

# Feature-based time series analysis

Rob J Hyndman

21 June 2018

# Outline

- 1 Time series feature spaces
- 2 Irish smart metre data
- 3 Quantiles conditional on time of week
- 4 Finding typical and unusual households
- 5 Visualization via embedding
- 6 Features and limitations

# M3 competition



ELSEVIER

International Journal of Forecasting 16 (2000) 451–476

international journal  
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## The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon\*

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### Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy

# M3 competition



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## etition: results, conclusions

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Abstr

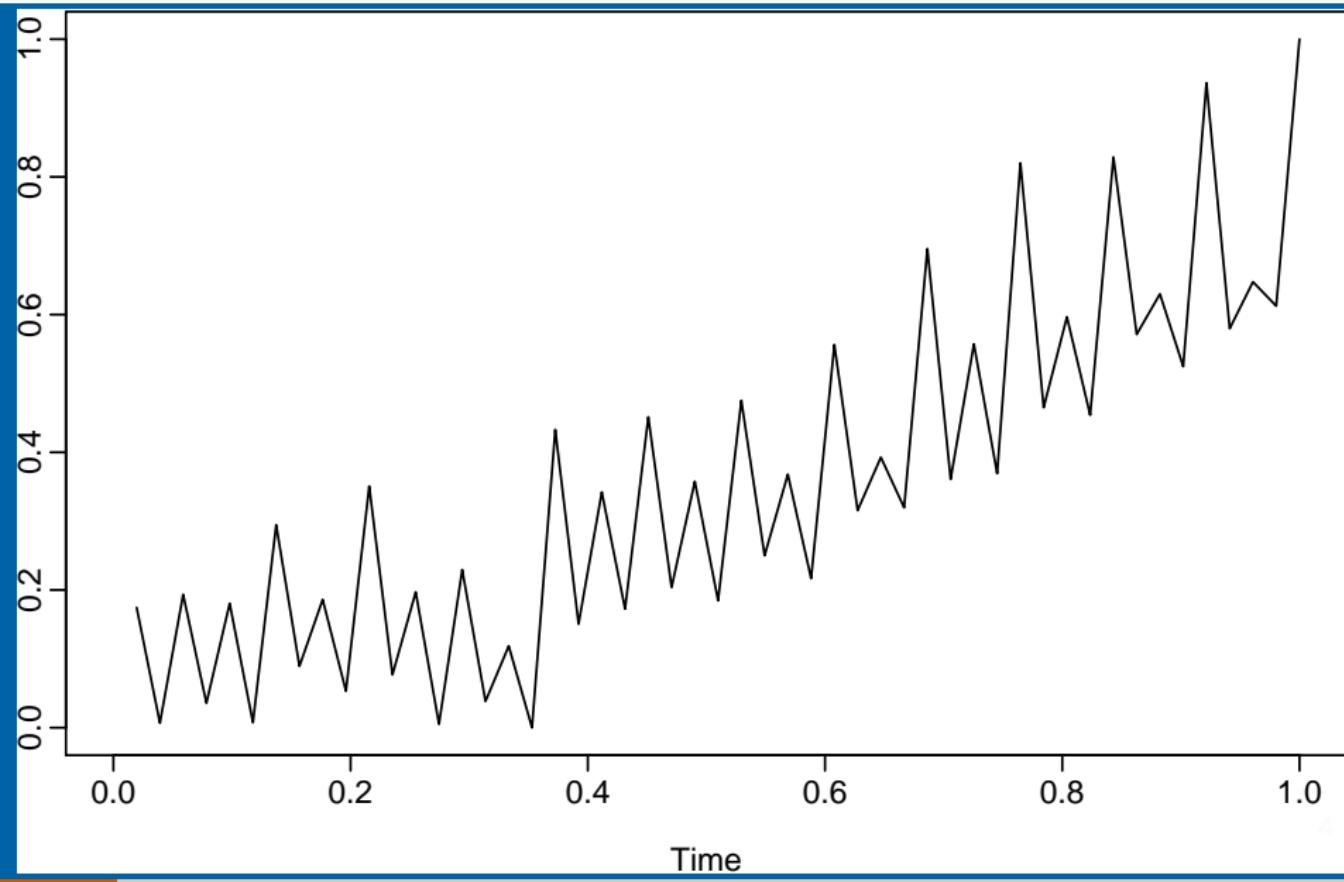


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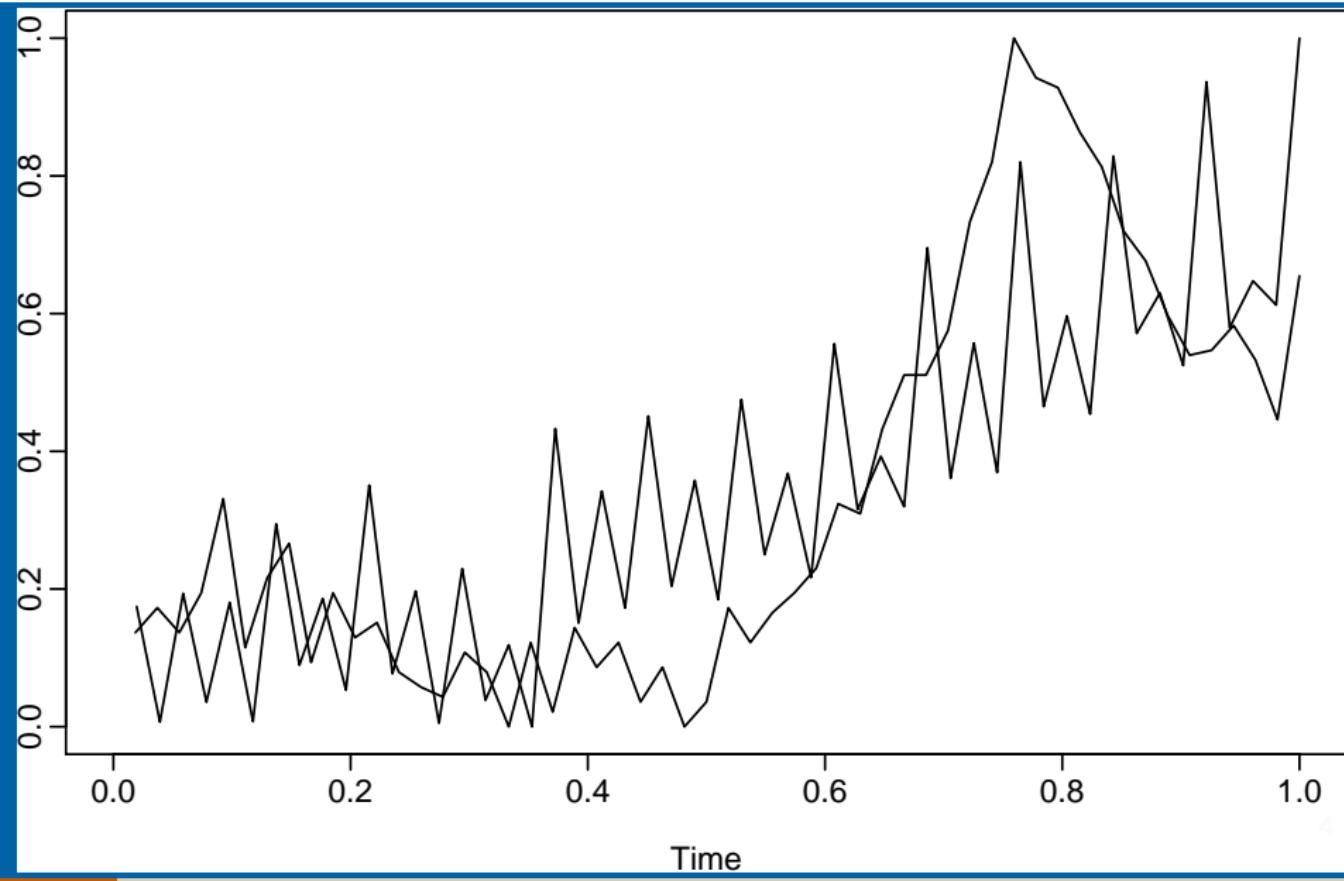
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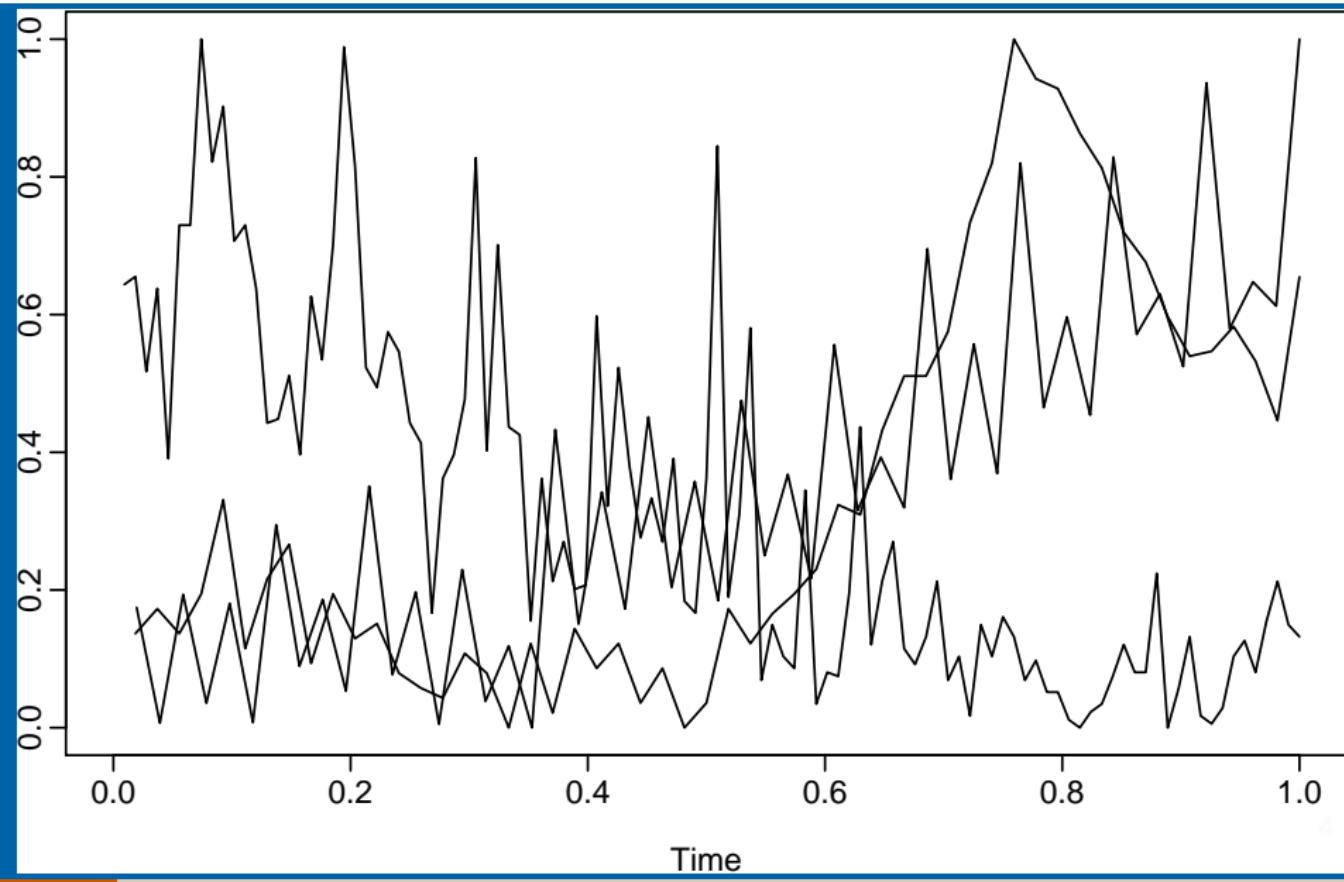
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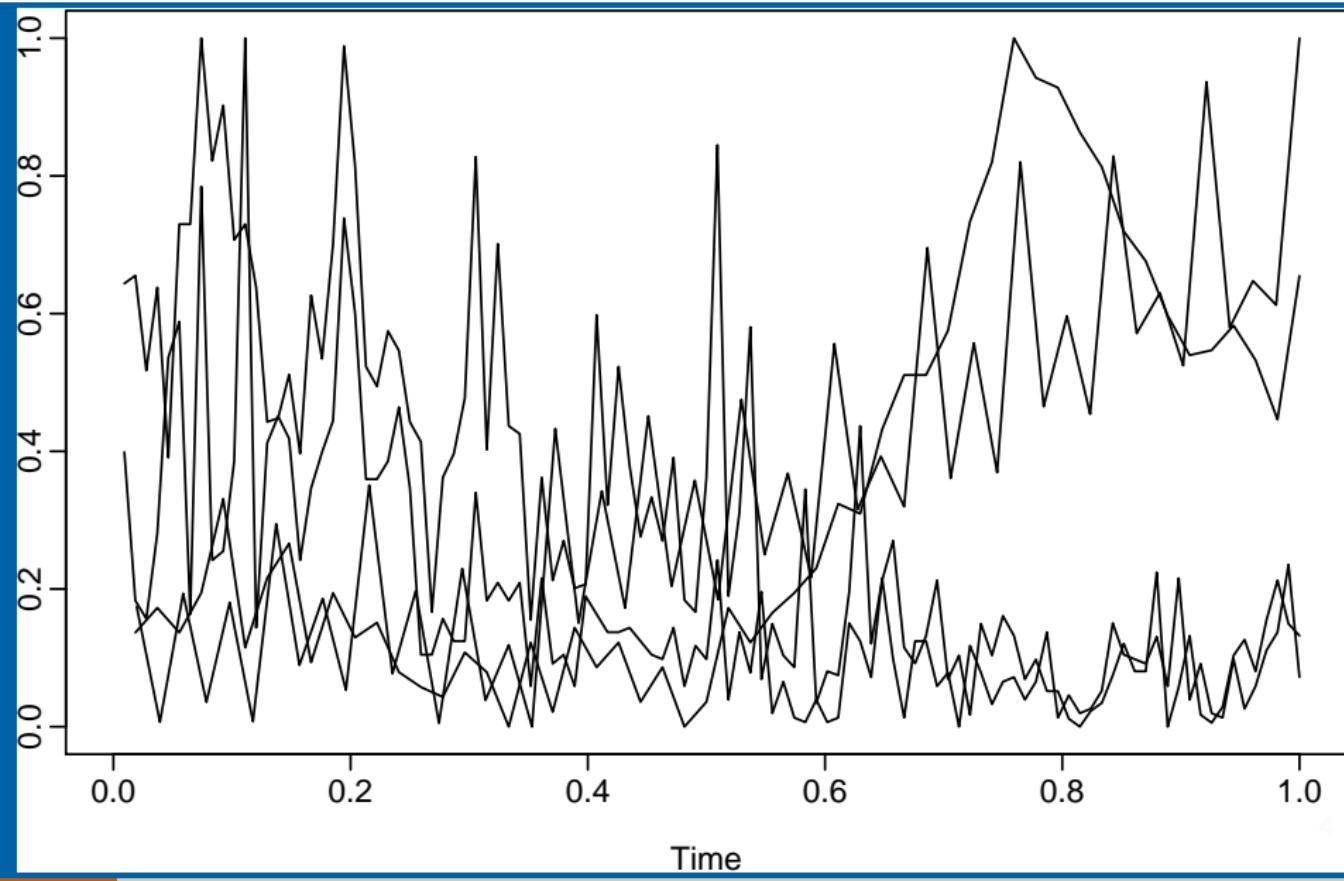
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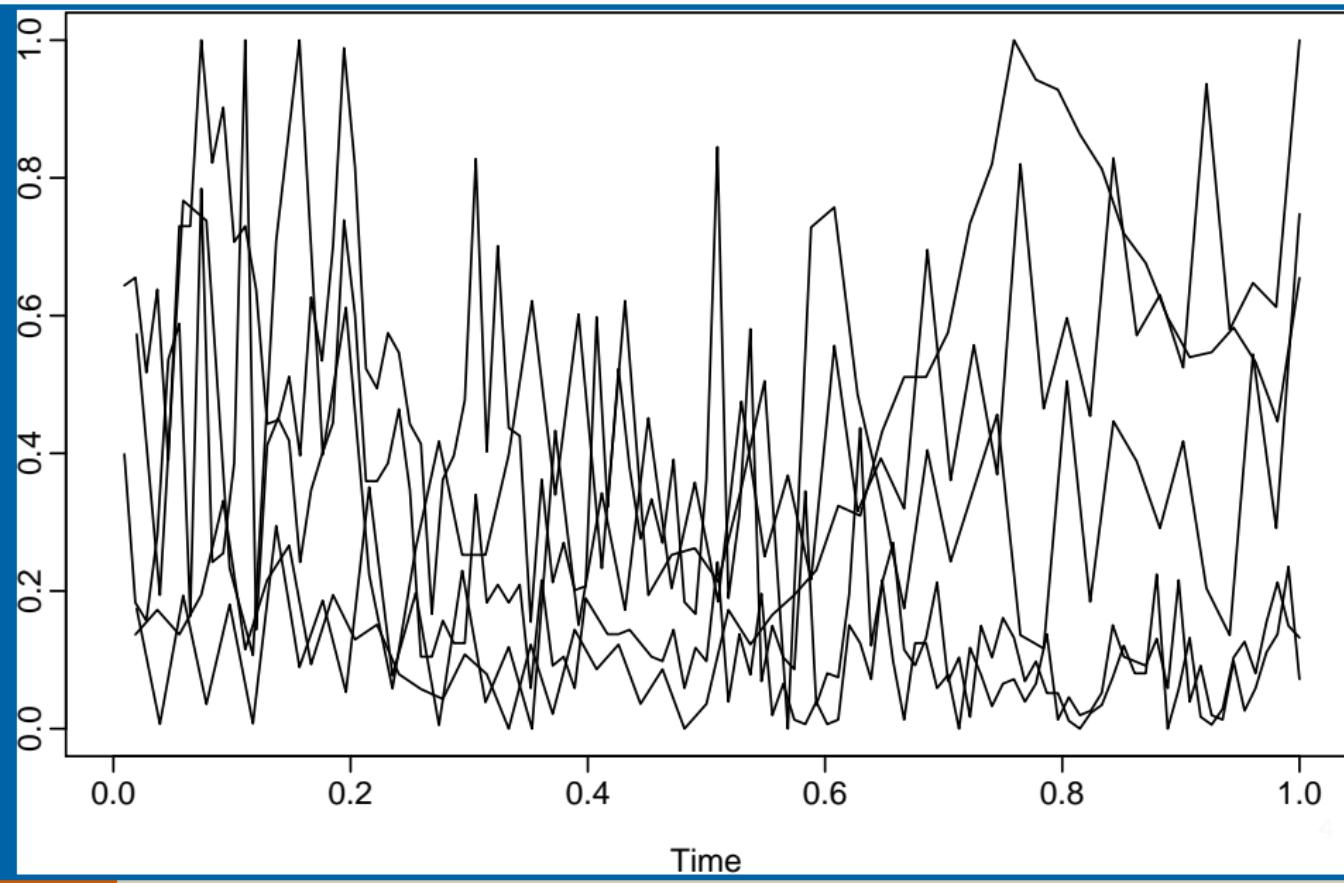
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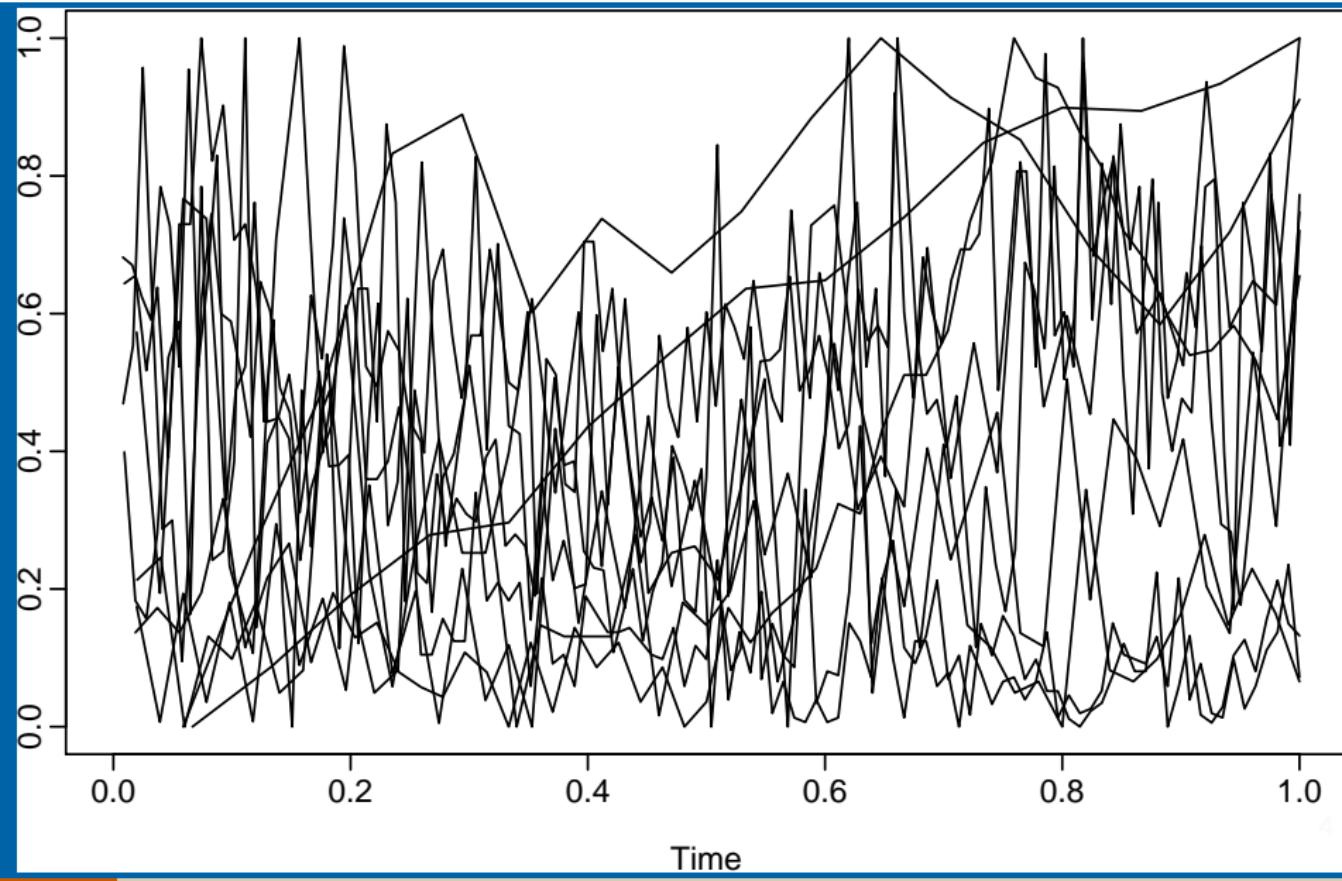
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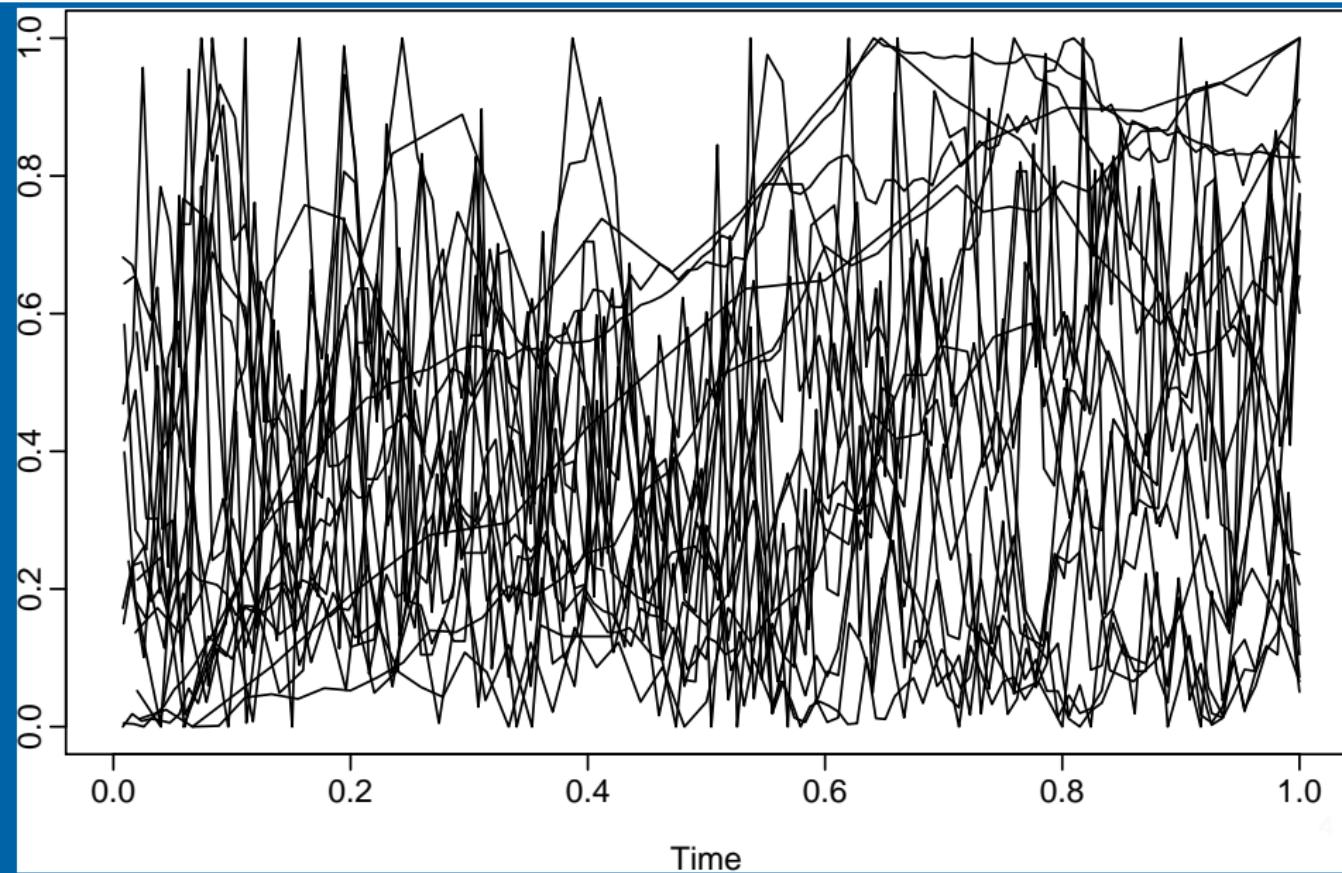
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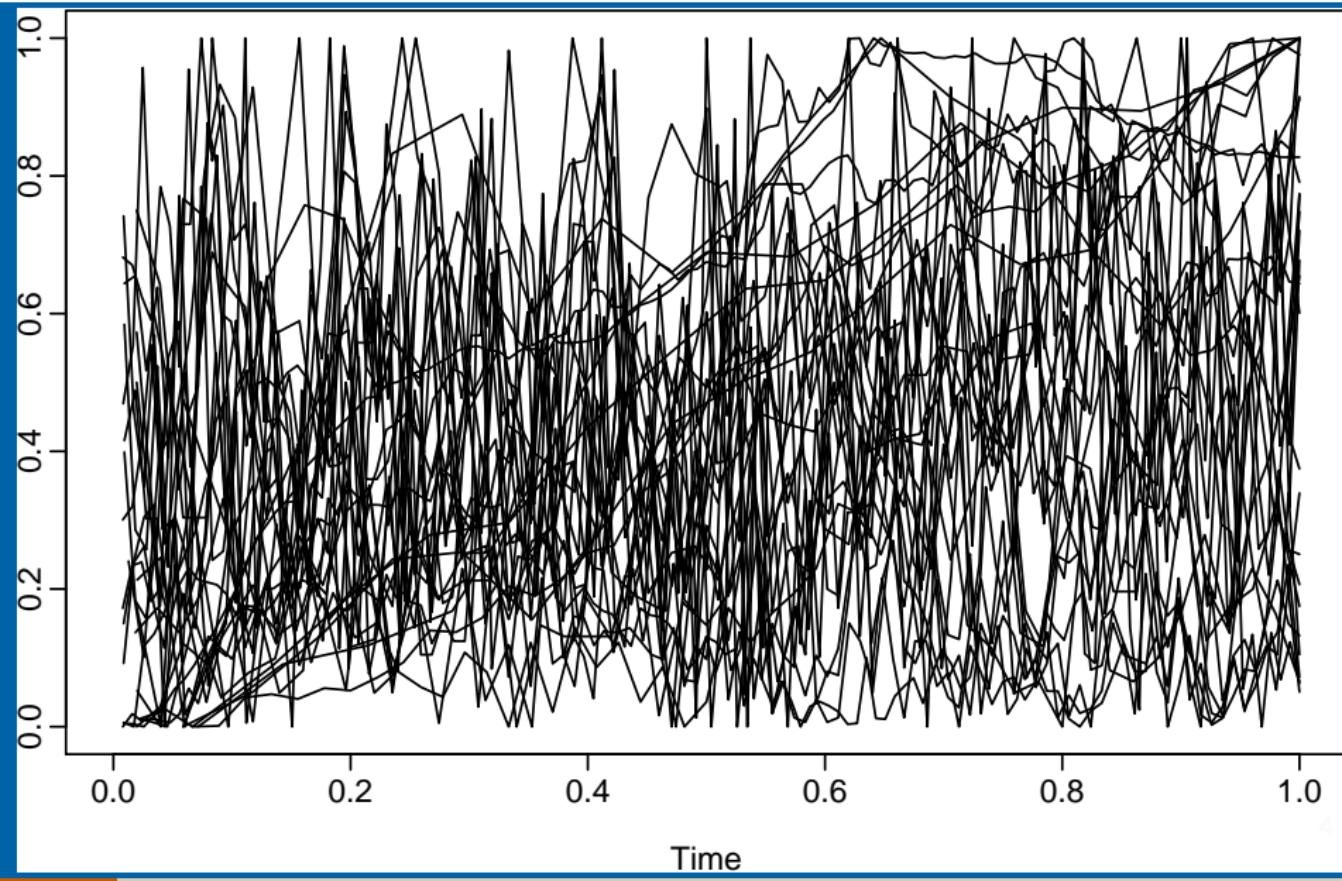
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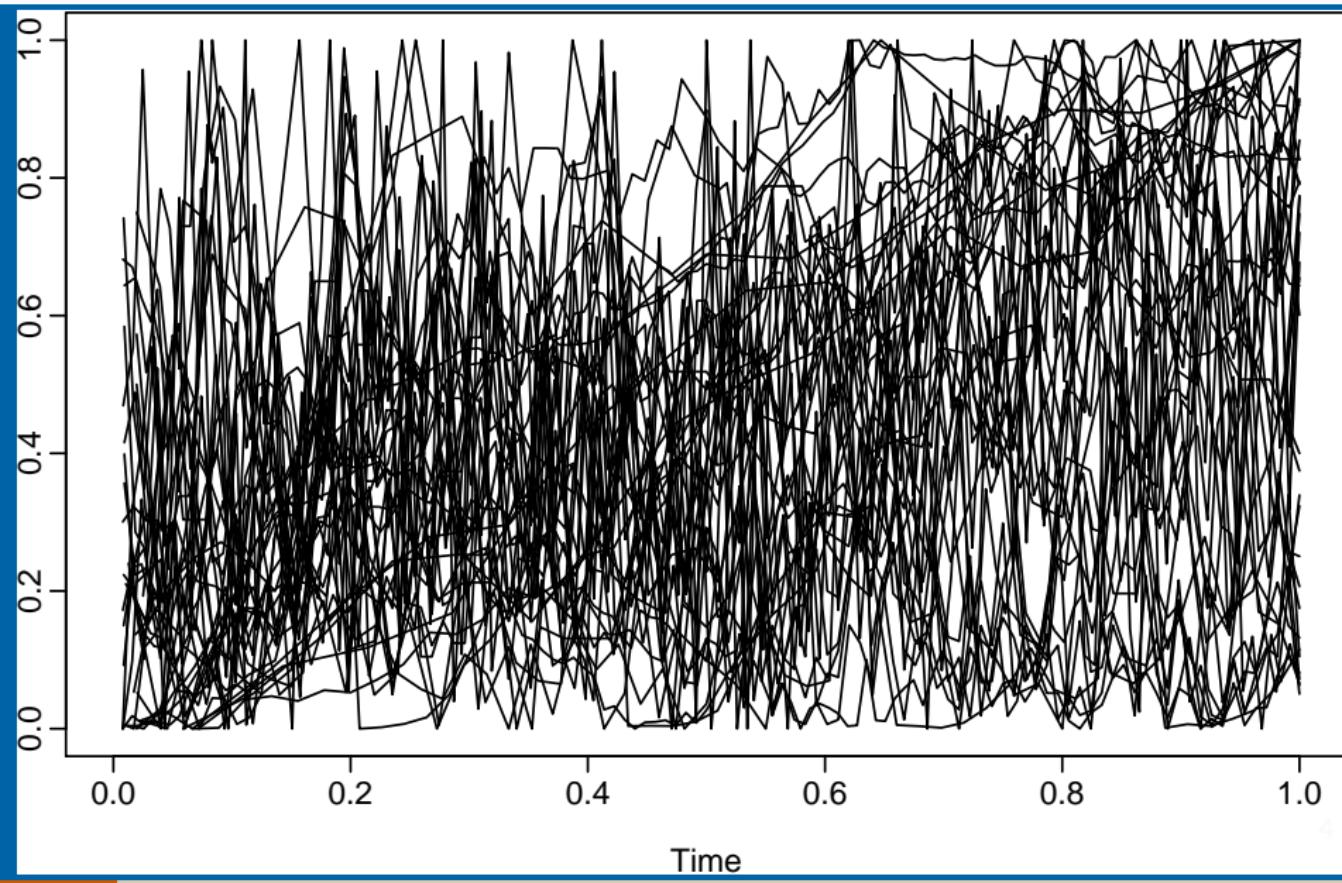
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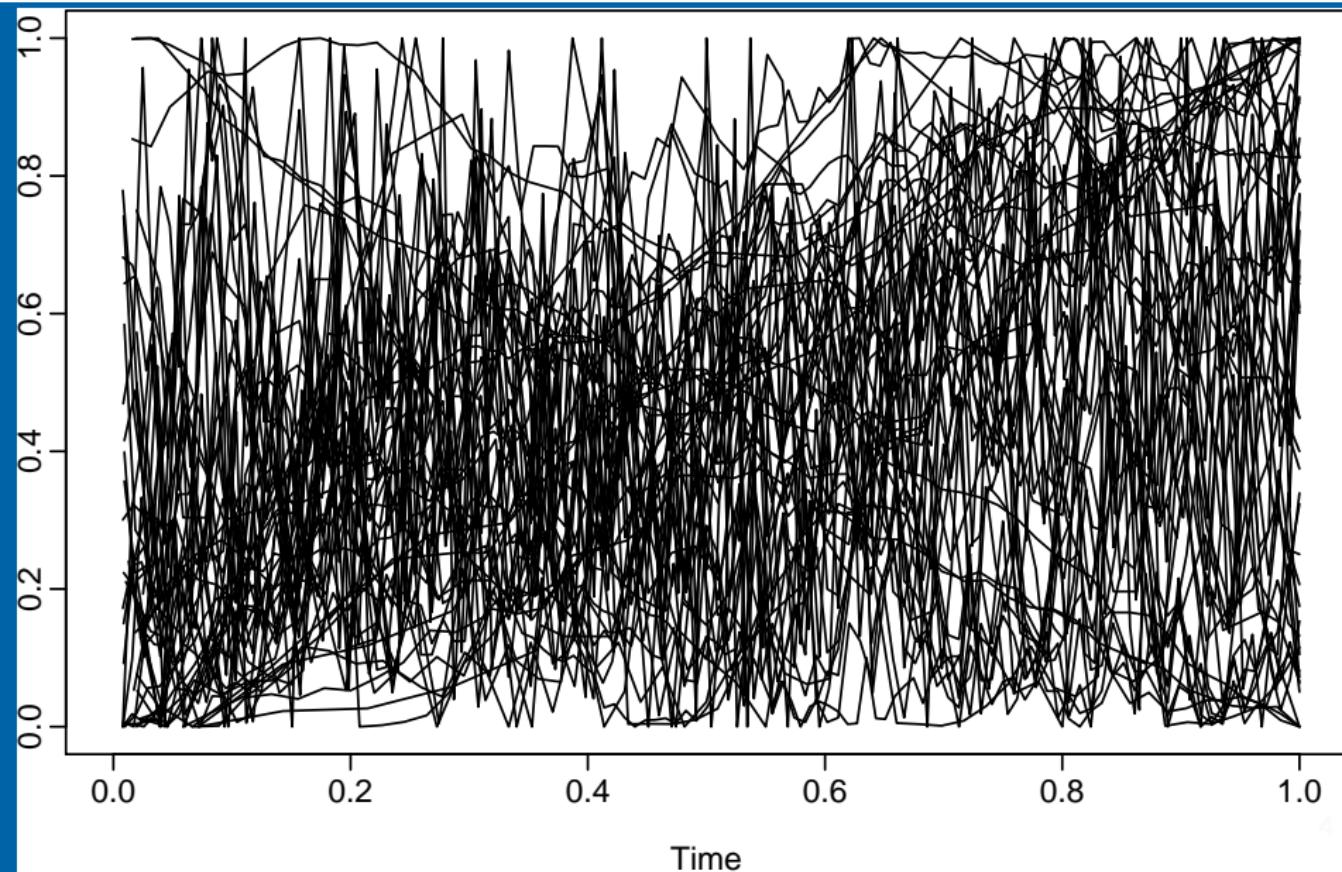
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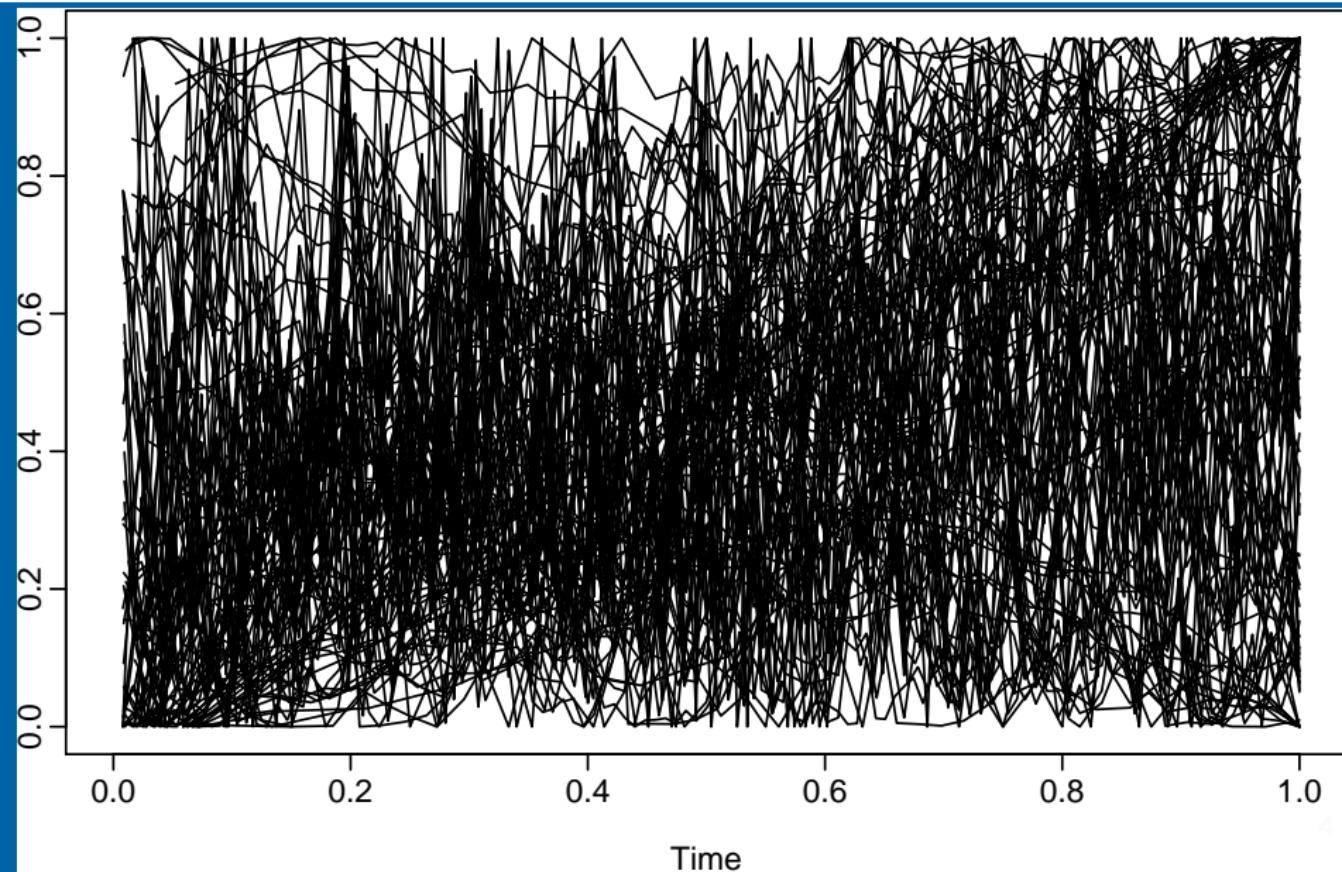
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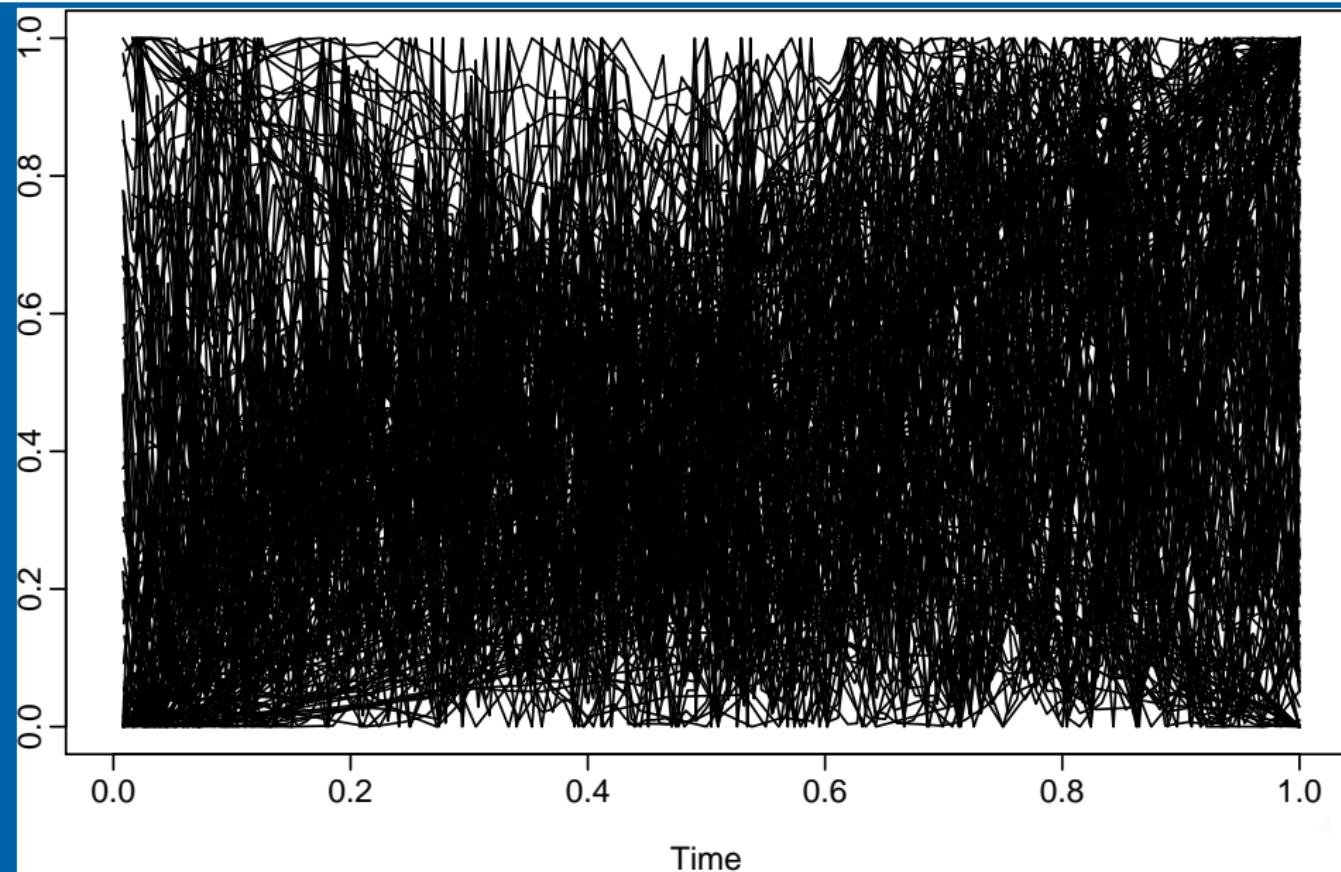
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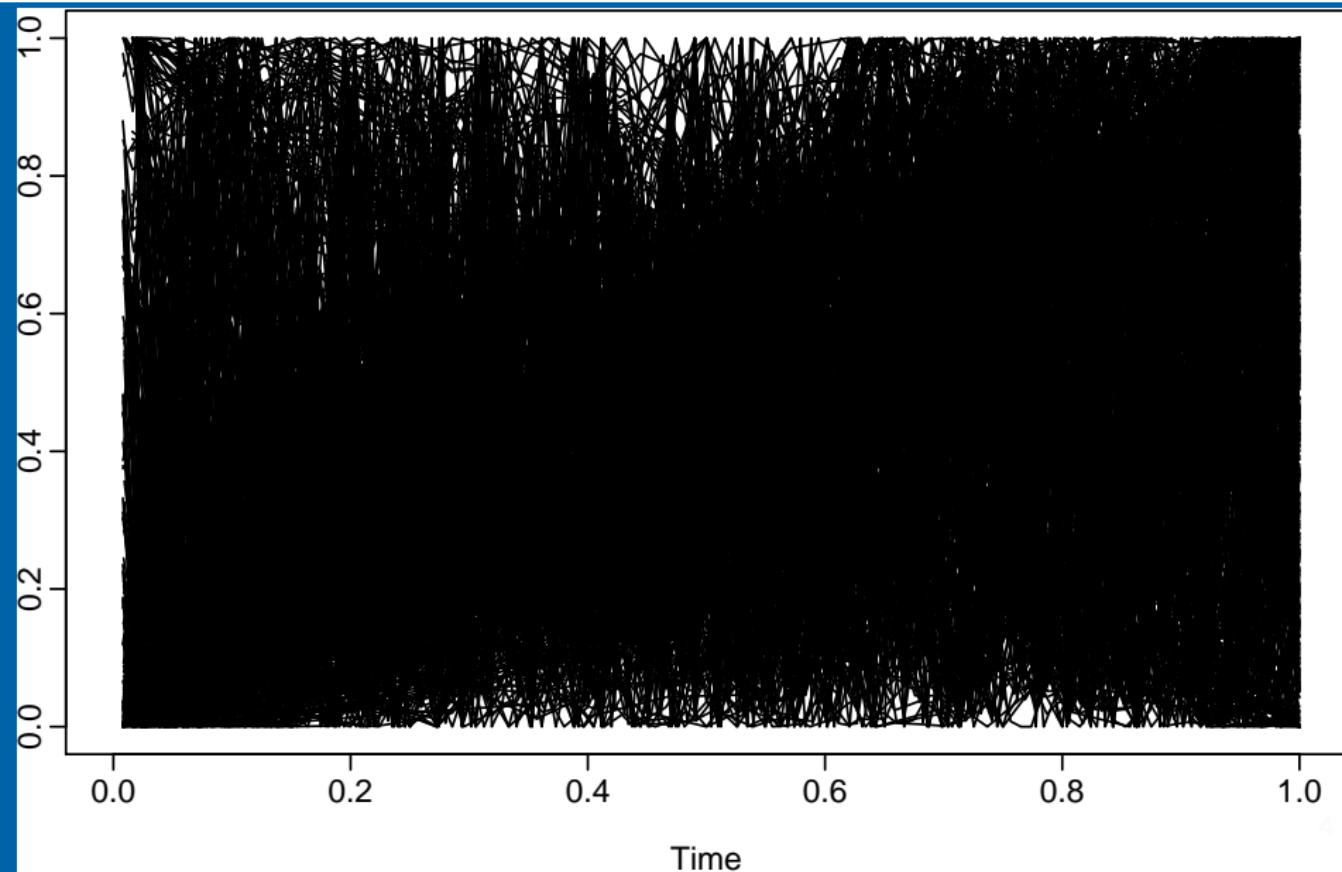
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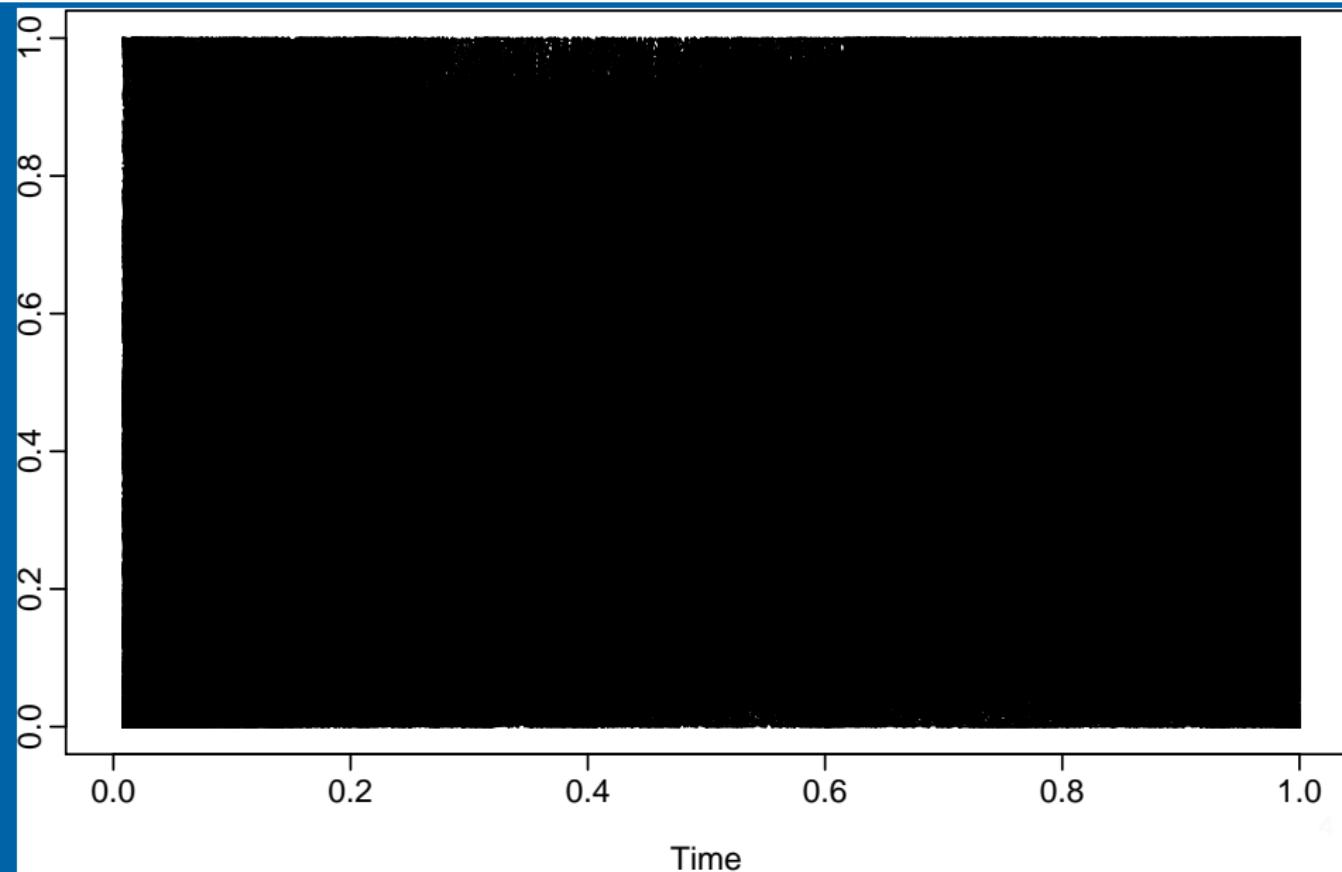
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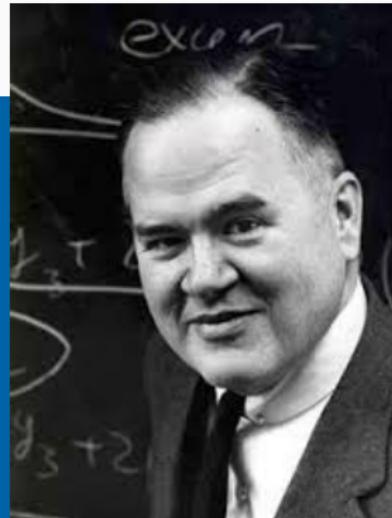
# How to plot lots of time series?



# Key idea

## Cognostics

Computer-produced diagnostics  
(Tukey and Tukey, 1985).

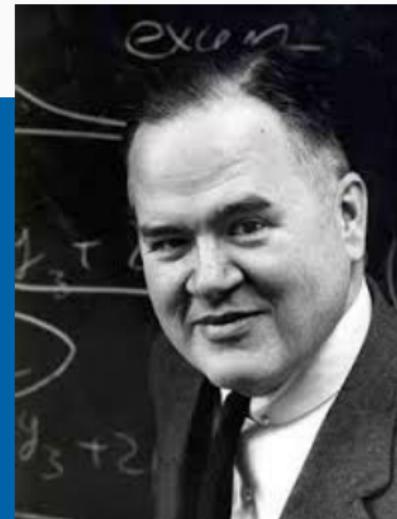


*John W Tukey*

# Key idea

## Cognostics

Computer-produced diagnostics  
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John W Tukey

## Examples for time series

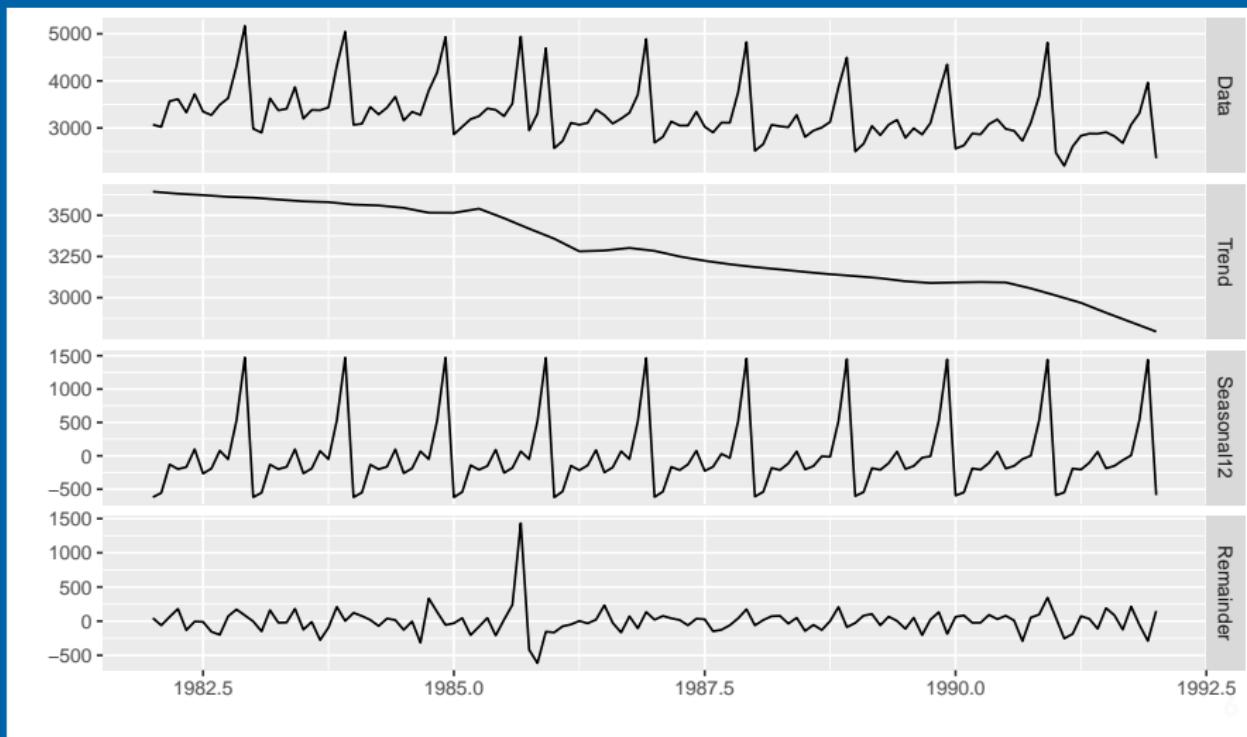
- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

Called “features” in the machine learning literature.

# An STL decomposition: N2096

$$Y_t = S_t + T_t + R_t$$

$S_t$  is periodic with mean 0



# Candidate features

## STL decomposition

$$Y_t = S_t + T_t + R_t$$

# Candidate features

## STL decomposition

$$Y_t = S_t + T_t + R_t$$

- Seasonal period
- Autocorrelations of data  $(Y_1, \dots, Y_T)$
- Autocorrelations of data  $(R_1, \dots, R_T)$
- Strength of seasonality:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)} \right)$
- Strength of trend:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)} \right)$
- Spectral entropy:  $H = - \int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$ , where  $f_y(\lambda)$  is spectral density of  $Y_t$ .  
Low values of  $H$  suggest a time series that is easier to forecast (more signal).
- Optimal Box-Cox transformation of data

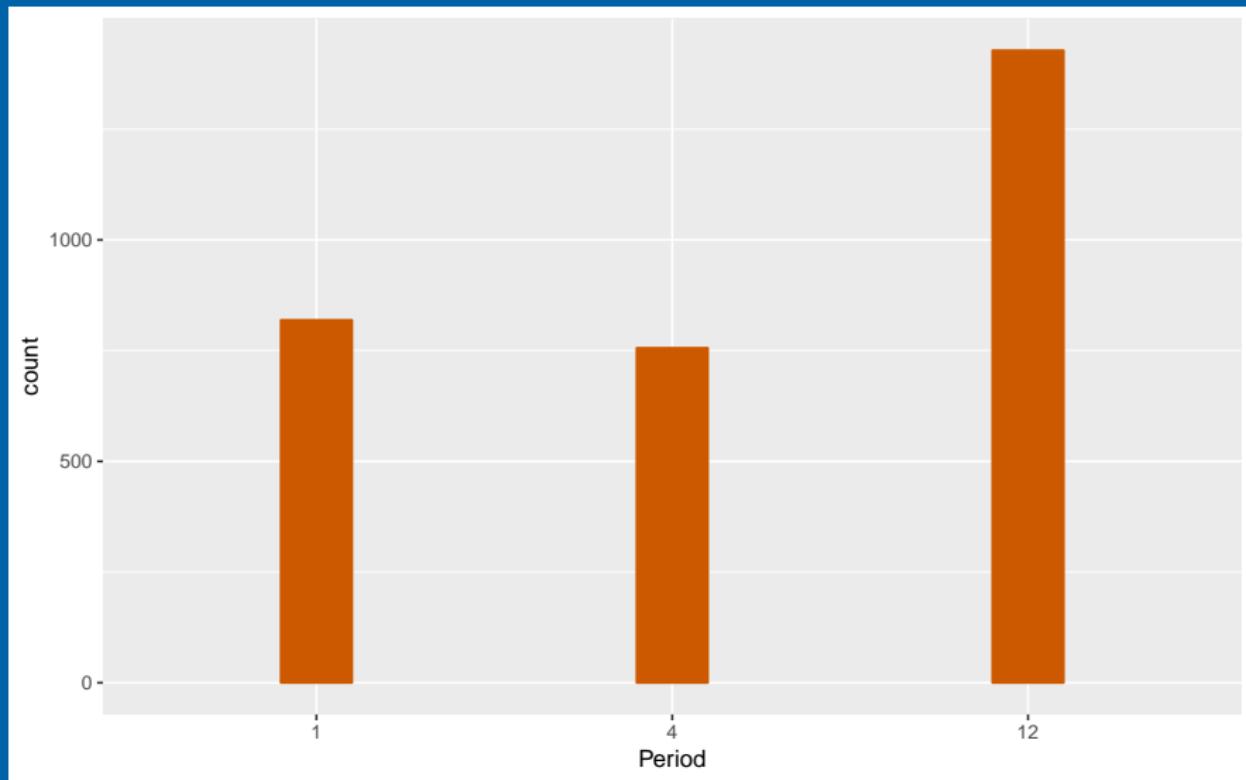
## tsfeatures package

```
library(tsfeatures)

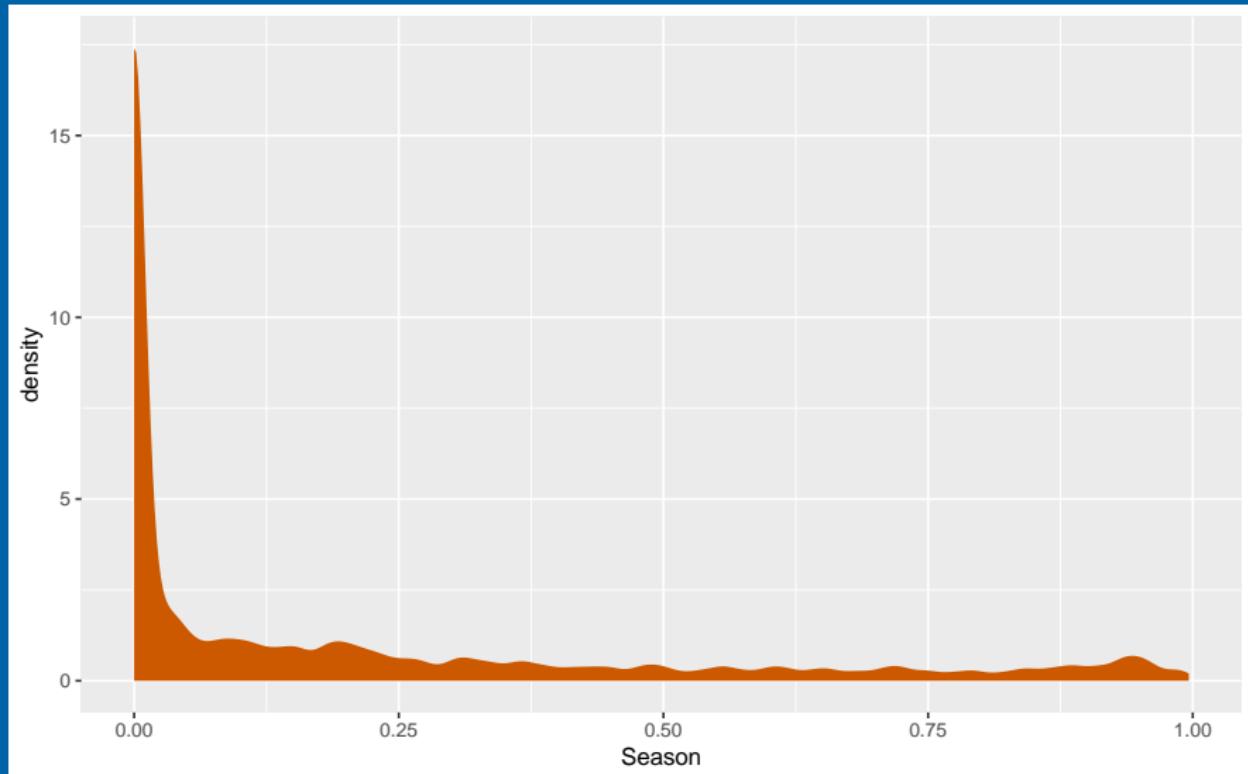
lambda_stl <- function(x,...)
{
  lambda <- forecast::BoxCox.lambda(x, lower=0)
  y <- forecast::BoxCox(x, lambda)
  c(stl_features(y,s.window='periodic', robust=TRUE),
    lambda_stl)
}

M3Features <- bind_cols(
  tsfeatures(M3data, c("frequency", "entropy")),
  tsfeatures(M3data, "lambda_stl", scale=FALSE),
  select(frequency, entropy, trend, seasonal_strength),
  replace_na(list(seasonal_strength=0))) %>%
```

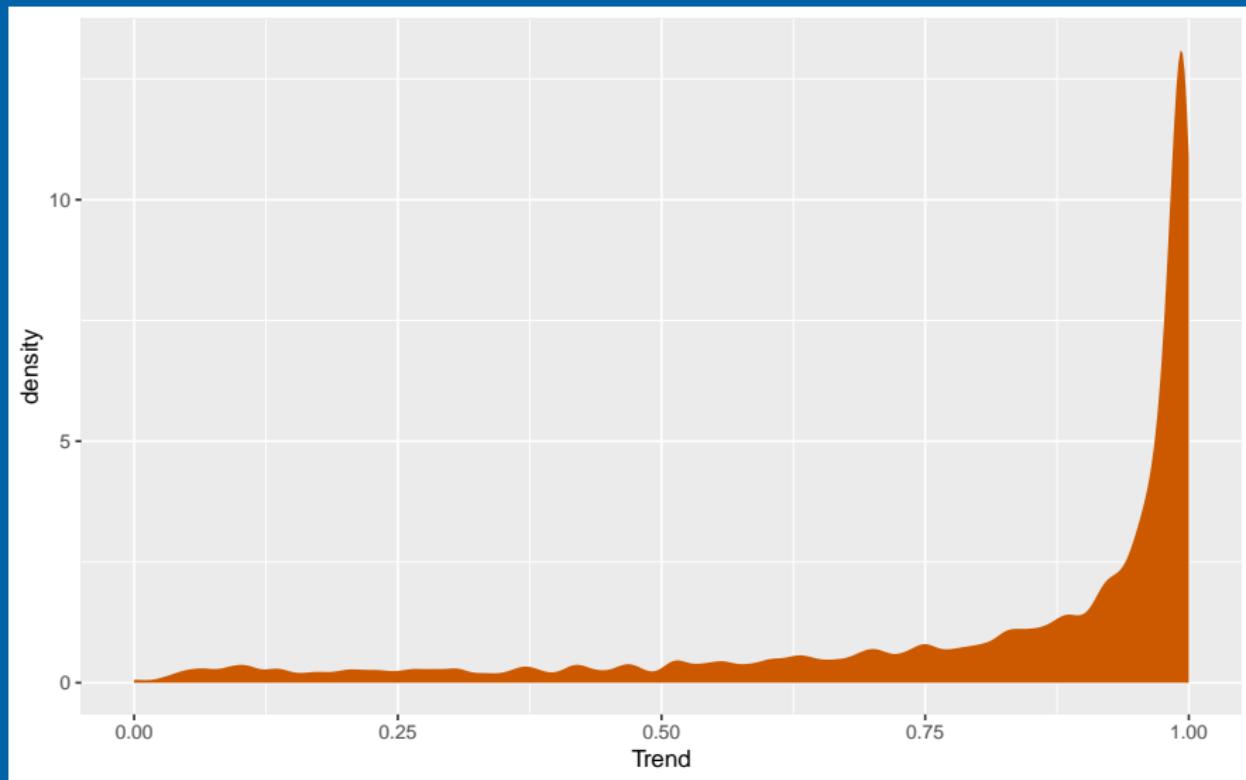
# Distribution of Period for M3



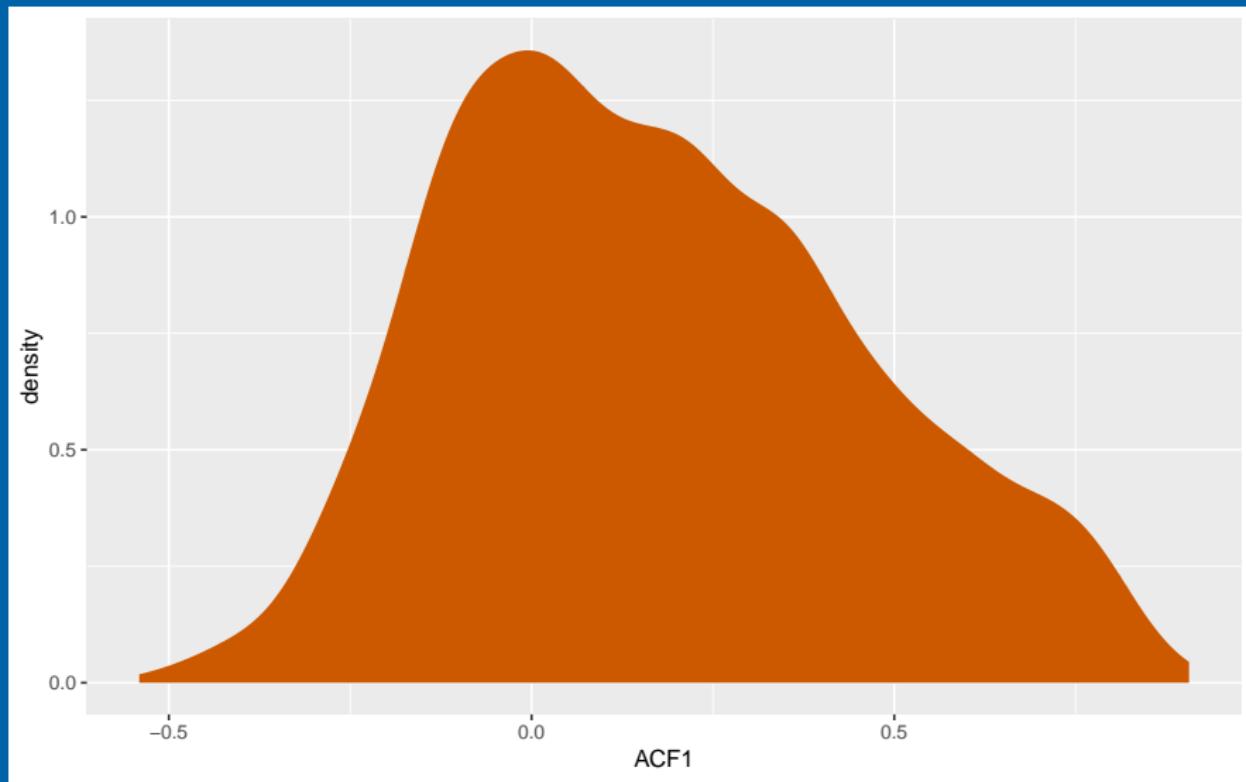
# Distribution of Seasonality for M3



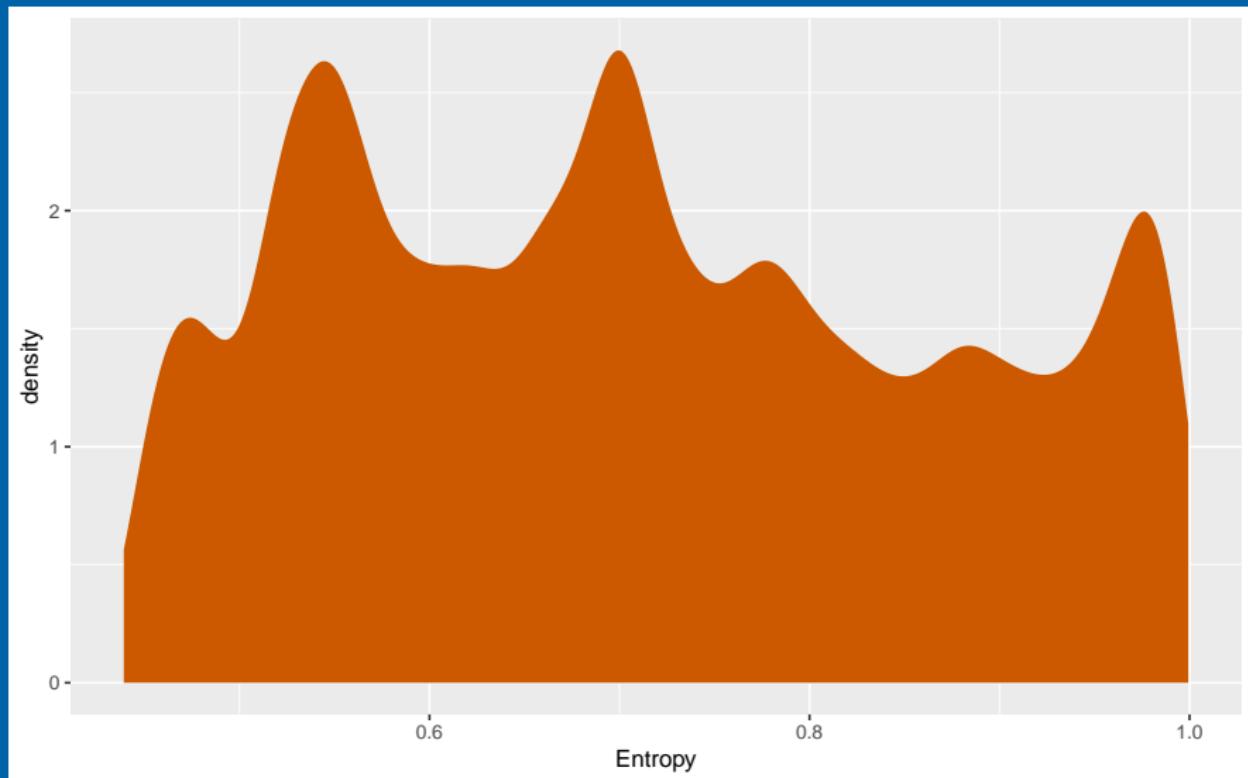
# Distribution of Trend for M3



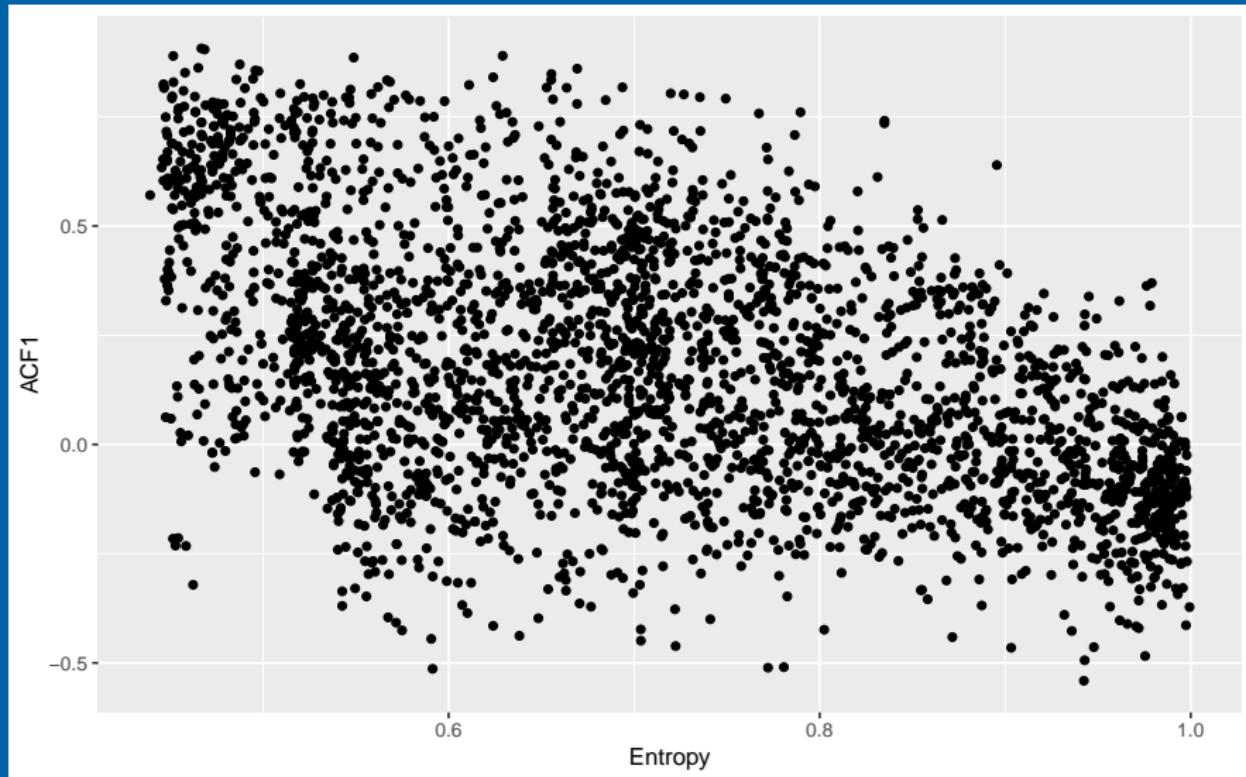
# Distribution of Residual ACF1 for M3



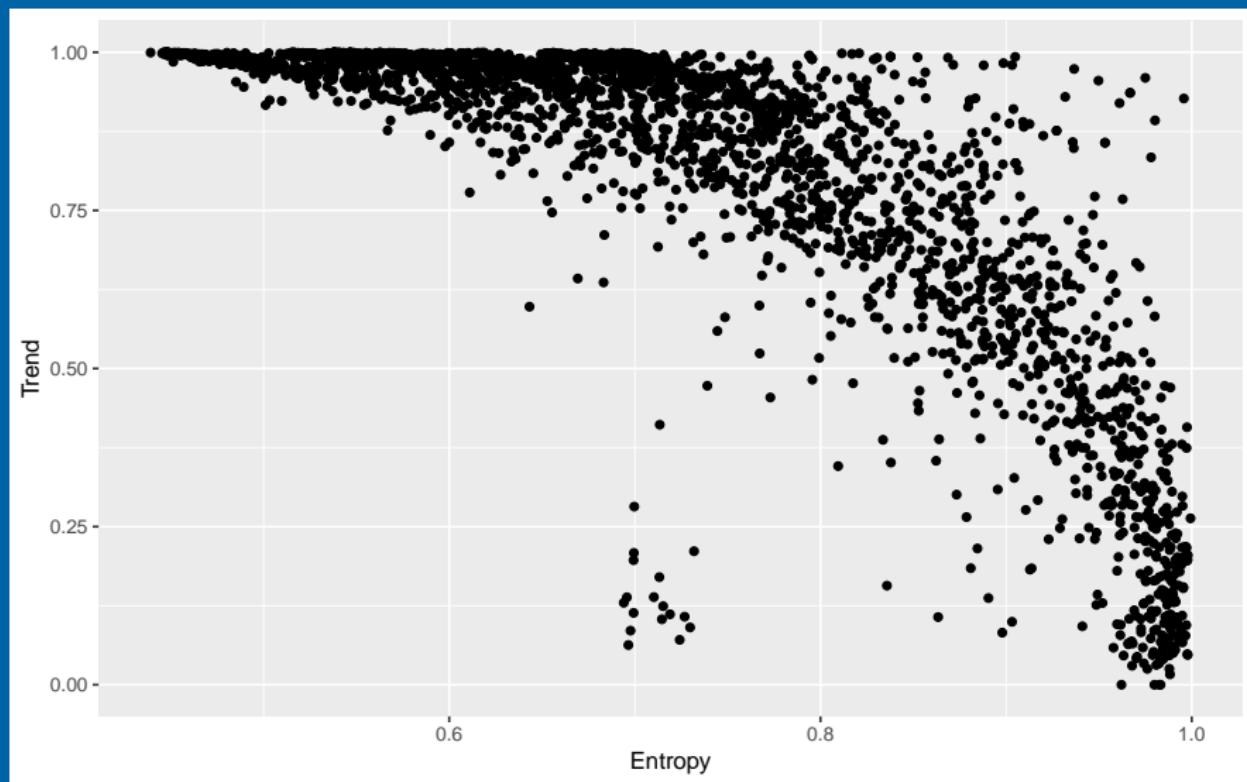
# Distribution of Spectral Entropy for M3



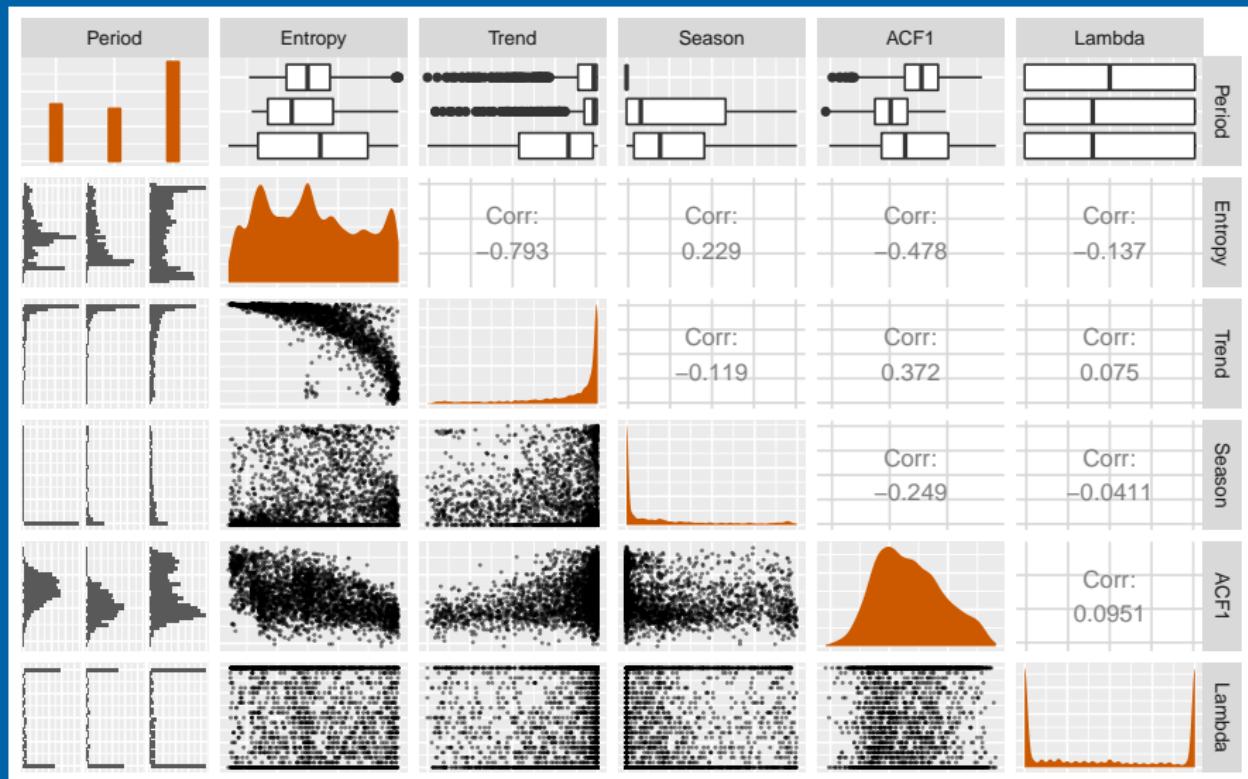
# Feature distributions



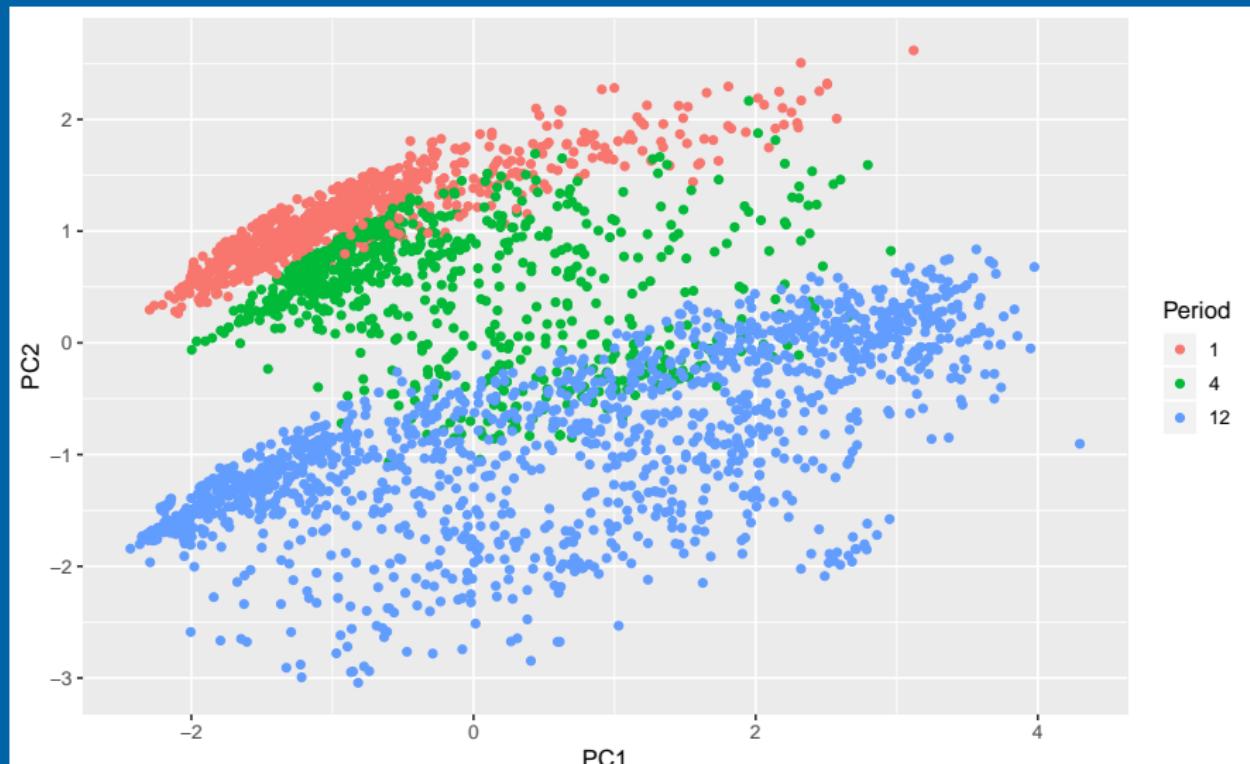
# Feature distributions



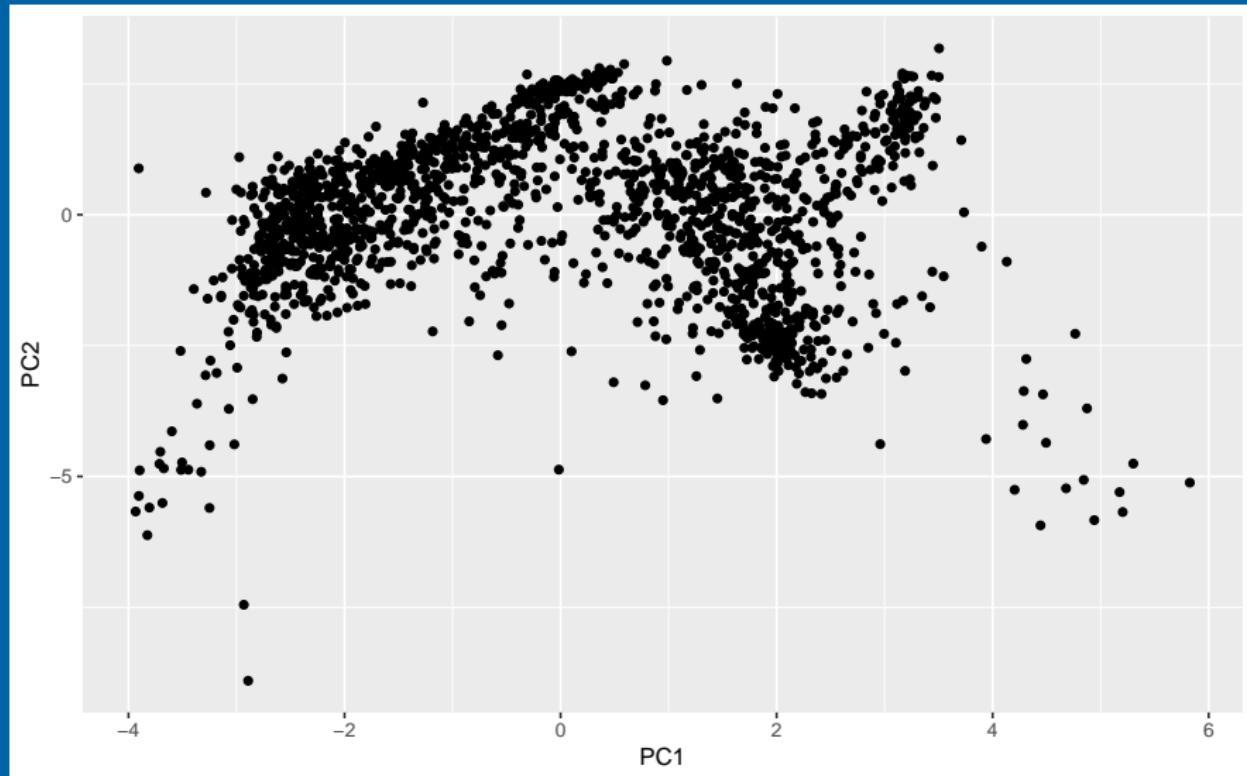
# Feature distributions



# Feature distributions



# Hyndman, Wang and Laptev (ICDM 2015)



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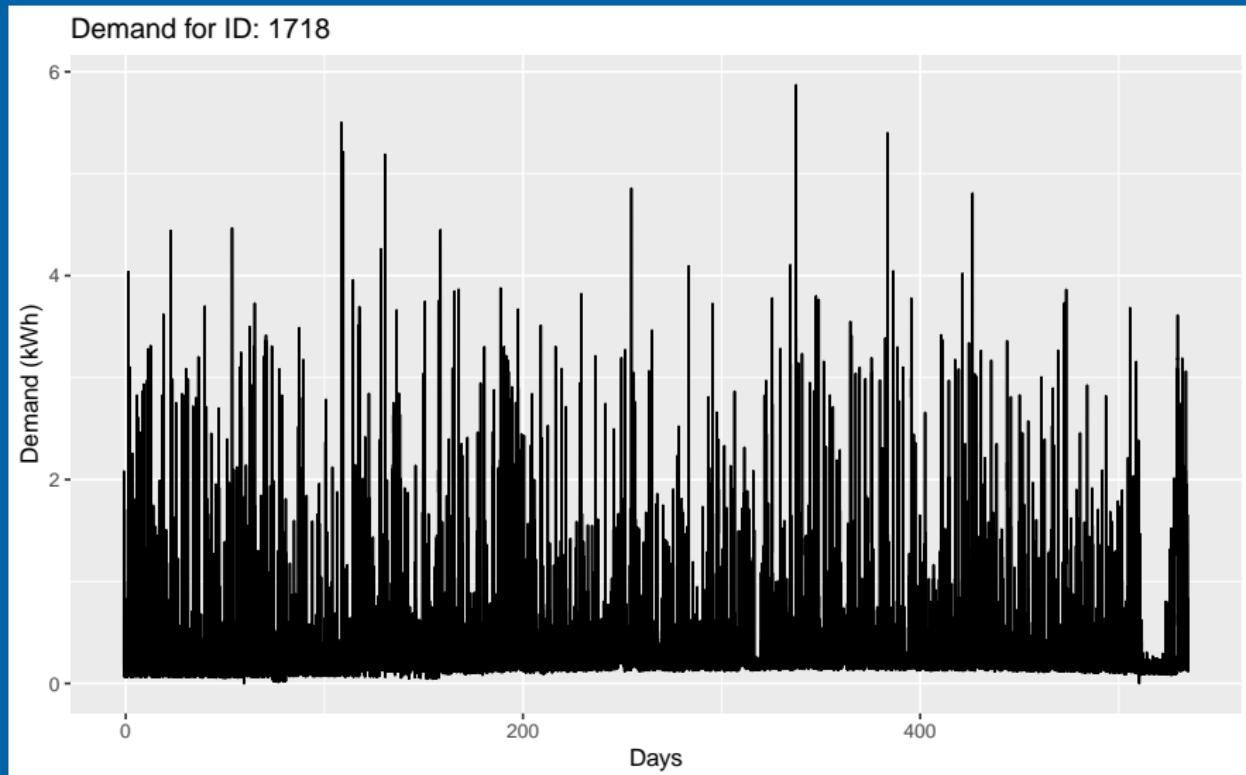
# Irish smart metre data



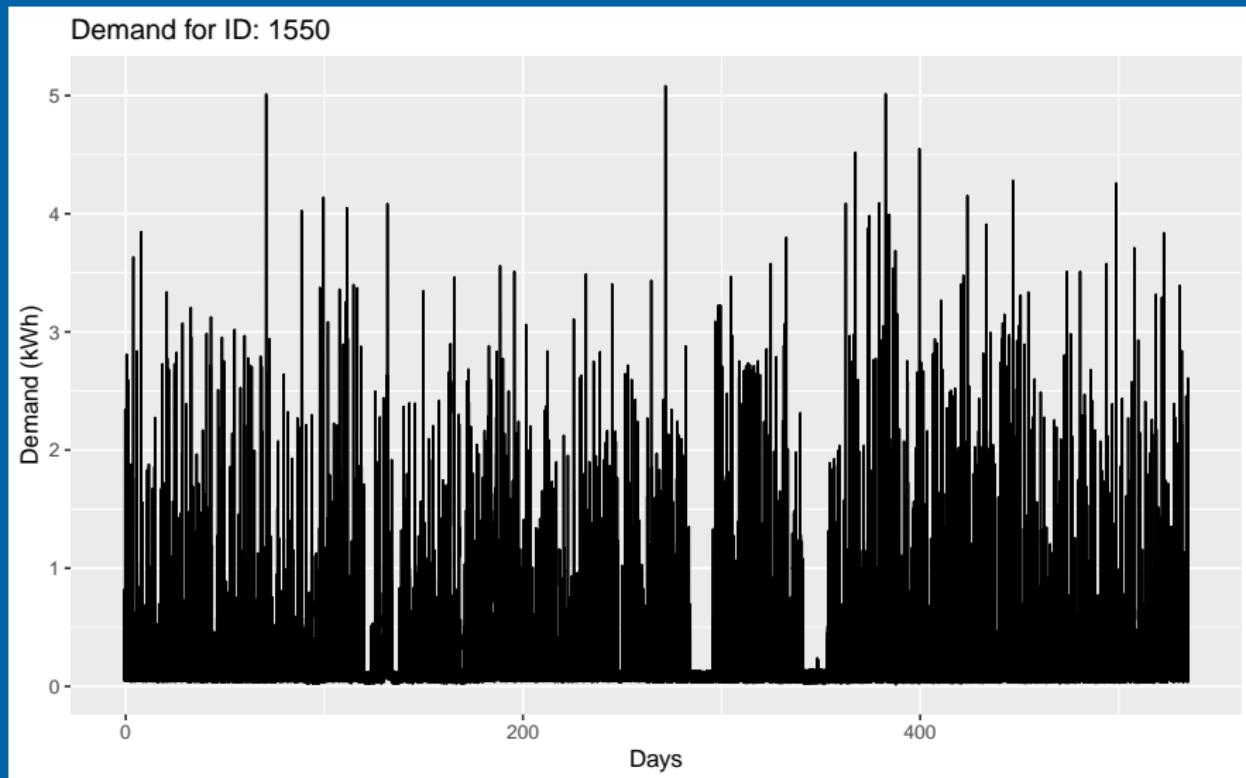
Figure: <http://solutions.3m.com>

- 500 households from smart metering trial
- Electricity consumption at 30-minute intervals between 14 July 2009 and 31 December 2010
- Heating/cooling energy usage excluded

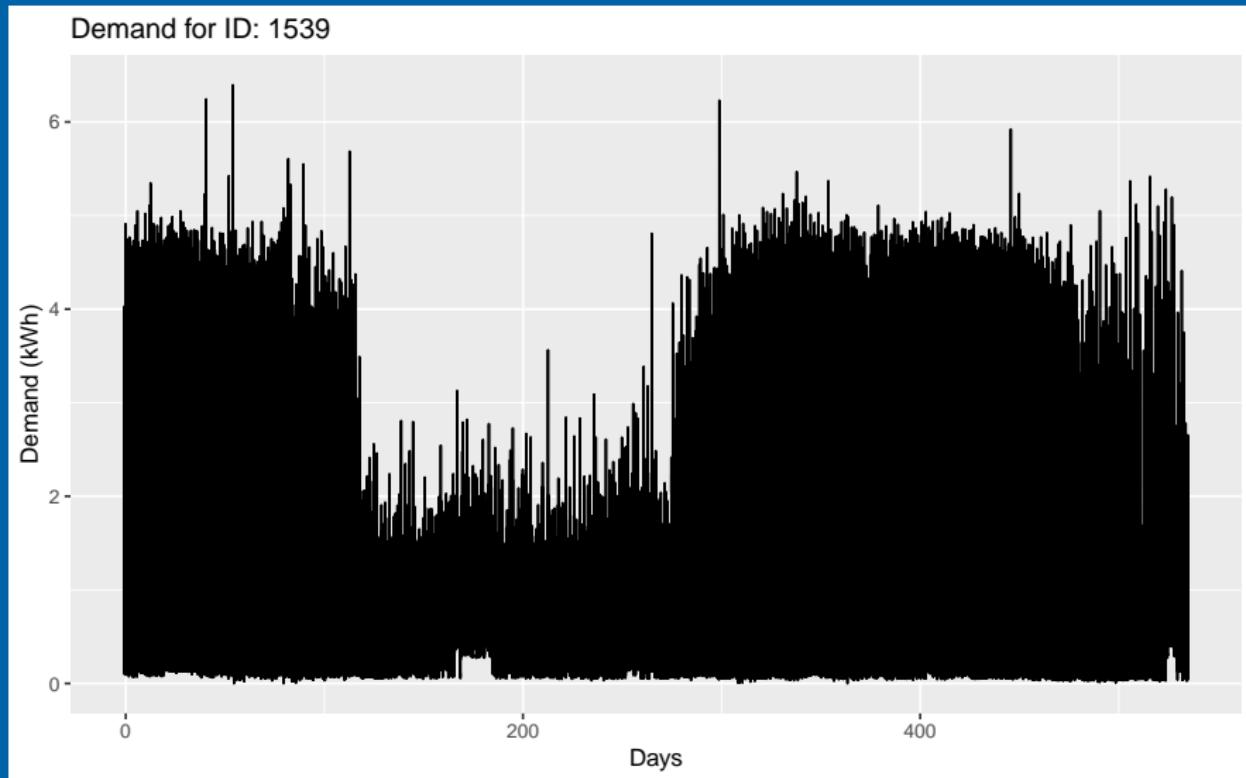
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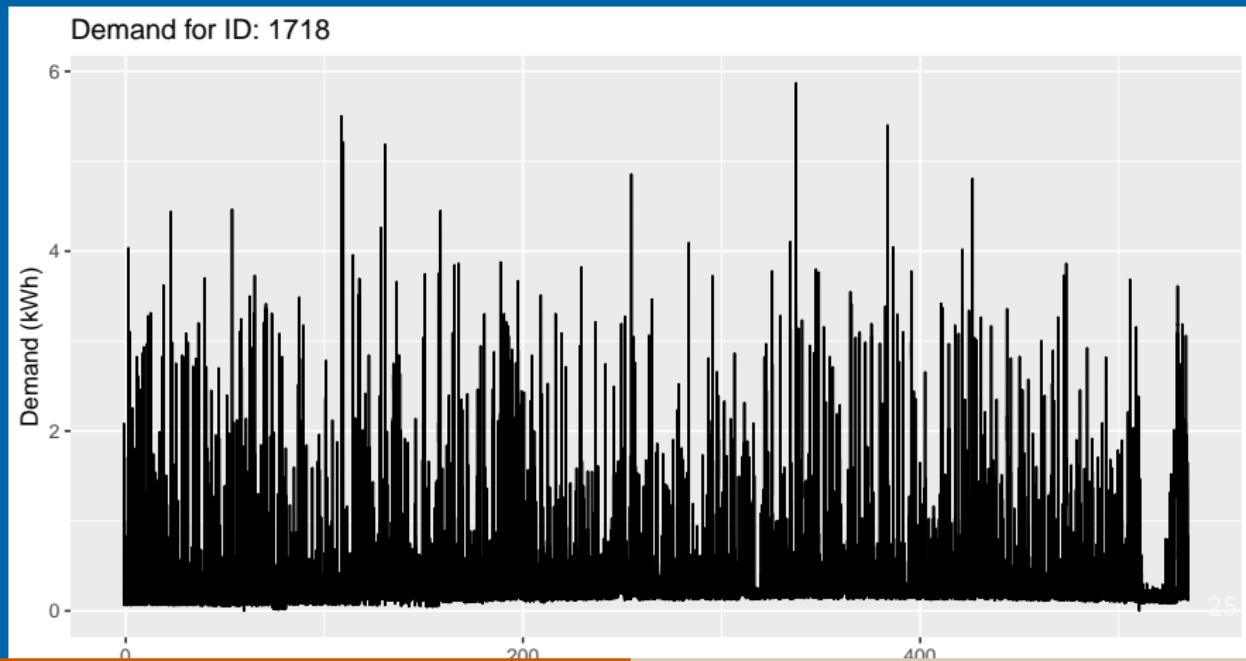


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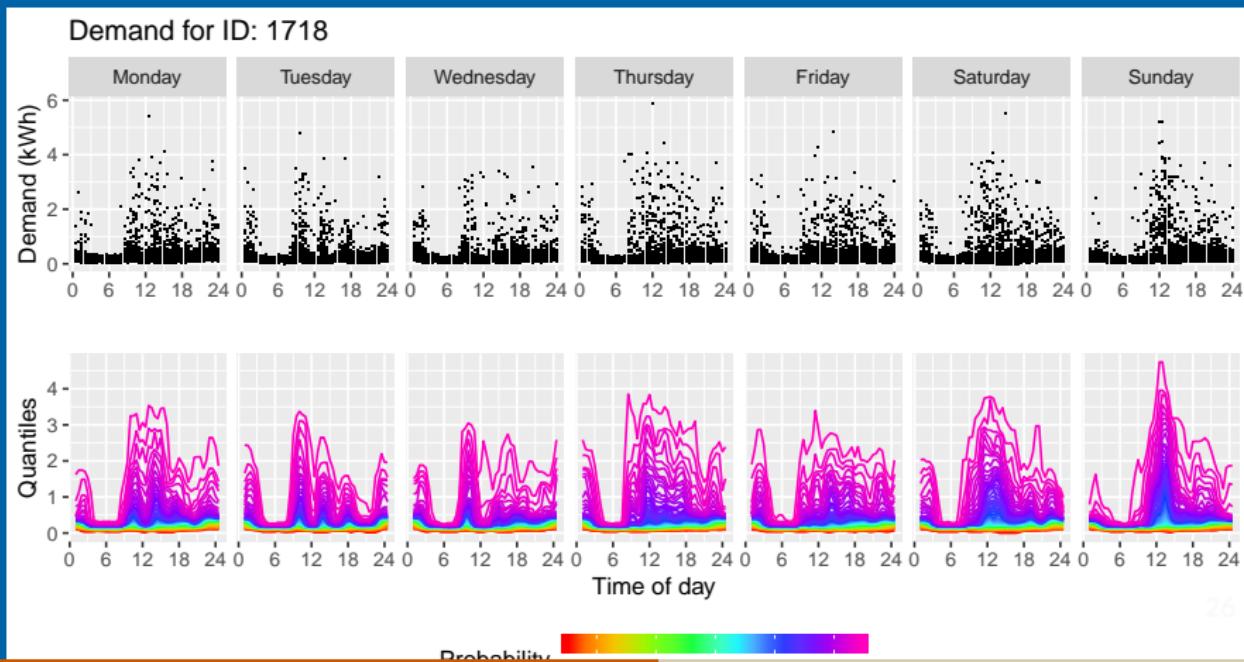
# Quantiles conditional on time of week

- Compute sample quantiles at  $p = 0.01, 0.02, \dots, 0.99$  for each household and each half-hour of the week.
- 336 probability distributions per household.



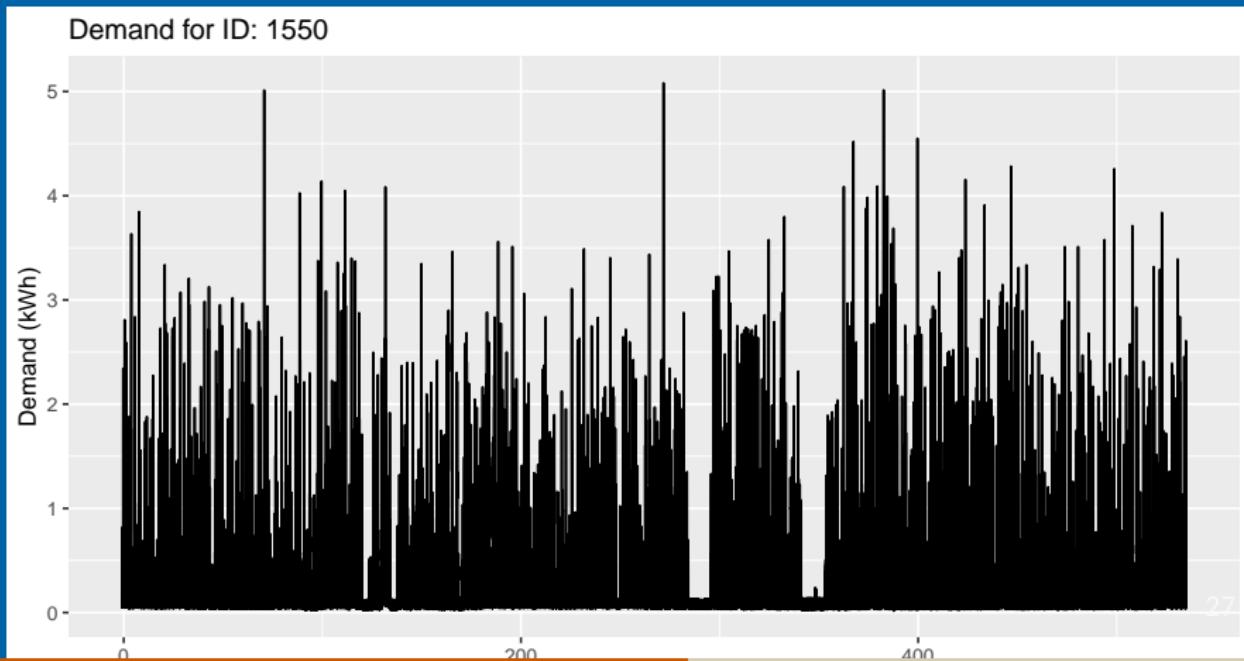
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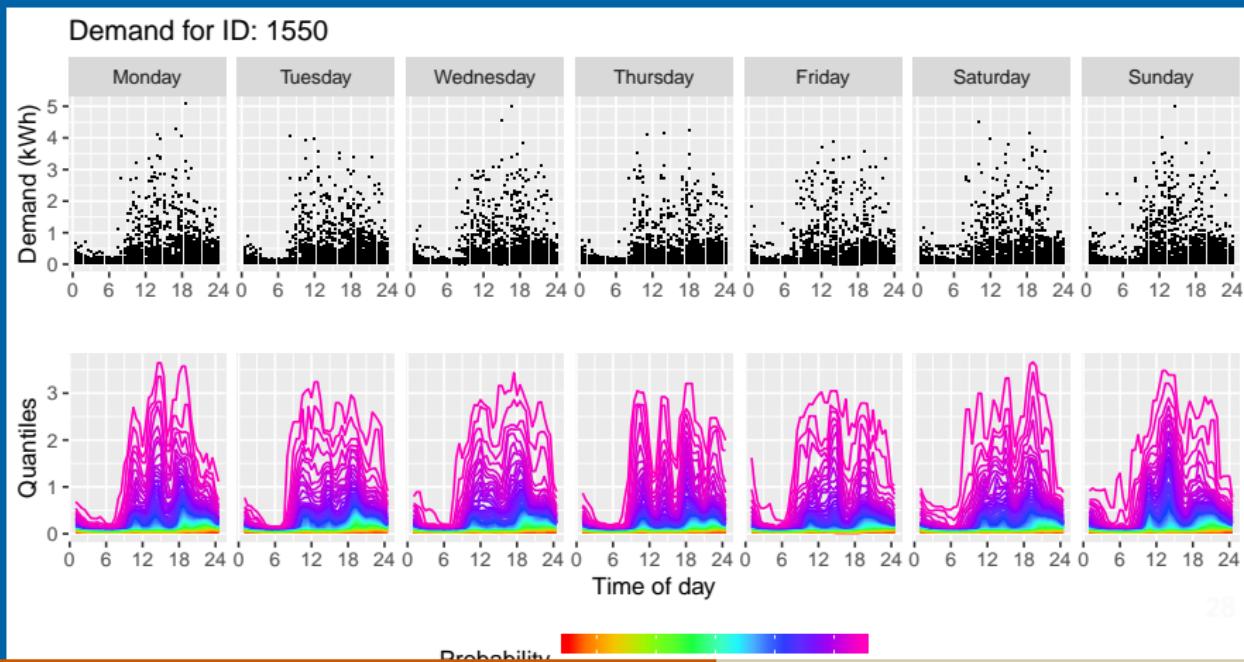
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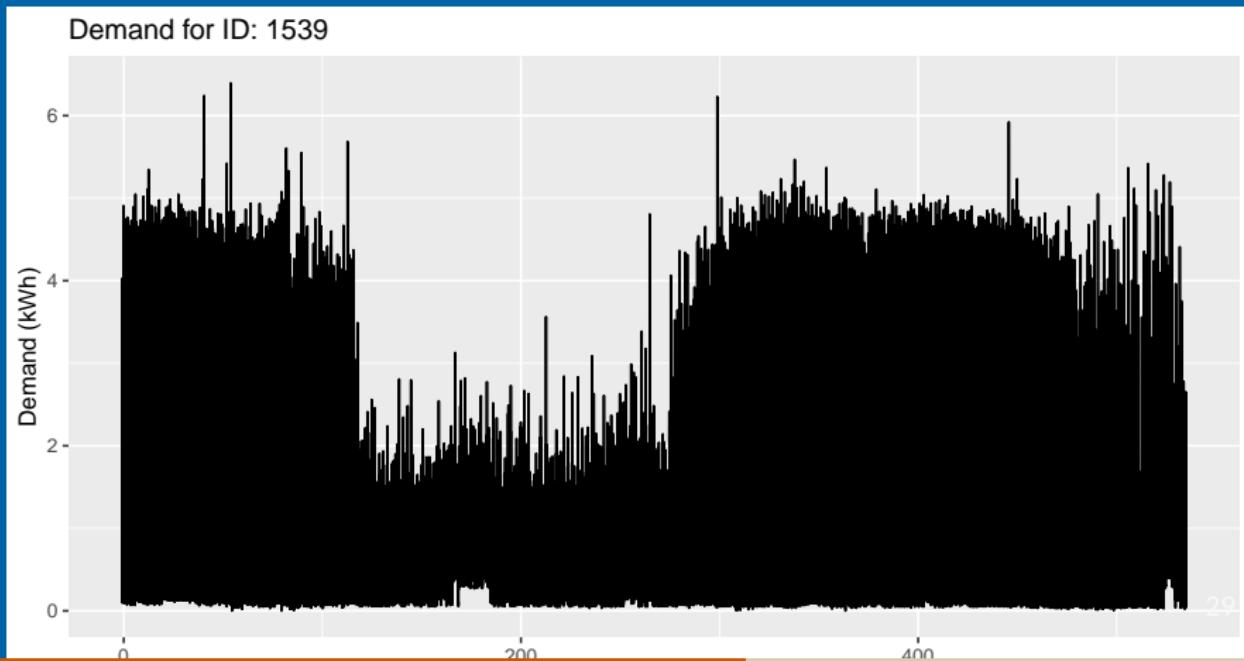
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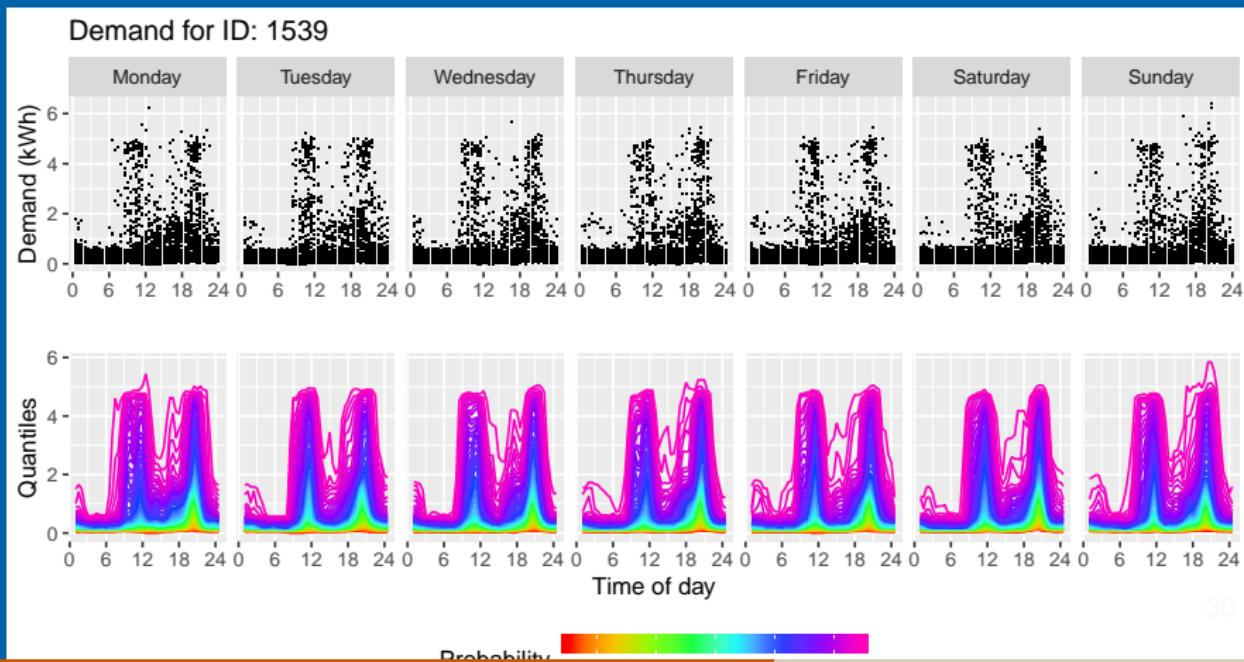
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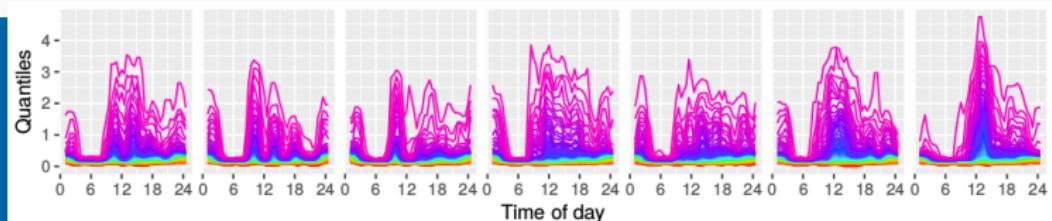


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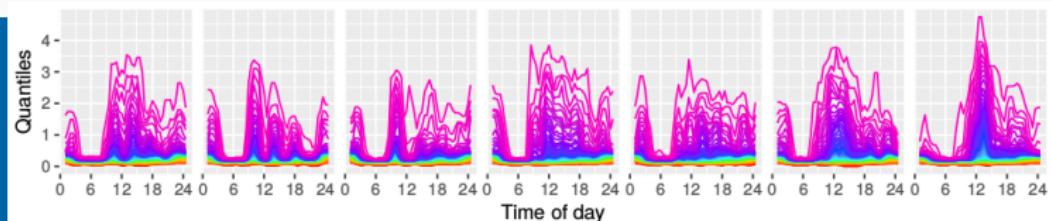


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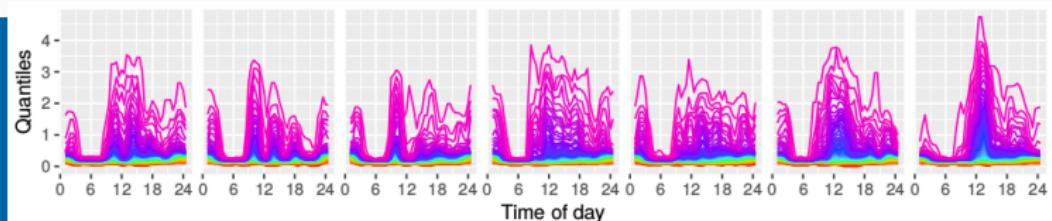
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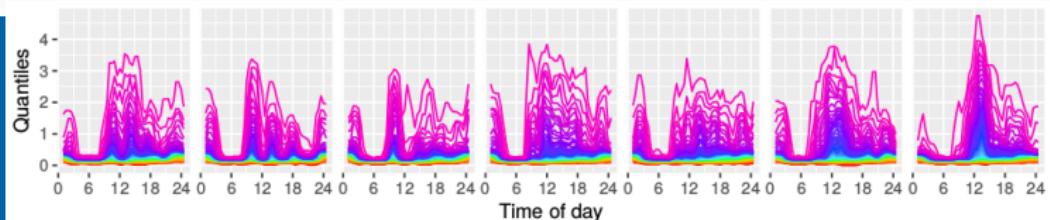
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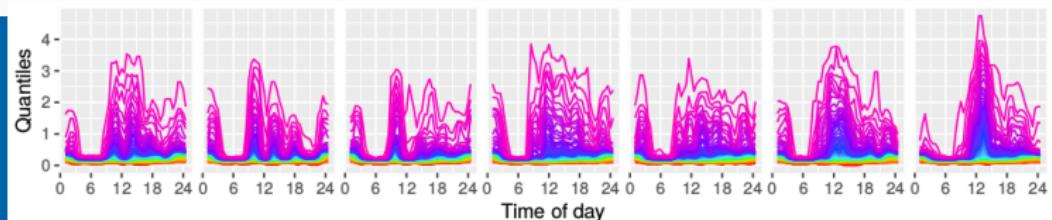
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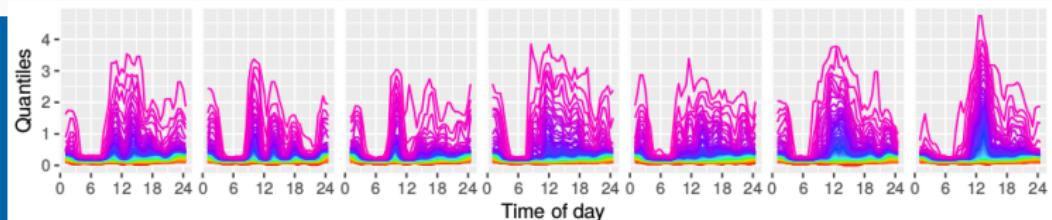
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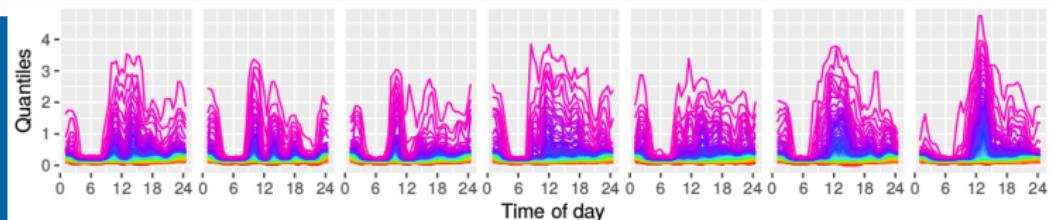
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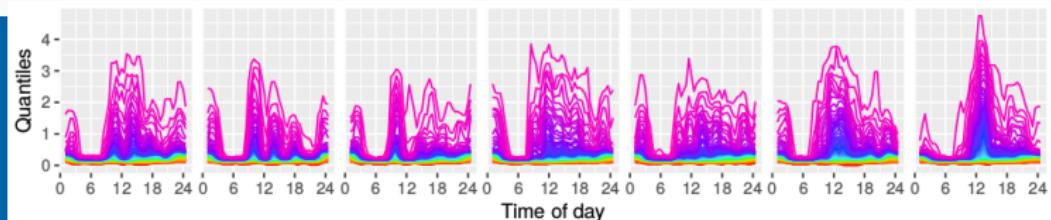
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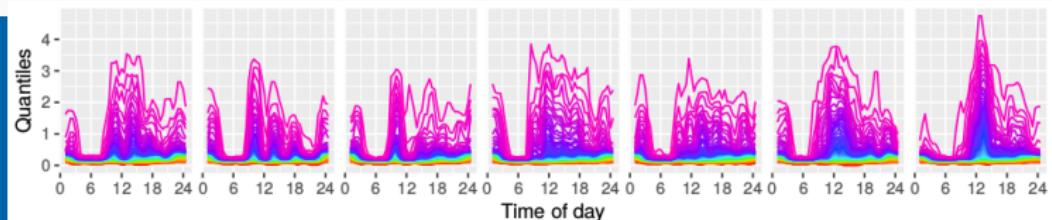
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- Allows identification of anomalous households.

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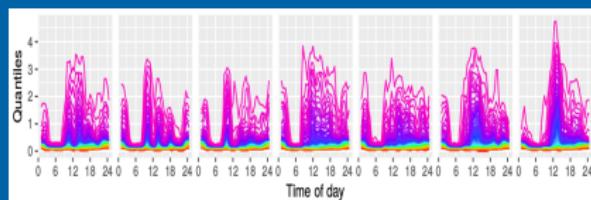
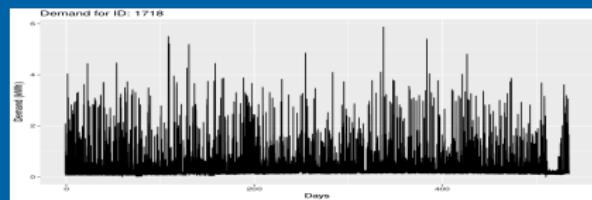


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- Allows identification of anomalous households.
- Allows estimation of typical household behaviour.

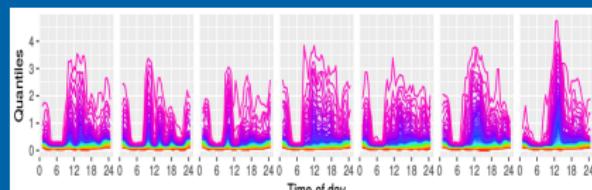
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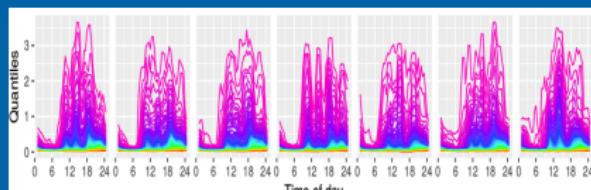
# Pairwise distances



- The time series of  $535 \times 48$  observations per household is mapped to a set of  $7 \times 48 \times 99$  quantiles giving a bivariate surface for each household.
- Can we compute pairwise distances between all households?



← ? →  
Distance



## Jensen-Shannon distances

### Kullback-Leibler divergence between two densities

$$D(p, q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

## Jensen-Shannon distances

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$$\text{JS}(p, q) = [D(p, r) + D(q, r)]/2 \quad \text{where } r = (p + q)/2$$

## Jensen-Shannon distances

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### Jensen-Shannon distance between two densities

$$\text{JS}(p, q) = [D(p, r) + D(q, r)]/2 \quad \text{where } r = (p + q)/2$$

### Distance between two households

$$\Delta_{ij} = \sum_{t=1}^{7 \times 48} \text{JS}(p_t, q_t)$$

# Kernel matrix and density ranking

## Similarity between two households

$$w_{ij} = \exp(-\Delta_{ij}^2/h^2).$$

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$$w_{ij} = \exp(-\Delta_{ij}^2/h^2).$$

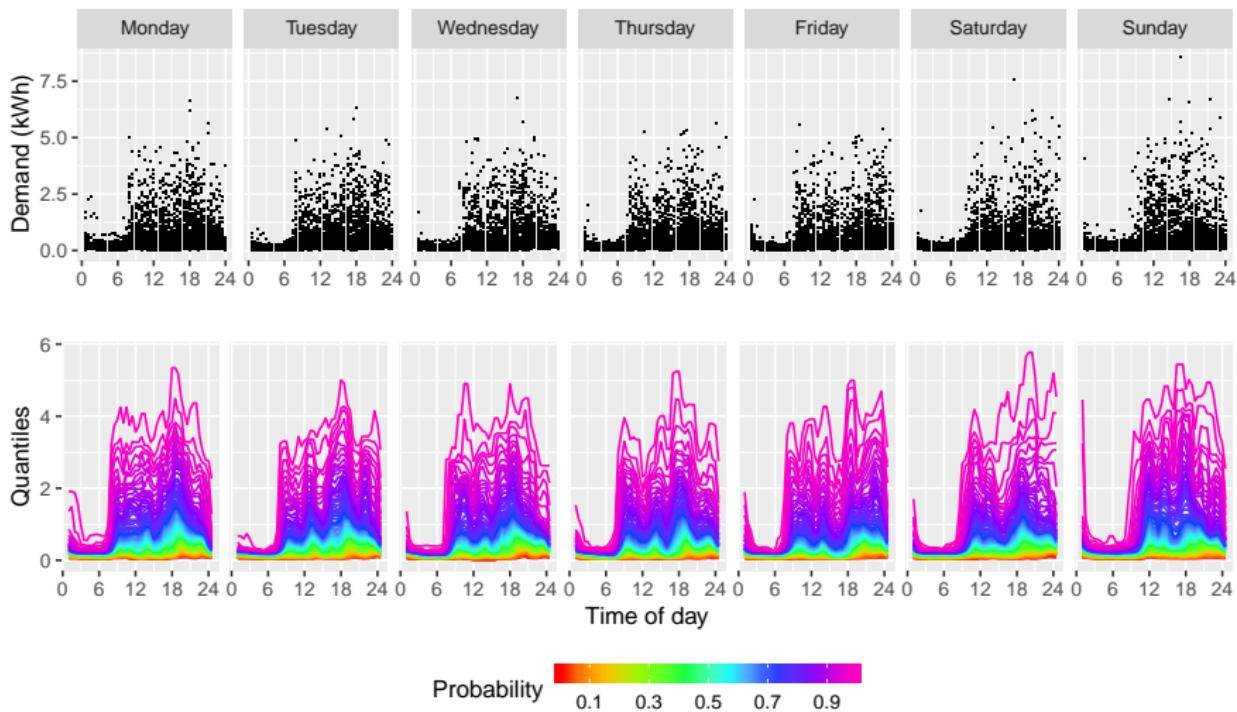
Row sums of the kernel matrix gives a scaled kernel density estimate of households:

$$\hat{f}_i = \sum_{j=1}^n w_{ij}$$

- $h$  is bandwidth in Gaussian kernel.
- Households can be ranked by density values.

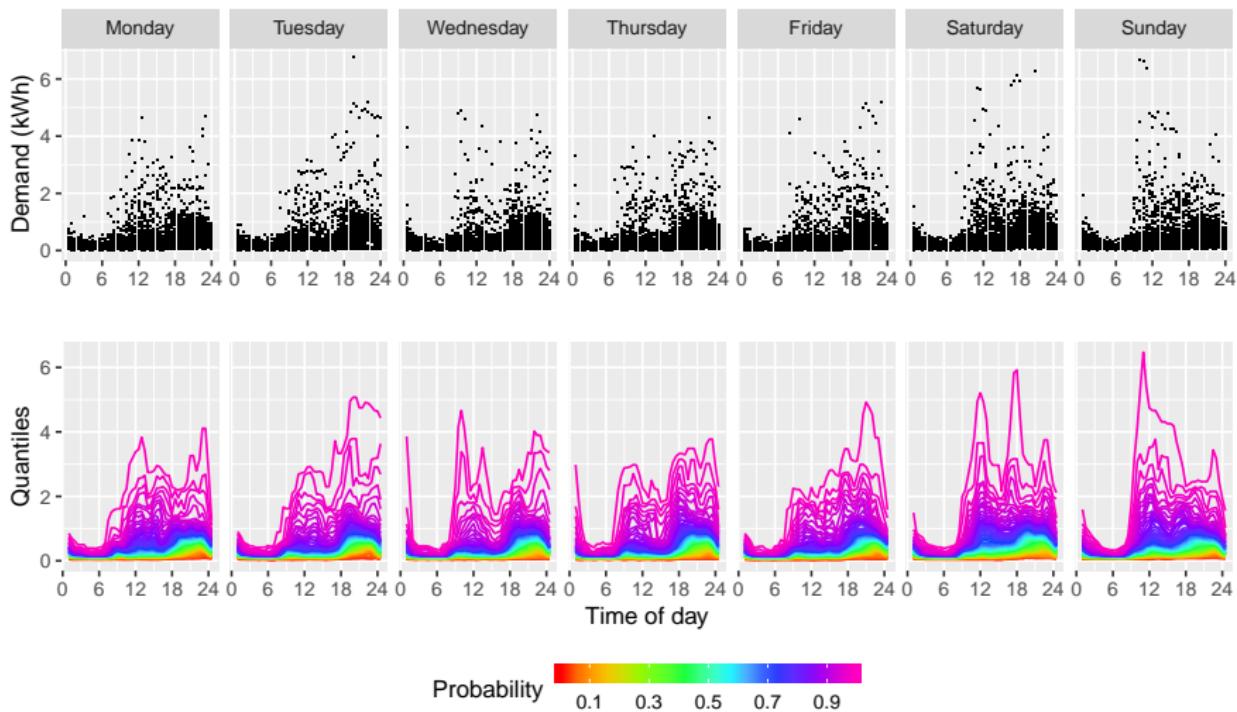
# Typical households

Demand for ID: 1672



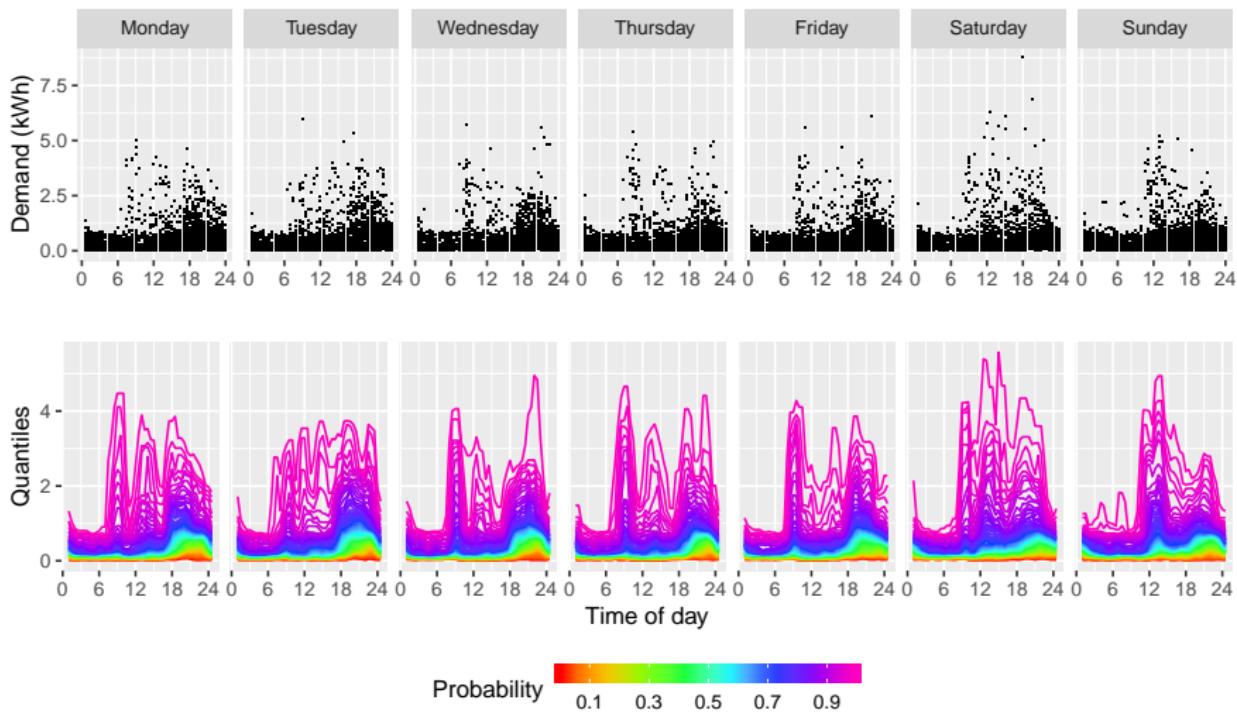
# Typical households

Demand for ID: 1058



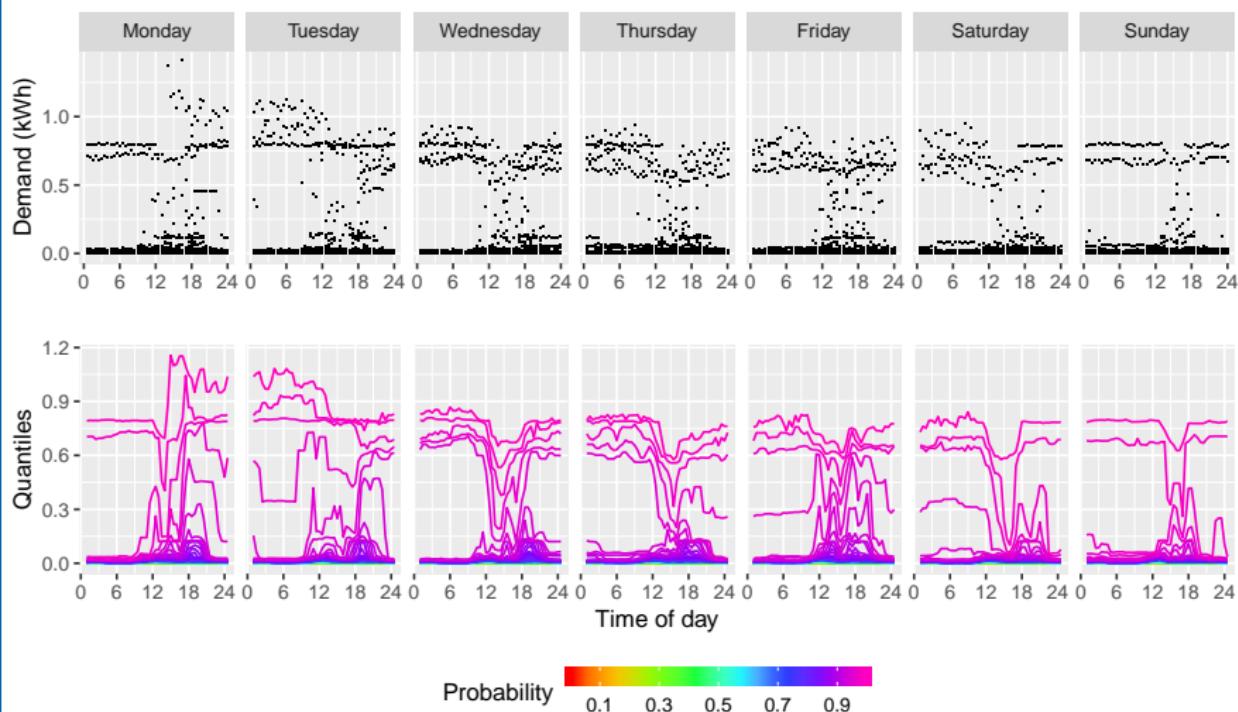
# Typical households

Demand for ID: 1183



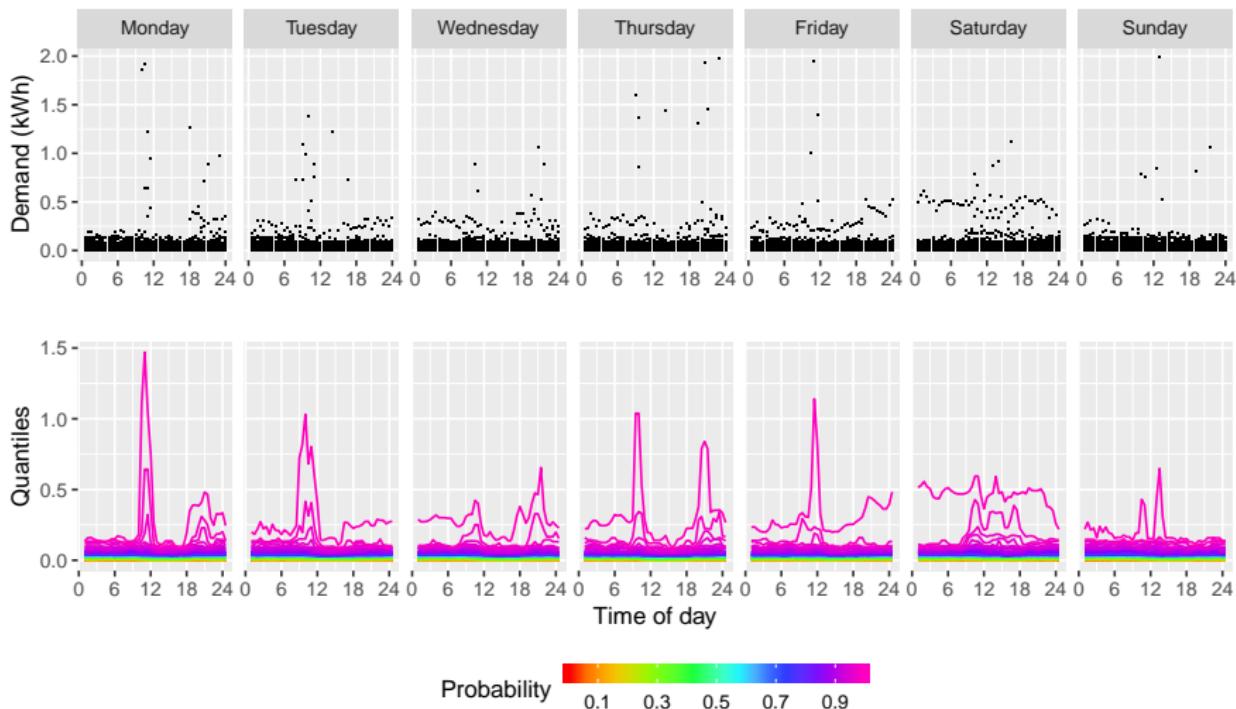
# Anomalous households

Demand for ID: 1881



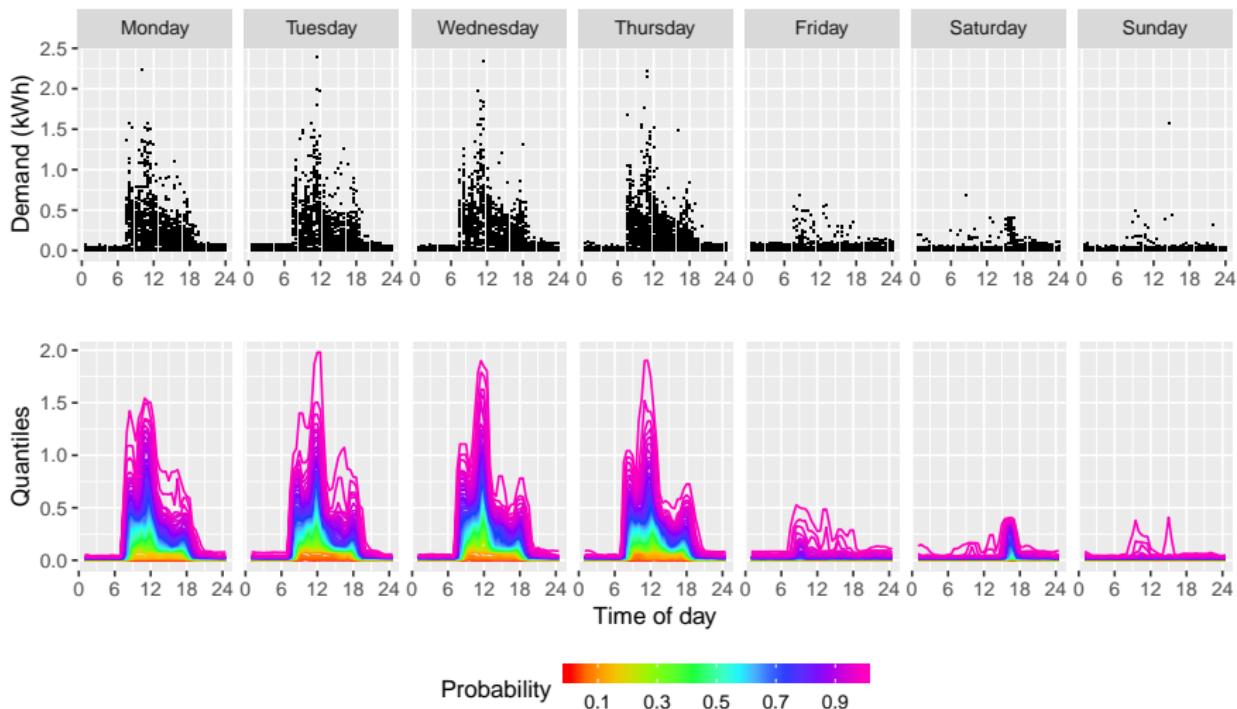
# Anomalous households

Demand for ID: 1607



# Anomalous households

Demand for ID: 1821



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# Laplacian eigenmaps

- **Idea:** Embed conditional densities in a 2d space where the distances are preserved “as far as possible”.

# Laplacian eigenmaps

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- Let  $\mathbf{W} = [w_{ij}]$  where  $w_{ij} = \exp(-\Delta_{ij}^2/h^2)$ .  
$$\mathbf{D} = \text{diag}(\hat{f}_i) \quad \text{where } \hat{f}_i = \sum_{j=1}^n w_{ij}$$
$$\mathbf{L} = \mathbf{D} - \mathbf{W} \quad (\text{the Laplacian matrix}).$$

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- Let  $\mathbf{e}_k$  be eigenvector corresponding to  $k$ th *smallest* eigenvalue.
- Then  $\mathbf{e}_2$  and  $\mathbf{e}_3$  create an embedding of households in 2d space.

## Key property of Laplacian embedding

Let  $y_i = (e_{2,i}, e_{3,i})$  be the embedded point corresponding to household  $i$ .

Then the Laplacian eigenmap minimizes

$$\sum_{ij} w_{ij}(y_i - y_j)^2 = \mathbf{y}' \mathbf{L} \mathbf{y} \quad \text{such that} \quad \mathbf{y}' \mathbf{D} \mathbf{y} = 1.$$

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- First eigenvalue is 0 due to translation invariance.
- Equivalent to optimal embedding using Laplace-Beltrami operator on manifolds.

## Outliers computed in embedded space:

# Outline

- 1 Time series feature spaces
- 2 Irish smart metre data
- 3 Quantiles conditional on time of week
- 4 Finding typical and unusual households
- 5 Visualization via embedding
- 6 Features and limitations

# Features and limitations

## Features of approach

- Converting time series to quantile surfaces conditional on time of week.
- Using pairwise distances between households
- Using kernel matrices for density ranking, embedding and clustering

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## Unresolved issues

- Need to select the bandwidth  $h$  in constructing the similarity matrix.
- Two different uses of bandwidth: density-ranking, embedding. Different bandwidth in each case?
- The use of pairwise distances makes it hard to scale this algorithm.