

# Feature-based time series analysis

Rob J Hyndman

2/3 February 2022

# Outline

- 1 Feature-based visualization
- 2 R packages
- 3 Feature-based anomaly detection
- 4 Feature-based forecasting

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# M3 competition: 2000



ELSEVIER

International Journal of Forecasting 16 (2000) 451–476

*international journal  
of forecasting*

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## The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon\*

*INSEAD, Boulevard de Constance, 77305 Fontainebleau, France*

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### Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting 4 accuracy

# M3 competition: 2000



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petition: results, conclusions a

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Abst



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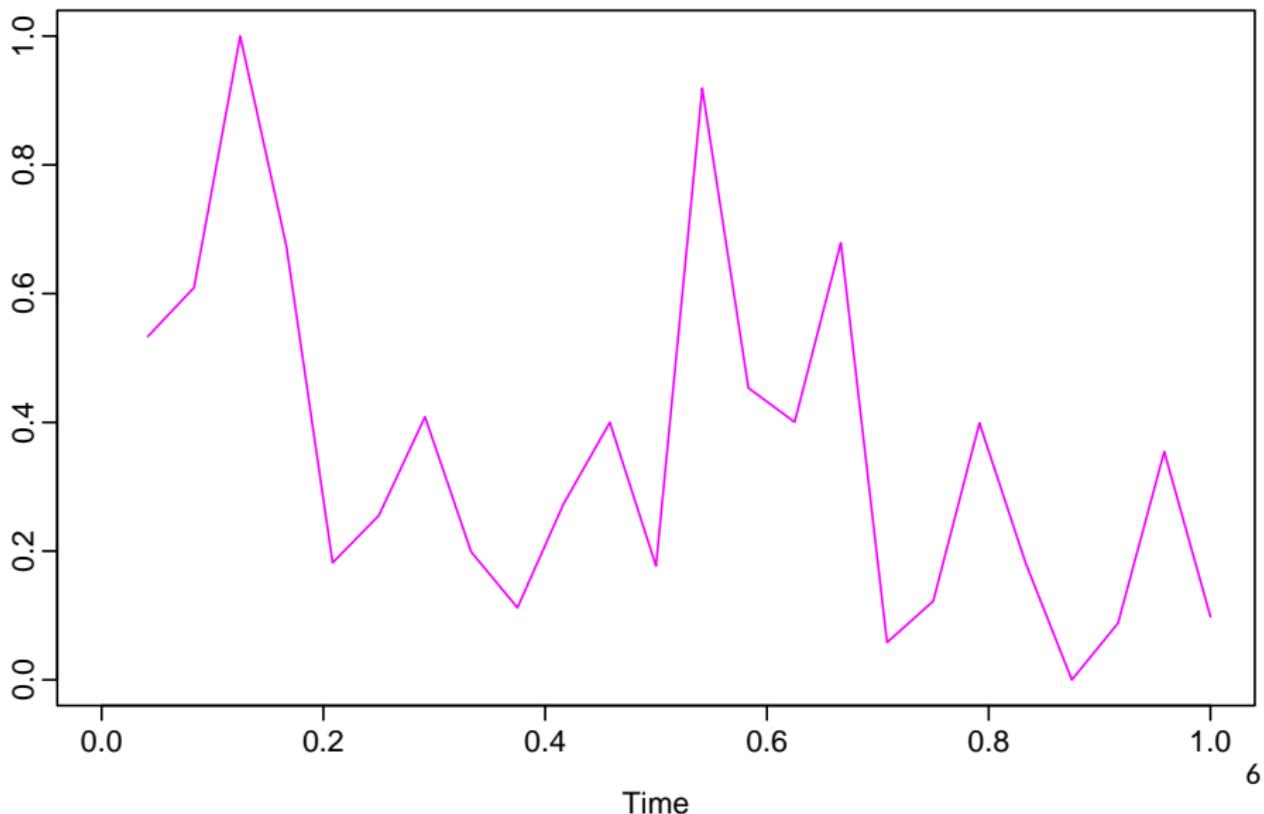
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## M3 competition: 2000

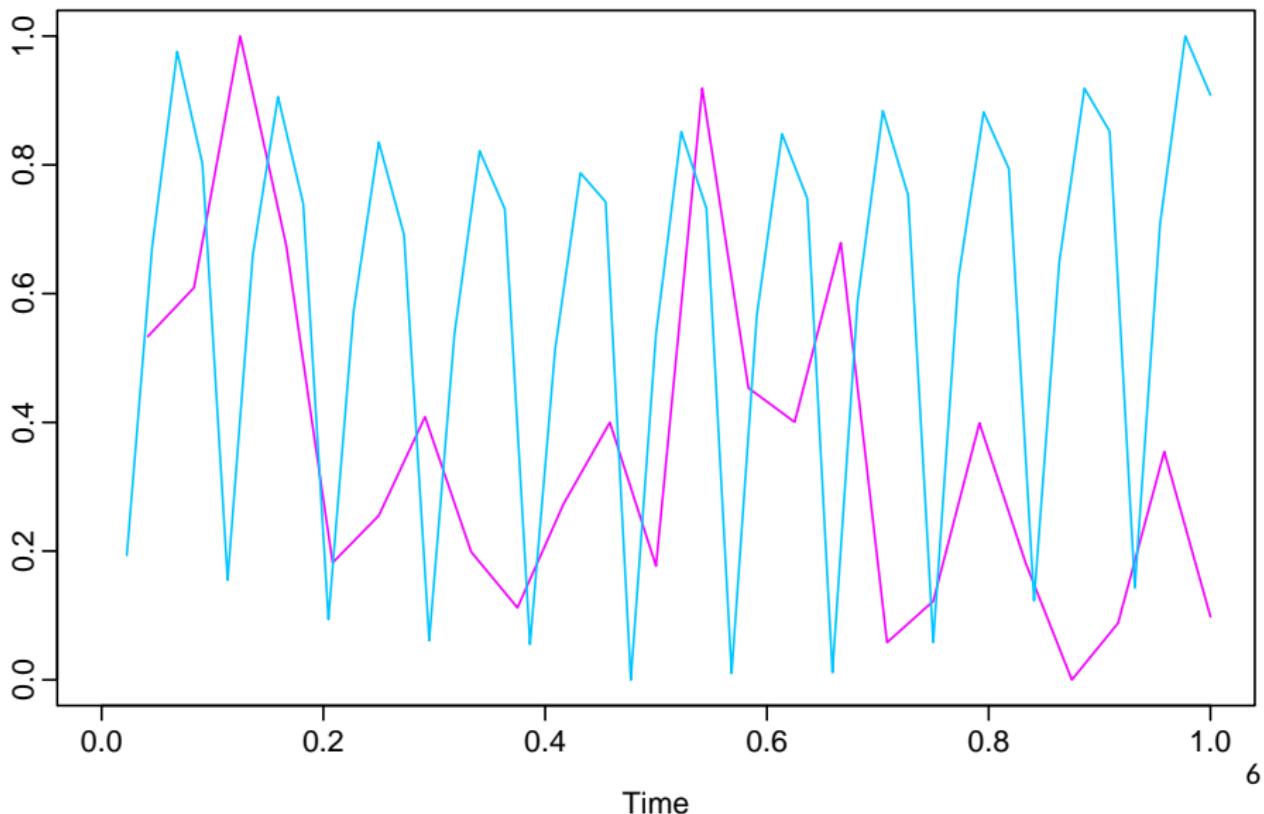
“The M3-Competition is a final attempt by the authors to settle the accuracy issue of various time series methods... The extension involves the inclusion of more methods/researchers (in particular in the areas of neural networks and expert systems) and more series.”

- 3003 series
- All data from business, demography, finance and economics.
- Series length between 14 and 126.
- Either non-seasonal, monthly or quarterly.
- All time series positive.

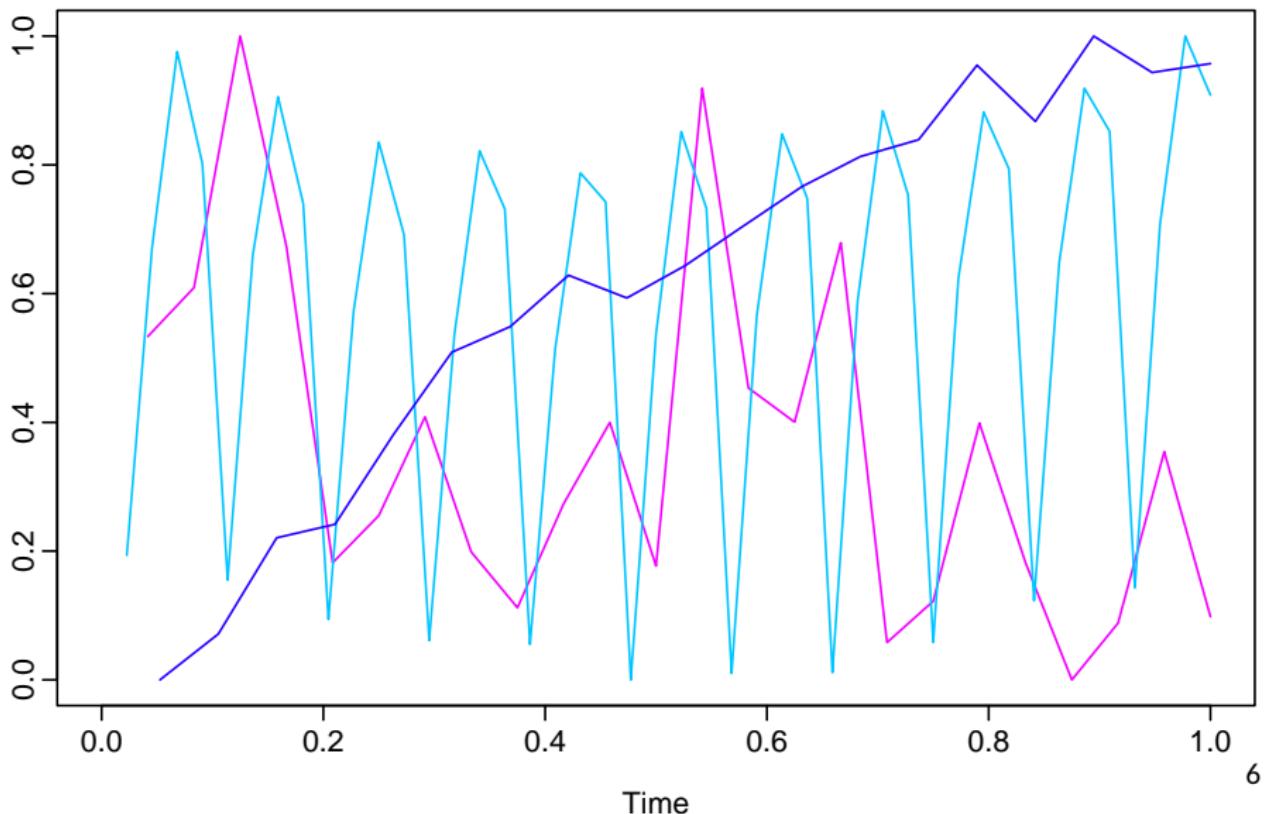
# How to plot lots of time series?



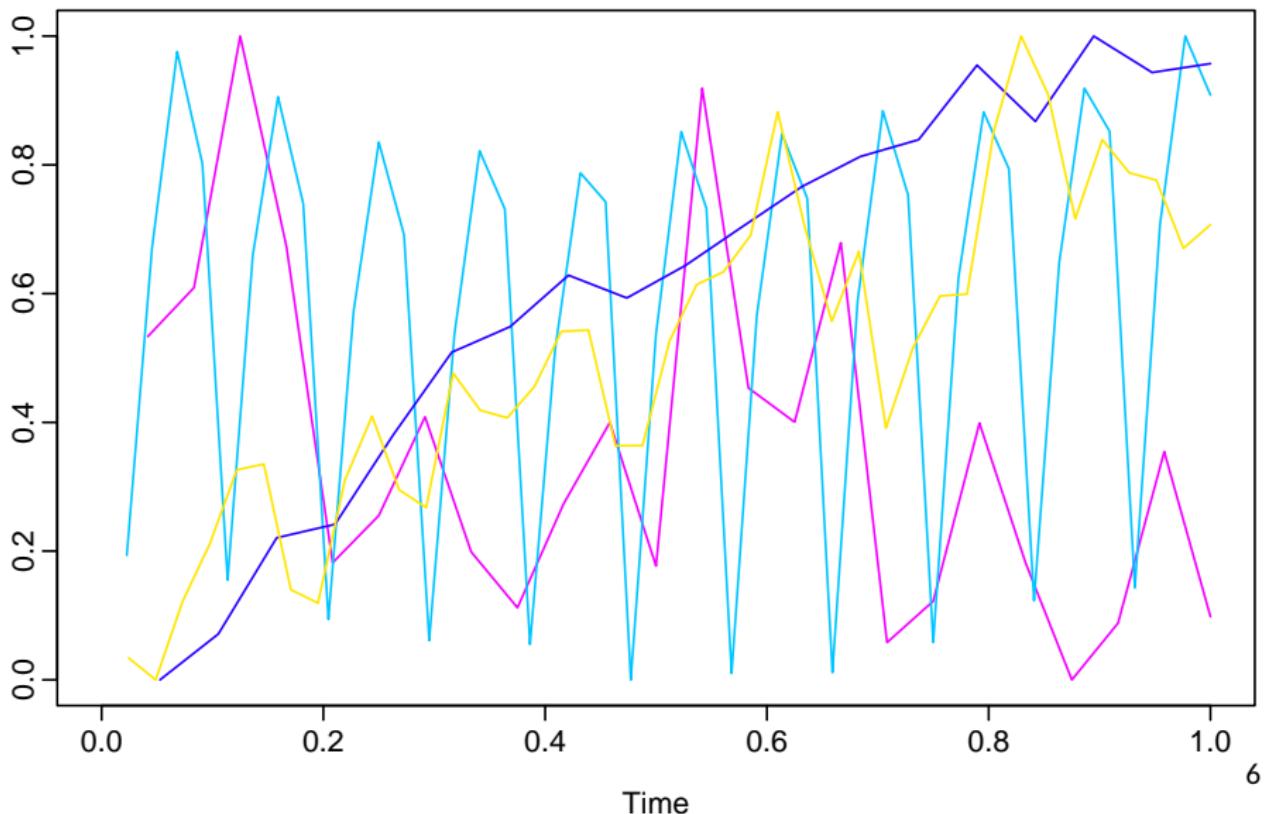
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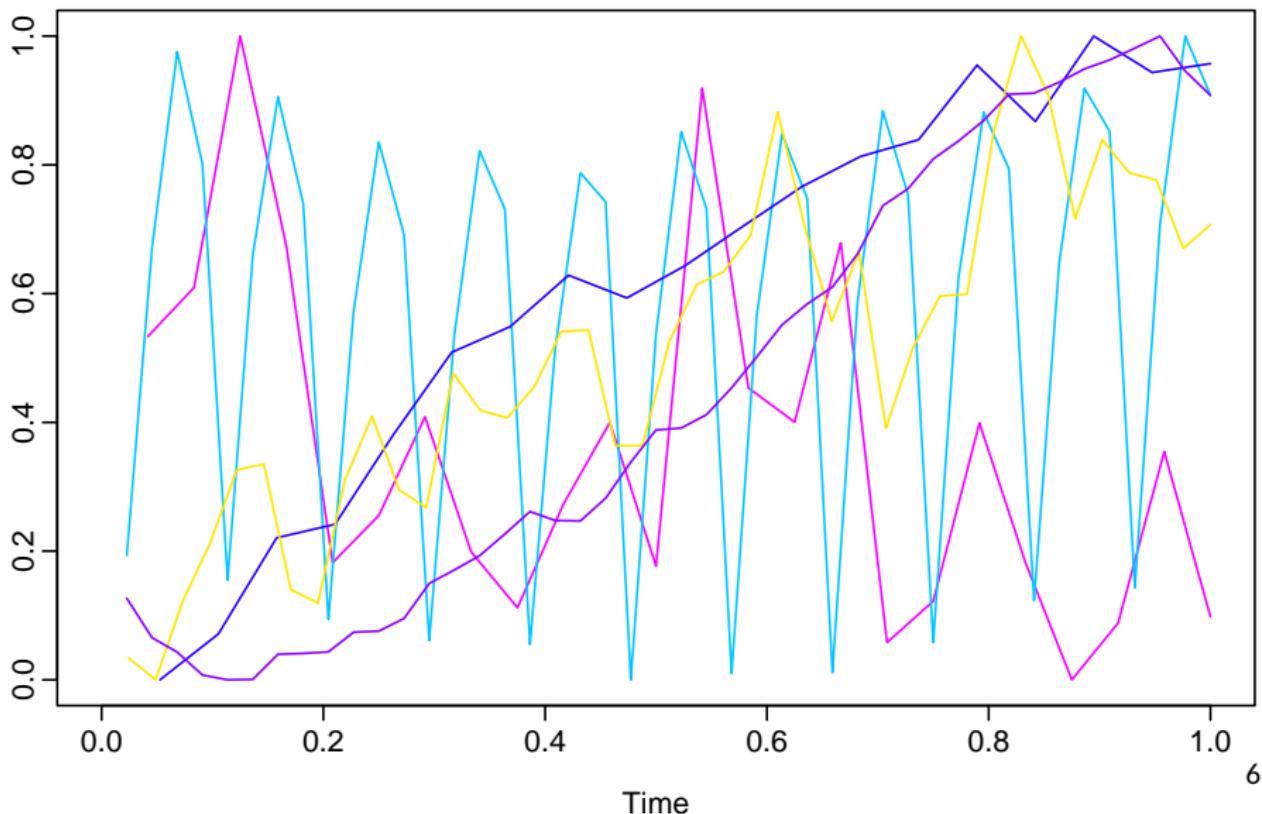
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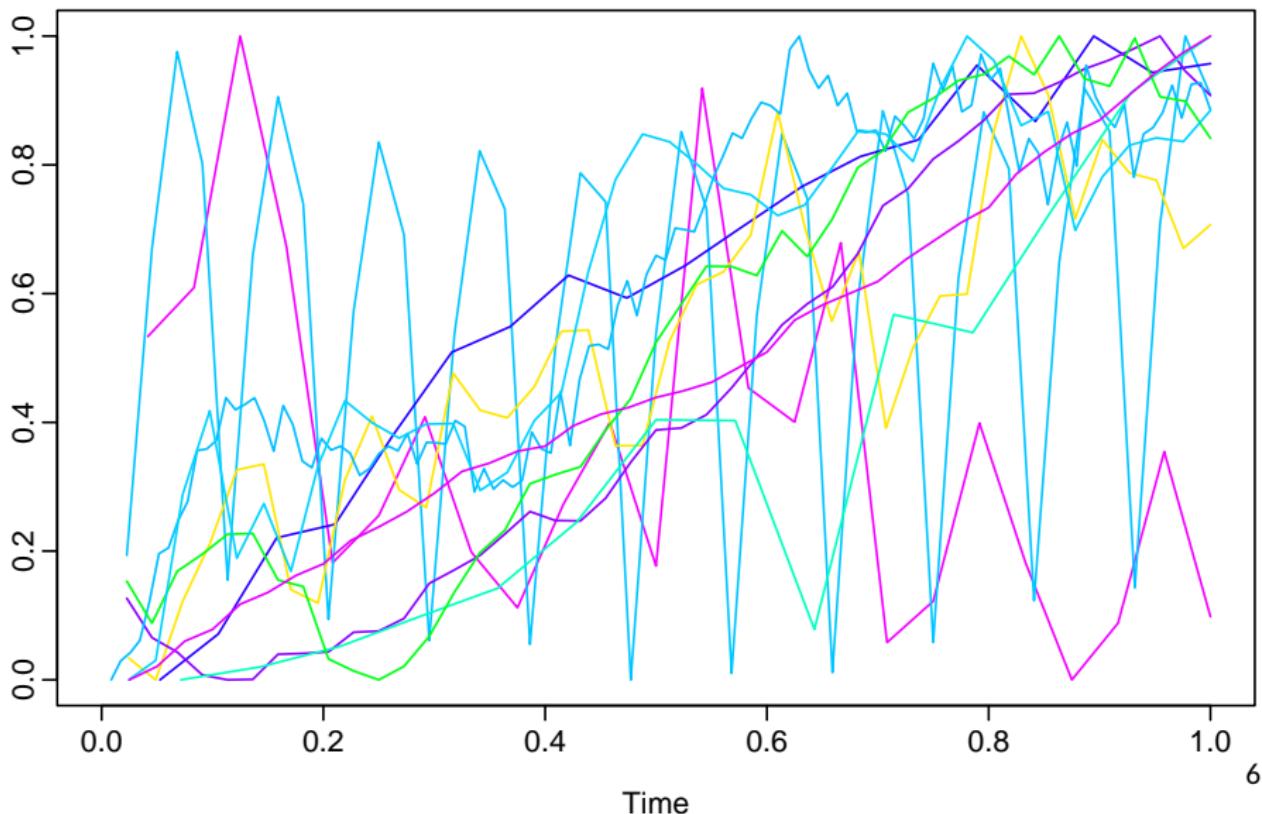
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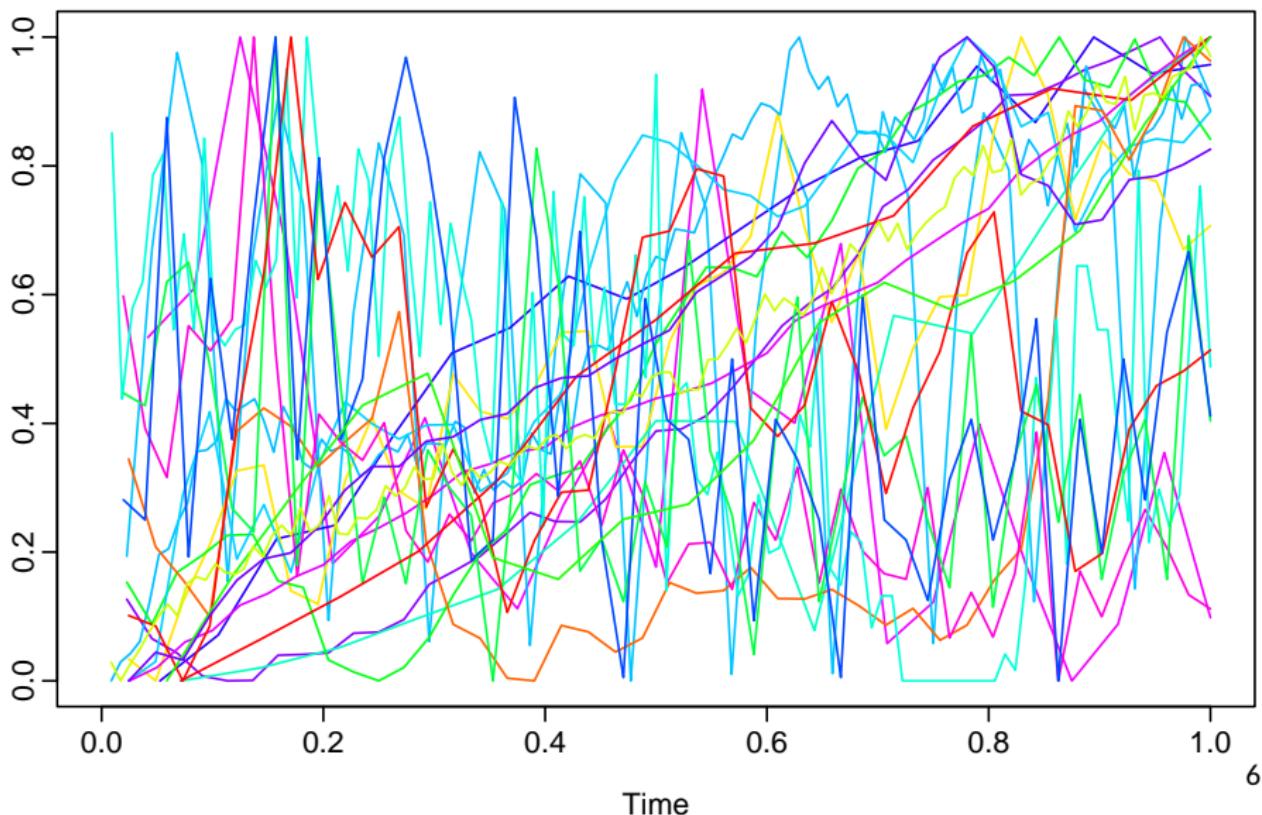
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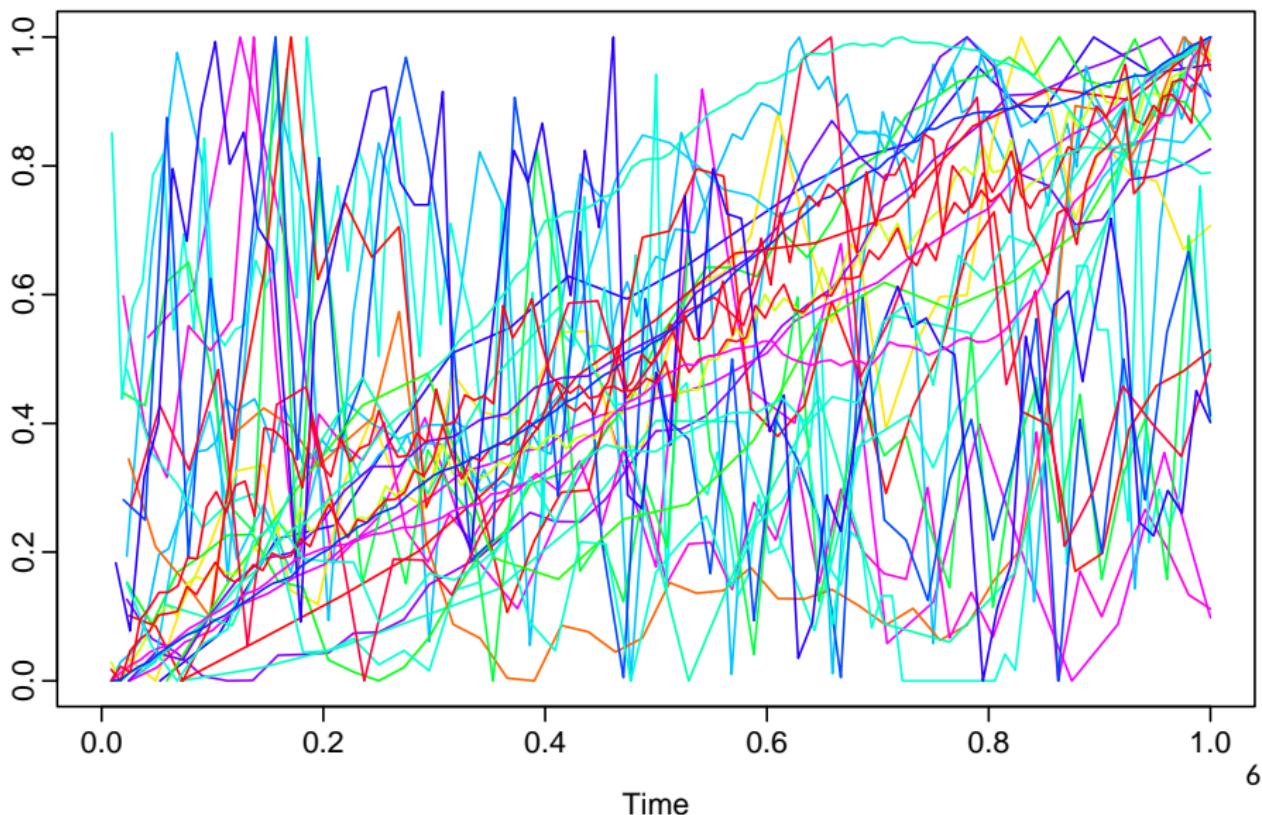
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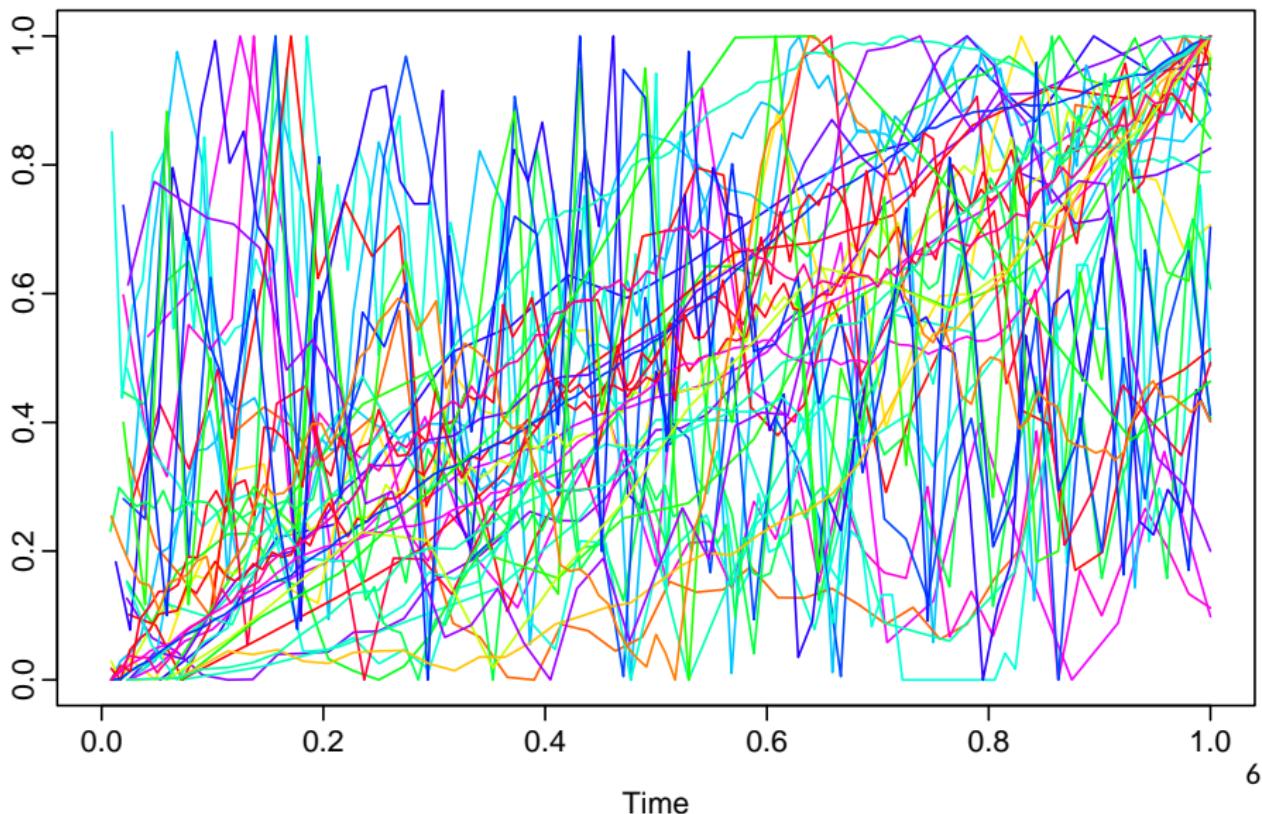
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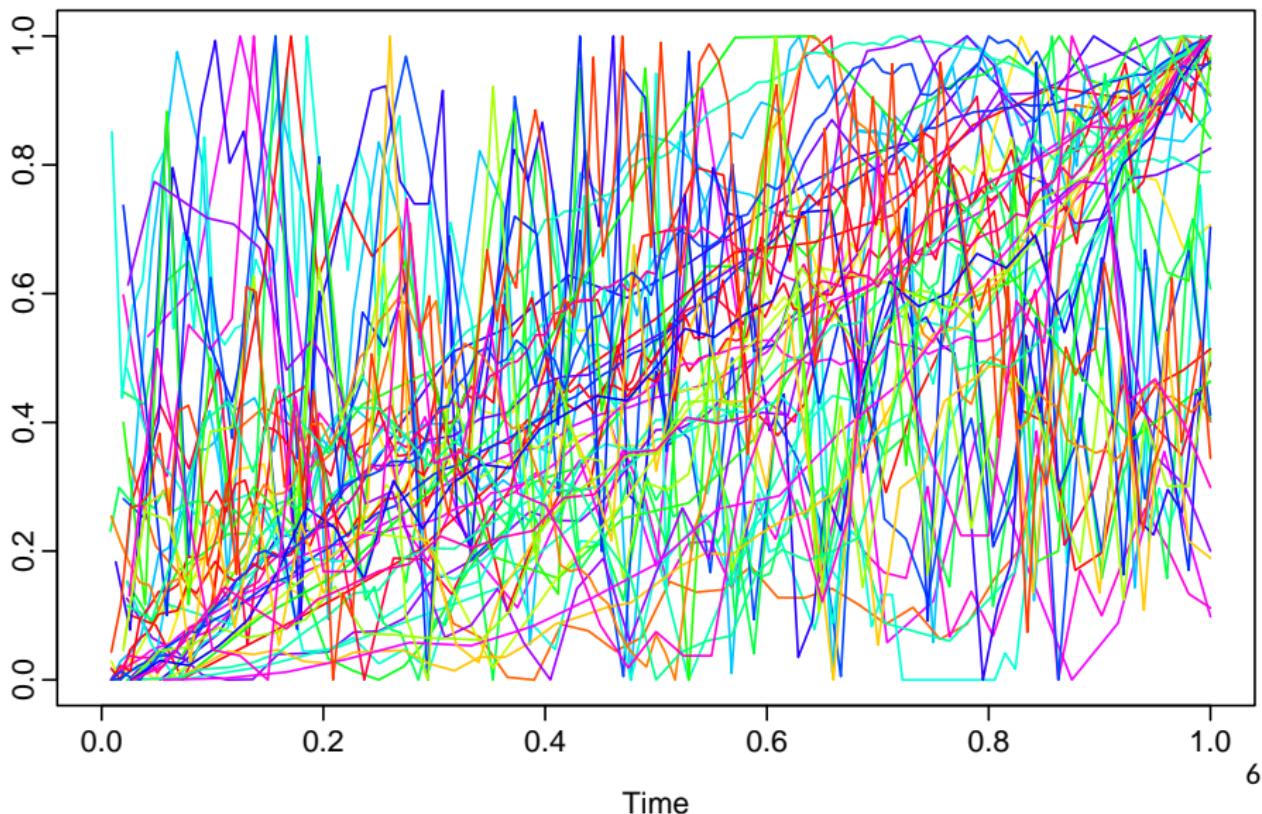
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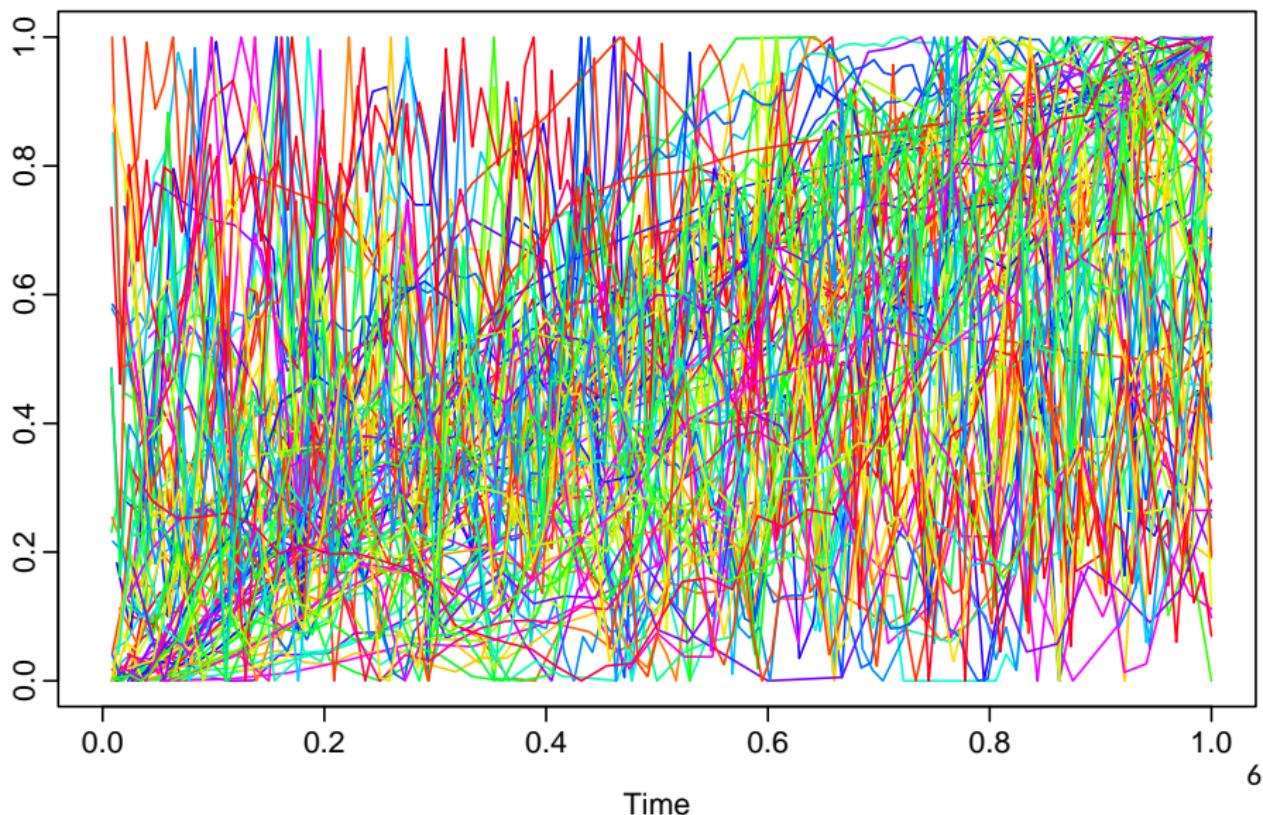
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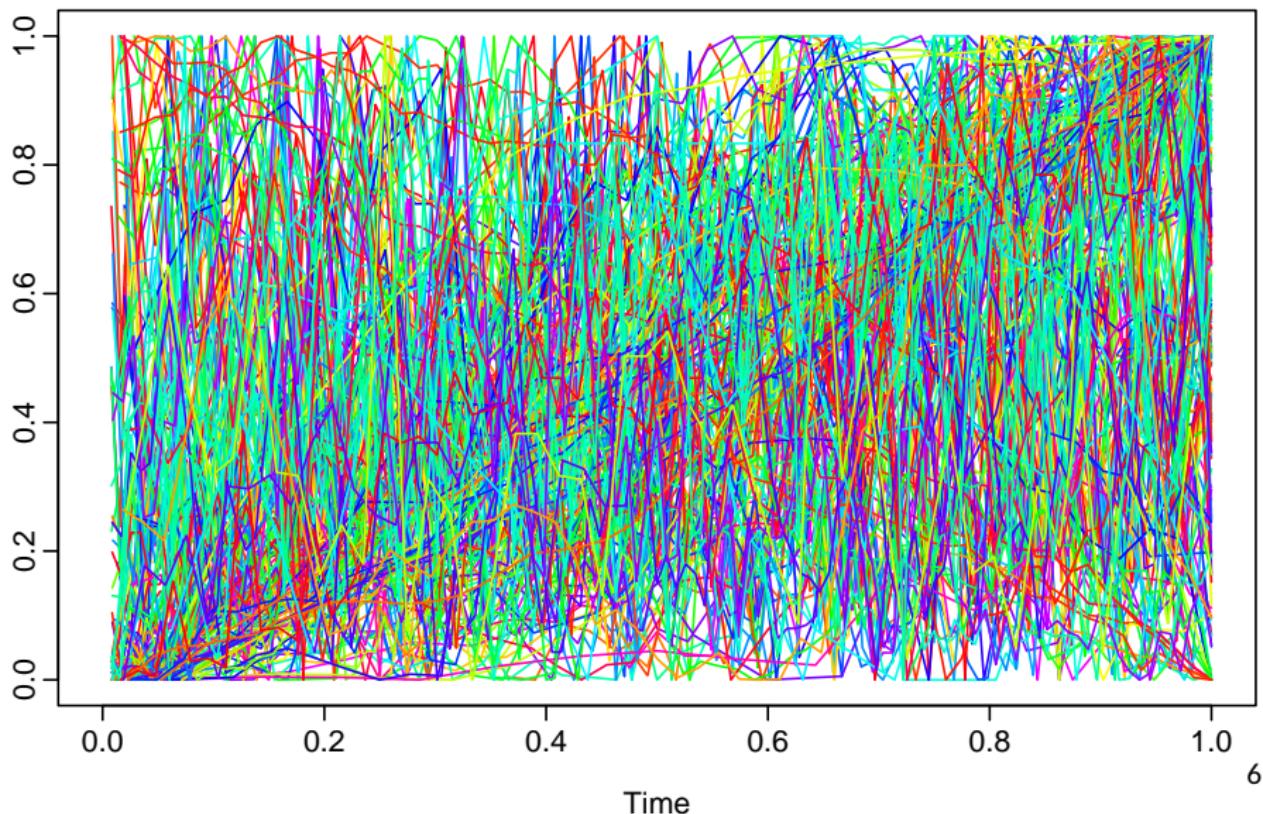
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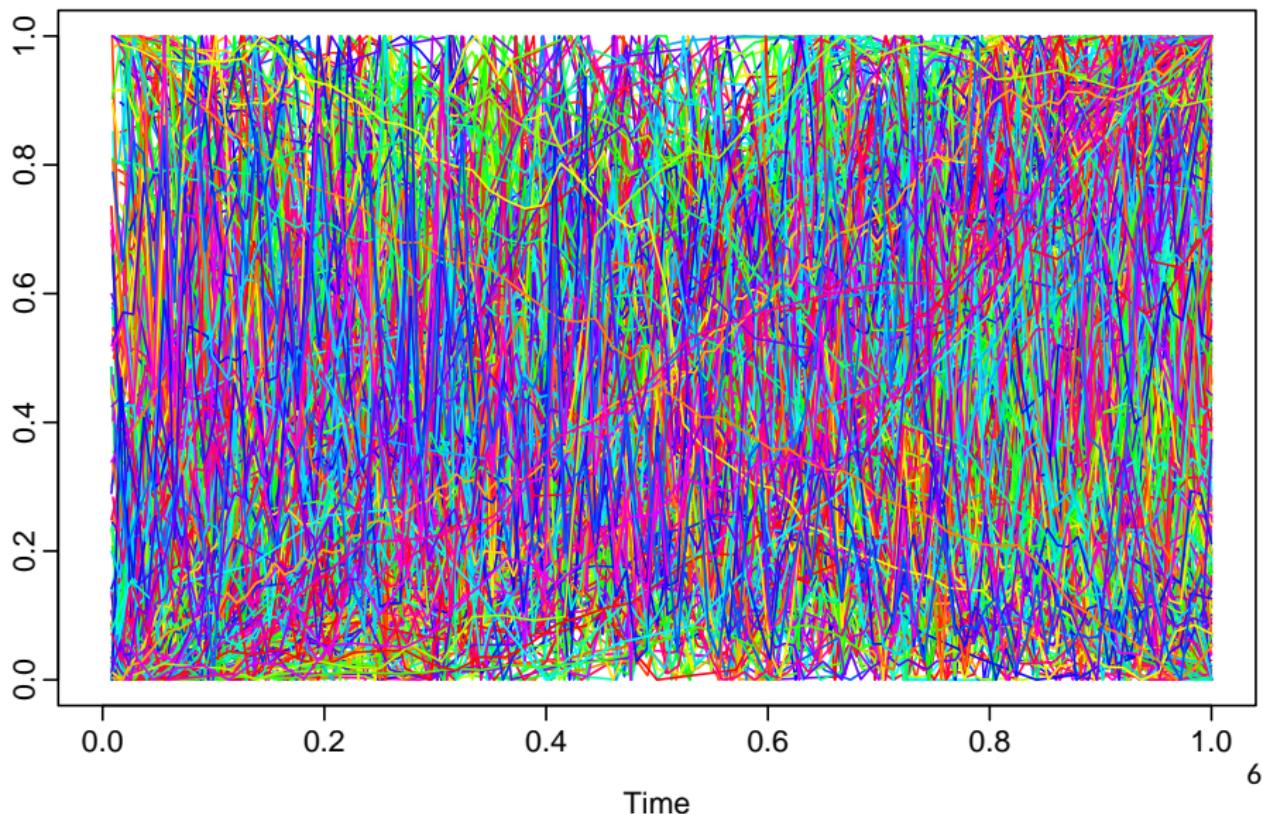
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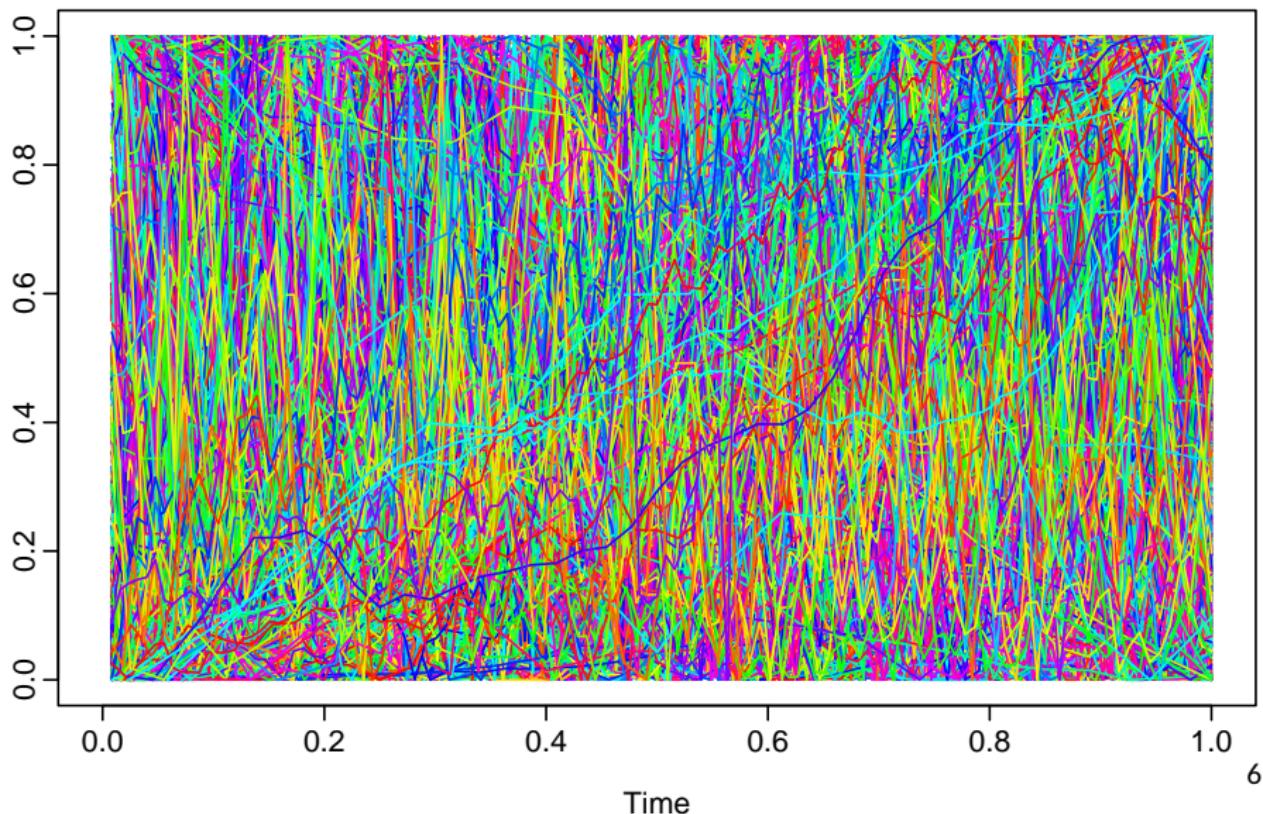
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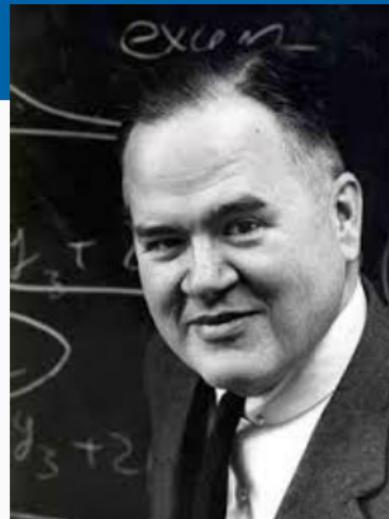
# How to plot lots of time series?



# Key idea

## Cognostics

Computer-produced diagnostics  
(Tukey and Tukey, 1985).

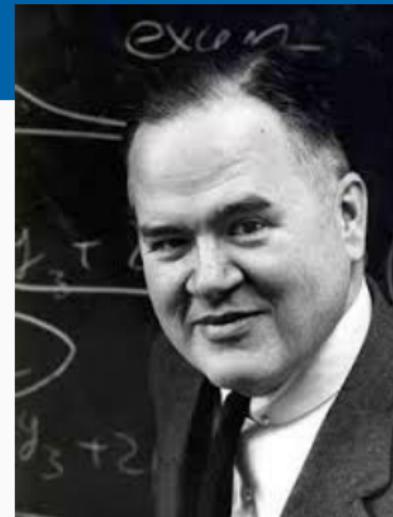


*John W Tukey*

# Key idea

## Cognostics

Computer-produced diagnostics  
(Tukey and Tukey, 1985).



John W Tukey

### Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

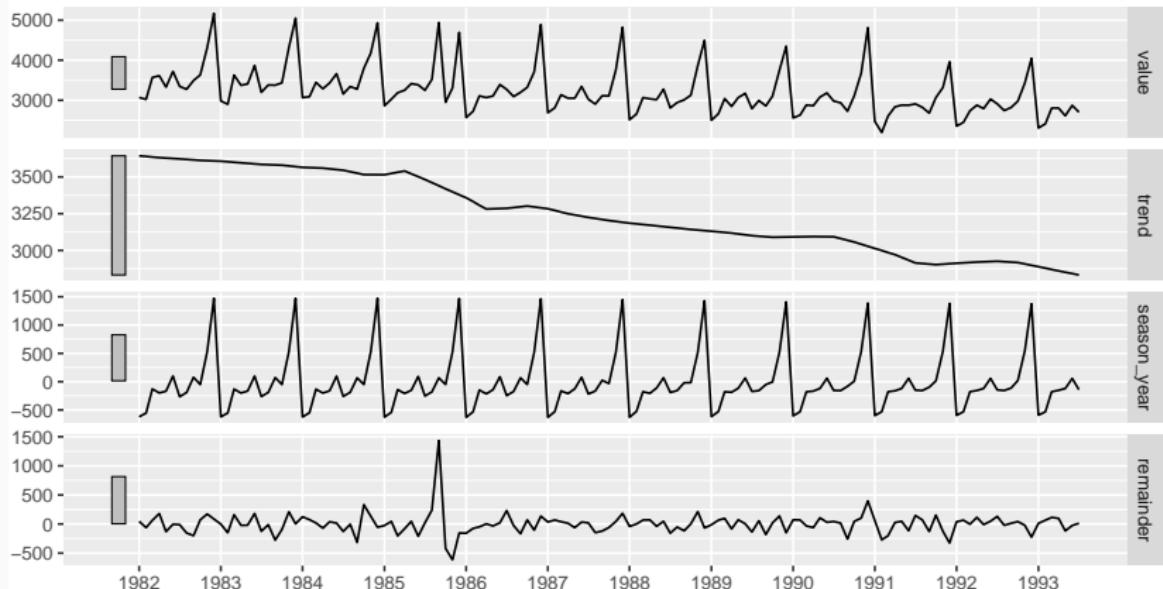
Called “features” in the machine learning literature  
and “statistics” in the statistics literature.

# An STL decomposition: N2096

$$Y_t = S_t + T_t + R_t \quad S_t \text{ is periodic with mean 0}$$

STL decomposition

value = trend + season\_year + remainder



# Candidate features

## STL decomposition

$$Y_t = S_t + T_t + R_t$$

# Candidate features

## STL decomposition

$$Y_t = S_t + T_t + R_t$$

- Seasonal period
- Autocorrelations of data  $(Y_1, \dots, Y_T)$
- Autocorrelations of data  $(R_1, \dots, R_T)$
- Strength of seasonality:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)} \right)$
- Strength of trend:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)} \right)$
- Spectral entropy:  $H = - \int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$ ,  
where  $f_y(\lambda)$  is spectral density of  $Y_t$ .  
Low values of  $H$  suggest a time series that is  
easier to forecast (more signal).
- Optimal Box-Cox transformation of data

# Feature properties

For series with different lengths, scales, domains, etc., we need features that are:

- scale-independent
- ergodic

# Feature properties

For series with different lengths, scales, domains, etc., we need features that are:

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- 1 Seasonal period
- 2 Strength of seasonality
- 3 Strength of trend
- 4 First autocorrelation of STL remainder series
- 5 Spectral entropy
- 6 Optimal MLE Box-Cox transformation of data

# Feature properties

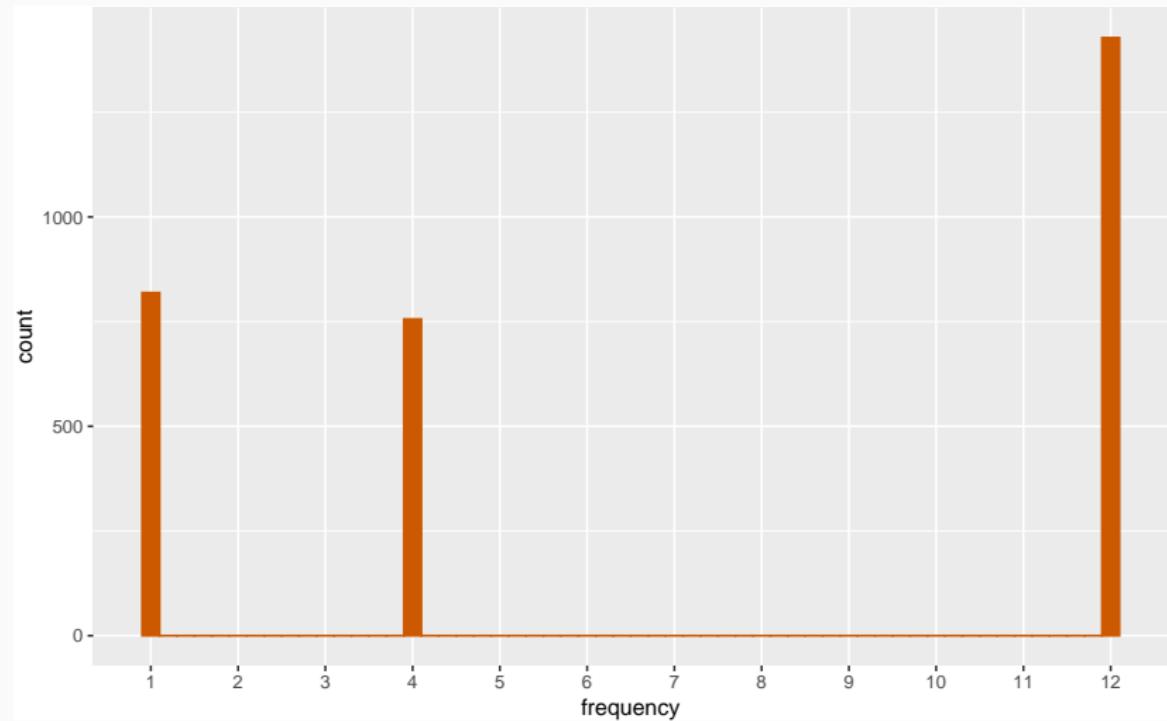
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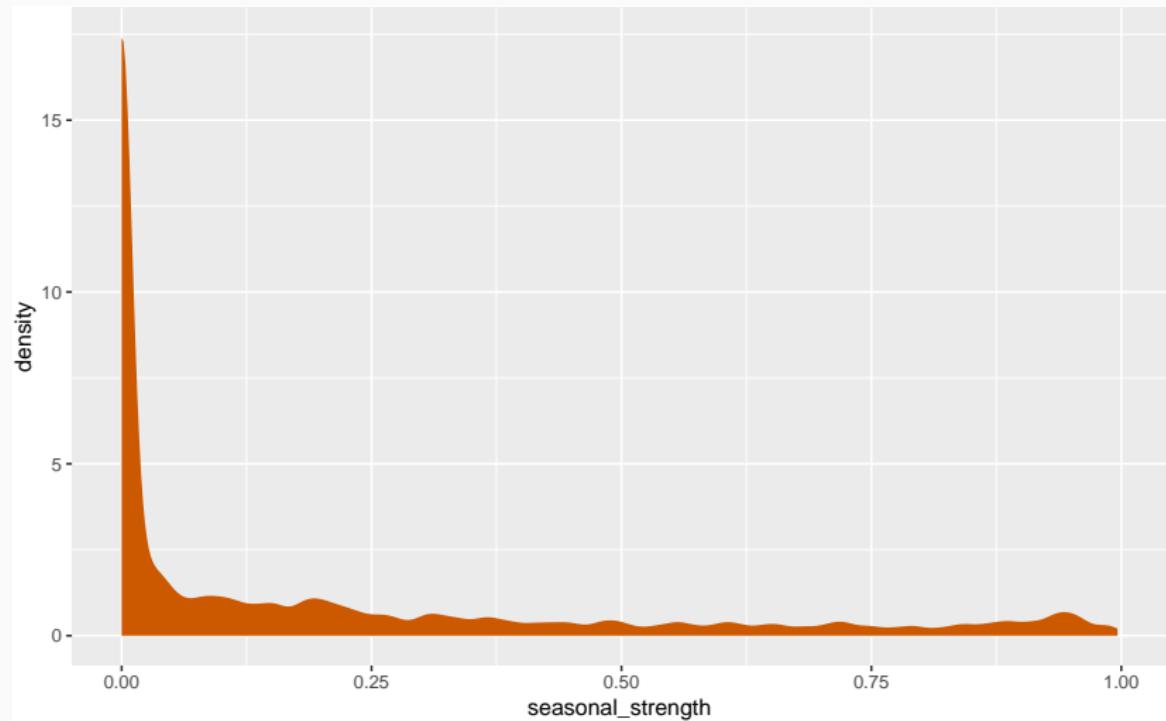
- 1 Seasonal period
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For other analyses, it may be appropriate to have different requirements.

# Distribution of Period for M3

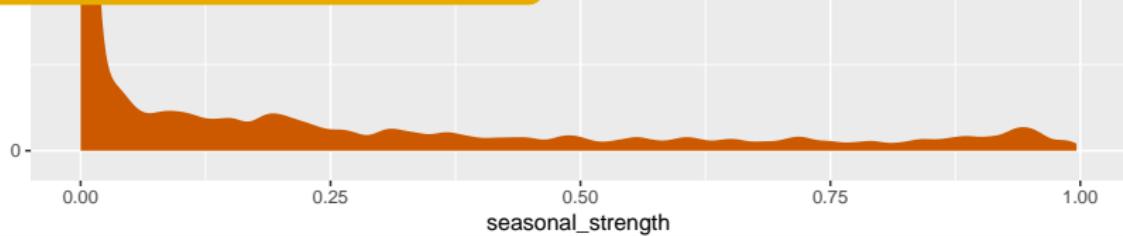
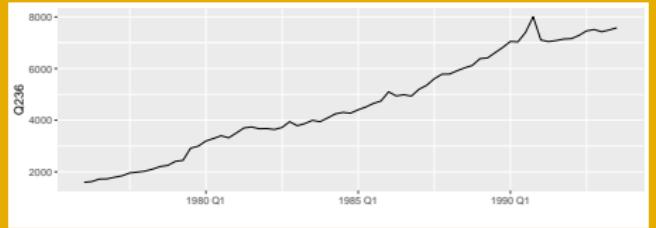


# Distribution of Seasonality for M3



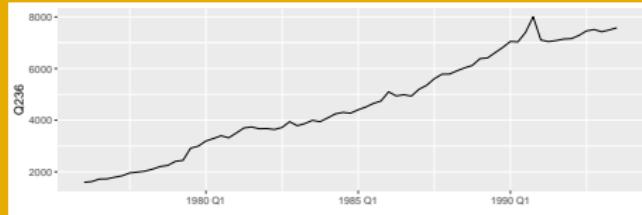
# Distribution of Seasonality for M3

Low Seasonality

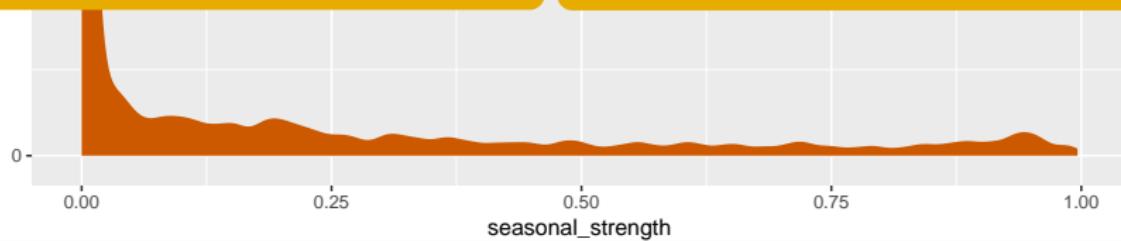
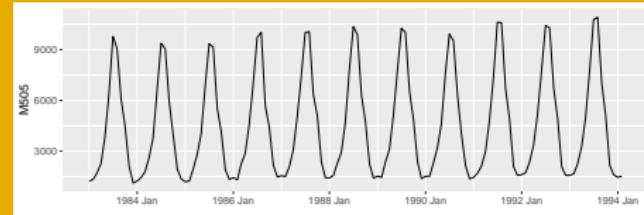


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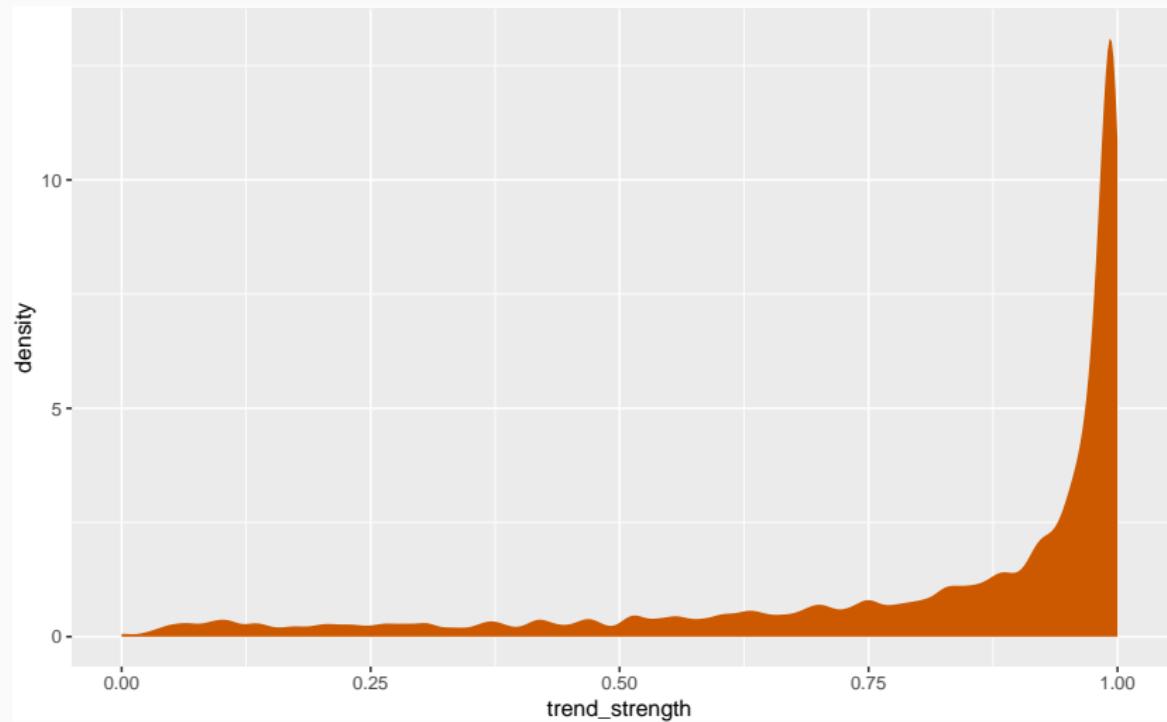
Low Seasonality



High Seasonality

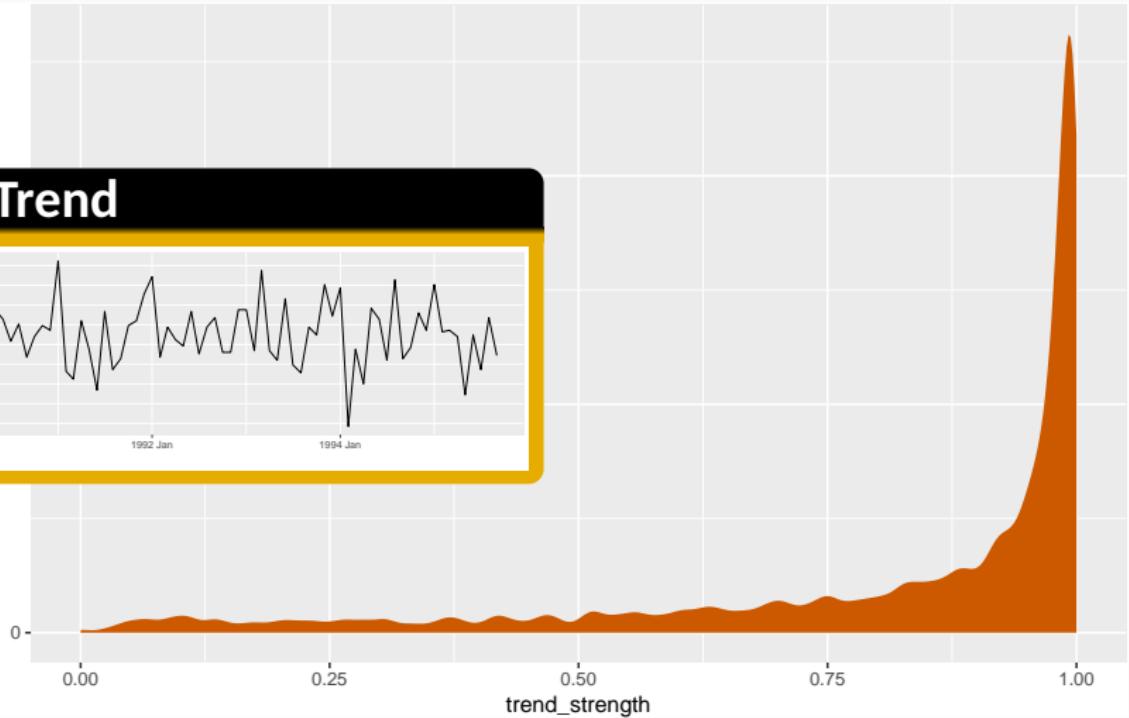
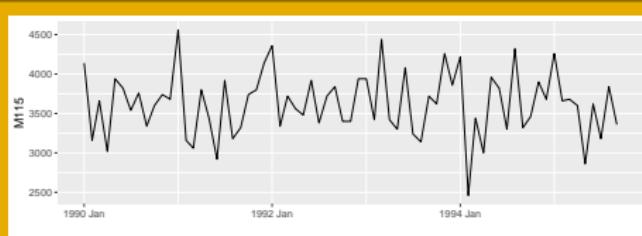


# Distribution of Trend for M3



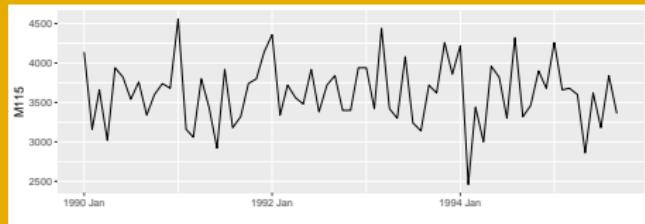
# Distribution of Trend for M3

Low Trend

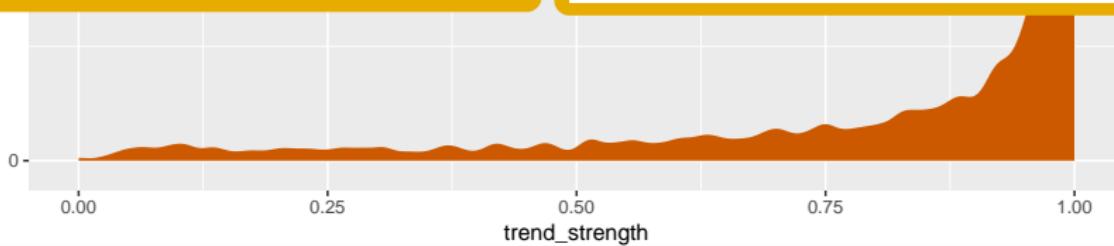
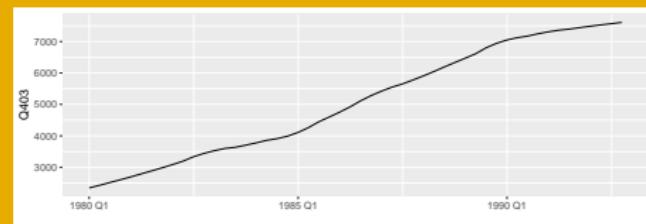


# Distribution of Trend for M3

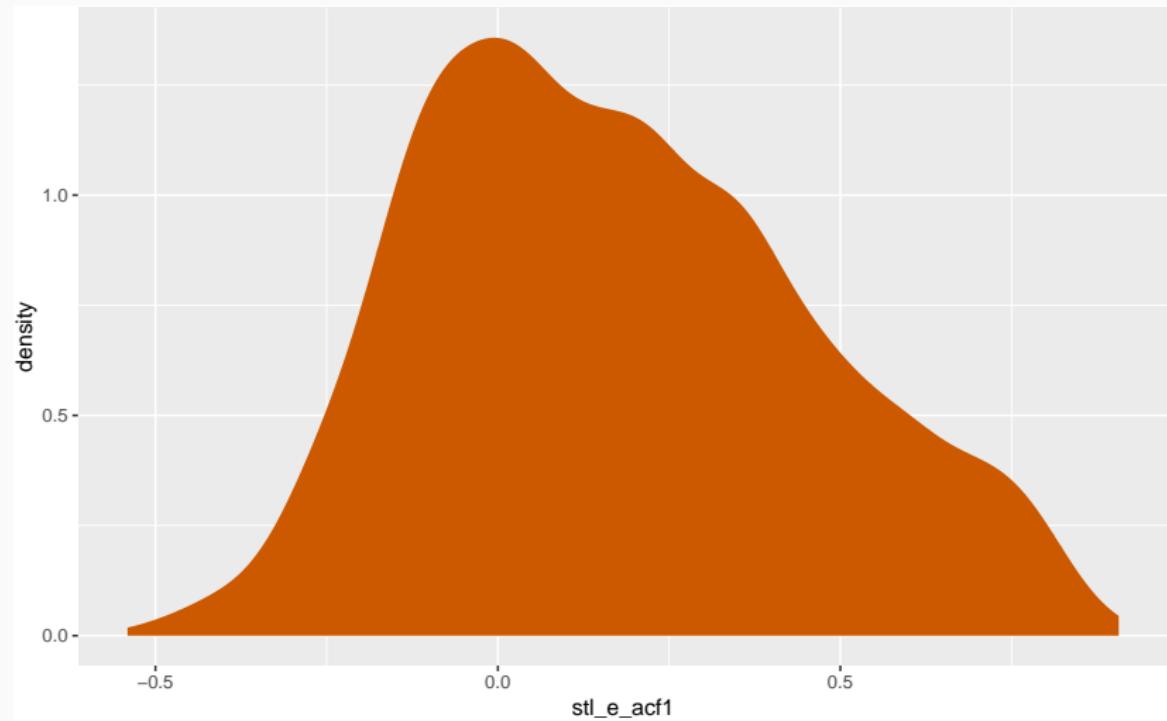
Low Trend



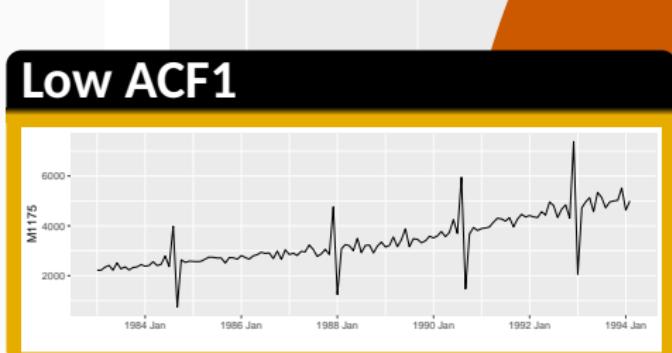
High Trend



# Distribution of Residual ACF1 for M3

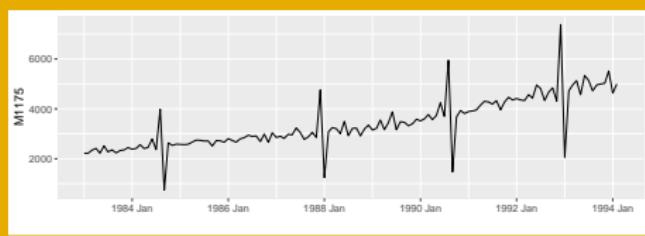


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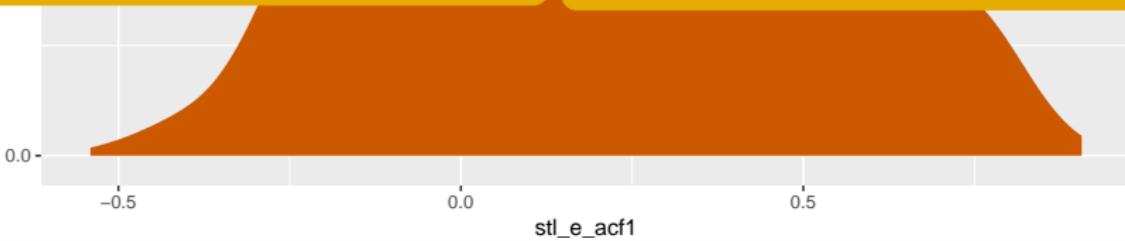
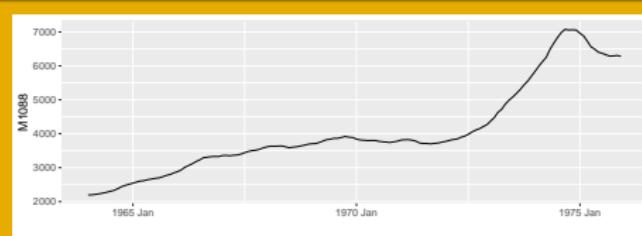


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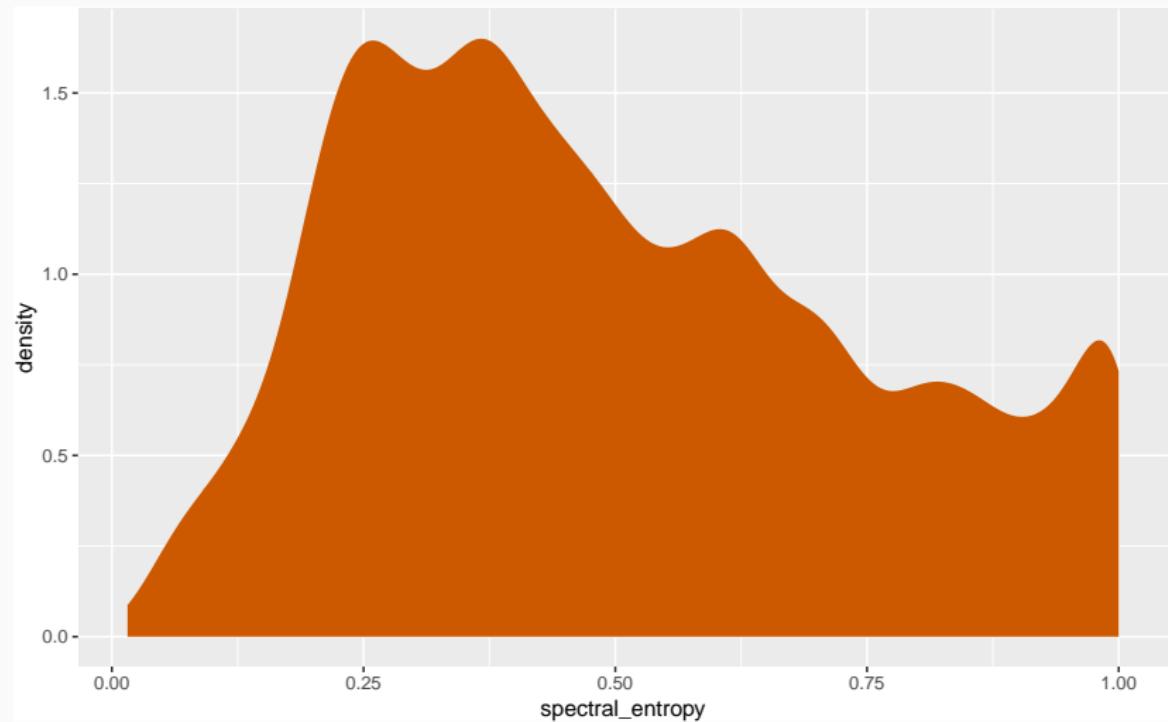
Low ACF1



High ACF1

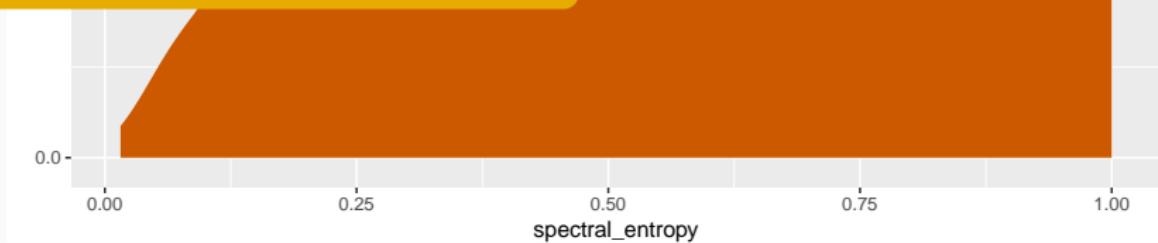
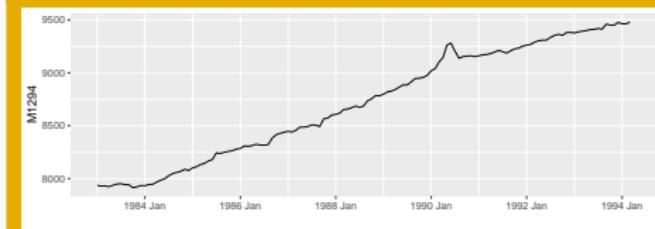


# Distribution of Spectral Entropy for M3

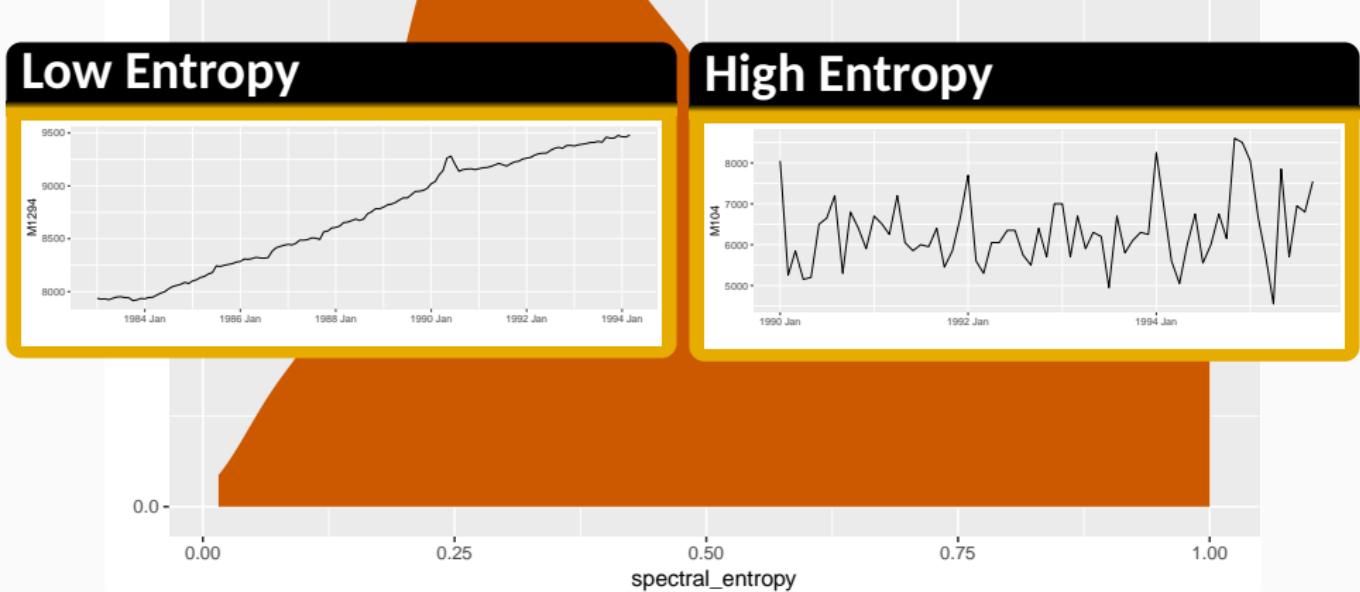


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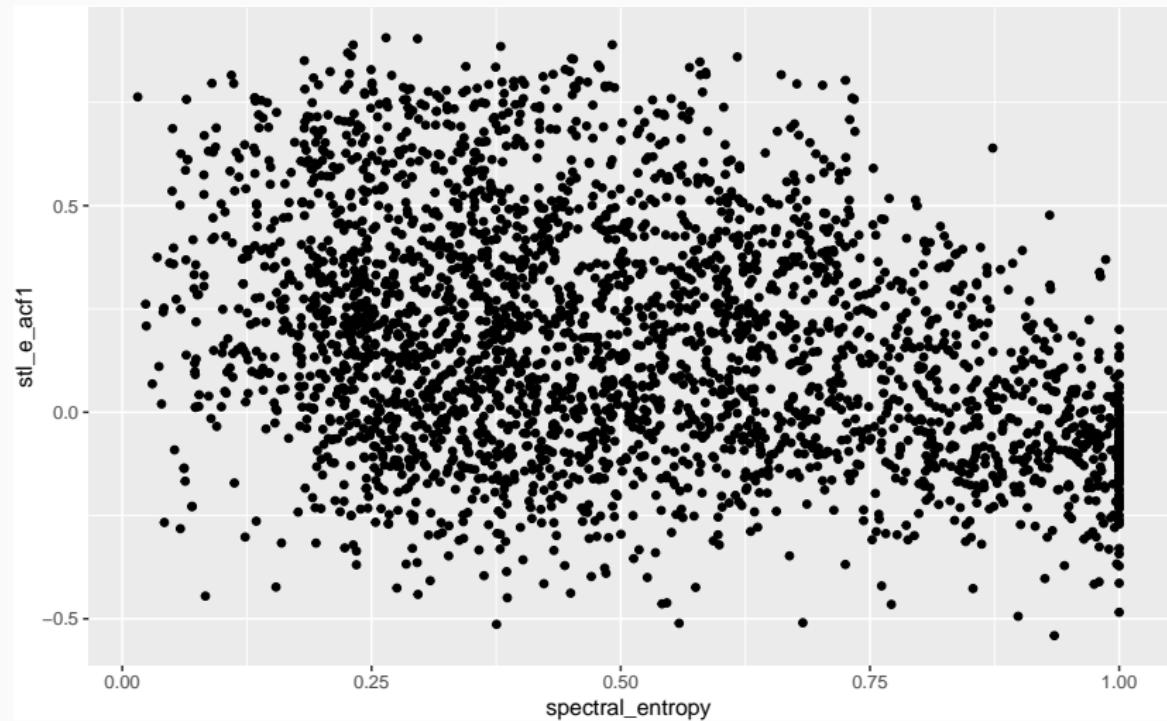
Low Entropy



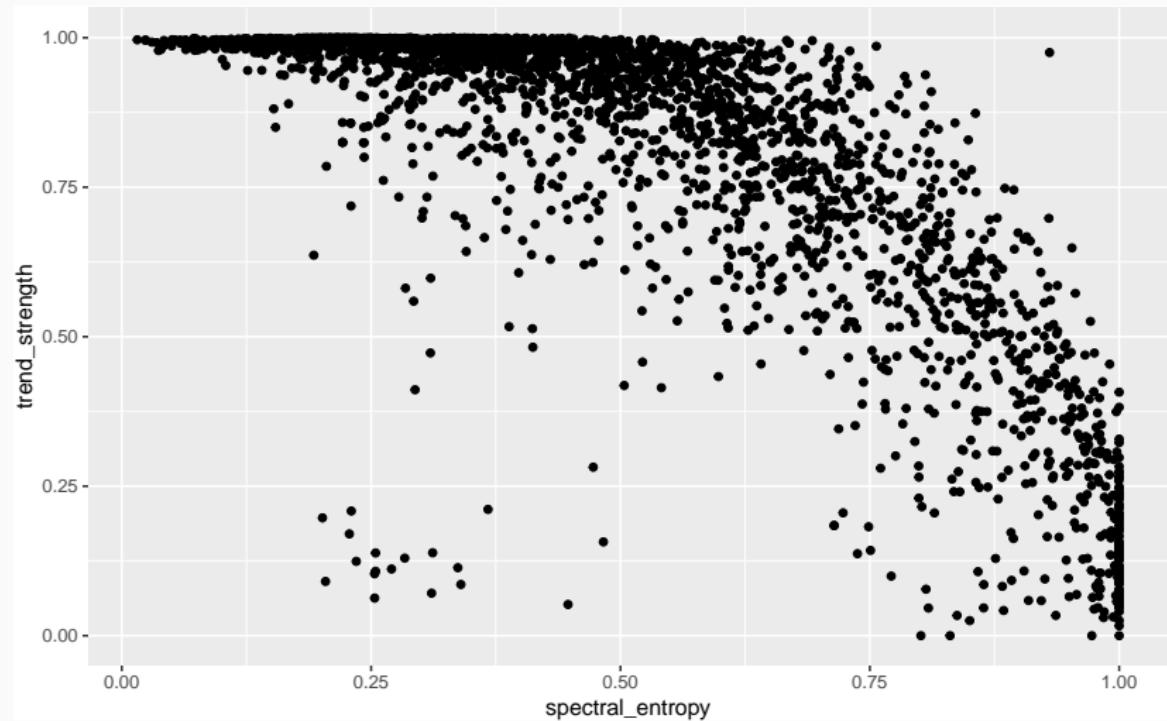
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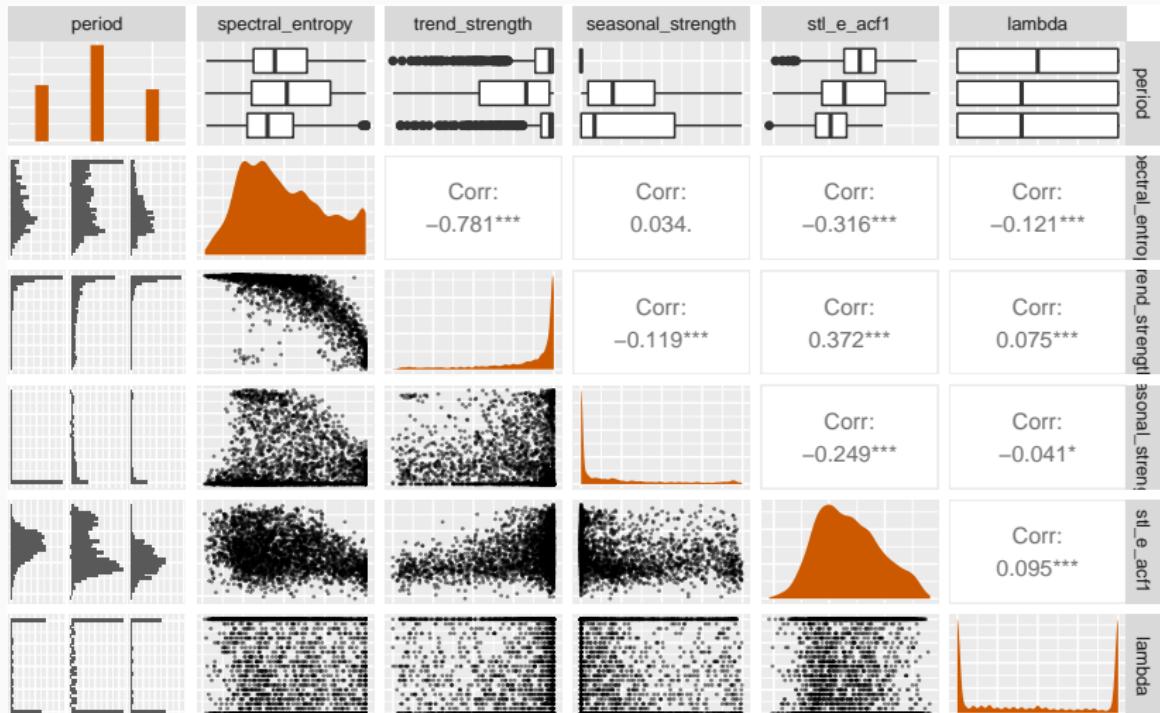
# Feature distributions



# Feature distributions

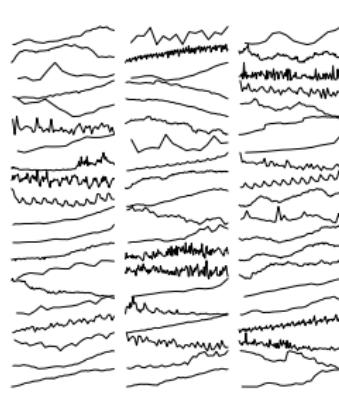


# Feature distributions

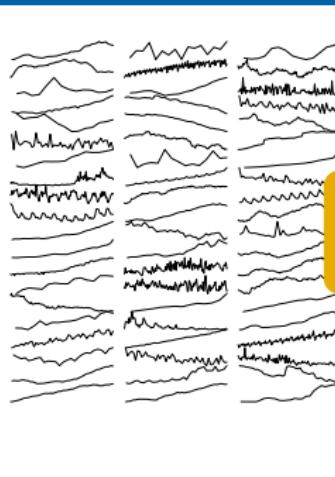


```
## pdf
```

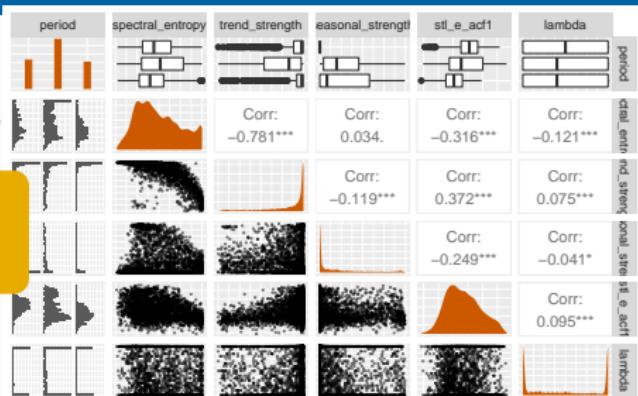
# Dimension reduction for time series



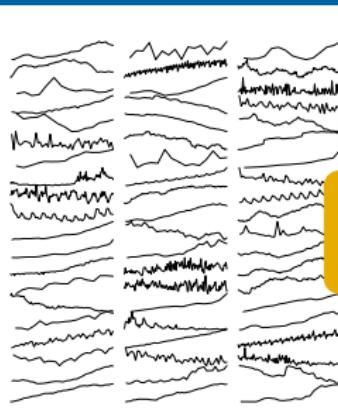
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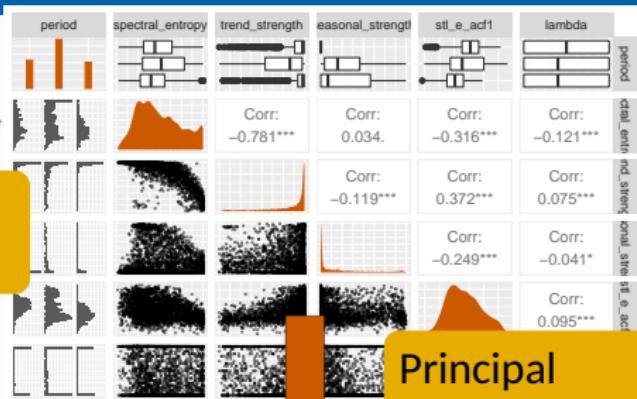
Feature  
calculation



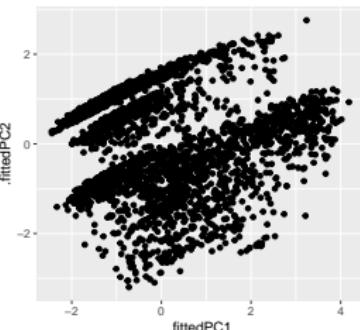
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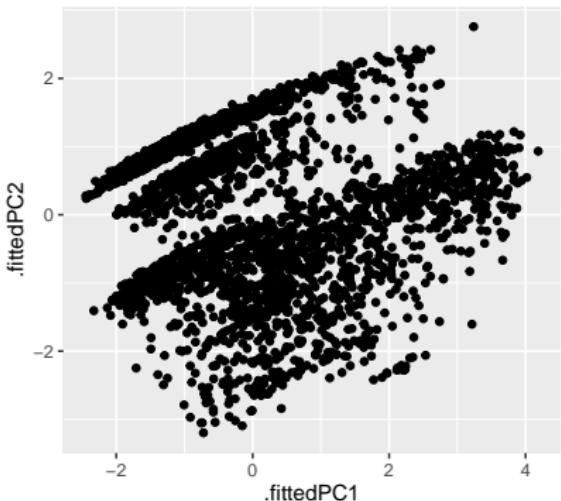


Principal  
component  
decomposition

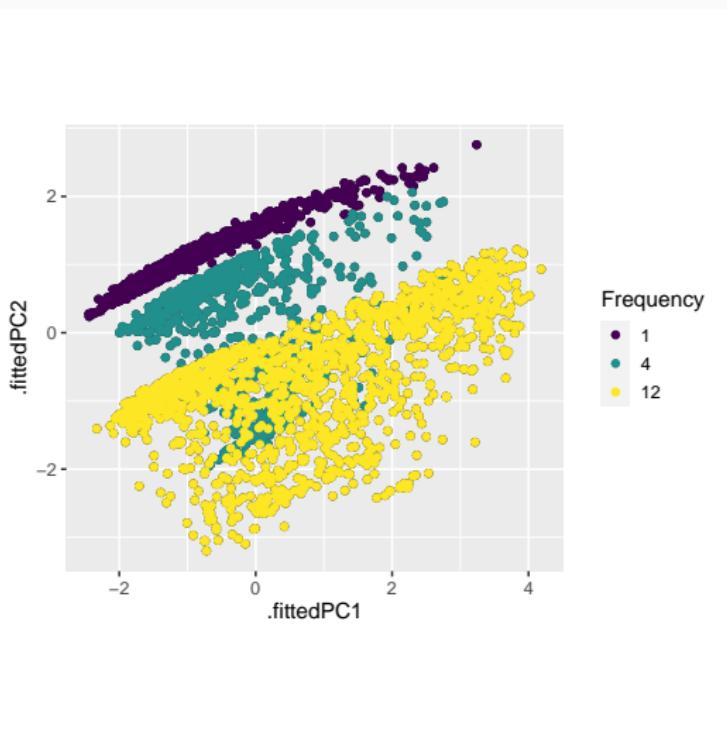


# M3 feature space

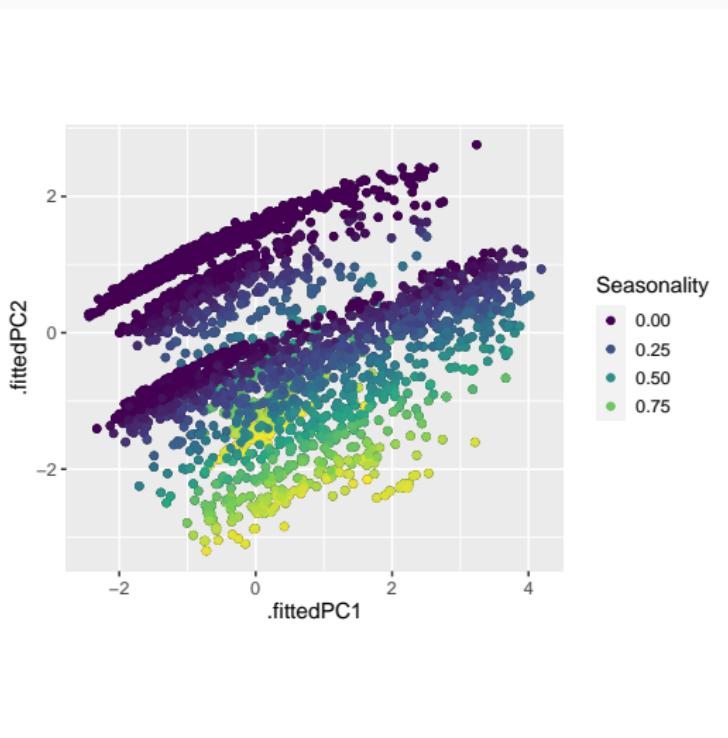
First two PCs explain  
58.5% of the variance.



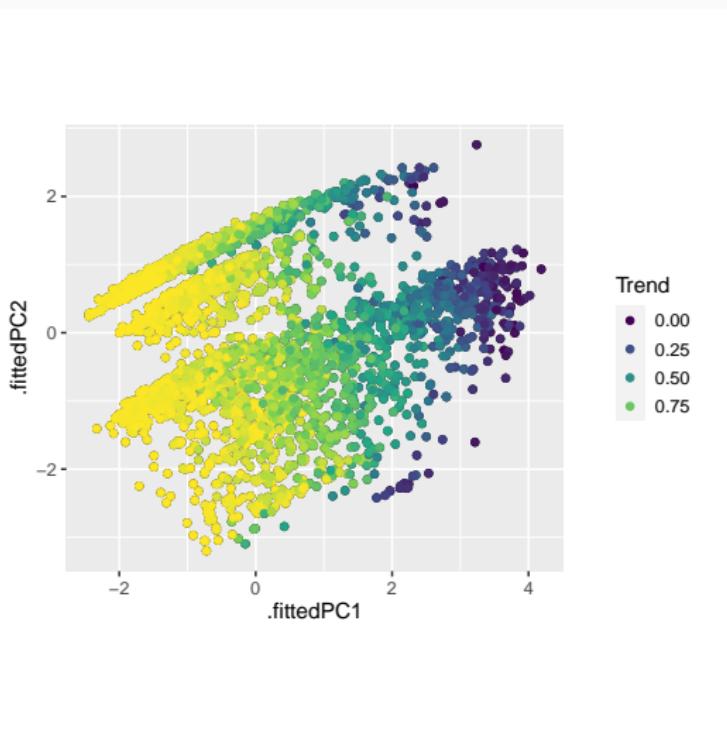
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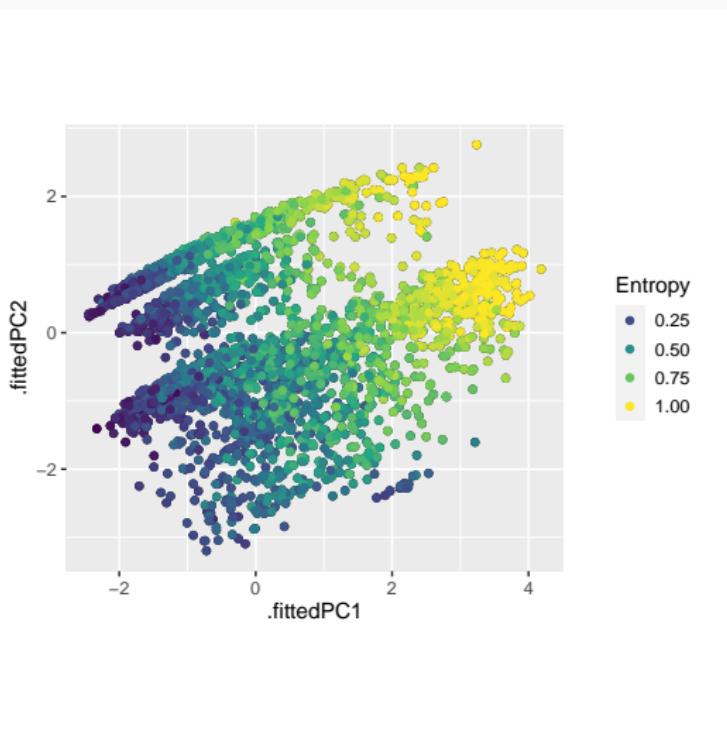
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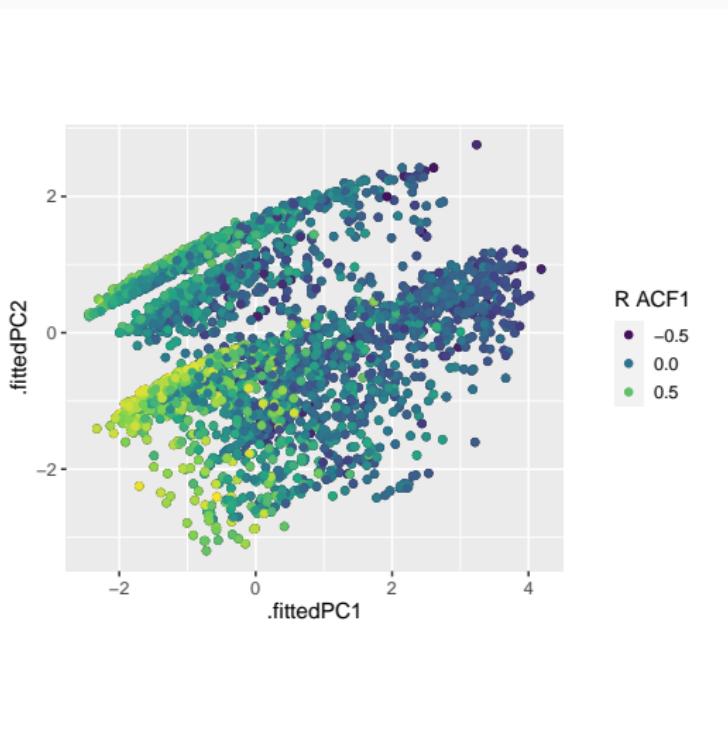
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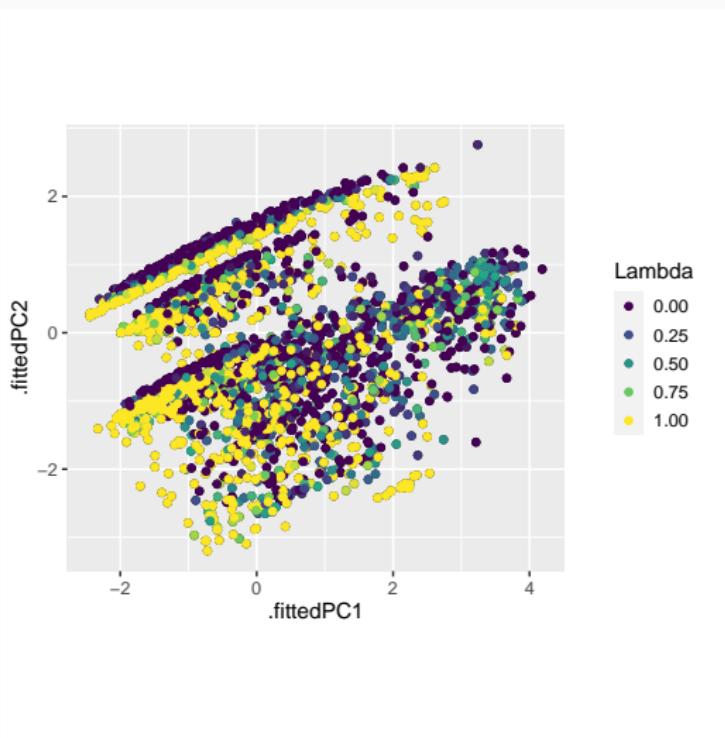
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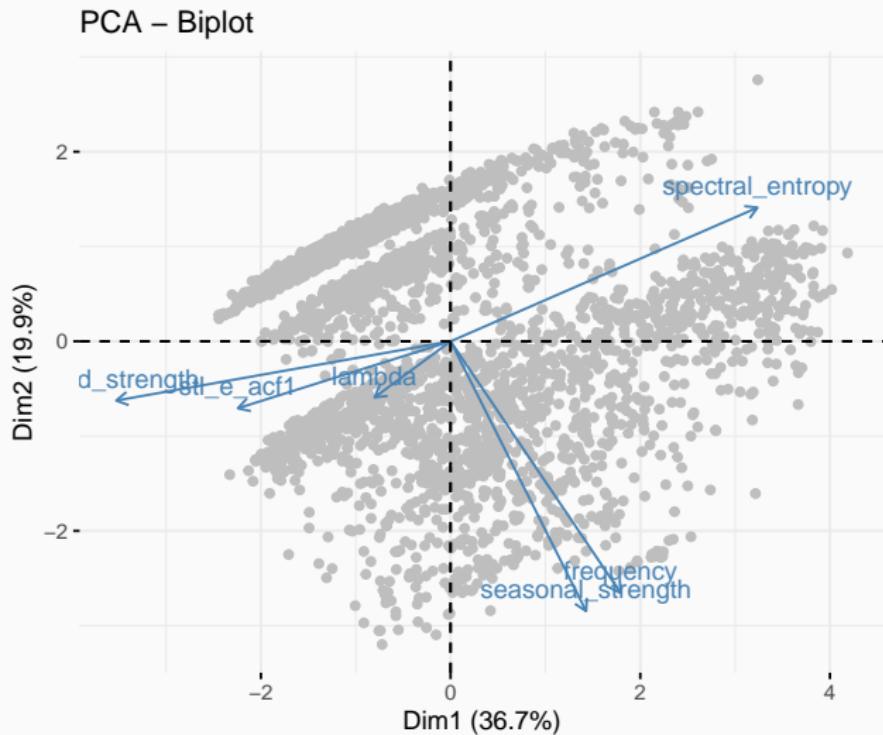
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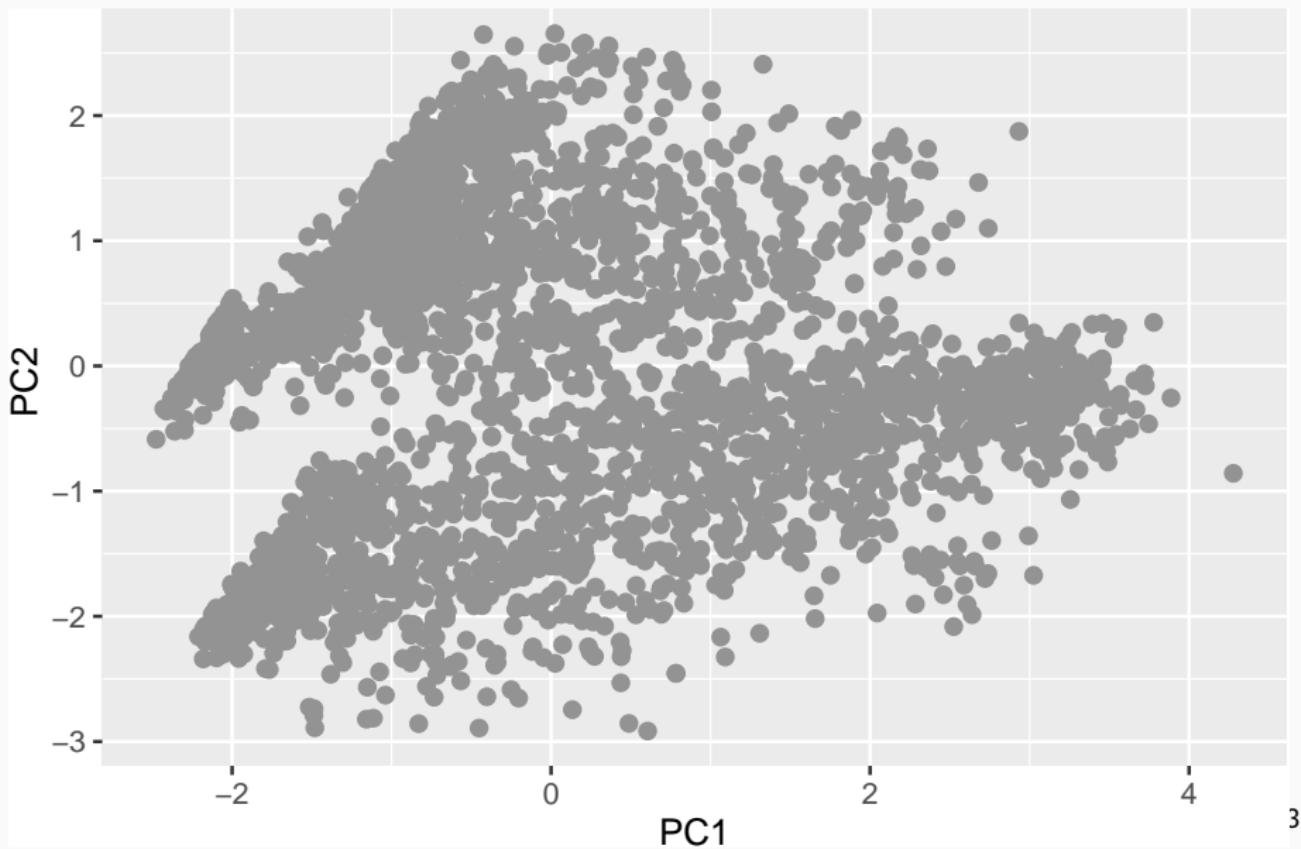
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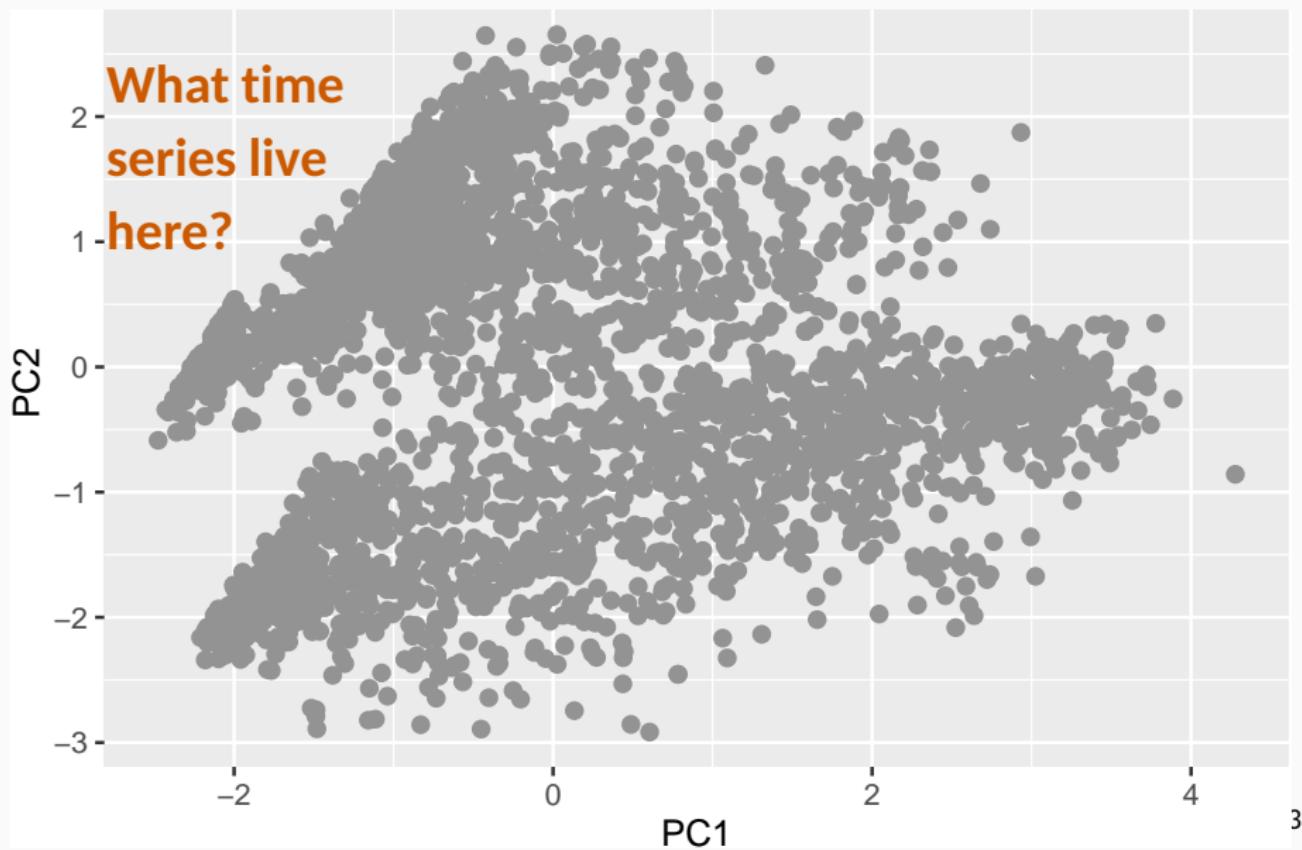
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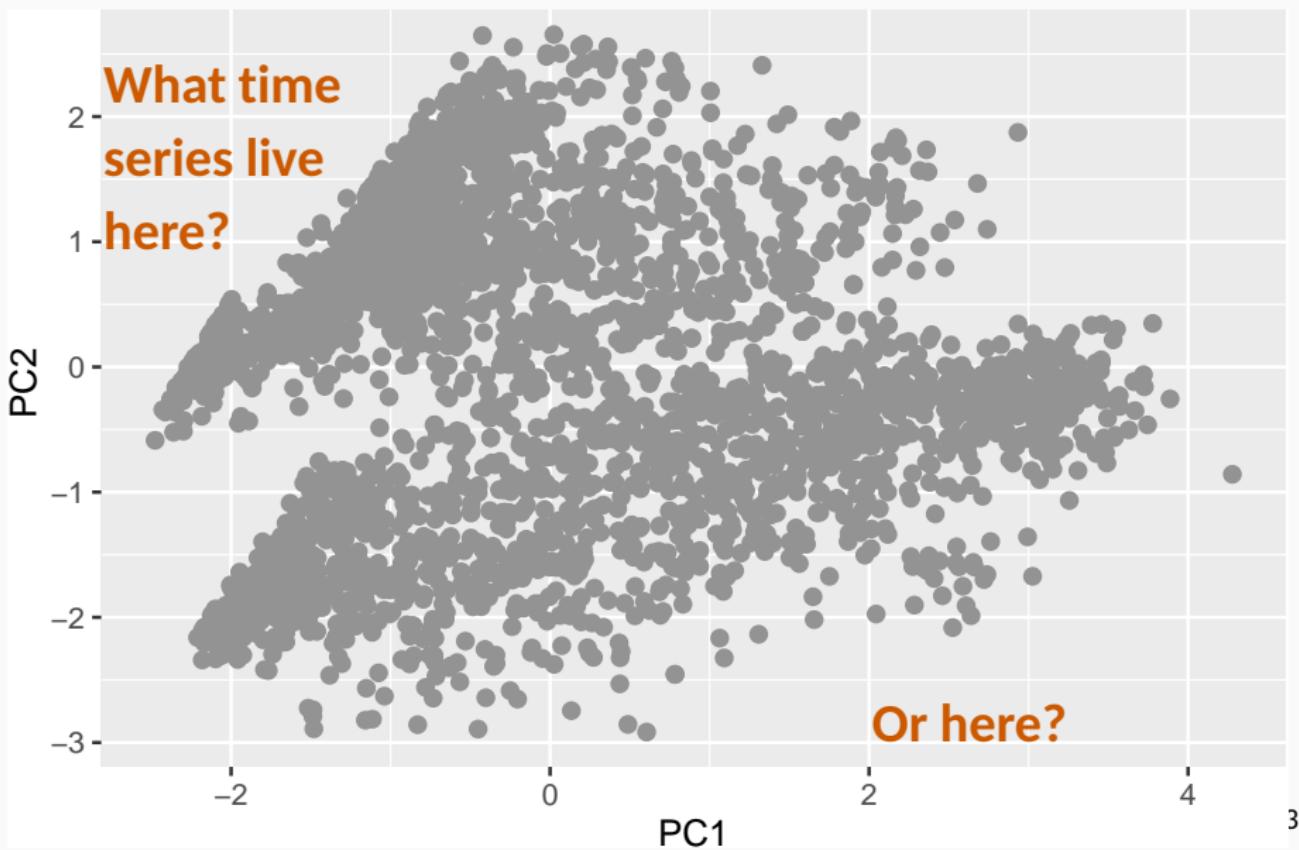
# What about the holes?



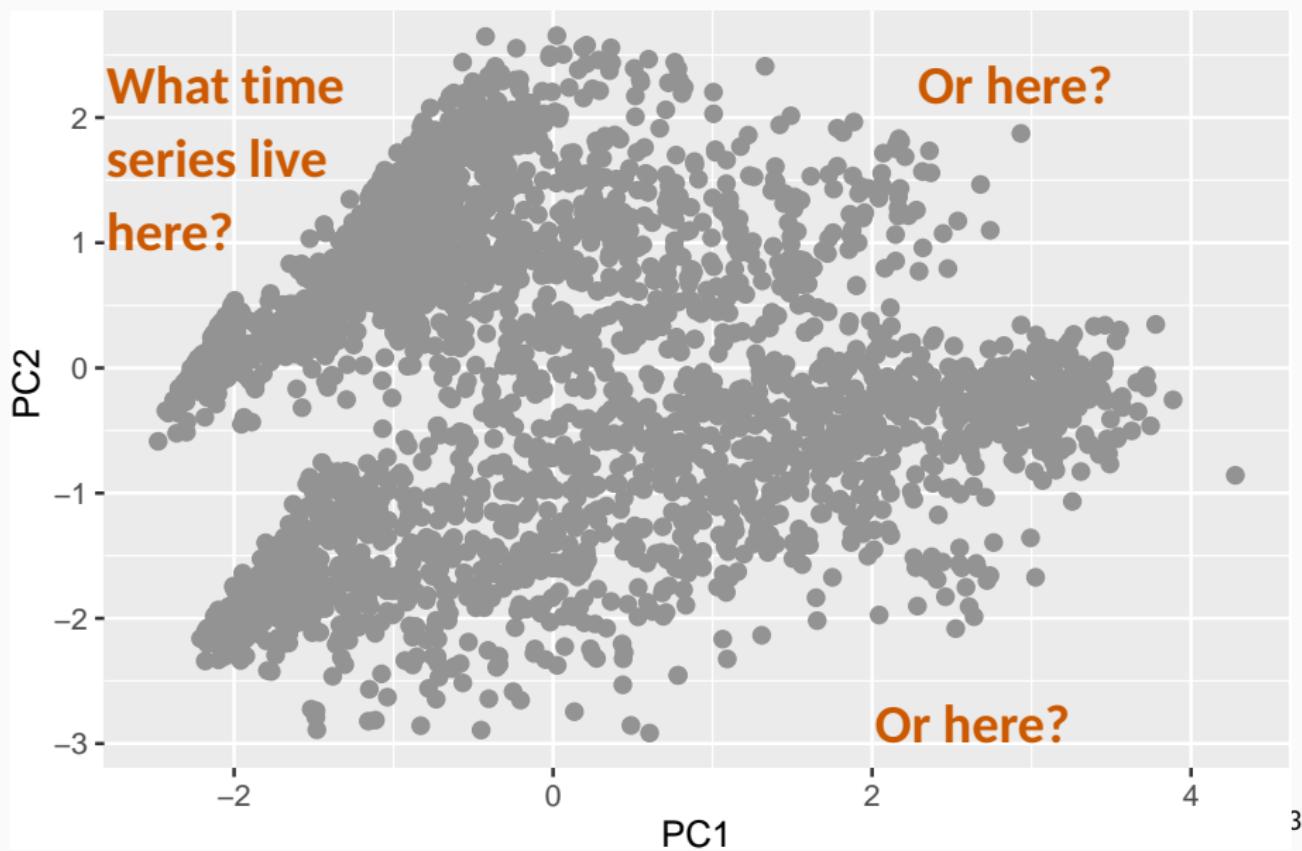
# What about the holes?



# What about the holes?



# What about the holes?



# Generating new time series

We can use the feature space to:

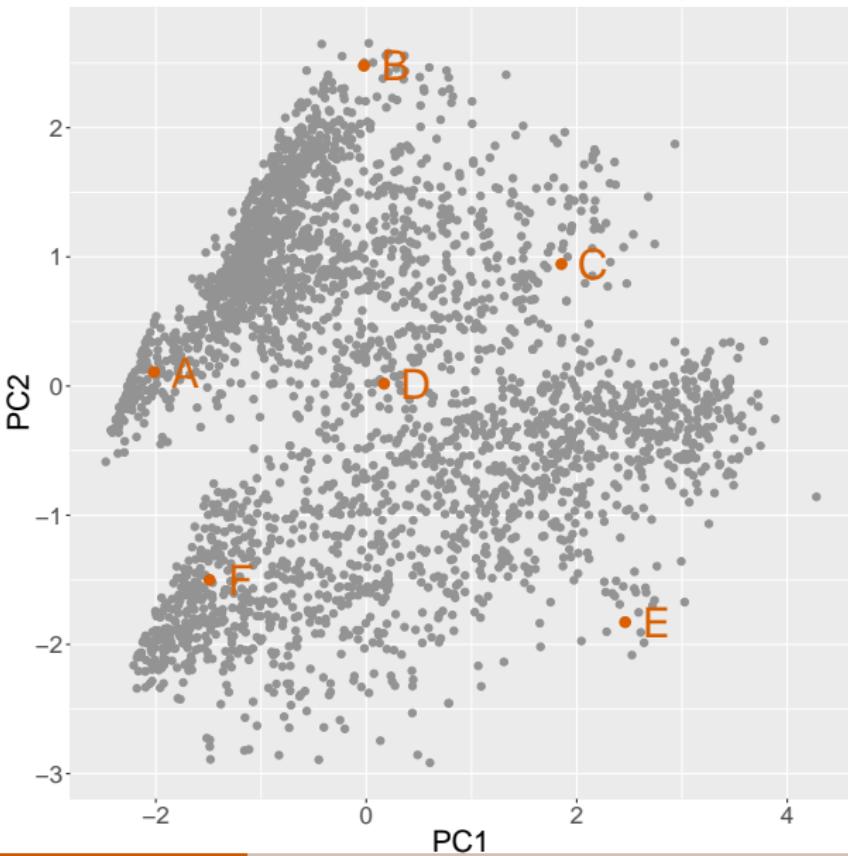
- Generate new time series with similar features to existing series
- Generate new time series where there are “holes” in the feature space.

# Generating new time series

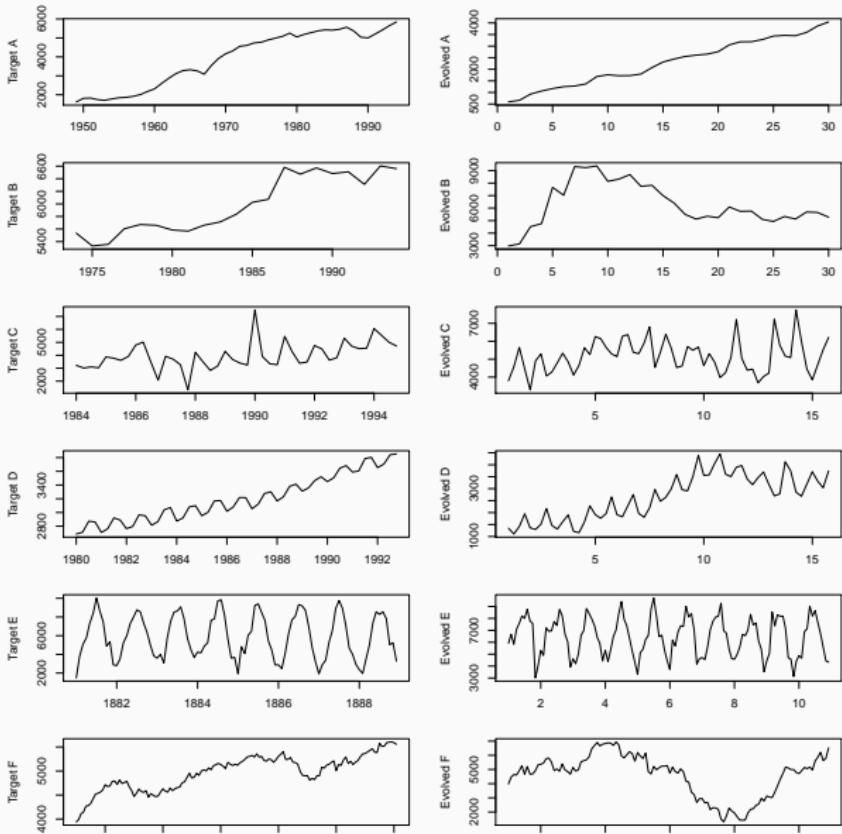
We can use the feature space to:

- ▶ Generate new time series with similar features to existing series
- ▶ Generate new time series where there are “holes” in the feature space.
- Let  $\{\text{PC}_1, \text{PC}_2, \dots, \text{PC}_n\}$  be a “population” of time series of specified length and period.
- Genetic algorithm uses a process of selection, crossover and mutation to evolve the population towards a target point  $T_i$ .
- Optimize: Fitness ( $\text{PC}_j$ ) =  $-\sqrt{(|\text{PC}_j - T_i|^2)}$ .
- Initial population random with some series in neighbourhood of  $T_i$ .

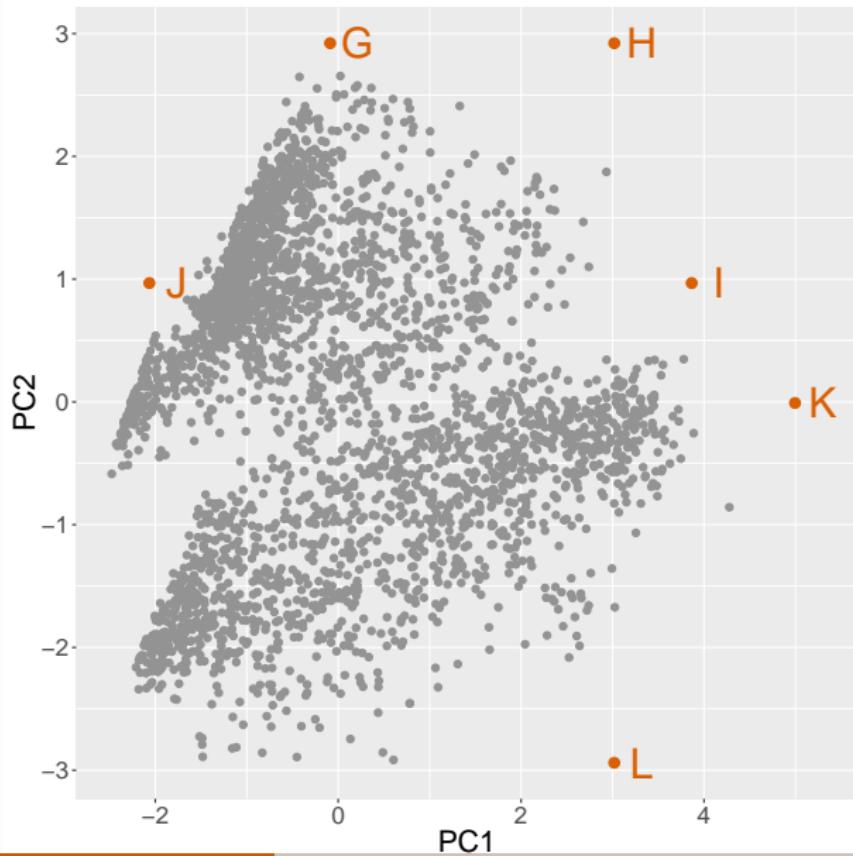
# Evolving new time series



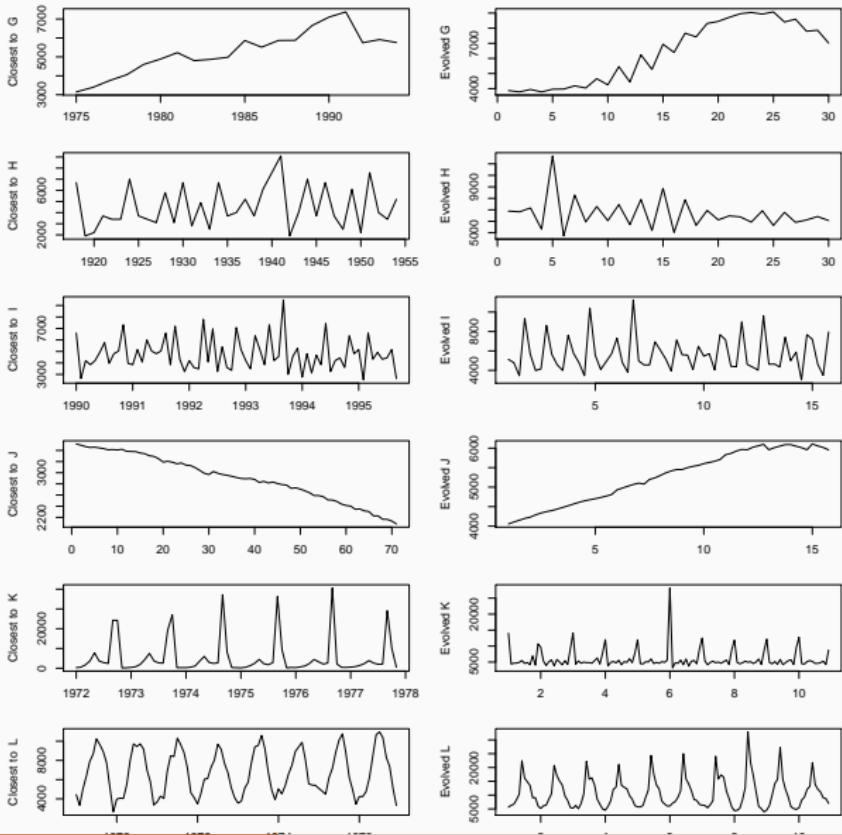
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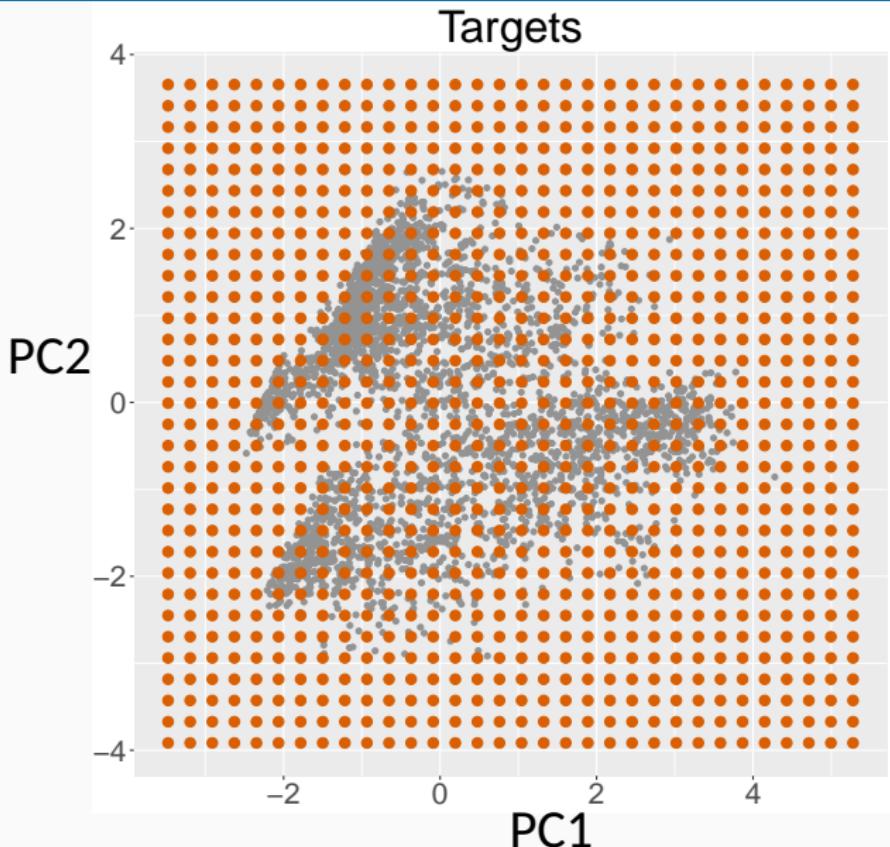
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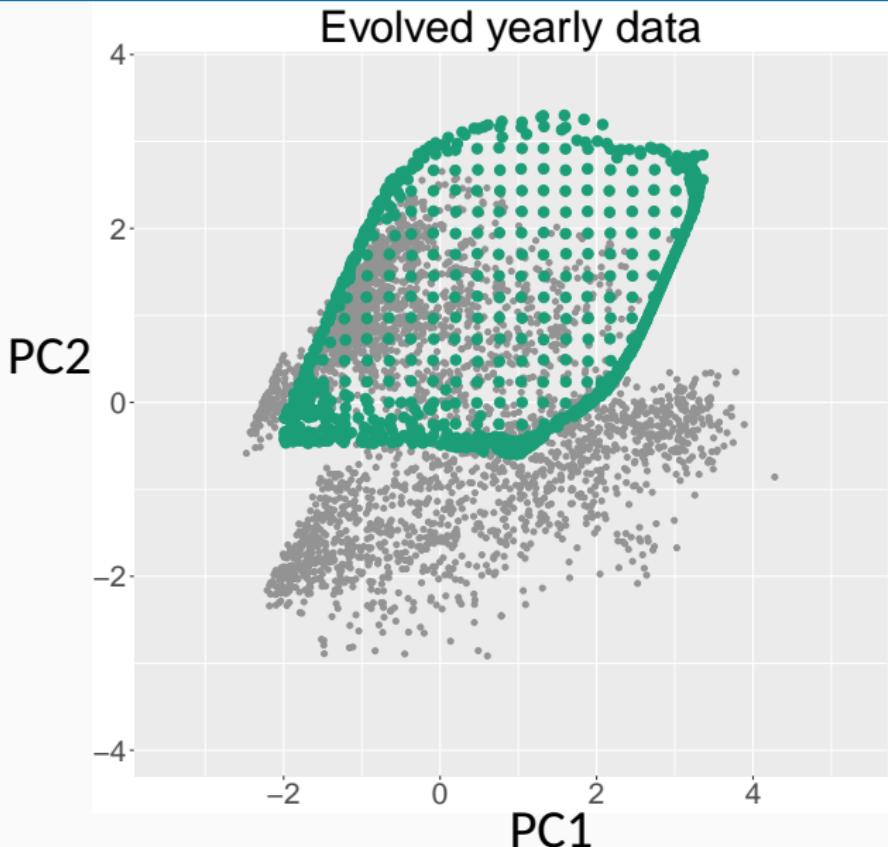
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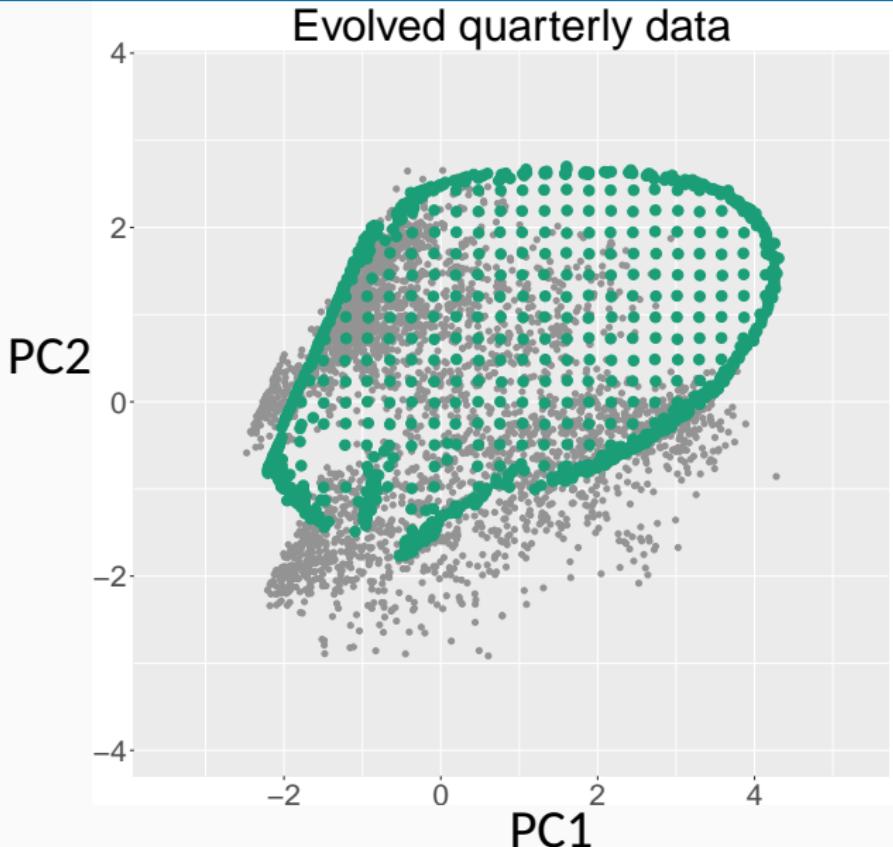
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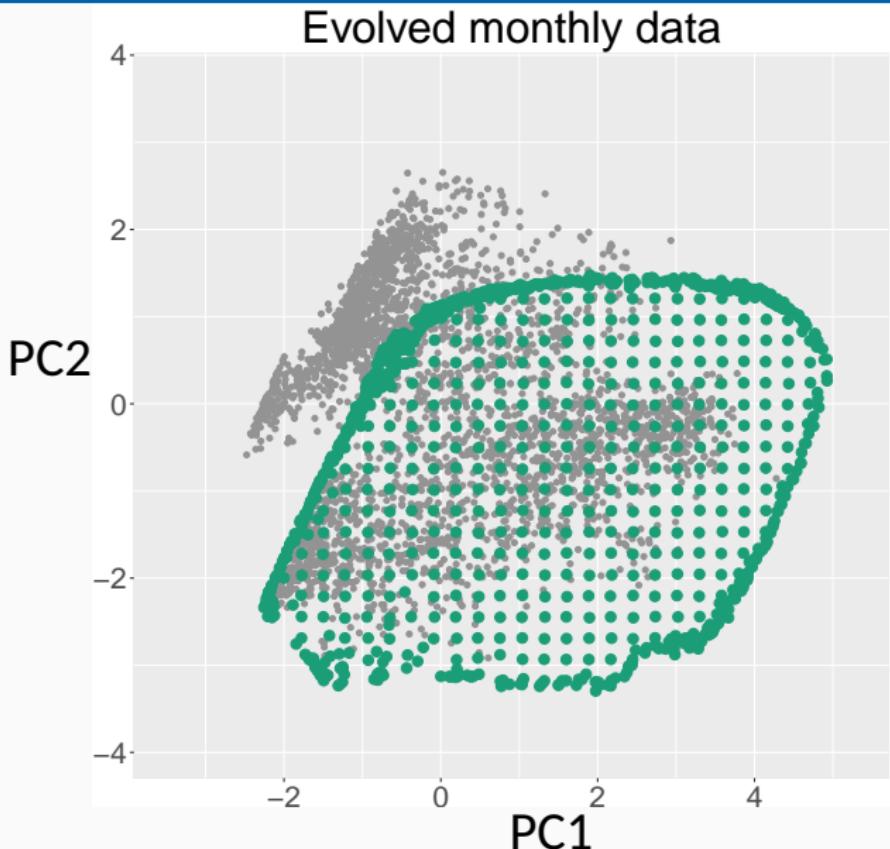
# Evolving new time series



# Evolving new time series



# Evolving new time series



# Outline

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- 4 Feature-based forecasting



# tsibble objects

```
library(tidyverse)
library(tsibble)
library(feasts)
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:      Region, State, Purpose [304]
##   Quarter Region  State Purpose  Trips
##       <qtr> <chr>    <chr> <chr>    <dbl>
## 1 1998   Q1 Adelaide SA  Business  135.
## 2 1998   Q2 Adelaide SA  Business  110.
## 3 1998   Q3 Adelaide SA  Business  166.
## 4 1998   Q4 Adelaide SA  Business  127.
## 5 1999   Q1 Adelaide SA  Business  137.
## 6 1999   Q2 Adelaide SA  Business  200.
## 7 1999   Q3 Adelaide SA  Business  169.
## 8 1999   Q4 Adelaide SA  Business  134.
```

# tsibble objects

```
library(tidyverse)
```

```
library(tsibble)
```

```
library(feasts)
```

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:      Region, State, Purpose [304]
##   Quarter Region  State Purpose Trips
##   Index    <chr>    <chr>  <chr>   <dbl>
## 1 1998   Q1 Adelaide SA   Business 135.
## 2 1998   Q2 Adelaide SA   Business 110.
## 3 1998   Q3 Adelaide SA   Business 166.
## 4 1998   Q4 Adelaide SA   Business 127.
## 5 1999   Q1 Adelaide SA   Business 137.
## 6 1999   Q2 Adelaide SA   Business 200.
## 7 1999   Q3 Adelaide SA   Business 169.
## 8 1999   Q4 Adelaide SA   Business 134.
```

# tsibble objects

```
library(tidyverse)
library(tsibble)
library(feasts)
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:      Region, State, Purpose [304]
## #   Quarter Region  State Purpose Trips
## #   Index     Keys          <dbl>
## # 1 1998 Q1 Adelaide SA Business 135.
## # 2 1998 Q2 Adelaide SA Business 110.
## # 3 1998 Q3 Adelaide SA Business 166.
## # 4 1998 Q4 Adelaide SA Business 127.
## # 5 1999 Q1 Adelaide SA Business 137.
## # 6 1999 Q2 Adelaide SA Business 200.
## # 7 1999 Q3 Adelaide SA Business 169.
## # 8 1999 Q4 Adelaide SA Business 134.
```

# tsibble objects

```
library(tidyverse)
library(tsibble)
library(feasts)
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:      Region, State, Purpose [304]
## #   Quarter Region  State Purpose Trips
## #   Index     Keys          Measure
## # 1 1998 Q1 Adelaide SA Business 135.
## # 2 1998 Q2 Adelaide SA Business 110.
## # 3 1998 Q3 Adelaide SA Business 166.
## # 4 1998 Q4 Adelaide SA Business 127.
## # 5 1999 Q1 Adelaide SA Business 137.
## # 6 1999 Q2 Adelaide SA Business 200.
## # 7 1999 Q3 Adelaide SA Business 169.
## # 8 1999 Q4 Adelaide SA Business 134.
```

# tsibble objects

```
library(tidyverse)
library(tsibble)
library(feasts)
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:      Region, State, Purpose [304]
## #   Quarter Region  State Purpose Trips
## #   Index     Keys          Measure
## # 1 1998 Q1 Adelaide SA Business 135.
## # 2 1998 Q2 Adelaide SA Business 110. Domestic visitor
## # 3 1998 Q3 Adelaide SA Business 166. nights in thousands
## # 4 1998 Q4 Adelaide SA Business 127. by state/region and
## # 5 1999 Q1 Adelaide SA Business 137. purpose.
## # 6 1999 Q2 Adelaide SA Business 200.
## # 7 1999 Q3 Adelaide SA Business 169.
## # 8 1999 Q4 Adelaide SA Business 134.
```

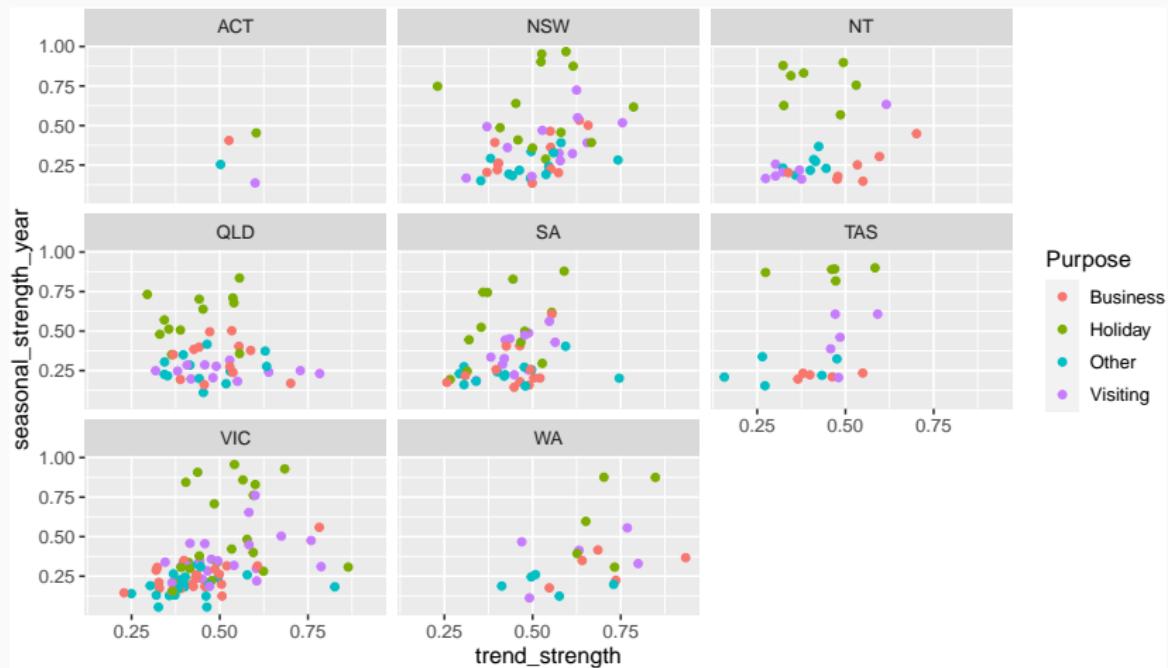
# Feature extraction and statistics

```
tourism %>% features(Trips, feature_set(tags="stl"))
```

```
## # A tibble: 304 x 12
##   Region      State Purpose trend_strength seasonal_streng~
##   <chr>        <chr>  <chr>    <dbl>            <dbl>
## 1 Adelaide    SA     Busine~    0.464            0.407
## 2 Adelaide    SA     Holiday   0.554            0.619
## 3 Adelaide    SA     Other    0.746            0.202
## 4 Adelaide    SA     Visiti~  0.435            0.452
## 5 Adelaide Hills SA     Busine~  0.464            0.179
## 6 Adelaide Hills SA     Holiday  0.528            0.296
## 7 Adelaide Hills SA     Other   0.593            0.404
## 8 Adelaide Hills SA     Visiti~  0.488            0.254
## 9 Alice Springs NT     Busine~  0.534            0.251
## 10 Alice Springs NT    Holiday  0.381            0.832
## # ... with 294 more rows, and 7 more variables:
## #   seasonal_peak_year <dbl>, seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>,
## #   stl_e_acf1 <dbl>, stl_e_acf10 <dbl>
```

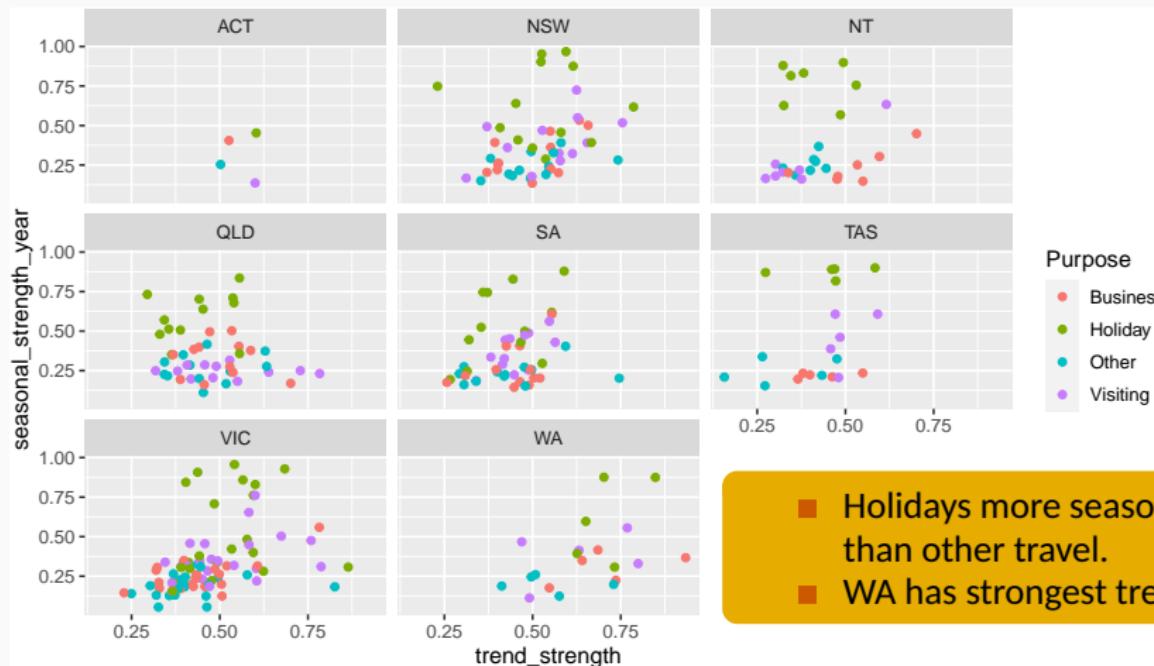
# Feature extraction and statistics

```
tourism %>% features(Trips, feature_set(tags="stl")) %>%
  ggplot(aes(x=trend_strength, y=seasonal_strength_year, col=Purpose)) +
  geom_point() + facet_wrap(vars(State))
```



# Feature extraction and statistics

```
tourism %>% features(Trips, feature_set(tags="stl")) %>%
  ggplot(aes(x=trend_strength, y=seasonal_strength_year, col=Purpose)) +
  geom_point() + facet_wrap(vars(State))
```



# Feature extraction and statistics

Find the most seasonal time series:

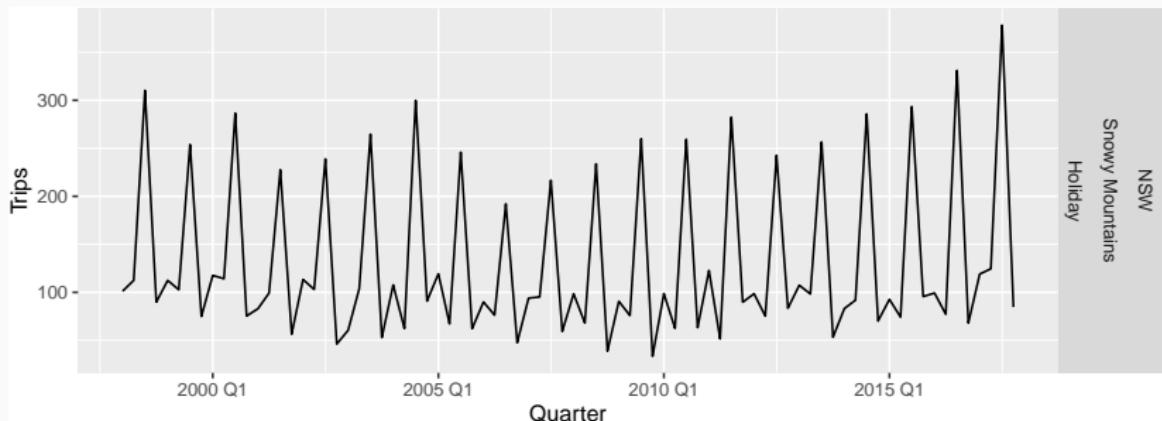
```
most_seasonal <- tourism %>%
  features(Trips, feature_set(tags="stl")) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

# Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism %>%
  features(Trips, feature_set(tags="stl")) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))

tourism %>%
  right_join(most_seasonal, by = c("State","Region","Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) + geom_line() +
  facet_grid(vars(State,Region,Purpose))
```



# Feature extraction and statistics

```
tourism_features <- tourism %>%
  features(Trips, feature_set(pkgs="feasts"))

## # A tibble: 304 x 51
##   Region      State Purpose trend_strength seasonal_streng~
##   <chr>        <chr>  <chr>     <dbl>           <dbl>
## 1 Adelaide    SA     Busine~    0.464          0.407
## 2 Adelaide    SA     Holiday    0.554          0.619
## 3 Adelaide    SA     Other     0.746          0.202
## 4 Adelaide    SA     Visiti~   0.435          0.452
## 5 Adelaide Hills SA     Busine~   0.464          0.179
## 6 Adelaide Hills SA     Holiday   0.528          0.296
## 7 Adelaide Hills SA     Other     0.593          0.404
## 8 Adelaide Hills SA     Visiti~   0.488          0.254
## 9 Alice Springs NT     Busine~   0.534          0.251
## 10 Alice Springs NT     Holiday   0.381          0.832
## # ... with 294 more rows, and 46 more variables:
## #   seasonal_peak_year <dbl>, seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>,
## #   stl_e_acf1 <dbl>, stl_e_acf10 <dbl>, acf1 <dbl>,
## #   acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   ...
```

All features from  
the feasts  
package

# Feature extraction and statistics

```
pcs <- tourism_features %>% select(-State, -Region, -Purpose) %>%  
  prcomp(scale=TRUE) %>% augment(tourism_features)
```

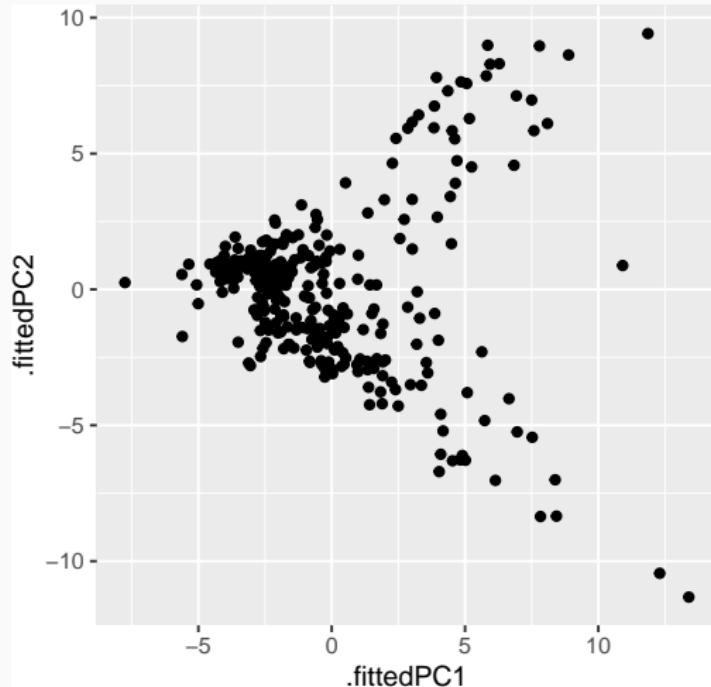
```
## # A tibble: 304 x 100  
##   .rownames Region      State Purpose trend_s  
##   <chr>     <chr>      <chr> <chr>  
## 1 1         Adelaide    SA    Business  
## 2 2         Adelaide    SA    Holiday  
## 3 3         Adelaide    SA    Other  
## 4 4         Adelaide    SA    Visiting  
## 5 5         Adelaide Hills SA    Business  
## 6 6         Adelaide Hills SA    Holiday      0.528  
## 7 7         Adelaide Hills SA    Other       0.593  
## 8 8         Adelaide Hills SA    Visiting    0.488  
## 9 9         Alice Springs NT    Business    0.534  
## 10 10        Alice Springs NT    Holiday     0.381  
## # ... with 294 more rows, and 95 more variables:  
## #   seasonal_strength_year <dbl>, seasonal_peak_year <dbl>,  
## #   seasonal_trough_year <dbl>, spikiness <dbl>,  
## #   linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,  
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>,  
## #   ...
```

Principal components based on all features from the feasts package

# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +  
  geom_point() + theme(aspect.ratio=1)
```

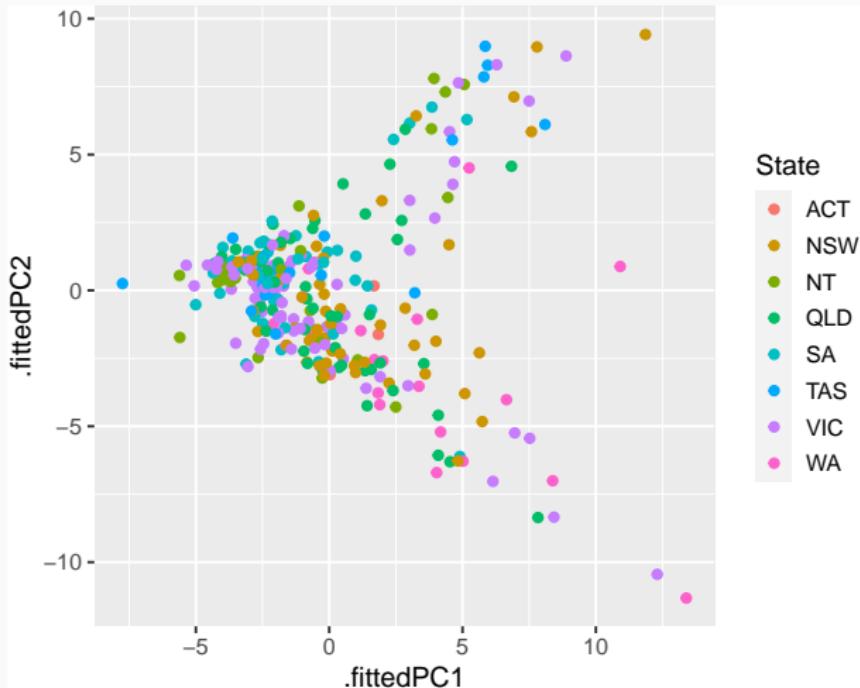
Principal components  
based on all features  
from the feasts  
package



# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +  
  geom_point() + theme(aspect.ratio=1)
```

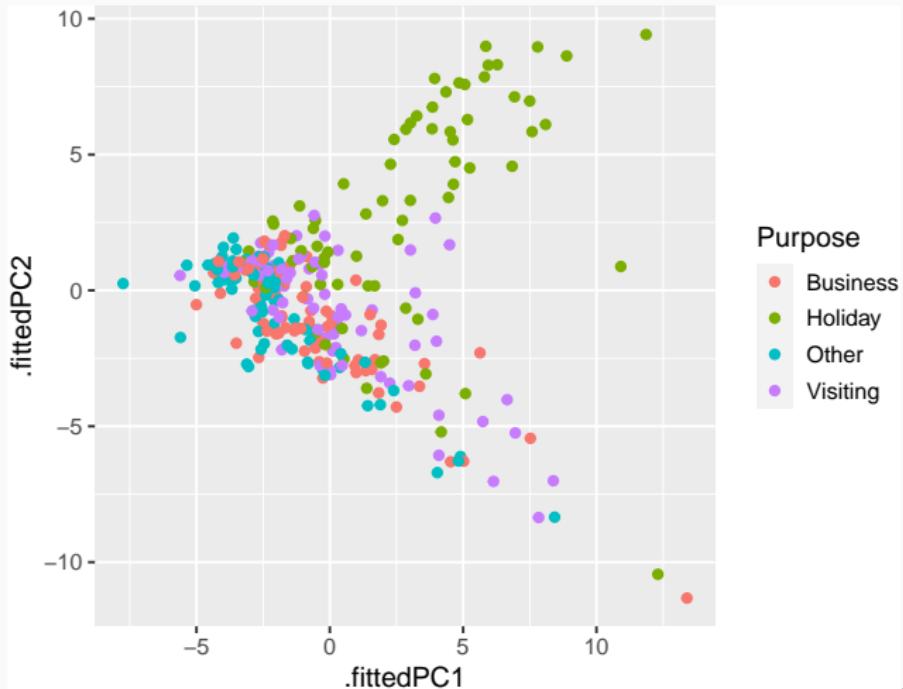
Principal components  
based on all features  
from the feasts  
package



# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +  
  geom_point() + theme(aspect.ratio=1)
```

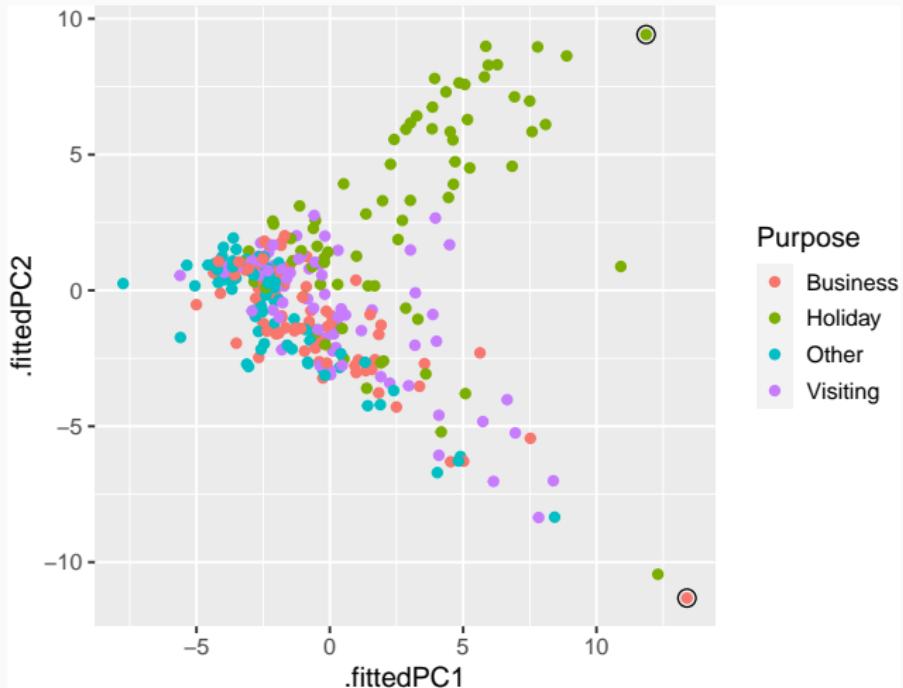
Principal components  
based on all features  
from the feasts  
package



# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +  
  geom_point() + theme(aspect.ratio=1)
```

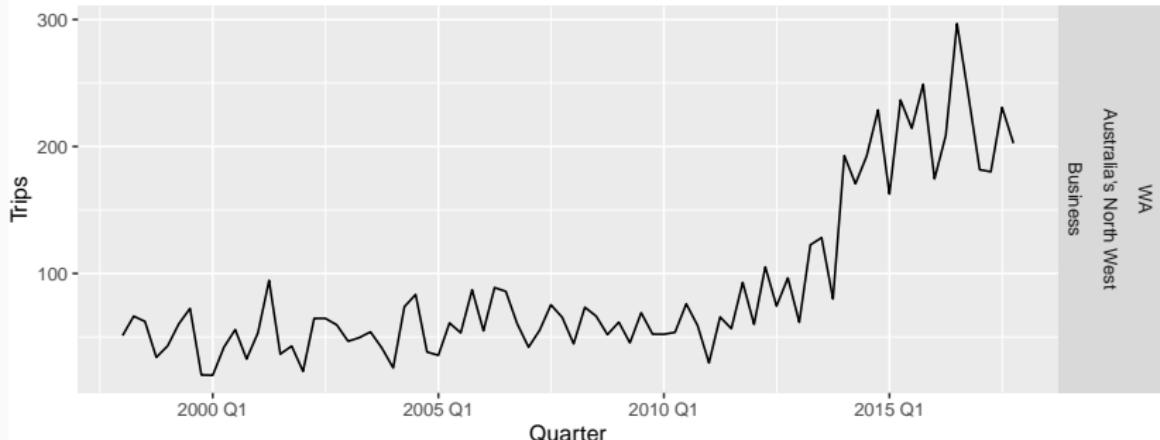
Principal components  
based on all features  
from the feasts  
package



# Feature extraction and statistics

```
pcs %>% filter(.fittedPC1 == max(.fittedPC1)) %>%
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose)) +
  ggtitle("Outlying time series in PC space") +
  theme(legend.position = "none")
```

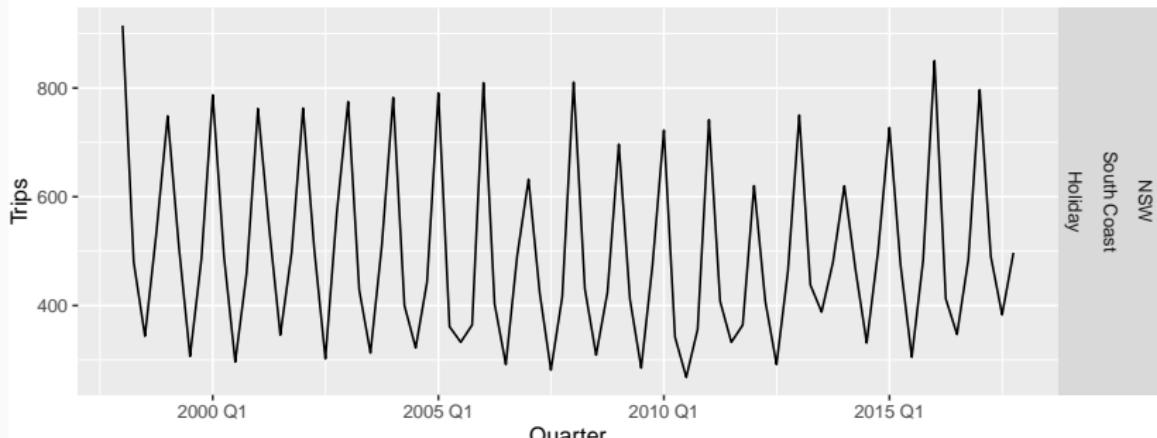
Outlying time series in PC space



# Feature extraction and statistics

```
pcs %>% filter(.fittedPC1 > 10 & .fittedPC2 > 2.5) %>%  
left_join(tourism, by = c("State", "Region", "Purpose")) %>%  
ggplot(aes(x = Quarter, y = Trips)) +  
  geom_line() +  
  facet_grid(vars(State, Region, Purpose)) +  
  ggtitle("Outlying time series in PC space") +  
  theme(legend.position = "none")
```

Outlying time series in PC space

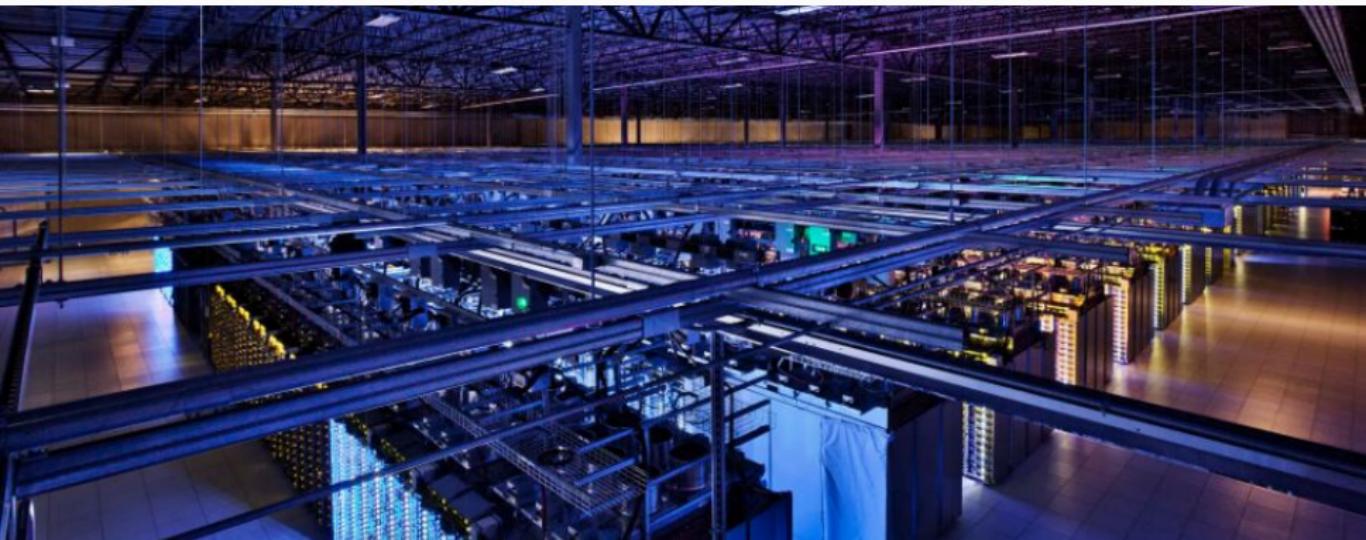


# Outline

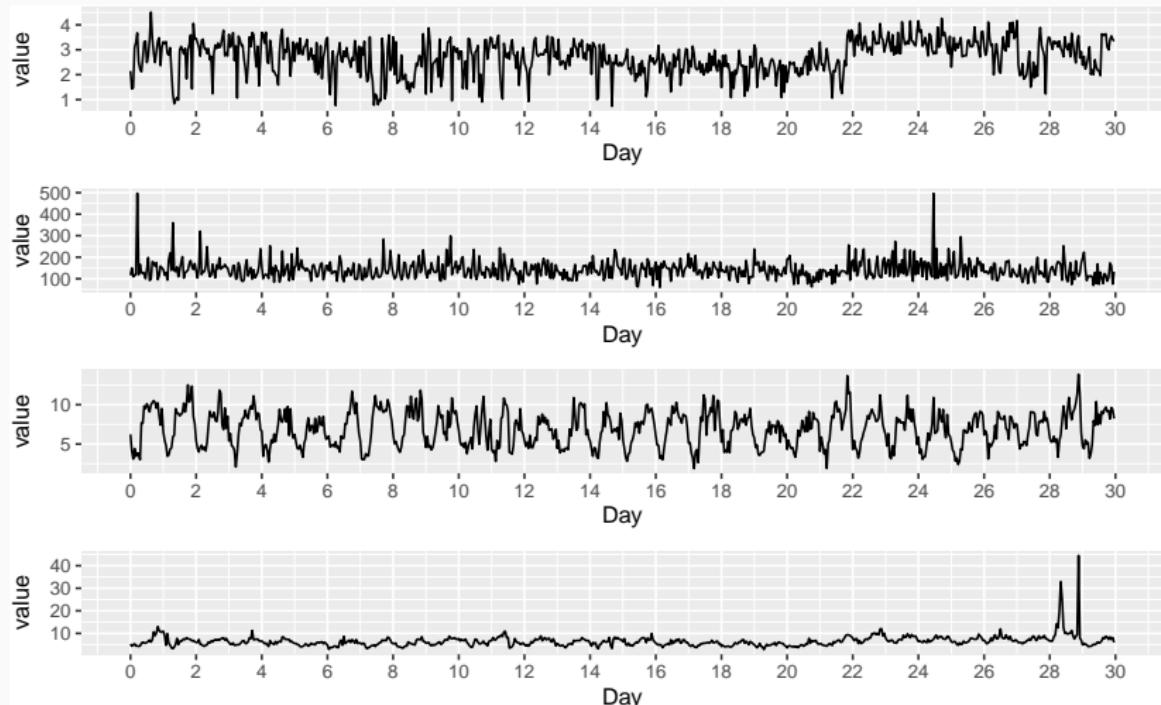
- 1 Feature-based visualization
- 2 R packages
- 3 Feature-based anomaly detection
- 4 Feature-based forecasting

# Yahoo server metrics

- Tens of thousands of time series collected at one-hour intervals over 1–2 months.
- Consisting of several server metrics (e.g. CPU usage and paging views) from many server farms globally.
- Aim: find unusual (anomalous) time series.



# Yahoo server metrics



# Yahoo server metrics

- **ACF1:** first order autocorrelation =  $\text{Corr}(Y_t, Y_{t-1})$
- Strength of **trend** and **seasonality** based on STL
- Size of seasonal **peak** and **trough**
- Spectral **entropy**
- **Lumpiness:** variance of block variances (block size 24).
- **Spikiness:** variances of leave-one-out variances of STL remainders.
- **Level shift:** Maximum difference in trimmed means of consecutive moving windows of size 24.
- **Variance change:** Max difference in variances of consecutive moving windows of size 24.
- **Flat spots:** Discretize sample space into 10 equal-sized intervals. Find max run length in any interval.
- Number of **crossing points** of mean line.
- **Kullback-Leibler score:** Maximum of  
$$D_{KL}(P\|Q) = \int P(x) \ln P(x)/Q(x)dx$$
 where  $P$  and  $Q$  are estimated by kernel density estimators applied to consecutive windows

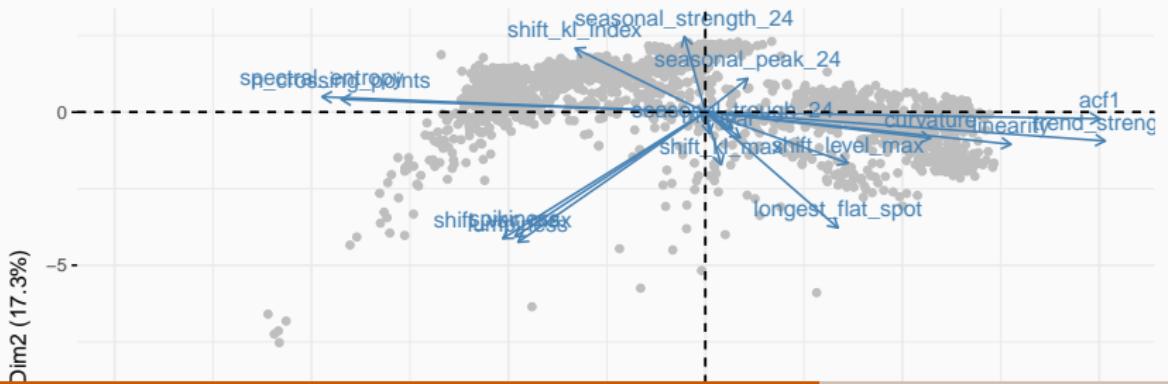
# Feature space

```
yahoo_features <- left_join(  
  yahoo %>%  
    features(value, features = list(  
      mean = ~ mean(., na.rm = TRUE),  
      var = ~ var(., na.rm = TRUE)  
    )),  
  yahoo %>%  
    features(scale(value), features = list(  
      ~ feat_acf(.),  
      ~ feat_spectral(.),  
      ~ n_flat_spots(.),  
      ~ n_crossing_points(.),  
      ~ var_tiled_var(., .period = 24, .size = 24),  
      ~ shift_level_max(., .period = 24, .size = 24),  
      ~ shift_var_max(., .period = 24, .size = 24),  
      ~ shift_kl_max(., .period = 24, .size = 48),  
      ~ feat_stl(., .period = 24, s.window = "periodic", 45b  
    ))
```

# Feature space

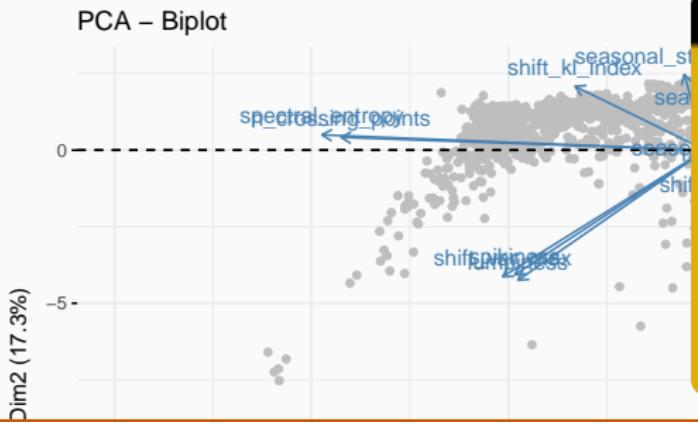
```
yahoo_features %>%  
  select(-key) %>%  
  na.omit() %>%  
  prcomp(scale = TRUE) %>%  
  factoextra::fviz_pca_biplot(geom = "point",
```

PCA – Biplot



# Feature space

```
yahoo_features %>%  
  select(-key) %>%  
  na.omit() %>%  
  prcomp(scale = TRUE) %>%  
  factoextra::fviz_pca_biplot(geom = "point",
```

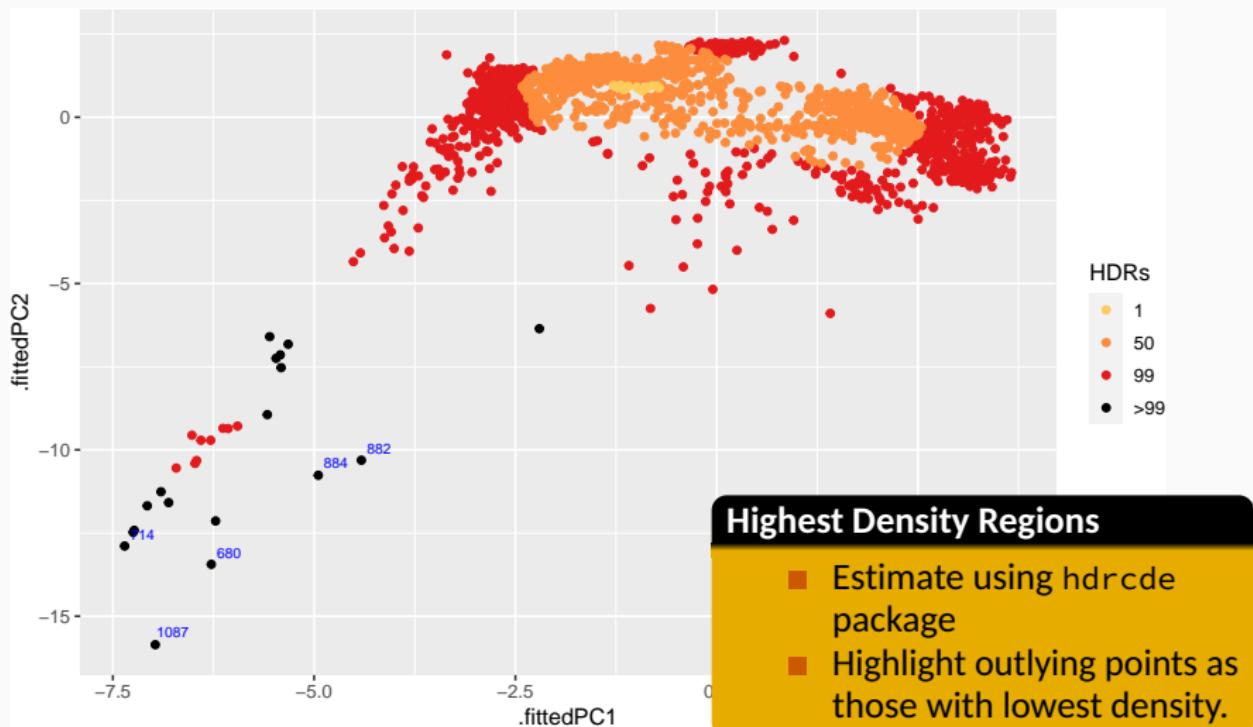


## What is “anomalous”?

- We need a measure of the “anomalousness” of a time series.
- Rank points based on their local density using a bivariate kernel density estimate.

# Finding weird time series

```
hdrcde:::hdrcscatterplot(hwl_pca$.fittedPC1, hwl_pca$.fittedPC2, nou
```



# Outline

- 1 Feature-based visualization
- 2 R packages
- 3 Feature-based anomaly detection
- 4 Feature-based forecasting

# M competition: 1982

*Journal of Forecasting, Vol. 1, 111–153 (1982)*

## The Accuracy of Extrapolation (Time Series) Methods: Results of a Forecasting Competition

S. MAKRIDAKIS  
INSEAD, Fontainebleau, France

A. ANDERSEN  
University of Sydney, Australia

R. CARBONE  
Université Laval, Quebec, Canada

R. FILDES  
Manchester Business School, Manchester, England

M. HIBON  
INSEAD, Fontainebleau, France

R. LEWANDOWSKI  
Marketing Systems, Essen, Germany

J. NEWTON  
E. PARZEN  
Texas A & M University, Texas, U.S.A.

R. WINKLER  
Indiana University, Bloomington, U.S.A.

### ABSTRACT

In the last few decades many methods have become available for forecasting. As always, when alternatives exist, choices need to be made so that an appropriate forecasting method can be selected and used for the specific situation being considered. This paper reports the results of a forecasting competition that provides information to facilitate such choice. Seven experts in each of the 24 methods forecasted up to 1001 series for six up to eighteen time horizons. The results of the competition are presented in this paper whose purpose is to provide empirical evidence about differences found to exist among the various extrapolative (time series) methods used in the competition.

## M-competition

- 1001 series from demography, industry, economics.
- Annual, quarterly, monthly data.
- Anyone could submit forecasts.
- Multiple forecast measures used.

# M3 competition: 2000



ELSEVIER

International Journal of Forecasting 16 (2000) 451–476

*international journal  
of forecasting*

[www.elsevier.com/locate/ijforecast](http://www.elsevier.com/locate/ijforecast)

## The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon\*

*INSEAD, Boulevard de Constance, 77305 Fontainebleau, France*

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### Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy 50

# M4 competition: 2018

International Journal of Forecasting 34 (2018) 802–808



Contents lists available at [ScienceDirect](#)

## International Journal of Forecasting

journal homepage: [www.elsevier.com/locate/ijforecast](http://www.elsevier.com/locate/ijforecast)



## The M4 Competition: Results, findings, conclusion and way forward



Spyros Makridakis <sup>a,b,\*</sup>, Evangelos Spiliotis <sup>c</sup>, Vassilios Assimakopoulos <sup>c</sup>

<sup>a</sup> University of Nicosia, Nicosia, Cyprus

<sup>b</sup> Institute For the Future (IFF), Nicosia, Cyprus

<sup>c</sup> Forecasting and Strategy Unit, School of Electrical and Computer Engineering, National Technical University of Athens, Zografou, Greece

### ARTICLE INFO

#### Keywords:

Forecasting competitions

M Competitions

### ABSTRACT

The M4 competition is the continuation of three previous competitions started more than 45 years ago whose purpose was to learn how to improve forecasting accuracy, and

## M4 competition: 2018

- January – May 2018
- 100,000 time series: yearly, quarterly, monthly, weekly, daily, hourly.
- Point forecast and prediction intervals assessed.
- Code must be public
- 248 registrations, 50 submissions.

# M4 competition: 2018

- January – May 2018
- 100,000 time series: yearly, quarterly, monthly, weekly, daily, hourly.
- Point forecast and prediction intervals assessed.
- Code must be public
- 248 registrations, 50 submissions.

## Winning methods

- 1 Hybrid of Recurrent Neural Network and Exponential Smoothing models
- 2 FFORMA: Feature-based forecast combinations using xgboost to find weights

# Features used to select a forecasting model

- length
- strength of seasonality
- strength of trend
- linearity
- curvature
- spikiness
- stability
- lumpiness
- parameter estimates of Holt's linear trend method
- spectral entropy
- Hurst exponent
- nonlinearity
- parameter estimates of Holt-Winters' additive method
- unit root test statistics
- crossing points, flat spots
- peaks, troughs
- ACF and PACF based features - calculated on raw, differenced, and remainder series.
- ARCH/GARCH statistics and ACF of squared series and residuals.

# Features used to select a forecasting model

## Why these features?

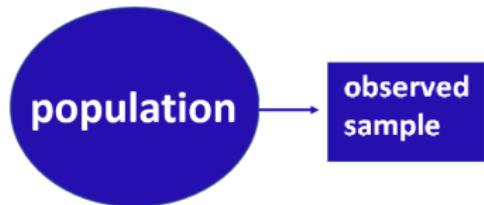
- Hyndman, Wang and Laptev. “Large scale unusual time series detection” (ICDM 2015).
- Kang, Hyndman & Smith-Miles. “Visualising forecasting algorithm performance using time series instance spaces” (IJF 2017).
- Talagala, Hyndman and Athanasopoulos. “Meta-learning how to forecast time series” (2018).
- Implemented in the **feasts** R package

# FFORMS: Feature-based FORcast Model Selection

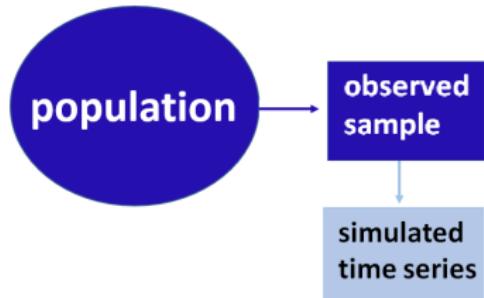


population

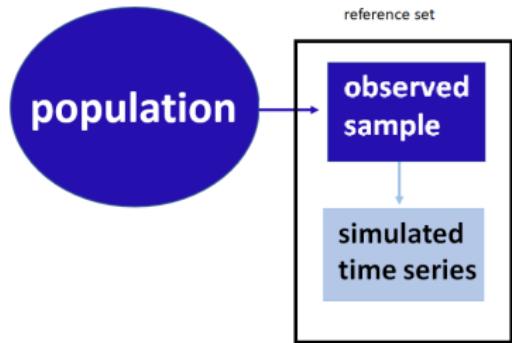
# FFORMS: Feature-based FORecast Model Selection



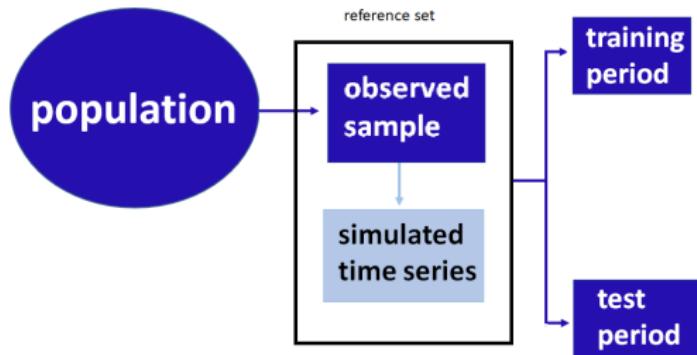
# FFORMS: Feature-based FORecast Model Selection



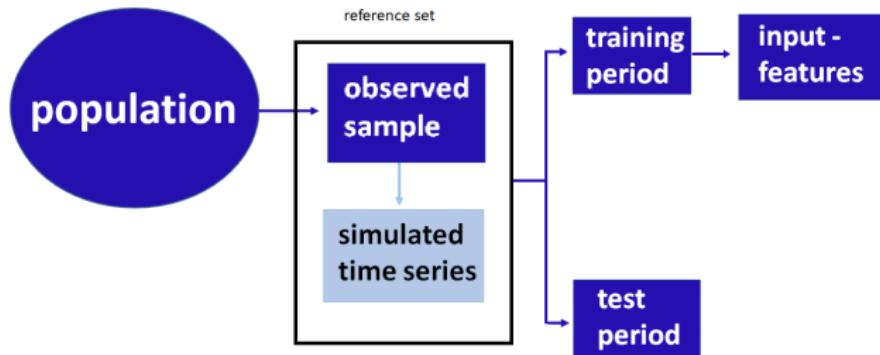
# FFORMS: Feature-based FORcast Model Selection



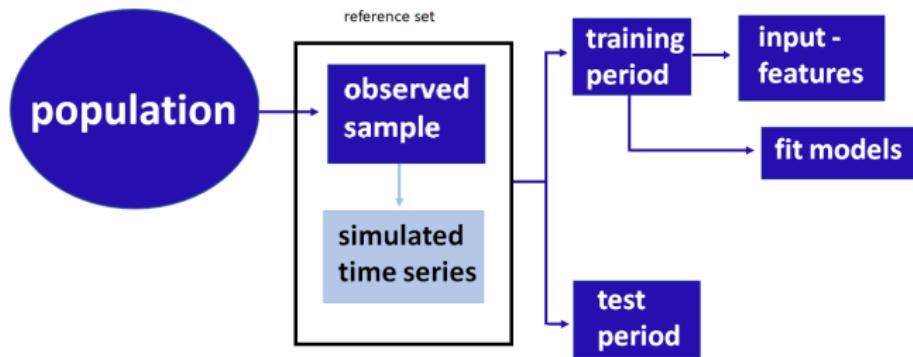
# FFORMS: Feature-based FORcast Model Selection



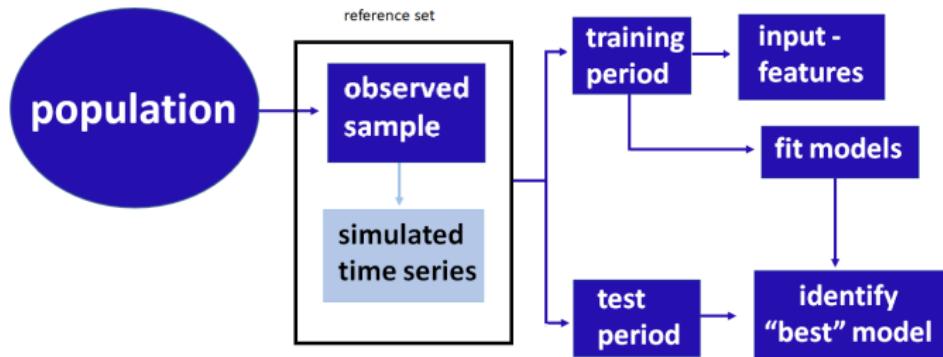
# FFORMS: Feature-based FORecast Model Selection



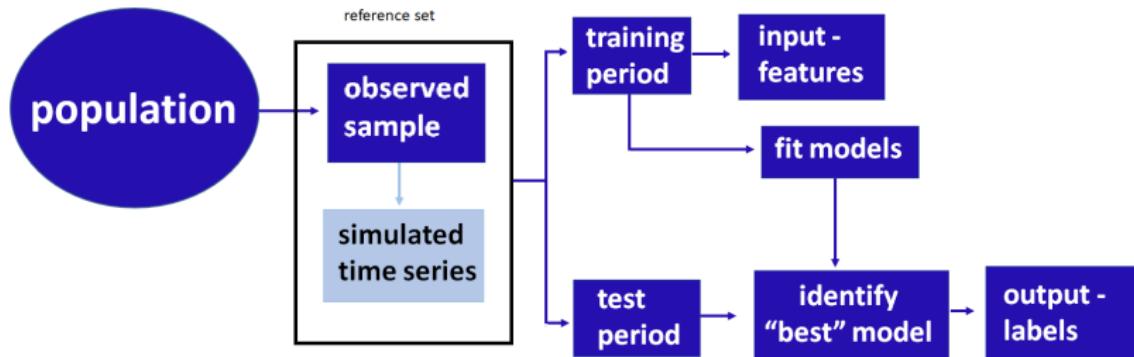
# FFORMS: Feature-based FORecast Model Selection



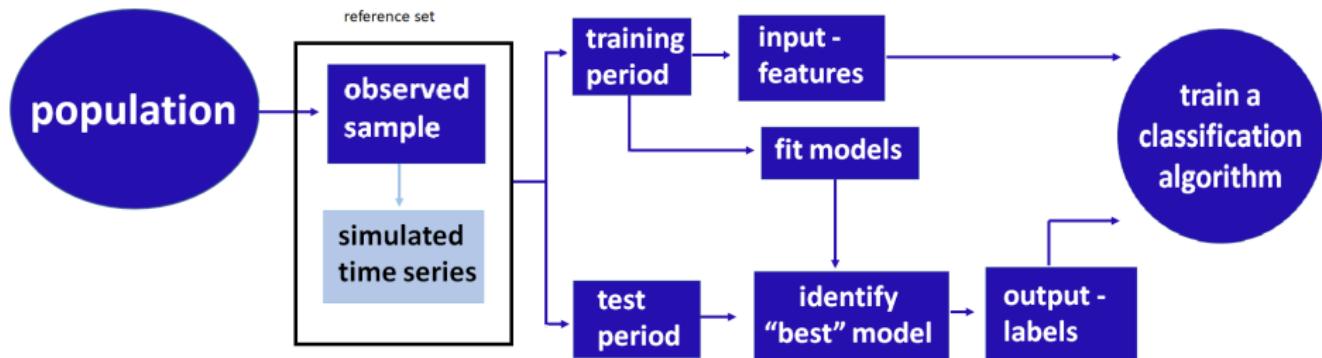
# FFORMS: Feature-based FORcast Model Selection



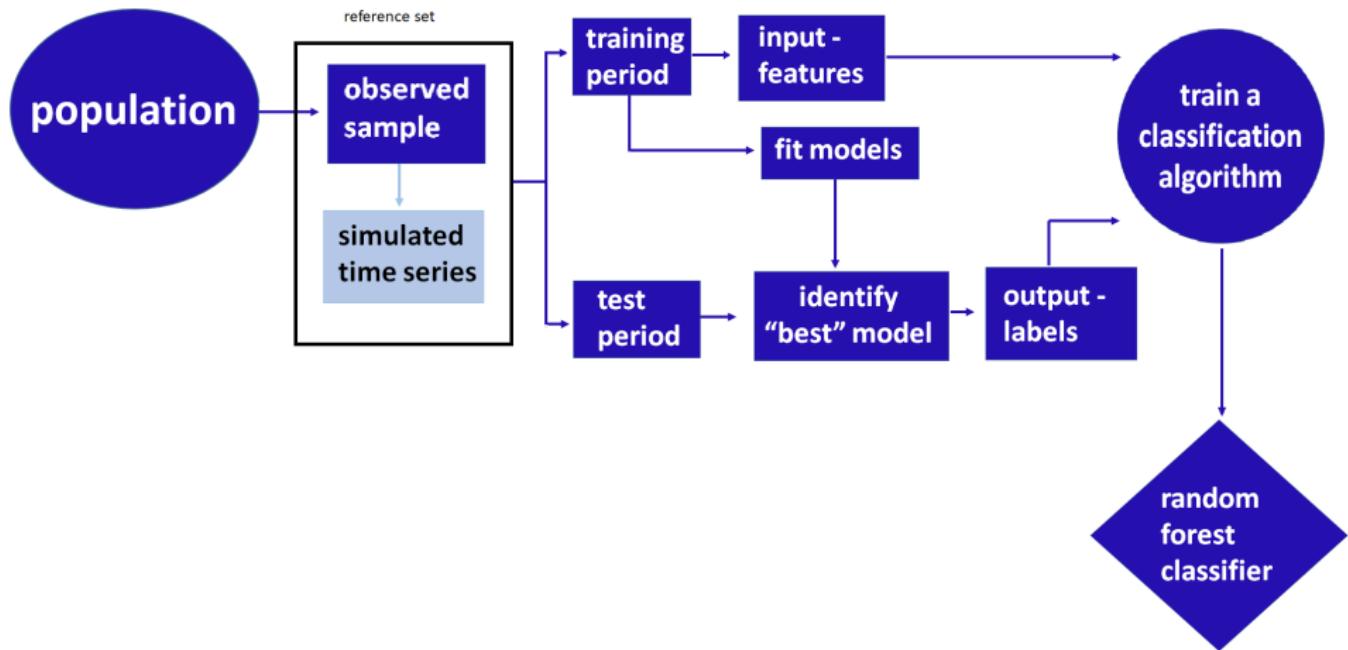
# FFORMS: Feature-based FORcast Model Selection



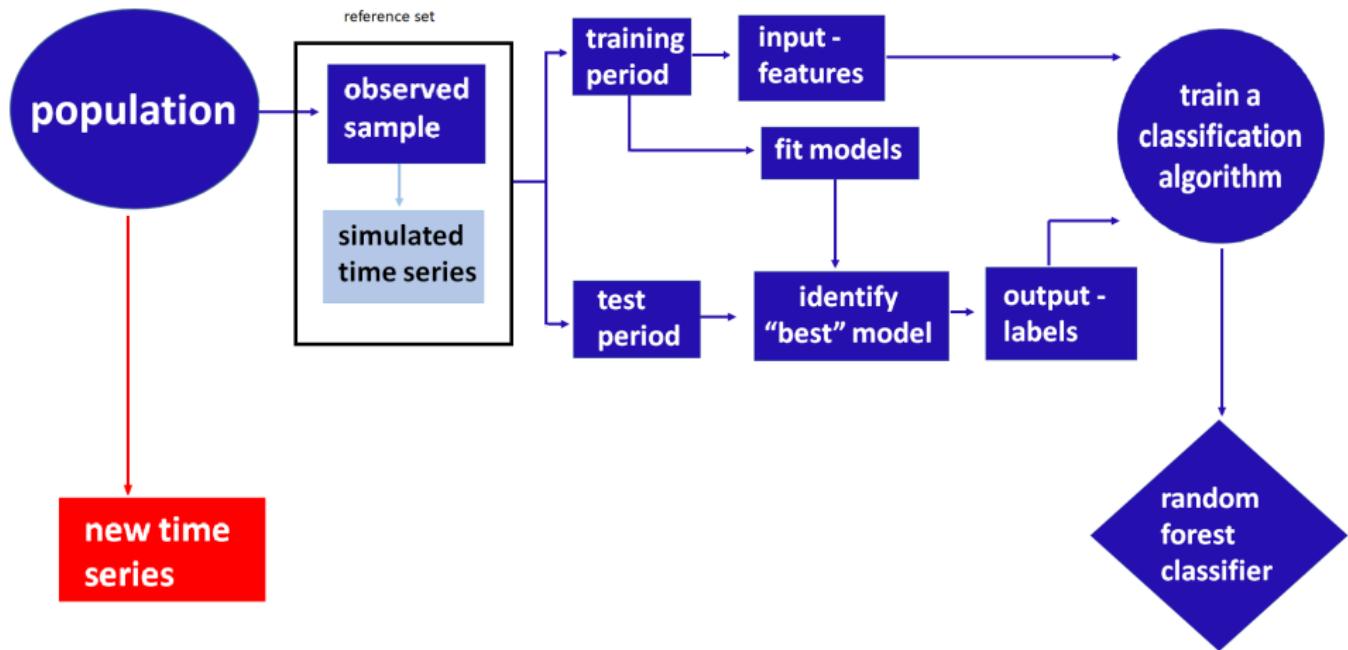
# FFORMS: Feature-based FORecast Model Selection



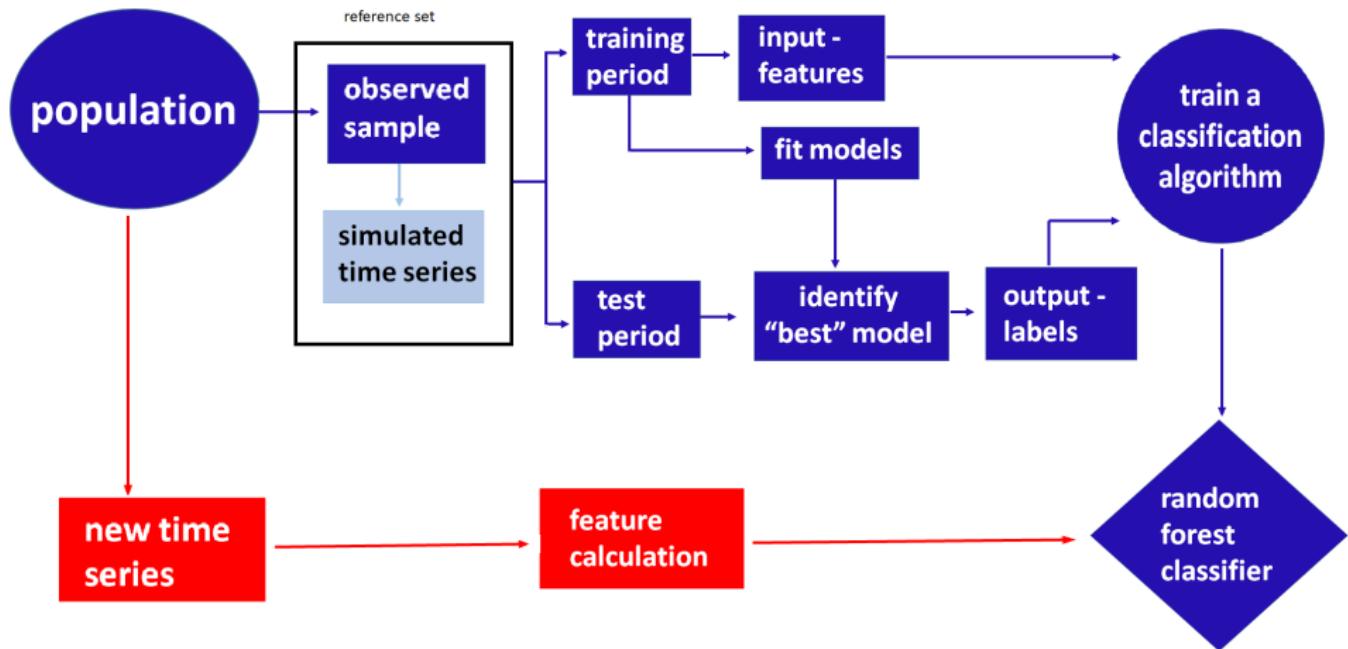
# FFORMS: Feature-based FORecast Model Selection



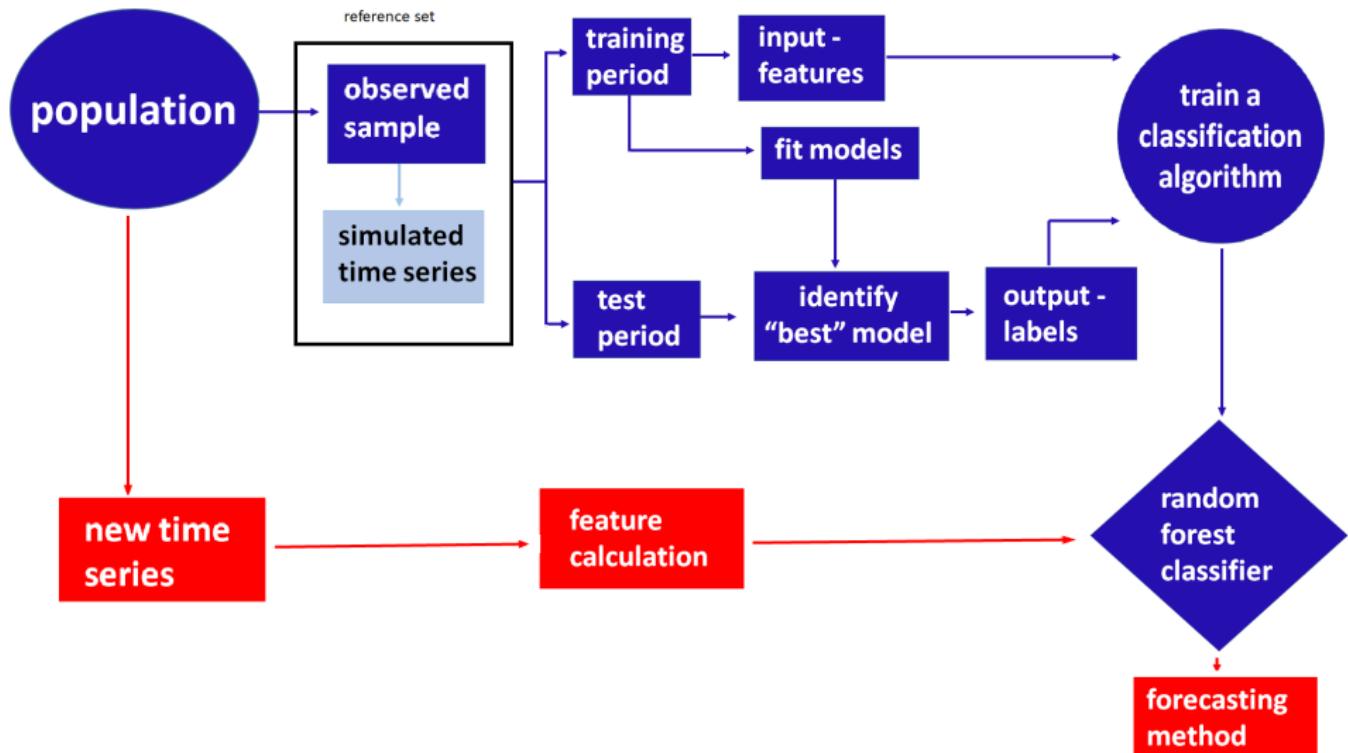
# FFORMS: Feature-based FORecast Model Selection



# FFORMS: Feature-based FORcast Model Selection



# FFORMS: Feature-based FORecast Model Selection



# Application to M competition data

## Experiment 1

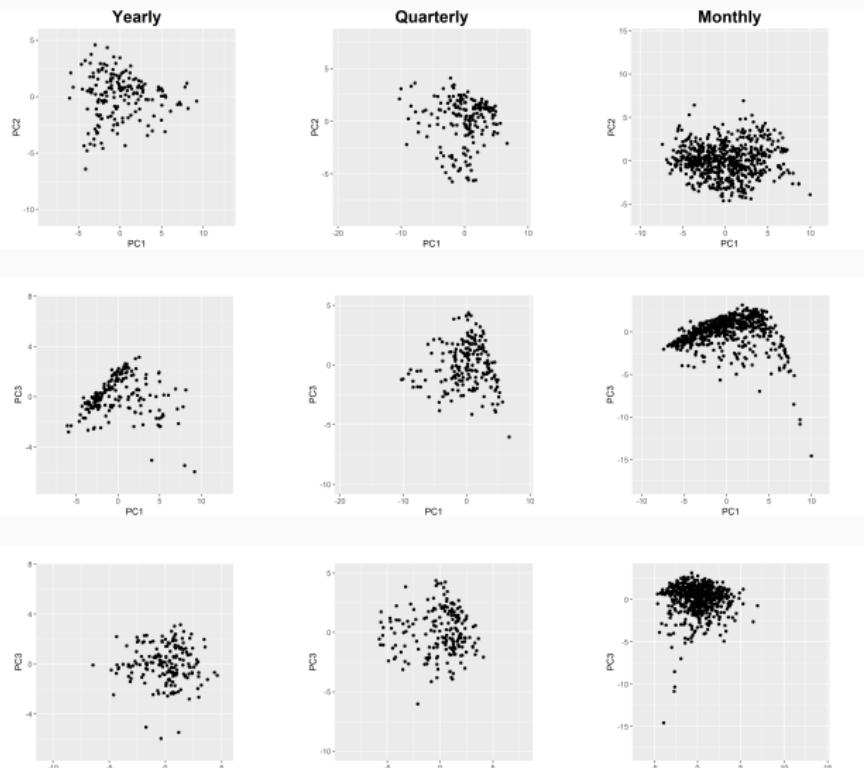
	Source	Y	Q	M
Observed series	M1	181	203	617
Simulated series		362000	406000	123400
New series	M3	645	756	1428

## Experiment 2

	Source	Y	Q	M
Observed series	M3	645	756	1428
Simulated series		1290000	1512000	285600
New series	M1	181	203	617

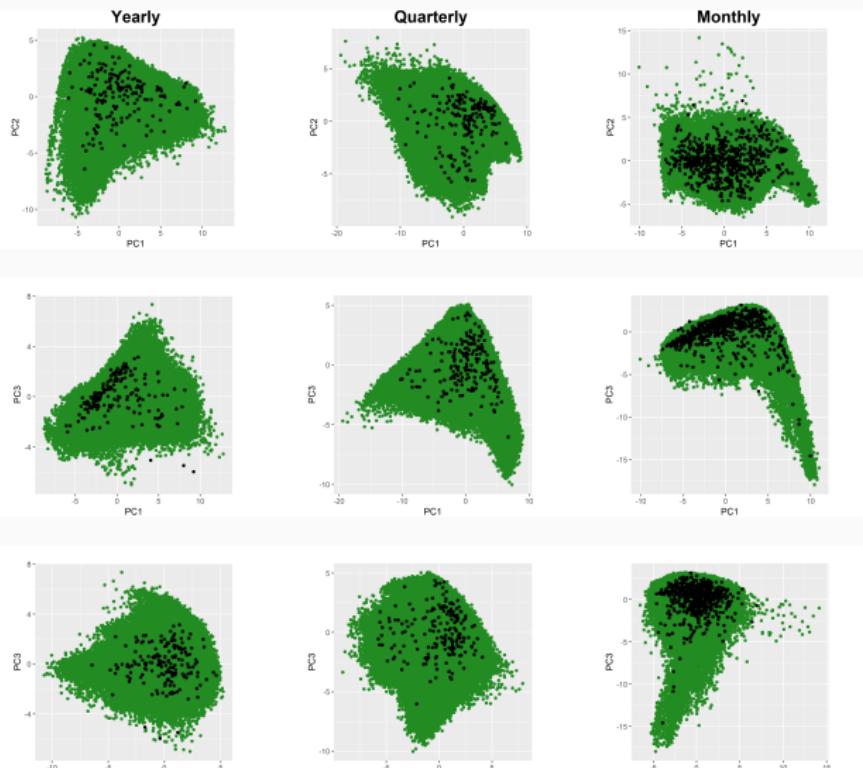
# Experiment 1: Distribution of time series in PCA space

observed - M1



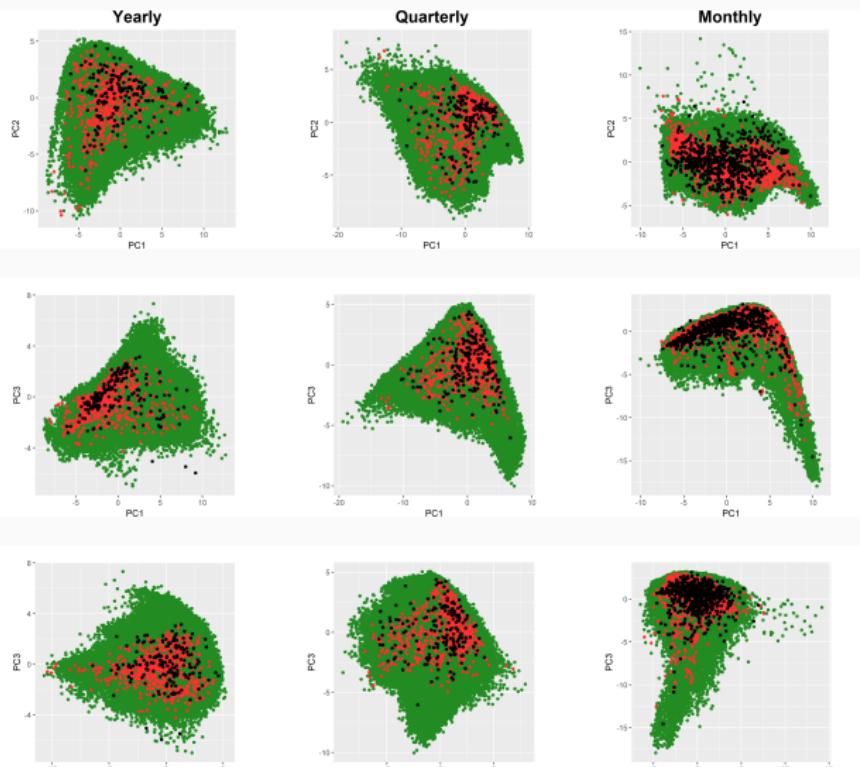
# Experiment 1: Distribution of time series in PCA space

observed - M1 | simulated



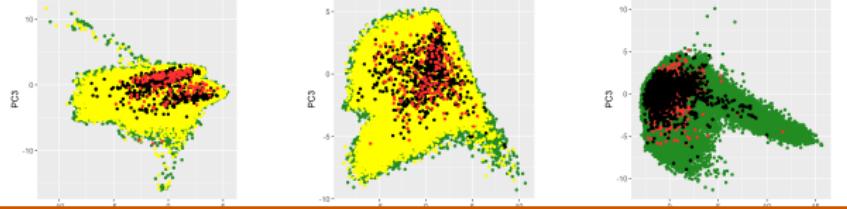
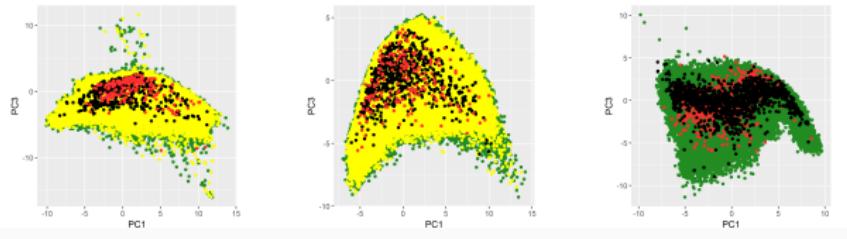
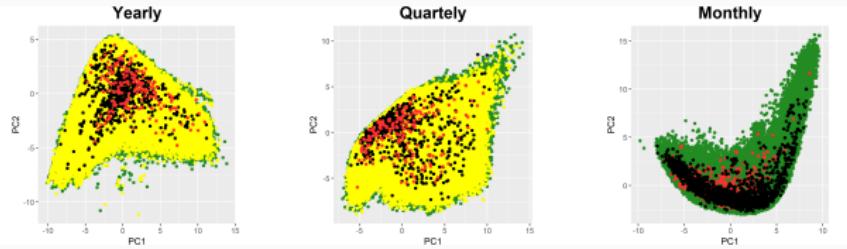
# Experiment 1: Distribution of time series in PCA space

observed - M1 | simulated | new - M3

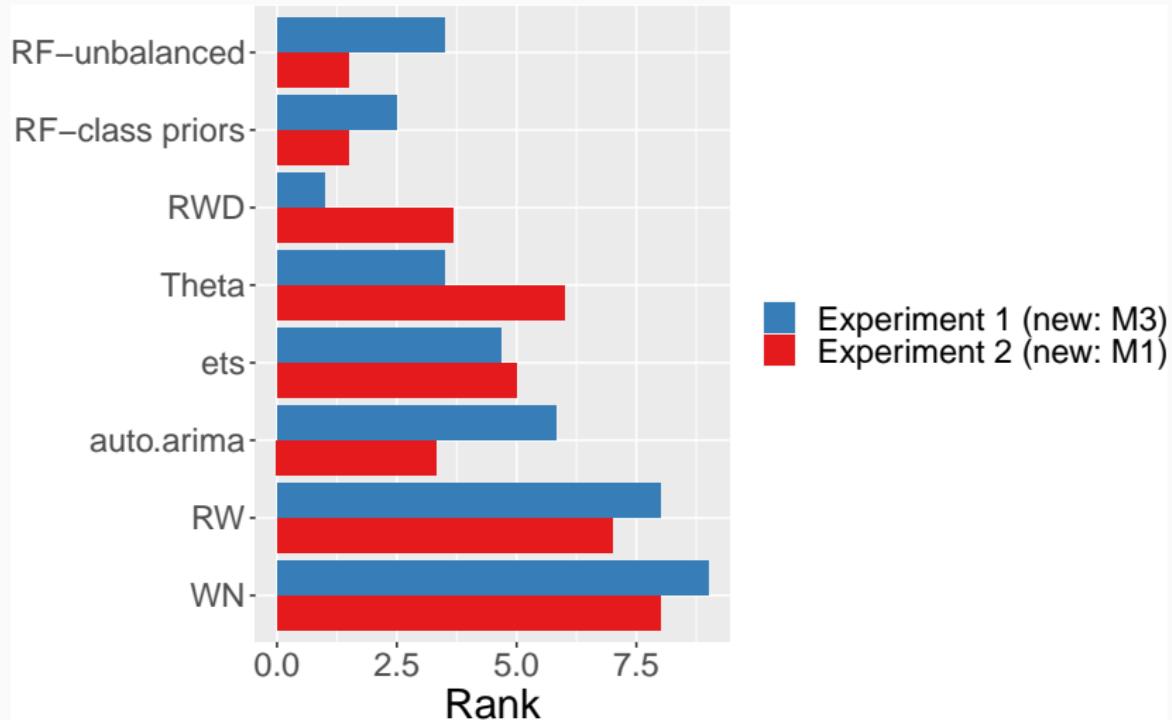


## Experiment 2: Distribution of time series in PCA space

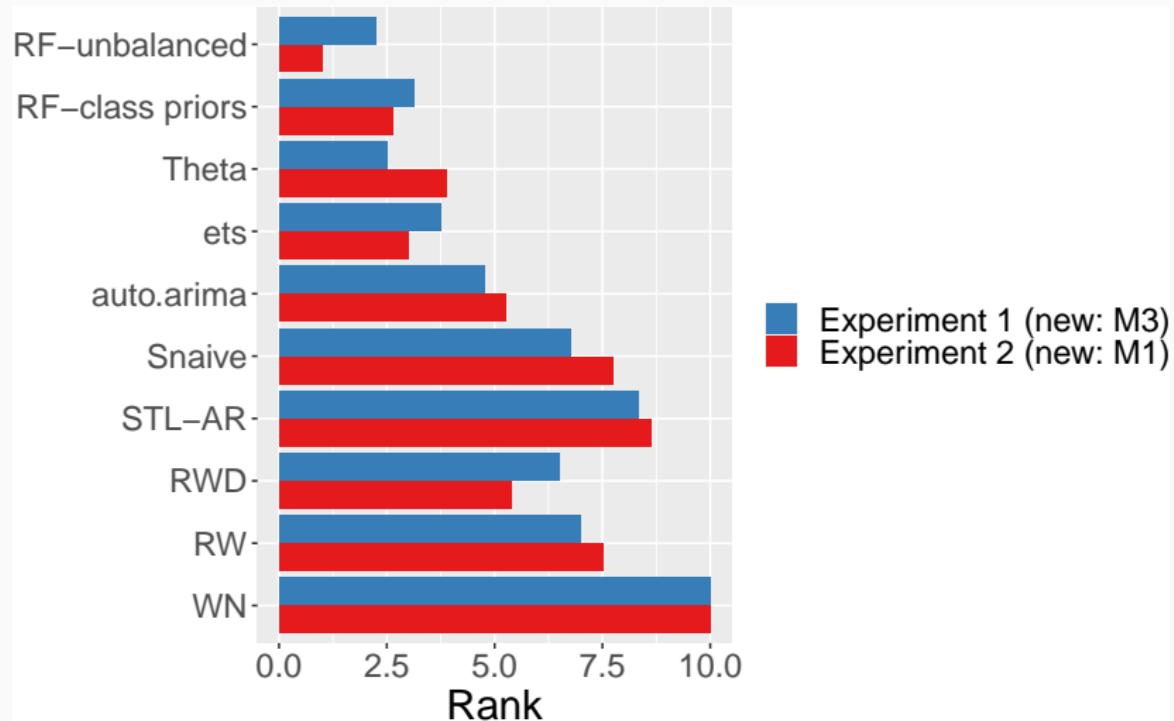
observed - M3   simulated   subset   new - M1



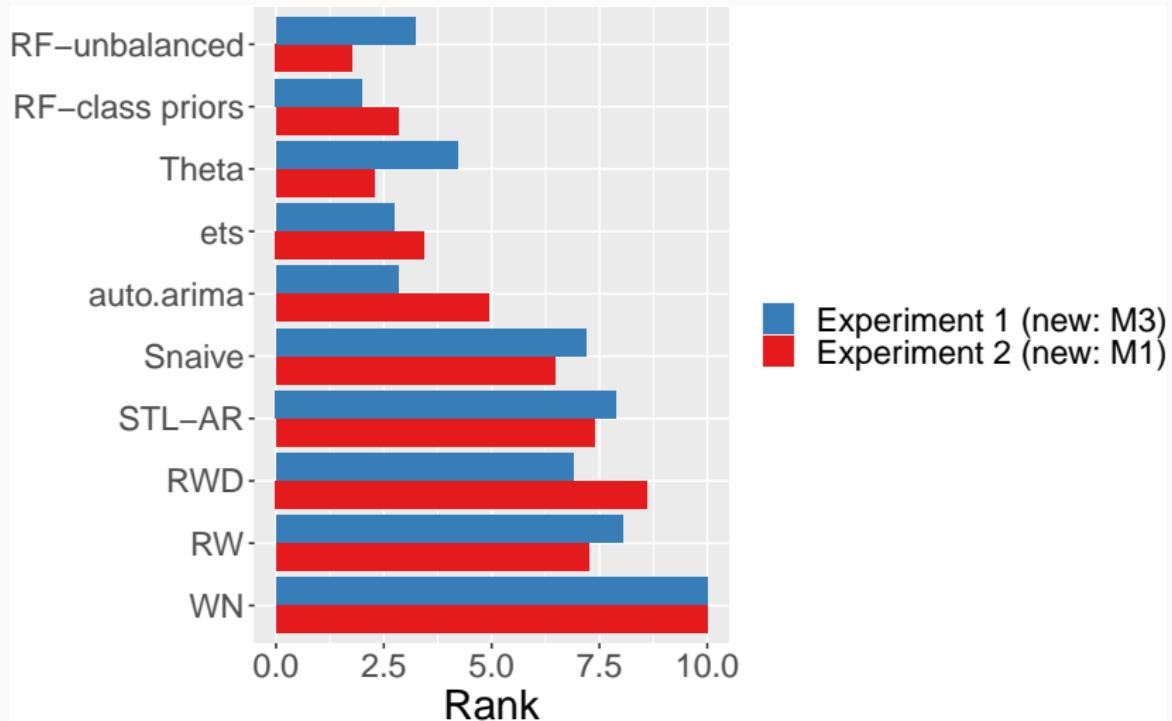
# Results: Yearly



# Results: Quarterly



# Results: Monthly



## FFORMA: Feature-based FOrecast Model Averaging

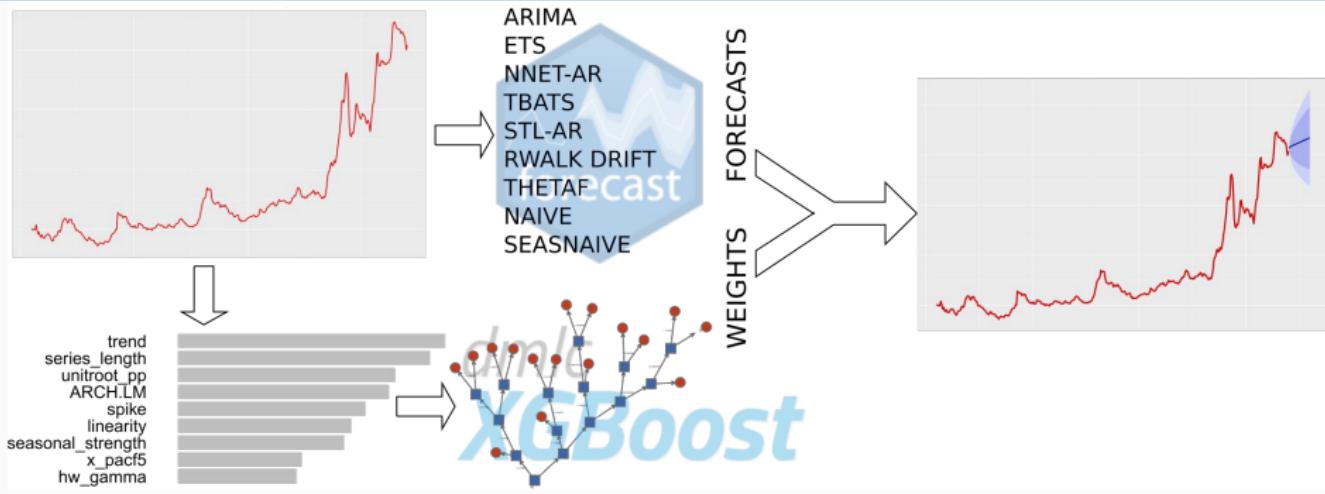
- Like FFORMS but using gradient boosted trees (xgboost) rather than random forest.
- Trained on temporal holdout version of M4 dataset, where size of test sets equal to required forecast horizons
- Optimization criterion: forecast accuracy not classification accuracy.
- Probability of each model being best is used to construct model weights for combination forecast.
- 5 days computing time.

# FFORMA: Feature-based FOrecast Model Averaging

## Models included

- 1 Naive
- 2 Seasonal naive
- 3 Random walk with drift
- 4 Theta method
- 5 ARIMA
- 6 ETS
- 7 TBATS
- 8 STL decomposition with AR for seasonally  
adjusted series
- 9 Neural network autoregression

# FFORMA: Feature-based FORcast Model Averaging

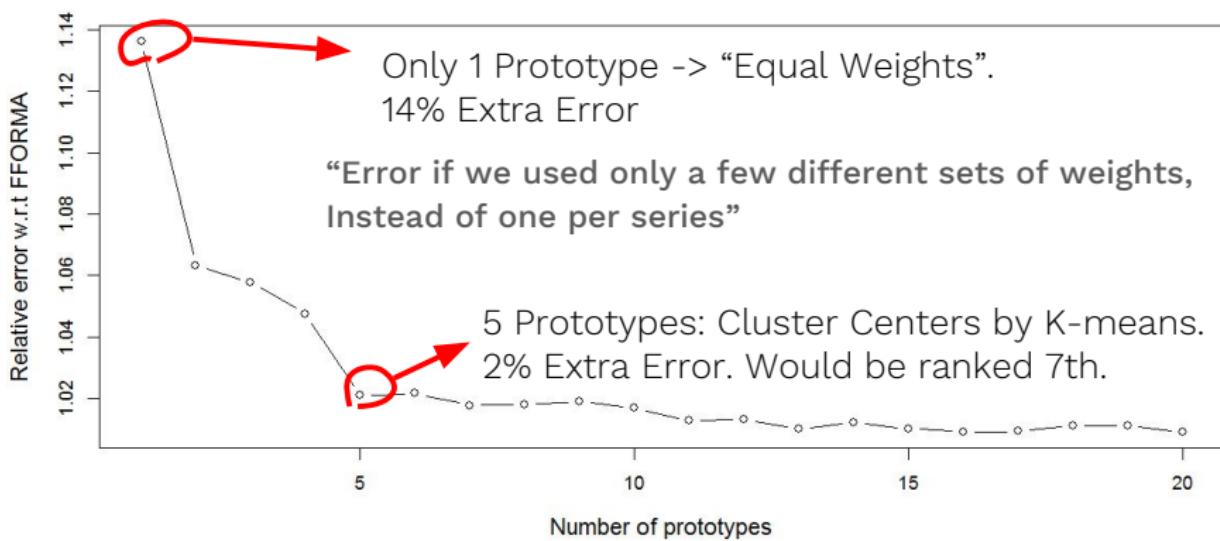


## M4 competition results (based on average OWA)

```
tribble(  
  ~Place, ~OWA, ~Method,  
  "1st", 0.821, NA,
```

# FFORMA: Feature-based FOrecast Model Averaging

## Looking for Prototypes in the weights



# FFORMA: Feature-based FOrecast Model Averaging

**“Roughly Equal Weights”. 40000 Series in M4**

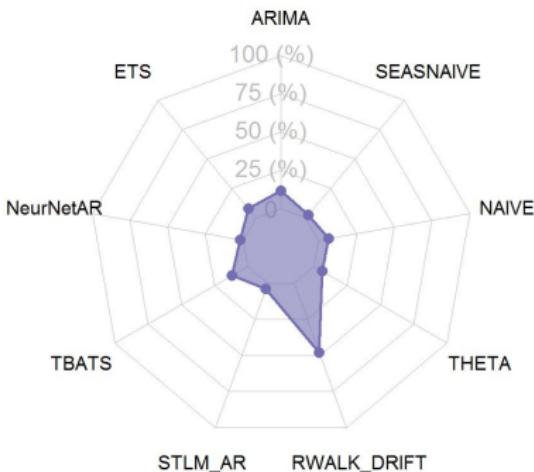
Weights of Prototype I



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly RandomWalk Drift”. 20000 Series in M4**

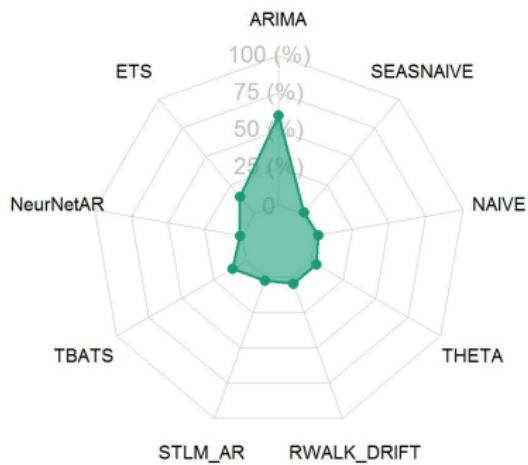
Weights of Prototype II



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly ARIMA”. 16000 Series in M4**

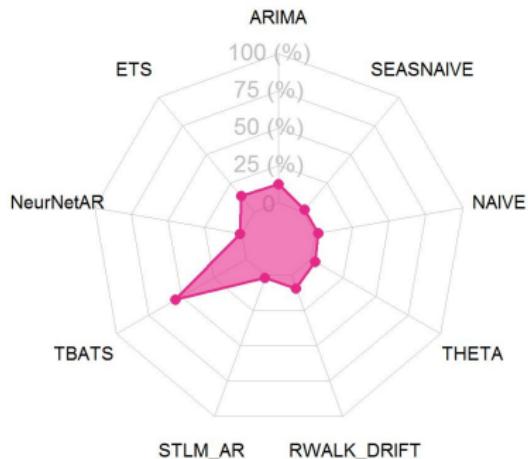
Weights of Prototype III



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly TBATS”. 13000 Series in M4**

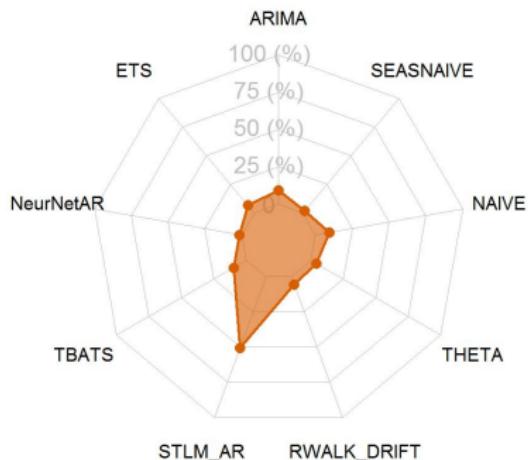
Weights of Prototype IV



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly STLM-AR”. 8000 Series in M4**

Weights of Prototype V



# Acknowledgments



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Yanfei Kang



Mitchell O'Hara-Wild



Pablo Montero-Manso