Forecasting the Population Age Structure and the Old-Aged Dependency Ratio in Australia: What is the Most Appropriate Pension Age Scheme?

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Abstract

The Australian government has been adjusting the pension age in recent years as a response to the ageing issue. This paper contributes to the discussion of this issue in three ways: (a) it provides a new method for forecasting the age structure of the population; (b) it provides forecasts of the old-aged-dependency ratio for different pension schemes; and (c) it provides a tool for finding an appropriate pension age scheme which is economically viable for Australia.

My approach involves developing a stochastic population forecasting method based on functional data models for mortality, fertility and net migration, and then simulating future age-structures of the population. I then use these simulations to forecast the old-aged dependency ratio, and to estimate a pension age scheme that will provide a stable old aged dependency ratio at a specified level.

The stochastic population forecasting method used is new, and involves a modification of the approach proposed by Hyndman and Booth (2008) adapted to constrain the mortality and migration components to be non-divergent. I also propose a new method for obtaining a confidence interval for the estimated pension age scheme. I apply these ideas to Australian historical demographic data and the results suggest that the pension age should be increased to 67 by 2021, 70 by 2032 and 75 by 2058 in order to maintain the old aged dependency ratio at the current level, which is 21%. This approach can easily be extended to other target levels of the old-aged dependency ratio and to different countries.

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1 Introduction

Over the last decade, the ageing population of Australia has become a critical demographic issue, significantly affecting both residents' retirement welfare and the government's political decisions. An ageing population is a common experience among developed countries, and is unavoidable, being a result of significant technological advances in medical care, coupled with a large decline in the fertility rate (Costello, 2004). The Australian Federal Government and other institutions have projected the population over the next 40 years using a variety of methods, with results suggesting that Australia is facing a demographic challenge which requires immediate actions. Australian intergenerational reports in 2002 (Australian Government, 2002) and in 2007 (Australia Government, 2007) highlighted that the age-related policies at that time will dramatically raise the burden of the future working generation.

To date, the Australian government has addressed the problem through a variety of approaches. These include constantly modifying superannuation schemes and instigating work incentives for aged people, and another more controversial action: increasing pension age[1]. Two big adjustments to the pension age have been announced recently by government in response to the ageing population issue. In 2009, the previous Labor government announced an increase in pension age from 65 to 67 by 2023 (Nielson, 2013). In May 2014, Federal Treasurer Joe Hockey proposed to further increase pension age to 70 by 2035 in order to maintain the current welfare system. This new proposal is in line with the recommendations from both the Productivity Commission (2013) and the National Commission of Audit(2014), both of which support increasing the pension age to avoid the potential financial dilemma faced by retirees and the government's future budget.

This proposal has brought much attentions to the ageing issue and the government's response. The extent to which the pension age should be increased is a controversial topic, and is now one of the main concerns of the Australian people. Heated debates about the issue indicate that we need a detailed investigation into the appropriateness of previous modifications and the recent proposal for a pension age increase. Most of the debated points are given from an economic standpoint. However, investigation into the economic impact on the Australian society is complex but replies crucially on proper forecasting of the age structure of the Australian population which is no trivial matter. The research presented in this paper contributes to the discussion with a primary objective

[1]The *pension age* is defined as the age at which eligible residents can access the age pension for the first time. A more detailed definition of age pension and corresponding terminologies will be explained in section 2.1

of forecasting the age structure of population, quantifying the effects of changing the pension age on financial burdens and hence finding an appropriate pension age scheme which is economically viable for Australia. The analysis of economic impact of increasing pension age can be very complicated. In this paper, we will focus on one major aspect: the impact on the burden of future working age generations.

In this paper, we use the old aged dependency ratio (OADR[2]) as a measure of the financial burden on the Australian workforce because it has been emphasized in literature that rise in old aged dependency ratio will increase strain on the ability of working age population to finance pensions and health costs (Apps, Rees & Wood, 2007; Australian Government, 2002; Australian Government, 2007). The OADR is defined as the ratio of the number of people aged over the pension age to the number of people of working age (Apps, Rees & Wood, 2007). This can be expressed using the formula below:

$$OADR = \frac{\text{number of people aged over pension age}}{\text{number of people aged 15 to pension age}} * 100\% \tag{1}$$

The research presented here forecasts future population age structures over the next 50 years. We then use those projections to compute the OADR associated with pension age both before and after the adjustments. One major contribution of the current study is the combined use of the Hyndman-Ullah forecasting method (Hyndman & Ullah, 2007) and the product-ratio method (Hyndman, Booth & Yasmeen, 2013) to forecast a population age structure. Forecasts of population structure have been built on forecasts of age and gender-specific mortality rates, fertility rates and net migration using this method. These forecasts presuppose that the historical trends of past years will continue. Historical data of those rates have thus been used as input to simulate future paths of age and gender-specific population structure.

Using these predicted population paths, we can calculate the average of OADR and predict the levels of old age dependency that correspond to different pension age schemes. By testing different pension age schemes, we aim to find the pension age with the desired OADR outcome. Therefore, other main contributions of this research are (1) to examine the effect of adjusting pension ages by comparing OADR before and after adjustment; and (2) to suggest a suitable pension age scheme to ensure that the economic burden will remain relatively constant at a desired level in the future. It should be noted that the desired level has been set at 21%, which is the estimated current level of OADR in 2014. This level was chosen because, while the current financial burden on tax payers is affordable, it is approaching the limits of affordability. However, whether or not the

[2]All abbreviations have been summarized in Appendix A.

current level is the most appropriate level to Australia remains to be examined and it is not the focus of this paper. This paper concentrates on providing a framework for policy makers to find a target pension age for a given desired level of OADR.

Some may argue that using OADR as a measure of financial burden for the next generation could be biased, with a large number of retirees able to rely on self-financed retirement income, such as superannuation. However, 80% of Australians over pension age fully or partly rely on the age pension (Power, 2014). Therefore, the proportion of people over working age is still a good representation of financial burden on future working generations. Nevertheless, it should be aware that using OADR as a single measurement of financial burden might overlook some important economic factors. Some have suggested to use adjusted OADR that considers those important factors in analyzing pension finance (Hu & Yang, 2012). This paper emphasizes how to undertake a better forecasting of economic burden for a given policy rather than the better choice of measurements. Therefore, the simple version of OADR was chosen to illustrate my approach (i.e. without adjustment). There is a ground to extend the approach to incorporate more complex measurement which takes into account of other important economic factors.

Furthermore, choosing the right model to forecast population age structure was another important step involved in the quantification process. I chose to combine the Hyndman-Ullah method and product-ratio method for the forecast. The combination of those two methods overcomes some of shortcomings of other popular demographic forecasting methods such as the Lee-Carter method (Lee & Carter, 1992) or the deterministic methods used by the Australian Bureau of Statistics (ABS). More details on forecasting methods are given in later sections.

The empirical analysis presented in this paper shows the adjustment of the pension age announced in 2009 was effective in reducing the financial burden to the desired level, which was set at current OADR-21%. However, it has been argued that a one-time adjustment is not sufficient to ensure a stable OADR in the long term. Therefore, this paper recommends consistently reviewing the pension age according to the presented statistical approach. This approach has been used to calculate the target pension age from now until 2060 and its 80% confidence interval. The target pension age found in this research increases to 67 by 2023, 70 by 2032 and 75 by 2058. This generates a stable OADR of around 21%. The rate of growth in the pension age as proposed by the government in 2014 is slightly lower than the target pension age found in this study, which suggests that the government may need to slightly accelerate the growth of the pension age to maintain an OADR at the current level.

The remainder of this paper proceeds as follows. Section 2 reviews key literature on population forecasting and background works on the pension age. Section 3 describes the data and methods used in this study. Section 4 discusses the empirical results of this research, and outlines accuracy tests performed on the empirical results. Finally, Section 6 gives concluding remarks as well as identifies the scope for future research.

2 Background and Literature Review

2.1 Background on Australian Age Pension

Age pension in Australia is an income support and provides access to a range of concessions for older Australians who have passed means testing. For example, it provides single home-owners with a real income of \$13653 p.a and additional allowances to cover expenses such as rent or pharmaceuticals[3]. The means test ensures that retirees with enough personal savings will become ineligible for public provision by reducing the pension at fixed levels of pay as personal wealth increases. The age pension in Australia is very generous, even with the means test. For instance, a couple holding assets of more than one million dollars is still eligible for a partial age pension.

Currently, the age pension is the primary source of income for 70% -80% of Australian residents over 64 years (ABS, 2006). Even though the government has promoted self-financed retirement by introducing the Superannuation Guarantee in 1992, the Superannuation Guarantee accumulations are still modest, with the current average less than \$100,000 at retirement (ASFA, 2007). It has been estimated by Petrichev and Thorp (2008) that a retiree will need to have \$450,000 at 65 in order to support a pension-equivalent income and this is more than eight times of the average retirement accumulation of Australians. The required wealth to comply with personal-funded retirement savings requirements is too high for most Australians, suggesting that retirees will continue to rely on the age pension in the coming decades.

The ageing trend in Australia, along with retirees' dependence on the age pension, has dramatically increased the fiscal cost of the age pension. For instance, the fiscal cost of the age pension is currently around at 2.5% of Gross Domestic Profit (GDP), but is expected to increase to 4.4% by the 2050s (Australian Government, 2007). The Productivity Commission (2013) also indicates the government will need to raise taxes by

[3]The pension amount is adjusted every 6 months for inflation

21% to pay for extra aged care and health costs, which is caused by longer life expectancies in newer generations.

The issue is not only a problem for the government, but also for future retirees. Given children born today are expected to live up to their 90s on average, the Productivity Commission (2013) suggests that Australians won't be able to afford to spend 35 years in retirement (i.e. retire around 65). They have predicted that a third of baby boomers are expected to spend all their money and assets before they die[4]. This problem might eventually deteriorate the sustainability of the age pension, although the welfare system works well at the moment, because demographic changes often have delayed effects (Australian Government, 2004).

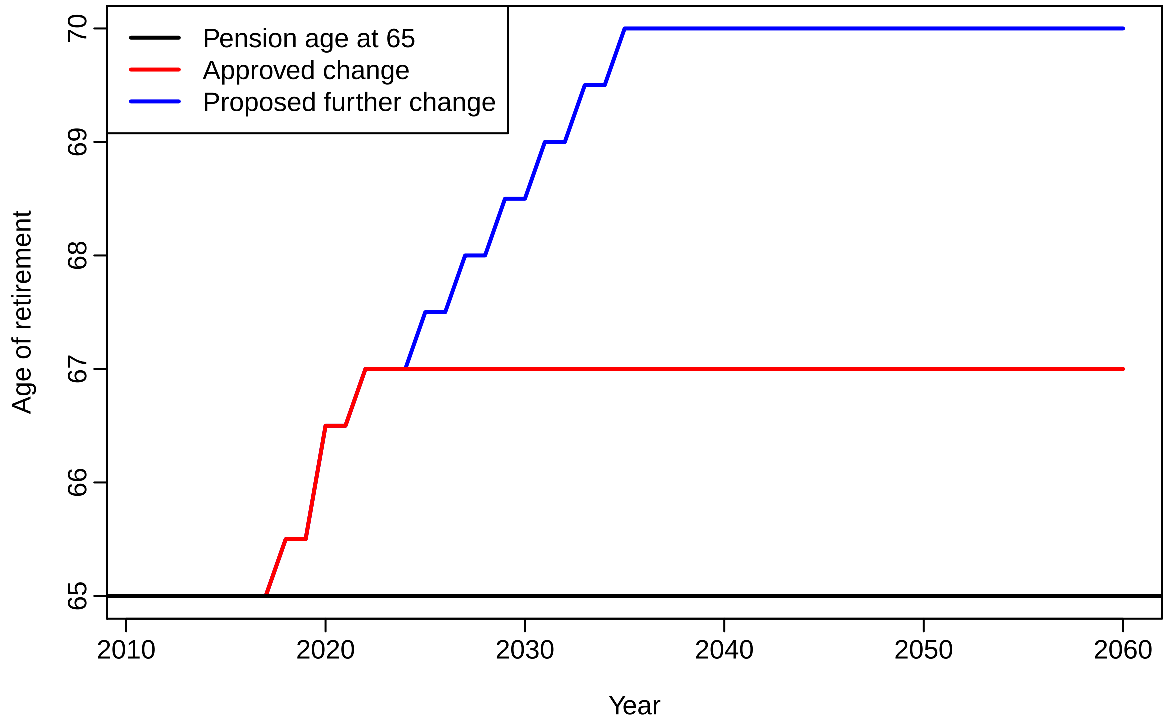

Figure 1: *Government's adjustments on pension age scheme.*

The Australian government is trying to take early intervention in relation to the pension age to balance between providing social support and fiscal cost. The pension age for males has been set at 65 for over a century. While the pension age for females was 60 years until 1994, it has steadily increased to 65 years, in line with that of males. The previous Labor government announced in 2009 that the pension age will increase six months every two years until 2017, capping at 67 in July 2023. This change is expected to have a significant impact on labor force participation and to decrease the retirement

[4]Baby Boomers are people born during the Post-World War II baby boom between the years 1946 and 1964

duration of residents because pension age is regarded as a normal age for retirement in Australia. However, the Productivity Commission (PC)(2013) proposed further lifting of the pension age to 70 on the grounds that increasing the pension age to 70 would save taxpayers $150 billion in welfare costs and health spending, and elderly people are capable of continuing work. The National Commission of Audit (NCOA)(2014) holds the same opinion, and recommended lifting the pension age to 70 from 2053. The current Treasurer Joe Hockey announced that the government would increase the pension age to 70 from 2035, which is proposed in the Federal Budget Statement for 2014-2015 to be confirmed by Australian Federal Government. Figure 1 illustrates the process of the adjustments on pension age scheme. This announcement received overwhelmingly negative responses in the media. Guest (2014) argues from an economic prospective that Australia currently has much better circumstances than other OECD countries in terms of age pension pressure, because spending on the age pension in Australia is less than half of the OECD average. He also claims that Australia has a higher consumption of all other goods and services, and so why are they not allowed to have more leisure time. Further, Power (2014) contends that many Australians are not physically able to work full-time to 70, so that recommendations made by the PC and NCOA are simplistic solutions to a more a complex social problem.

In fact, all supportive arguments for increasing the pension age in Australia have only justified the need to increase the pension age. No justification on the pace or degree of the change has been provided. Also, the government has not disclosed the method it used to determine the new pension age. It is therefore not surprising that the adjustment of the pension age has been questioned by the public. A consistent method to determine the appropriate pension age would be more persuasive in general. For example, it is preferred by OECD countries that pension age should be linked to life expectancy (OECD, 2012). There is a practical issue as to how to link life expectancy with pension ages given that there is no information on future mortality rates (Productivity Commission, 2013). Dr Knox proposed directly linking pension age with life expectancy in report of Committee for Economic Development of Australia (2007, p7) in the way that "pension age should increase by approximately 50% of any increase in life expectancy". However, using a rule of thumb approach might not be ideal, as 50% of the change in life expectancy may not be a good adjustment to pension age in the long term. My study suggests an approach, linking the pension age with the the aged dependency ratio, based on forecasts of future mortality and fertility rates. My approach has the advantage that it can be used to justify the change in pension age and directly links the pension age with our ultimate goal of a stable financial burden for the public.

2.2 Mathematical Preliminaries

2.2.1 Principal Component Analysis

Principal component analysis (PCA) is the main estimation method used in population forecasting. It is essential for understanding the models introduced in the following subsection. Therefore, a brief introduction of PCA is given here.

Principal components analysis reorients multivariate data in a way that ensures that the first few dimensions (called components) summarize as much of the available information as possible (Lattin, Carroll & Green, 2003). More precisely, it uses an orthogonal transformation to convert a set of possibly correlated variables into a set of linearly uncorrelated variables called *principal components*. It is possible to reduce the dimensions (new uncorrelated variables) if substantial redundancy is present in the data set. The rearrangement of data allows researchers to decide "how many principal components to retain for subsequent analysis, trading off simplicity (i.e., a small number of dimensions is easier to manage) against completeness (i.e., a large number of dimensions captures a greater amount of available information) (Lattin, Carroll & Green, 2003, p83).

The mathematical derivation of PCA is explained in detail by Lattin, Carroll and Green (2003). I have summarized the main ideas here. In PCA, the objective is to find a linear combination of original data with maximum variance, because the aim is to derive the component (which is the new variable after linear transformation) that summarizes as much information as possible. Suppose we have an $m \times n$ matrix $\boldsymbol{X}$ that contains m observations of n variables (dimensions) with a rank of k. If we denote linear combinations by the matrix $\boldsymbol{U} = [\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, ..., \boldsymbol{u_k}]$ where $\boldsymbol{u_i} = (u_{1i}, u_{2i}, u_{3i}, ..., u_{ni})'$ and denote the matrix of principal component scores by $\boldsymbol{Z} = [\boldsymbol{z_1}, \boldsymbol{z_2}, \boldsymbol{z_3}, ..., \boldsymbol{z_k}]$ where $\boldsymbol{z_i} = (z_{1i}, z_{2i}, z_{3i}, ..., z_{mi})'$, then our goal is to choose $\boldsymbol{u_i}$ to maximize the variance of the elements of $\boldsymbol{z_i} = \boldsymbol{X}\boldsymbol{u_i}$, which can be written as:

$$var(\boldsymbol{z_i}) = \frac{1}{m-1}\boldsymbol{u}_i'\boldsymbol{X}'\boldsymbol{X}\boldsymbol{u_i}. \tag{2}$$

This maximization will ensure that first principal component $\boldsymbol{z_1}$ captures the most information (variance) in the original data and every subsequent principal component captures the most information in the remaining variance. However, this maximization does not have a unique solution because the magnitude of $\boldsymbol{u}$ is arbitrary, thereby making $var(\boldsymbol{z_i})$ infinity. To achieve the maximization, we impose a constraint of unit magnitude on the vector $\boldsymbol{u_i}$ $\forall i$. Thus, the completed maximization problem is as follows:

$$\text{choose } \boldsymbol{u} \text{ to maximize } \frac{1}{m-1}\boldsymbol{u}'\boldsymbol{X}'\boldsymbol{X}\boldsymbol{u} \tag{3}$$

$$\text{subject to the constraint } \boldsymbol{u}'\boldsymbol{u} = 1.$$

We can solve this constrained optimization problem using the Lagrangian multiplier method (see Lattin, Carroll and Green (2003) for a detailed mathematical proof). These authors demonstrate that the solved vectors $\boldsymbol{z_i}$ and $\boldsymbol{u_i}$ are eigenvectors of the matrices $\boldsymbol{X}\boldsymbol{X}'$ and $\boldsymbol{X}'\boldsymbol{X}$ respectively, and the corresponding Lagrangian multiplier λ_i is the eigenvalue of $\boldsymbol{X}\boldsymbol{X}'$. The numbers $\lambda_1, \lambda_2, ...\lambda_k$ are singular values of $\boldsymbol{X}$ and vectors $\boldsymbol{z_1}, \boldsymbol{z_2}, ..., \boldsymbol{z_k}$ and $\boldsymbol{u_1}, \boldsymbol{u_2}, ..., \boldsymbol{u_k}$ are right and left singular vectors. Also, it can be shown that λ_i is equal to $var(\boldsymbol{z_i})$, which implies that variance-covariance matrix of $\boldsymbol{Z}$ (denoted by Σ) is a diagonal matrix with $\Sigma_{ii} = \lambda_i$. Therefore, the vector $\boldsymbol{z}$ associated with largest λ is the first principal component, which capture the most variance in original data.

The relationship $\boldsymbol{z_i} = \boldsymbol{X}\boldsymbol{u_i}$ can be written in matrix form as follows:

$$\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{U} \tag{4}$$

With a little algebra, we can rearrange the above equation to express $\boldsymbol{X}$ as a function of $\boldsymbol{Z}$. We post multiplying $\boldsymbol{U}'$ of both sides (note that we have $\boldsymbol{U}\boldsymbol{U}' = \boldsymbol{I}$ because of the mutual orthogonality of eigenvectors) and the new equation is as follows:

$$\boldsymbol{X} = \boldsymbol{Z}\boldsymbol{U}' \tag{5}$$

Further, we can express $\boldsymbol{X}$ as a function of standardized matrix of principal components (denoted $\boldsymbol{Z_s}$) by decomposing $\boldsymbol{Z}$ into $\boldsymbol{Z^s}\Sigma^{1/2}$. Then, the equation becomes:

$$\boldsymbol{X} = \boldsymbol{Z^s}\Sigma^{1/2}\boldsymbol{U}' \tag{6}$$

Alternatively, it can be expressed as:

$$\boldsymbol{X} = \boldsymbol{z_1^s}\sqrt{\lambda_1}\boldsymbol{u_1'} + \boldsymbol{z_2^s}\sqrt{\lambda_2}\boldsymbol{u_2'} + ... + \boldsymbol{z_k^s}\sqrt{\lambda_k}\boldsymbol{u_k'} \tag{7}$$

This way of expressing $\boldsymbol{X}$ is known as a singular value decomposition (SVD). It indicates that any data matrix $\boldsymbol{X}$ can be decomposed into three simpler matrices. $\boldsymbol{Z^s}$ is a standardized matrix of uncorrelated variables (each with unit variance), $\Sigma^{1/2}$ is a diagonal matrix which perform a stretching transformation of $\boldsymbol{Z^s}$ and $\boldsymbol{U'}$ performs an orthogonal rotation. Therefore, we can use singular value decomposition to undertake a PCA.

2.2.2 ARIMA and ARFIMA Model

ARIMA and ARIFMA are essential mathematical models that is required to understand to perform the population forecasting method. I give a brief introduction of them based on the work of Hyndman and Athanasopoulos (2013).

AR(p) model Autoregression models forecast the dependent variable using a linear combination of its past values. An autoregression model the order of p (referred as AR(p) model) can be written as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2}, ..., +\phi_p y_{p-1} + e_t \tag{8}$$

where c is a constant and e_t is white noise.

MA(q) model A moving average model supposes that the variable of interest can be explained by past forecast errors. It can be written as:

$$y_t = c + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + ... + \theta_q e_{t-q} \tag{9}$$

where e_t is white noise.

Non-seasonal ARIMA models A non-seasonal ARIMA model is a combination of differencing, autoregression and moveing average models. ARIMA is an acronym for AutoRegressive Integrated Moving Average model. An ARIMA model can be written as:

$$y'_t = c + \phi_1 y'_{t-1} + ... + \phi_p y'_{t-p} + \theta_1 e_{t-1} + ... + \theta_q e_{t-q} + e_t \tag{10}$$

where y'_t is the differenced series, which is differenced d times. This regression is referred to as an ARIMA(p, d, q) model, where p is the order of the auotoregression part; d is the degree of first differencing and q is the order of the moving average part.

In fact, the AR(p) and MA(q) models are ARIMA (p,0,0) and ARIMA (0,0,q) models respectively. ARIMA models can be written in backshift notation (i.e.$By_t = y_{t-1}$)

as follows:

$$(1 - \phi_1 B - ... - \phi_p B^p)(1 - B)^d y_t = c + (1 + \theta_1 B + ... + \theta_q B^q)e_t \tag{11}$$

ARFIMA model We can allow d in the ARIMA model to be fractional in order to capture the long-memory property of some series. The ARIMA model with fractional differencing (i.e. $-\frac{1}{2} < d < \frac{1}{2}$) is called the ARFIMA model. The range $0 < d < \frac{1}{2}$ gives long-memory process (Cowpertwait & Metcalfe, 2009).

Akaike's Information Criterion AIC is a model-selection procedure which is useful to determine the order of an ARIMA model. It can be written as:

$$AIC = -2log(L) + 2(p + q + k + 1) \tag{12}$$

where L is the likelihood of the data, $k = 1$ if $c \neq 0$ and $k = 0$ if $c = 0$.

For ARIMA models, the corrected AIC (Hyndman & Athanasopoulos, 2013) can be written as

$$AIC_c = \frac{AIC + 2(p + q + k + 1)(p + q + k + 2)}{T - p - q - k - 2} \tag{13}$$

where the correction is to reduce small sample bias. The ARIMA model with minimum AIC or minimum AIC_c is the optimal model. I prefer to use AIC_c to reduce sample bias in this research.

2.3 Literature Review on Population forecasting methods

In population forecasting, two very different approaches: a deterministic approach (sometimes called a quasi-statistical approach) and a stochastic approach (a purely statistical approach) are frequently used. For instance, the Australian Bureau of Statistics is using a deterministic approach for its demographic projection, while some other countries or institutes forecast demographic trends based on purely statistical approaches such as Statistics New Zealand and the United States Census Bureau. In the deterministic approach, experts project future demographic changes using judgement-based scenarios. It ignores the non-demographic factors such as major pandemics or wars, although historically they are important factors that affect demographic behaviour (Productivity Commission, 2013). In stochastic approaches, the Lee-Carter (LC) method (Lee & Carter, 1992) is the most prominent demographic forecasting method, which now has several variants and extensions that relax some of the assumptions of the basic Lee-Carter method. Although statistical forecasting models are rigorous, they still leave a basis for judgment-based adaptation, as they sometimes produce unrealistic projections of demographic factors such as long-run projections of mortality rates (Production Commission, 2013).

Therefore, some official agencies decide to mix the two approaches in their forecasts in order to provide a supplement to both approaches. For instance, the most recent forecast of population in Australia by the Productivity Commission (2013) was based on basic Lee-Carter methods plus some judgement-based adaptations to address the characteristics of demographic trends that the Lee-Carter method failed to capture. The approach of forecasting used in this study is very similar to the Productivity Commission(2013). The Hyndman-Ullah method (2007) is used as a basic forecasting approach, which is a comprehensive extension of Lee-Carter method. In addition, an adapted product-ratio method is used to eliminate the potential long-term divergence between female and male mortality rates. A more detailed review of forecasting methods is given below.

2.3.1 Deterministic Methods Used by the ABS

The deterministic method is judgment-based. The Australian Bureau of Statistics formulates assumptions on the basis of demographic trends, in conjunction with advice from individual experts or government departments. The projections are not predictions or forecasts, "but are simply illustrations of the growth and change in population which would occur if certain assumptions about future levels of fertility, mortality, internal migration and overseas migration were to prevail over the projection period" (ABS, 2012, p.2). In addition, they do not attempt to take into account of non-demographic factors, such as major government policy changes and catastrophes. Therefore, they normally report multiple (usually three) series based on different levels of assumptions of demographic factors to allow for variety of population projections.

2.3.2 Lee-Carter Method

The Lee-Carter method (Lee & Carter, 1992) is a significant milestone in demographic forecasting. It has been widely used to forecast mortality rates across various countries, including Australia.

The model proposed by Lee and Carter (1992) is as follows:

$$\log(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t} \tag{14}$$

where $m_{x,t}$ is the central death rate[5] for age x in year t. This model involves fitting a matrix of log central rates with sets of age-specific constants, a_x and b_x, and a time-varying index k_t. b_x is a set of parameters that tell us the rate at which each age responds to changes in k_t. k_t measures the general level of the log death rates; a_x is the general pattern across age of the log mortality rate and $\epsilon_{x,t}$ is an error term with zero mean and constant variance, reflecting the randomness that can't be captured by the model.

The model is under-determined in terms of estimation of parameters. For example, if the vector a, b, k is one solution, then for any scalar $c, a - bc, b, k + bc$ also must be one solution. Therefore, Lee and Carter (1992) impose the following constraints:

$$\sum_{t=1}^{n} k_t = 0, \sum_{x=x_1}^{x_m} b_x = 1 \tag{15}$$

Also, we normally seek the least squares solution when estimating parameters. However, in this case, the model cannot be estimated using ordinary regression methods because there is no regressor on the right side of the equation. Lee and Carter (1992) choose a singular value decomposition (SVD) method to undertake PCA, and hence find the least squares solution. It has been shown that applying SVD methods to a matrix of the logarithms of the rates after subtracting the average over time of the log age-specific rates can find the least square solution (Good, 1969). Therefore, the estimations of b_x and k_t are the first principal component and scores of $\log(m_{x,t}) - a_x$ (i.e. $b_x = \boldsymbol{z_1}$ and $k_t = \boldsymbol{u_1'}$ when $\boldsymbol{X} = \log(m_{x,t}) - a_x$). However, the empirical results suggest that the resulting fitted number of deaths are biased estimates of actual number of deaths because of the log transformation. To overcome this problem, the LC method adjusts k_t such that for each year, the implied number of deaths will be equal to the actual number of deaths. The adjusted k_t is then extrapolated using random walk with drift models, which is equivalent to ARIMA $(0, 1, 0)$ model with $c \neq 0$.

2.3.3 Hyndman-Ullah Method

Hyndman & Ullah (2007) proposed a nonparametric method for demographic modelling and forecasting. It extended the LC model in four ways: First, the rates are smoothed using penalized regression splines (Booth, Hyndman & Tickle, 2014) before modeling. Second, it uses functional principal components analysis, which is a continuous version of PCA. Further, it uses more than one principal component, which address the main

[5] The central death rate is properly defined in Section.3

weakness of the LC method. Finally, the forecasting models for the principal component scores are more complex than the Random walk with drift model used in LC model.

The model proposed by Hyndman and Ullah (2007) is as follows:

$$y_t(x) = s_t(x) + \sigma_t(x)\epsilon_{t,x} \tag{16}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{t,j}\phi_j(x) + e_t(x) \tag{17}$$

The first equation describes the smoothing process. $y_t(x)$ is the observed data in year t and age x, and $s_t(x)$ is the smoothed function of $y_t(x)$, estimated by constrained weighted penalized regression splines (Booth, Hyndman & Tickle, 2014). $\sigma_t(x)$ allows the variance to be a function of time and age; and $\epsilon_{t,x}$ are independent identical distributed standard normal error terms. The second equation has a similar structure to the LC method, except that it incorporates more than one principal component and score. It describes the dynamics of $s_t(x)$ over time. The $\beta_{t,j}$ for every j and $\mu(x)$ are sets of age-specific constants, and $\phi_j(x)$ for every j is a time-varying index. The $e_t(x)$ is the model error, which is the residual function with mean zero and no serial correlation. The $\mu(x)$ is estimated by the average of $s_t(x)$ across years, which measures the general shape of the observed data $y_t(x)$. The estimates of $\phi_j(x)$ and $\beta_{t,j}$ are the jth principal component and scores of $s_t(x) - \mu(x)$ respectively. In other words, we take singular value decomposition of $s_t(x) - \mu(x)$, then equation (7) now becomes

$$s_t(x) - \mu(x) = \phi_1(x)\beta_{t,1} + \phi_2(x)\beta_{t,2} + ... + \phi_k(x)\beta_{t,k} \tag{18}$$

We chose J in a way that we believe the majority of information for $s_t(x) - \mu(x)$ has been captured by the first J components.

In terms of forecasting, by conditioning on the observed data $\boldsymbol{I} = \{y_1(x), ..., y_n(x)\}$ and the set of functional principal components $\boldsymbol{B} = \{\phi_1(x), \phi_2(x), ..., \phi_j(x)\}$, the h-step-ahead forecast of log rate $y_{n+h}(x)$ can be acquired by:

$$\hat{y}_{n+h|n}(x) = \hat{s}_{n+h|n}(x) = E[\hat{s}_{n+h|n}(x) \mid \boldsymbol{I}, \boldsymbol{B}] = \hat{\mu}(x) + \sum_{j=1}^{J} \phi_j(x)\hat{\beta}_{n+h|n,j} \tag{19}$$

where $\hat{\beta}_{n+h|n,j}$ denotes the h-step-ahead forecast of $\beta_{n+h|n,j}$. Because of the time-varying property of $\{\beta_{t,j}\}$, it controls the dynamics of the process. For every component series $\{\beta_{t,j}\}$, we fit them with an $ARIMA(p_j, d_j, q_j)$ process, where p_j, d_j, q_j are determined by minimizing the Akaike Information Criterion (Booth, Hyndman & Tickle, 2014). The fitted process as follows:

$$(1-\phi_1 B - ... - \phi_{p_j} B^{p_j})(1-B)^{d_j}\beta_{t,j} = c + (1+\theta_1 B + ... + \theta_{q_j} B^{q_j})e_t \ \forall j = 1, 2.., J \quad (20)$$

where the parameters are estimated using maximum likelihood estimation.

3 Data and Methodology

3.1 Data

We use the same notation and similar procedures to construct data as Hyndman and Booth (2008), because the fundamental model for forecasting population for this study is the Hyndman and Ullah method. The data used to construct population age structure included: age and gender-specific birth and death numbers of each calendar year, age and gender-specific population members on 1 January of each year, and age and gender-specific exposures to risk (i.e. population of age x at 30 June) for each year. The notations are as follows:

$$\begin{aligned}
B_t(x) &= \text{Births in calendar year } t \text{ to females of age } x \\
D_t(x) &= \text{Deaths in calendar year } t \text{ of persons of age } x \\
P_t(x) &= \text{Population of age } x \text{ at 1 January of year } t \\
E_t(x) &= \text{Population of age } x \text{ exposed to risk at 30 June of year } t
\end{aligned}$$

where x denotes age and $x = 0, 1, 2, ..., m^+$. m^+ is the open-ended upper age group, which is 100 in this study. t denotes year and $t = 1, 2, ..., n$, where n is the horizon of historical data used. Therefore, all of those data points can be recorded in an $m \times n$ matrix. The super-scripts M and F denote male and female respectively.

We can obtain historical mortality rates $m_t(x)$ (also called central death rates) and fertility rates $f_t(x)$ as follows:

$$\begin{aligned}
m_t(x) &= \frac{D_t(x)}{E_t(x)} = \text{age-sex-specific central death rates in calendar year t} \\
f_t(x) &= B_t(x)/E_t^F(x) = \text{age-specific fertility rates in calendar year t}
\end{aligned}$$

In addition, the net migration[6] (denoted G) is estimated using a demographic growth-balance equation, which is an equation to summarize the relationship between population change and three factors: net migration, births and deaths. The equation is:

$$\begin{aligned} G_t(x, x+1) &= P_{t+1}(x+1) - P_t(x) + D_t(x, x+1) \text{ for } x = 0, 1, 2, ..., p-2, \\ G_t(p-1^+, p^+) &= P_{t+1}(p^+) - P_t(p^+) - P_t(p-1) + D_t(p-1^+, p^+) \\ G_t(B, 0) &= P_{t+1}(0) - B_t + D_t(B, 0) \end{aligned}$$

where $G_t(x, x+1)$ refers to net migration in calendar year t of persons aged x at beginning of year t, $G_t(p-1^+, p)$ refers to migration in calendar year t of persons aged $p-1$ and older at beginning of year t, and $G_t(B, 0)$ refers to deaths in calendar year t of births during year t; and similarly for deaths, $D_t(x, x+1), D_t(p-1^+, p^+), D_t(B, 0)$. The deaths are estimated using the standard life table approach of population projection. It should be noted that the estimated net migration includes errors in data recording.

Historical central death rates and start-year and mid-year populations of residents in Australia by sex and age in single years for age group 0-99 and 100+ have been sourced from the Human Mortality Database (2013) for 1921-2009. The start-year population for 2010 is also used to estimate net migration. Data for age-specific annual fertility rates by single years of age for 15-49 over the period 1950-2009 was obtained from the Australian Bureau of Statistics (2012). Even though annual mortality rates are available from 1921 to 2009, only data collected after 1950 is used for two reasons. First, the period between 1921-1950 has very different mortality patterns from later years because of wars and epidemics (Hyndman & Booth, 2008). Since the model for this study is valid under the assumption that extrapolative methods can be used, it seemed best to delete the less relevant data, which might influence estimations of parameters if we include them. Secondly, fertility rates are only available from 1950, so the mortality rate was cut to fit the same period.

3.2 Methodology

3.2.1 Forecasting Population Age Structure and OADR

In this study, three steps were taken to simulate OADR associated with a specific pension age scheme. A stochastic functional model was applied to five components of population

[6] Net migration is the difference between immigration and emigration in a certain area during a specified time frame.

change: female mortality rates, male mortality rates, female net migration, male net migration and fertility rates to simulate these five components over the prediction horizon (which is 50 years in this study). Age and gender-specific population paths over the prediction horizon were simulated using the demographic growth-balance equation. The Monte Carlo simulation method was used consistently in this research. Finally, calculation of OADR associated with a specific pension age scheme was performed by applying the OADR formula. Even though the process seems complicated, it is relatively resy to implement using *R*. The detailed explanation of each step is as follows. The corresponding R code is attached in Appendix B.

Step 1 The main procedure for forecasting the population age structure follows Hyndman and Booth(2008), except that the data is updated, and we incorporate the product-ratio method (Hyndman, Booth & Yasmeen, 2013) to improve the accuracy of the forecast. The Hyndman and Ullah (2007) model, introduced in section 2.3.2, is applied to the mortality rate, fertility rate and net migration separately, based on the assumption that they behave independently. Therefore, to fit the model to historical fertility rates, the $y_t(x)$ in equation (16) denotes the observed historical fertility data. However, for mortality rate by sex, the product-ratio method is used to ensure the forecast of females and males are non-divergent or coherent (Hyndman, Booth & Yasmeen, 2013). In the product-ratio method, after smoothing the female and male mortality rates, two other series were created: product series (denoted by $p_t(x)$) and ratio series (denoted by $r_t(x)$), which are defined as follows:

$$p_t(x) = \sqrt{s_{t,M}(x)s_{t,F}(x)}, r_t(x) = \sqrt{\frac{s_{t,M}(x)}{s_{t,F}(x)}}$$

Then, Hyndman and Ullah (2007) was applied to these two new variables instead of male and female mortality rate as follows:

$$log[p_t(x)] = \mu_p(x) + \sum_{j=1}^{J} \beta_{t,j}\phi_j(x) + e_t(x)$$
$$log[r_t(x)] = \mu_r(x) + \sum_{l=1}^{L} \gamma_{t,l}\psi_l(x) + z_t(x)$$

where time series $\{\beta_{t,j}\}$ are fitted with an ARIMA model while $\{\gamma_{t,l}\}$ are fitted by stationary ARFIMA model (i.e. d in ARIMA(p, d, q) constrained to $0 < d < \frac{1}{2}$). It is the stationary constraint on $\gamma_{t,l}$ that ensures the forecast of male and female do not diverge. Then, simulation h-step-ahead forecast values of product variable and ratio variable was performed (denoted by $p_{n+h|n}(x)$ and $r_{n+h|n}(x)$ respectively) according to equation (18) and (19). From here, obtaining h-step-ahead forecast of sex-specific coherent mortality rates (denoted $m_{n+h|n,M}(x)$ and $m_{n+h|n,F}(x)$) was performed using a back-transformation:

$$m_{n+h|n,M}(x) = p_{n+h|n}(x)r_{n+h|n}(x)$$
$$m_{n+h|n,F}(x) = p_{n+h|n}(x)/r_{n+h|n}(x)$$

The method for simulating h-step-ahead values of net migration by sex is similar to the above method used on mortality rates except no log scale is applied to net migration. A stationary constraint was imposed on the ARIMA model for net migration to make sure the difference between sexes did not diverge.

Step 2 After simulating paths for age-specific fertility rate, age and gender-specific mortality rate and age and gender-specific net migration, future population value were simulated using the demographic growth-balance equations over the prediction horizon. The future deaths and births in this equation are assumed to follow Poisson distribution, with parameters as a function of future mortality and fertility rates (see Hyndman and Booth, 2008 for the details of the functions). Hence, we can simulate h-step-ahead age and gender-specific births and deaths based on simulated mortality and fertility rates. According to the demographic growth-balance equation, one-step-ahead net migrations, deaths, births and population in the last year were simulated, allowing for the generation of one-step-ahead population predictions by sex and age. This simulation was repeated for years $t = n+1, ..., n+h$ to obtain a path of population. The process was repeated and simulated i paths of population based on Monte Carlo simulation method were created (we set $i = 1000$ in these research). It should be noted that there are three types of randomness in the simulation: randomness from the second equation in the Hyndman-Ullah specification (i.e. $e_t(x)$ in equation (17)), randomness from the ARIMA model fitted to principal component scores and randomness from the Poisson distribution. It should be noted that the simulated population will be an array with 3 dimensions: age, year and simulation.

Step 3 Even though the OADR is a simple function of the population age structure and pension age, the formula can't be directly applied to the simulated population array because the pension age changes over time. To overcome this problem, we assume that the pension age can only be adjusted at the beginning of a year for these calculations. Therefore, one pension age is applicable for a whole year and hence OADR is generated at the beginning of each year to show the trend. I will show the process of obtaining a one-step-ahead OADR (i.e. forecast of OADR in year 2011), given a specific pension age. The same method can be applied to all years over the prediction horizon, given their corresponding pension age. Further, the fractional pension age causes practical issues in calculating OADR, since ages are recorded as integers. I will also show how this issue was handled in the example below.

Suppose the pension age for $n+1$ is $a.r$ where a is the integer and and r is the reminder of pension age. Recall that for $h=1$, we can express the simulated population paths as a matrix with age in the row and simulation in the column as follows:

$$\boldsymbol{p}_{n+1} = [p_{mi}] = \begin{bmatrix} p_{1,1} & \cdots\cdots & p_{1,1000} \\ \vdots & \ddots & \vdots \\ p_{100^+,1} & \cdots\cdots & p_{100^+,1000} \end{bmatrix} \tag{21}$$

When $r=0$, then the simulation of a population aged 15 to pension age is a subset of the above matrix from 15^{th} row to $a-1^{th}$ row, which is:

$$\begin{bmatrix} p_{15,1} & \cdots\cdots & p_{15,1000} \\ \vdots & \ddots & \vdots \\ p_{a-1,1} & \cdots\cdots & p_{a-1,1000} \end{bmatrix} \tag{22}$$

To calculate the total number of working age, we take a column sum of the trimmed matrix (22). The simulations of total number of work age become a vector:

$$\boldsymbol{p}_{n+1}^{work} = [\sum_{m=15}^{a-1} p_{m,1}, \sum_{m=15}^{a-1} p_{m,2}, ..., \sum_{m=15}^{a-1} p_{m,1000}] \tag{23}$$

Using the same method, the simulations of total number of aged pensioners in the population can be shown as:

$$\boldsymbol{p}_{n+1}^{aged} = [\sum_{m=a}^{100^+} p_{m,1}, \sum_{m=a}^{100^+} p_{m,2}, ..., \sum_{m=a}^{100^+} p_{m,1000}] \tag{24}$$

When $r \neq 0$, the vector of total number of work age (23) is added, and the vecor of total number of aged population is subtracted with an adjustment:

$$\text{adjustment} = 0.r \times 100\% [p_{a,1}, p_{a,2}, ..., p_{a,1000}] \tag{25}$$

This adjustment is based on the assumption that the birthday of population is uniformly distributed over the year. Then, $r \times 100\%$ of population with a recorded integer age of a

at beginning of the year is actually younger than $a.r$, so they are still under the pension age. In other words,$(1 - r \times 100\%)$ of population with recorded integer age a at beginning of the year is actually older than $a.r$, so they should be counted as the aged population. The simulation vectors of total work age population and total aged population should then be:

$$\begin{aligned}
\boldsymbol{p}_{n+1}^{work} &= [p_{n+1,1}^{work}, p_{n+1,2}^{work}, ..., p_{n+1,1000}^{work}] \\
&= [\sum_{m=15}^{a-1} p_{m,1}, \sum_{m=15}^{a-1} p_{m,2}, ..., \sum_{m=15}^{a-1} p_{m,1000}] + 0.r \times 100\%[p_{a,1}, p_{a,2}, ..., p_{a,1000}] \\
\boldsymbol{p}_{n+1}^{aged} &= [p_{n+1,1}^{aged}, p_{n+1,2}^{aged}, ..., p_{n+1,1000}^{aged}] \\
&= [\sum_{m=a}^{100^+} p_{m,1}, \sum_{m=a}^{100^+} p_{m,2}, ..., \sum_{m=a}^{100^+} p_{m,1000}] - 0.r \times 100\%[p_{a,1}, p_{a,2}, ..., p_{a,1000}]
\end{aligned}$$

After the adjustment, we can generate simulation vectors of the one-step-ahead OADR by dividing every element of $\boldsymbol{p}_{n+1}^{work}$ by the corresponding element of $\boldsymbol{p}_{n+1}^{aged}$, i.e.:

$$\begin{aligned}
\boldsymbol{OADR}_{n+1} &= [OADR_{n+1,1}, OADR_{n+1,2}, ..., OADR_{n+1,1000}] \\
&= \left[\frac{p_{n+1,1}^{work}}{p_{n+1,1}^{aged}}, ..., \frac{p_{n+1,1000}^{work}}{p_{n+1,1000}^{aged}}\right]
\end{aligned}$$

The average value of elements of $\boldsymbol{OADR}_{n+1}$ is the mean prediction of one-step-ahead OADR (i.e mean forecast of OADR in 2011).

The same algorithm is applied to the simulation matrix of the population age structure to obtain h-step-ahead simulation vectors of OADR for $h = 1, 2, ..., H$, where $H = 50$ for this study (i.e. a prediction horizon of 2011 to 2060). Simulations of OADR over a 50 year prediction horizon by the matrix is summarised below, with the forecast year in rows and simulation in the columns:

$$\boldsymbol{OADR}^{forecast} = [OADR_{h,i}] = \begin{bmatrix} OADR_{n+1,1} & \cdots\cdots & OADR_{n+50,1000} \\ \vdots & \ddots & \vdots \\ OADR_{n+50,1} & \cdots\cdots & OADR_{n+50,1000} \end{bmatrix} \tag{26}$$

3.2.2 Finding the Target Pension Age Scheme

The goal of this research is to keep the pension age low, as long as OADR is lower than or equal to the desired OADR level. According to the formula of OADR, OADR at a given time can be written as a function of pension age, when the population age structure is given. Suppose we denote pension age schemes over the prediction horizon by $\boldsymbol{PA} = [PA_{n+1}, PA_{n+2}, ..., PA_{n+H}]$. The OADR function of pension age over the prediction horizon is denoted as $OADR_h(PA_{n+h})$ for $h = 1, 2, 3, ..., H$. There is a negative relationship between pension age and OADR. The negative relationship implies that the higher the pension age, the lower the OADR. Therefore, our goal becomes finding the minimum pension age which has an OADR lower than desired OADR level. Several practical issues need to be considered to achieve this goal. First, only simulated forecasting values of future OADR are available, so it is assumed that the mean value of the simulated OADR is a good estimate of true underlying OADR. Second, pension age cannot be increased by more than one within a year or decreased because either one of the two actions will force some retirees back to the workforce, which is unreasonable in practical applications. Therefore, the target pension age scheme $\boldsymbol{PA}^{target}$ can be defined in the following way: for $h = 1, 2, ..., H$

$$\text{minimise } PA_{n+h} \text{ such that } OADR_h(PA_{n+h}) \leq OADR^{desired} \tag{27}$$

$$\text{subject to the constraint } 0 \leq PA_{n+h} - PA_{n+h-1} \leq 1$$

In order to achieve this maximization subject to constraint, these calculations assume that the adjustment unit of pension age is one month for practical convenience. It is also supposed that the set starting value of pension age is at 65 for all years [7], which is:

$$\boldsymbol{PA} = [PA_{n+1}, PA_{n+2}, ..., PA_{n+H}] = [65, 65, ..., 65]$$

We take the following algorithm: For $h = 1, 2, ..., H$, we keep increasing pension age vector $[PA_{n+h}, PA_{n+h+1}, ..., PA_{n+H}]$ by a month at a time, unless we find that

$$OADR_h^{mean}(PA_{n+h}) > OADR^{desired} \text{ or } PA_{n+h} - PA_{n+h-1} \geq 1 \tag{28}$$

where $OADR_h^{mean}(PA_{n+h})$ is the mean value of h-step-ahead forecast of OADR with a pension age at the time $n + h$ equal to PA_{n+h}.

In addition to finding the target pension age based on the mean value of OADR, the aim is to find plausible pension age schemes that could give the desired OADR level. To be more specific, "plausible", means finding a range of pension age schemes that could possibly lead to the desired OADR, contained by 80% prediction intervals of simulated

[7] Even though pension age for males and females is 65 and 64.5 respectively. The same pension age for males and females was set to simplify the calculations. This simplification is reasonable because the pension age will be same in 2017 onward.

OADR. Because of the monotonic relationship between pension age and OADR, we only need to find the upper (lower) boundary of plausible pension age schemes, at which the upper (lower) limit of 80% prediction intervals of OADR is equal to the desired OADR level. Hence, any pension age scheme within those two boundaries is plausible. The range of plausible pension age schemes can be thought of as 80% confidence interval of pension age scheme where we are 80% sure that there will be a desired outcome. In other words, any pension age scheme outside the range is undesirable to use, because we are 80% certain that the corresponding OADR will deviate from the desired level of OADR.

The algorithm to find the upper and lower boundary of plausible pension age schemes is similar to the algorithm to find the target pension age scheme, except that the mean value of OADR in equation (28) is replaced by the boundary of the 80% prediction interval. Mean is replaced by the upper (lower) limit of the 80% prediction interval to find upper (lower) boundary for pension age schemes.

4 Empirical Results

4.1 Forecast in Population Age Structure and OADR

I use the fitted model of fertility rates as an example to interpret the results of fitted models. It should be noted that models fitted to product and ratio series are less likely to be interpreted in an economic sense. Figure 2 above shows the mean age pattern ($\mu(x)$), the first two basis functions ($\phi_1(x)$ and $\phi_2(x)$) and first two coefficients (i.e. principal component scores $\beta_{t,1}$ and $\beta_{t,2}$) for fertility rates. The first term ($\phi_1(x) * \beta_{t,1}$) accounts for the most variation in fertility, so a relatively sensible interpretation can be given to the first term. In the plot of coefficient 1, estimated historical first principal component scores $\beta_{t,1}$ are shown in black for 1950-2009. It captures the general trend of fertility change over time. It indicates a sharp increase between 1950-1960, followed by a rapid decline of fertility rates from 1961. This increase of fertility is linked to the Post-World War II baby boom[8] in Australia, resulting from the recovery from war and the Great Depression[9]. The dramatic decrease from 1960 was due to the availability of contraception pills in Australia. A noticeable further reduction of fertility rates in late 1960s may be due to a change in the law allowing abortion in some states in Australia.

[8]between the years 1946 and 1964

[9]The Great Depression was a severe economic depression in the decade before World War II

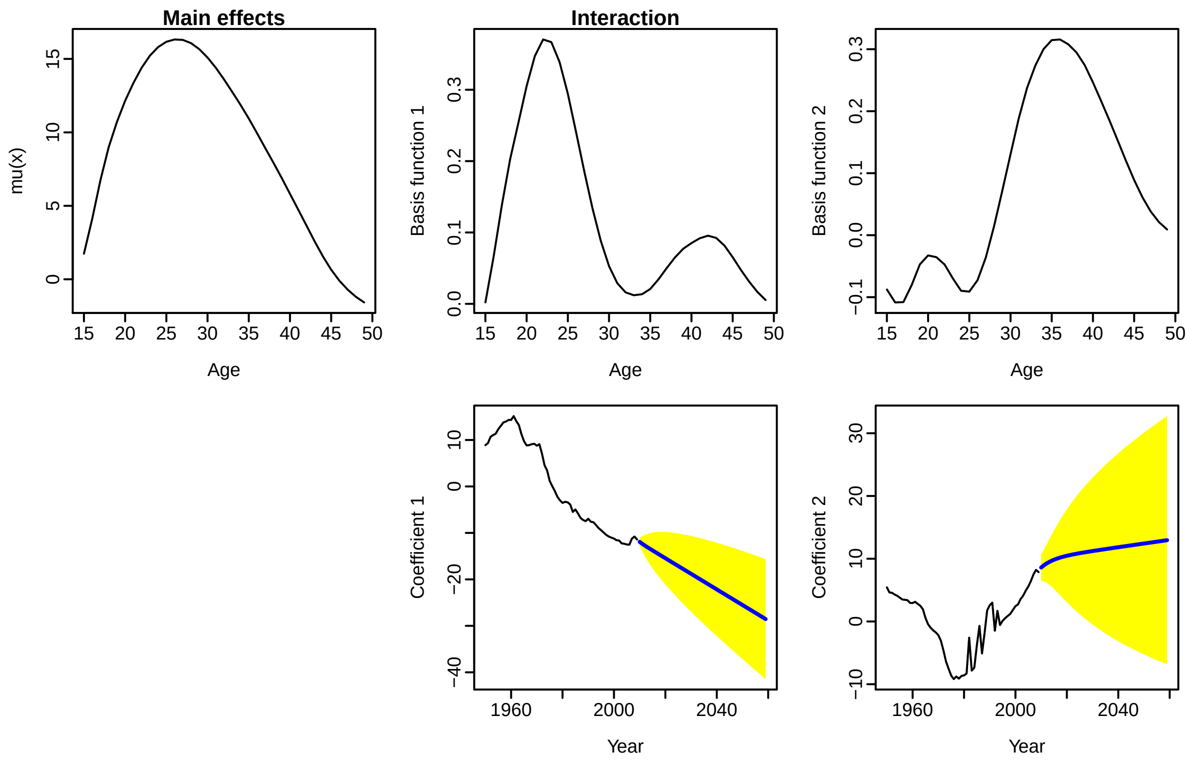

Figure 2: *Fitted basis functions and coefficients for Australian fertility rates*

The first basis function $\phi_1(x)$ measures the different effects of the trend over time across ages. It indicates the general pattern of fertility has very little effect on females of age 15. This makes intuitive sense because most early pregnancies are accidental, therefore, any of previously discussed events has no effect on early pregnancies. In addition, there are two spikes in the first basic function, which suggests that the historical trend has a distinct impact on females aged around their 20s and 40s. Furthermore, the blue line is the fitted value of future coefficients (i.e. $\beta_{n+h,1}$, for $h = 1, 2, ..., 50$) and the yellow shading shows their 80% prediction interval according to the fitted ARIMA model of first coefficients $\{\beta_{t,1}\}$. Other terms further explain variation that has not been captured by first term, but less economic explanations can be given for them.

Both Figure 3 and Figure 4 show the forecast results of population. Figure 3 shows the forecast of total population by gender over the prediction horizon, while Figure 4 shows the forecast of population age structure in a specific year, 2028. Figure 3 indicates that both the female and male population are expected to increase to 18 million by 2060. Since the fertility rate is expected to remain low, the possible reason of the significant increase in population size is net migration. We can also observe that the further the projected year is, the bigger the forecast variation of population. This increase in variation shows that we have less relevant information for further apart prediction horizons.

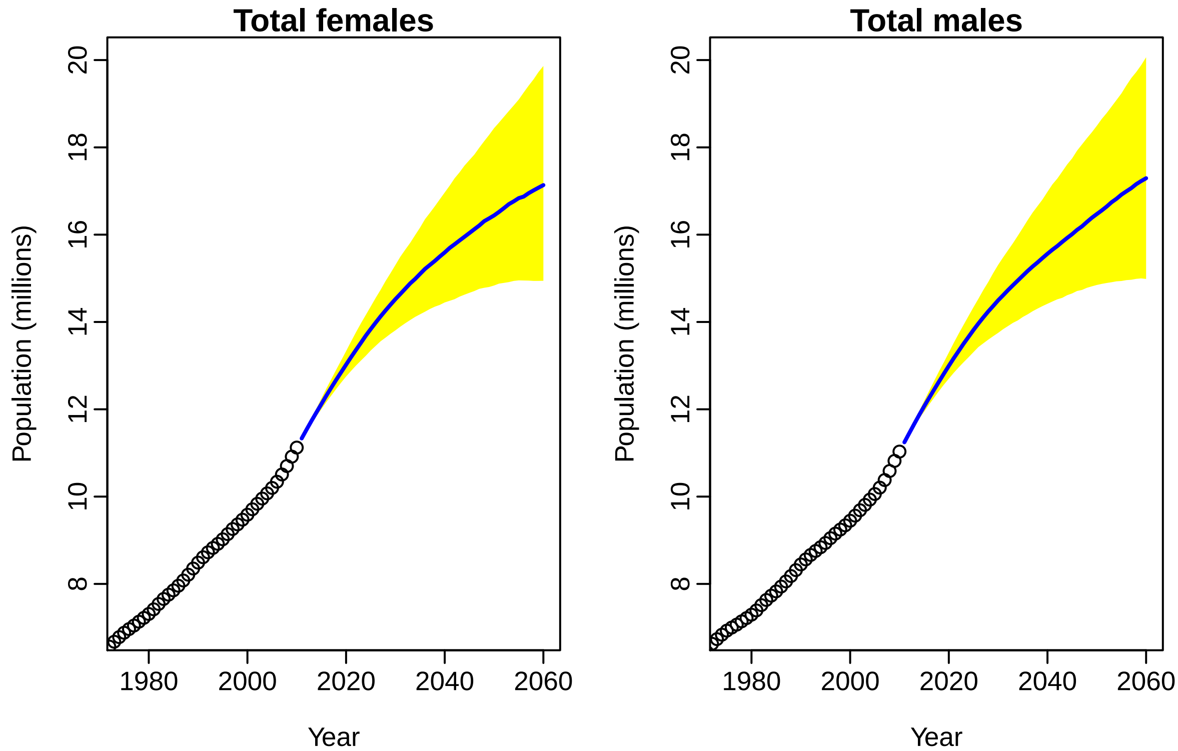

Figure 3: *Fifth-year forecasts of total population for each sex, along with 80% prediction intervals. The actually population from 1950 to 2010 is shown using circles.*

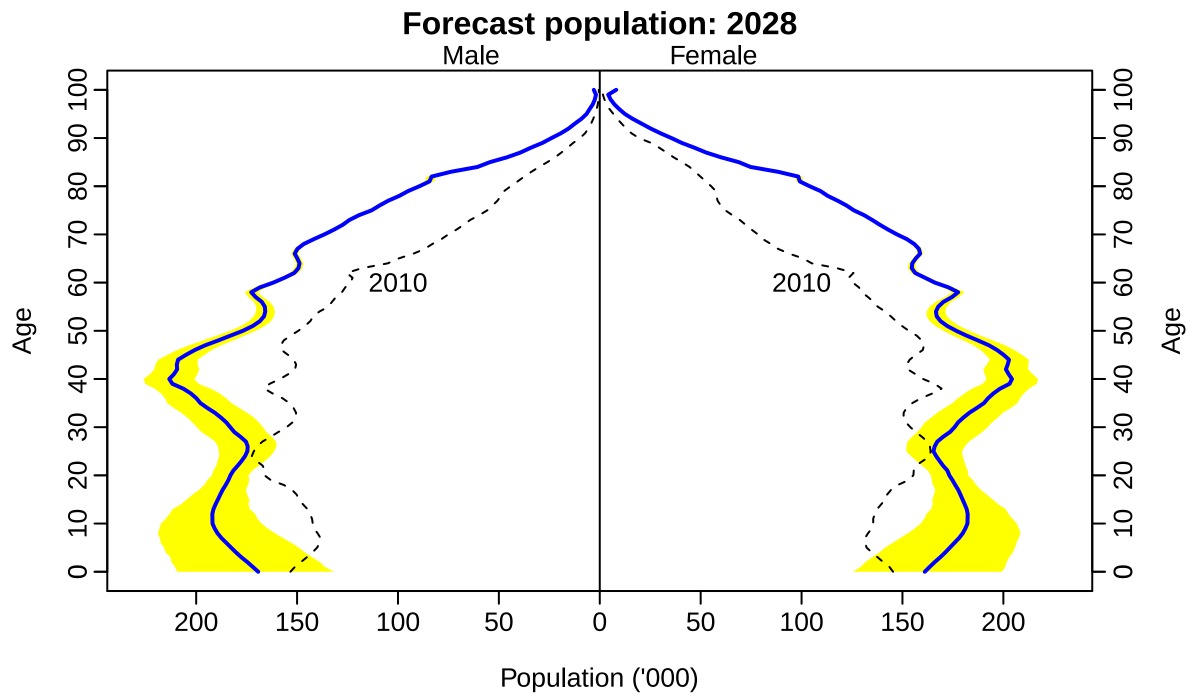

Figure 4: *Forecast population pyramid for 2028, along with 80% prediction intervals. The actual pyramid for 2010 is shown using dashed lines.*

Figure 4 shows the mean and the 80% prediction interval of simulated population paths for year 2028, along with the 2010 base population. It can be observed that overall population size is expected to increase substantially. The uncertainty of population of infant is much higher than other ages, which reflects the higher forecast variation of fertility rates than mortality rates. The uncertainty of the young age population is mainly due to migration (Hyndman & Booth, 2008).

Analysis was performed by first generating an OADR associated with a fixed pension age of 65 and an OADR associated with pension age scheme with approved change, gradually increasing the pension age from 65 to 67 by mid-2023. Even though the government might further increase the pension age in future, I kept the pension age at 67 from 2024 to see what would happen if no further action was taken. Figure 5 shows the mean forecast and prediction interval of the old aged dependency ratio with a pension age of 65, along with the historical OADR for 1920-2010. A strong positive trend was observed, which indicates an increasing burden on the next working-age generation if the pension age is fixed at 65. This is consistent with the prediction by inter-generation reports in 2002 (Australian Government, 2002) and in 2007 (Australian Government, 2007). According to former Australian Treasurer Costello (2004), this could be a result of better health care and low fertility rates. These two reasons could be regarded as permanent

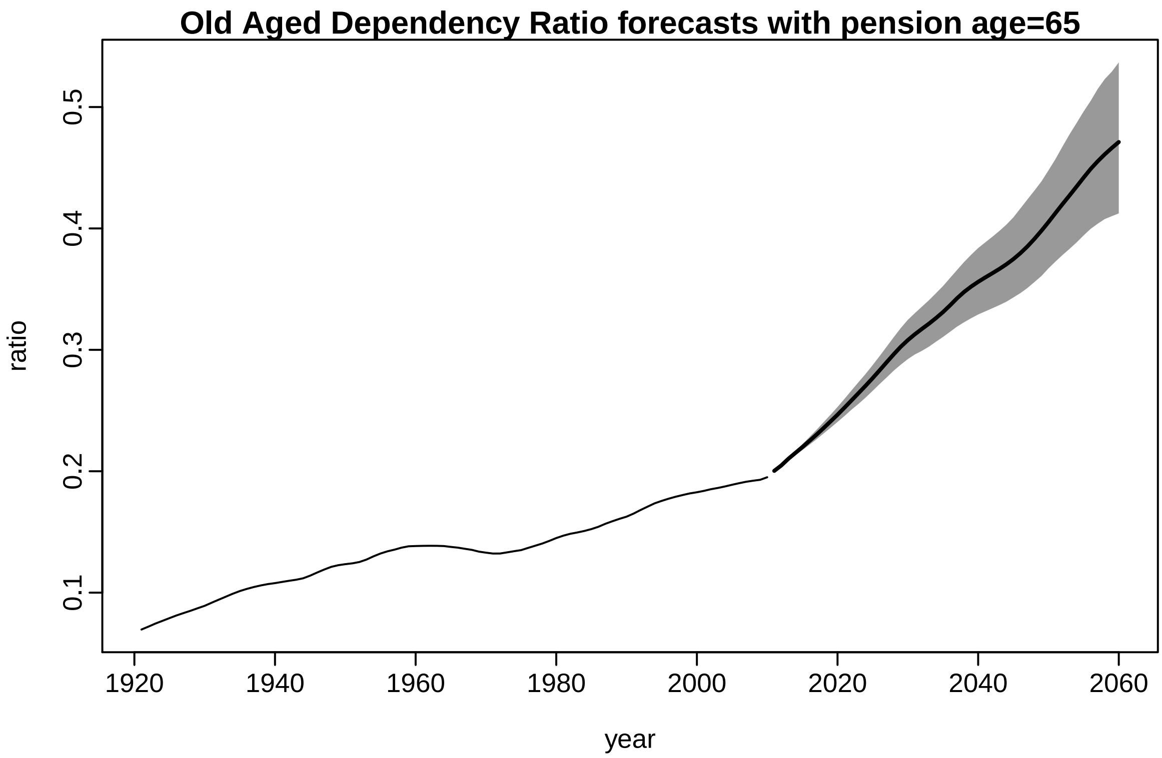

Figure 5: *Fifty-year forecasts of the old aged dependency ratio with assumption of pension age equal to 65 for both sexes, along with 80% prediction interval.*

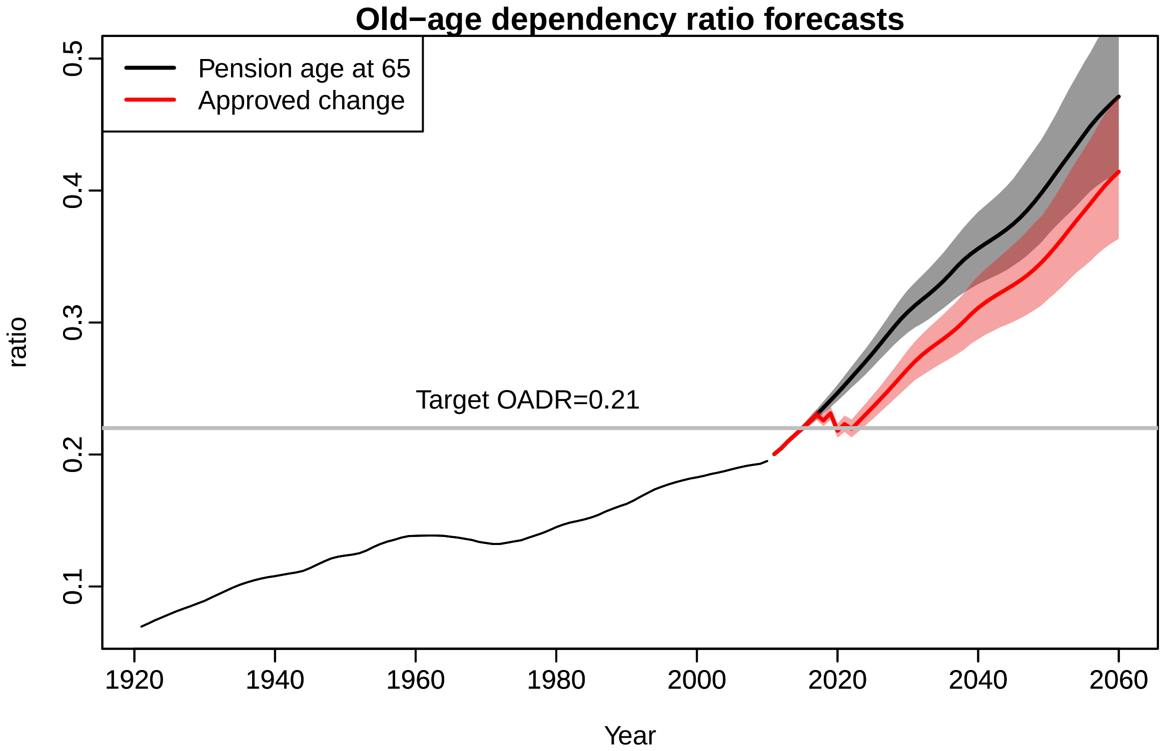

Figure 6: *Fifty-year forecasts of the old aged dependency ratio associated with a pension age equal to 65 (black line) and equal to the pension age scheme with approved change (red line) respectively, along with the desired OADR level in grey (i.e. mean forecast of OADR in 2014). The actual OADRs for 1950-2010 are shown in black.*

change and therefore a persistent trend. In addition, there is a big jump from actual data to predicted data. This jump is possibly a result of the Baby Boomers retirement. Those born in 1946, the start of the baby boom, would begin retiring in 2011 when the pension age was 65.

Figure 6 presents a comparison of the mean forecast of OADR with the pension age equal to 65 and a pension age equal to the pension age scheme with approved change, along with comparison of both forecasts with desired level of OADR, which has been set at 21%. It should be noted that OADR corresponding to pension age scheme with approved change is approaching the desired level, which suggests that the approved change does achieve the aim of providing an affordable financial burden, at least for a moment. However, once the pension age reaches 67, the OADR of the pension scheme with approved adjustment grows at nearly same rate as the OADR of a pension age equal of 65. This is consistent with the idea that there is a persistent positive trend in OADR associated with a constant pension age, due to low fertility rates and better health care. Therefore, Figure 6 demonstrates that the pension age scheme requires constant review to ensure a stable OADR over the long term. However, the questions of when the action needs to be taken and how much of an adjustment needs to be made remain to be answered. This answer will allow us to respond to the recent proposal of further increasing pension age to 70 by 2035.

4.2 Plausible Pension Age Schemes

Table 1 shows the target pension age scheme, which is properly defined in section 3.2.2. It is assumed that it is impractical to change the pension age during 2011-2015, so the pension age has to be set at 65 for those years. Figure 7 provides a visible comparison of the desired OADR level of 21% and the mean forecast of the OADR corresponding to the target pension age. Note that the OADR associated with target pension age scheme has nearly overlapped the desired OADR level, which indicates that we have successfully found the pension age with stable OADR at the desired levels.

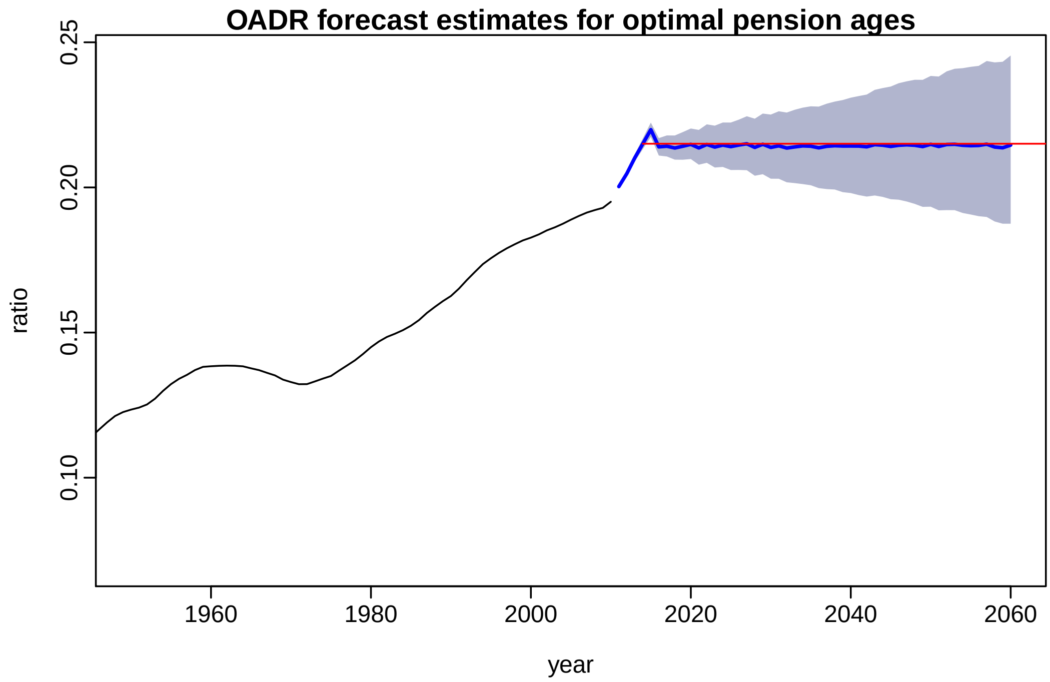

Figure 7: *The mean forecast of old aged dependency ratio associated with target pension age scheme (listed in Table 1) shown in blue, along with the desired OADR level in red (i.e. mean forecast of OADR in 2014). The 80% prediction interval of the forecast of OADR is shown as blue shading.*

The upper and lower boundaries of the plausible pension age schemes (also called the 80 % confidence interval of the target pension age scheme in this paper) are reported in Table 2, and their corresponding OADR simulation results are shown in Figure 8. It can be seen that the lower (upper) limit of the 80% prediction interval of OADR associated with lower (upper) boundary of the 80% confidence interval pension age scheme

Year	2011	2012	2013	2014	2015	2016	2017	2018
Age	65Y	65Y	65Y	65Y	65Y	65Y7M	65Y10M	66Y2M
Year	2019	2020	2021	2022	2023	2024	2025	2026
Age	66Y5M	66Y8M	67Y	67Y3M	67Y7M	67Y10M	68Y2M	68Y5M
Year	2027	2028	2029	2030	2031	2032	2033	2034
Age	68Y8M	69Y	69Y3M	69Y7M	69Y10M	70Y2M	70Y5M	70Y8M
Year	2035	2036	2037	2038	2039	2040	2041	2042
Age	70Y11M	71Y2M	71Y3M	71Y6M	71Y8M	71Y10M	72Y	72Y2M
Year	2043	2044	2045	2046	2047	2048	2049	2050
Age	72Y5M	72Y8M	72Y11M	73Y1M	73Y3M	73Y5M	73Y7M	73Y8M
Year	2051	2052	2053	2054	2055	2056	2057	2058
Age	73Y10M	73Y11M	74Y1M	74Y3M	74Y5M	74Y7M	74Y9M	75Y
Year	2059	2060						
Age	75Y3M	75Y4M						

Table 1: *Fifty-year target pension age scheme with stable OADR around desired OADR. Y denotes years and M denotes months.*

Year	2011	2012	2013	2014	2015	2016	2017	2018
Lower	65Y	65Y	65Y	65Y	65Y	65Y5M	65Y8M	66Y11M
Upper	65Y	65Y	65Y	65Y	65Y	65Y9M	66Y	66Y4M
Year	2019	2020	2021	2022	2023	2024	2025	2026
Lower	66Y2M	66Y5M	66Y8M	66Y11M	67Y2M	67Y5M	67Y8M	67Y11M
Upper	66Y8M	67Y	67Y4M	67Y8M	67Y11M	68Y4M	68Y7M	68Y11M
Year	2027	2028	2029	2030	2031	2032	2033	2034
Lower	68Y2M	68Y5M	68Y8M	68Y11M	69Y4M	69Y5M	69Y8M	69Y11M
Upper	69Y4M	69Y6M	69Y10M	70Y2M	70Y6M	70Y9M	71Y1M	71Y4M
Year	2035	2036	2037	2038	2039	2040	2041	2042
Lower	70Y1M	70Y4M	70Y5M	70Y6M	70Y8M	70Y10M	71Y	71Y4M
Upper	71Y8M	71Y11M	72Y2M	72Y4M	72Y7M	72Y9M	73Y	73Y4M
Year	2043	2044	2045	2046	2047	2048	2049	2050
Lower	71Y5M	71Y8M	71Y10M	71Y11M	72Y	72Y2M	72Y3M	72Y4M
Upper	73Y5M	73Y8M	74Y	74Y3M	74Y5M	74M8M	74Y10M	75Y
Year	2051	2052	2053	2054	2055	2056	2057	2058
Lower	72Y5M	72Y7M	72Y8M	72Y10M	73Y	73Y2M	73Y4M	73Y7M
Upper	75Y2M	75Y4M	75Y6M	75Y8M	75Y10M	76Y	76Y3M	76Y5M
Year	2059	2060						
Lower	73Y9M	73Y11M						
Upper	76Y8M	76Y10M						

Table 2: *Fifth-years lower and higher boundary of pension age scheme with one boundary of 80 % OADR around desired OADR. Y denotes years and M denotes months.*

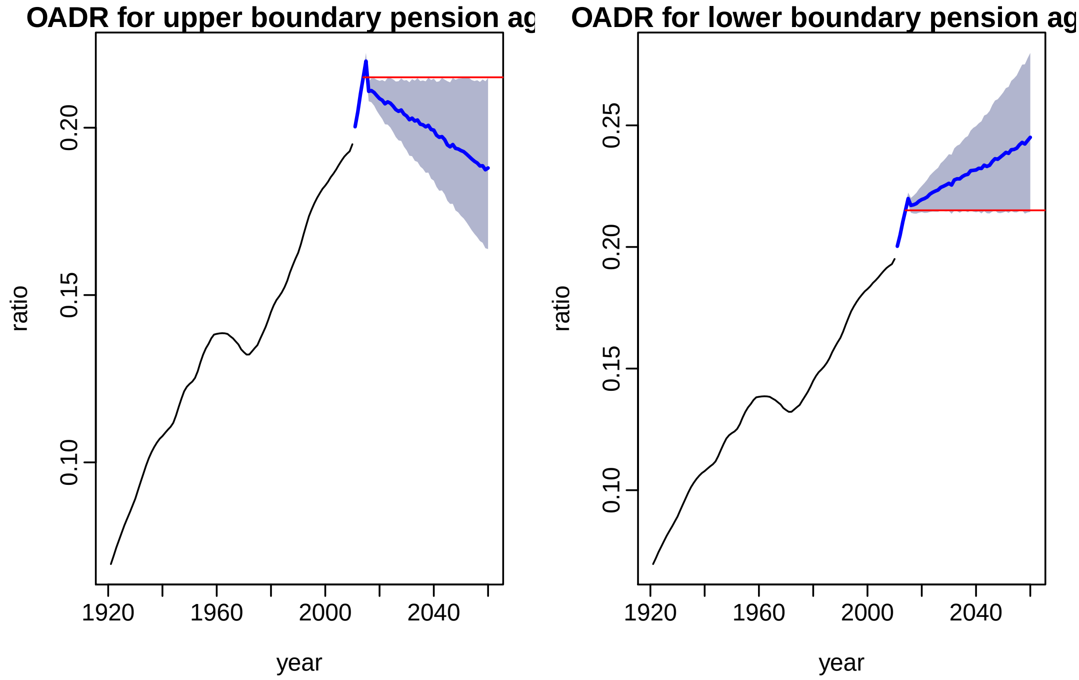

Figure 8: *The 80% prediction interval forecast of OADR (in blue) associated with upper and lower boundary of plausible pension age schemes (listed in Table.2), along with desired OADR level (in red).*

is entirely stable around the desired level. Note the negative relationship between OADR and pension age. It suggests, on one hand, that any pension age higher than the upper boundary in table 2 is deemed to be undesirable because the age is too high to allow the age pension to provide enough welfare support or insurance to the aged. On the other hand, any pension age scheme lower than the lower boundary of the pension age reported in Table 2 will also be regarded as unreasonable, because it sacrifices the interests of the next generation to provide a very generous age pension immediately.

Figure 9 compares the target pension age scheme and its 80% confidence interval with the pension age scheme with proposed further change. It was found that the target pension age increases at higher rate than the proposed new pension age scheme. Reported in Table 1, the target pension age is supposed to reach 67 by 2021 and 70 by 2032, while pension age has been proposed to change to 67 by 2023 and 70 by 2035. Moreover, the proposed pension age scheme is below the lower boundary of plausible pension age schemes, which means we are 80% sure that it will cause an OADR higher than desired level. From this result, it is concluded that government will need to adjust pension age slightly quicker than proposed pace to ensure we have a stable OADR around desired level at 21%.

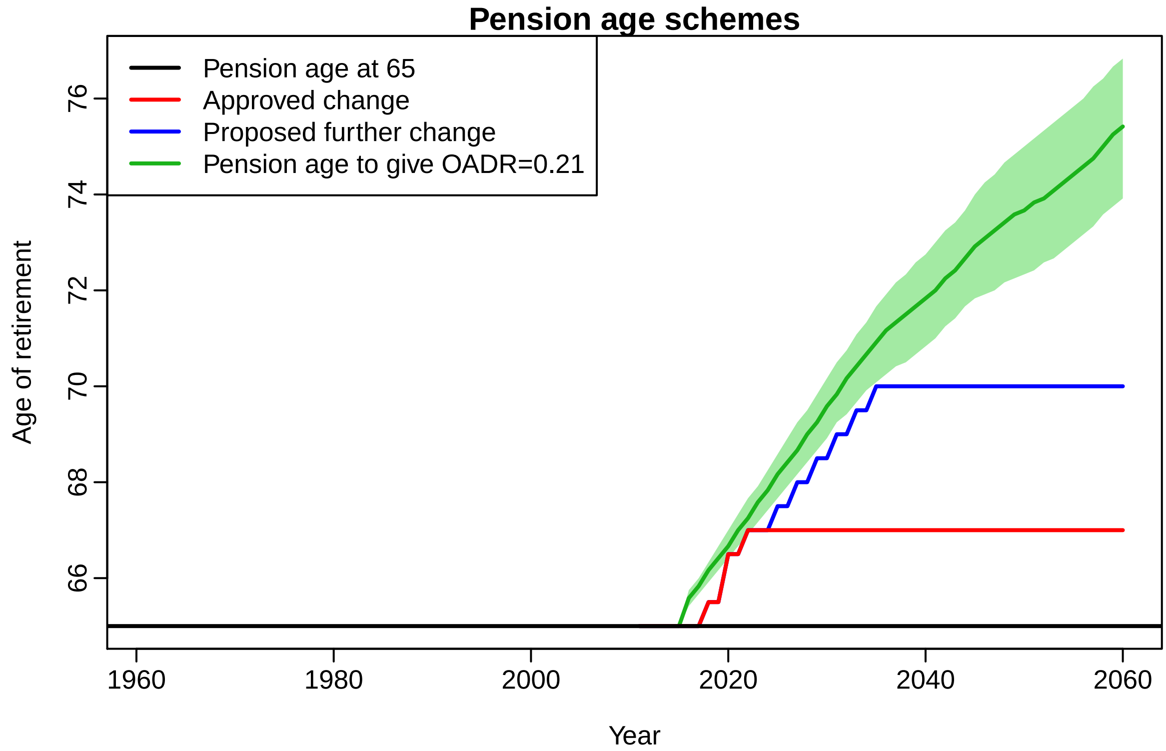

Figure 9: *The comparison between the target pension age scheme (in green) and real pension age scheme that has been set or proposed by government, along with the 80% confidence interval of the target pension age shown in green shade. The red solid line indicates the changes to the pension age from 65 to 67, announced in 2009.*

However, it is interesting to note that the proposed pension age scheme does not diverge too far from the plausible pension age schemes, suggesting proposed new pension age scheme may be appropriate if we can afford slightly higher OADR. Therefore, it is worth performing a sensitivity test on the desired level here. Using the approach suggested in this paper, the target pension age associated with desired OADR equal to current level plus 1% has been found. We have shown the comparison of it and its 80% confidence level with real pension age scheme in Figure 10. When we allow 1% more in desired OADR, the real pension age becomes plausible. Figure 11 shows simulated OADR associated with proposed pension age scheme[10]. It confirms that OADR is stable around a level higher than current level of OADR before 2035. This results suggest that pension age scheme with proposed change could be suitable if the government are targeting a slightly higher financial pressure than current level.

[10]Since we don't know the change rate set by government for 2023-2035, we assume it follows increase rate for 2017-2023, which increase half year pension age in every two years

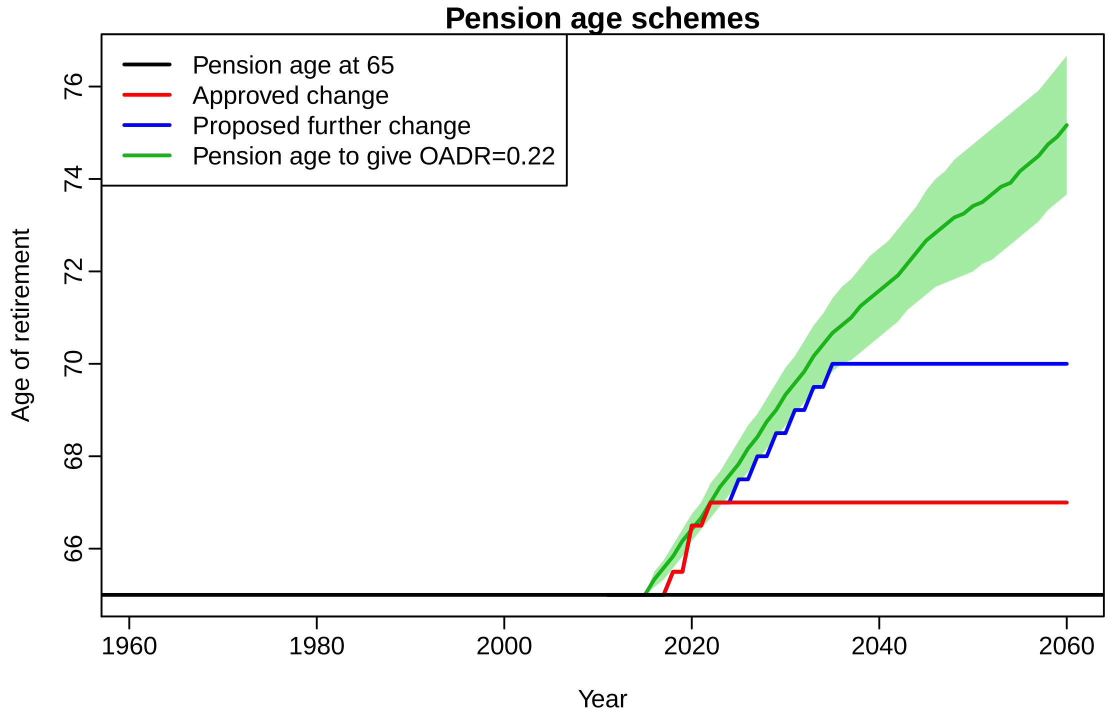

Figure 10: *The comparison between the target pension age scheme (in green) associated with desired level equal to current level plus* 1% *(i.e. 22%) and real pension age scheme that has been set or proposed by government, along with the 80% confidence interval of the target pension age shown in green shade.*

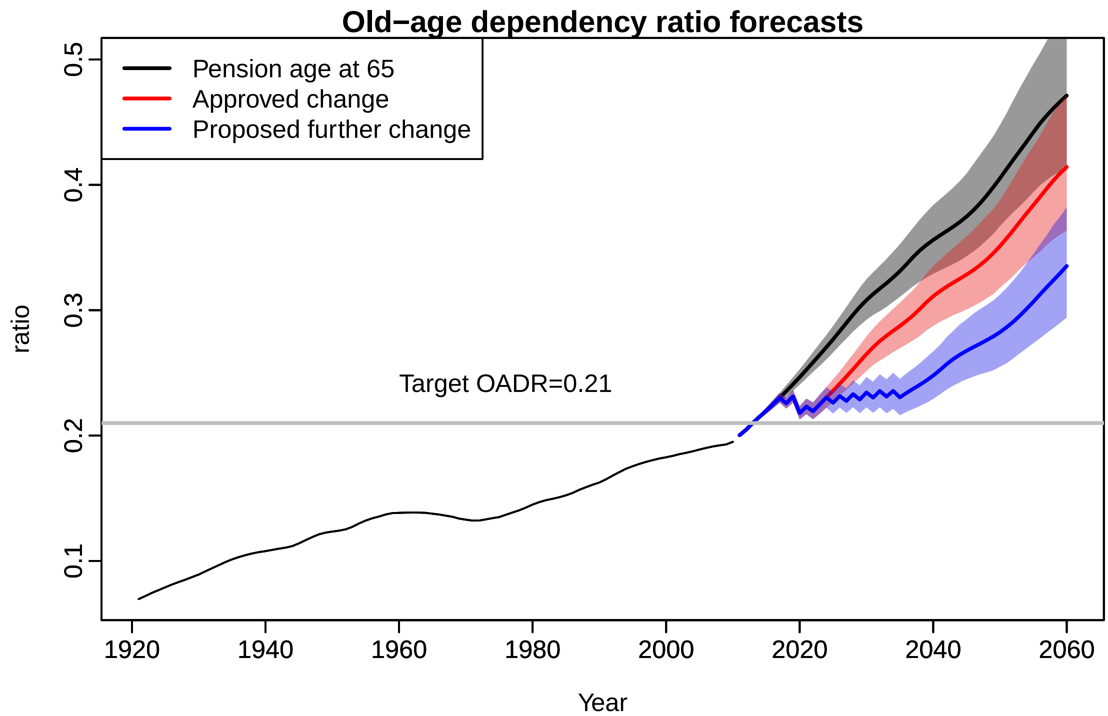

Figure 11: *The mean forecast of old aged dependency ratio associated with different pension age schemes under investigation, along with the desired OADR level in grey (i.e. mean forecast of OADR in 2014). The 80% prediction interval of the forecast of OADR is shown as shading.*

4.3 Evaluating Forecast Accuracy

Since all the results and implications are dependent on the forecast accuracy, it is important to evaluate the accuracy of our forecast. It is not valid to assess the model's fit in terms of historical data. We are concerned with how well a model performs on out-of-sample data, which is known but not used to fit the model. Therefore, we will use a Cross-Validation procedure (Hyndman & Athanasopoulos, 2013) to evaluate the accuracy of the models used in this paper.

The logic of a Cross-Validation test is to use a portion of available data for fitting, and use the remaining data to test. Suppose we need k years' observations to produce a reliable forecast[11], we are interested in the F prediction horizon. Then we are allowed to repeat the test $n - F - k + 1$ times, where n is the number of total years of observation. The procedure is as follows:

Let $i = 1$, we first select historical data for a time interval of $[i, i+1, ..., i+k-1]$ as "in sample data" to fit the model, and then forecast F years' OADR[12]. Then, we can calculate the difference of forecasted OADR over the period $[i+k, i+k+1, ..., i+k+F-1]$ from the real OADR for the same period, which is known as absolute error. We repeat the following steps for $i = 1, 2, ..., n-k-F+1$. Then, we can summary the information into a matrix as follows:

$$\boldsymbol{Absolute\ Error} = [AE_{f,i}] = \begin{bmatrix} AE_{1,1} & \cdots\cdots & AE_{1,n-k-F+1} \\ \vdots & \ddots & \vdots \\ AE_{F,1} & \cdots\cdots & AE_{F,n-k-F+1} \end{bmatrix} \tag{29}$$

where f denotes for the forecast horizon and $AE_{f,i} = |OADR_{f,i}^{mean} - OADR_{f,i}^{real}|$. This process is also known as "rolling forecasting origin", because the "origin" $(k+i-1)$ at which the forecast is based rolls forward in time. We take the average of each row to obtain an absolute mean error of f-step-ahead forecast of OADR where $f = 1, 2, ..., F$.

In our sample, $n = 2010 - 1950 + 1 = 61$ and we set $k = 25$ and $F = 25$ to perform the test. The mean absolute error of f-step-ahead forecast of OADR for $f = 1, 2, ..., 25$ is shown in Figure 12. From this figure, we observe that the absolute mean error is increasing as f increases, which is reasonable as we have less relevant information to make a forecast for further apart OADR. Overall, the mean absolute error is very small. For example, the mean absolute error for 25-step-ahead forecast is 1.5%, which is very small if compared to the desired level of OADR, which is at 21%.

[11] k can't be too small

[12] the OADR is generated based on a pension age equal to 65 for the purpose of test

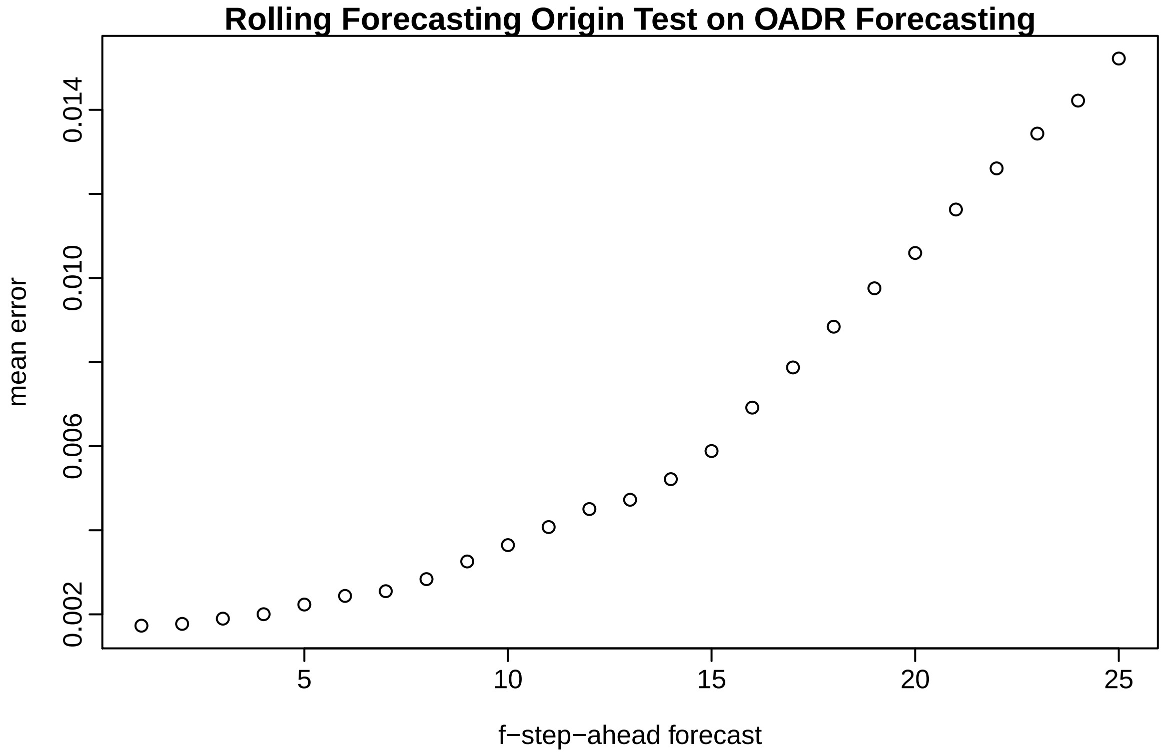

Figure 12: *The mean absolute error of f-step-ahead forecast for f=1,2,. . . ,25*

It would normally be ideal to compare the mean absolute error of the forecast from my model to other available models to measure the accuracy. For example, suppose we can take the population projections from ABS reports over the last few decades, their projection of population age structure reported in different years can be used to mimic the rolling origin test of OADR, because we now have the real OADR for those projected years. However, we are not able to actually perform it because full projection data is no longer available online and ABS projections have not used a consistent and replicate methodology.

5 Conclusions

The ageing of population is expected to become a critical issue in Australia due to the increase in life expectancy and the retirement of baby boomers. The fiscal cost of the age pension is projected to sharply increase as a result. For the purpose of sustainability of the welfare system in Australia, the government has announced an increase in the pension age to 67 in 2023 and 70 in 2035 to reduce the financial burden on society.

In this paper, I have suggested a statistical and demographic approach to address the issue. I attempt to quantify the effects of changes in the pension age on financial burdens using an old aged dependency ratio as a measure of financial burden on Australian society. The quantification process involves applying stochastic models to forecast future population age structure and then using those future age structure projections to find the most appropriate pension age scheme that will ensure stable old age dependency ratio around the desired level. To forecast population age structure, three demographic components of population were modelled separately using Hyndman and Ullah (2007) functional time series method, with an extension of the product-ratio method (Hyndman, Booth & Yasmeen, 2013). Next, future age structures were used to simulate OADR associated with different pension age schemes until the minimum pension age scheme was found, which ensures the old age dependency ratio no greater than desired level. This approach is proposed to find target pension age scheme based on a given desired level of OADR.

I undertake the approach based on assuming the desired level of OADR is at current level of 21%. Using the historical data from 1950 to 2010, the forecast of future populations suggests a substantial increase in total population, and confirms the inherent ageing trend. It was found that pension age should not be fixed, but instead should grow at a rate that will ensure a stable old aged dependency ratio at a level which can be affordable to Australians of working age. Findings indicate that the pension age scheme with proposed change was not too high, as it is still lower than the target pension age scheme and outside its confidence interval. It was concluded that the pension age set by government should be adjusted slight quicker to be in line with our target pension age, which ensures a stable OADR of around 21%.

Although my results are promising, there are still grounds for further research. First, my research results are based on forecasts from one particular model. Further studies can apply the same approach but with different models of population forecasting, which allows for enhanced credibility of results. Moreover, my research addresses the issue from a purely demographic perspective. It is possible to include an economic perspective. For example, the economic implications of retaining older workers in workforce can be considered. Second, my research uses an old age dependency ratio as a single measurement of financial burden. Further research should allow the effect of other economic factors to be taken into account. For instance, the effect of growth in superannuation can be considered in a way that the effects of high OADR can be offset by the growth in superannuation.

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Appendix A: Abbreviations

ABS: Australian Bureau of Statistics
AIC: Akaikes Information Criterion
ARFIMA model: AutoRegressive fractionally Integrated Moving Average model
ARIMA model: AutoRegressive Integrated Moving Average model
CI: Confidence Interval
GDP: Gross Domestic Profit
HU method: Hyndman-Ullah method
LC method: Lee-Carter method
NCOA: National Commission of Audit
OADR: Old-Aged Dependency Ratio
PC: Production Commission
PCA: Principal Component Analysis
SVD: Singular Value Decomposition

Appnedix B: R code

B.1. Code to Collect Data

```
############################
## GET ALL THE DATA
############################

##### MORTALITY

# Username and password for the www.mortality.org website.
username <- "username"
password <- "password"

# Get population data for 1 Jan 2010
pop2010 <- hmd.pop("AUS",username,password,"Australia")

pop2010 <- extract.ages(pop2010, age=0:100)

# Get mortality data
aus.mort <- hmd.mx("AUS",username,password,"Australia")
```

```
aus.mort <- extract.years(extract.ages(aus.mort, 0:100, combine.upper=FALSE),
                          year=1950:max(aus.mort$year))

##### FERTILITY

# Get fertility data:
# Need to install addb package from http://robjhyndman.com/software/addb/
library(addb)
# Contains fertility data for 1921-2006
aus.fert <- aus.fertility

# Update fertility data to 2009
ausfertility <- read.csv("http://robjhyndman.com/data/ausfertility.csv",
header=TRUE,colClasses="numeric")
aus.fert$rate$female <- as.matrix(cbind(aus.fert$rate$female[,30:54],
ausfertility[,2:36]))
aus.fert$pop$female <- cbind(aus.fert$pop$female[,30:86],
aus.mort$pop$female[16:50,58:60])
colnames(aus.fert$rate$female) <- colnames(aus.fert$pop$female)
<- aus.fert$year <- 1950:2009
rm(ausfertility)


##### MIGRATION
aus.mig <- netmigration(aus.mort, aus.fert, mfratio = 1.05)
```

B.2. Forecasting Population Age Structure

```
################ SMOOTHING STEP ################
ausmort.sm <- smooth.demogdata(aus.mort, obs.var="theoretical")
ausfert.sm <- smooth.demogdata(aus.fert, obs.var="theoretical")
ausmig.sm <- smooth.demogdata(aus.mig)

################ MODELLING STEP ################
# MORTALITY
mort.fit <- coherentfdm(ausmort.sm)
mortf <- forecast(mort.fit, h=50)
plot(mortf$product, 'c', comp=2)
```

```
plot(mortf$ratio$male, 'c', comp=2)

# FERTILITY
fert.fit <- fdm(ausfert.sm)
fertf <- forecast(fert.fit, h=50)
# Adjust fertility forecasts (d=2 for second coefficient causes problems)
fertf$coeff[[3]] <- forecast(auto.arima(fert.fit$coeff[,3],d=1),h=50, level=80)
fertf <- update(fertf)

plot(fertf, 'c', comp=2,mean.lab="mu(x)")

# MIGRATION
mig.fit <- coherentfdm(ausmig.sm)
migf <- forecast(mig.fit, h=50, stationary=TRUE)
plot(migf$product, 'c', comp=3)
plot(migf$ratio$male, 'c', comp=3)

############### SIMULATION STEP #################
# This takes a while!
aus.sim <- pop.sim(mortf, fertf, migf, firstyearpop=pop2010, N=100)

## Means and intervals
popm.mean <- apply(aus.sim$male,c(1,2),mean)
popm.lo <- apply(aus.sim$male,c(1,2),quantile,p=.1)
popm.hi <- apply(aus.sim$male,c(1,2),quantile,p=.9)
popf.mean <- apply(aus.sim$female,c(1,2),mean)
popf.lo <- apply(aus.sim$female,c(1,2),quantile,p=.1)
popf.hi <- apply(aus.sim$female,c(1,2),quantile,p=.9)

## TOTAL POPULATION COMPARISONS
totfpop <- apply(aus.sim$female,c(2,3),sum)
totmpop <- apply(aus.sim$male,c(2,3),sum)
totfpop.q <- apply(totfpop,1,quantile,p=c(0.025,.1,.5,.9,.975),na.rm=TRUE)/1e6
totmpop.q <- apply(totmpop,1,quantile,p=c(0.025,.1,.5,.9,.975),na.rm=TRUE)/1e6
```

B.3. Forecasting OADR

```
# Function to produce old age dependency ratio
oadr <- function(x, sim, pension.age=65,
 historical.pension.age=65,
 level=80)
{
  adjustment<-
(historical.pension.age%%1)*colSums(extract.ages
(x,as.integer(historica
l.pension.age)+1,FALSE)$pop$total,na.rm=TRUE)
  workers <-
colSums(extract.ages(x,15:as.integer
(historical.pension.age),FALSE)$pop
$total,na.rm=TRUE)+adjustment
  aged <-
colSums(extract.ages
(x,as.integer(historical.pension.age):max(x$age),TR
UE)$pop$total,na.rm=TRUE)-adjustment
  xhistory <- ts(aged/workers,s=x$year[1],f=1)
  ##ts is "time series" s is starting f is frequency

  h <- dim(sim[[1]])[2]
  N <- dim(sim[[1]])[3]
  if(length(pension.age)==1)
    pension.age <- rep(pension.age, h)
  if(length(pension.age) != h)
    stop("length of pension.age should be equal to forecast horizon")
  # Check that pension.age does not decrease
  if(max(diff(pension.age)) < 0)
    stop("Pension ages should not decrease")
  # Check that pension.age does not increase too quickly
  if(max(diff(pension.age)) > 1)
    stop("Pension age should not rise more than 1 year at a time")

  simages <- as.numeric(dimnames(sim[[1]])[[1]])
# Ages vector from sim
  # Set up matrices to store populations from each forecast horizon
 and each simulation
  male.workers <- female.workers <- male.aged <- female.aged
<- matrix(NA,nrow=h,ncol=N)
```

```
  for (i in (1:h))
  {
    worker.rows <- (simages >= 15 &
     simages < as.integer(pension.age[i]))
    pension.rows <- (simages >=
     as.integer(pension.age[i]))
adjustment.male<-
(pension.age[i]%%1)*sim$male[as.integer(pension.age[i]),i,]
adjustment.female<-
(pension.age[i]%%1)*sim$female[as.integer(pension.age[i]),i,]
male.workers[i,] <-
colSums(sim$male[worker.rows,i,])+adjustment.male
female.workers[i,] <-
colSums(sim$female[worker.rows,i,])+adjustment.female
male.aged[i,] <-
colSums(sim$male[pension.rows,i,])-adjustment.male
female.aged[i,] <-
colSums(sim$female[pension.rows,i,])- adjustment.female

  }

  oadp.f <- (male.aged+female.aged)/(male.workers+female.workers)
  oadp.lo <- apply(oadp.f,1,quantile,prob=(0.5-level/200))
  oadp.hi <- apply(oadp.f,1,quantile,prob=1-(0.5-level/200))

  firstyear <- min(as.numeric(dimnames(sim[[1]])[[2]]))
  oadp.f <-
structure(list(mean=ts(rowMeans(oadp.f),s=firstyear+1),x=xhistory,
upper=ts(oadp.hi,s=firstyear+1),
lower=ts(oadp.lo,s=firstyear+1),

level=level),class="forecast") #s=firstyear+1?
  return(oadp.f)
}
```

B.4. Finding the Target Pension Age Scheme

```
###find the optimal scheme of pension.age based on mean foreast
```

```
#target <- tail(oadp.f$x,1)
target <- oadp.f$mean[4]

pension.age.optimal <- rep(65,50)
for(i in 6:50)
{
  current <- oadr(pop2010,aus.sim,
pension.age=pension.age.optimal)$mean[i]
  while
  ((current > target) & (max(diff(pension.age.optimal[(i-1):50]))
<=(1-1/12)))
  {
    pension.age.optimal[i:50] <- pension.age.optimal[i:50] + 1/12

      current <- oadr(pop2010,aus.sim,
pension.age=pension.age.optimal)$mean[i]
  }
}
oadp.f.optimal<- oadr(pop2010,
aus.sim,pension.age=pension.age.optimal)

###find the upper boundary of pension ages whose
prediction interval contains target
pension.age.upper <- rep(65,50)
for(i in 6:50)
{
  current.upper <- oadr(pop2010,aus.sim,
pension.age=pension.age.upper)$upper[i]
  while
  ((current.upper > target) & (max(diff(pension.age.upper[(i-1):50]))
<=(1-1/12)))
  {
    pension.age.upper[i:50] <- pension.age.upper[i:50] + 1/12

current.upper <- oadr(pop2010,aus.sim,
pension.age=pension.age.upper)$upper[i]
  }
}
oadp.f.upper<- oadr(pop2010, aus.sim,pension.age=pension.age.upper)
```

```
#####find the lower boundary of pension ages whose
prediction interval contains target
pension.age.lower <- rep(65,50)
for(i in 6:50)
{
  current.lower <- oadr(pop2010,aus.sim,
pension.age=pension.age.lower)$lower[i]
  while
  ((current.lower > target) &
(max(diff(pension.age.lower[(i-1):50]))<=(1-1/12)))
  {
    pension.age.lower[i:50] <- pension.age.lower[i:50] + 1/12

current.lower <- oadr(pop2010,aus.sim,
pension.age=pension.age.lower)$lower[i]
  }
}
oadp.f.lower<- oadr(pop2010, aus.sim,pension.age=pension.age.lower)
```