



Forecasting is not prophecy

Dealing with high-dimensional probabilistic forecasts in practice

Rob J Hyndman

What is it?



What is it?

Clay model of sheep's liver

Used by Babylonian forecasters approximately 600 B.C.



Now in British Museum.

Modern prophecy

A Timeline of

Very Bad Future Predictions

1800



Rail travel at high speed is not possible. because passengers, unable to breathe, would die of asphyxia.

Dr. Dionysys Larder, Professor of Natural Philosophy & Astronomy, University College

1859

1876



Drill for oil? You mean drill into the ground to try and find oil? You're crazy!

Associates of Edwin L. Drake refusing his suggestion to drill for oil in 1859 (Later that year. Drake succeeded in drilling the first oil well.)



This telephone has too many shortcomings to be seriously considered as a means of communication .

Western Union internal memo



Everyone acquainted with the subject will recognize it as a conspicuous failure. Henry Morton, president of the Stevens Institute of Technology, on Edison's light bulb

1902



Flight by machines heavier than air is unpractical and insignificant, if not utterly impossible.

Simon Newcomb Canadian-American astronomer and mathematician, 18 months before the Wright Brothers' flight at Kittyhawk

The president of the Michigan Savings Bank,

advising Henry Ford's lawyer not to invest in

the Ford Motor Company

1916



The idea that cavalry will be replaced by these iron coaches is absurd. It is little short of treasonous

Comment of Aide-de-camp to Field Marshal Haig, at tank demonstration

1916



The cinema is little more than a fad. It's canned drama. What audiences really want to see is flesh and blood on the stage. Charlie Chaplin, actor, producer, director, and

studio founder

1921



The wireless music box has no imaginable commercial value. Who would pay for a message sent to no one in particular?

Associates of commercial radio and television pioneer, David Sarnoff, responding to his call for investment in the radio

1946



Television won't last because people will soon get tired of staring at a plywood box every night.

Darryl Zanuck, movie producer, 20th Century



There is no reason for any individual to have a computer in his home.

Ken Olson, president, chairman and founder of Digital Equipment Corporation

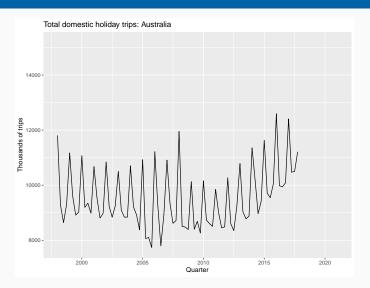
1995



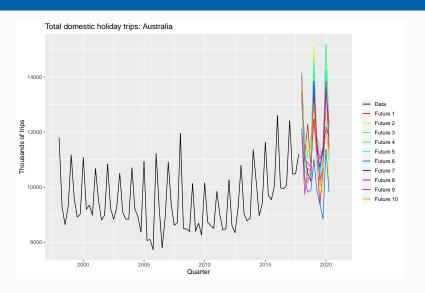
The truth is no online database will replace your daily newspaper...

Clifford Stoll Newsweek article entitled The Internet? Rah!

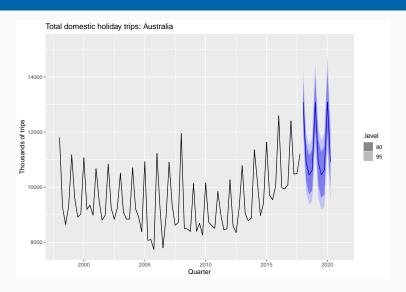
Probabilistic forecasting



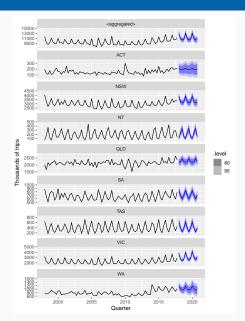
Probabilistic forecasting



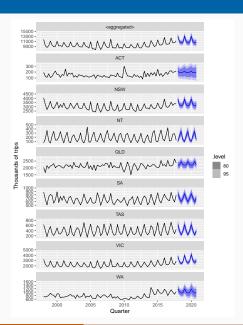
Probabilistic forecasting



Multivariate probabilistic forecasting



Multivariate probabilistic forecasting



These show the marginal distributions of the 9-dimensional distribution. In practice, we also want forecast for smaller geographic areas as well.

Who needs multivariate probabilistic forecasts?

- Tourism authorities forecasting visitor numbers to plan facilities.
- Manufacturing companies forecasting product demand to plan their supply chains.
- Call centres forecasting call volume to inform staff scheduling.
- Technology companies forecasting web traffic to maintain service levels.
- Energy companies forecasting electricity demand to prevent blackouts.

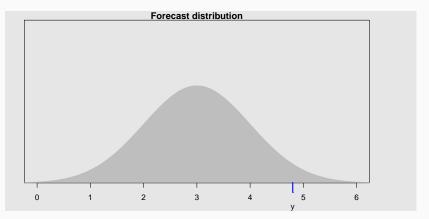
Challenges in multivariate probabilistic forecasting

- How to produce high-dim probabilistic forecasts?
- 2 How to use and interpret high-dim probabilistic forecasts?
- How to measure accuracy of high-dim probabilistic forecasts?

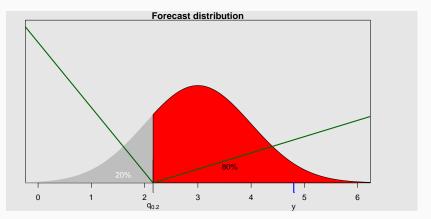
Challenges in multivariate probabilistic forecasting

- How to produce high-dim probabilistic forecasts?
- 2 How to use and interpret high-dim probabilistic forecasts?
- How to measure accuracy of high-dim probabilistic forecasts?
 - Users care about marginal distributions, but we need multivariate distribution to compute them.
 - Forecast reconciliation provides a way to efficiently compute marginal distributions.
 - Probability scoring allows us to measure distributional forecast accuracy.

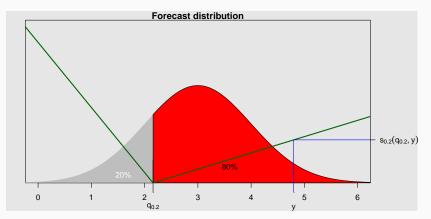
- y = actual value
- F = forecast distribution
- $q_p = F^{-1}(p)$ = quantile with probability p.



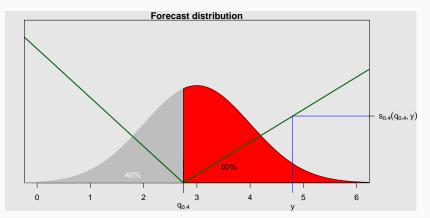
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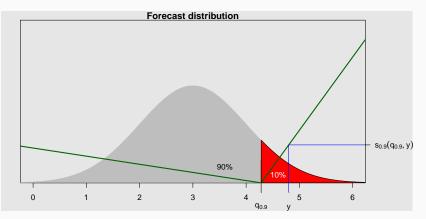
- y = actual value
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Quantile Score (pinball loss)

$$s_p(q_p, y) = \begin{cases} (1-p)(q_p - y), & \text{if } y < q_p; \\ p(y - q_p), & \text{if } y \ge q_p. \end{cases}$$

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Quantile Score (pinball loss)

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Continuous Rank Probability Score

CRPS(F, y) =
$$2 \int_0^1 s_p(q_p, y) dp$$
.

- y = actual value
- F = forecast distribution
- $q_p = F^{-1}(p) = \text{quantile with probability } p.$

Continuous ranked probability scores

CRPS(F, y) =
$$2 \int_0^1 s_p(q_p, y) dp$$
.
= $\int_{-\infty}^{\infty} (F(x) - 1\{y \le x\})^2 dx$

- y = actual value
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Continuous ranked probability scores

CRPS(F, y) =
$$2 \int_0^1 s_p(q_p, y) dp$$
.
= $\int_{-\infty}^{\infty} (F(x) - 1\{y \le x\})^2 dx$

- CRPS in same units as observations
- Expected CRPS minimized when observation *y* from same distribution as *F*.
- Maximizes sharpness subject to calibration

Tidyverts R packages

tidyverts.org



```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [10]
## # Kev:
               Region, State [76]
     Quarter Region State Trips
##
##
       <qtr> <chr> <chr> <chr> <chr> <chr> <dbl>
##
   1 1998 Q1 Adelaide SA
                             224.
   2 1998 02 Adelaide SA 130.
##
   3 1998 Q3 Adelaide SA 156.
##
##
   4 1998 Q4 Adelaide SA 182.
##
   5 1999 Q1 Adelaide SA
                             185.
   6 1999 Q2 Adelaide SA
##
                             135.
   7 1999 Q3 Adelaide SA
                             136.
##
   8 1999 Q4 Adelaide SA
##
                             169.
##
   9 2000 Q1 Adelaide SA
                             184.
```

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```
## # A tsibble: 6,080 x 4 [10]
## # Key:
               Region, State [76]
##
      Index
             Region State Trips
##
        <qtr> <chr> <chr> <chr> <chr> <chr> <dbl>
##
    1 1998 Q1 Adelaide SA
                             224.
   2 1998 Q2 Adelaide SA
                             130.
##
   3 1998 Q3 Adelaide SA 156.
##
##
   4 1998 Q4 Adelaide SA
                             182.
##
    5 1999 Q1 Adelaide SA
                             185.
    6 1999 Q2 Adelaide SA
##
                             135.
   7 1999 Q3 Adelaide SA
                             136.
##
    8 1999 Q4 Adelaide SA
##
                             169.
##
   9 2000 Q1 Adelaide SA
                             184.
```

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## # A tsibble: 6,080 x 4 [10]
##
  # Kev:
                Region, State [76]
      Index
##
              Keys
                             Trips
        <qtr> <chr> <chr> <chr> <chr> <dbl>
##
##
    1 1998 Q1 Adelaide SA
                              224.
   2 1998 Q2 Adelaide SA
                              130.
##
   3 1998 Q3 Adelaide SA
                             156.
##
##
   4 1998 Q4 Adelaide SA
                              182.
##
    5 1999 Q1 Adelaide SA
                              185.
    6 1999 Q2 Adelaide SA
##
                              135.
   7 1999 Q3 Adelaide SA
                              136.
##
    8 1999 O4 Adelaide SA
##
                              169.
##
   9 2000 Q1 Adelaide SA
                              184.
```

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```

```
## # A tsibble: 6,080 x 4 [10]
##
  # Kev:
                Region, State [76]
      Index
##
              Keys
                             Measure
        <qtr> <chr> <chr> <chr> <chr> <dbl>
##
##
    1 1998 Q1 Adelaide SA
                              224.
   2 1998 Q2 Adelaide SA
                              130.
##
   3 1998 Q3 Adelaide SA
                             156.
##
##
   4 1998 Q4 Adelaide SA
                              182.
##
   5 1999 Q1 Adelaide SA
                              185.
    6 1999 Q2 Adelaide SA
##
                              135.
   7 1999 Q3 Adelaide SA
                              136.
##
    8 1999 O4 Adelaide SA
##
                              169.
##
   9 2000 Q1 Adelaide SA
                              184.
```

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library(tsibble)
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```

```
# A tsibble: 6,080 x 4 [10]
   # Key:
##
                 Region, State [76]
      Index
                               Measure
##
               Kevs
        <qtr> <chr> <chr> <chr> <chr> <dbl>
##
##
    1 1998 Q1 Adelaide SA
                                224.
    2 1998 02 Adelaide SA
                                130.
##
    3 1998 Q3 Adelaide SA
                                156.
##
##
    4 1998 Q4 Adelaide SA
                                182.
##
    5 1999 Q1 Adelaide SA
                                185.
    6 1999 O2 Adelaide SA
                                135.
##
    7 1999 03 Adelaide SA
                                136.
##
    8 1999 Q4 Adelaide SA
##
                                169.
##
    9 2000 Q1 Adelaide SA
                                184.
```

Domestic overnight holiday trips in thousands by state/region.

Compute all aggregates

```
aus_holidays_agg <- aus_holidays %>%
aggregate_key(State/Region, Trips=sum(Trips))
aus_holidays_agg
```

```
## # A tsibble: 6,800 x 4 [10]
##
  # Kev:
               State, Region [85]
##
     State
                  Region
                               Quarter Trips
##
     <chr>>
                  <chr>
                                  <atr>
                                         <dbl>
##
    1 <aggregated> <aggregated> 1998 Q1 11806.
##
    2 <aggregated> <aggregated> 1998 Q2
                                        9276.
##
    3 <aggregated> <aggregated> 1998 Q3
                                        8642.
##
   4 <aggregated> <aggregated> 1998 Q4
                                        9300.
##
    5 <aggregated> <aggregated> 1999 Q1 11172.
    6 <aggregated> <aggregated> 1999 Q2
##
                                        9608.
##
    7 <aggregated> <aggregated> 1999 Q3
                                        8914.
    8 <aggregated> <aggregated> 1999 Q4
                                        9026.
##
##
    9 <aggregated> <aggregated> 2000 Q1 11071.
```

High-dimensional time series data

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) # 2 year test set</pre>
```

```
## # A tsibble: 6,120 x 4 [1Q]
  # Key:
                State, Region [85]
##
##
      State
                   Region
                                Quarter Trips
##
      <chr>>
                   <chr>
                                  <atr>
                                          <dbl>
##
    1 <aggregated> <aggregated> 1998 Q1 11806.
##
    2 <aggregated> <aggregated> 1998 Q2
                                         9276.
##
    3 <aggregated> <aggregated> 1998 Q3
                                         8642.
##
   4 <aggregated> <aggregated> 1998 Q4
                                          9300.
##
    5 <aggregated> <aggregated> 1999 Q1 11172.
                                         9608.
##
    6 <aggregated> <aggregated> 1999 Q2
                                         8914.
##
    7 <aggregated> <aggregated> 1999 Q3
                                         9026.
##
    8 <aggregated> <aggregated> 1999 Q4
    9 <aggregated> <aggregated> 2000 Q1 11071.
##
   10 <aggregated> <aggregated> 2000 Q2
                                         9196.
```

Fit univariate models

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
model(mean = MEAN(Trips), ets = ETS(Trips))
```

```
## # A mable: 85 x 4
## # Key: State, Region [85]
      State Region
##
                                     mean
                                             ets
      <chr> <chr>
                                     <model>
                                             <model>
##
##
    1 ACT
            Canberra
                                     <MEAN> <ETS(M,N,A)>
    2 ACT <aggregated>
                                     <MEAN>
                                             <ETS(M,N,A)>
##
##
    3 NSW
            Blue Mountains
                                     <MEAN>
                                             <ETS(M,N,M)>
##
    4 NSW
            Capital Country
                                     <MEAN>
                                             <ETS(A,N,N)>
    5 NSW
            Central Coast
                                     <MEAN>
                                             <ETS(M,N,M)>
##
            Central NSW
                                     <MEAN>
                                             <ETS(M,N,A)>
##
    6 NSW
                                     <MEAN>
                                             \langle ETS(A,N,A) \rangle
##
    7 NSW
            Hunter
    8 NSW
            New England North West <MEAN>
                                             <ETS(M,N,N)>
##
##
    9 NSW
            North Coast NSW
                                     <MEAN>
                                             <ETS(A,N,A)>
```

Reconcile results

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
reconcile(ets = min_trace(ets))
```

```
## # A mable: 85 x 4
##
   # Key: State, Region [85]
      State Region
##
                                             ets
                                     mean
##
      <chr> <chr>
                                     <model> <model>
##
    1 ACT
            Canberra
                                     <MEAN> <ETS(M,N,A)>
    2 ACT <aggregated>
##
                                     <MEAN>
                                             <ETS(M,N,A)>
##
    3 NSW
            Blue Mountains
                                     <MEAN>
                                             <ETS(M,N,M)>
    4 NSW
            Capital Country
                                     <MEAN>
                                             <ETS(A,N,N)>
##
    5 NSW
            Central Coast
                                     <MEAN>
                                             <ETS(M,N,M)>
##
    6 NSW
            Central NSW
                                     <MEAN>
                                             <ETS(M,N,A)>
##
                                             \langle ETS(A,N,A) \rangle
##
    7 NSW
            Hunter
                                     <MEAN>
            New England North West <MEAN>
                                             <ETS(M,N,N)>
##
    8 NSW
```

Produce probabilistic forecasts

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
reconcile(ets = min_trace(ets)) %>%
forecast(h = "2 years")
```

```
## # A fable: 1,360 x 6 [10]
##
  # Key: State, Region, .model [170]
##
     State Region .model Quarter Trips .distribution
##
     <chr> <chr>
                       <chr> <qtr> <dbl> <dist>
##
   1 ACT
           Canberra
                              2016 Q1 146. N(146, 1521)
                       mean
##
   2 ACT Canberra
                              2016 Q2 146. N(146, 1521)
                       mean
   3 ACT Canberra
                              2016 Q3 146. N(146, 1521)
##
                       mean
   4 ACT Canberra
                              2016 Q4 146. N(146, 1521)
##
                       mean
   5 ACT Canberra
                              2017 Q1 146. N(146, 1521)
##
                       mean
   6 ACT
           Canberra
                              2017 Q2 146. N(146, 1521)
##
                       mean
##
   7 ACT
           Canberra
                       mean
                              2017 Q3 146. N(146, 1521)
```

Compare against test set

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
aggregate_key(State/Region, Trips=sum(Trips)) %>%
model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
reconcile(coherent = min_trace(ets)) %>%
forecast(h = "2 years") %>%
accuracy(aus_holidays_agg, measures = list(CRPS = CRPS))
```

```
## # A tibble: 170 x 5
     .model State Region
                                        .type CRPS
##
##
     <chr> <chr> <chr> <chr>
                                        <chr> <dbl>
##
   1 ets ACT
                  Canberra
                                        Test 36.0
##
   2 ets
            ACT <aggregated>
                                        Test 36.0
                  Blue Mountains
                                        Test 15.2
##
   3 ets
            NSW
##
   4 ets
            NSW
                  Capital Country
                                       Test 15.0
                  Central Coast
                                        Test 11.5
##
   5 ets
            NSW
##
   6 ets
            NSW
                  Central NSW
                                        Test 15.9
```

```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
aggregate_key(State/Region, Trips=sum(Trips)) %>%
model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
reconcile(coherent = min_trace(ets)) %>%
forecast(h = "2 years") %>%
accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
spread(key=.model, value=CRPS)
```

```
## # A tibble: 85 x 5
##
     State Region
                                 .type ets
                                             mean
##
     <chr> <chr>
                                 <chr> <dbl> <dbl>
   1 ACT Canberra
                                 Test 36.0 34.4
##
                                 Test 36.0 34.4
##
   2 ACT <aggregated>
   3 NSW Blue Mountains
                                 Test 15.2 35.8
##
   4 NSW
           Capital Country
                                 Test 15.0 16.2
##
##
   5 NSW
           Central Coast
                                 Test 11.5 23.3
```

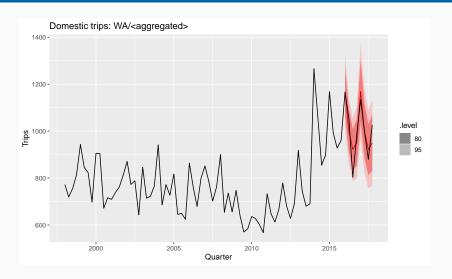
```
aus_holidays_agg %>%
  filter(Quarter <= yearquarter("2015 Q4")) %>%
  aggregate_key(State/Region, Trips=sum(Trips)) %>%
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
  reconcile(coherent = min_trace(ets)) %>%
  forecast(h = "2 years") %>%
  accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
  spread(key=.model, value=CRPS) %>%
  mutate(RelCRPS = ets/mean)
```

```
## # A tibble: 85 x 6
##
     State Region
                                 .type ets mean RelCRPS
     <chr> <chr>
                                 <chr> <dbl> <dbl>
                                                    <dbl>
##
   1 ACT
           Canberra
                                 Test 36.0 34.4
                                                    1.05
##
   2 ACT <aggregated>
                                Test 36.0 34.4
                                                    1.05
##
   3 NSW Blue Mountains
                                                   0.424
##
                                Test 15.2 35.8
                                Test 15.0 16.2
                                                   0.925
##
   4 NSW
           Capital Country
```

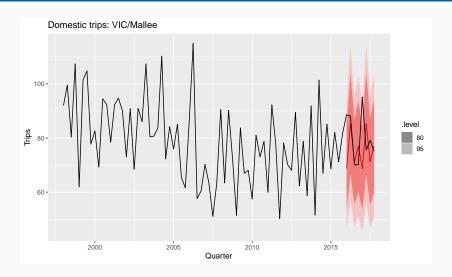
```
aus_holidays_agg %>%
filter(Quarter <= yearquarter("2015 Q4")) %>%
aggregate_key(State/Region, Trips=sum(Trips)) %>%
model(mean = MEAN(Trips), ets = ETS(Trips)) %>%
reconcile(coherent = min_trace(ets)) %>%
forecast(h = "2 years") %>%
accuracy(aus_holidays_agg, measures = list(CRPS = CRPS)) %>%
spread(key=.model, value=CRPS) %>%
mutate(RelCRPS = ets/mean) %>%
arrange(RelCRPS)
```

```
## # A tibble: 85 x 6
                                             ets mean RelCRPS
##
     State Region
                                      .type
     <chr> <chr>
                                      <chr> <dbl> <dbl>
                                                        <dbl>
##
                                     Test 34.8 177.
                                                        0.196
##
   1 WA <aggregated>
   2 NSW South Coast
                                     Test 26.2 101.
                                                        0.261
##
                                                        0.265^{27}
##
   3 WA Australia's South West
                                     Test 27.1 102.
```

Best forecast (RelCRPS)



Worst forecast (RelCRPS)

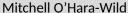


Challenges in multivariate probabilistic forecasting

- The fable package produces high-dim probabilistic forecasts?
- We can focus on marginal distributions for interpretation, while needing the full distribution for calculation.
- Forecast reconciliation provides a way to efficiently compute marginal distributions.
- Probability scoring allows us to measure distributional forecast accuracy.

Acknowledgements







Earo Wang

tidyverts.org robjhyndman.com