MONASH University

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Forecasting is not prophecy

Dealing with high-dimensional probabilistic forecasts in practice

Rob J Hyndman

What is it?

What is it?

Clay model of sheep's liver

Used by Babylonian forecasters approximately 600 B.C.

Now in British Museum.

Modern prophecy

A Timeline of Very Bad Future Predictions

1800

"Rail travel at high speed is not possible, because passengers, unable to breathe, would die of asphyxia."

Dr. Dionysys Larder, Professor of Natural Philosophy & Astronomy, University College London

1859

"Drill for oil? You mean drill into the ground to try and find oil? You're crazy!"

Associates of Edwin L. Drake refusing his suggestion to drill for oil in 1859 (Later that year, Drake succeeded in drilling the first oil well.)

1876

"This telephone has too many shortcomings to be seriously considered as a means of communication."

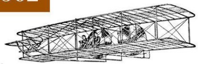
Western Union internal memo

1880

"Everyone acquainted with the subject will recognize it as a conspicuous failure."

Henry Morton, president of the Stevens Institute of Technology, on Edison's light bulb

1902



"Flight by machines heavier than air is unpractical and insignificant, if not utterly impossible."

Simon Newcomb, Canadian-American astronomer and mathematician, 18 months before the Wright Brothers' flight at Kittyhawk

1903

"The horse is here to stay, but the automobile is only a novelty, a fad."

The president of the Michigan Savings Bank, advising Henry Ford's lawyer not to invest in the Ford Motor Company

1916

"The idea that cavalry will be replaced by these iron coaches is absurd. It is little short of treasonous."

Comment of Aide-de-camp to Field Marshal Haig, at tank demonstration

1916

"The cinema is little more than a fad. It's canned drama. What audiences really want to see is flesh and blood on the stage."

Charlie Chaplin, actor, producer, director, and studio founder

1921

"The wireless music box has no imaginable commercial value. Who would pay for a message sent to no one in particular?"

Associates of commercial radio and television pioneer, David Sarnoff, responding to his call for investment in the radio

1946

"Television won't last because people will soon get tired of staring at a plywood box every night."

Darryl Zanuck, movie producer, 20th Century Fox

1977

"There is no reason for any individual to have a computer in his home."

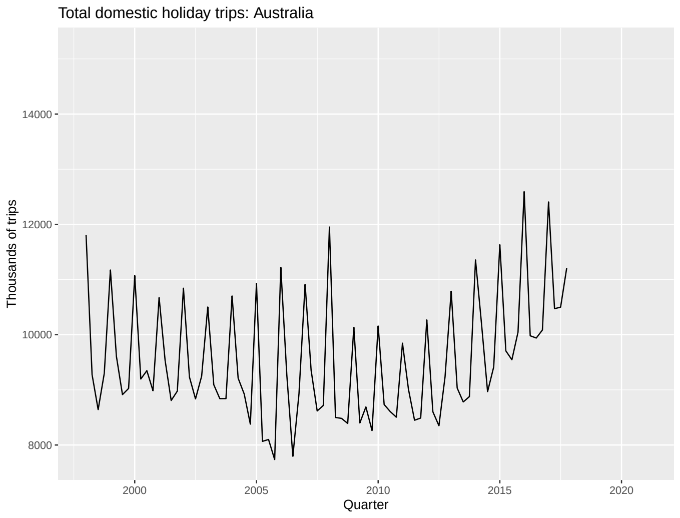
Ken Olson, president, chairman and founder of Digital Equipment Corporation

1995

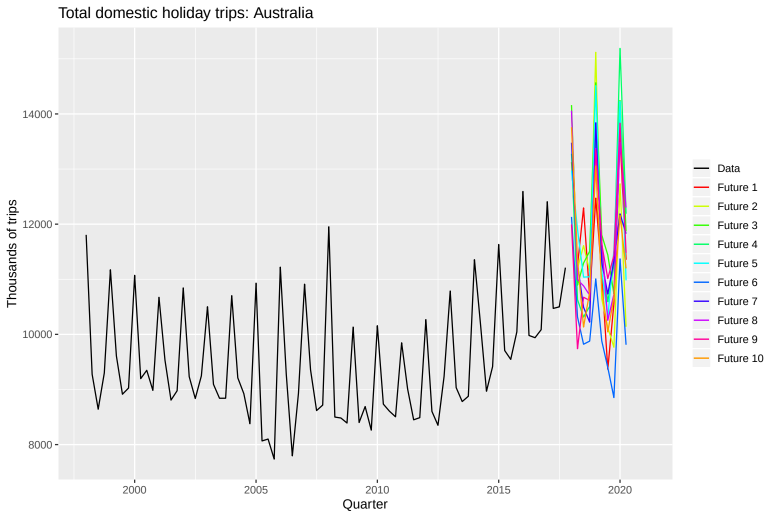
"The truth is no online database will replace your daily newspaper…"

Clifford Stoll, Newsweek article entitled ***The Internet? Bah!***

Probabilistic forecasting



Probabilistic forecasting



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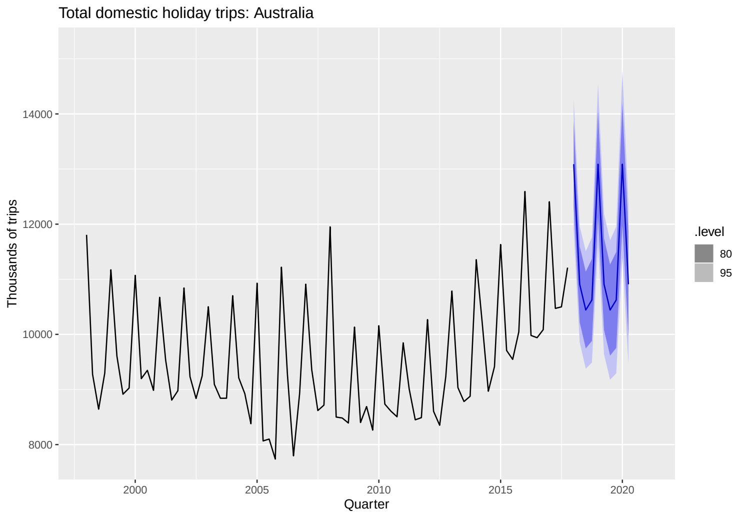
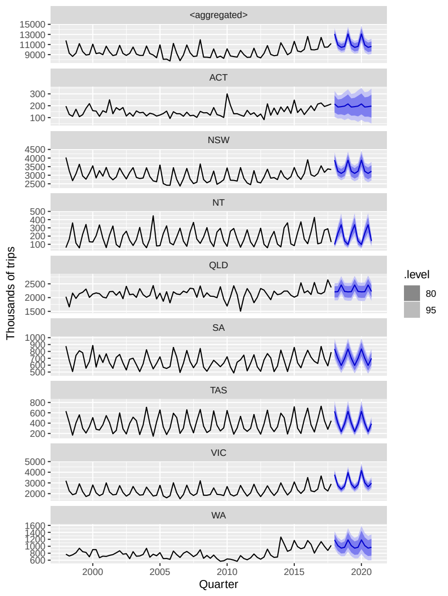
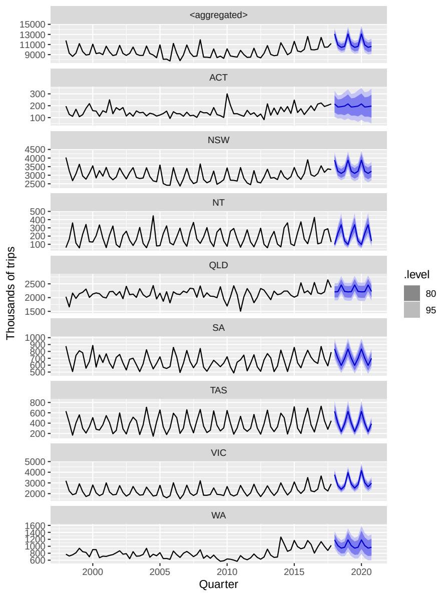


Multivariate probabilistic forecasting



Multivariate probabilistic forecasting



These show the marginal distributions of the 9-dimensional distribution.

In practice, we also want forecast for smaller geographic areas as well.

Who needs multivariate probabilistic forecasts?

- Tourism authorities forecasting visitor numbers to plan facilities.
- Manufacturing companies forecasting product demand to plan their supply chains.
- Call centres forecasting call volume to inform staff scheduling.
- Technology companies forecasting web traffic to maintain service levels.
- Energy companies forecasting electricity demand to prevent blackouts.

Challenges in multivariate probabilistic forecasting

1. How to produce high-dimensional probabilistic forecasts?
2. How to use and interpret high-dimensional probabilistic forecasts?
3. How to measure accuracy of high-dimensional probabilistic forecasts?

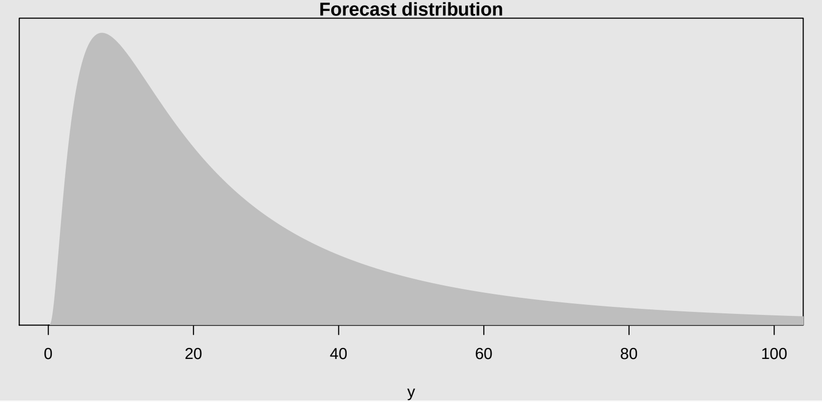
Challenges in multivariate probabilistic forecasting

1. How to produce high-dimensional probabilistic forecasts?
2. How to use and interpret high-dimensional probabilistic forecasts?
3. How to measure accuracy of high-dimensional probabilistic forecasts?

- Users care about marginal distributions, but we need multivariate distribution to compute them.
- Forecast reconciliation provides a way to efficiently compute marginal distributions.
- Probability scoring allows us to measure distributional forecast accuracy.

Probability scoring

- y = actual value
- F = forecast distribution
- $q_p = F^{-1}(p)$ = quantile with probability p.



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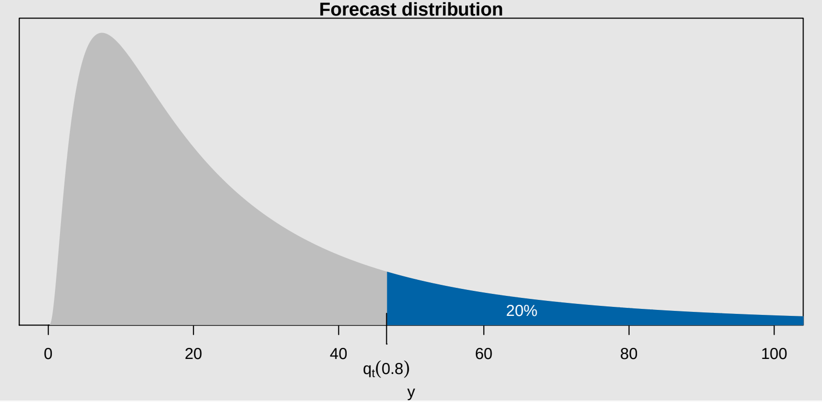

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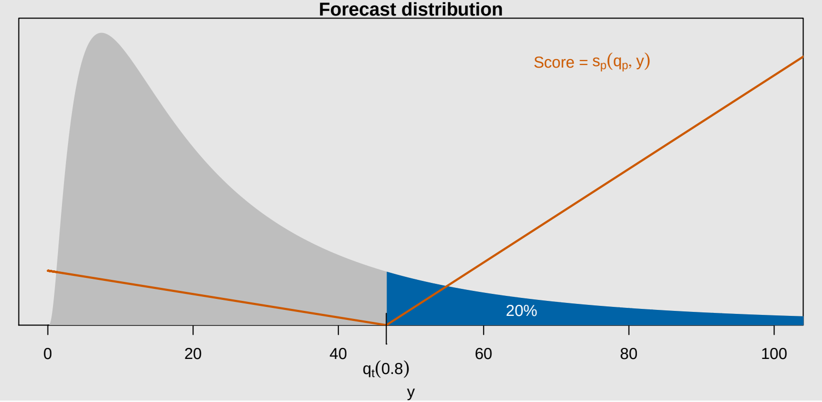

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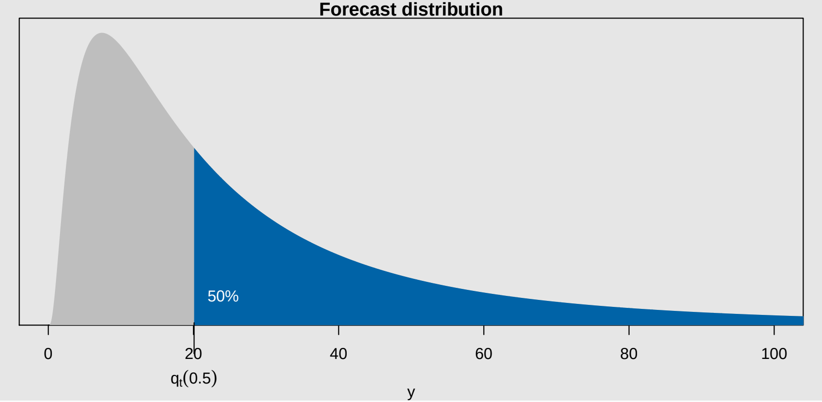

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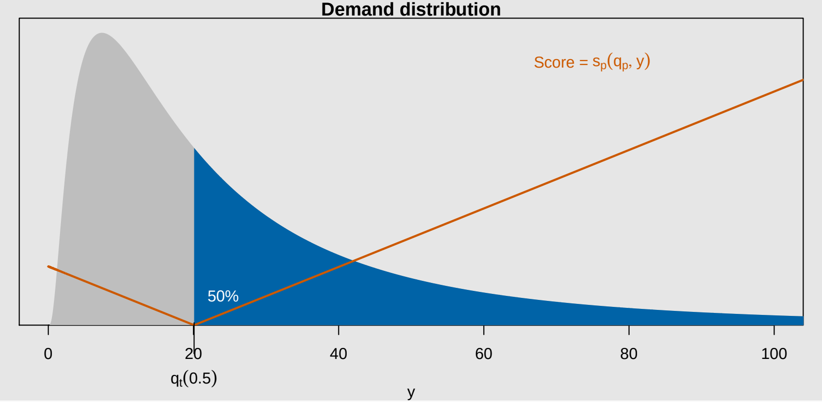

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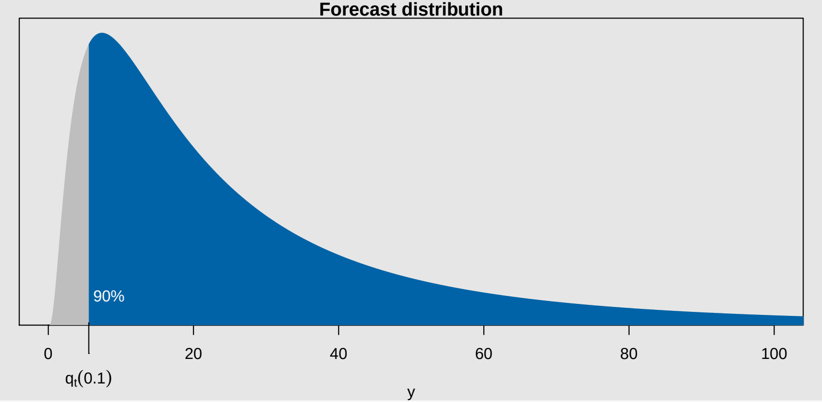

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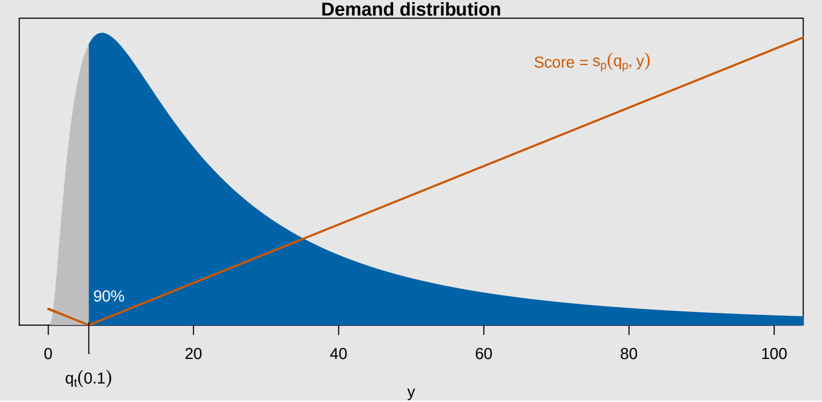

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Quantile Score (pinball loss)

$$s_p(q_p, y) = \begin{cases} (1-p)(q_p - y), & \text{if } y < q_p; \\ p(y - q_p), & \text{if } y \geq q_p. \end{cases}$$

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Continuous Rank Probability Score

$$\text{CRPS}(F, y) = 2\int_0^1 s_p(q_p, y)dp.$$

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Continuous ranked probability scores

$$\begin{aligned}\text{CRPS}(F, y) &= 2\int_0^1 s_p(q_p, y)dp. \\ &= \int_{-\infty}^{\infty}(F(x) - 1\{y \leq x\})^2 dx\end{aligned}$$

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- CRPS in same units as observations
- Expected CRPS minimized when observation y from same distribution as F.
- Maximizes sharpness subject to calibration.

Tidyverts R packages

tidyverts.org

tsibble

tsibbledata

feasts

fable

tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##    Quarter Region   State Trips
##      <qtr> <chr>    <chr> <dbl>
##  1 1998 Q1 Adelaide SA     224.
##  2 1998 Q2 Adelaide SA     130.
##  3 1998 Q3 Adelaide SA     156.
##  4 1998 Q4 Adelaide SA     182.
##  5 1999 Q1 Adelaide SA     185.
##  6 1999 Q2 Adelaide SA     135.
##  7 1999 Q3 Adelaide SA     136.
##  8 1999 Q4 Adelaide SA     169.
##  9 2000 Q1 Adelaide SA     184.
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```

tsibble objects

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## # A tsibble: 6,080 x 4 [1Q]
## # Key:       Region, State [76]
##     Index    Keys              Trips
##       <qtr> <chr>     <chr> <dbl>
##  1 1998 Q1 Adelaide SA     224.
##  2 1998 Q2 Adelaide SA     130.
##  3 1998 Q3 Adelaide SA     156.
##  4 1998 Q4 Adelaide SA     182.
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```
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## # Key:       Region, State [76]
##     Index    Keys            Measure
##       <qtr> <chr>      <chr> <dbl>
##  1 1998 Q1 Adelaide SA      224.
##  2 1998 Q2 Adelaide SA      130.
##  3 1998 Q3 Adelaide SA      156.
##  4 1998 Q4 Adelaide SA      182.
##  5 1999 Q1 Adelaide SA      185.
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##      Index  Keys          Measure
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##  6 1999 Q2 Adelaide SA     135.
##  7 1999 Q3 Adelaide SA     136.
##  8 1999 Q4 Adelaide SA     169.
##  9 2000 Q1 Adelaide SA     184.
## 10 2000 Q2 Adelaide SA     134.
```

Domestic overnight holiday trips in thousands by state/region.

Compute all aggregates

```r
aus_holidays_agg <- aus_holidays %>%
  aggregate_key(State/Region, Trips=sum(Trips))
aus_holidays_agg
```

```
## # A tsibble: 6,800 x 4 [1Q]
## # Key:       State, Region [85]
##    State        Region       Quarter  Trips
##    <chr>        <chr>          <qtr>  <dbl>
##  1 <aggregated> <aggregated> 1998 Q1 11806.
##  2 <aggregated> <aggregated> 1998 Q2  9276.
##  3 <aggregated> <aggregated> 1998 Q3  8642.
##  4 <aggregated> <aggregated> 1998 Q4  9300.
##  5 <aggregated> <aggregated> 1999 Q1 11172.
##  6 <aggregated> <aggregated> 1999 Q2  9608.
##  7 <aggregated> <aggregated> 1999 Q3  8914.
##  8 <aggregated> <aggregated> 1999 Q4  9026.
##  9 <aggregated> <aggregated> 2000 Q1 11071.
## 10 <aggregated> <aggregated> 2000 Q2  9196.
```

Create training/test sets

```
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) # 2 year test set
```

```
## # A tsibble: 6,120 x 4 [1Q]
## # Key:       State, Region [85]
##    State        Region       Quarter  Trips
##    <chr>        <chr>          <qtr>  <dbl>
##  1 <aggregated> <aggregated> 1998 Q1 11806.
##  2 <aggregated> <aggregated> 1998 Q2  9276.
##  3 <aggregated> <aggregated> 1998 Q3  8642.
##  4 <aggregated> <aggregated> 1998 Q4  9300.
##  5 <aggregated> <aggregated> 1999 Q1 11172.
##  6 <aggregated> <aggregated> 1999 Q2  9608.
##  7 <aggregated> <aggregated> 1999 Q3  8914.
##  8 <aggregated> <aggregated> 1999 Q4  9026.
##  9 <aggregated> <aggregated> 2000 Q1 11071.
## 10 <aggregated> <aggregated> 2000 Q2  9196.
## # ... with 6,110 more rows
```

Fit univariate models

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips))
```

```
## # A mable: 85 x 4
## # Key:     State, Region [85]
##    State Region                mean    ets
##    <chr> <chr>                 <model> <model>
##  1 ACT   Canberra              <MEAN>  <ETS(M,N,A)>
##  2 ACT   <aggregated>          <MEAN>  <ETS(M,N,A)>
##  3 NSW   Blue Mountains        <MEAN>  <ETS(M,N,M)>
##  4 NSW   Capital Country       <MEAN>  <ETS(A,N,N)>
##  5 NSW   Central Coast         <MEAN>  <ETS(M,N,M)>
##  6 NSW   Central NSW           <MEAN>  <ETS(M,N,A)>
##  7 NSW   Hunter                <MEAN>  <ETS(A,N,A)>
##  8 NSW   New England North West <MEAN> <ETS(M,N,N)>
##  9 NSW   North Coast NSW       <MEAN>  <ETS(A,N,A)>
## 10 NSW   Outback NSW           <MEAN>  <ETS(M,N,A)>
```

Reconcile results

```
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets))
```

```
## # A mable: 85 x 4
## # Key:     State, Region [85]
##    State Region                     mean    ets
##    <chr> <chr>                   <model> <model>
##  1 ACT   Canberra                 <MEAN>  <ETS(M,N,A)>
##  2 ACT   <aggregated>             <MEAN>  <ETS(M,N,A)>
##  3 NSW   Blue Mountains           <MEAN>  <ETS(M,N,M)>
##  4 NSW   Capital Country          <MEAN>  <ETS(A,N,N)>
##  5 NSW   Central Coast            <MEAN>  <ETS(M,N,M)>
##  6 NSW   Central NSW              <MEAN>  <ETS(M,N,A)>
##  7 NSW   Hunter                   <MEAN>  <ETS(A,N,A)>
##  8 NSW   New England North West   <MEAN>  <ETS(M,N,N)>
##  9 NSW   North Coast NSW          <MEAN>  <ETS(A,N,A)>
```

Produce probabilistic forecasts

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years")
```

```
## # A fable: 1,360 x 6 [1Q]
## # Key:     State, Region, .model [170]
##    State Region        .model Quarter Trips .distribution
##    <chr> <chr>         <chr>    <qtr> <dbl> <dist>
##  1 ACT   Canberra      mean   2016 Q1  146. N(146, 1521)
##  2 ACT   Canberra      mean   2016 Q2  146. N(146, 1521)
##  3 ACT   Canberra      mean   2016 Q3  146. N(146, 1521)
##  4 ACT   Canberra      mean   2016 Q4  146. N(146, 1521)
##  5 ACT   Canberra      mean   2017 Q1  146. N(146, 1521)
##  6 ACT   Canberra      mean   2017 Q2  146. N(146, 1521)
##  7 ACT   Canberra      mean   2017 Q3  146. N(146, 1521)
##  8 ACT   Canberra      mean   2017 Q4  146. N(146, 1521)
```

Compare against test set

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS))
```

```
## # A tibble: 170 x 5
##    .model State Region                   .type  CRPS
##    <chr>  <chr> <chr>                    <chr> <dbl>
##  1 ets    ACT   Canberra                 Test   36.0
##  2 ets    ACT   <aggregated>             Test   36.0
##  3 ets    NSW   Blue Mountains           Test   15.2
##  4 ets    NSW   Capital Country          Test   15.0
##  5 ets    NSW   Central Coast            Test   11.5
##  6 ets    NSW   Central NSW              Test   15.9
##  7 ets    NSW   Hunter                   Test   48.4
##  8 ets    NSW   New England North West   Test   11.1
```

Reorganize results

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS)
```

```
## # A tibble: 85 x 5
##    State Region                   .type   ets  mean
##    <chr> <chr>                    <chr> <dbl> <dbl>
##  1 ACT   Canberra                 Test   36.0  34.4
##  2 ACT   <aggregated>             Test   36.0  34.4
##  3 NSW   Blue Mountains           Test   15.2  35.8
##  4 NSW   Capital Country          Test   15.0  16.2
##  5 NSW   Central Coast            Test   11.5  23.3
##  6 NSW   Central NSW              Test   15.9  17.0
##  7 NSW   Hunter                   Test   48.4  44.0
```

Scale CRPS values

```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS) %>%
  mutate(RelCRPS = ets/mean)
```

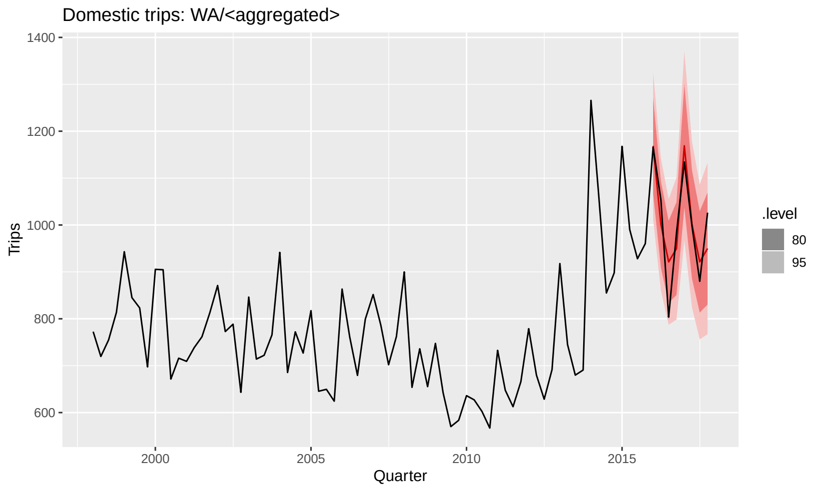
```
## # A tibble: 85 x 6
##    State Region                    .type   ets  mean RelCRPS
##    <chr> <chr>                     <chr> <dbl> <dbl>   <dbl>
##  1 ACT   Canberra                  Test   36.0  34.4   1.05
##  2 ACT   <aggregated>              Test   36.0  34.4   1.05
##  3 NSW   Blue Mountains            Test   15.2  35.8   0.424
##  4 NSW   Capital Country           Test   15.0  16.2   0.925
##  5 NSW   Central Coast             Test   11.5  23.3   0.494
##  6 NSW   Central NSW               Test   15.9  17.0   0.935
```

Sort by scaled CRPS

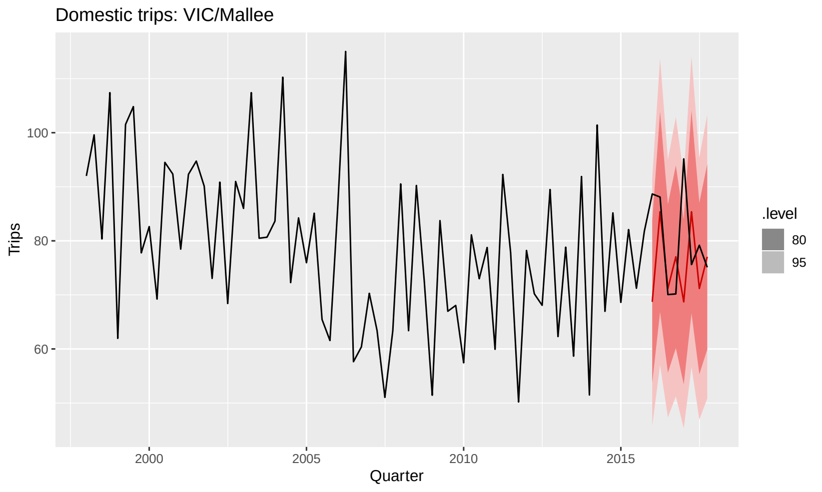
```r
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(ets = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS) %>%
  mutate(RelCRPS = ets/mean) %>%
  arrange(RelCRPS)
```

```
## # A tibble: 85 x 6
##    State Region                       .type   ets   mean RelCRPS
##    <chr> <chr>                        <chr> <dbl>  <dbl>   <dbl>
##  1 WA    <aggregated>                 Test  34.8  177.     0.196
##  2 NSW   South Coast                  Test  26.2  101.     0.261
##  3 WA    Australia's South West       Test  27.1  102.     0.265
##  4 NT    Katherine Daly               Test   3.61  11.8    0.305
##  5 NSW   Snowy Mountains              Test  22.3   64.8    0.344
```

Best forecast (RelCRPS)



Worst forecast (RelCRPS)



Challenges in multivariate probabilistic forecasting

1. The fable package produces high-dimensional probabilistic forecasts.
2. We can focus on marginal distributions for interpretation, while needing the full distribution for calculation.
3. Forecast reconciliation provides a way to efficiently compute marginal distributions.
4. Probability scoring allows us to measure distributional forecast accuracy.

Acknowledgements

Mitchell O'Hara-Wild

Earo Wang

tidyverts.org

robjhyndman.com