MONASH University

MONASH BUSINESS SCHOOL

Forecasting is not prophecy

Dealing with high-dimensional probabilistic forecasts in practice

Rob J Hyndman

What is it?

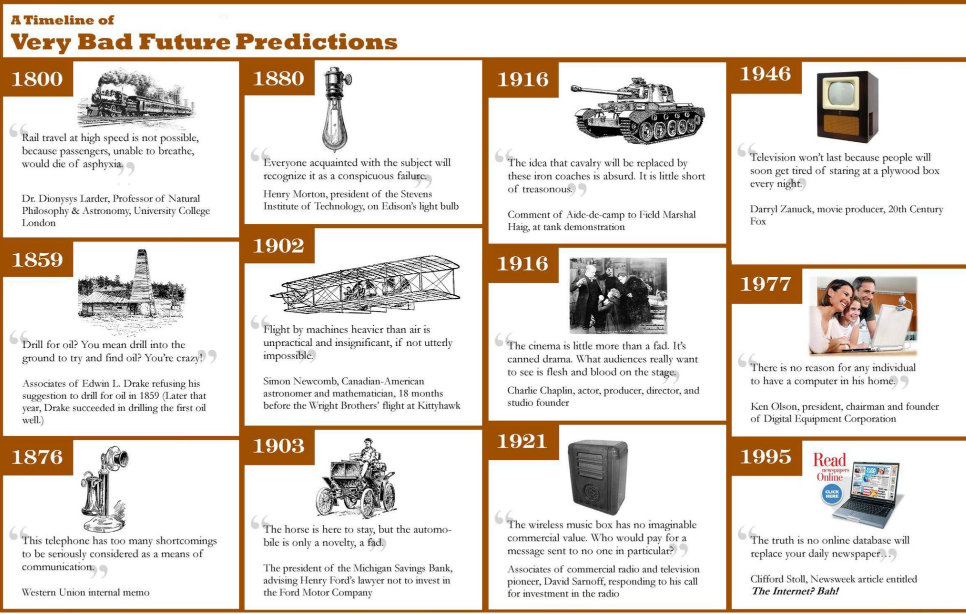
What is it?

Clay model of sheep's liver

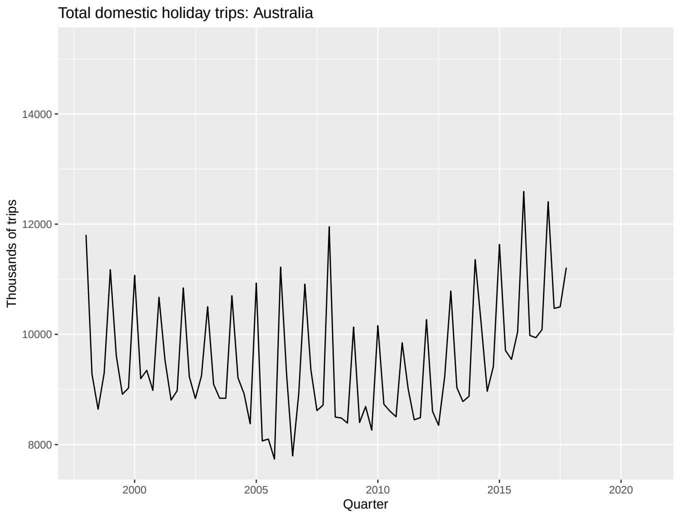
Used by Babylonian forecasters approximately 600 B.C.

Now in British Museum.

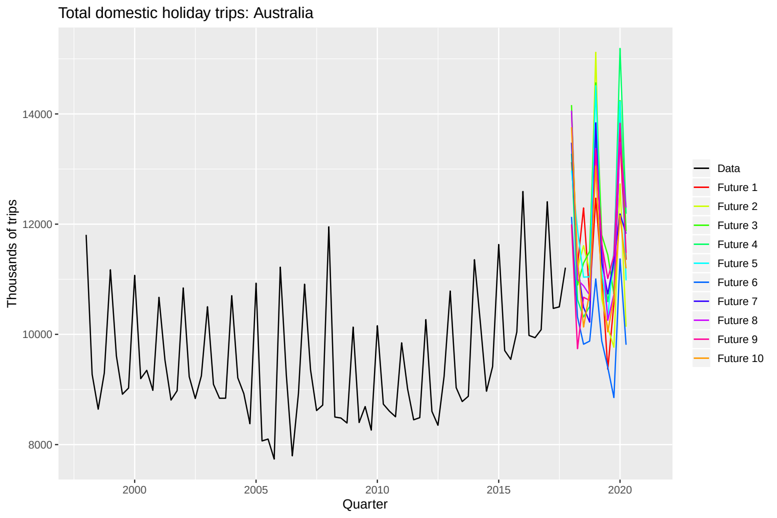
Modern prophecy



Probabilistic forecasting



Probabilistic forecasting



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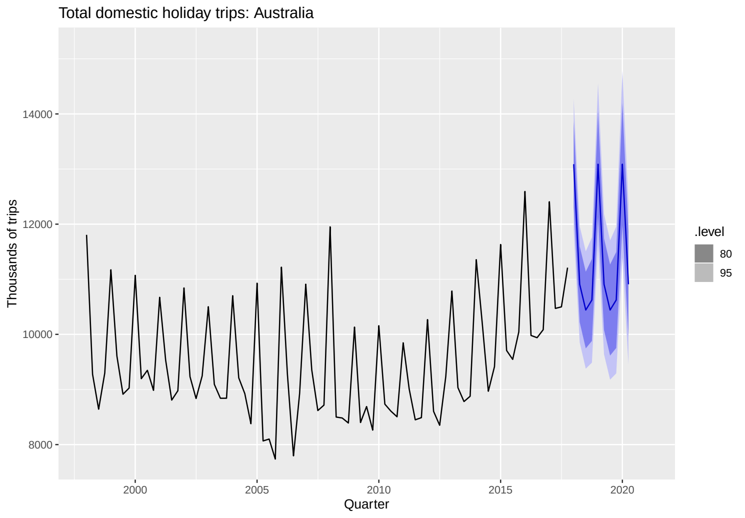
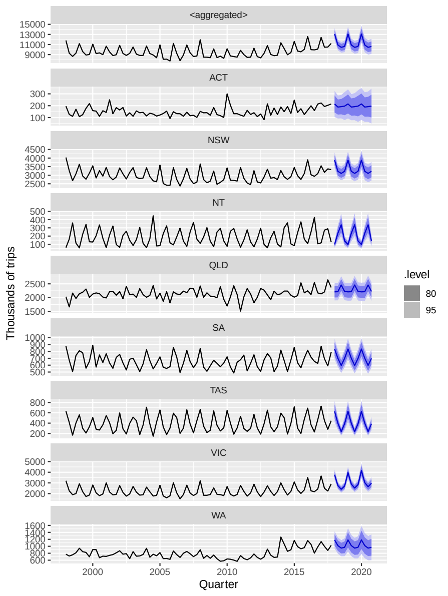
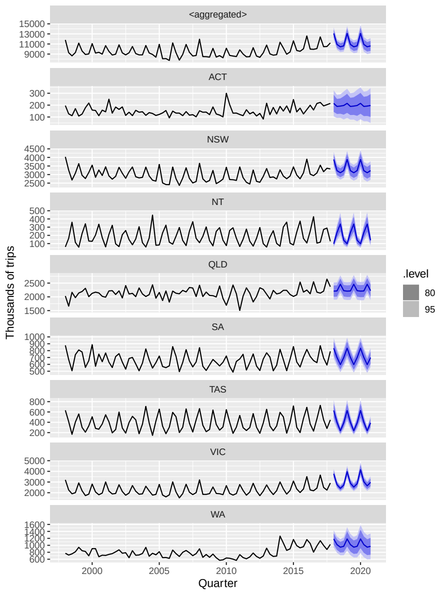


Multivariate probabilistic forecasting



Multivariate probabilistic forecasting



These show the marginal distributions of the 9-dimensional distribution. In practice, we also want forecast for smaller geographic areas as well.

Who needs multivariate probabilistic forecasts?

- Tourism authorities forecasting visitor numbers to plan facilities.
- Manufacturing companies forecasting product demand to plan their supply chains.
- Call centres forecasting call volume to inform staff scheduling.
- Technology companies forecasting web traffic to maintain service levels.
- Energy companies forecasting electricity demand to prevent blackouts.

Challenges in multivariate probabilistic forecasting

1. How to produce high-dim probabilistic forecasts?
2. How to use and interpret high-dim probabilistic forecasts?
3. How to measure accuracy of high-dim probabilistic forecasts?

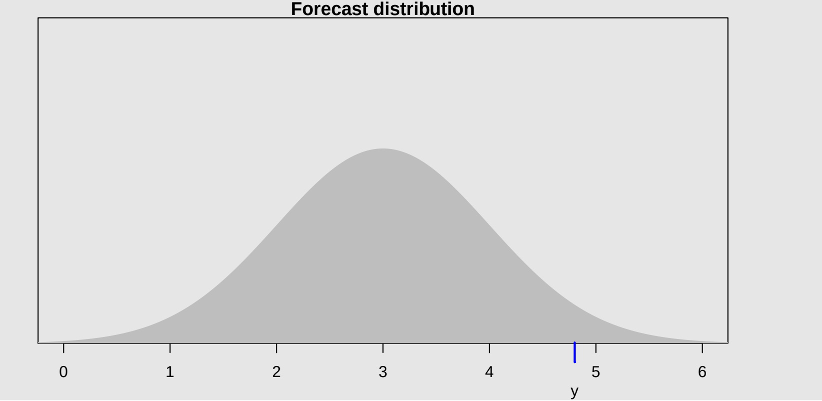
Challenges in multivariate probabilistic forecasting

1. How to produce high-dim probabilistic forecasts?
2. How to use and interpret high-dim probabilistic forecasts?
3. How to measure accuracy of high-dim probabilistic forecasts?

- Users care about marginal distributions, but we need multivariate distribution to compute them.
- Forecast reconciliation provides a way to efficiently compute marginal distributions.
- Probability scoring allows us to measure distributional forecast accuracy.

Probability scoring

- y = actual value
- F = forecast distribution
- $q_p = F^{-1}(p)$ = quantile with probability p.



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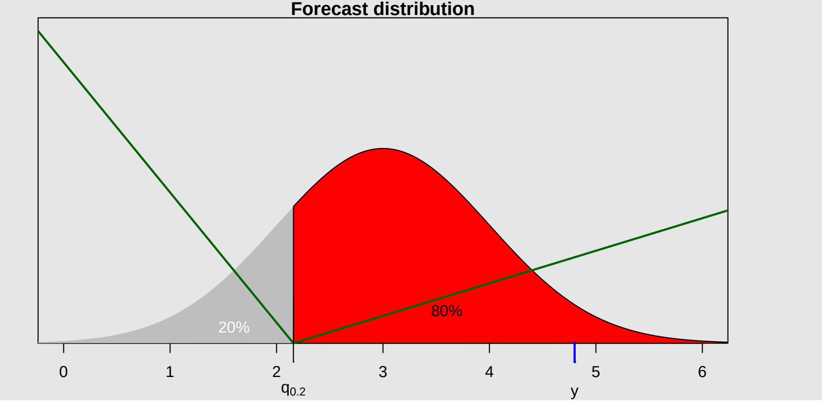

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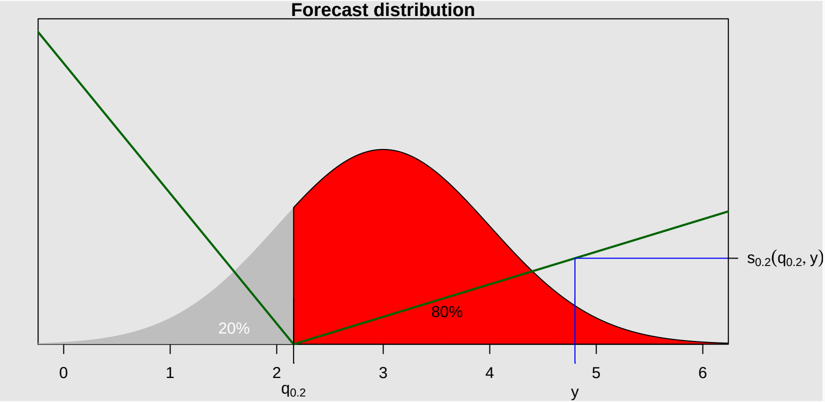

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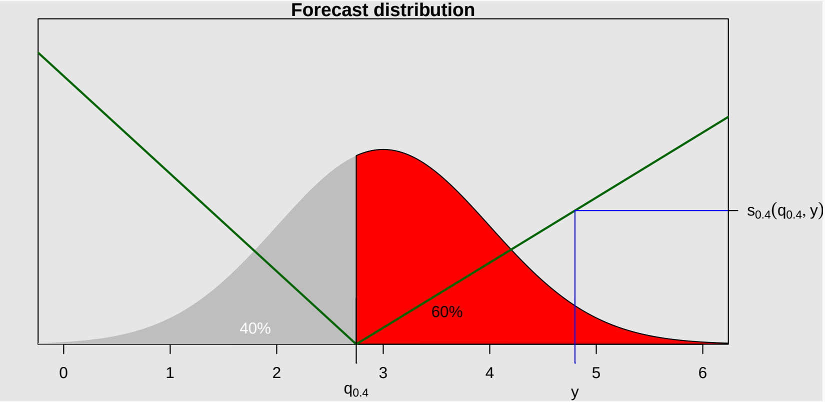

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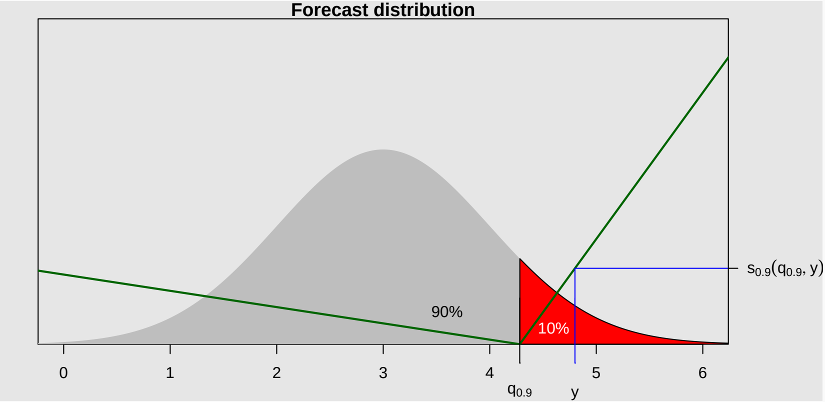

Probability scoring

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Quantile Score (pinball loss)

$$s_p(q_p, y) = \begin{cases} (1-p)(q_p - y), & \text{if } y < q_p; \\ p(y - q_p), & \text{if } y \geq q_p. \end{cases}$$

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Continuous Rank Probability Score

$$\text{CRPS}(F, y) = 2\int_0^1 s_p(q_p, y)dp.$$

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Continuous ranked probability scores

$$\begin{aligned}\text{CRPS}(F, y) &= 2\int_0^1 s_p(q_p, y)dp. \\ &= \int_{-\infty}^{\infty} (F(x) - 1\{y \le x\})^2 dx\end{aligned}$$

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Continuous ranked probability scores

$$\begin{aligned}\text{CRPS}(F, y) &= 2\int_0^1 s_p(q_p, y)dp.\\ &= \int_{-\infty}^{\infty}(F(x) - 1\{y \leq x\})^2 dx\end{aligned}$$

- CRPS in same units as observations
- Expected CRPS minimized when observation y from same distribution as F.
- Maximizes sharpness subject to calibration

Tidyverts R packages

tidyverts.org

tsibble

tsibbledata

feasts

fable

tsibble objects

```r
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##    Quarter Region   State Trips
##      <qtr> <chr>    <chr> <dbl>
##  1 1998 Q1 Adelaide SA     224.
##  2 1998 Q2 Adelaide SA     130.
##  3 1998 Q3 Adelaide SA     156.
##  4 1998 Q4 Adelaide SA     182.
##  5 1999 Q1 Adelaide SA     185.
##  6 1999 Q2 Adelaide SA     135.
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## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##      Index   Keys            Trips
##        <qtr> <chr>     <chr> <dbl>
##   1 1998 Q1 Adelaide SA      224.
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```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##      Index    Keys              Measure
##      <qtr> <chr>     <chr> <dbl>
##  1 1998 Q1 Adelaide  SA     224.
##  2 1998 Q2 Adelaide  SA     130.
##  3 1998 Q3 Adelaide  SA     156.
##  4 1998 Q4 Adelaide  SA     182.
##  5 1999 Q1 Adelaide  SA     185.
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```

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## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##
##        <qtr> <chr>    <chr> <dbl>
##  1 1998 Q1 Adelaide SA     224.
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##  3 1998 Q3 Adelaide SA     156.
##  4 1998 Q4 Adelaide SA     182.
##  5 1999 Q1 Adelaide SA     185.
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##  7 1999 Q3 Adelaide SA     136.
##  8 1999 Q4 Adelaide SA     169.
##  9 2000 Q1 Adelaide SA     184.
```

Index

Keys

Measure

Domestic overnight holiday trips in thousands by state/region.

Compute all aggregates

```
aus_holidays_agg <- aus_holidays %>%
  aggregate_key(State/Region, Trips=sum(Trips))
aus_holidays_agg
```

```
## # A tsibble: 6,800 x 4 [1Q]
## # Key:       State, Region [85]
##    State        Region       Quarter  Trips
##    <chr>        <chr>          <qtr>  <dbl>
##  1 <aggregated> <aggregated> 1998 Q1 11806.
##  2 <aggregated> <aggregated> 1998 Q2  9276.
##  3 <aggregated> <aggregated> 1998 Q3  8642.
##  4 <aggregated> <aggregated> 1998 Q4  9300.
##  5 <aggregated> <aggregated> 1999 Q1 11172.
##  6 <aggregated> <aggregated> 1999 Q2  9608.
##  7 <aggregated> <aggregated> 1999 Q3  8914.
##  8 <aggregated> <aggregated> 1999 Q4  9026.
##  9 <aggregated> <aggregated> 2000 Q1 11071.
```

High-dimensional time series data

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) # 2 year test set
```

```
## # A tsibble: 6,120 x 4 [1Q]
## # Key:       State, Region [85]
##    State        Region       Quarter  Trips
##    <chr>        <chr>          <qtr>  <dbl>
##  1 <aggregated> <aggregated> 1998 Q1 11806.
##  2 <aggregated> <aggregated> 1998 Q2  9276.
##  3 <aggregated> <aggregated> 1998 Q3  8642.
##  4 <aggregated> <aggregated> 1998 Q4  9300.
##  5 <aggregated> <aggregated> 1999 Q1 11172.
##  6 <aggregated> <aggregated> 1999 Q2  9608.
##  7 <aggregated> <aggregated> 1999 Q3  8914.
##  8 <aggregated> <aggregated> 1999 Q4  9026.
##  9 <aggregated> <aggregated> 2000 Q1 11071.
## 10 <aggregated> <aggregated> 2000 Q2  9196.
```

Fit univariate models

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips))
```

```
## # A mable: 85 x 4
## # Key:     State, Region [85]
##    State Region                 mean    ets
##    <chr> <chr>                  <model> <model>
##  1 ACT   Canberra               <MEAN>  <ETS(M,N,A)>
##  2 ACT   <aggregated>           <MEAN>  <ETS(M,N,A)>
##  3 NSW   Blue Mountains         <MEAN>  <ETS(M,N,M)>
##  4 NSW   Capital Country        <MEAN>  <ETS(A,N,N)>
##  5 NSW   Central Coast          <MEAN>  <ETS(M,N,M)>
##  6 NSW   Central NSW            <MEAN>  <ETS(M,N,A)>
##  7 NSW   Hunter                 <MEAN>  <ETS(A,N,A)>
##  8 NSW   New England North West <MEAN>  <ETS(M,N,N)>
##  9 NSW   North Coast NSW        <MEAN>  <ETS(A,N,A)>
```

Reconcile results

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets))
```

```
## # A mable: 85 x 4
## # Key:     State, Region [85]
##    State Region                    mean    ets
##    <chr> <chr>                    <model> <model>
##  1 ACT   Canberra                 <MEAN>  <ETS(M,N,A)>
##  2 ACT   <aggregated>             <MEAN>  <ETS(M,N,A)>
##  3 NSW   Blue Mountains           <MEAN>  <ETS(M,N,M)>
##  4 NSW   Capital Country          <MEAN>  <ETS(A,N,N)>
##  5 NSW   Central Coast            <MEAN>  <ETS(M,N,M)>
##  6 NSW   Central NSW              <MEAN>  <ETS(M,N,A)>
##  7 NSW   Hunter                   <MEAN>  <ETS(A,N,A)>
##  8 NSW   New England North West   <MEAN>  <ETS(M,N,N)>
```

Produce probabilistic forecasts

```
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years")
```

```
## # A fable: 1,360 x 6 [1Q]
## # Key:     State, Region, .model [170]
##    State Region        .model Quarter Trips .distribution
##    <chr> <chr>         <chr>    <qtr> <dbl> <dist>
##  1 ACT   Canberra      mean   2016 Q1  146. N(146, 1521)
##  2 ACT   Canberra      mean   2016 Q2  146. N(146, 1521)
##  3 ACT   Canberra      mean   2016 Q3  146. N(146, 1521)
##  4 ACT   Canberra      mean   2016 Q4  146. N(146, 1521)
##  5 ACT   Canberra      mean   2017 Q1  146. N(146, 1521)
##  6 ACT   Canberra      mean   2017 Q2  146. N(146, 1521)
##  7 ACT   Canberra      mean   2017 Q3  146. N(146, 1521)
```

Compare against test set

```
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  aggregate_key(State/Region, Trips=sum(Trips)) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(coherent = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS))
```

```
## # A tibble: 170 x 5
##    .model State Region                    .type  CRPS
##    <chr>  <chr> <chr>                     <chr> <dbl>
##  1 ets    ACT   Canberra                  Test   36.0
##  2 ets    ACT   <aggregated>              Test   36.0
##  3 ets    NSW   Blue Mountains            Test   15.2
##  4 ets    NSW   Capital Country           Test   15.0
##  5 ets    NSW   Central Coast             Test   11.5
##  6 ets    NSW   Central NSW               Test   15.9
```

Probability scoring

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  aggregate_key(State/Region, Trips=sum(Trips)) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(coherent = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS)
```

```
## # A tibble: 85 x 5
##     State Region                    .type   ets  mean
##     <chr> <chr>                     <chr> <dbl> <dbl>
##   1 ACT   Canberra                  Test   36.0  34.4
##   2 ACT   <aggregated>              Test   36.0  34.4
##   3 NSW   Blue Mountains            Test   15.2  35.8
##   4 NSW   Capital Country           Test   15.0  16.2
##   5 NSW   Central Coast             Test   11.5  23.3
```

Probability scoring

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  aggregate_key(State/Region, Trips=sum(Trips)) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(coherent = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS) %>%
  mutate(RelCRPS = ets/mean)
```

```
## # A tibble: 85 x 6
##    State Region                     .type   ets  mean RelCRPS
##    <chr> <chr>                      <chr> <dbl> <dbl>   <dbl>
##  1 ACT   Canberra                   Test   36.0  34.4   1.05
##  2 ACT   <aggregated>               Test   36.0  34.4   1.05
##  3 NSW   Blue Mountains             Test   15.2  35.8   0.424
##  4 NSW   Capital Country            Test   15.0  16.2   0.925
```

Probability scoring

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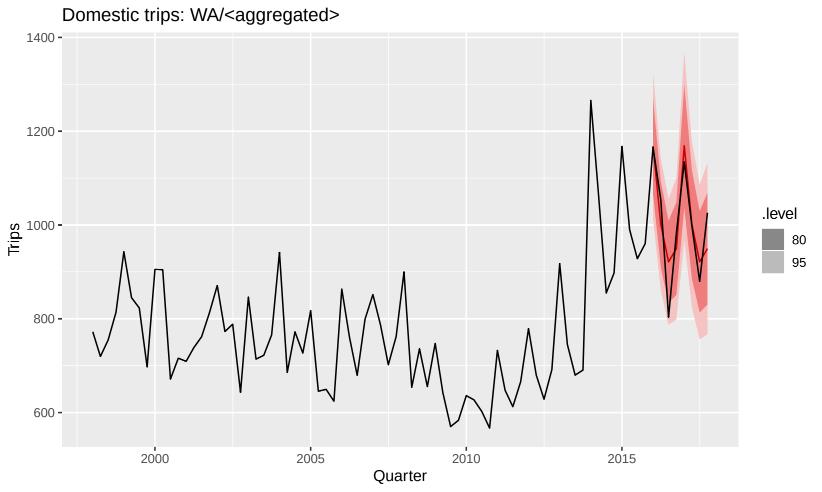
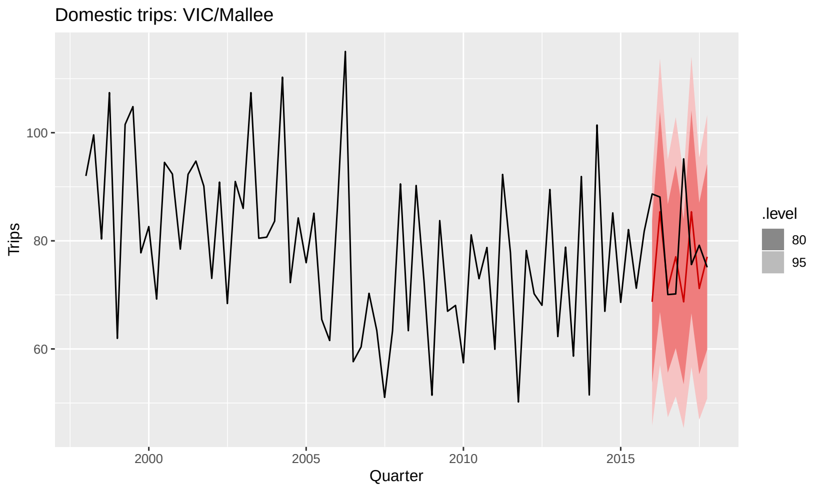
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  aggregate_key(State/Region, Trips=sum(Trips)) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(coherent = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS) %>%
  mutate(RelCRPS = ets/mean) %>%
  arrange(RelCRPS)
```

```
## # A tibble: 85 x 6
##    State Region                        .type   ets  mean RelCRPS
##    <chr> <chr>                         <chr> <dbl> <dbl>   <dbl>
##  1 WA    <aggregated>                  Test  34.8  177.    0.196
##  2 NSW   South Coast                   Test  26.2  101.    0.261
##  3 WA    Australia's South West        Test  27.1  102.    0.265
```

Best forecast (RelCRPS)



Worst forecast (RelCRPS)



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1. The fable package produces high-dim probabilistic forecasts?
2. We can focus on marginal distributions for interpretation, while needing the full distribution for calculation.
3. Forecast reconciliation provides a way to efficiently compute marginal distributions.
4. Probability scoring allows us to measure distributional forecast accuracy.

Acknowledgements

Mitchell O'Hara-Wild

Earo Wang

tidyverts.org

robjhyndman.com