



# Forecasting is not prophecy

Dealing with  
high-dimensional  
probabilistic forecasts  
in practice

# What is it?



# What is it?

## Clay model of sheep's liver

Used by  
Babylonian  
forecasters  
approximately  
600 B.C.



Now in British Museum.

# Modern prophecy

## A Timeline of Very Bad Future Predictions

1800



“Rail travel at high speed is not possible, because passengers, unable to breathe, would die of asphyxia.”

Dr. Dionysys Larder, Professor of Natural Philosophy & Astronomy, University College London

1859



“Drill for oil? You mean drill into the ground to try and find oil? You're crazy!”

Associates of Edwin L. Drake refusing his suggestion to drill for oil in 1859 (Later that year, Drake succeeded in drilling the first oil well.)

1876



“This telephone has too many shortcomings to be seriously considered as a means of communication.”

Western Union internal memo

1880



“Everyone acquainted with the subject will recognize it as a conspicuous failure.”

Henry Morton, president of the Stevens Institute of Technology, on Edison's light bulb

1902



“Flight by machines heavier than air is impractical and insignificant, if not utterly impossible.”

Simon Newcomb, Canadian-American astronomer and mathematician, 18 months before the Wright Brothers' flight at Kittyhawk

1903



“The horse is here to stay, but the automobile is only a novelty, a fad.”

The president of the Michigan Savings Bank, advising Henry Ford's lawyer not to invest in the Ford Motor Company

1916



“The idea that cavalry will be replaced by these iron coaches is absurd. It is little short of treasonous.”

Comment of Aide-de-camp to Field Marshal Haig, at tank demonstration

1916



“The cinema is little more than a fad. It's canned drama. What audiences really want to see is flesh and blood on the stage.”

Charlie Chaplin, actor, producer, director, and studio founder

1921



“The wireless music box has no imaginable commercial value. Who would pay for a message sent to no one in particular?”

Associates of commercial radio and television pioneer, David Sarnoff, responding to his call for investment in the radio

1946



“Television won't last because people will soon get tired of staring at a plywood box every night.”

Darryl Zanuck, movie producer, 20th Century Fox

1977



“There is no reason for any individual to have a computer in his home.”

Ken Olson, president, chairman and founder of Digital Equipment Corporation

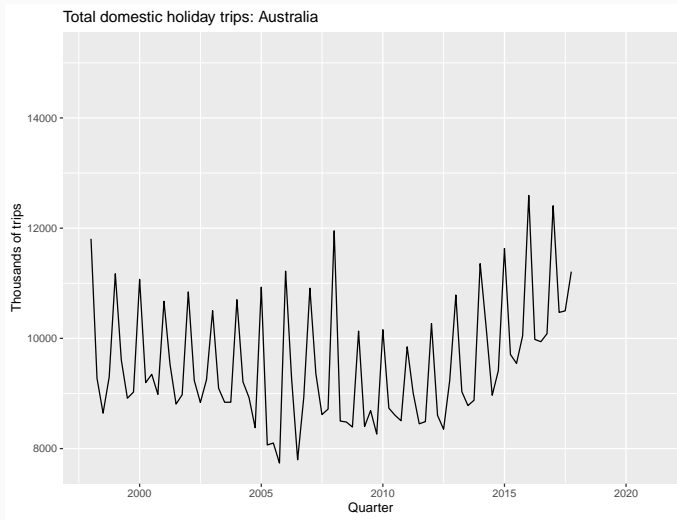
1995



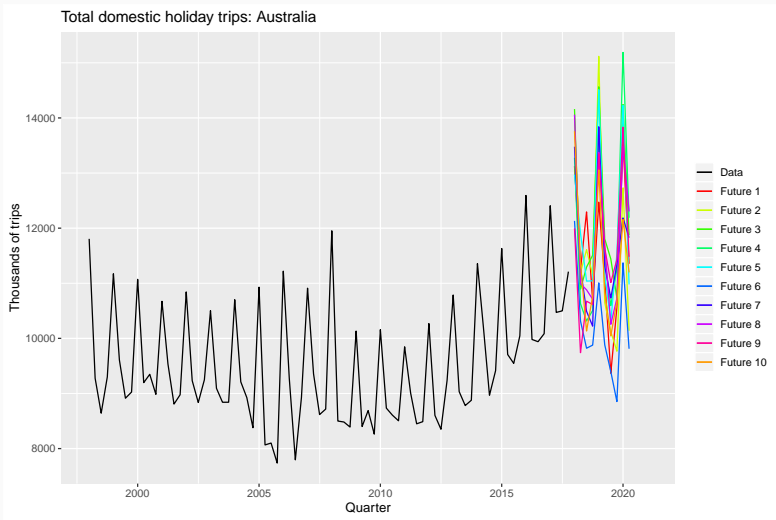
“The truth is no online database will replace your daily newspaper.”

Clifford Stoll, Newsweek article entitled *The Internet? Bah!*

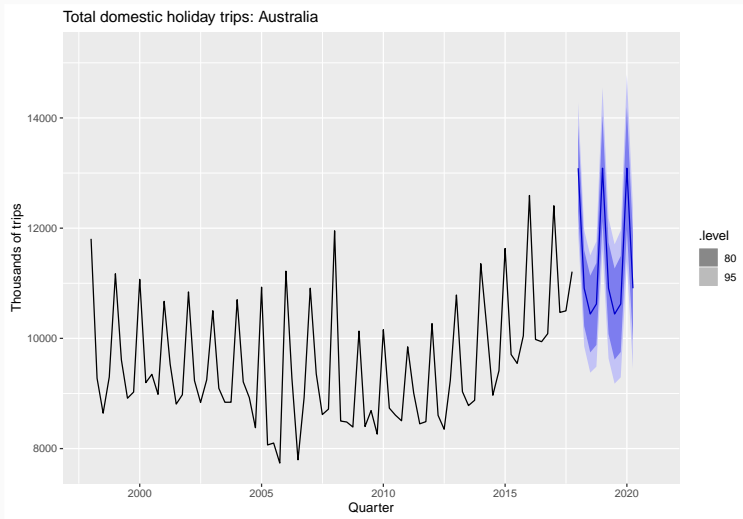
# Probabilistic forecasting



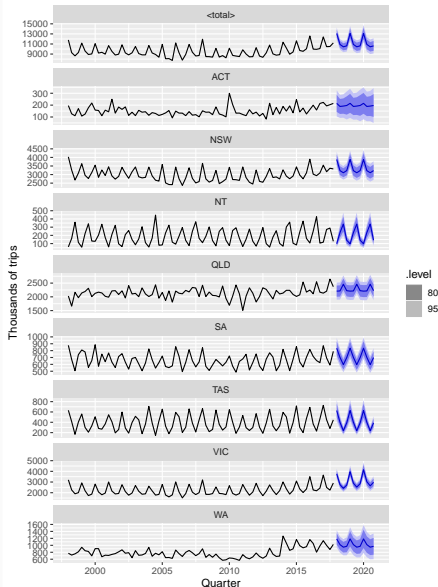
# Probabilistic forecasting



# Probabilistic forecasting

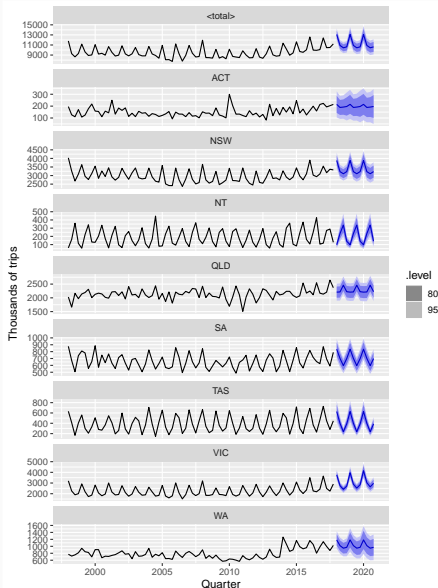


# Multivariate probabilistic forecasting





# Multivariate probabilistic forecasting



These show the marginal distributions of the 9-dimensional distribution. In practice, we also want forecast for smaller geographic areas as well.

# Who needs multivariate probabilistic forecasts?

- Tourism authorities forecasting visitor numbers to plan facilities.
- Manufacturing companies forecasting product demand to plan their supply chains.
- Call centres forecasting call volume to inform staff scheduling.
- Technology companies forecasting web traffic to maintain service levels.
- Energy companies forecasting electricity demand to prevent blackouts.

# Challenges in multivariate probabilistic forecasting

- 1 How to produce high-dim probabilistic forecasts?
- 2 How to use and interpret high-dim probabilistic forecasts?
- 3 How to measure accuracy of high-dim probabilistic forecasts?

# Challenges in multivariate probabilistic forecasting

1 How to produce high-dim probabilistic forecasts?

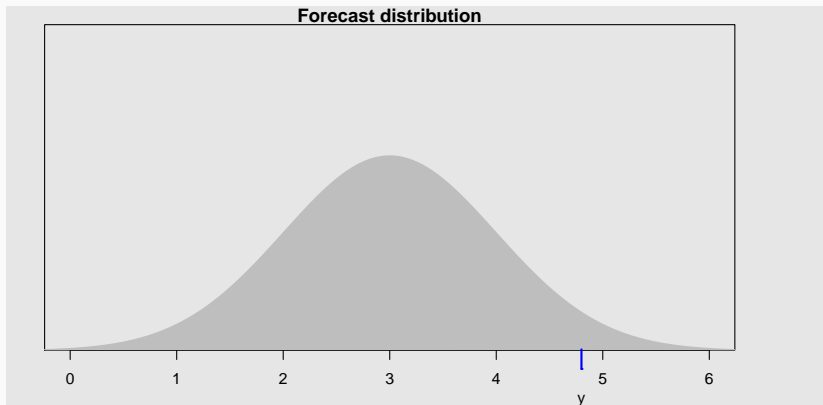
2 How to use and interpret high-dim probabilistic forecasts?

3 How to measure accuracy of high-dim probabilistic forecasts?

- Users care about marginal distributions, but we need multivariate distribution to compute them.
- Forecast reconciliation provides a way to efficiently compute marginal distributions.
- Probability scoring allows us to measure distributional forecast accuracy.

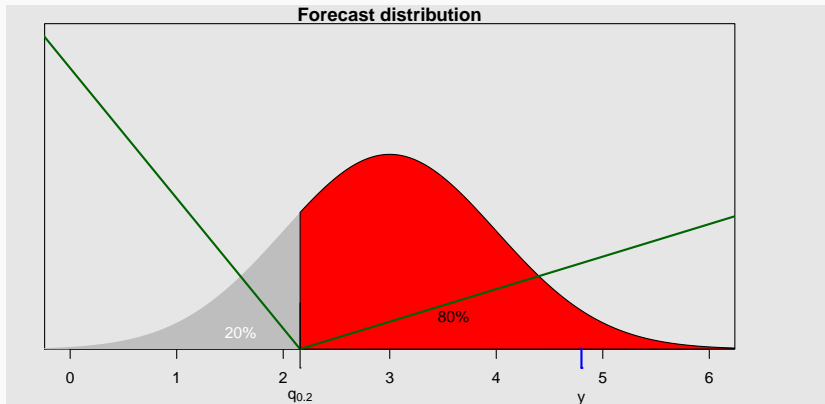
# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p)$  = quantile with probability  $p$ .



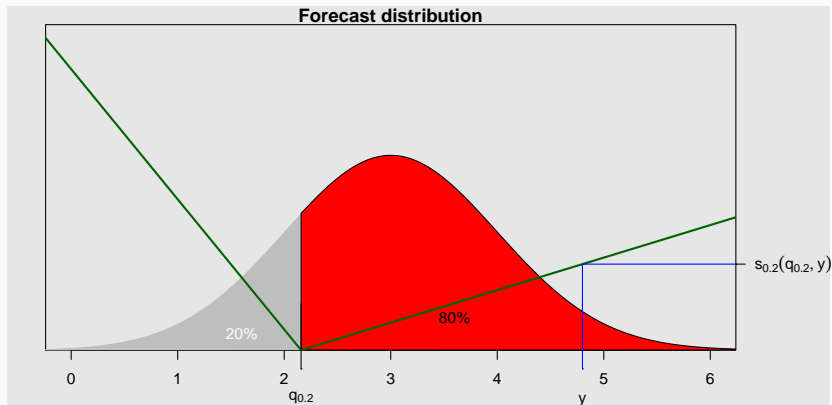
# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p) =$  quantile with probability  $p$ .



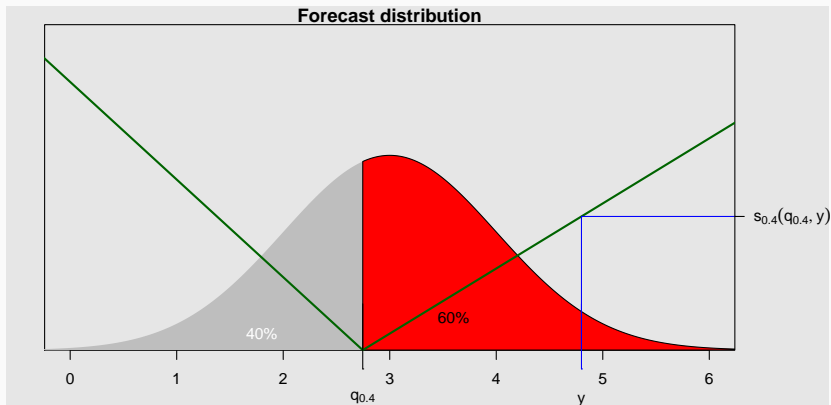
# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p) =$  quantile with probability  $p$ .



# Probability scoring

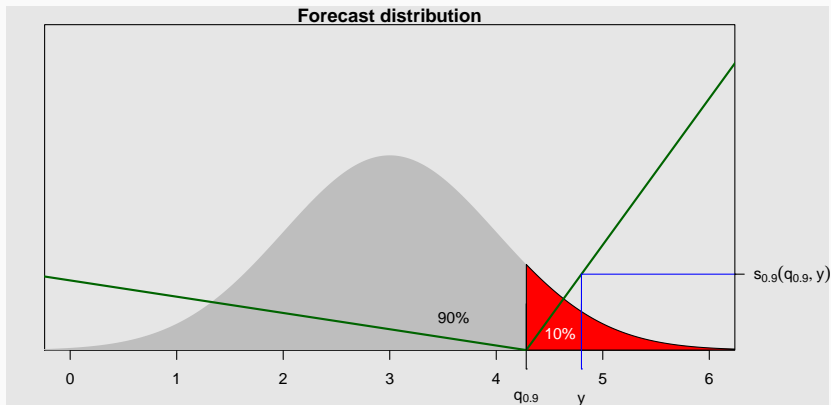
- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p) =$  quantile with probability  $p$ .





# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p) =$  quantile with probability  $p$ .



# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p)$  = quantile with probability  $p$ .

## Quantile Score (pinball loss)

$$s_p(q_p, y) = \begin{cases} (1 - p)(q_p - y), & \text{if } y < q_p; \\ p(y - q_p), & \text{if } y \geq q_p. \end{cases}$$

# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p)$  = quantile with probability  $p$ .

## Quantile Score (pinball loss)

$$s_p(q_p, y) = \begin{cases} (1 - p)(q_p - y), & \text{if } y < q_p; \\ p(y - q_p), & \text{if } y \geq q_p. \end{cases}$$

## Continuous Rank Probability Score

$$\text{CRPS}(F, y) = 2 \int_0^1 s_p(q_p, y) dp.$$

# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p)$  = quantile with probability  $p$ .

## Continuous ranked probability scores

$$\begin{aligned}\text{CRPS}(F, y) &= 2 \int_0^1 s_p(q_p, y) dp. \\ &= \int_{-\infty}^{\infty} (F(x) - 1\{y \leq x\})^2 dx\end{aligned}$$

# Probability scoring

- $y$  = actual value
- $F$  = forecast distribution
- $q_p = F^{-1}(p)$  = quantile with probability  $p$ .

## Continuous ranked probability scores

$$\begin{aligned}\text{CRPS}(F, y) &= 2 \int_0^1 s_p(q_p, y) dp. \\ &= \int_{-\infty}^{\infty} (F(x) - 1\{y \leq x\})^2 dx\end{aligned}$$

- CRPS in same units as observations
- Expected CRPS minimized when observation  $y$  from same distribution as  $F$ .
- Maximizes sharpness subject to calibration.



**tsibble**



**tsibbledata**



**feasts**



**Sable**

# tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:           Region, State [76]
##   Quarter Region   State Trips
##   <qtr> <chr>      <chr> <dbl>
## 1 1998 Q1 Adelaide SA      224.
## 2 1998 Q2 Adelaide SA      130.
## 3 1998 Q3 Adelaide SA      156.
## 4 1998 Q4 Adelaide SA      182.
## 5 1999 Q1 Adelaide SA      185.
## 6 1999 Q2 Adelaide SA      135.
## 7 1999 Q3 Adelaide SA      136.
## 8 1999 Q4 Adelaide SA      169.
## 9 2000 Q1 Adelaide SA      184.
## 10 2000 Q2 Adelaide SA      134.
```

# tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:           Region, State [76]
##   Index Region State Trips
##   <qtr> <chr>   <chr> <dbl>
## 1 1998 Q1 Adelaide SA      224.
## 2 1998 Q2 Adelaide SA      130.
## 3 1998 Q3 Adelaide SA      156.
## 4 1998 Q4 Adelaide SA      182.
## 5 1999 Q1 Adelaide SA      185.
## 6 1999 Q2 Adelaide SA      135.
## 7 1999 Q3 Adelaide SA      136.
## 8 1999 Q4 Adelaide SA      169.
## 9 2000 Q1 Adelaide SA      184.
## 10 2000 Q2 Adelaide SA      134.
```



# tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:           Region, State [76]
##   Index      Keys      Trips
##   <qtr> <chr>    <chr> <dbl>
## 1 1998 Q1 Adelaide SA      224.
## 2 1998 Q2 Adelaide SA      130.
## 3 1998 Q3 Adelaide SA      156.
## 4 1998 Q4 Adelaide SA      182.
## 5 1999 Q1 Adelaide SA      185.
## 6 1999 Q2 Adelaide SA      135.
## 7 1999 Q3 Adelaide SA      136.
## 8 1999 Q4 Adelaide SA      169.
## 9 2000 Q1 Adelaide SA      184.
## 10 2000 Q2 Adelaide SA      134.
```

# tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:           Region, State [76]
```

|    |  | Index | Keys  | Measure |       |
|----|--|-------|-------|---------|-------|
| ## |  | <qtr> | <chr> | <chr>   | <dbl> |

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 1 | 1998 | Q1 | Adelaide SA | 224. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 2 | 1998 | Q2 | Adelaide SA | 130. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 3 | 1998 | Q3 | Adelaide SA | 156. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 4 | 1998 | Q4 | Adelaide SA | 182. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 5 | 1999 | Q1 | Adelaide SA | 185. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 6 | 1999 | Q2 | Adelaide SA | 135. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 7 | 1999 | Q3 | Adelaide SA | 136. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 8 | 1999 | Q4 | Adelaide SA | 169. |
|----|---|------|----|-------------|------|

|    |   |      |    |             |      |
|----|---|------|----|-------------|------|
| ## | 9 | 2000 | Q1 | Adelaide SA | 184. |
|----|---|------|----|-------------|------|

|    |    |      |    |             |      |
|----|----|------|----|-------------|------|
| ## | 10 | 2000 | Q2 | Adelaide SA | 134. |
|----|----|------|----|-------------|------|

# tsibble objects

```
library(tsibble)
library(tsibbledata)
aus_holidays
```

```
## # A tsibble: 6,080 x 4 [1Q]
## # Key:           Region, State [76]
##   Index      Keys      Measure
##   <qtr> <chr>    <chr> <dbl>
## 1 1998 Q1 Adelaide SA    224.
## 2 1998 Q2 Adelaide SA    130.
## 3 1998 Q3 Adelaide SA    156.
## 4 1998 Q4 Adelaide SA    182.
## 5 1999 Q1 Adelaide SA    185.
## 6 1999 Q2 Adelaide SA    135.
## 7 1999 Q3 Adelaide SA    136.
## 8 1999 Q4 Adelaide SA    169.
## 9 2000 Q1 Adelaide SA    184.
## 10 2000 Q2 Adelaide SA    134.
```

Domestic overnight  
holiday trips in  
thousands by  
state/region.

# High-dimensional time series data

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) # 2 year test set
```

```
## # A tsibble: 5,472 x 4 [1Q]  
## # Key:           Region, State [76]  
##   Quarter Region   State Trips  
##   <qtr> <chr>    <chr> <dbl>  
## 1 1998 Q1 Adelaide SA      224.  
## 2 1998 Q2 Adelaide SA      130.  
## 3 1998 Q3 Adelaide SA      156.  
## 4 1998 Q4 Adelaide SA      182.  
## 5 1999 Q1 Adelaide SA      185.  
## 6 1999 Q2 Adelaide SA      135.  
## 7 1999 Q3 Adelaide SA      136.  
## 8 1999 Q4 Adelaide SA      169.  
## 9 2000 Q1 Adelaide SA      184.  
## 10 2000 Q2 Adelaide SA      134.  
## # with 5,462 more rows
```

# Compute all aggregates

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips))
```

```
## # A tsibble: 6,120 x 4 [1Q]  
## # Key:      State, Region [85]  
##   State   Region Quarter Trips  
##   <chr>   <chr>    <qtr>  <dbl>  
## 1 <total> <total> 1998 Q1 11806.  
## 2 <total> <total> 1998 Q2  9276.  
## 3 <total> <total> 1998 Q3  8642.  
## 4 <total> <total> 1998 Q4  9300.  
## 5 <total> <total> 1999 Q1 11172.  
## 6 <total> <total> 1999 Q2  9608.  
## 7 <total> <total> 1999 Q3  8914.  
## 8 <total> <total> 1999 Q4  9026.  
## 9 <total> <total> 2000 Q1 11071.  
## 10 <total> <total> 2000 Q2  8186.
```

# Fit univariate models

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips))
```

```
## # A mable: 85 x 4  
## # Key:      State, Region [85]  
##   State Region          mean    ets  
##   <chr> <chr>          <model> <model>  
## 1 ACT   Canberra      <MEAN> <ETS(M,N,A)>  
## 2 ACT   <total>         <MEAN> <ETS(M,N,A)>  
## 3 NSW   Blue Mountains <MEAN> <ETS(M,N,M)>  
## 4 NSW   Capital Country <MEAN> <ETS(A,N,N)>  
## 5 NSW   Central Coast  <MEAN> <ETS(M,N,M)>  
## 6 NSW   Central NSW    <MEAN> <ETS(M,N,A)>  
## 7 NSW   Hunter         <MEAN> <ETS(A,N,A)>  
## 8 NSW   New England North West <MEAN> <ETS(M,N,N)>  
## 9 NSW   North Coast NSW <MEAN> <ETS(A,N,A)>
```

# Reconcile results

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(ets = min_trace(ets))
```

```
## # A mable: 85 x 4  
## # Key:      State, Region [85]  
##   State Region          mean    ets  
##   <chr> <chr>          <model> <model>  
## 1 ACT   Canberra      <MEAN> <ETS(M,N,A)>  
## 2 ACT   <total>         <MEAN> <ETS(M,N,A)>  
## 3 NSW   Blue Mountains <MEAN> <ETS(M,N,M)>  
## 4 NSW   Capital Country <MEAN> <ETS(A,N,N)>  
## 5 NSW   Central Coast   <MEAN> <ETS(M,N,M)>  
## 6 NSW   Central NSW     <MEAN> <ETS(M,N,A)>  
## 7 NSW   Hunter         <MEAN> <ETS(A,N,A)>  
## 8 NSW   New England North West <MEAN> <ETS(M,N,N)>
```

# Produce probabilistic forecasts

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(ets = min_trace(ets)) %>%  
  forecast(h = "2 years")
```

```
## # A fable: 1,360 x 6 [1Q]  
## # Key:      State, Region, .model [170]  
##   State Region   .model Quarter Trips .distribution  
##   <chr> <chr>    <chr>    <qtr> <dbl> <dist>  
## 1 ACT   Canberra mean     2016 Q1  146. N(146, 1521)  
## 2 ACT   Canberra mean     2016 Q2  146. N(146, 1521)  
## 3 ACT   Canberra mean     2016 Q3  146. N(146, 1521)  
## 4 ACT   Canberra mean     2016 Q4  146. N(146, 1521)  
## 5 ACT   Canberra mean     2017 Q1  146. N(146, 1521)  
## 6 ACT   Canberra mean     2017 Q2  146. N(146, 1521)  
## 7 ACT   Canberra mean     2017 Q3  146. N(146, 1521)
```



# Compare against test set

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(coherent = min_trace(ets)) %>%  
  forecast(h = "2 years") %>%  
  accuracy(aus_holidays)
```

```
## # A tibble: 170 x 4  
##   .model Region          State CRPS  
##   <chr>   <chr>          <chr> <dbl>  
## 1 ets    Canberra          ACT    34.6  
## 2 ets    <total>            ACT    34.6  
## 3 ets    Blue Mountains     NSW    15.1  
## 4 ets    Capital Country     NSW    15.0  
## 5 ets    Central Coast       NSW    11.5  
## 6 ets    Central NSW         NSW    15.8  
## 7 ets    Hunter              NSW    17.9
```

# Probability scoring

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(coherent = min_trace(ets)) %>%  
  forecast(h = "2 years") %>%  
  accuracy(aus_holidays) %>%  
  spread(key=.model, value=CRPS)
```

```
## # A tibble: 85 x 4
```

| ## | Region                       | State | ets   | mean  |
|----|------------------------------|-------|-------|-------|
| ## | <chr>                        | <chr> | <dbl> | <dbl> |
| ## | 1 Adelaide                   | SA    | 24.1  | 24.4  |
| ## | 2 Adelaide Hills             | SA    | 2.90  | 4.18  |
| ## | 3 Alice Springs              | NT    | 6.08  | 10.4  |
| ## | 4 Australia's Coral Coast    | WA    | 14.9  | 35.0  |
| ## | 5 Australia's Golden Outback | WA    | 9.93  | 17.2  |
| ## | 6 Australia's North West     | WA    | 7.99  | 18.0  |

# Probability scoring

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(coherent = min_trace(ets)) %>%  
  forecast(h = "2 years") %>%  
  accuracy(aus_holidays) %>%  
  spread(key=.model, value=CRPS) %>%  
  mutate(RelCRPS = coherent/mean)
```

```
## # A tibble: 85 x 5
```

| ## | Region                       | State | ets   | mean  | RelCRPS |
|----|------------------------------|-------|-------|-------|---------|
| ## | <chr>                        | <chr> | <dbl> | <dbl> | <dbl>   |
| ## | 1 Adelaide                   | SA    | 24.1  | 24.4  | 0.988   |
| ## | 2 Adelaide Hills             | SA    | 2.90  | 4.18  | 0.694   |
| ## | 3 Alice Springs              | NT    | 6.08  | 10.4  | 0.586   |
| ## | 4 Australia's Coral Coast    | WA    | 14.9  | 35.0  | 0.427   |
| ## | 5 Australia's Golden Outback | WA    | 9.93  | 17.2  | 0.579   |

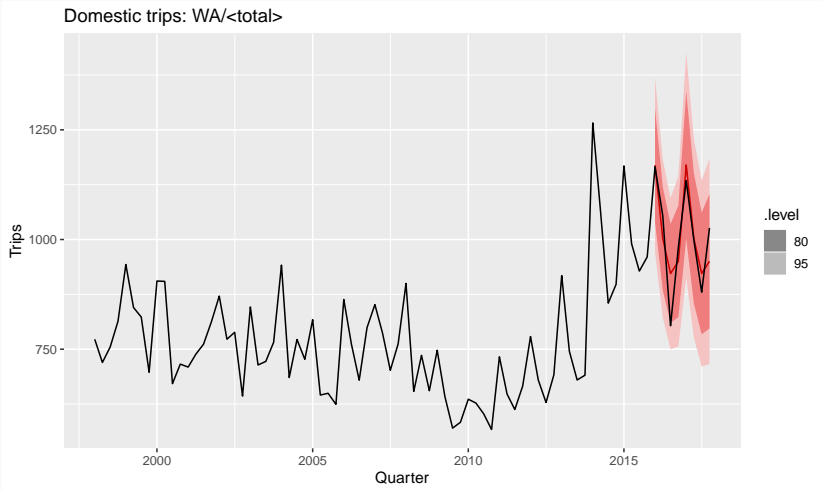
# Probability scoring

```
aus_holidays %>%  
  filter(Quarter <= yearquarter("2015 Q4")) %>%  
  aggregate_key(State/Region, Trips=sum(Trips)) %>%  
  model(mean = MEAN(Trips), ets = ETS(Trips)) %>%  
  reconcile(coherent = min_trace(ets)) %>%  
  forecast(h = "2 years") %>%  
  accuracy(aus_holidays) %>%  
  spread(key=.model, value=CRPS) %>%  
  mutate(RelCRPS = coherent/mean) %>%  
  arrange(RelCRPS)
```

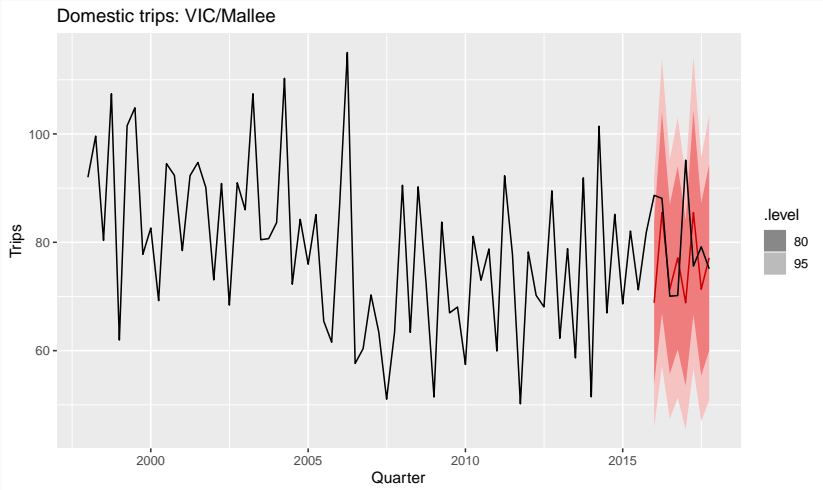
```
## # A tibble: 85 x 5
```

| ## | Region                   | State | ets   | mean  | RelCRPS |
|----|--------------------------|-------|-------|-------|---------|
| ## | <chr>                    | <chr> | <dbl> | <dbl> | <dbl>   |
| ## | 1 <total>                | WA    | 37.6  | 177.  | 0.212   |
| ## | 2 South Coast            | NSW   | 26.1  | 101.  | 0.259   |
| ## | 3 Australia's South West | WA    | 27.5  | 102.  | 0.268   |
| ## | 4 Katherine Daly         | NT    | 3.59  | 11.8  | 0.304   |

# Best forecast (RelCRPS)



# Worst forecast (RelCRPS)



# Challenges in multivariate probabilistic forecasting

- 1 The fable package produces high-dim probabilistic forecasts?
- 2 We can focus on marginal distributions for interpretation, while needing the full distribution for calculation.
- 3 Forecast reconciliation provides a way to efficiently compute marginal distributions.
- 4 Probability scoring allows us to measure distributional forecast accuracy.

# Acknowledgements



Mitchell O'Hara-Wild



Earo Wang

**[tidyverts.org](https://tidyverts.org)**  
**[robjhyndman.com](https://robjhyndman.com)**