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Seasonal functional autoregressive models

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Outline

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Examples

Notation

$f_t(x)$ where $t = 1, \dots, T$ indexes regularly spaced time and x is a continuous variable.

1. $f_t(x)$ = vegetation index at location x in month t, measured by average satellite observations.
2. $f_t(x)$ = mortality rate for people aged x at day t.

Sometimes x may denote a second time variable.

3. $f_t(x)$ = pollution level observed every 30 minutes. x denotes time-of-day, t denotes day.

Seasonality

Notation

$f_t(x)$ where $t = 1, \ldots, T$ indexes regularly spaced time and x is a continuous variable.

Seasonality occurs when $f_t(x)$ is influenced by seasonal factors (e.g., the quarter of the year, the month, the day of the week, etc.).

A possibly de-trended series is seasonal of period S if

$$\mathrm{E}(f_t(x)) = \mathrm{E}(f_{t+S}(x)).$$

Functional autoregression

FAR process introduced by Bosq (2000):

- popular for functional time series
- cannot handle seasonality

Seasonal autoregression

For univariate time series, $\{X_t\}$, seasonal autoregressive processes satisfy

$$X_t = \phi_1 X_{t-S} + \phi_2 X_{t-2S} + \cdots + \phi_P X_{t-PS} + \varepsilon_t.$$

where $\varepsilon_t \sim$ white noise. For stationarity, the roots of $\phi(x) = \phi_1 x^S - \phi_2 x^{2S} - \cdots - \phi_P x_{PS}$ must lie outside unit circle.

In this paper, we propose a class of seasonal