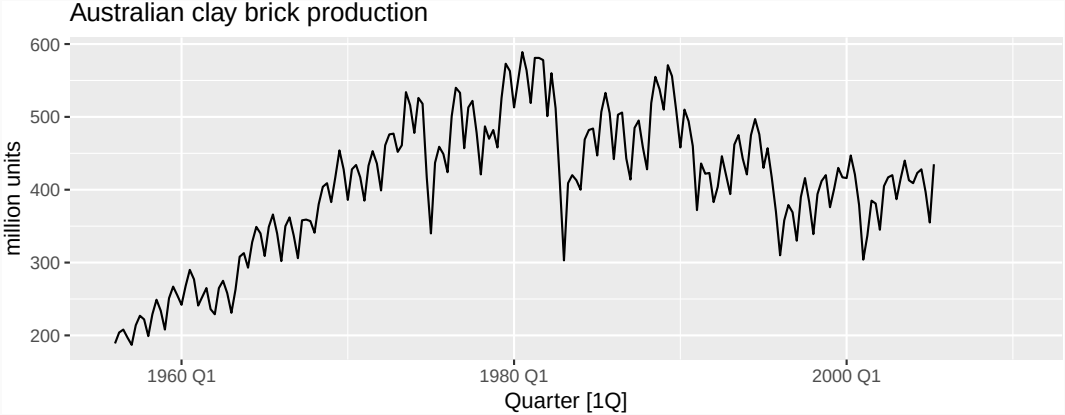
Time series patterns

```
aus_production |>
  filter(year(Quarter) >= 1980) |>
  autoplot(Electricity) +
  labs(y = "GWh", title = "Australian electricity production")
```



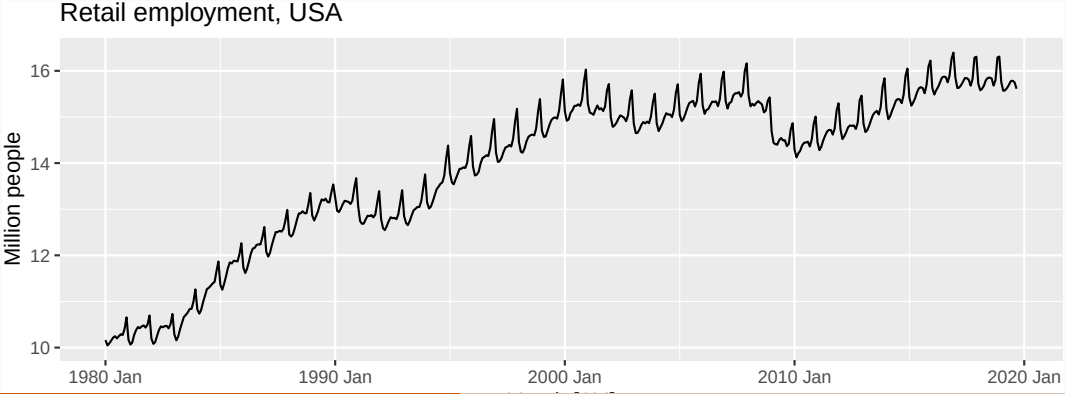
Time series patterns

```
aus_production |>
  autoplot(Bricks) +
  labs(y = "million units", title = "Australian clay brick production")
```



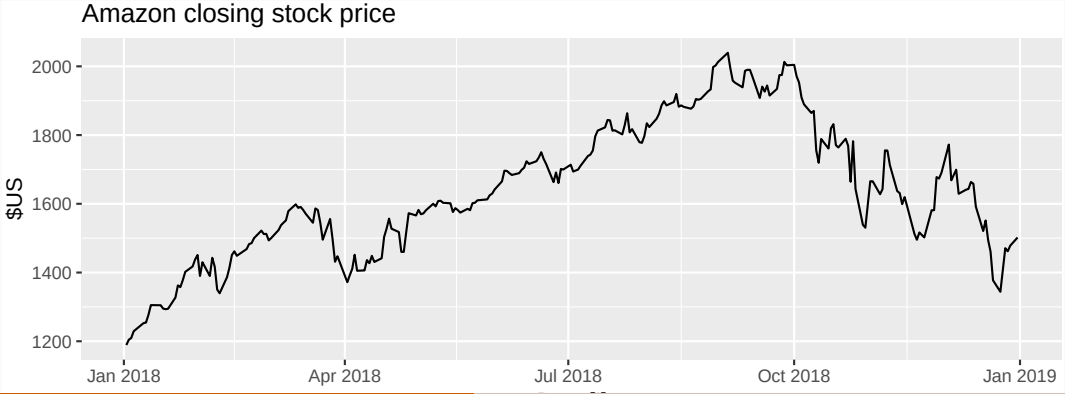
Time series patterns

```
us_employment |>
  filter(Title == "Retail Trade", year(Month) >= 1980) |>
  autoplot(Employed / 1e3) +
  labs(y = "Million people", title = "Retail employment, USA")
```



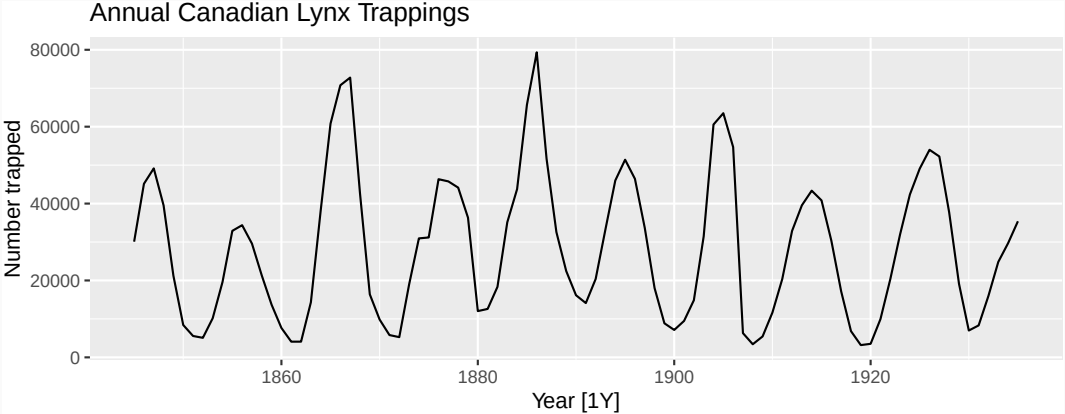
Time series patterns

```
gafa_stock |>
  filter(Symbol == "AMZN", year(Date) >= 2018) |>
  autoplot(Close) +
  labs(y = "$US", title = "Amazon closing stock price")
```



Time series patterns

```
pelt |>
  autoplot(Lynx) +
  labs(y = "Number trapped", title = "Annual Canadian Lynx Trappings")
```



Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

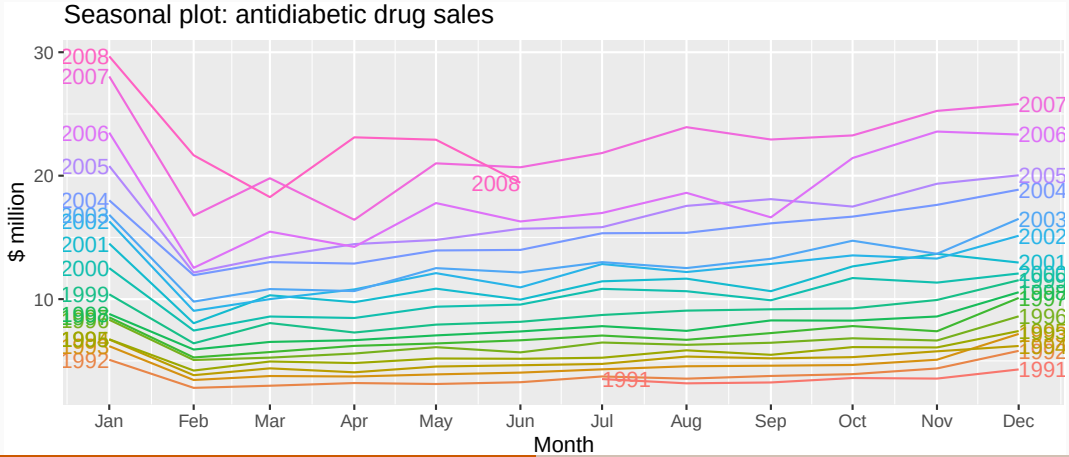
- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

Outline

Seasonal plots

```
a10 |> gg_season(total_cost, labels = "both") +
  labs(y = "$ million", title = "Seasonal plot: antidiabetic drug sales")
```

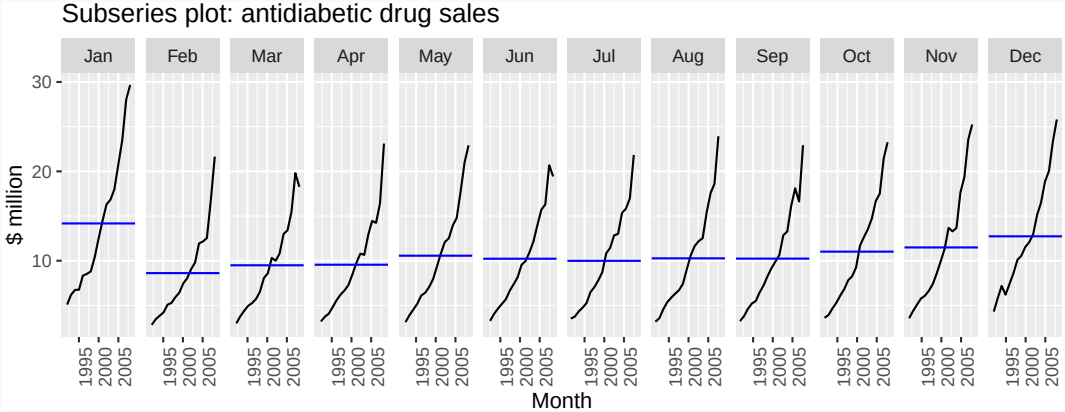


Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `gg_season()`

Seasonal subseries plots

```
a10 |>
  gg_subseries(total_cost) +
  labs(y = "$ million", title = "Subseries plot: antidiabetic drug sales")
```

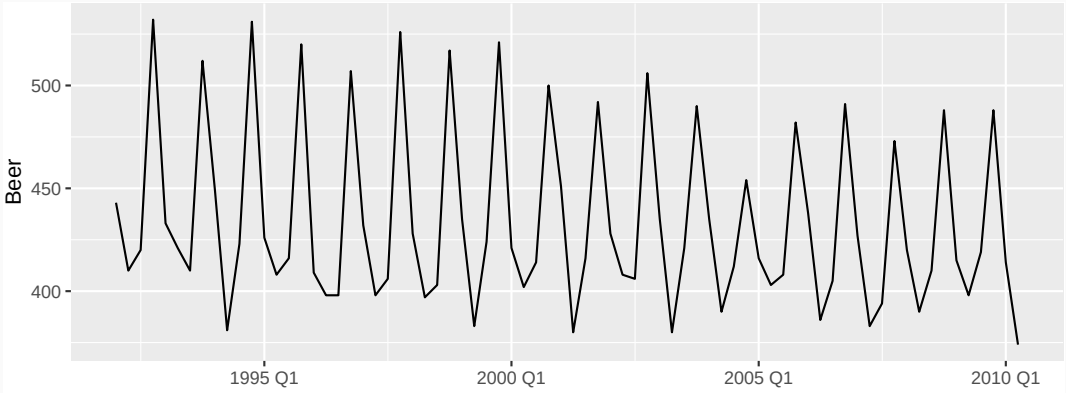


Seasonal subseries plots

- Data for each season collected together in time plot as separate time series.
- Enables the underlying seasonal pattern to be seen clearly, and changes in seasonality over time to be visualized.
- In R: `gg_subseries()`

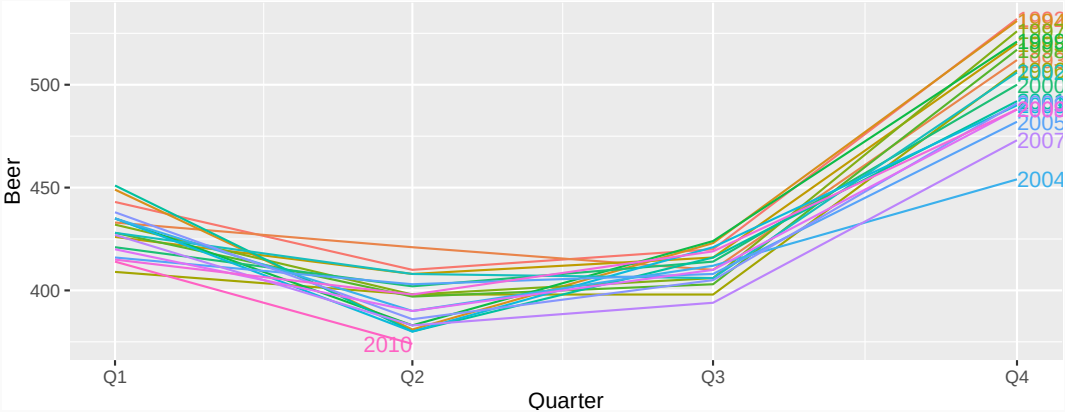
Quarterly Australian Beer Production

```
beer <- aus_production |>
  select(Quarter, Beer) |>
  filter(year(Quarter) >= 1992)
beer |> autoplot(Beer)
```



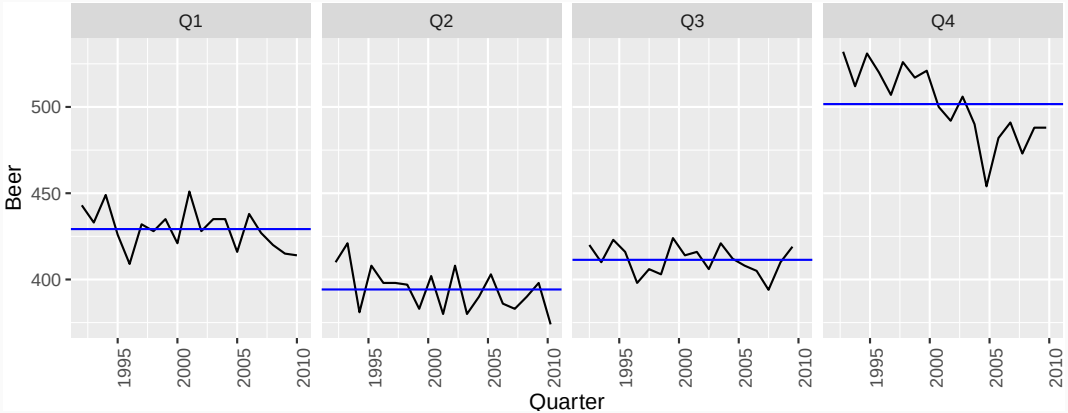
Quarterly Australian Beer Production

```
beer |> gg_season(Beer, labels = "right")
```



Quarterly Australian Beer Production

```
beer |> gg_subseries(Beer)
```



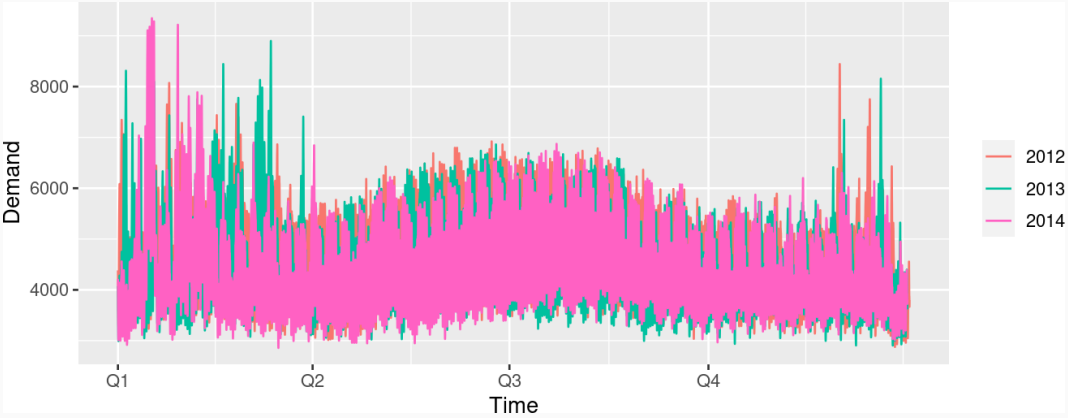
Multiple seasonal periods

```
vic_elec
```

```
## # A tsibble: 52,608 x 5 [30m] <Australia/Melbourne>
##    Time                Demand Temperature Date       Holiday
##    <dttm>               <dbl>       <dbl> <date>     <lgl>
##  1 2012-01-01 00:00:00  4383.        21.4 2012-01-01 TRUE
##  2 2012-01-01 00:30:00  4263.        21.0 2012-01-01 TRUE
##  3 2012-01-01 01:00:00  4049.        20.7 2012-01-01 TRUE
##  4 2012-01-01 01:30:00  3878.        20.6 2012-01-01 TRUE
##  5 2012-01-01 02:00:00  4036.        20.4 2012-01-01 TRUE
##  6 2012-01-01 02:30:00  3866.        20.2 2012-01-01 TRUE
##  7 2012-01-01 03:00:00  3694.        20.1 2012-01-01 TRUE
##  8 2012-01-01 03:30:00  3562.        19.6 2012-01-01 TRUE
##  9 2012-01-01 04:00:00  3433.        19.1 2012-01-01 TRUE
## 10 2012-01-01 04:30:00  3359.        19.0 2012-01-01 TRUE
## # ... with 52,598 more rows
```

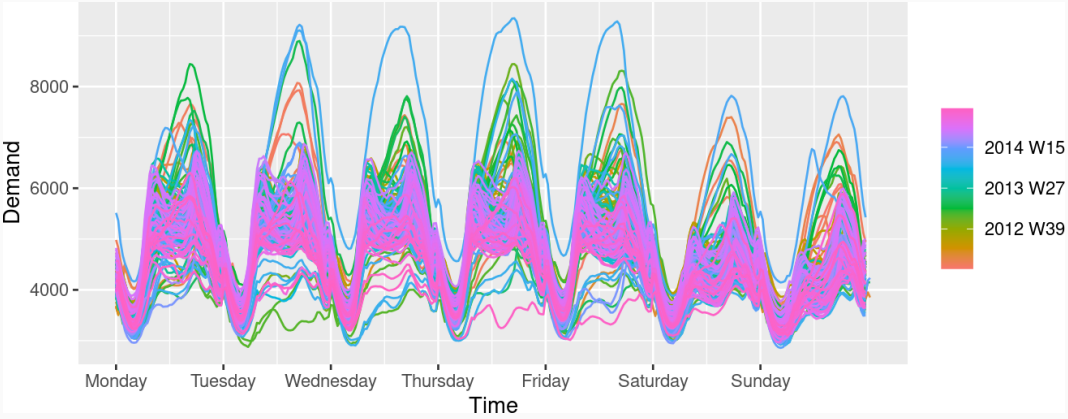
Multiple seasonal periods

```
vic_elec |> gg_season(Demand)
```



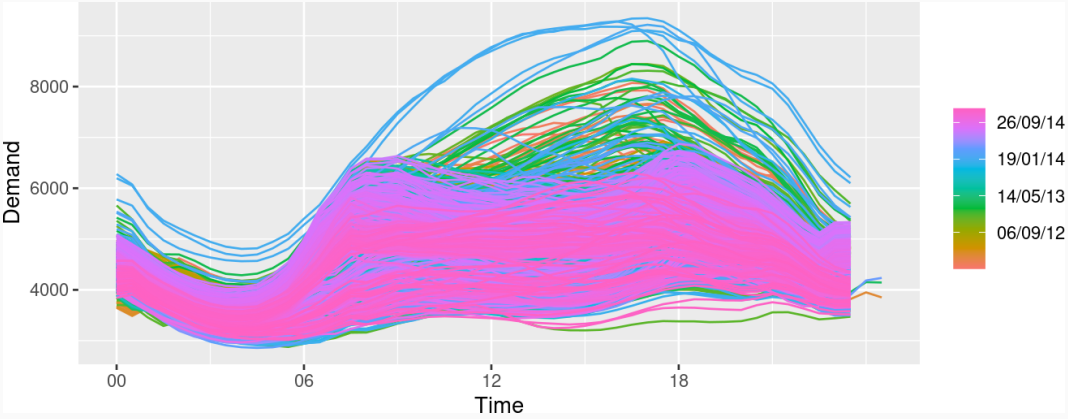
Multiple seasonal periods

```
vic_elec |> gg_season(Demand, period = "week")
```



Multiple seasonal periods

```
vic_elec |> gg_season(Demand, period = "day")
```



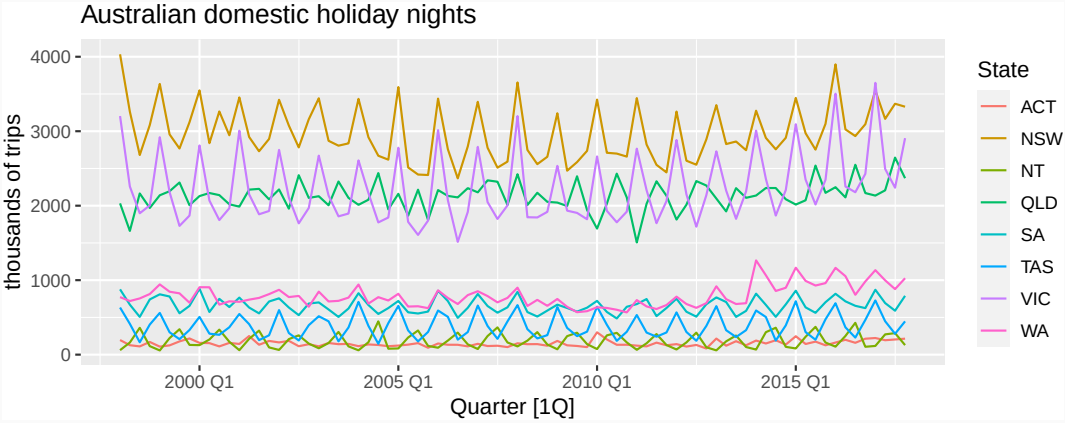
Australian holidays

```
holidays <- tourism |>
  filter(Purpose == "Holiday") |>
  group_by(State) |>
  summarise(Trips = sum(Trips))
```

```
## # A tsibble: 640 x 3 [1Q]
## # Key:       State [8]
##    State Quarter Trips
##    <chr>   <qtr> <dbl>
##  1 ACT   1998 Q1  196.
##  2 ACT   1998 Q2  127.
##  3 ACT   1998 Q3  111.
##  4 ACT   1998 Q4  170.
##  5 ACT   1999 Q1  108.
##  6 ACT   1999 Q2  125.
##  7 ACT   1999 Q3  178.
##  8 ACT   1999 Q4  218.
##  9 ACT   2000 Q1  158.
```

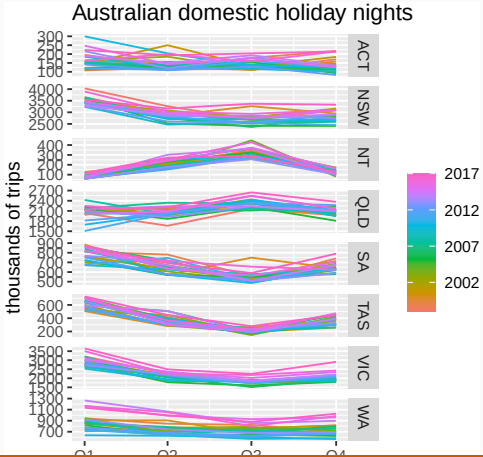
Australian holidays

```
holidays |> autoplot(Trips) +
  labs(y = "thousands of trips", title = "Australian domestic holiday nights")
```



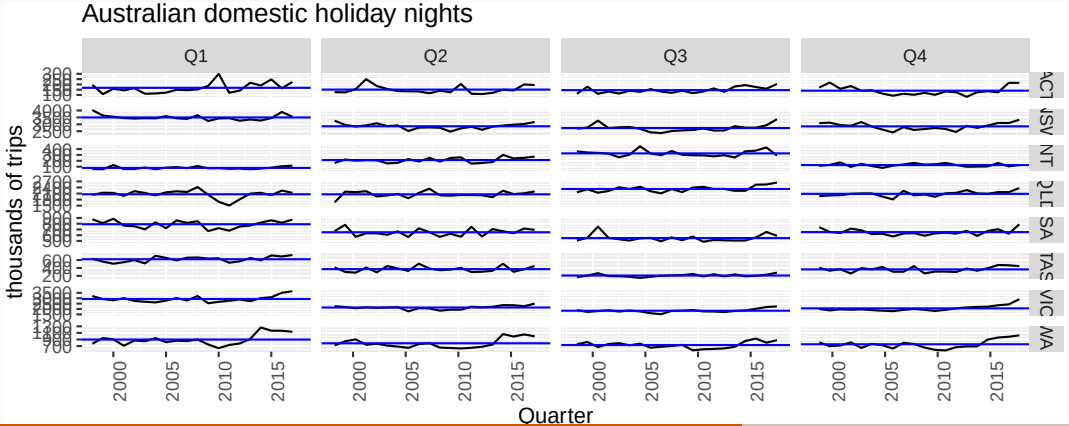
Seasonal plots

```
holidays |> gg_season(Trips) +
  labs(y = "thousands of trips", title = "Australian domestic holiday nights")
```



Seasonal subseries plots

```
holidays |>
  gg_subseries(Trips) +
  labs(y = "thousands of trips", title = "Australian domestic holiday nights")
```



Outline

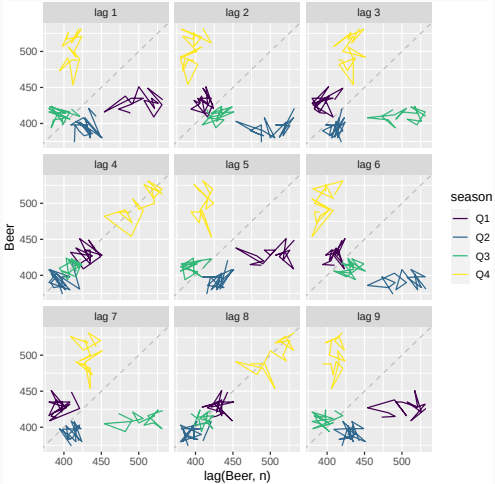
Example: Beer production

```
new_production <- aus_production |>
  filter(year(Quarter) >= 1992)
new_production
```

```
## # A tsibble: 74 x 7 [1Q]
##    Quarter  Beer Tobacco Bricks Cement Electricity   Gas
##      <qtr> <dbl>   <dbl>  <dbl>  <dbl>       <dbl> <dbl>
##  1 1992 Q1   443    5777    383   1289       38332   117
##  2 1992 Q2   410    5853    404   1501       39774   151
##  3 1992 Q3   420    6416    446   1539       42246   175
##  4 1992 Q4   532    5825    420   1568       38498   129
##  5 1993 Q1   433    5724    394   1450       39460   116
##  6 1993 Q2   421    6036    462   1668       41356   149
##  7 1993 Q3   410    6570    475   1648       42949   163
##  8 1993 Q4   512    5675    443   1863       40974   138
##  9 1994 Q1   449    5311    421   1468       40162   127
## 10 1994 Q2   381    5717    475   1755       41199   159
## # ... with 64 more rows
```

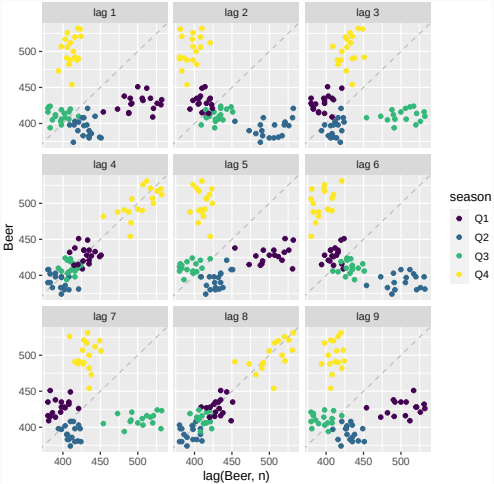
Example: Beer production

```
new_production |> gg_lag(Beer)
```



Example: Beer production

```
new_production |> gg_lag(Beer, geom = "point")
```



Lagged scatterplots

- Each graph shows y_t plotted against y_{t-k} for different values of k.
- The autocorrelations are the correlations associated with these scatterplots.
- ACF (autocorrelation function):
  - r_1 = Correlation(y_t, y_{t-1})
  - r_2 = Correlation(y_t, y_{t-2})
  - r_3 = Correlation(y_t, y_{t-3})
  - etc.

Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocorrelation

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Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

We measure the relationship between:

- y_t and y_{t-1}
- y_t and y_{t-2}
- y_t and y_{t-3}
- etc.

Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T} \sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k / c_0$$

Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k/c_0$$

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

Autocorrelation

Results for first 9 lags for beer data:

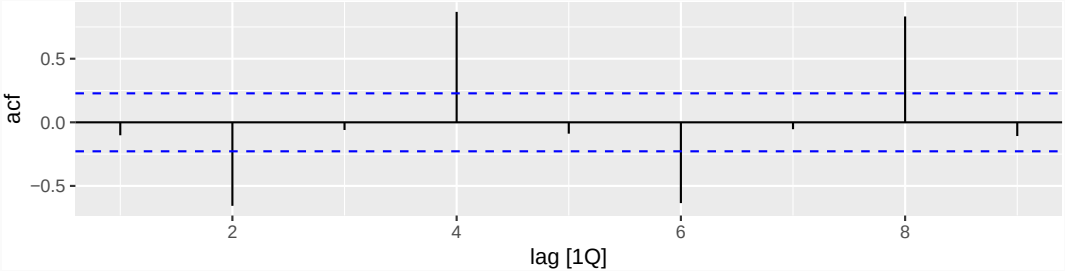
```
new_production |> ACF(Beer, lag_max = 9)
```

```
## # A tsibble: 9 x 2 [1Q]
##         lag     acf
##   <cf_lag>   <dbl>
## 1       1Q -0.102
## 2       2Q -0.657
## 3       3Q -0.0603
## 4       4Q  0.869
## 5       5Q -0.0892
## 6       6Q -0.635
## 7       7Q -0.0542
```

Autocorrelation

Results for first 9 lags for beer data:

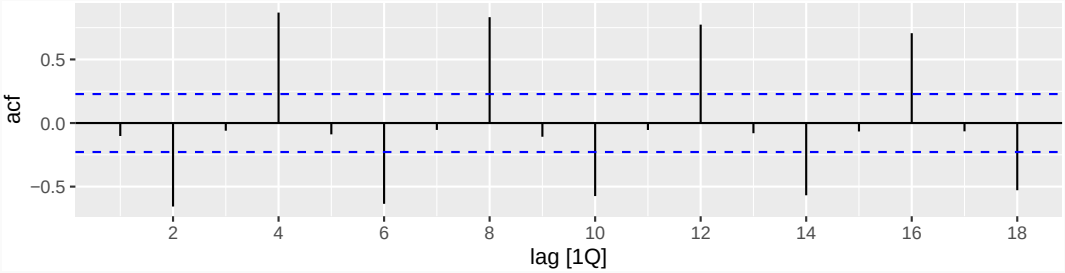
```
new_production |>
  ACF(Beer, lag_max = 9) |>
  autoplot()
```



- Together, the autocorrelations at lags 1, 2, ..., make up the *autocorrelation* or ACF.
- The plot is known as a **correlogram**

Autocorrelation

```
new_production |>
  ACF(Beer) |>
  autoplot()
```

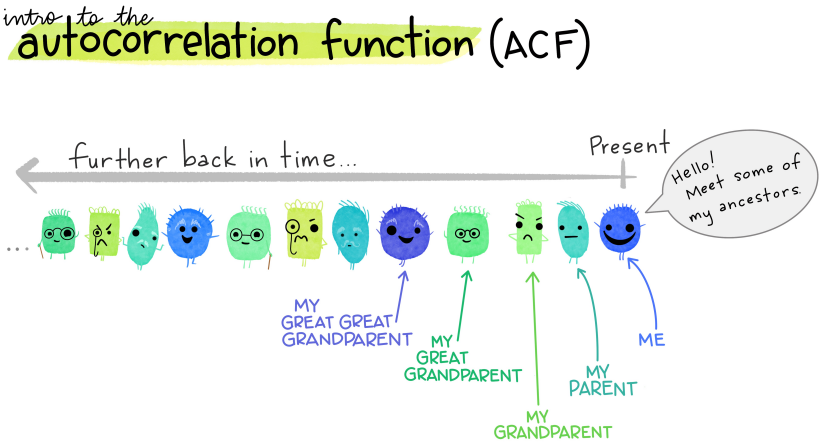


- r_4 higher than for the other lags. This is due to **the seasonal pattern in the data**: the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters

Trend and seasonality in ACF plots

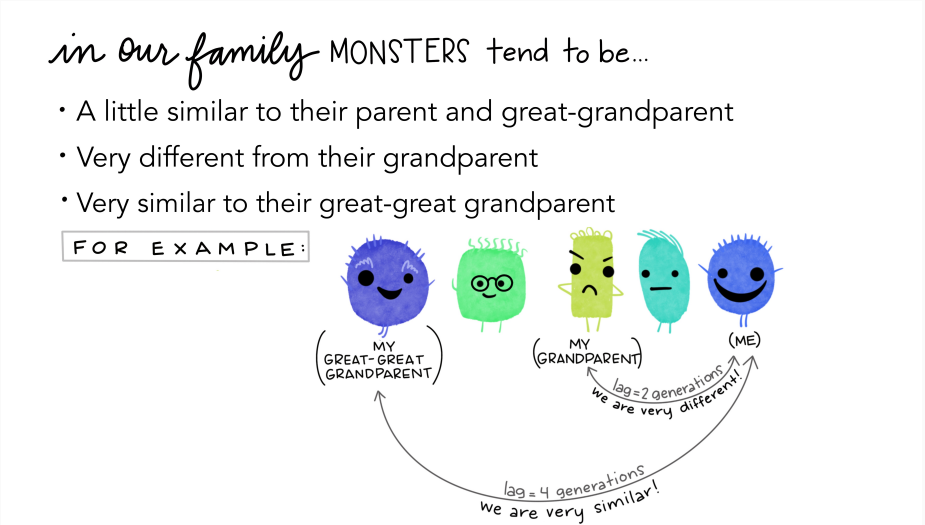
- When data have a trend, the autocorrelations for small lags tend to be large and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects.

Autocorrelation functions



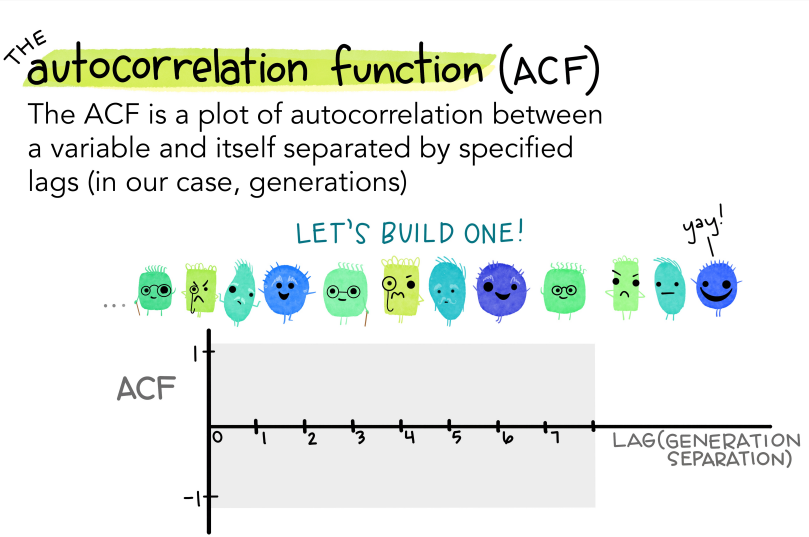
Artwork by @allison_horst

Autocorrelation functions



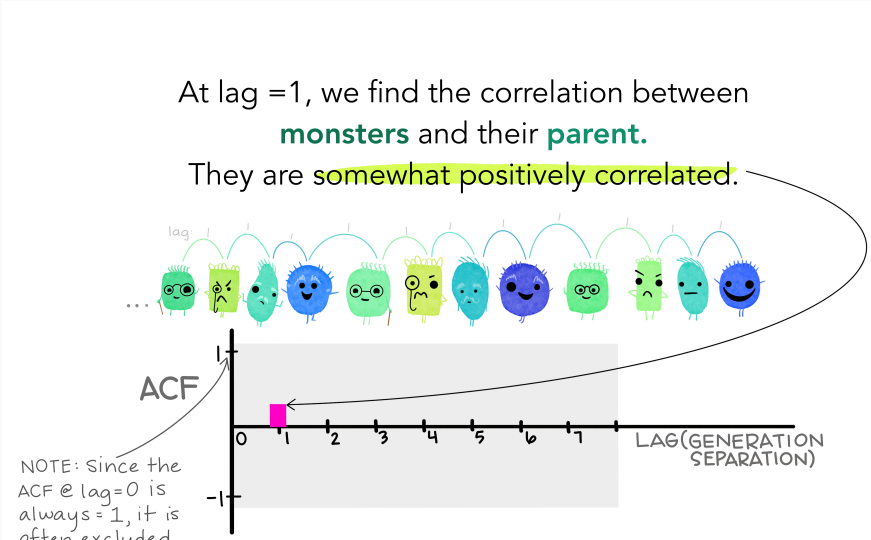
Artwork by @allison_horst

Autocorrelation functions



Artwork by @allison_horst

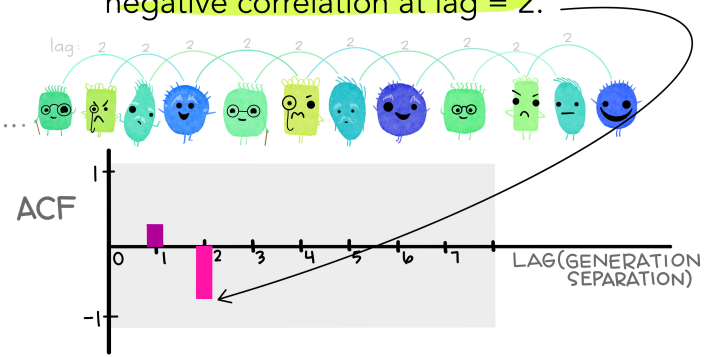
Autocorrelation functions



Artwork by @allison_horst

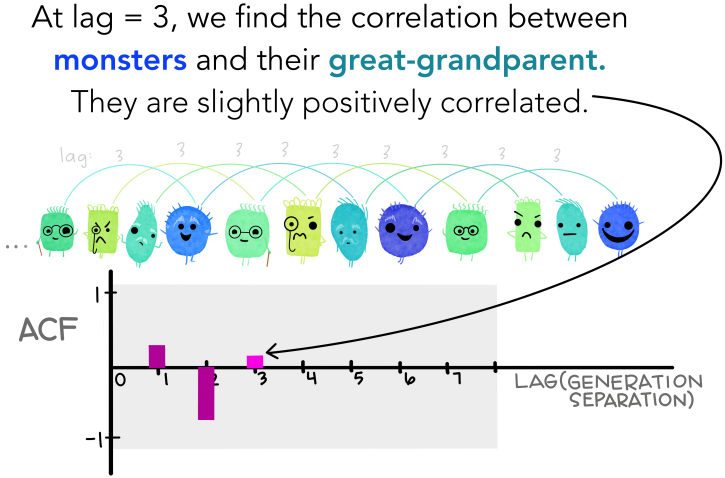
Autocorrelation functions

At lag = 2, we find the correlation between **monsters** and their **grandparent**.
Since they tend to be very different, we find a negative correlation at lag = 2.



Artwork by @allison_horst

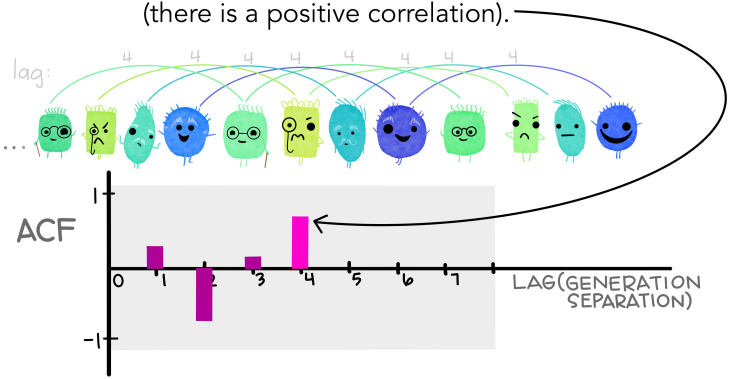
Autocorrelation functions



Artwork by @allison_horst

Autocorrelation functions

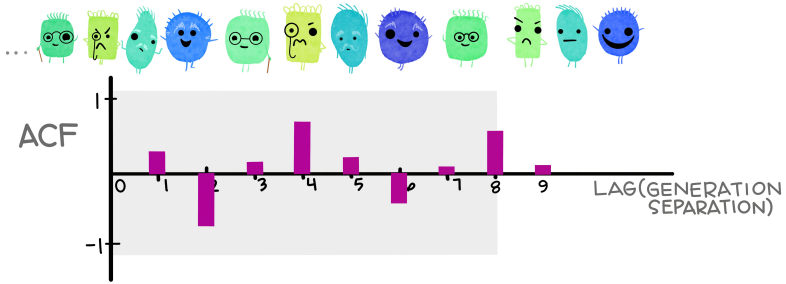
At lag = 4, we find the correlation between **monsters** and their **great-great grandparent**. They tend to be very similar (there is a positive correlation).



Artwork by @allison_horst

Autocorrelation functions

...and we continue finding the correlations as we increase the lag (generations) between the monsters...

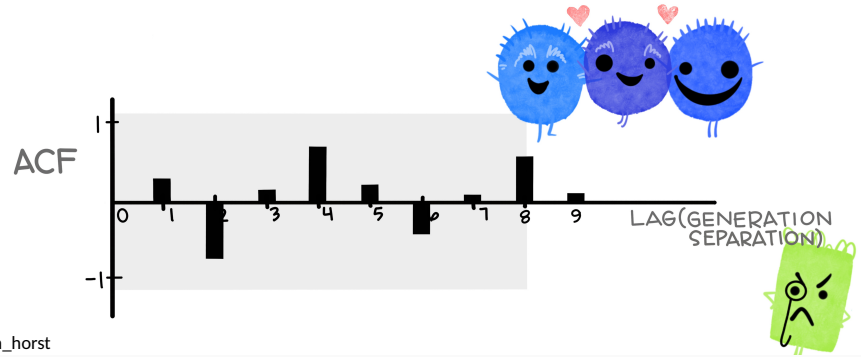


Artwork by @allison_horst

Autocorrelation functions

in Summary:

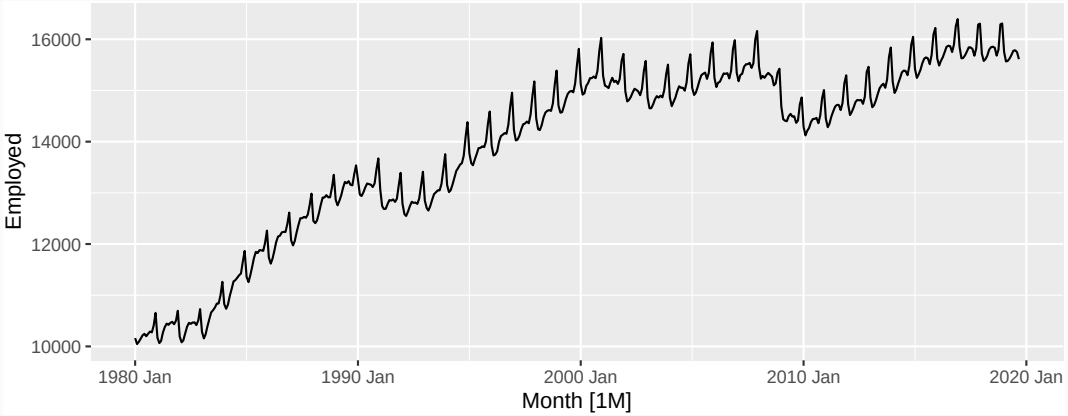
The autocorrelation function (ACF) tells us the correlation between observations and those that came before them, separated by different lags (here, monster generations)!



Artwork by @allison_horst

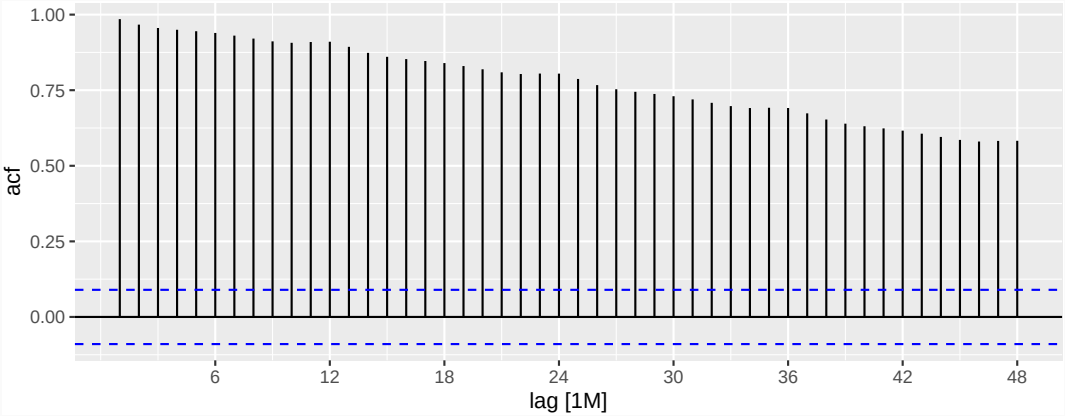
US retail trade employment

```
retail <- us_employment |>
  filter(Title == "Retail Trade", year(Month) >= 1980)
retail |> autoplot(Employed)
```



US retail trade employment

```
retail |>
  ACF(Employed, lag_max = 48) |>
  autoplot()
```



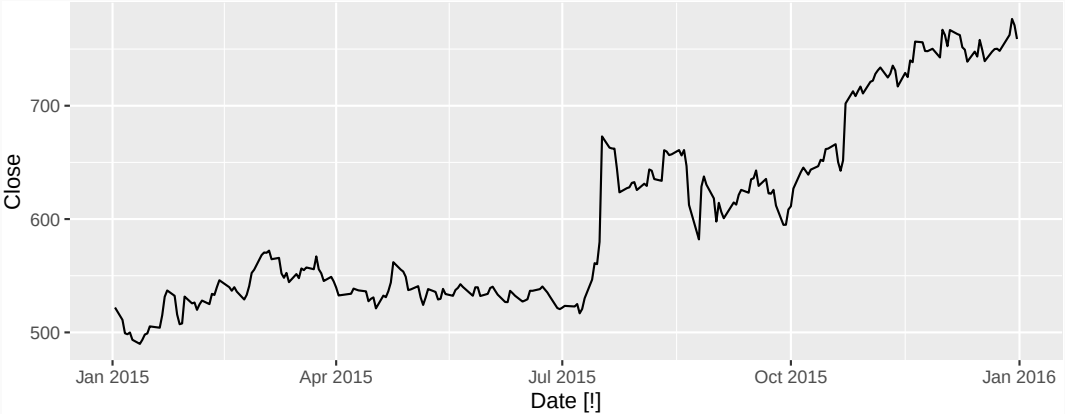
Google stock price

```
google_2015 <- gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2015) |>
  select(Date, Close)
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##    Date       Close
##    <date>     <dbl>
##  1 2015-01-02  522.
##  2 2015-01-05  511.
##  3 2015-01-06  499.
##  4 2015-01-07  498.
##  5 2015-01-08  500.
##  6 2015-01-09  493.
##  7 2015-01-12  490.
##  8 2015-01-13  493.
##  9 2015-01-14  498.
## 10 2015-01-15  499.
```

Google stock price

```
google_2015 |> autoplot(Close)
```



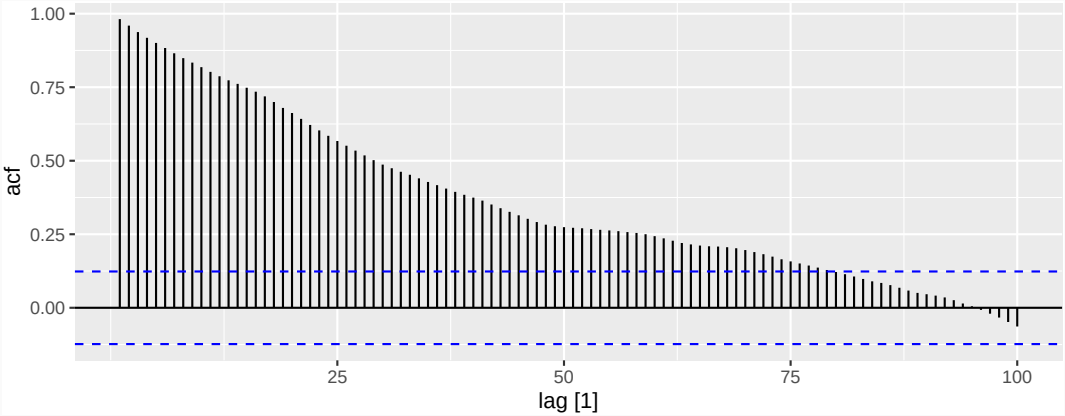
Google stock price

```
google_2015 |>
  ACF(Close, lag_max = 100)
```

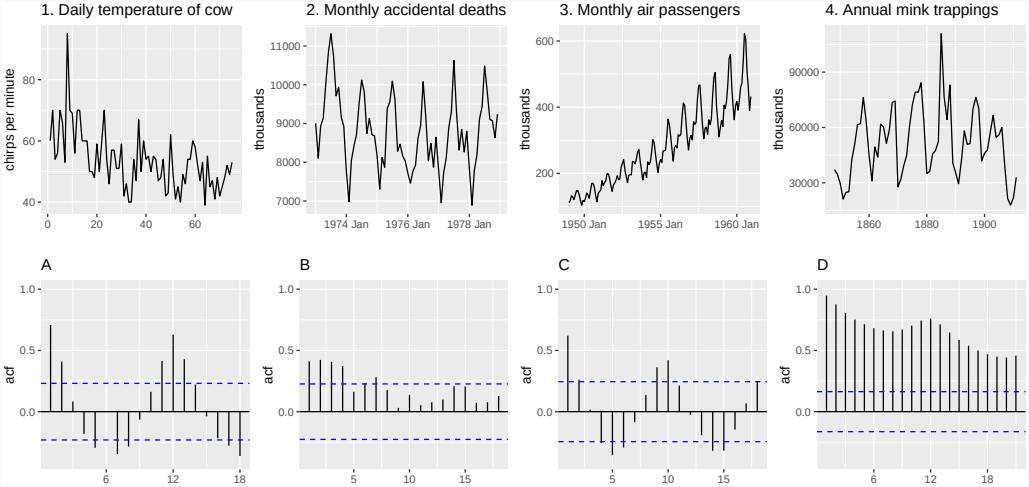
```
## # A tsibble: 100 x 2 [1]
##          lag   acf
##    <cf_lag> <dbl>
##  1        1 0.982
##  2        2 0.959
##  3        3 0.937
##  4        4 0.918
##  5        5 0.901
##  6        6 0.883
##  7        7 0.865
##  8        8 0.849
##  9        9 0.834
## 10       10 0.818
## # ... with 90 more rows
```

Google stock price

```
google_2015 |>
  ACF(Close, lag_max = 100) |>
  autoplot()
```



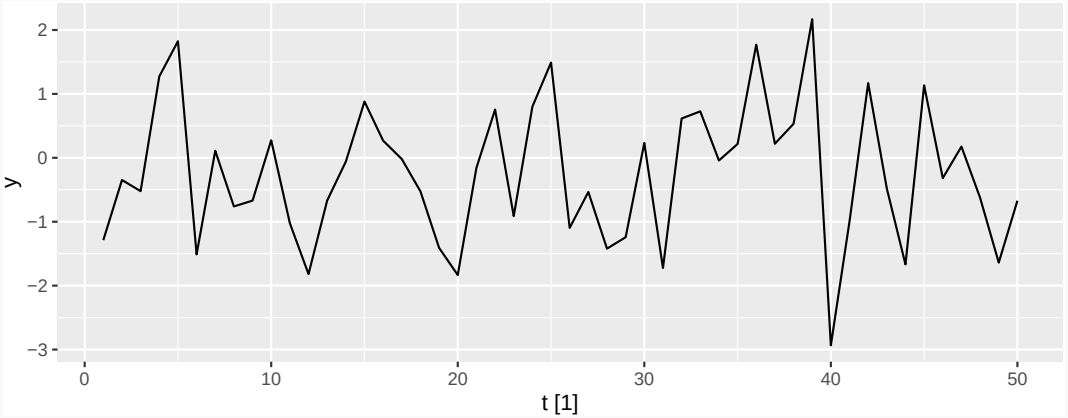
Which is which?



Outline

Example: White noise

```
set.seed(30)
wn <- tsibble(t = 1:50, y = rnorm(50), index = t)
wn |> autoplot(y)
```



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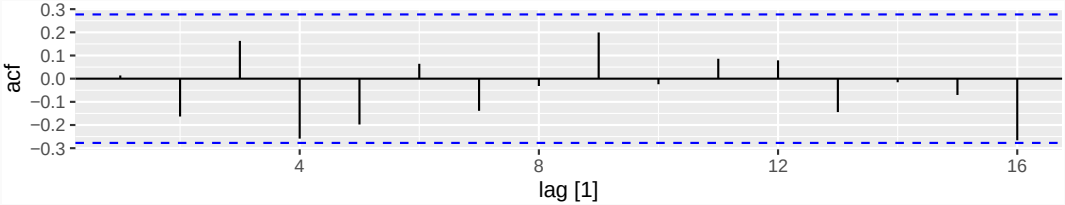
wn |> autoplot(y)
```

White noise data is uncorrelated across time with zero mean and constant variance.
(Technically, we require independence as well.)

Example: White noise

```
wn |> ACF(y)
```

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
0.014	-0.163	0.163	-0.259	-0.198	0.064	-0.139	-0.032	0.199	-0.024



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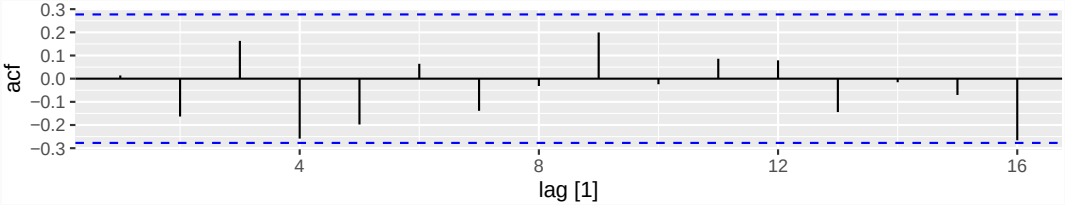


- Sample autocorrelations for white noise series.
- Expect each autocorrelation to be close to zero.
- Blue lines show 95% critical values.

Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $N(0,1/T)$.

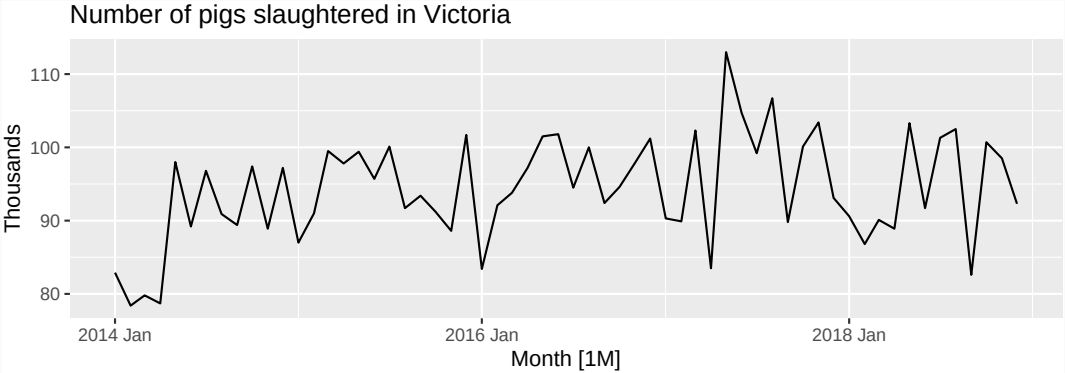
Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $\text{N}(0,1/T)$.

- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the **critical values**.

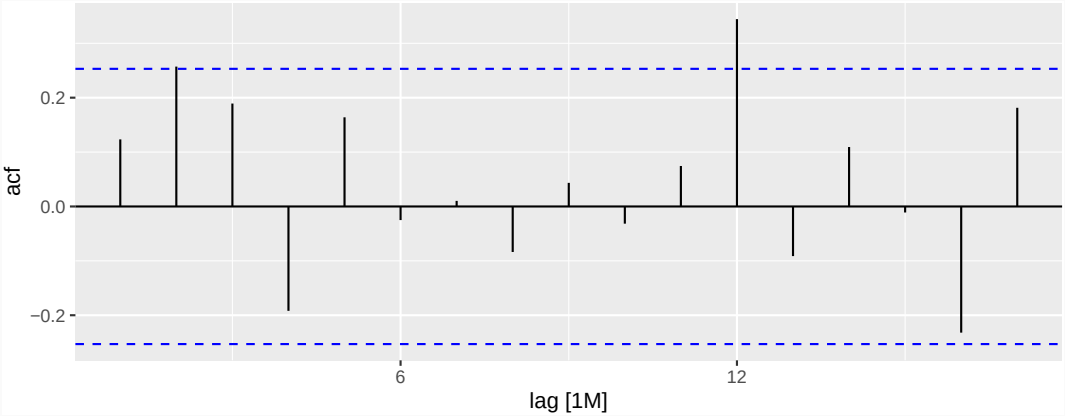
Example: Pigs slaughtered

```
pigs <- aus_livestock |>
  filter(State == "Victoria", Animal == "Pigs", year(Month) >= 2014)
pigs |> autoplot(Count / 1e3) +
  labs(y = "Thousands", title = "Number of pigs slaughtered in Victoria")
```



Example: Pigs slaughtered

```
pigs |>
  ACF(Count) |>
  autoplot()
```



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Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 2014 through December 2018 (Source: Australian Bureau of Statistics.)

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- Indicate some slight seasonality.

Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 2014 through December 2018 (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows significant autocorrelation for lag 2 and 12.
- Indicate some slight seasonality.

These show the series is **not a white noise series**.

Your turn

You can compute the daily changes in the Google stock price in 2018 using

```
dgoog <- gafa_stock |>
  filter(Symbol == "GOOG", year(Date) >= 2018) |>
  mutate(diff = difference(Close))
```

Does `diff` look like white noise?