

Feature-based time series analysis

Rob J Hyndman

21 June 2018

Outline

- 1 Time series feature spaces
- 2 Finding anomalies
- 3 Irish smart metre data
- 4 Quantiles conditional on time of week
- 5 Finding typical and unusual households
- 6 Visualization via embedding
- 7 Features and limitations

M3 competition



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International Journal of Forecasting 16 (2000) 451–476

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The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon*

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Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy

M3 competition



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etition: results, conclusions

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Abstr

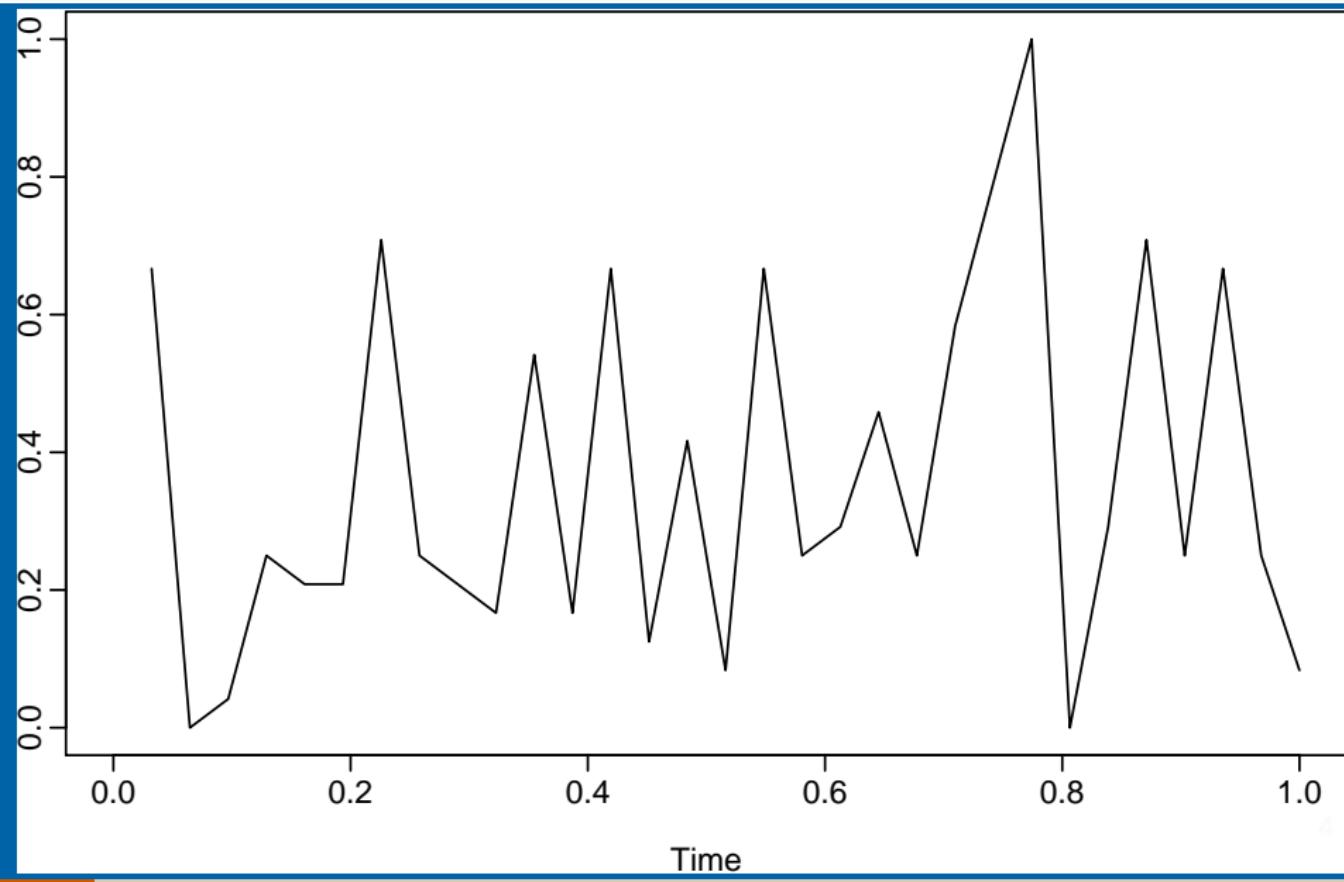


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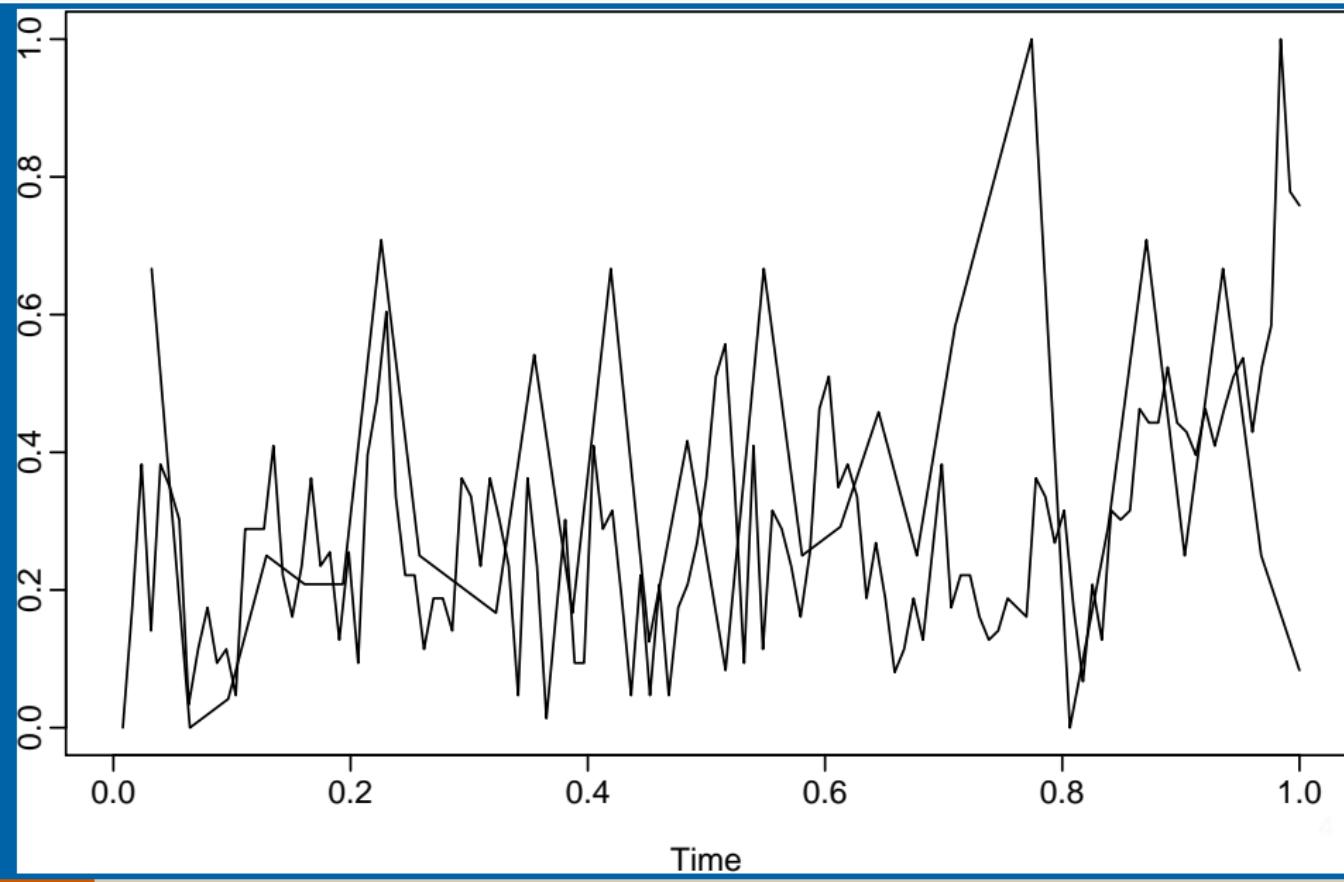
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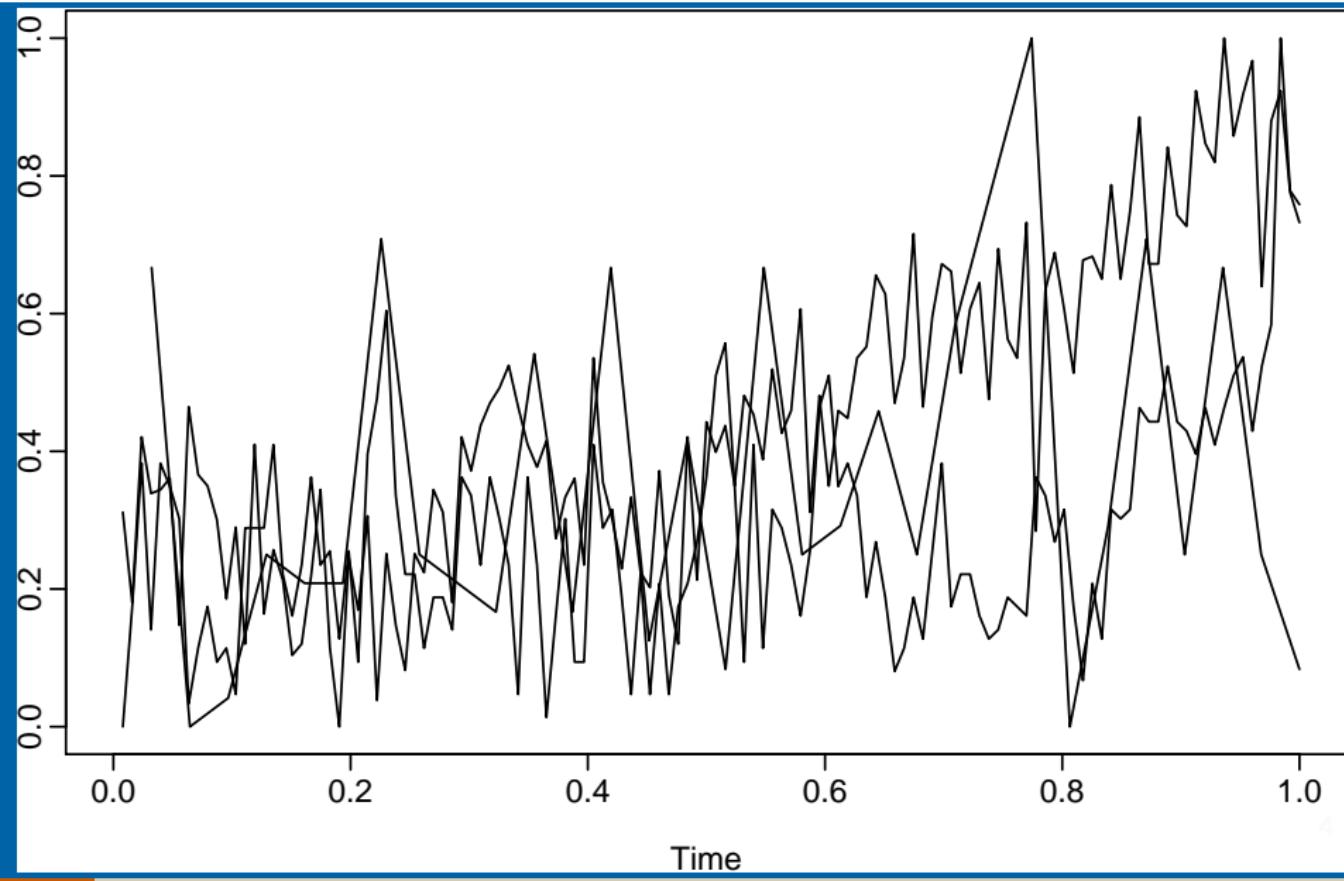
How to plot lots of time series?



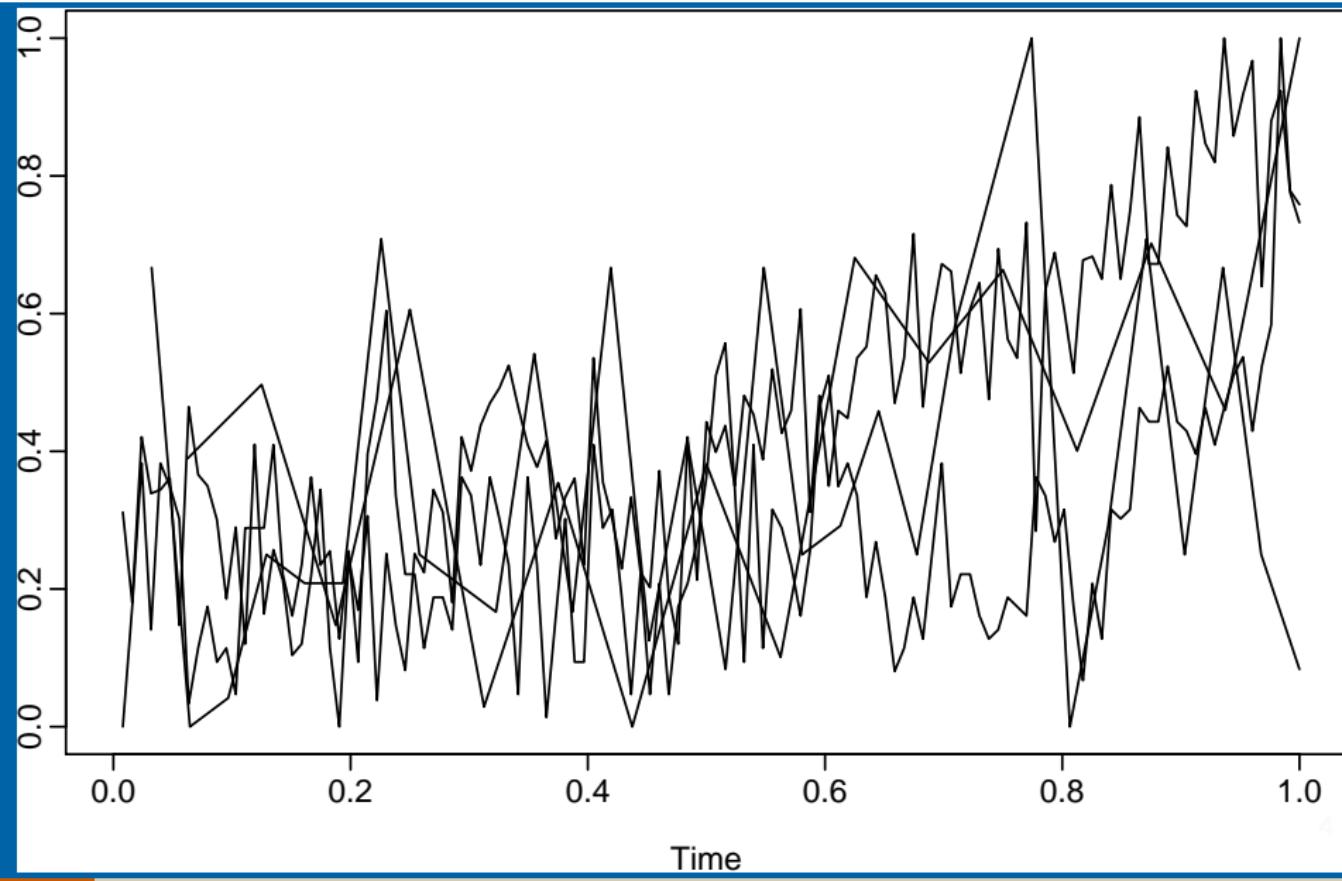
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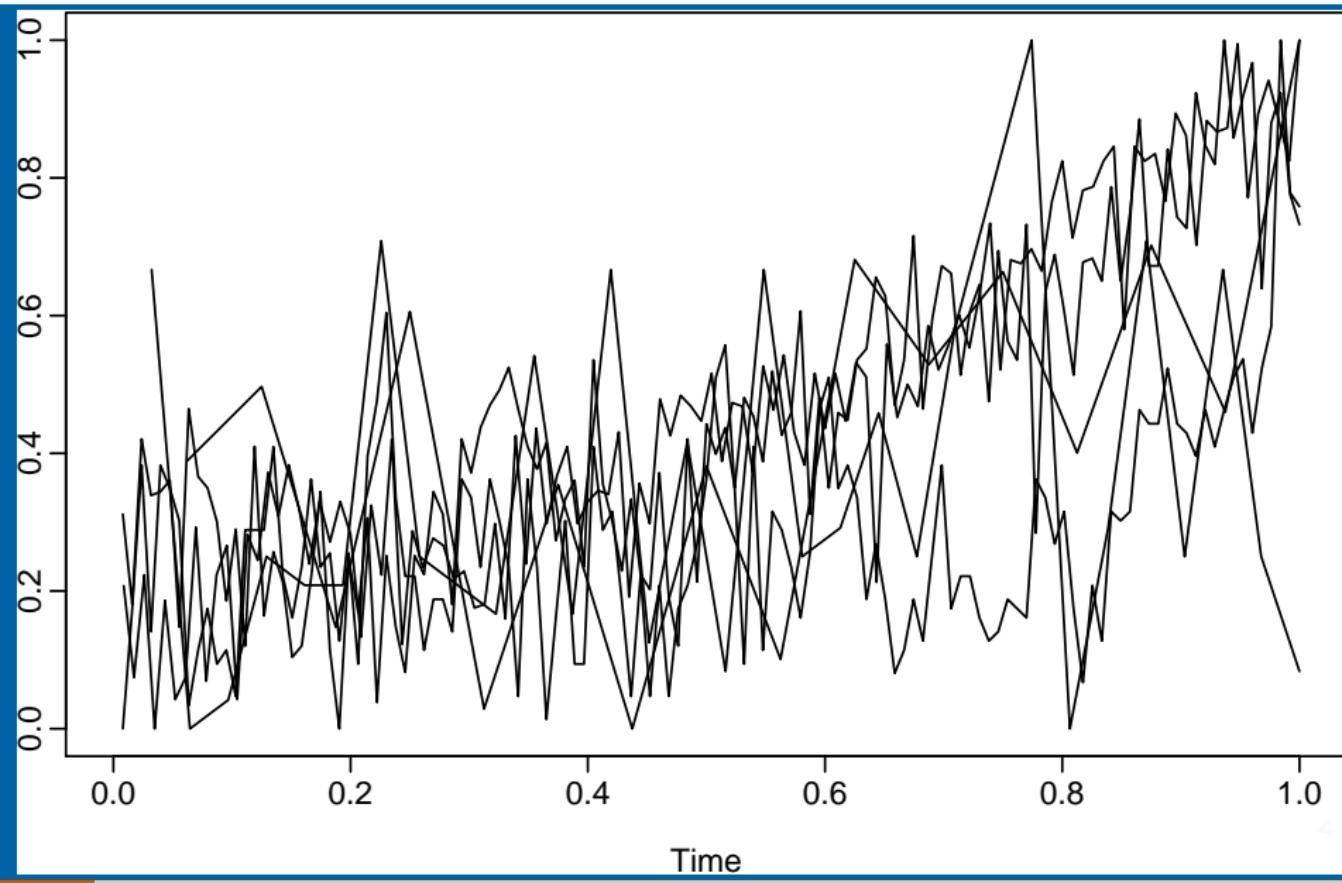
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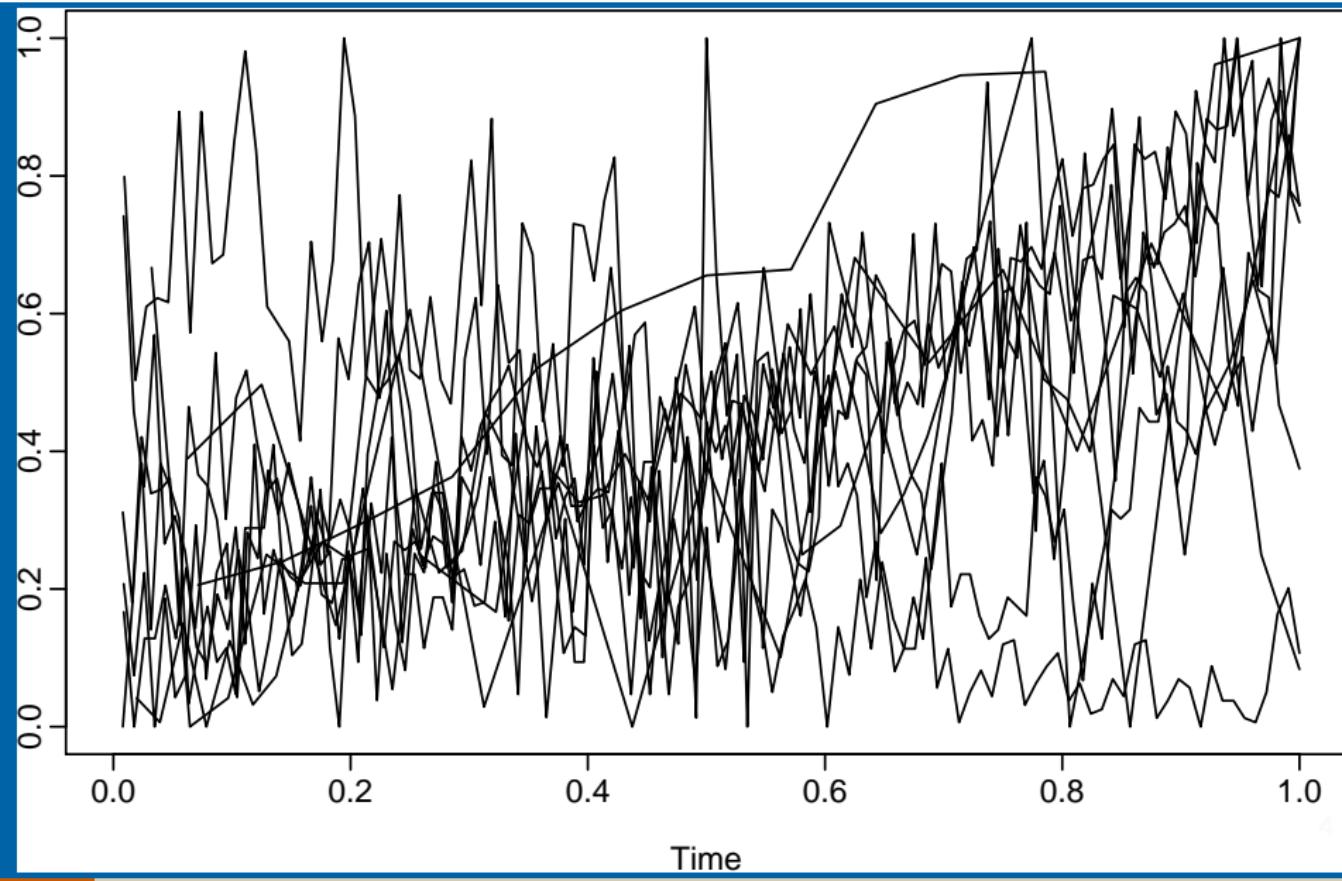
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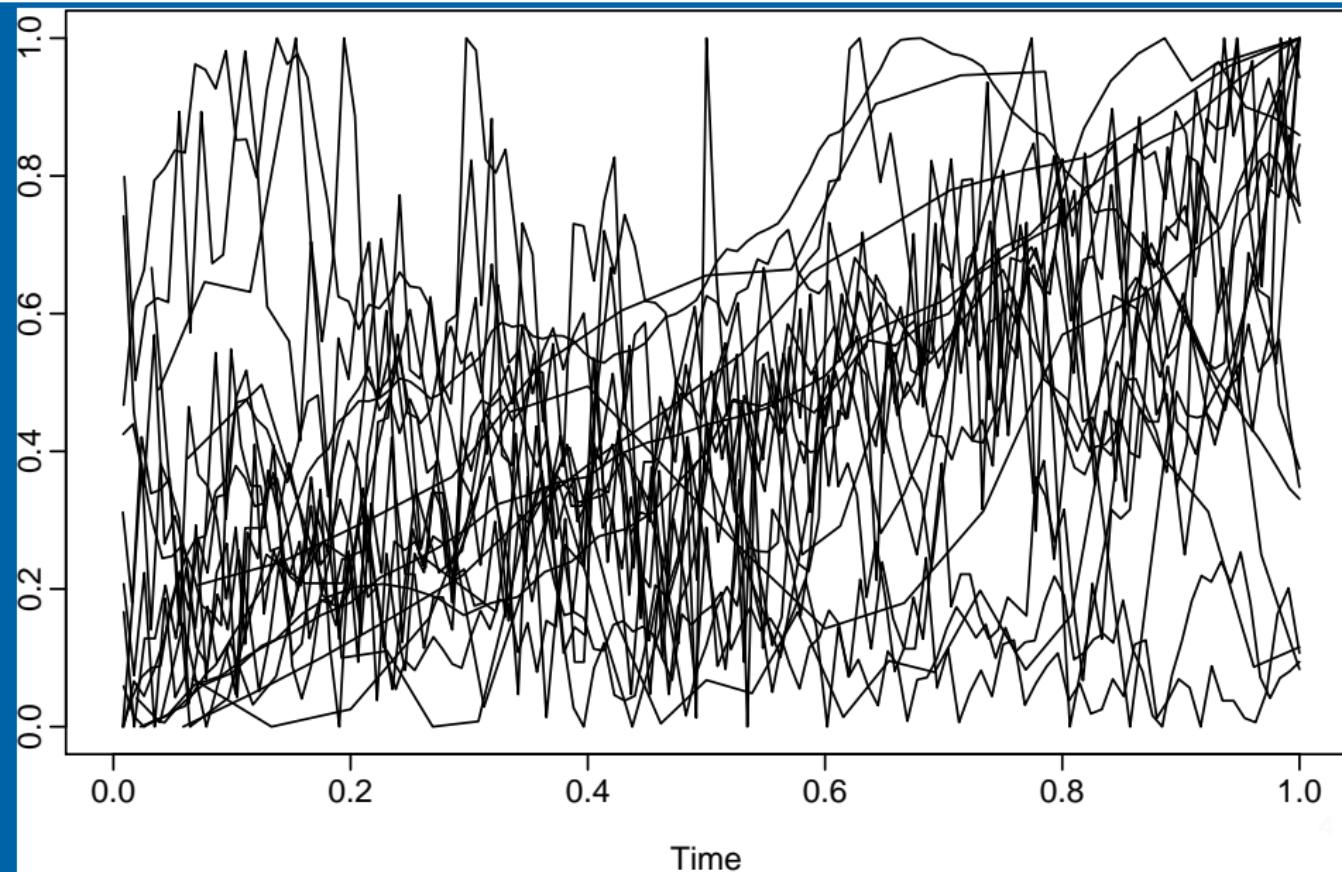
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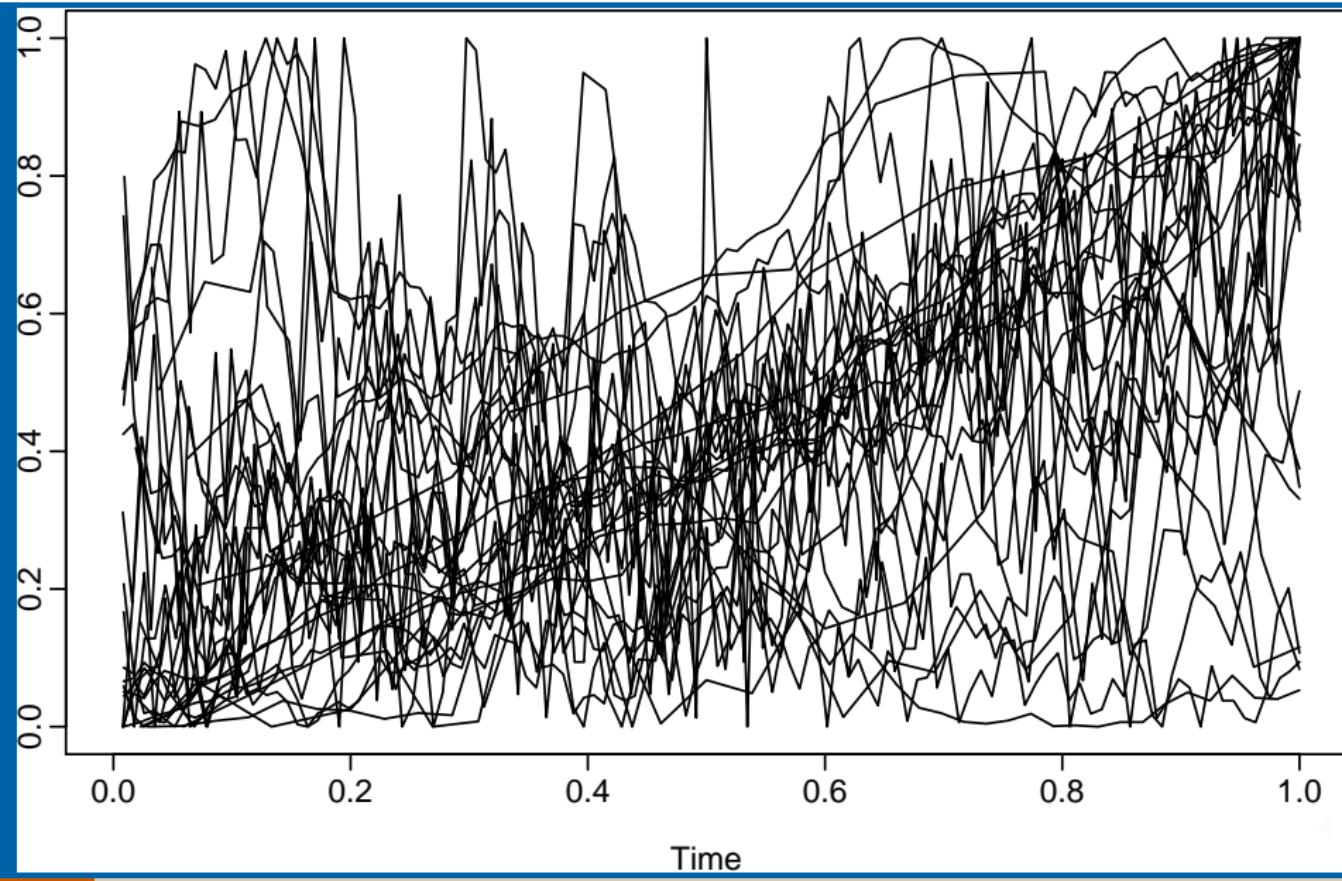
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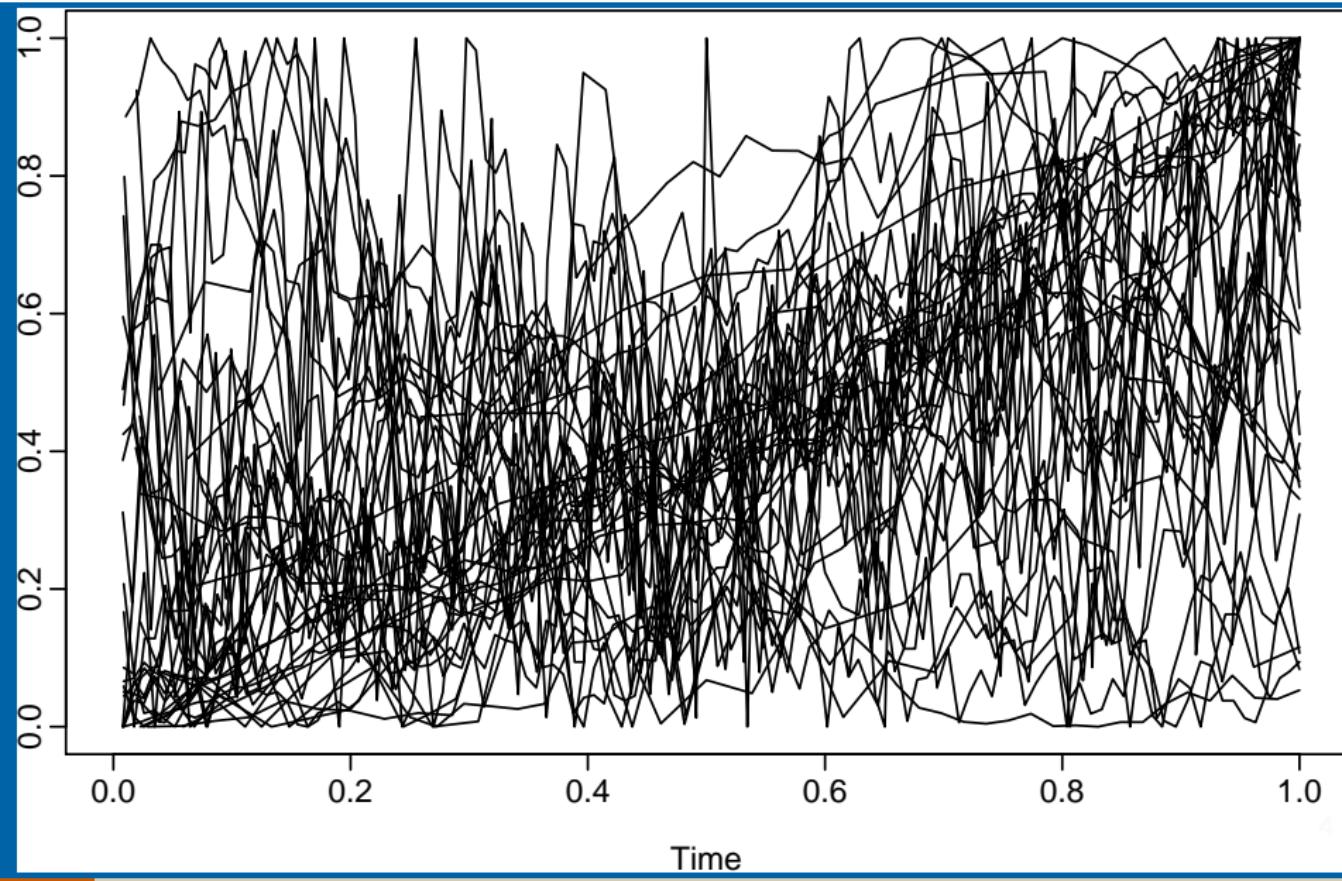
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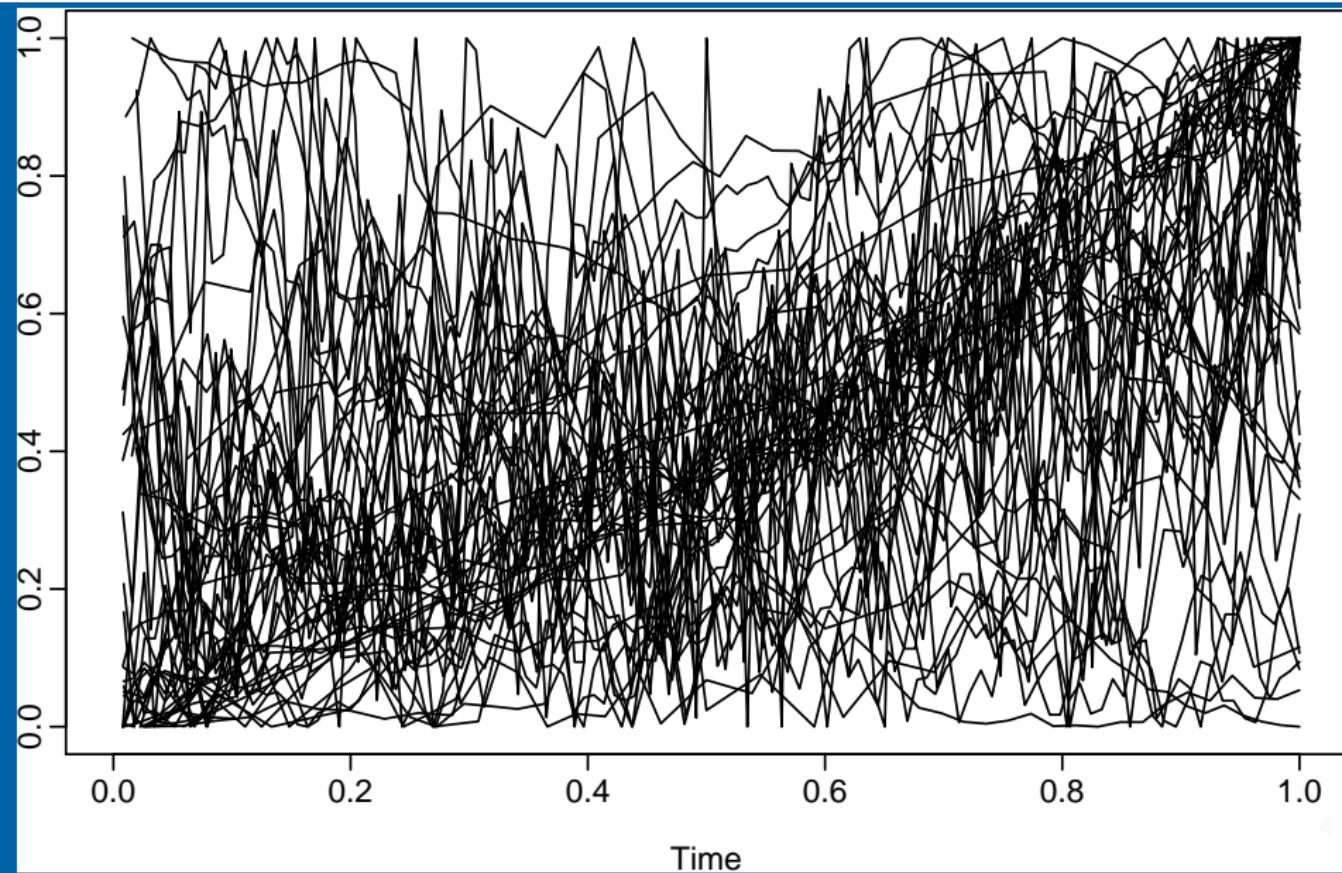
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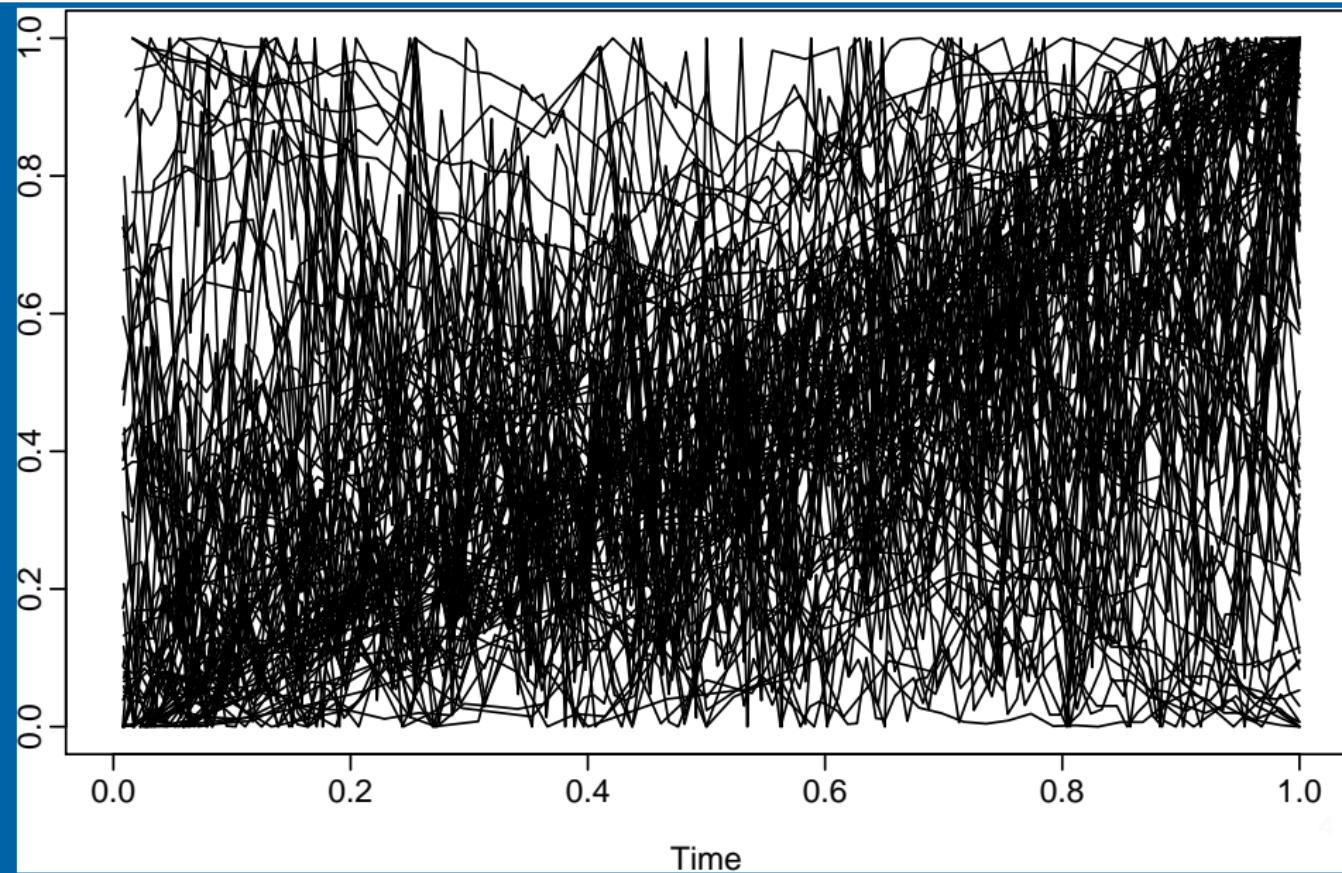
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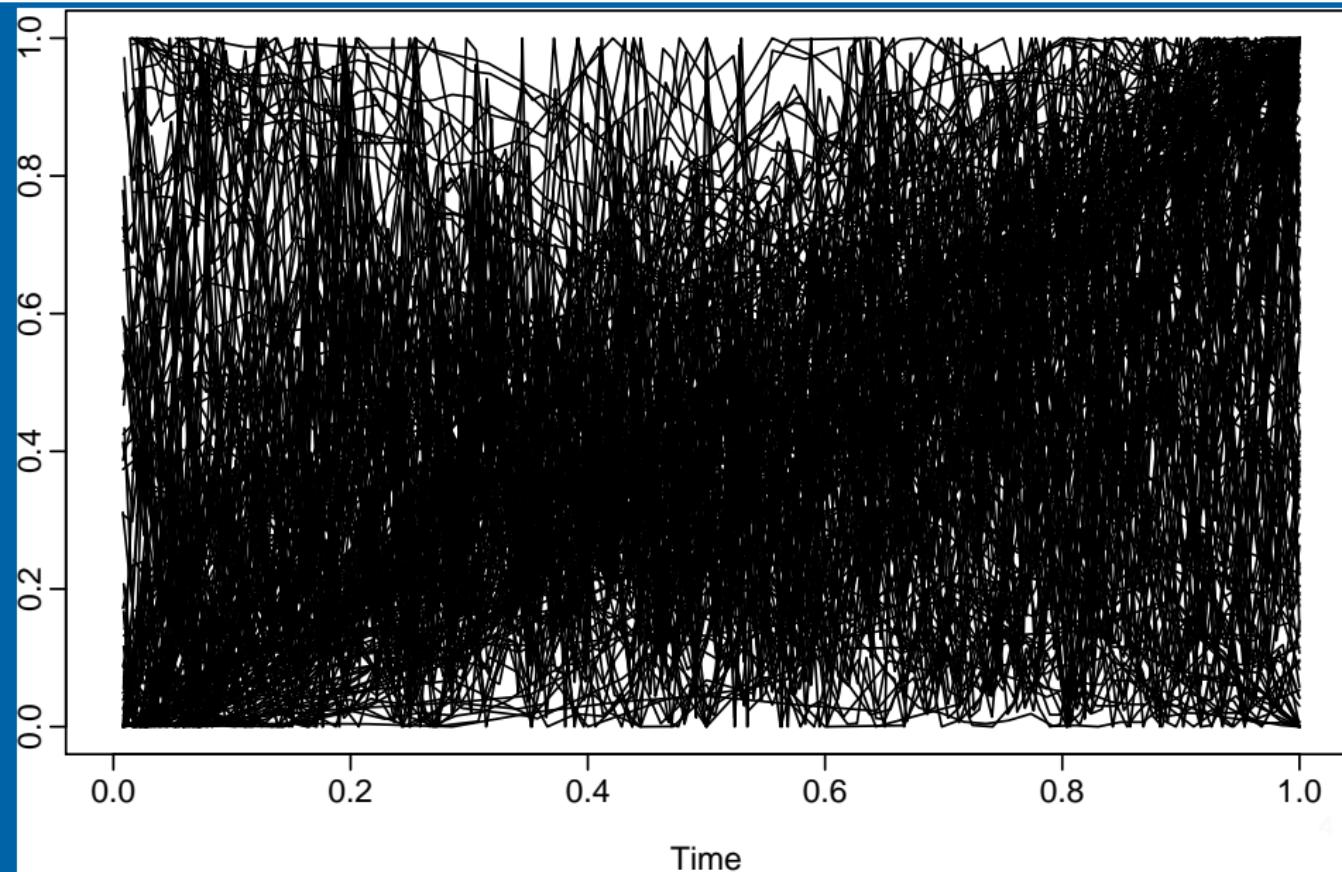
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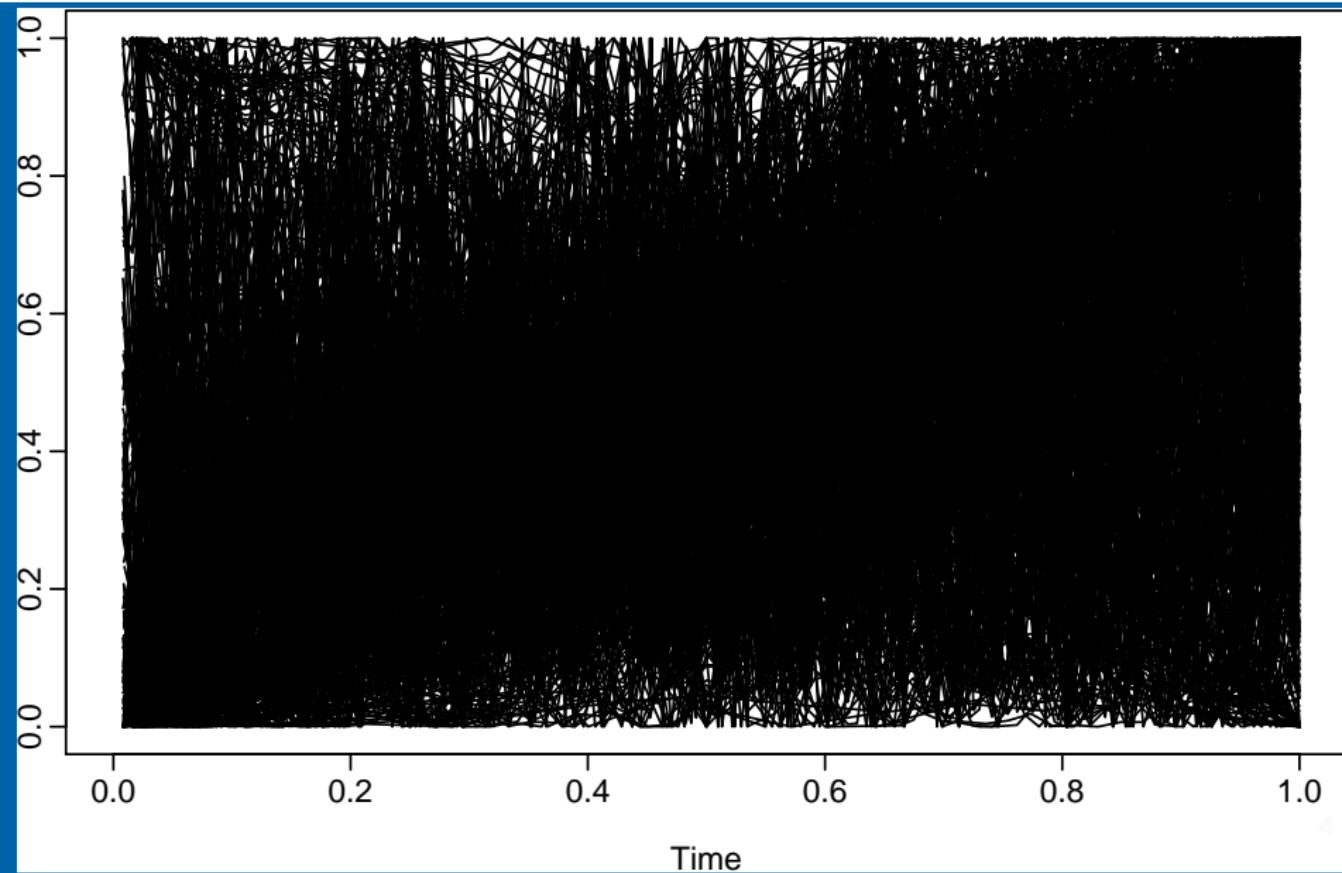
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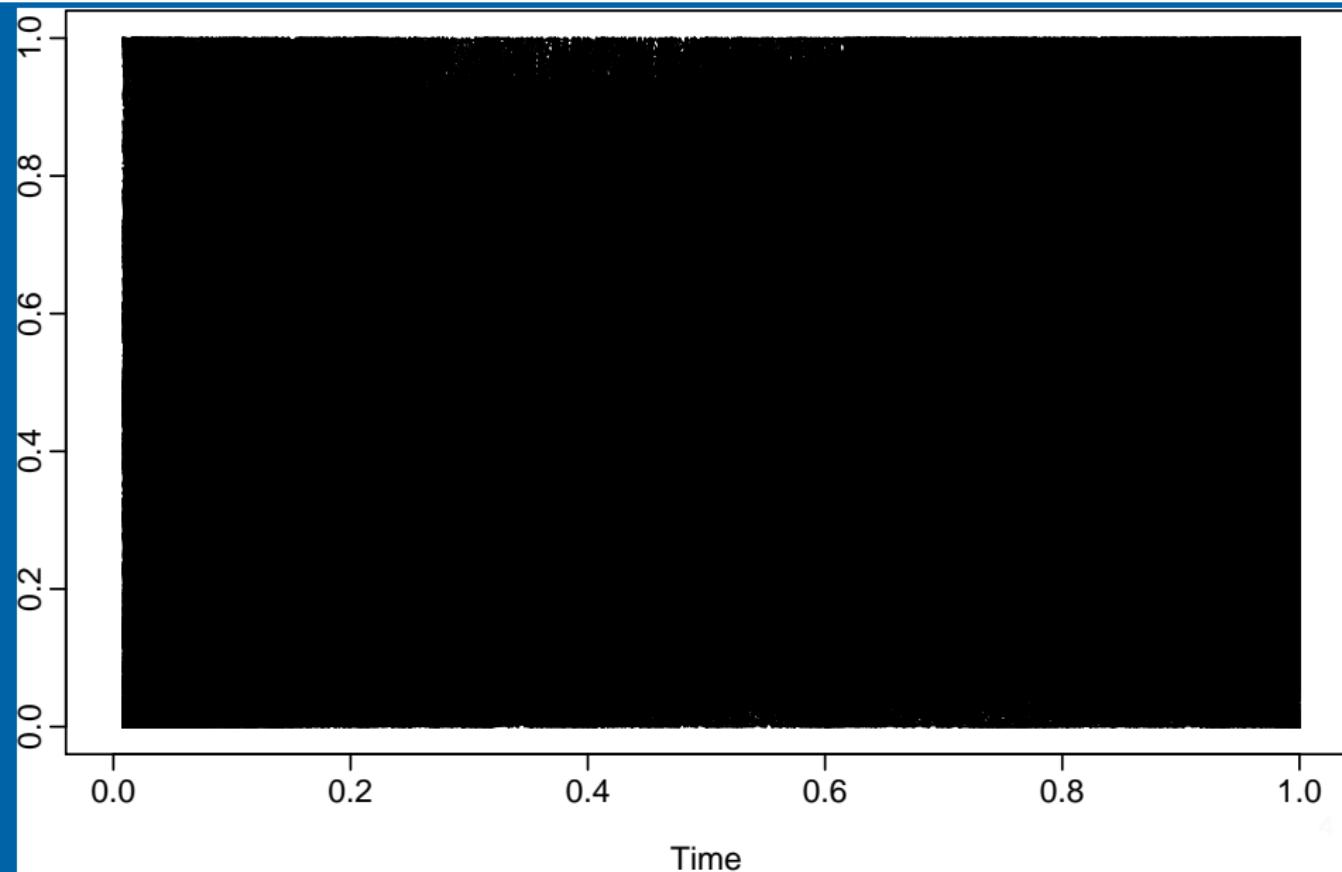
How to plot lots of time series?



How to plot lots of time series?



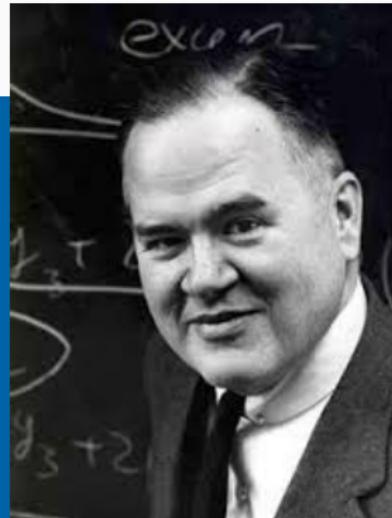
How to plot lots of time series?



Key idea

Cognostics

Computer-produced diagnostics
(Tukey and Tukey, 1985).



John W Tukey

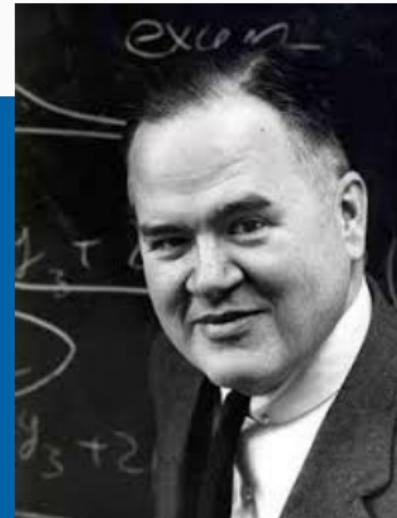
Key idea

Cognostics

Computer-produced diagnostics
(Tukey and Tukey, 1985).

Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy



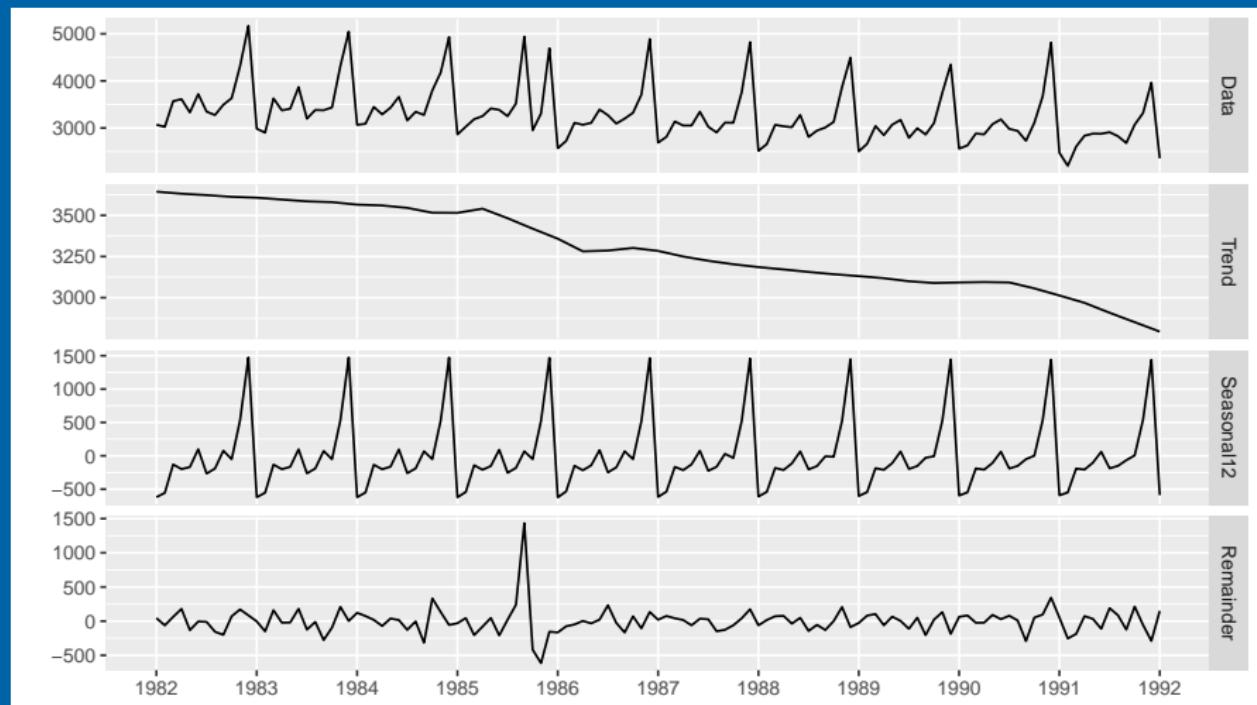
John W Tukey

Called “features” in the machine learning literature.

An STL decomposition: N2096

$$Y_t = S_t + T_t + R_t$$

S_t is periodic with mean 0



Candidate features

STL decomposition

$$Y_t = S_t + T_t + R_t$$

Candidate features

STL decomposition

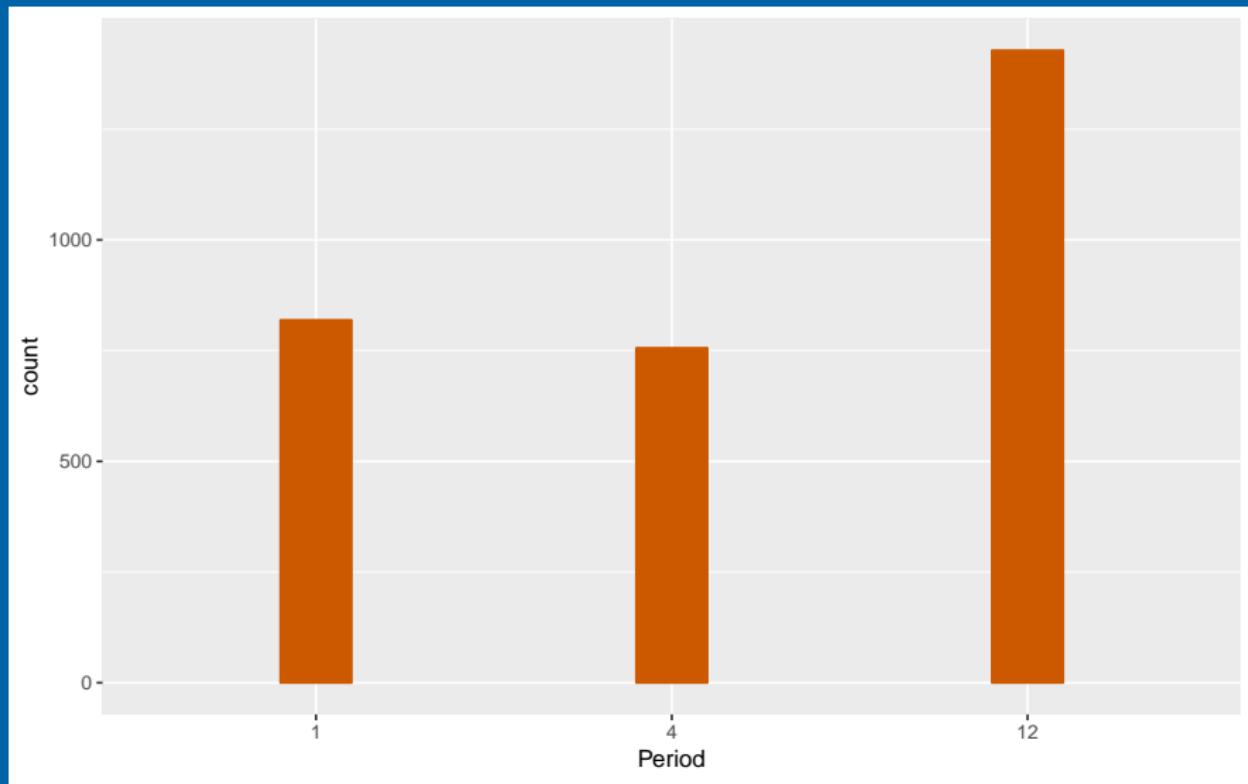
$$Y_t = S_t + T_t + R_t$$

- Seasonal period
- Autocorrelations of data (Y_1, \dots, Y_T)
- Autocorrelations of data (R_1, \dots, R_T)
- Strength of seasonality: $\max \left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)} \right)$
- Strength of trend: $\max \left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)} \right)$
- Spectral entropy: $H = - \int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$, where $f_y(\lambda)$ is spectral density of Y_t .
Low values of H suggest a time series that is easier to forecast (more signal).
- Optimal Box-Cox transformation of data

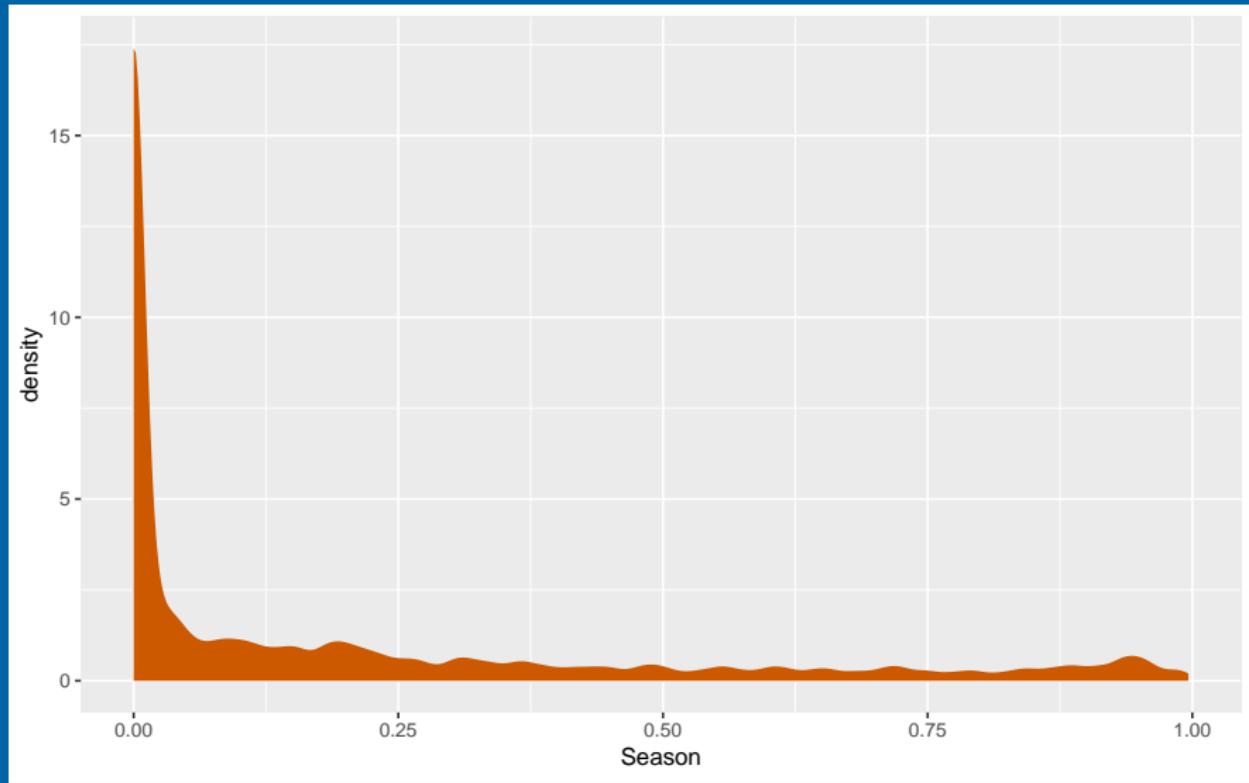
tsfeatures package

```
library(tsfeatures)
lambda_stl <- function(x,...) {
  lambda <- forecast::BoxCox.lambda(x,
    lower=0, upper=1, method='loglik')
  y <- forecast::BoxCox(x, lambda)
  c(stl_features(y,s.window='periodic', robust=TRUE, ...),
    lambda=lambda)
}
M3Features <- bind_cols(
  tsfeatures(M3data, c("frequency", "entropy")),
  tsfeatures(M3data, "lambda_stl", scale=FALSE)) %>%
  select(frequency, entropy, trend, seasonal_strength,
    e_acf1, lambda) %>%
  replace_na(list(seasonal_strength=0)) %>%
  rename(
    Frequency = frequency,
    Entropy = entropy,
    Trend = trend,
    Season = seasonal_strength,
    ACF1 = e_acf1,
    Lambda = lambda) %>%
  mutate(Period = as.factor(Frequency))
```

Distribution of Period for M3

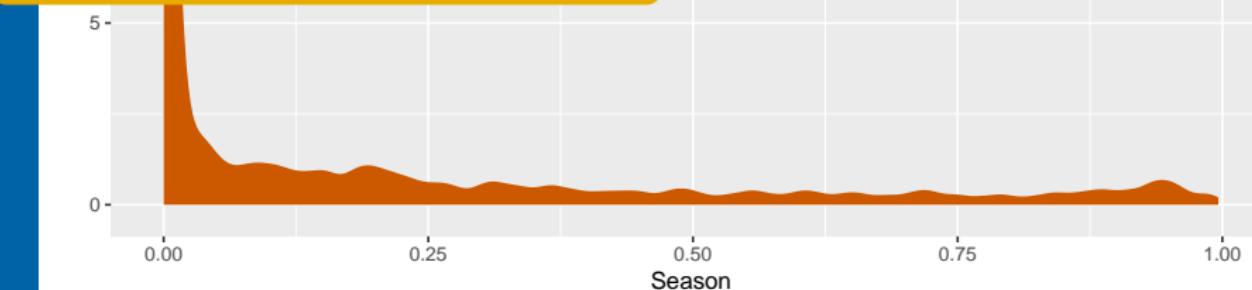
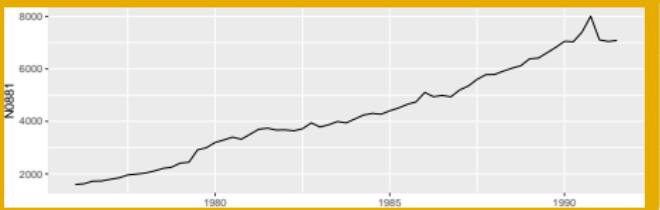


Distribution of Seasonality for M3

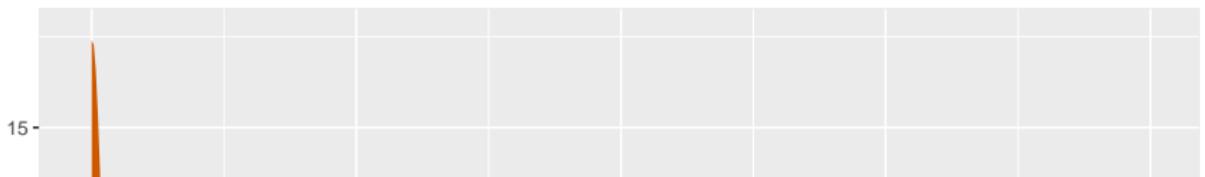


Distribution of Seasonality for M3

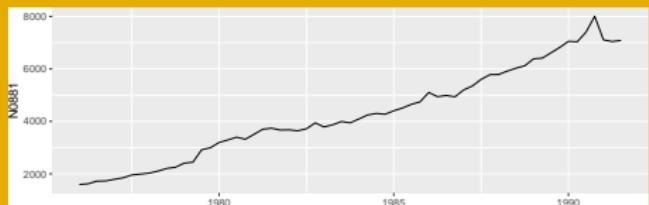
Low Seasonality



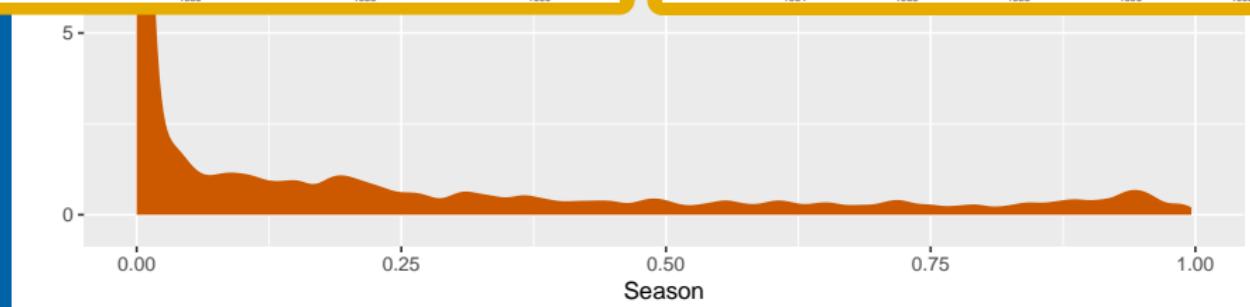
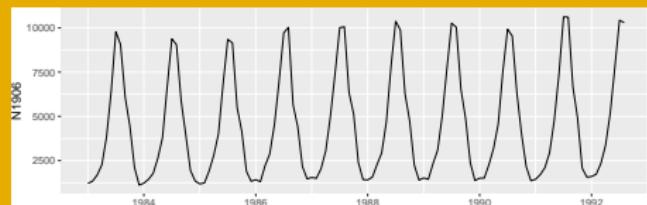
Distribution of Seasonality for M3



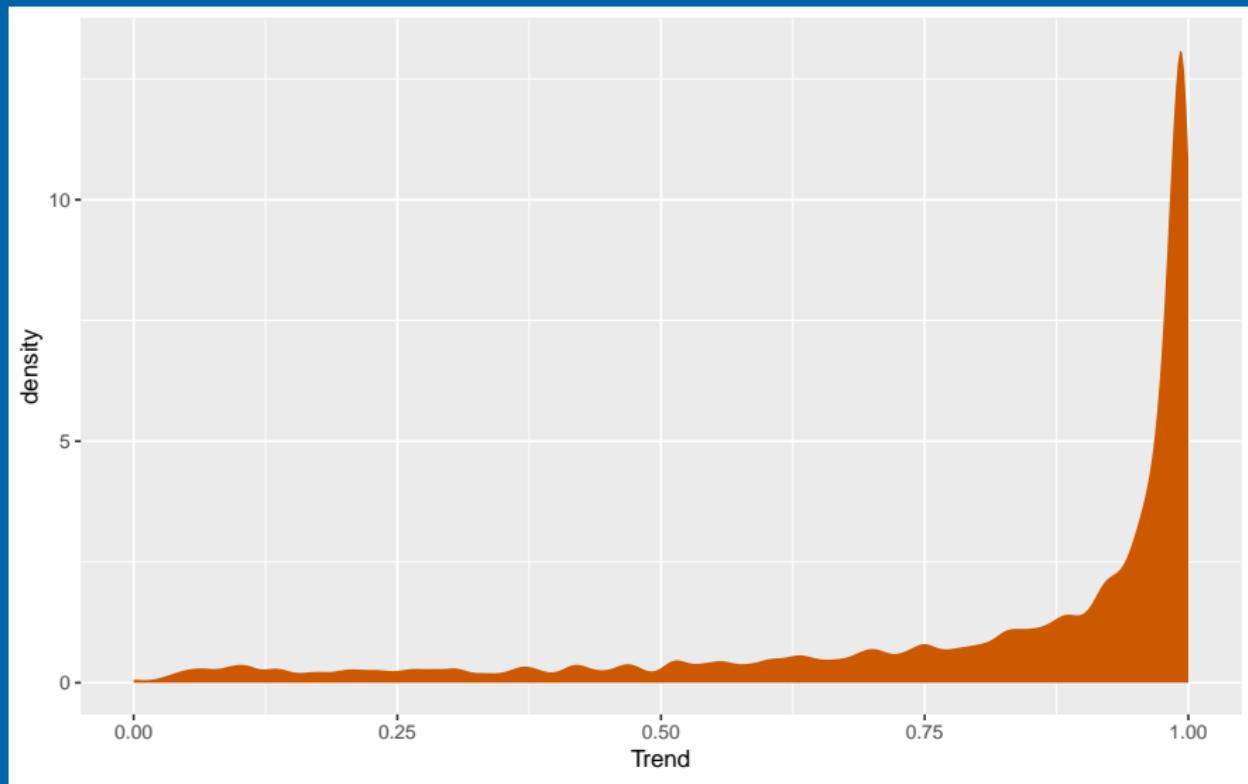
Low Seasonality



High Seasonality

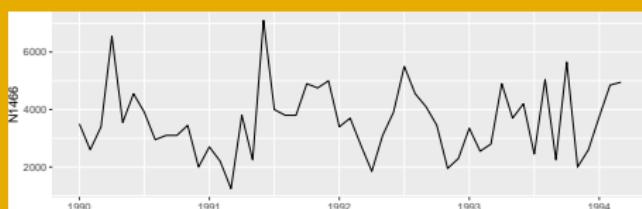


Distribution of Trend for M3



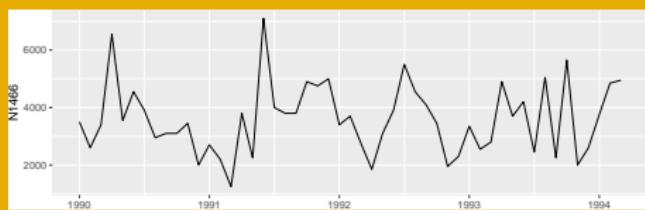
Distribution of Trend for M3

Low Trend

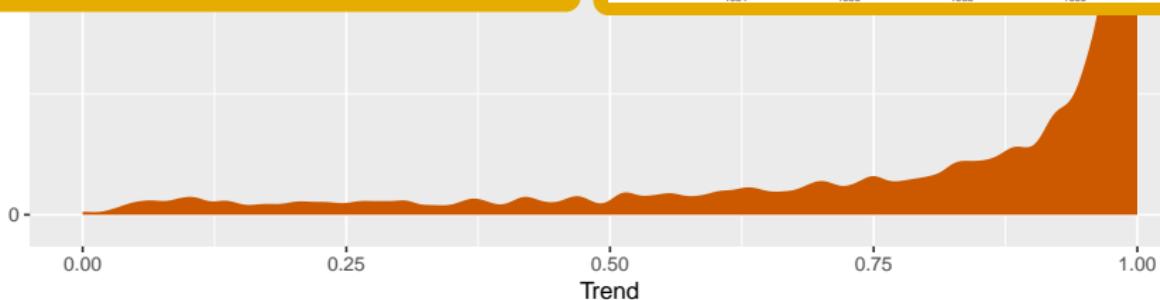
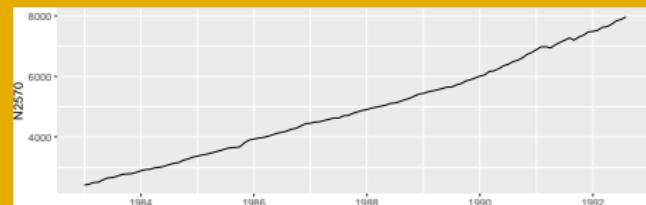


Distribution of Trend for M3

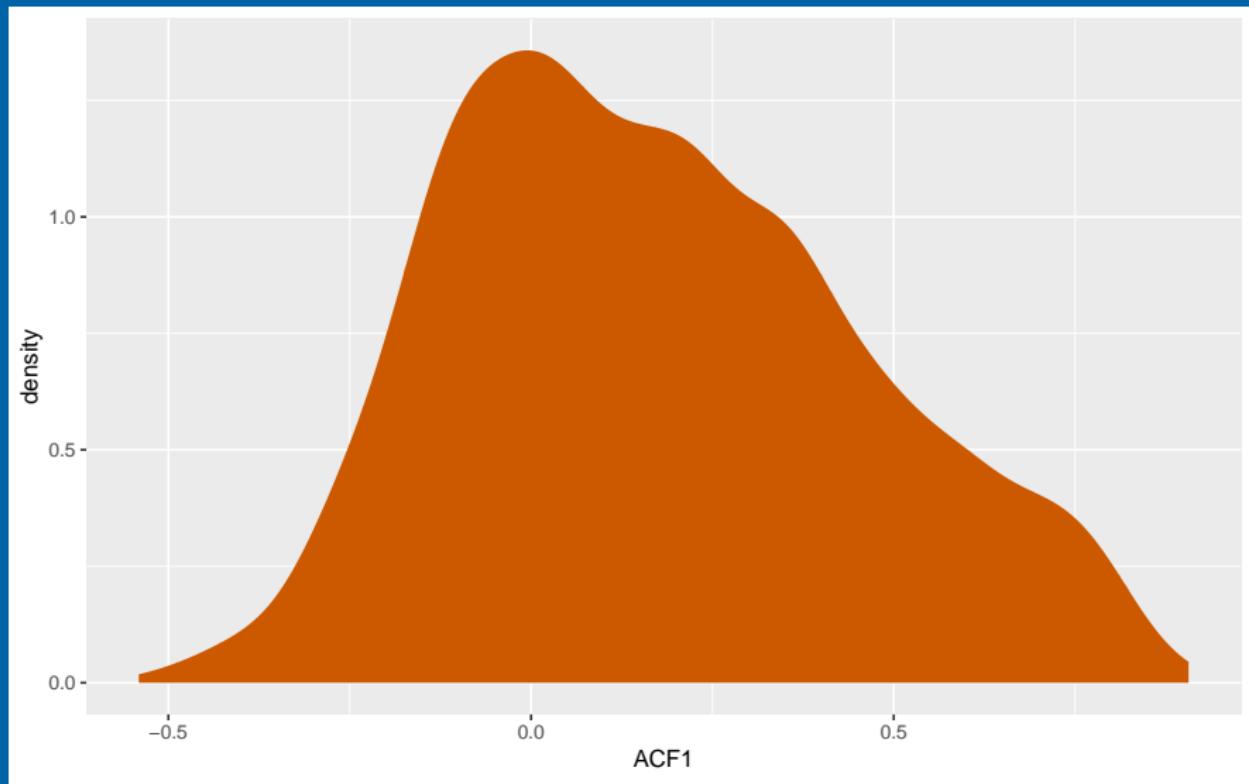
Low Trend



High Trend

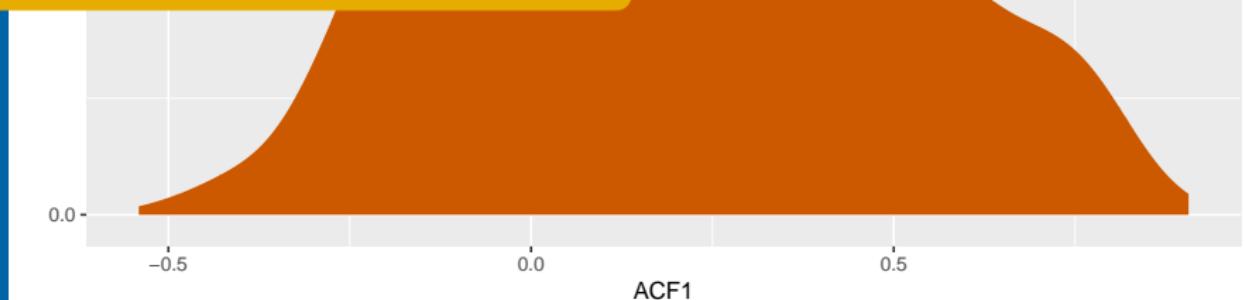
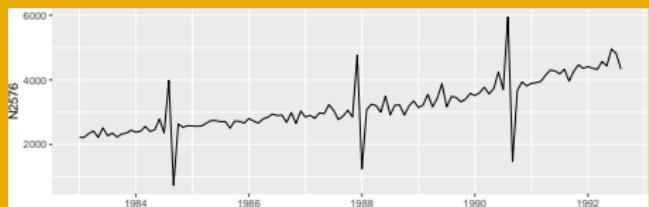


Distribution of Residual ACF1 for M3



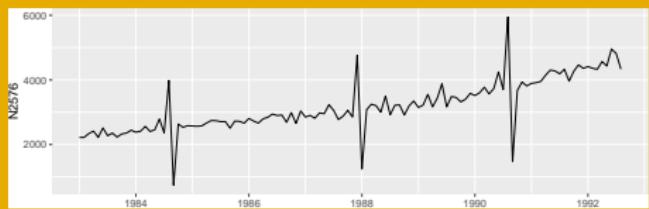
Distribution of Residual ACF1 for M3

Low ACF1

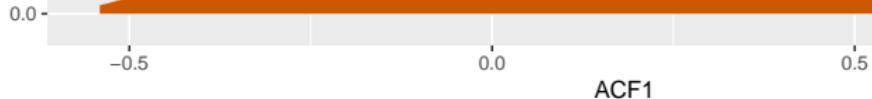
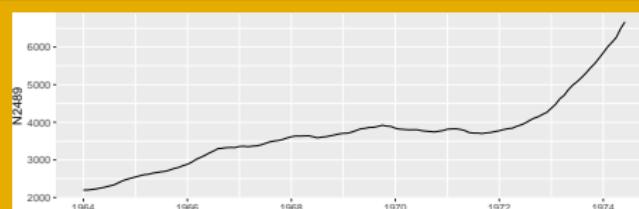


Distribution of Residual ACF1 for M3

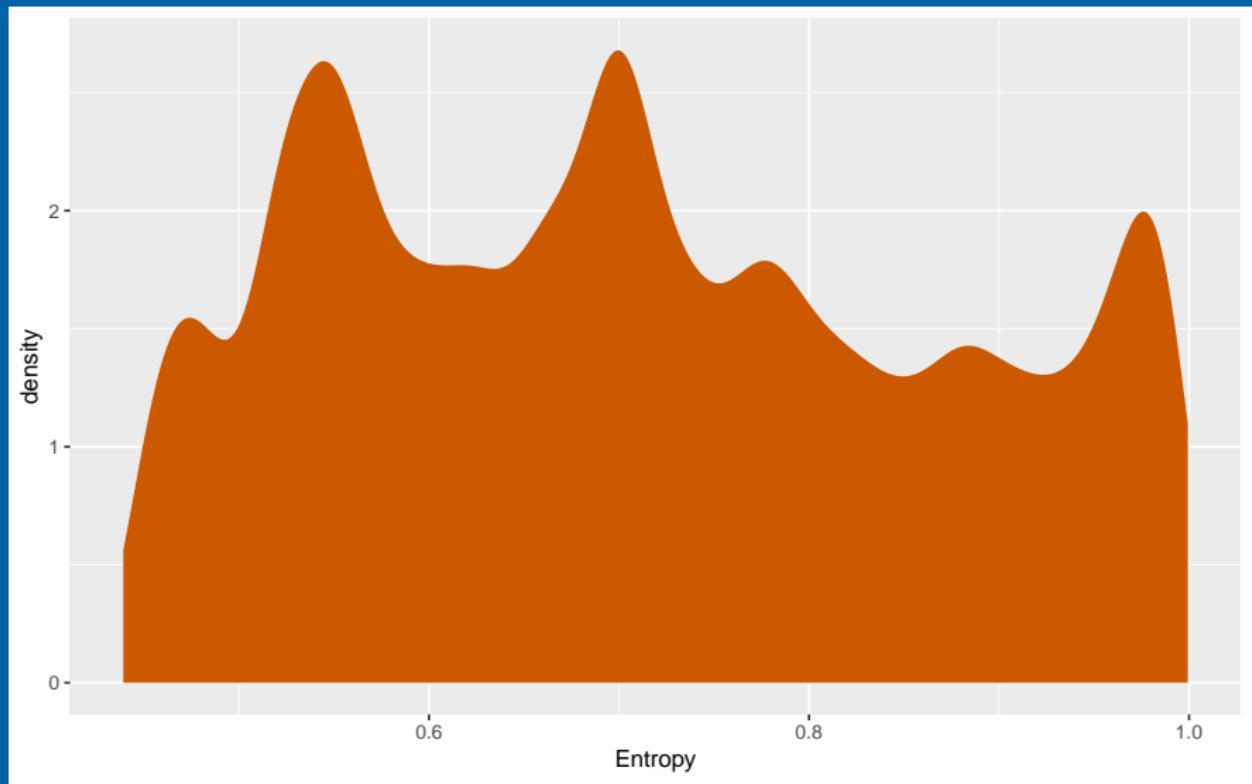
Low ACF1



High ACF1

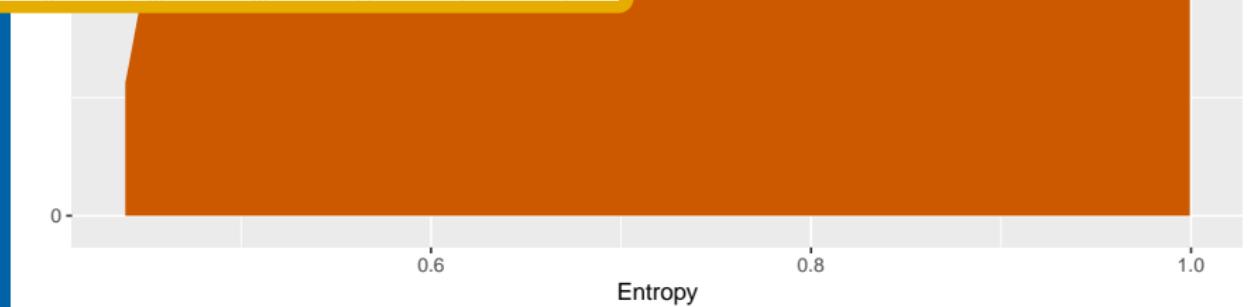
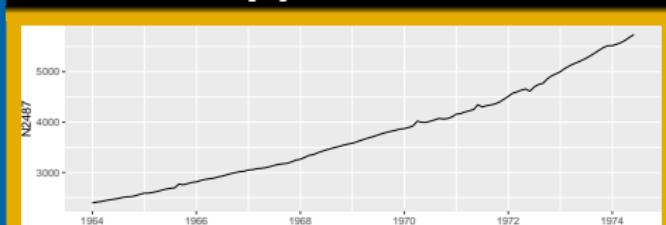


Distribution of Spectral Entropy for M3



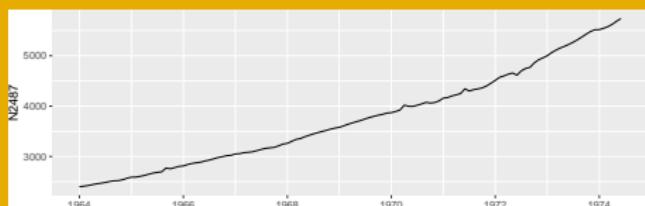
Distribution of Spectral Entropy for M3

Low Entropy

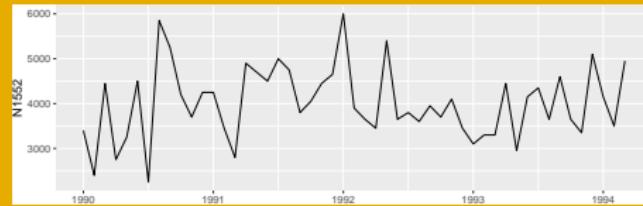


Distribution of Spectral Entropy for M3

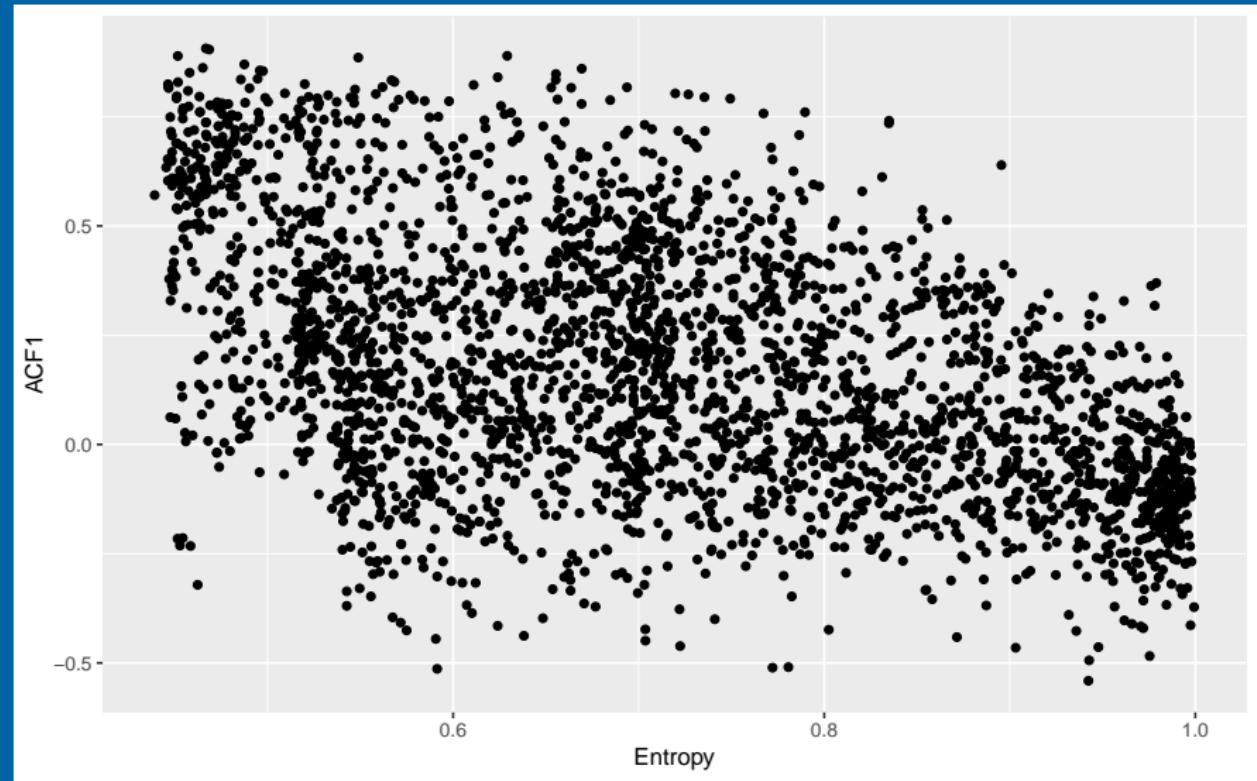
Low Entropy



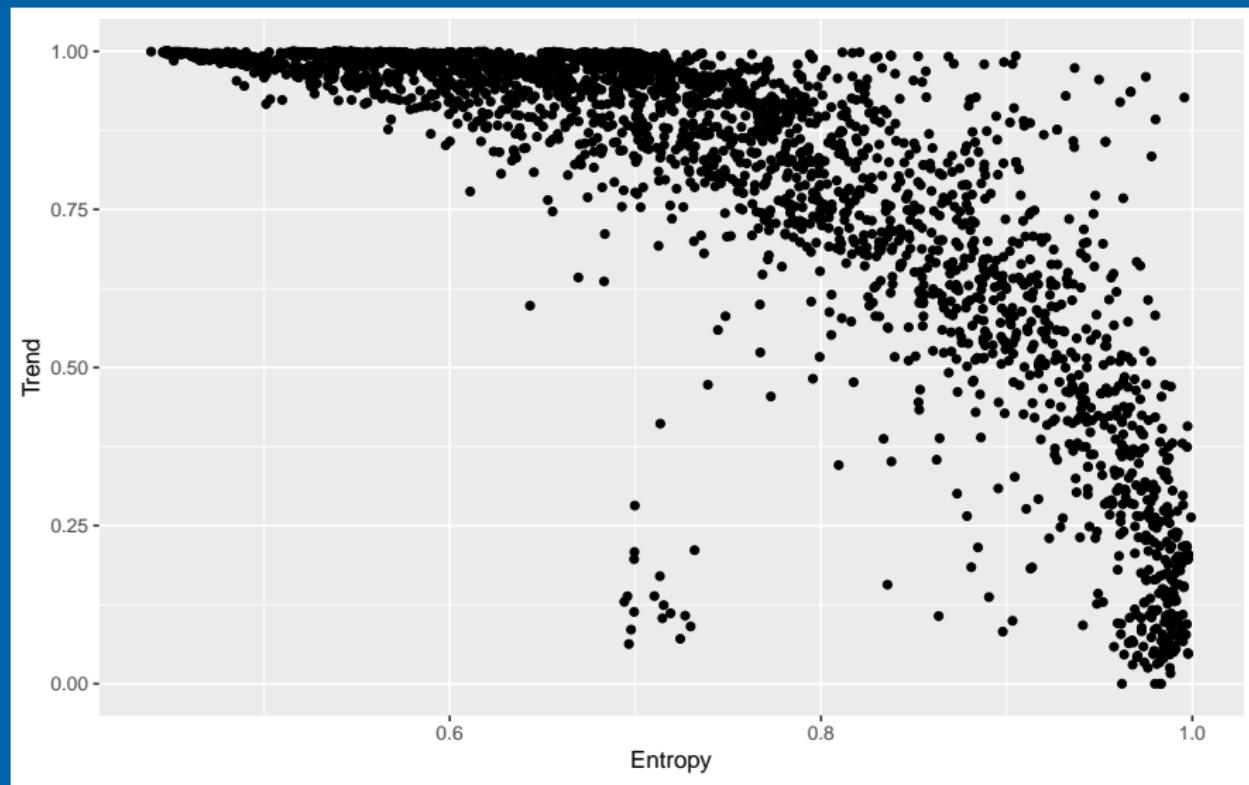
High Entropy



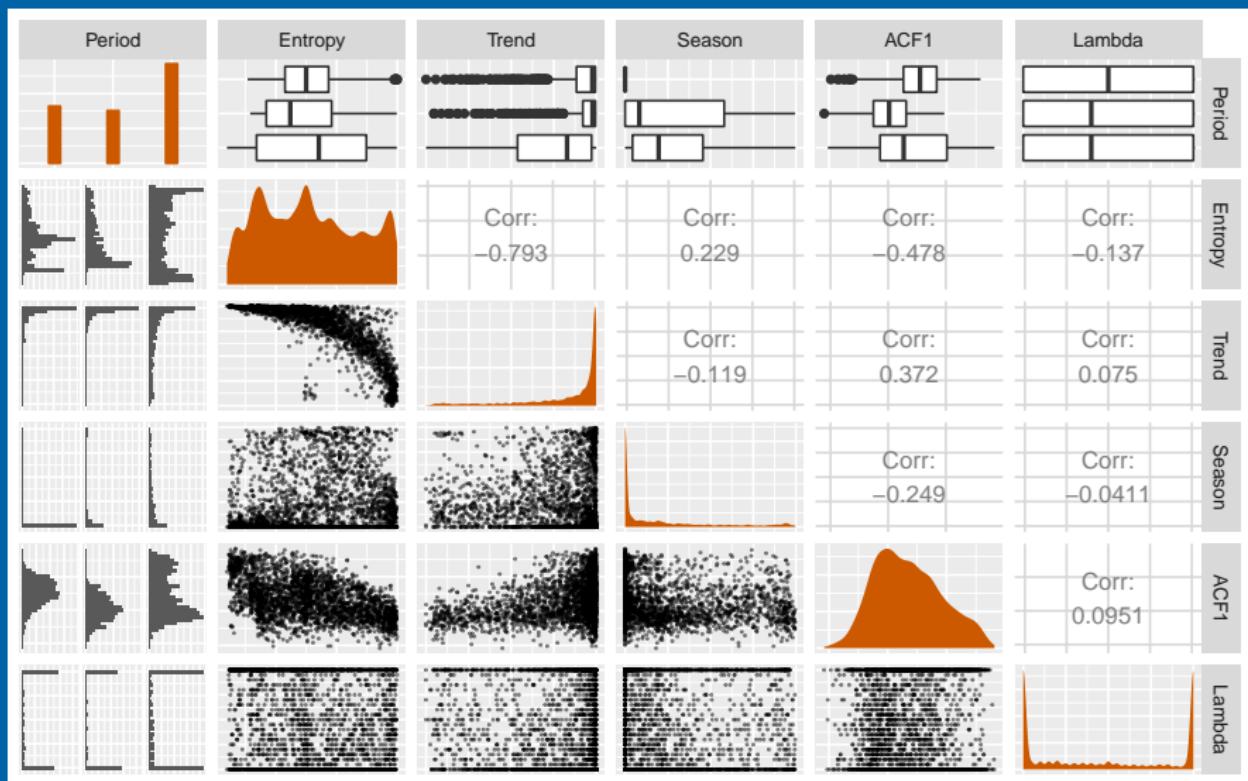
Feature distributions



Feature distributions



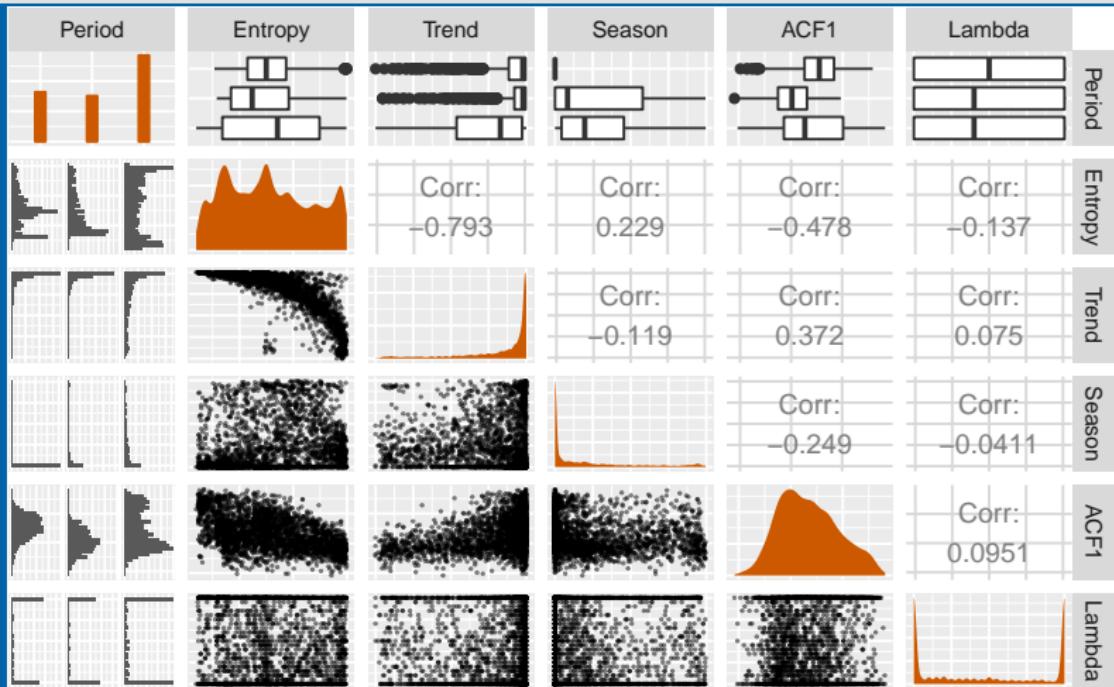
Feature distributions



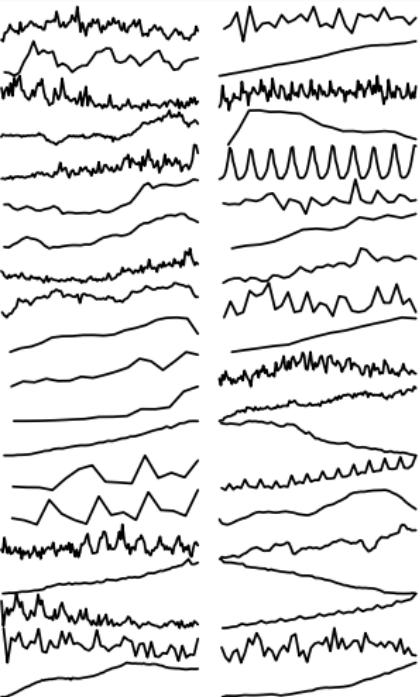
Feature distributions

M3Features %>%

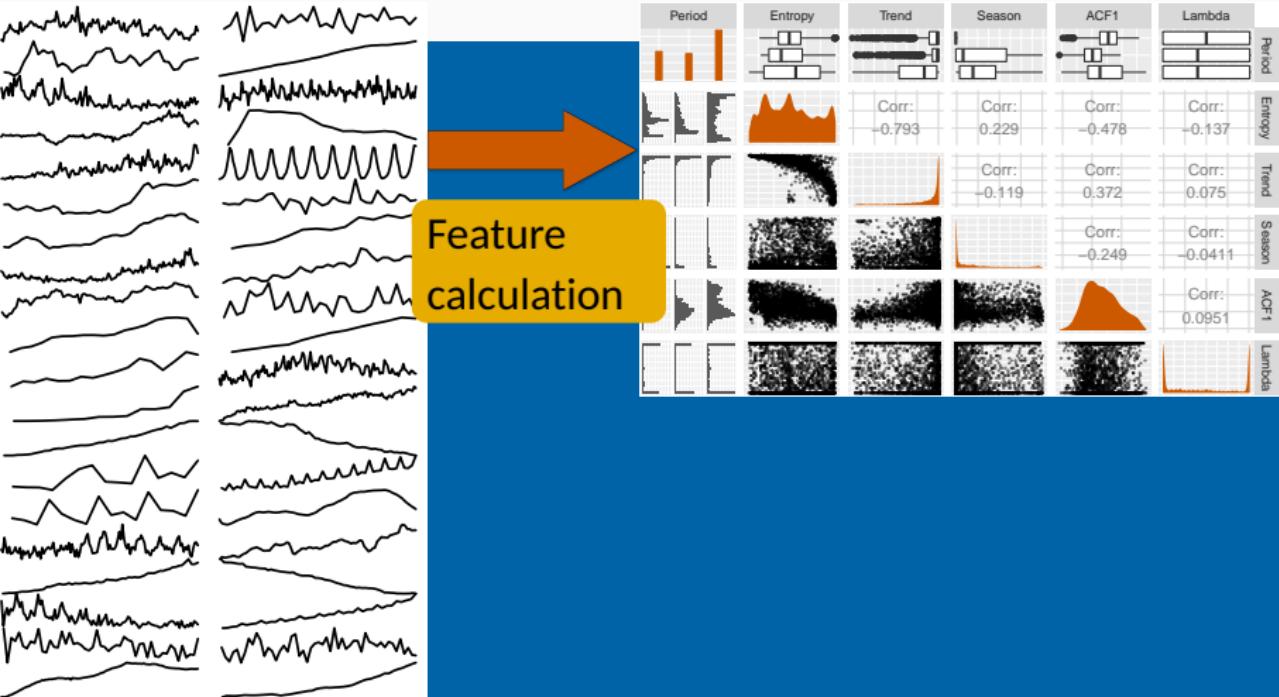
```
select(Period, Entropy, Trend, Season, ACF1, Lambda) %>%  
GGally::ggpairs()
```



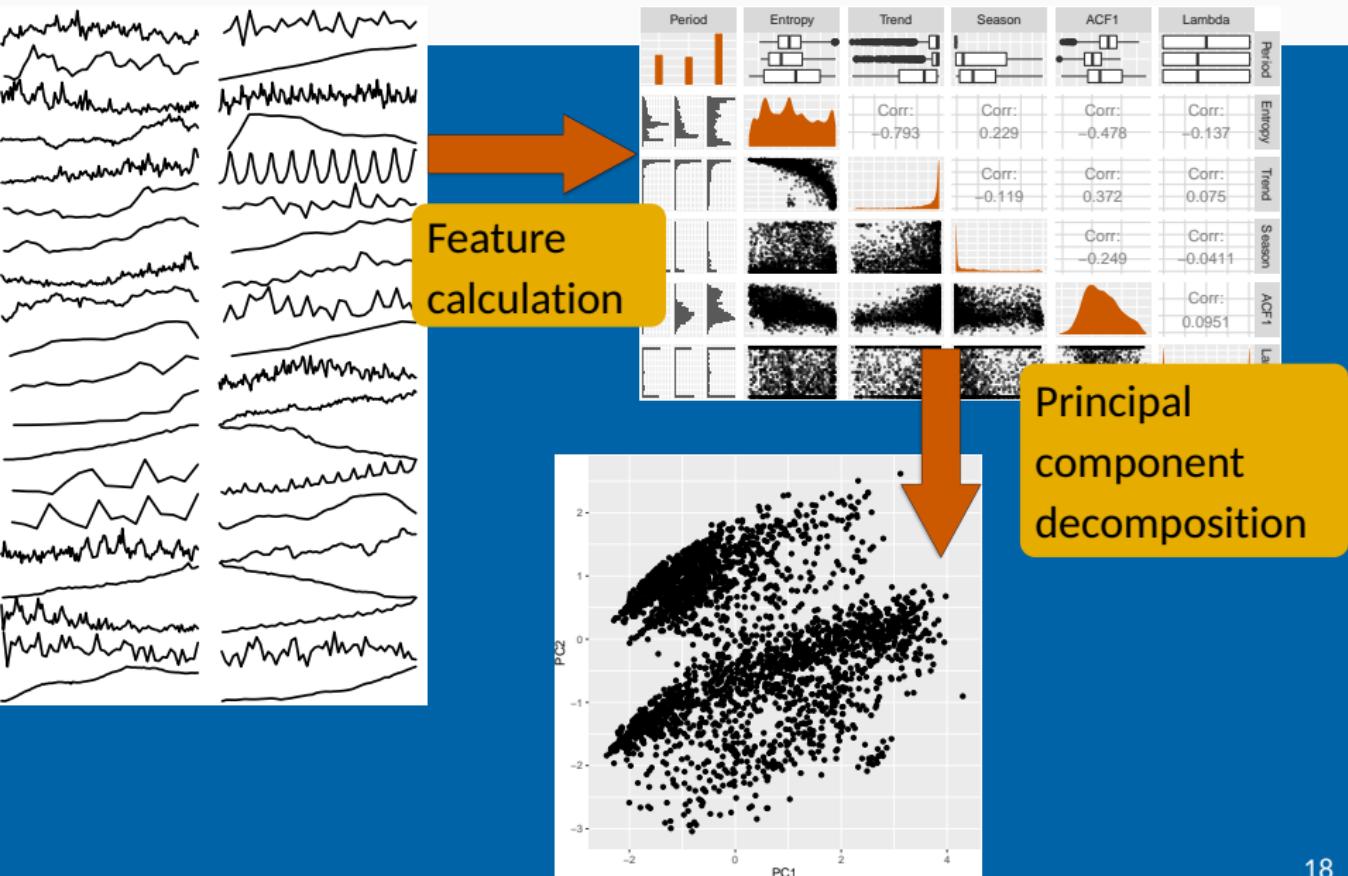
Dimension reduction for time series



Dimension reduction for time series

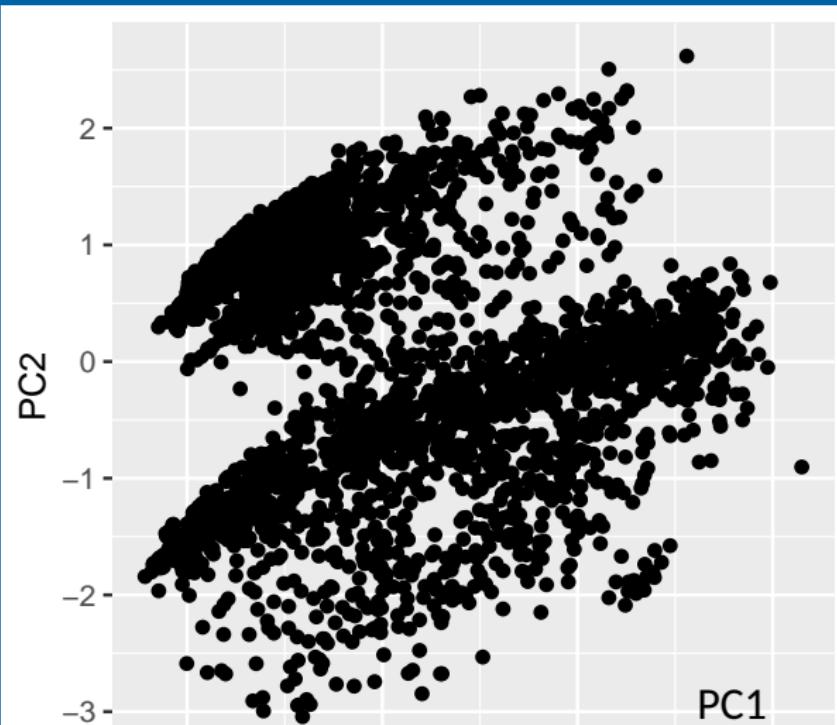


Dimension reduction for time series



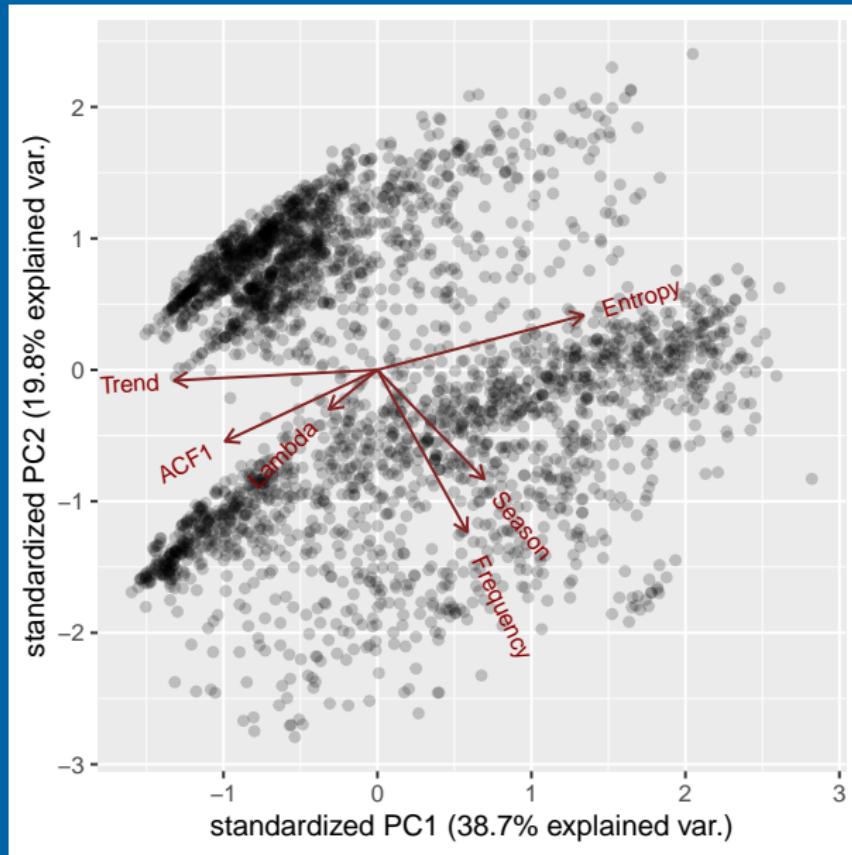
M3 feature space

```
prcomp(select(M3Features, -Period), scale=TRUE)$x %>%  
  ggplot(aes(x=PC1, y=PC2))
```

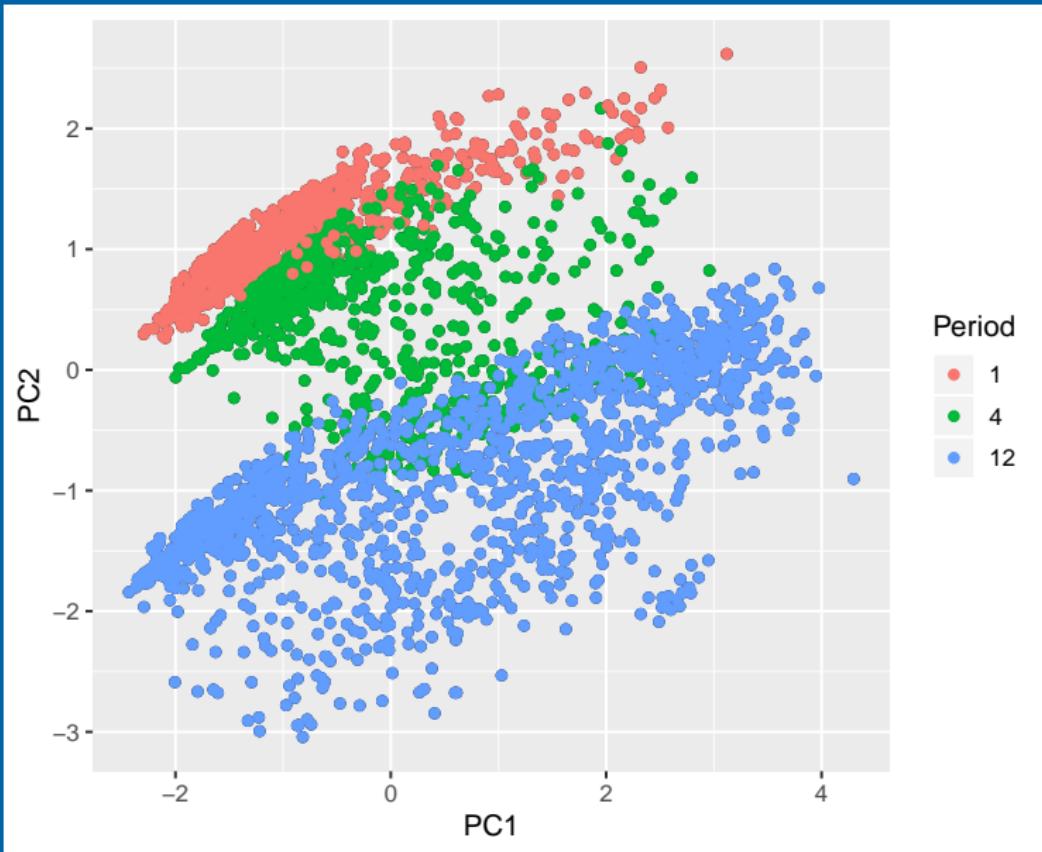


First two PCs
explain 58.5% of
the variance.

M3 feature space



M3 feature space



Papers and packages



Kang, Hyndman, & Smith-Miles, K. (2017)
Visualising forecasting algorithm performance using time series instance spaces.
IJF, 33(2) 345–358.



Hyndman, Wang, Kang, Talagala &
Montero-Manso (2018). **tsfeatures**: Time
Series Feature Extraction.
[github.com/robjhyndman/tsfeatures/](https://github.com/robjhyndman/tsfeatures)

Outline

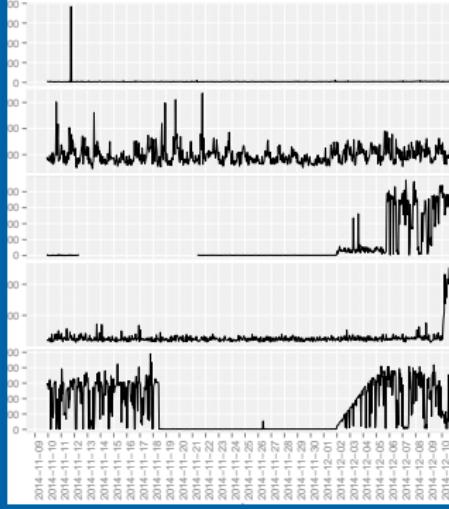
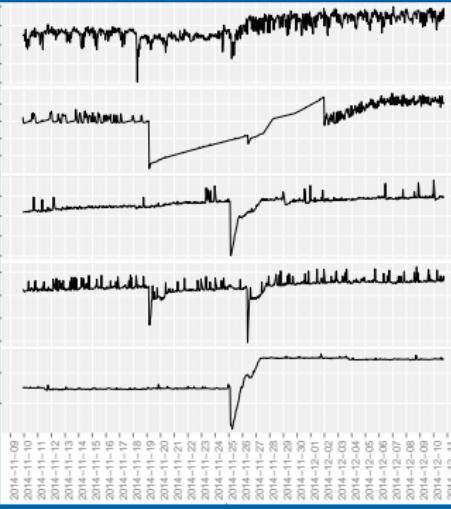
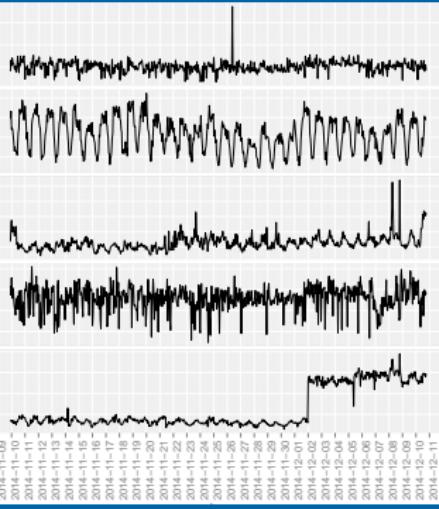
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Yahoo web-traffic

- Tens of thousands of time series collected at one-hour intervals over one month.
- Consisting of several server metrics (e.g. CPU usage and paging views) from many server farms globally.
- Aim: find unusual (anomalous) time series.



Yahoo web-traffic



Feature space

- ACF1: first order autocorrelation = $\text{Corr}(Y_t, Y_{t-1})$
- Strength of **trend** and **seasonality** based on STL
- Trend **linearity** and **curvature**
- Size of seasonal **peak** and **trough**
- Spectral **entropy**
- **Lumpiness**: variance of block variances (block size 24).
- **Spikiness**: variances of leave-one-out variances of STL remainders.
- **Level shift**: Maximum difference in trimmed means of consecutive moving windows of size 24.
- **Variance change**: Max difference in variances of consecutive moving windows of size 24.
- **Flat spots**: Discretize sample space into 10 equal-sized intervals. Find max run length in any interval.
- Number of **crossing points** of mean line.
- **Kullback-Leibler score**: Maximum of $D_{KL}(P||Q) = \int P(x) \ln P(x)/Q(x)dx$ where P and Q are estimated by kernel density estimators applied to consecutive windows of size 48.
- **Change index**: Time of maximum KL score

What is “anomalous”?

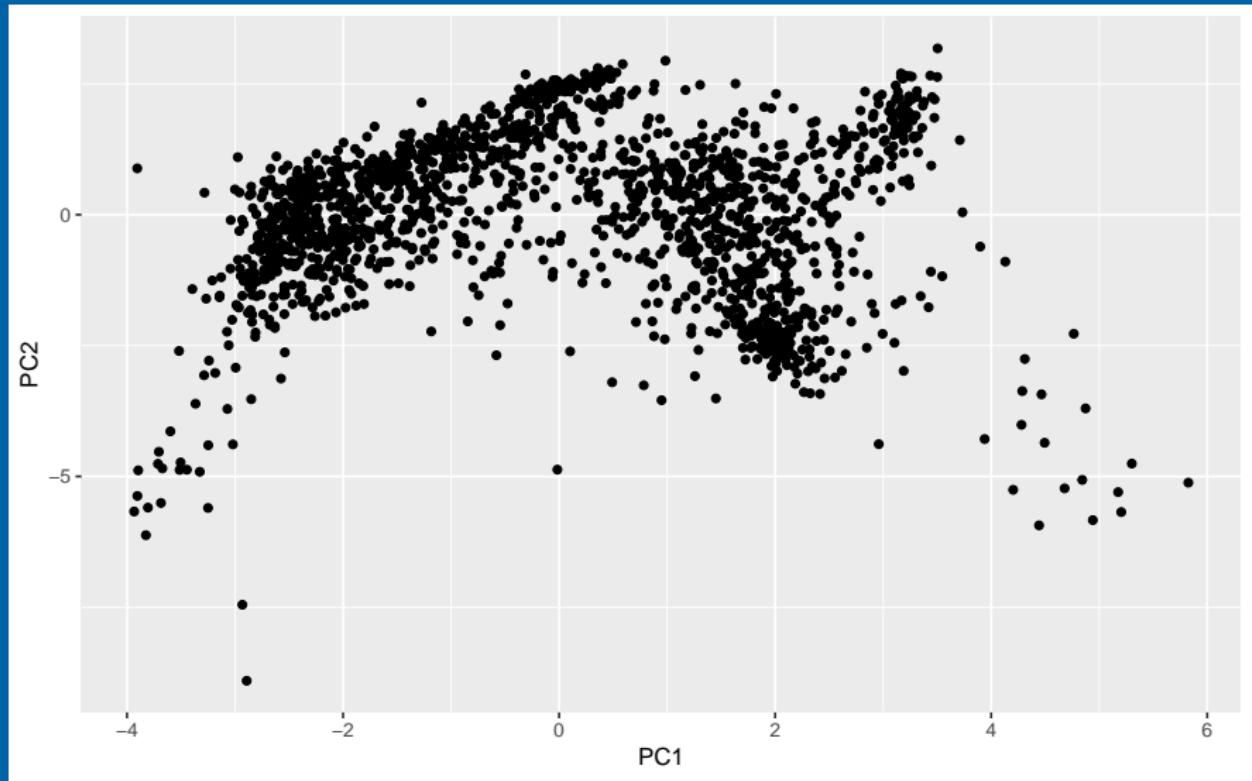
- We need a measure of the “anomalousness” of a time series.
- Idea: We rank points based on their local density using a bivariate kernel density estimate.

Bivariate kernel density

$$\hat{f}(\mathbf{x}; \mathbf{H}) = \frac{1}{n} \sum_{i=1}^n K_{\mathbf{H}}(\mathbf{x} - \mathbf{X}_i)$$

- \mathbf{X}_i ∈ a bivariate random sample $\{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n\}$
- $K_{\mathbf{H}}(\mathbf{x})$ is the standard normal kernel function
- \mathbf{H} estimated by minimizing the sum of AMISE
- Rank points based on \hat{f} values in 2d PCA space.

Hyndman, Wang and Laptev (ICDM 2015)



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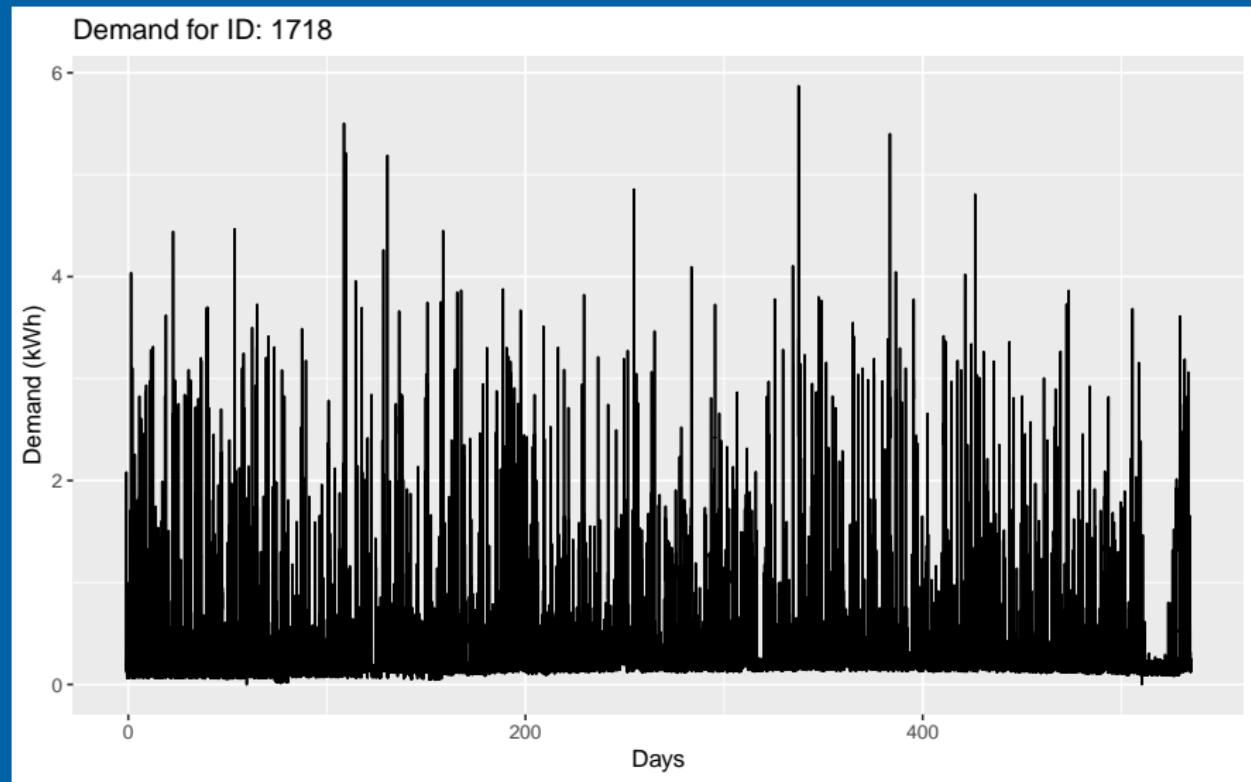
Irish smart metre data



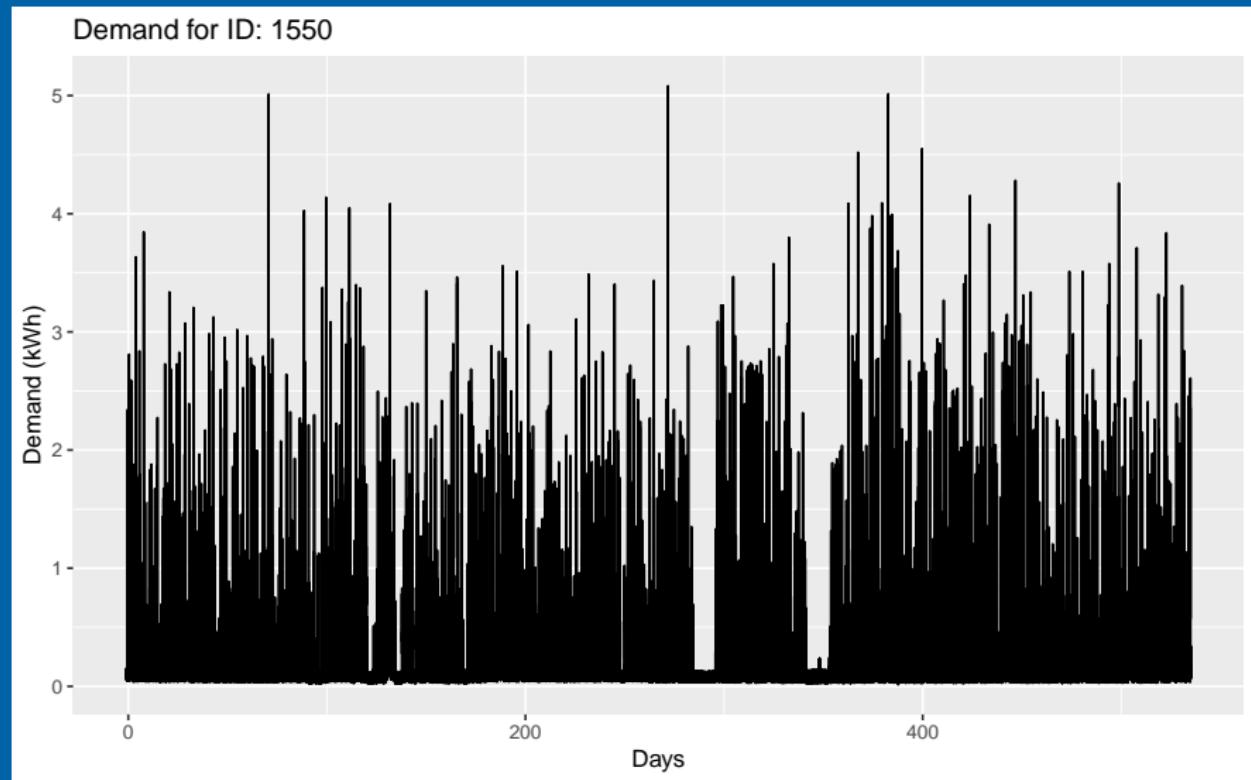
Figure: <http://solutions.3m.com>

- 500 households from smart metering trial
- Electricity consumption at 30-minute intervals between 14 July 2009 and 31 December 2010
- Heating/cooling energy usage excluded

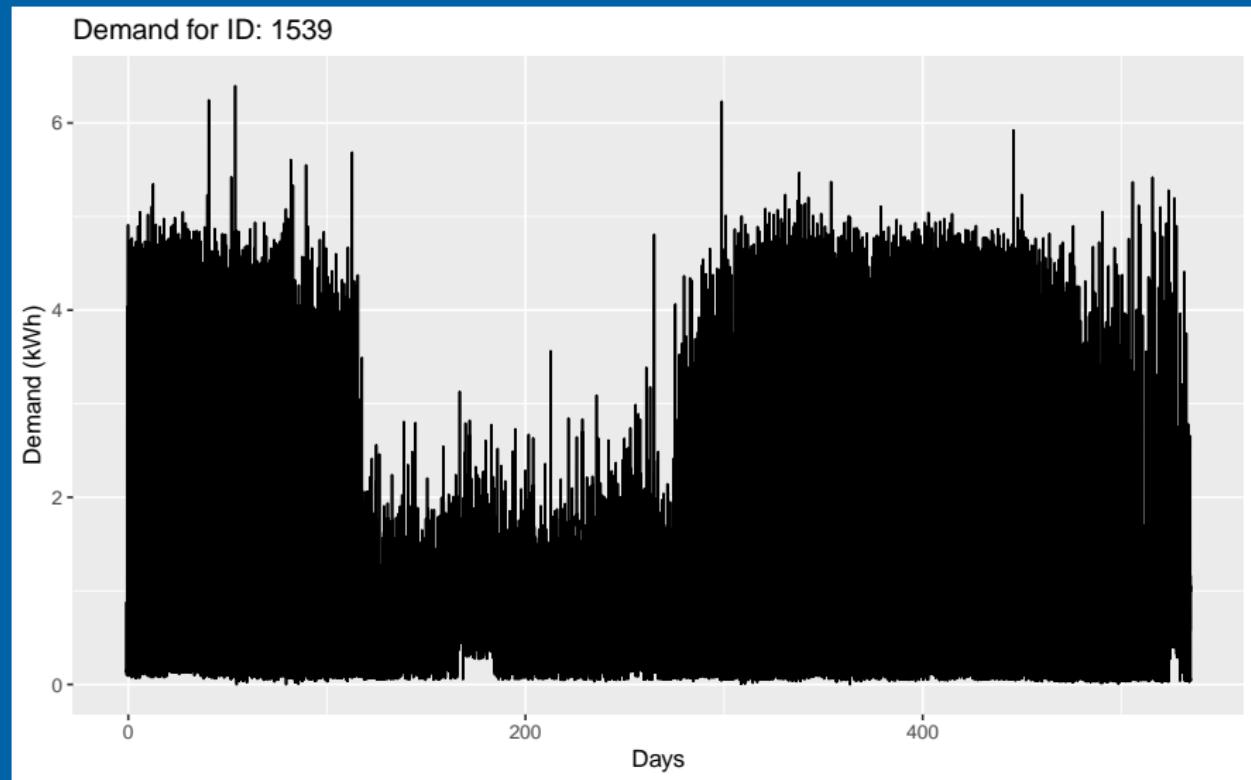
Irish smart metre data



Irish smart metre data



Irish smart metre data

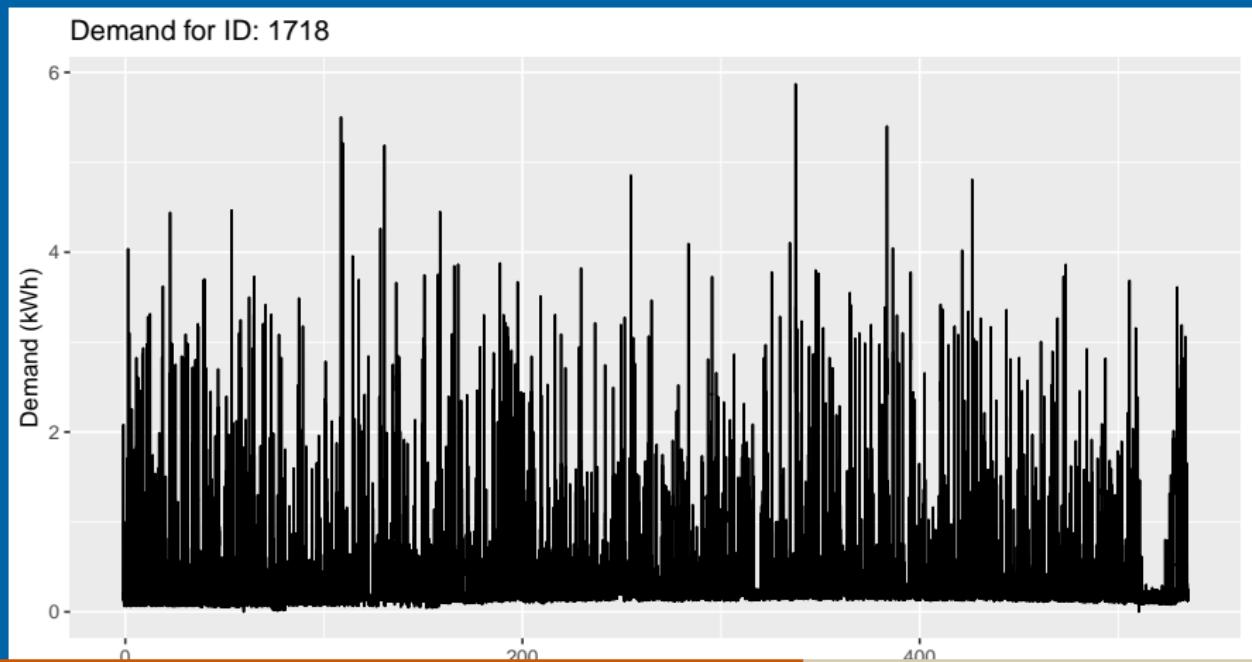


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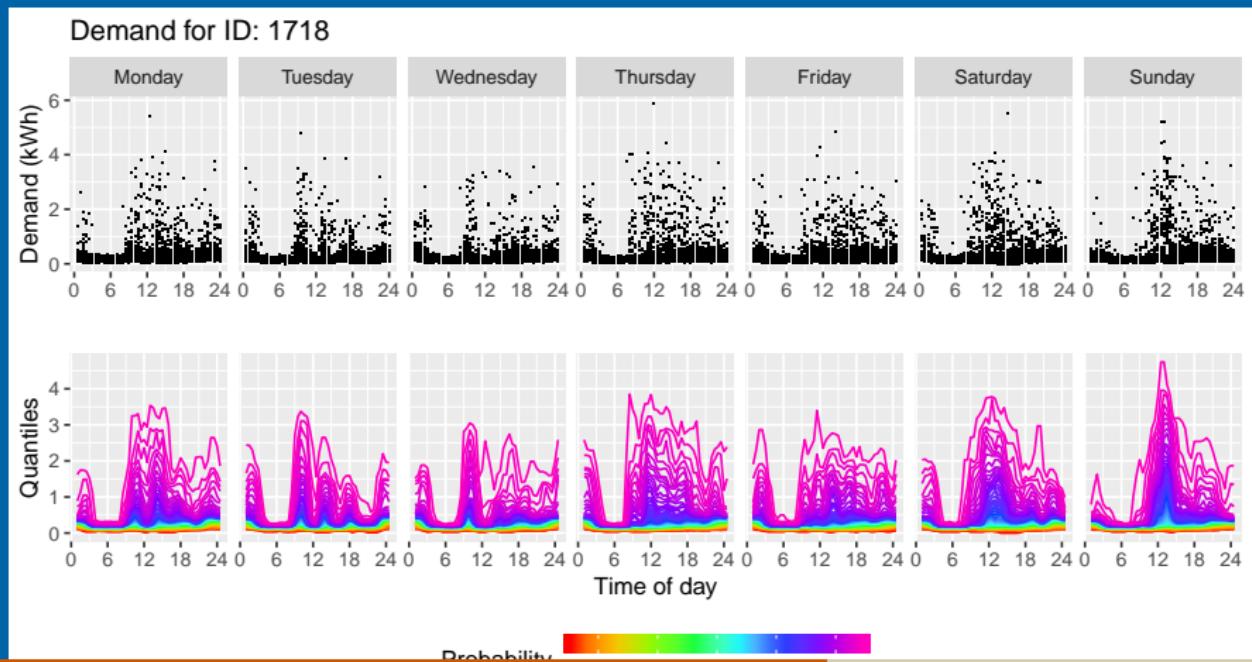
Quantiles conditional on time of week

- Compute sample quantiles at $p = 0.01, 0.02, \dots, 0.99$ for each household and each half-hour of the week.
- 336 probability distributions per household.



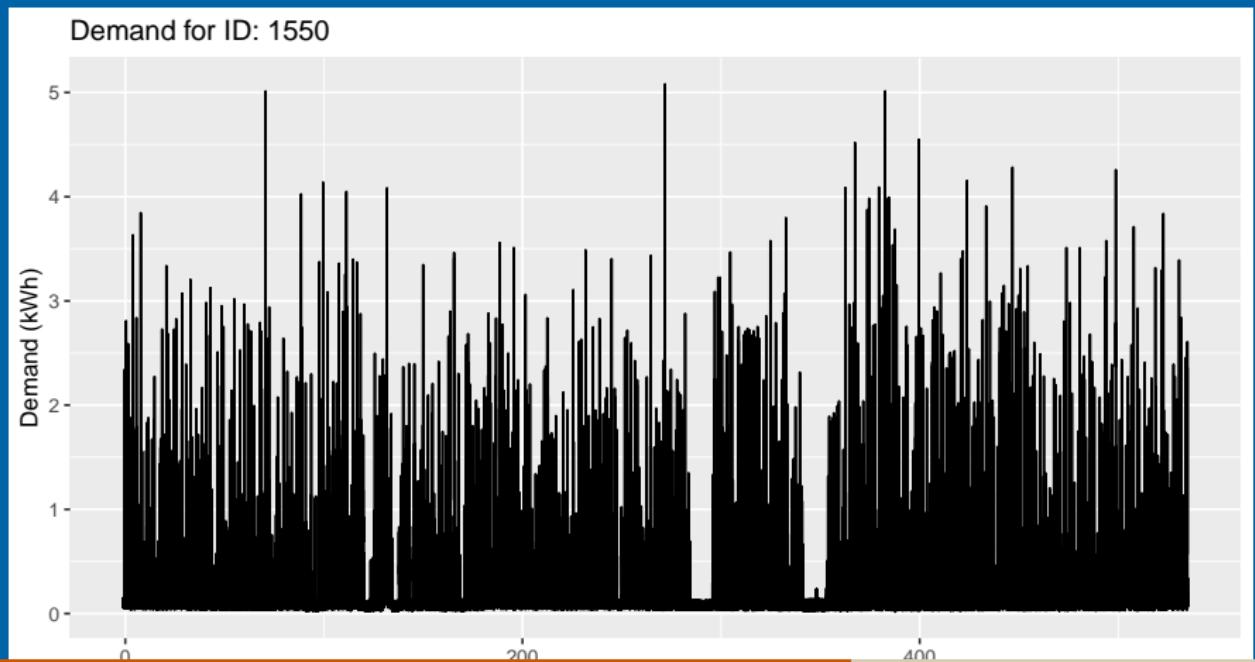
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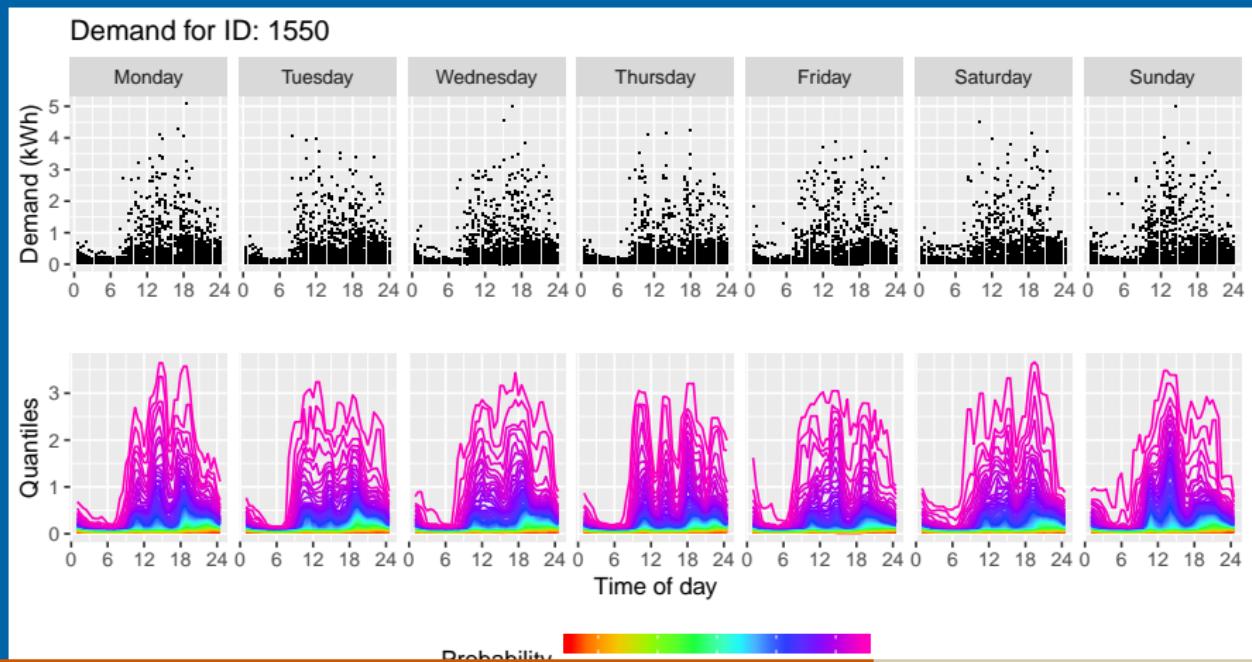
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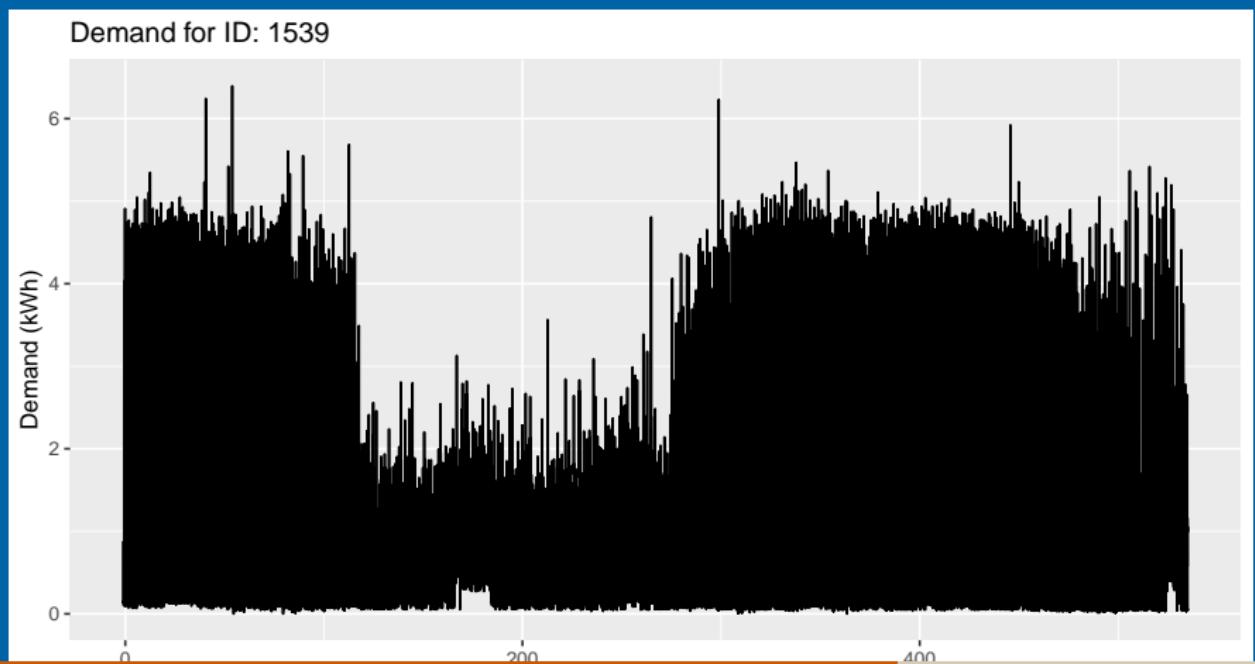
Quantiles conditional on time of week

- Compute sample quantiles at $p = 0.01, 0.02, \dots, 0.99$ for each household and each half-hour of the week.
- 336 probability distributions per household.



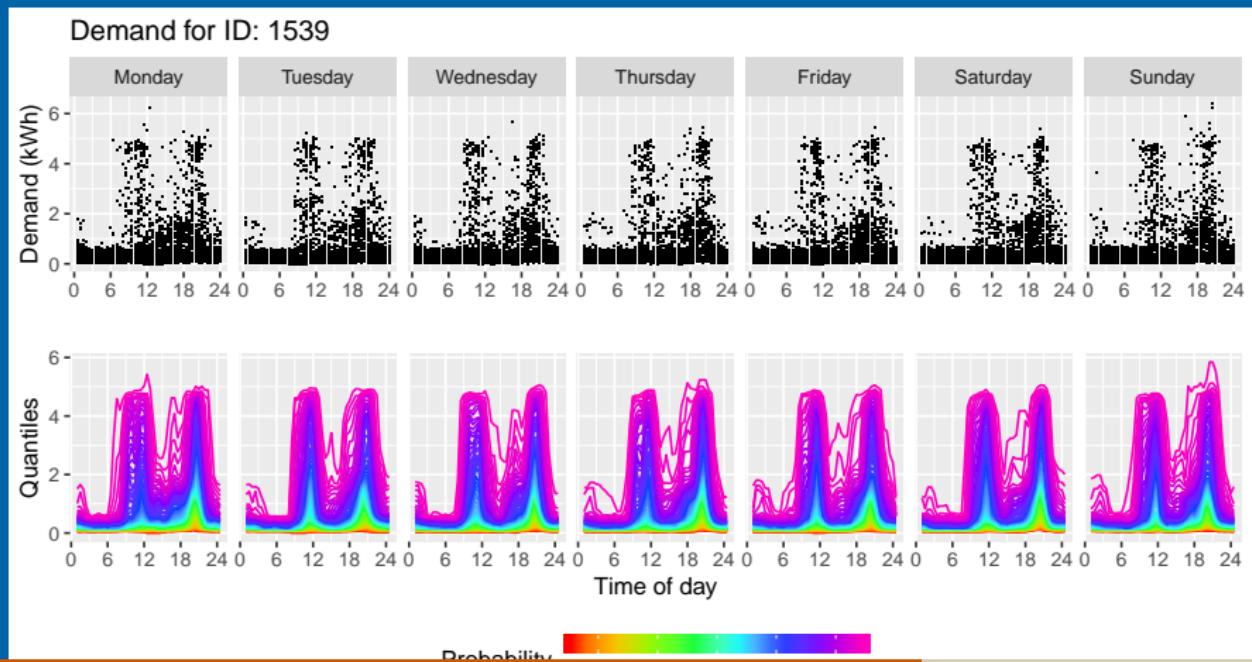
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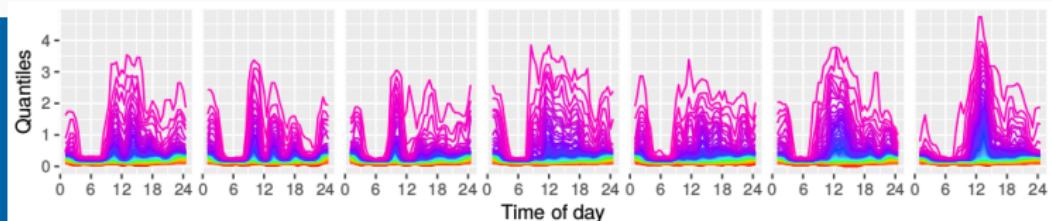


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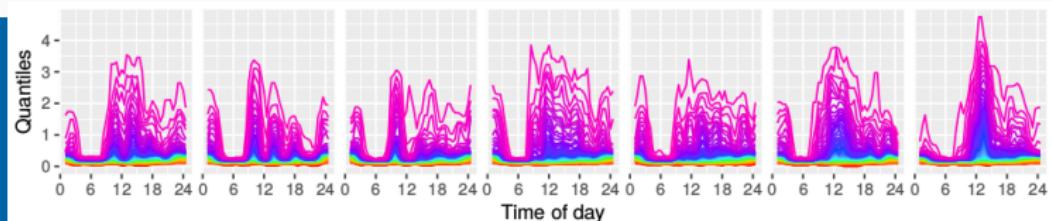


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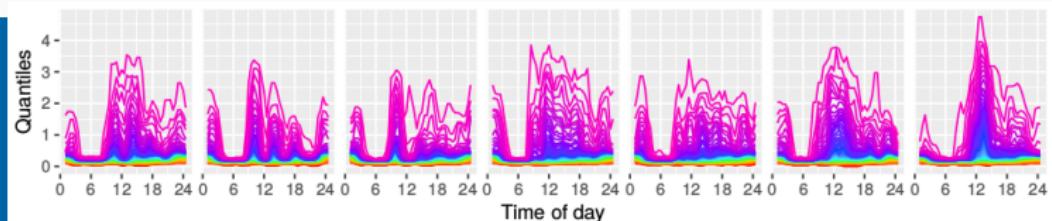
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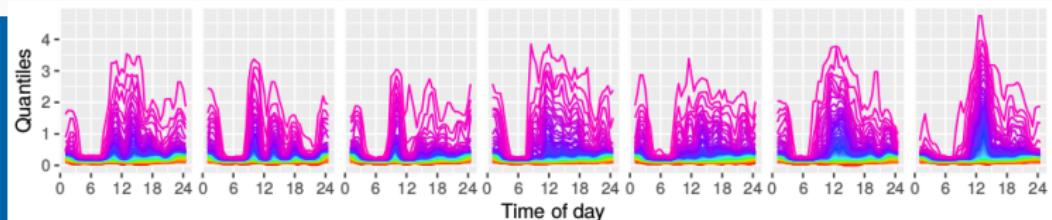
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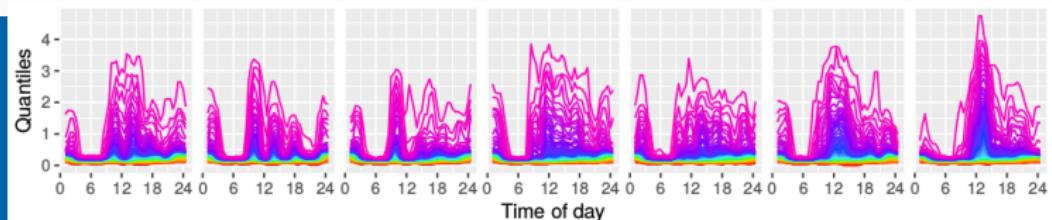
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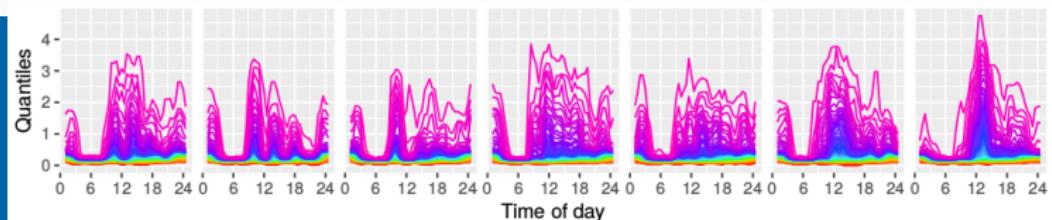
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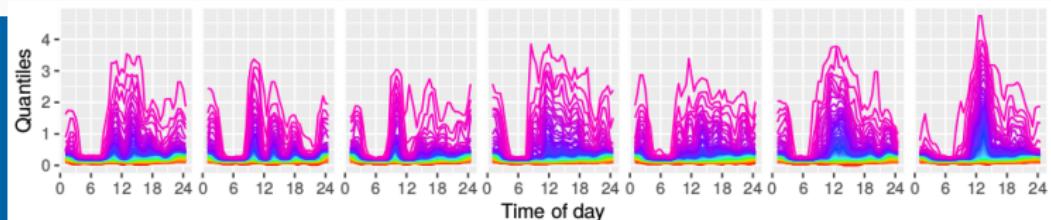
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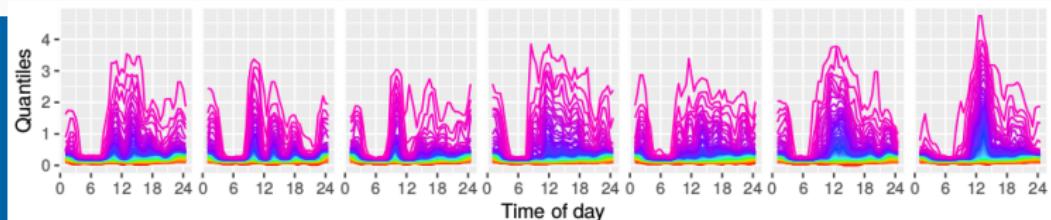
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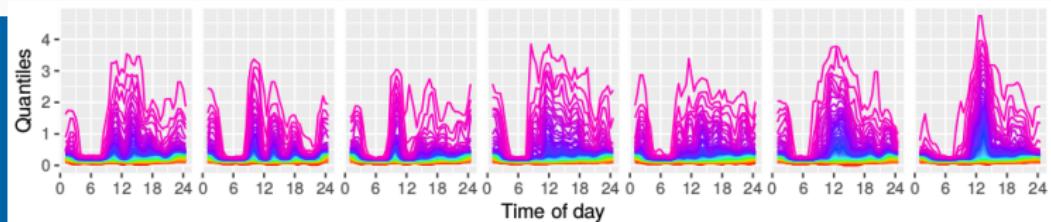
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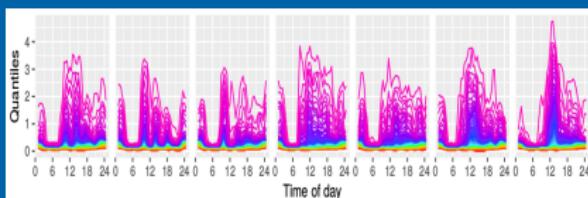
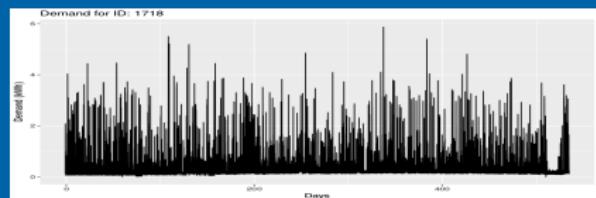


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- Allows estimation of typical household behaviour.

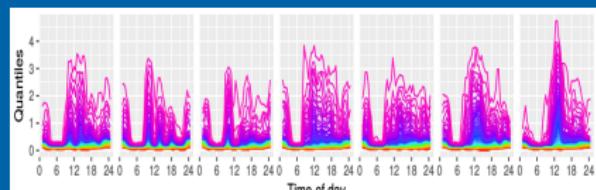
Outline

- 1 Time series feature spaces
- 2 Finding anomalies
- 3 Irish smart metre data
- 4 Quantiles conditional on time of week
- 5 Finding typical and unusual households
- 6 Visualization via embedding
- 7 Features and limitations

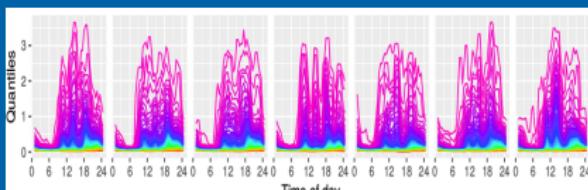
Pairwise distances



- The time series of 535×48 observations per household is mapped to a set of $7 \times 48 \times 99$ quantiles giving a bivariate surface for each household.
- Can we compute pairwise distances between all households?



← ? →
Distance



Jensen-Shannon distances

Kullback-Leibler divergence between two densities

$$D(p, q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

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Distance between two households

$$\Delta_{ij} = \sum_{t=1}^{7 \times 48} \text{JS}(p_t, q_t)$$

Kernel matrix and density ranking

Similarity between two households

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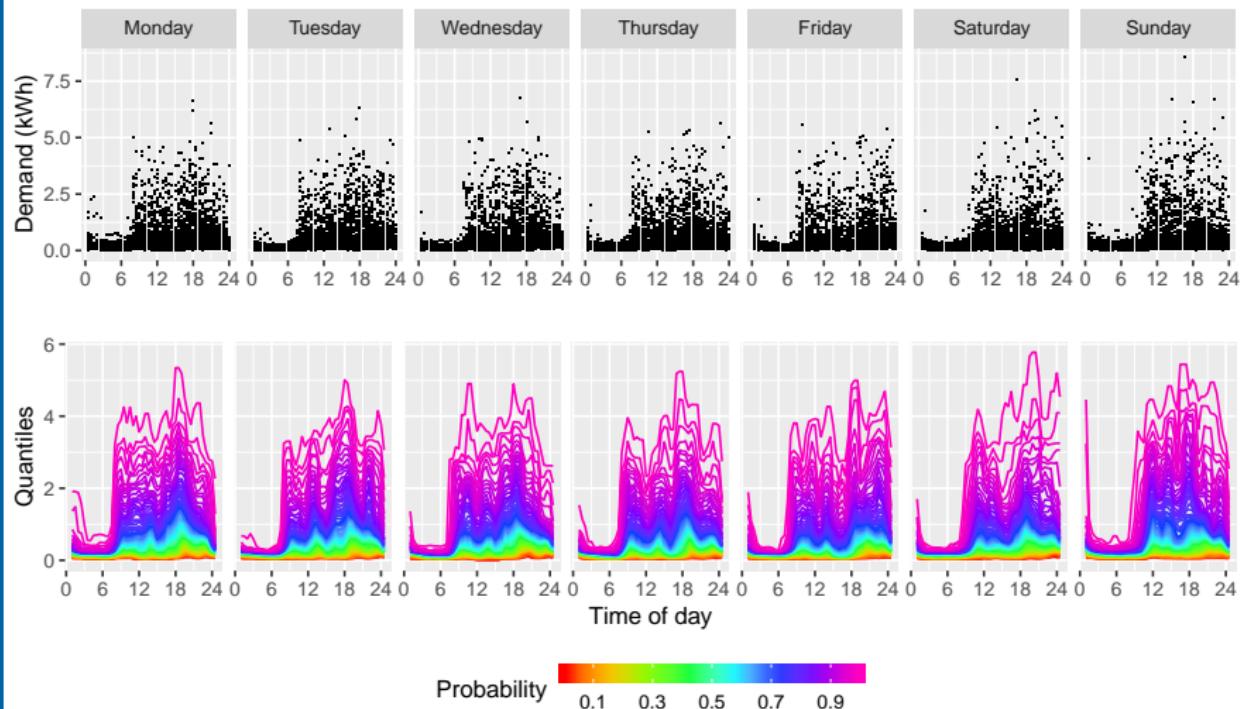
Row sums of the kernel matrix gives a scaled kernel density estimate of households:

$$\hat{f}_i = \sum_{j=1}^n w_{ij}$$

- h is bandwidth in Gaussian kernel.
- Households can be ranked by density values.

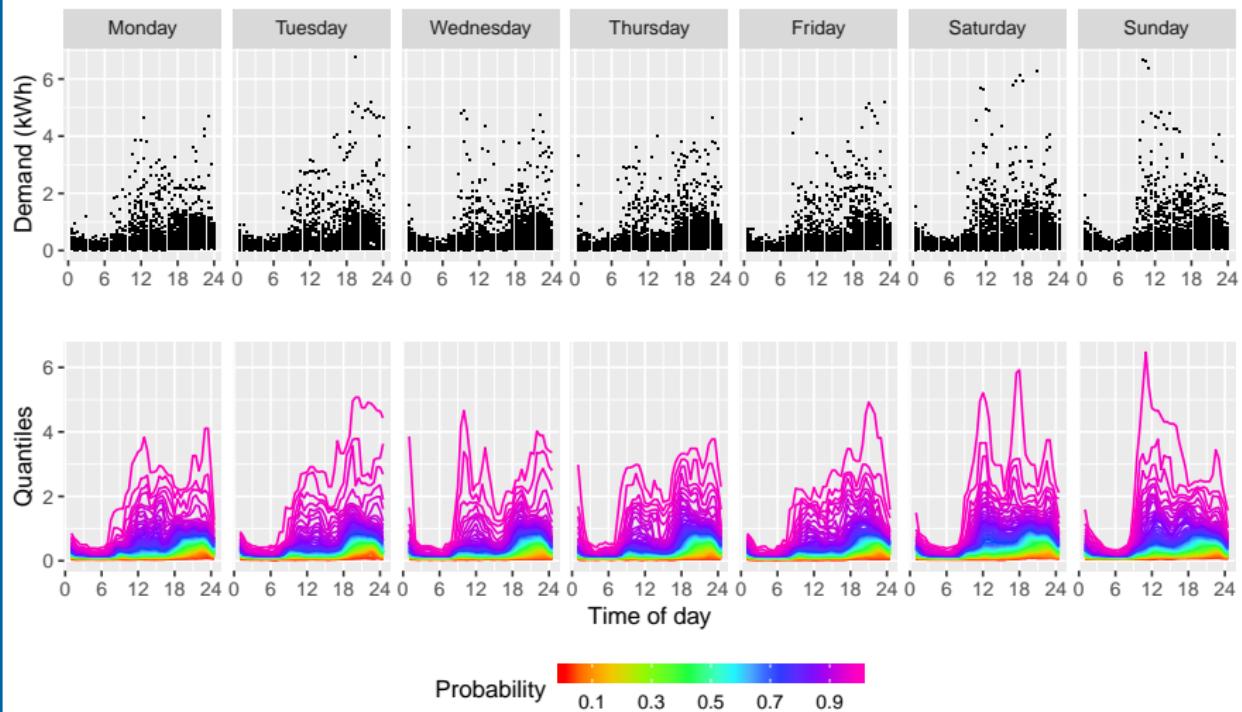
Typical households

Demand for ID: 1672



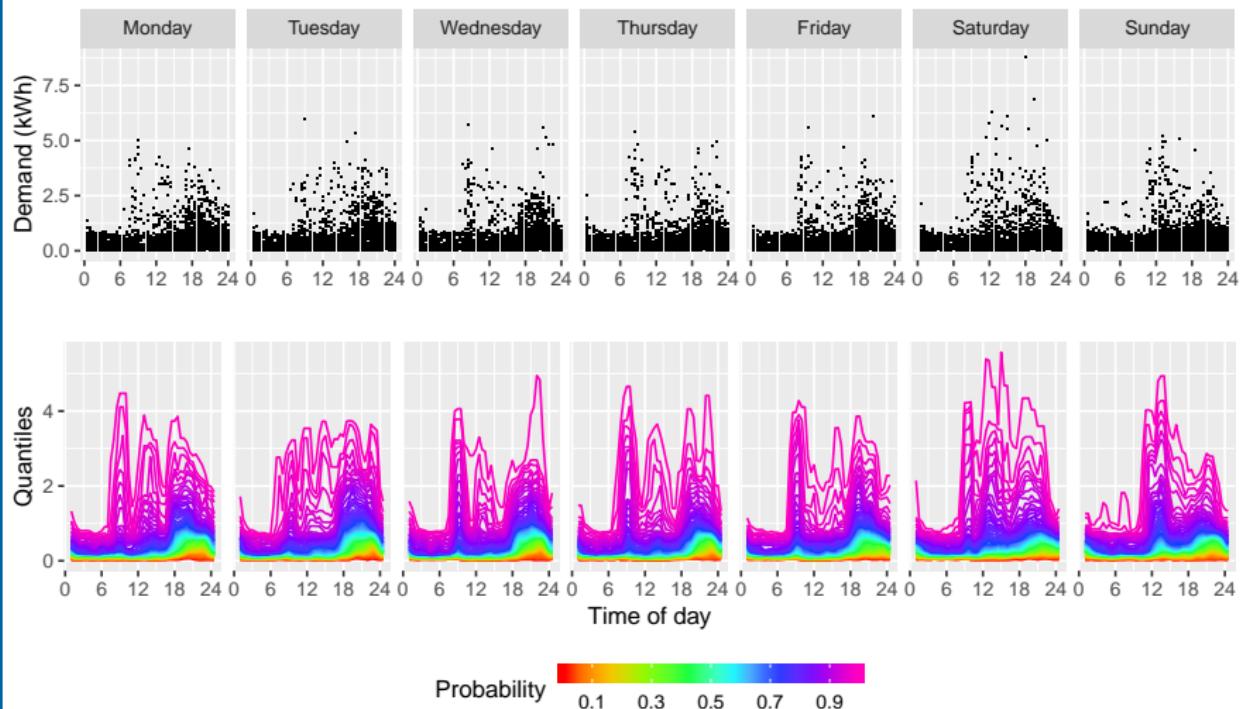
Typical households

Demand for ID: 1058



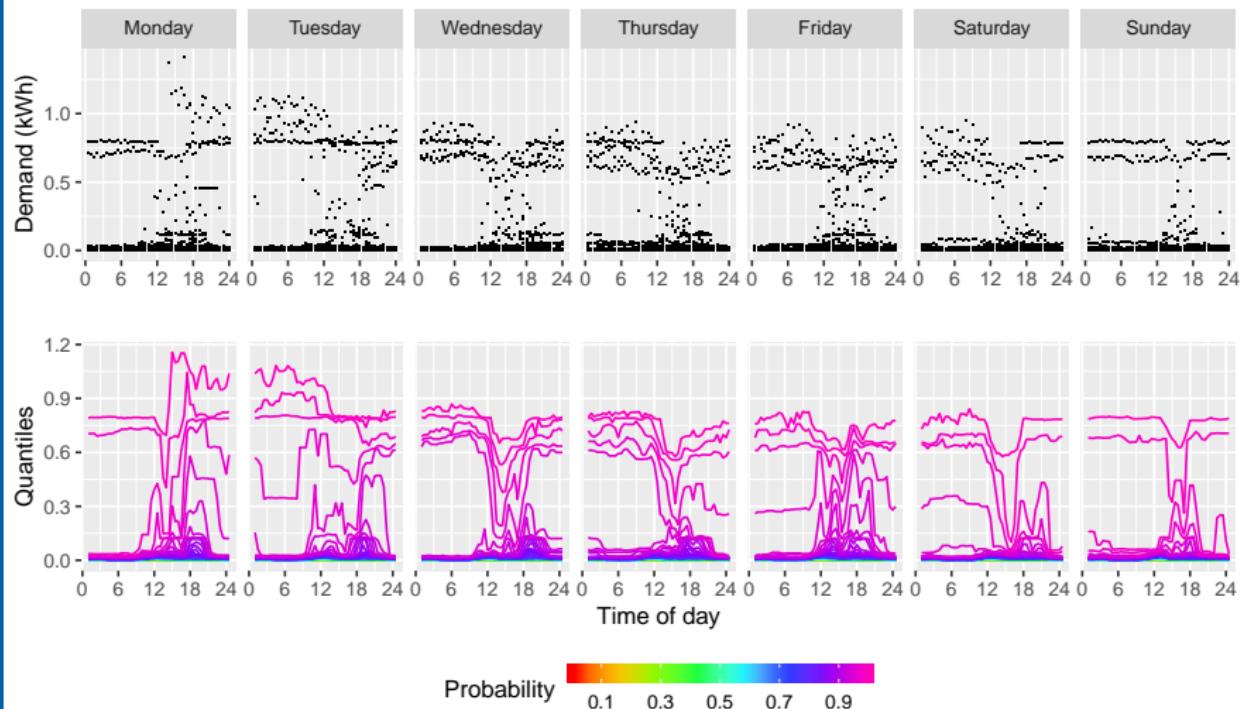
Typical households

Demand for ID: 1183

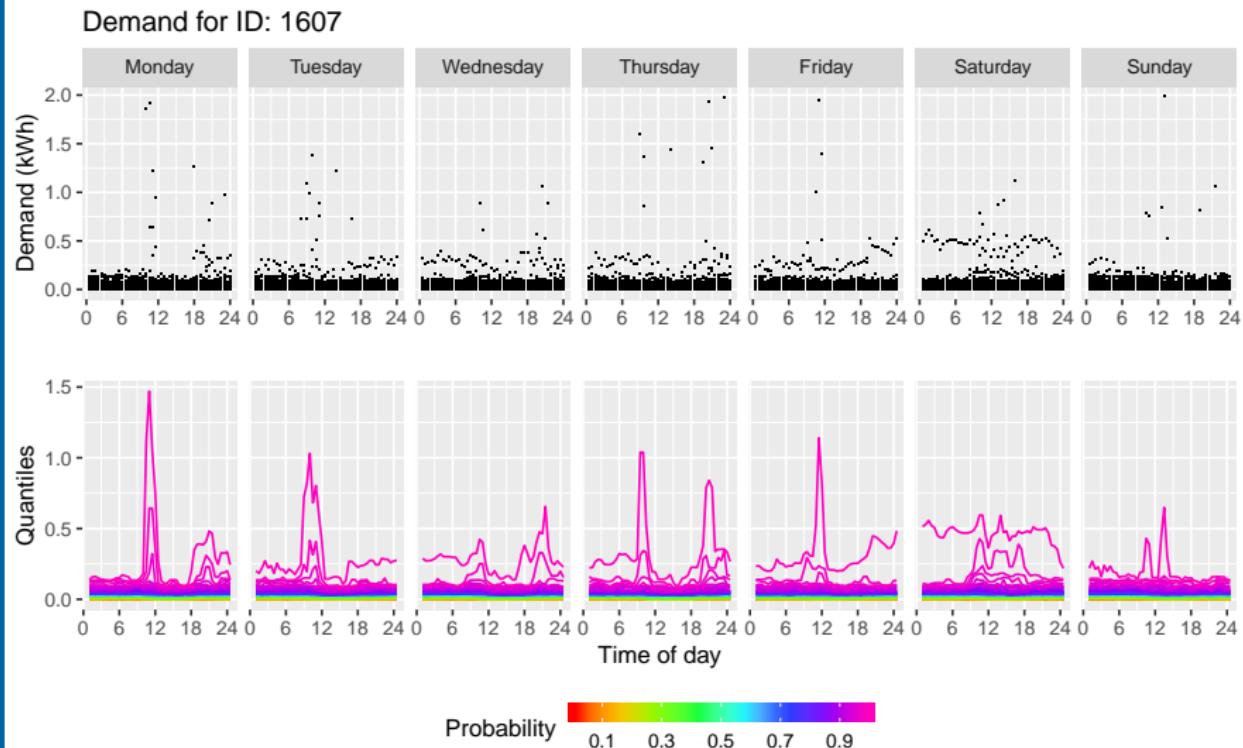


Anomalous households

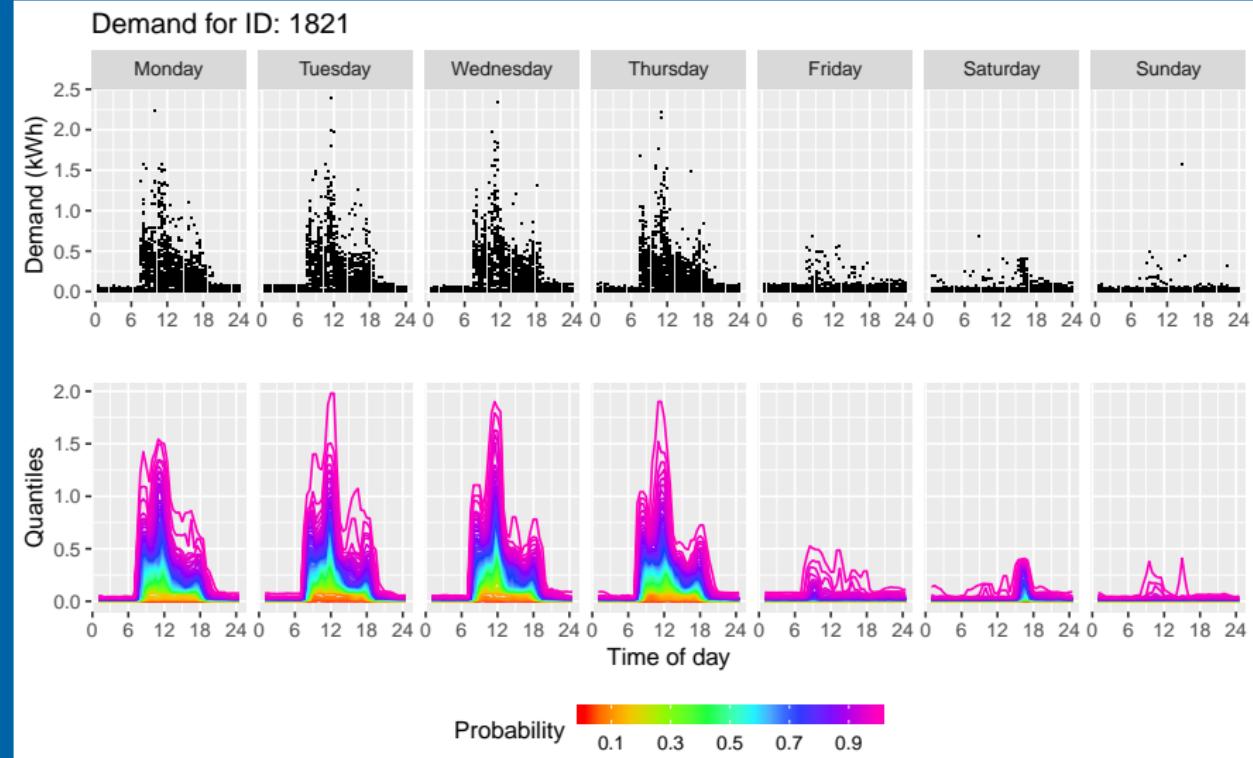
Demand for ID: 1881



Anomalous households



Anomalous households



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- Then \mathbf{e}_2 and \mathbf{e}_3 create an embedding of households in 2d space.

Key property of Laplacian embedding

Let $y_i = (e_{2,i}, e_{3,i})$ be the embedded point corresponding to household i .

Then the Laplacian eigenmap minimizes

$$\sum_{ij} w_{ij} (y_i - y_j)^2 = \mathbf{y}' \mathbf{L} \mathbf{y} \quad \text{such that} \quad \mathbf{y}' \mathbf{D} \mathbf{y} = 1.$$

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- Equivalent to optimal embedding using Laplace-Beltrami operator on manifolds.

Outliers computed in embedded space:

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Unresolved issues

- Need to select the bandwidth h in constructing the similarity matrix.
- Two different uses of bandwidth: density-ranking, embedding. Different bandwidth in each case?
- The use of pairwise distances makes it hard to scale this algorithm.