MONASH University

MONASH BUSINESS SCHOOL

Feature-based time series analysis

Rob J Hyndman

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Outline

M3 competition

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The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon*

INSEAD, Boulevard de Constance, 77305 Fontainebleau, France

Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated.

Keywords: Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy

M3 competition

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etition: results, conclusions

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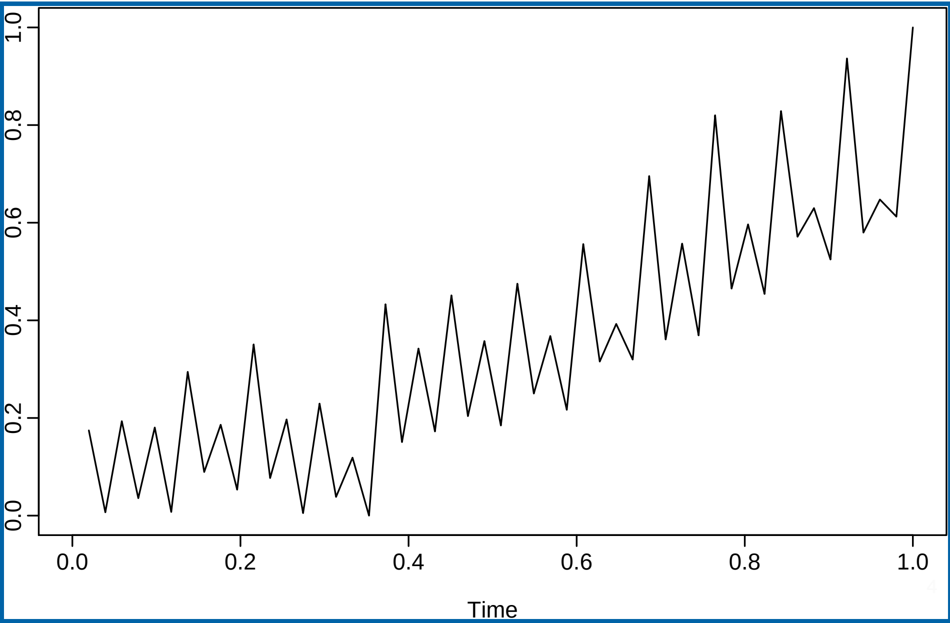

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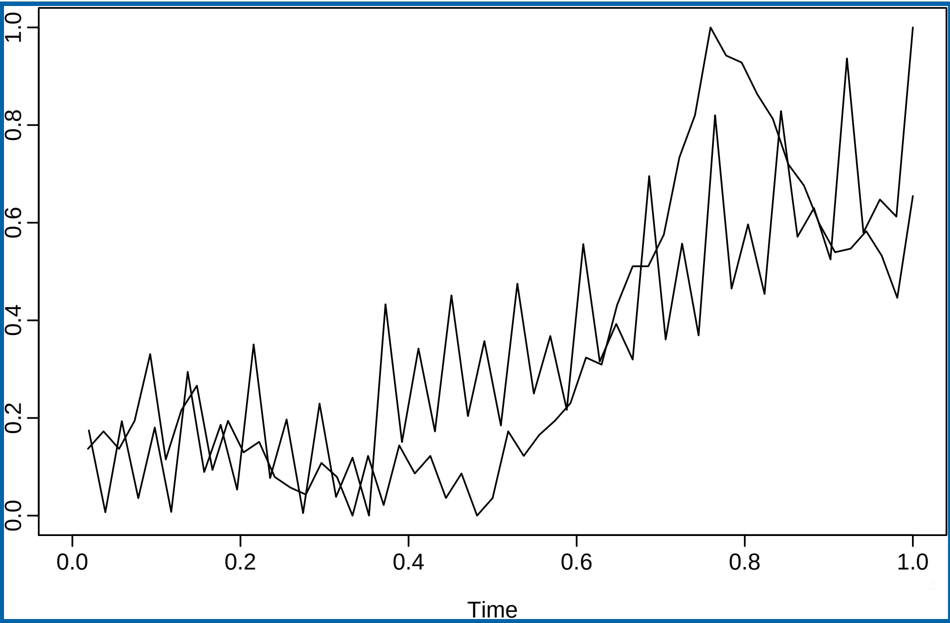

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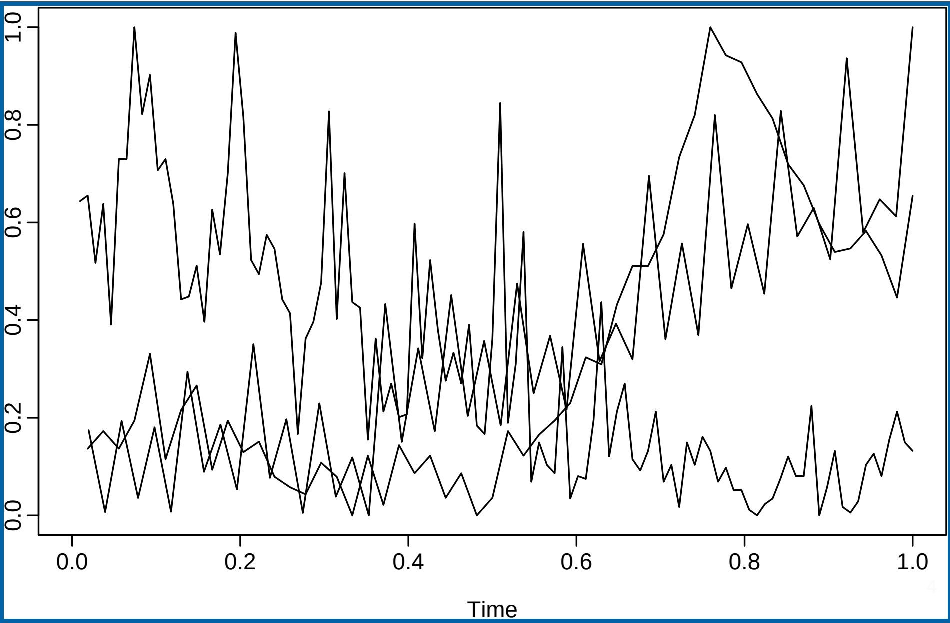

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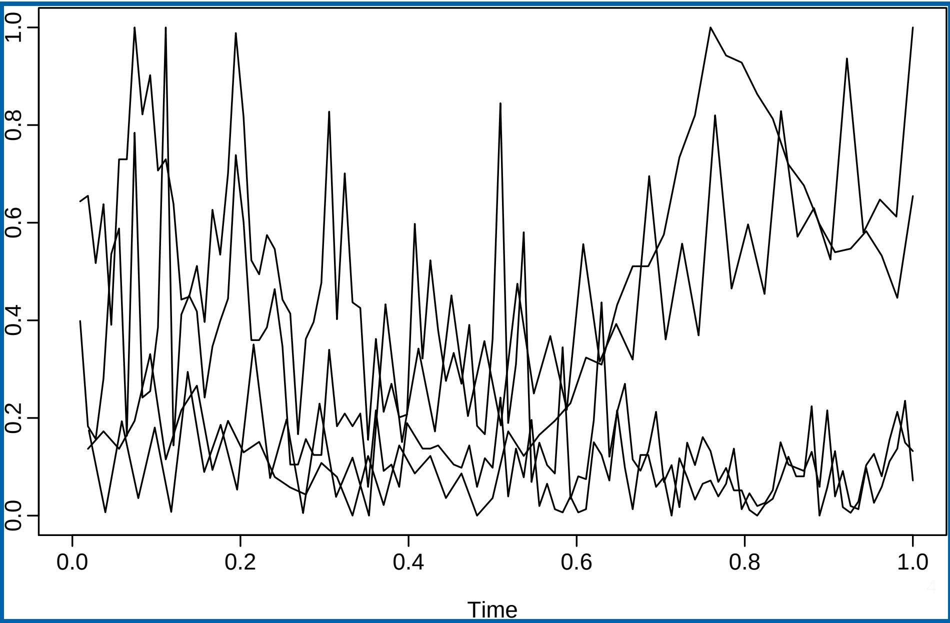

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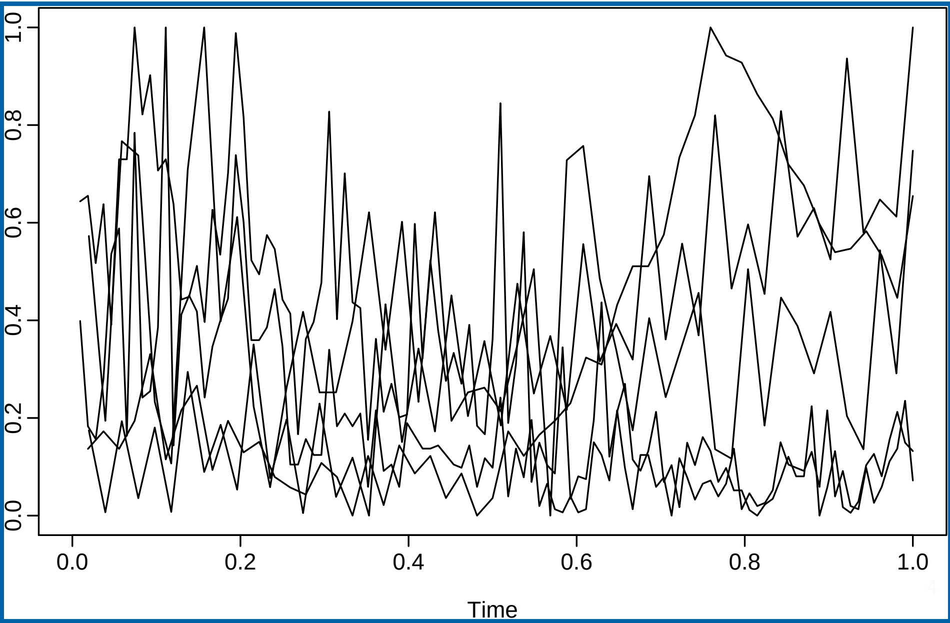

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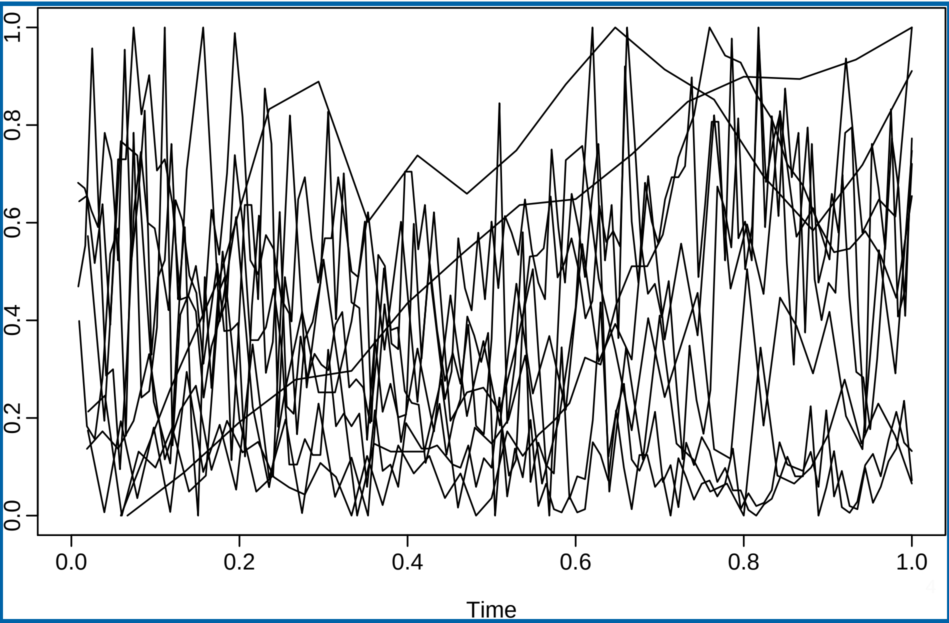

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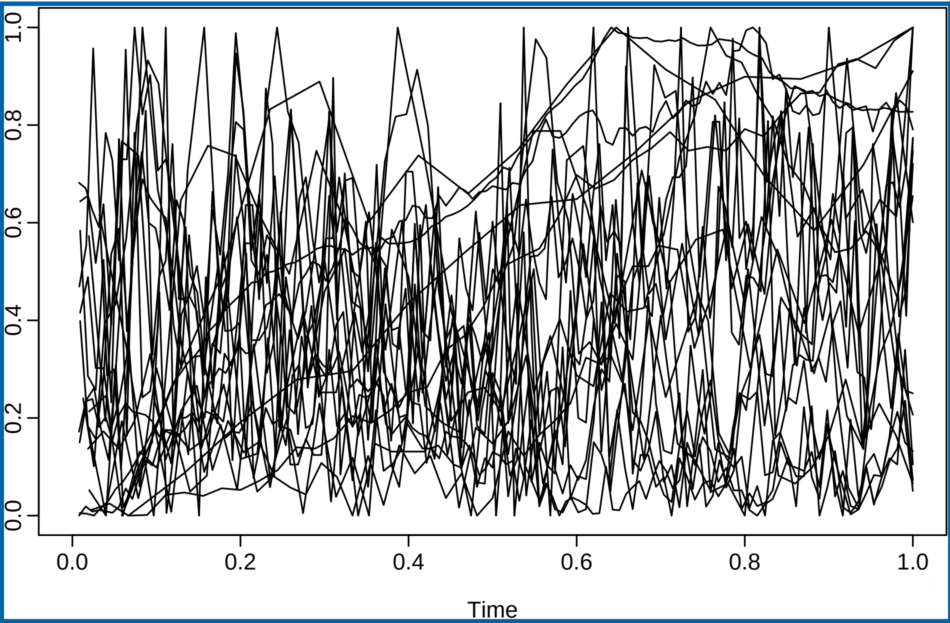

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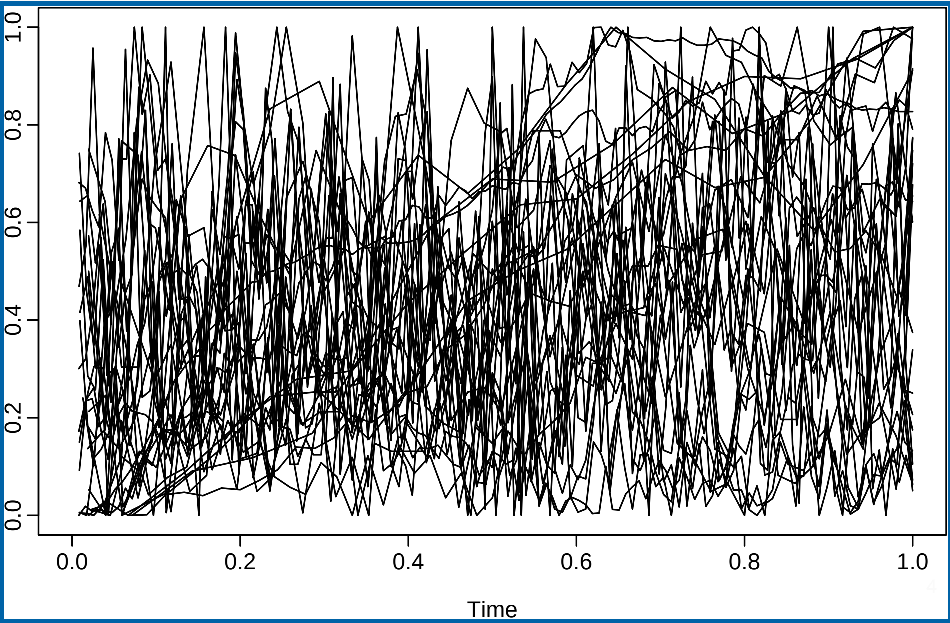

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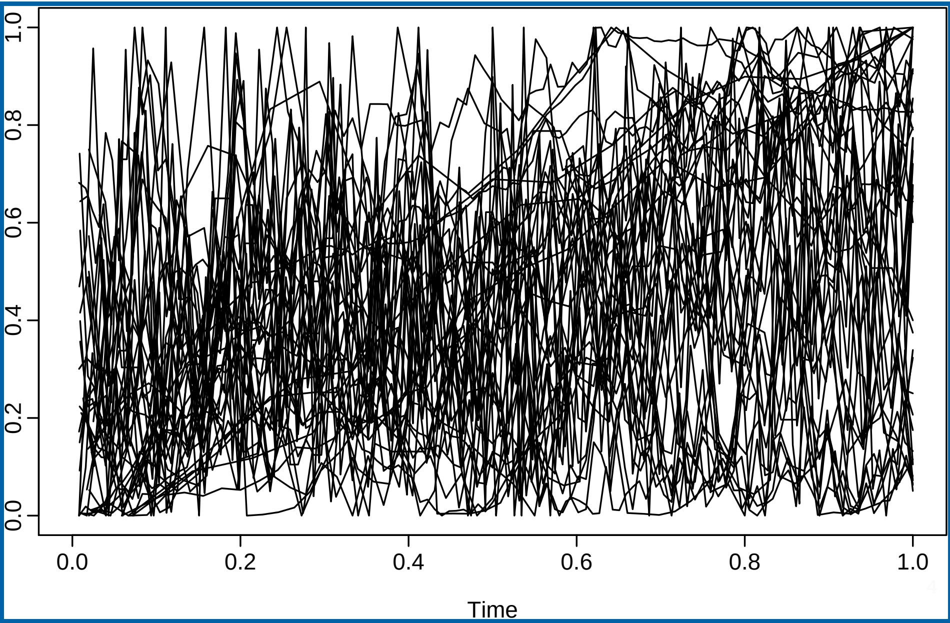

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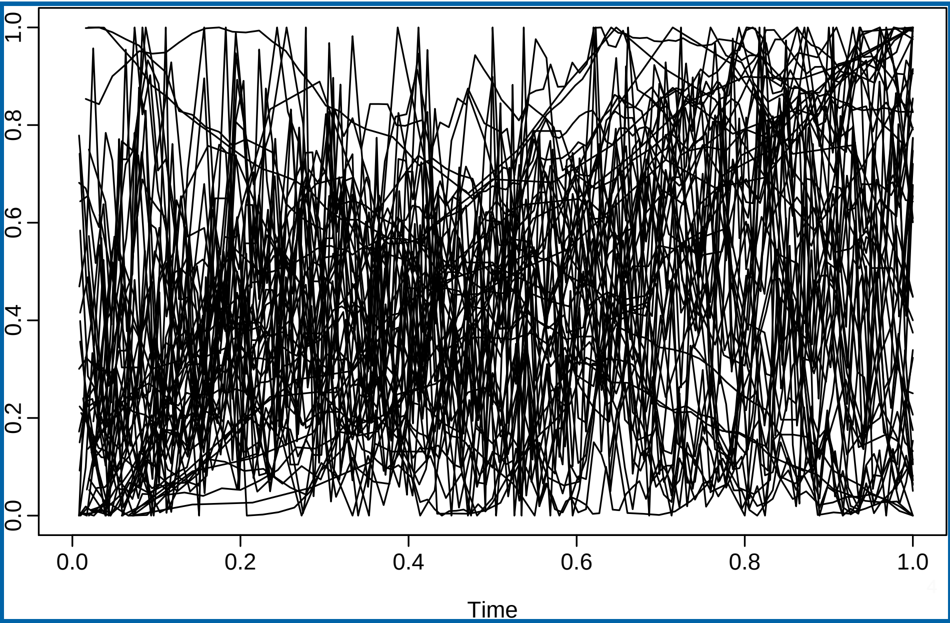

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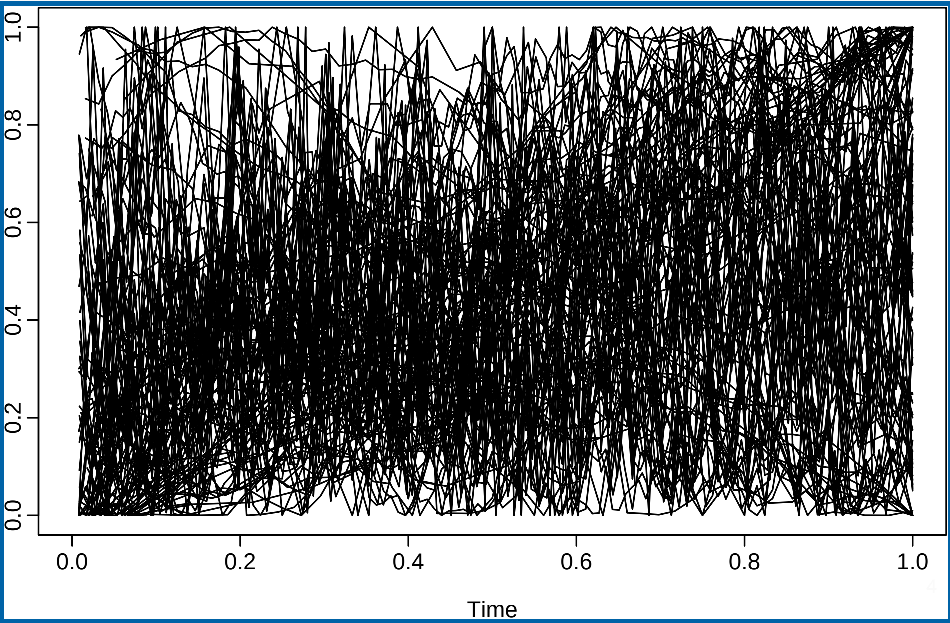

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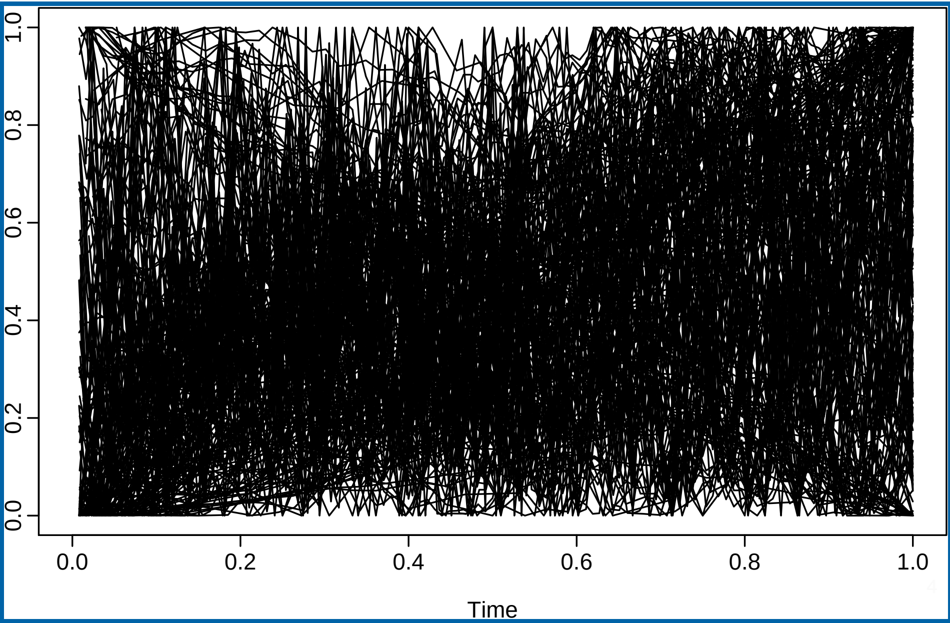

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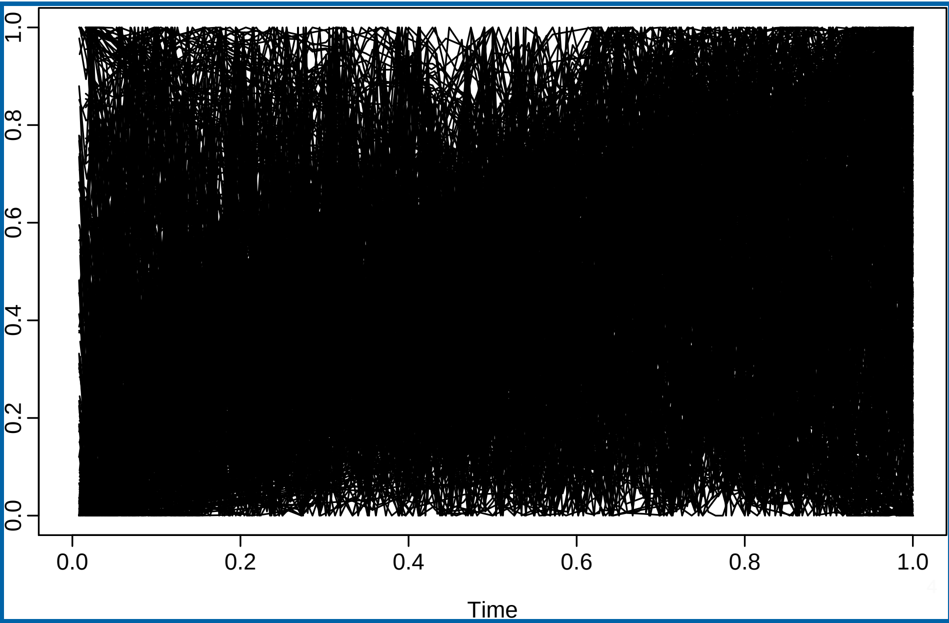

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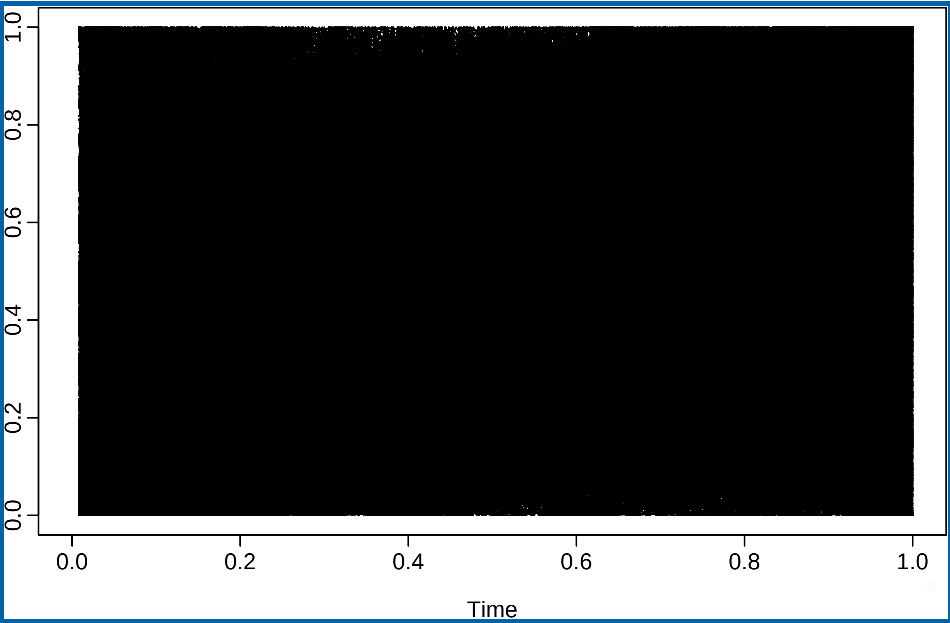
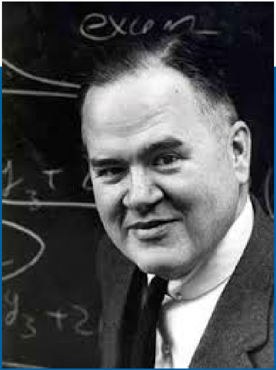


Key idea

Cognostics

Computer-produced diagnostics (Tukey and Tukey, 1985).



John W Tukey

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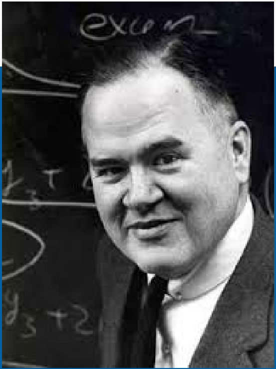


John W Tukey

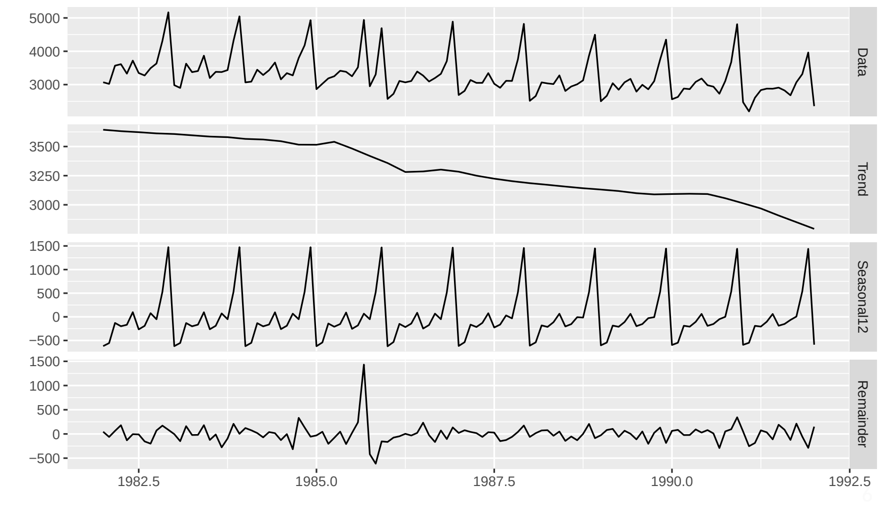
Examples for time series

- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

Called "features" in the machine learning literature.

An STL decomposition: N2096

$Y_t = S_t + T_t + R_t$ $\quad$ S_t is periodic with mean 0



Candidate features

STL decomposition

$$Y_t = S_t + T_t + R_t$$

Candidate features

STL decomposition

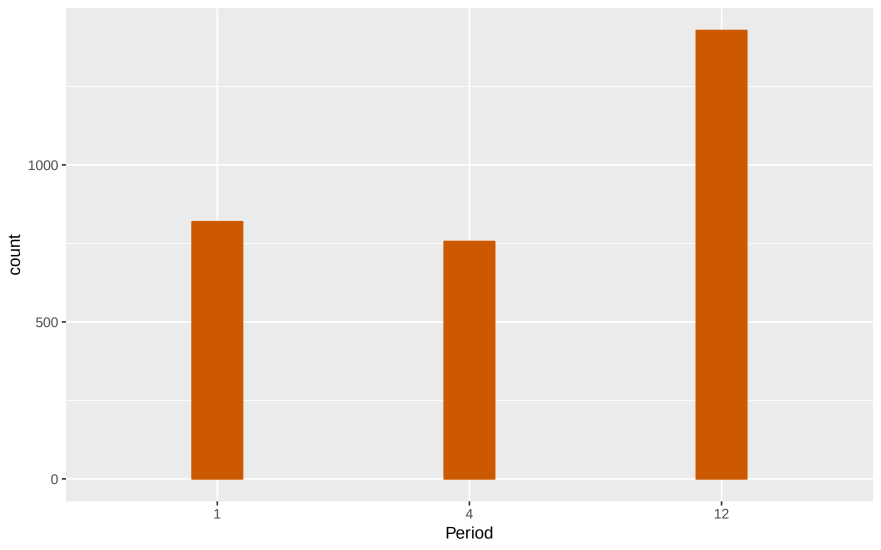
$$Y_t = S_t + T_t + R_t$$

- Seasonal period
- Autocorrelations of data $(Y_1, \dots, Y_T)$
- Autocorrelations of data $(R_1, \dots, R_T)$
- Strength of seasonality: $\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)}\right)$
- Strength of trend: $\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)}\right)$
- Spectral entropy: $H = -\int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$, where $f_y(\lambda)$ is spectral density of Y_t.\newline Low values of H suggest a time series that is easier to forecast (more signal).
- Optimal Box-Cox transformation of data

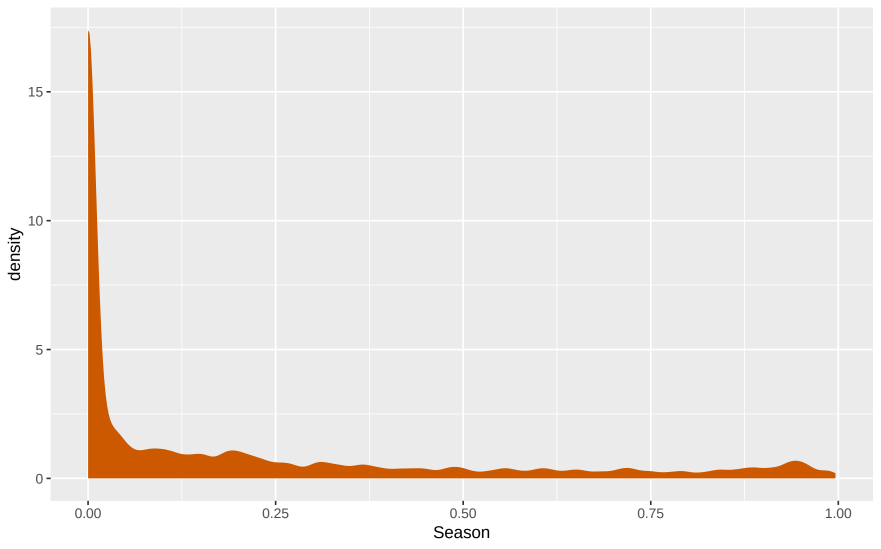
tsfeatures package

```
library(tsfeatures)
lambda_stl <- function(x,...) {
  lambda <- forecast::BoxCox.lambda(x,
    lower=0, upper=1, method='loglik')
  y <- forecast::BoxCox(x, lambda)
  c(stl_features(y,s.window='periodic', robust=TRUE, ...),
    lambda=lambda)
}
M3Features <- bind_cols(
  tsfeatures(M3data, c("frequency", "entropy")),
  tsfeatures(M3data, "lambda_stl", scale=FALSE)) %>%
  select(frequency, entropy, trend, seasonal_strength,
    e_acf1, lambda) %>%
  replace_na(list(seasonal_strength=0)) %>%
  rename(
    Frequency = frequency,
    Entropy = entropy,
    Trend = trend,
    Season = seasonal_strength,
    ACF1 = e_acf1,
    Lambda = lambda) %>%
  mutate(Period = as.factor(Frequency))
```

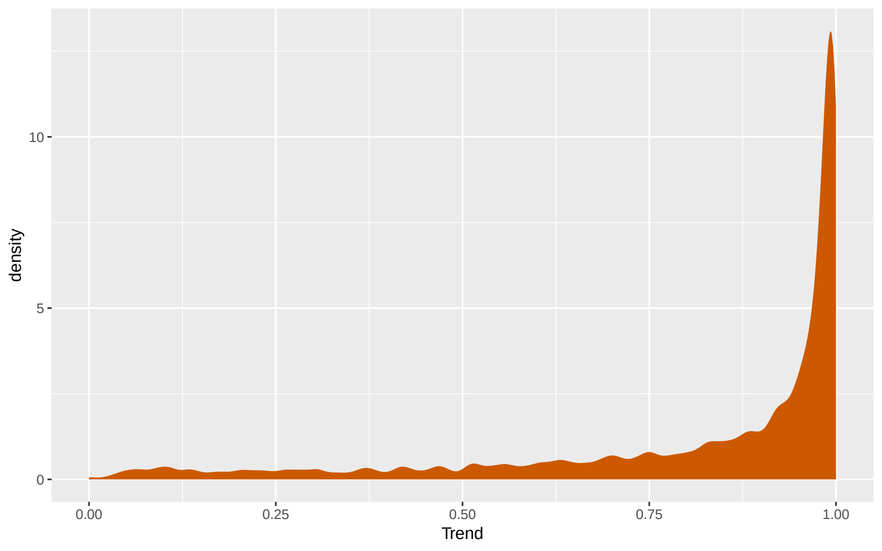
Distribution of Period for M3



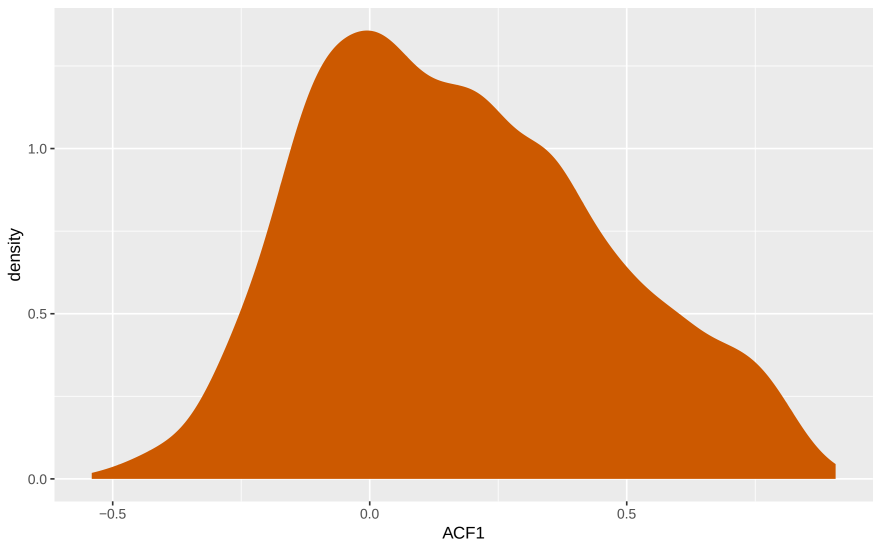
Distribution of Seasonality for M3



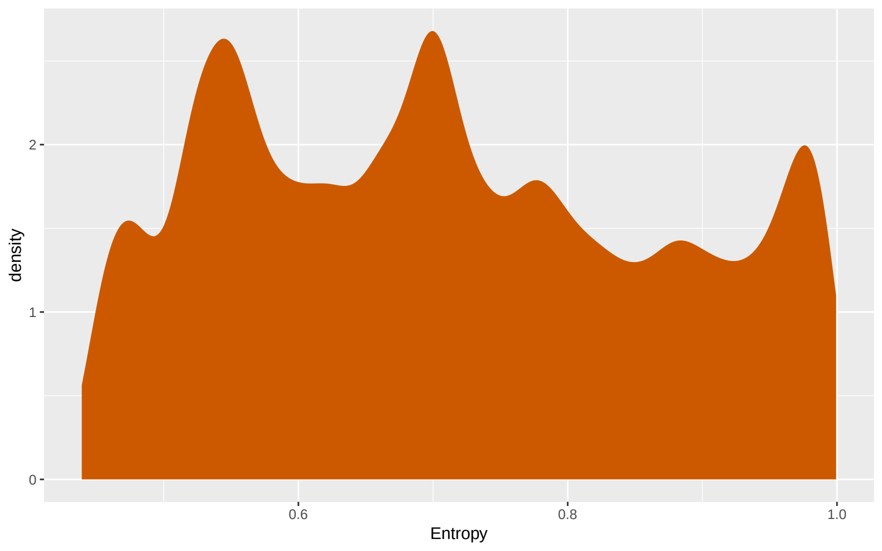
Distribution of Trend for M3



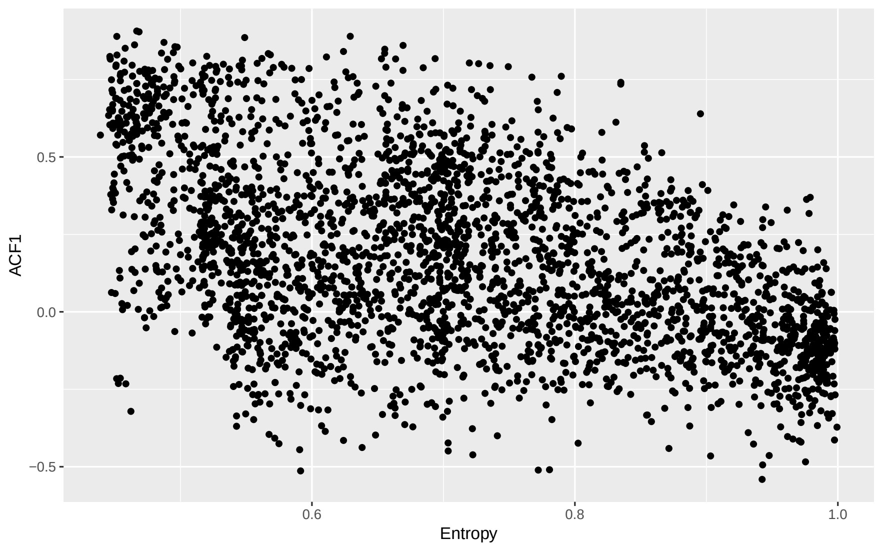
Distribution of Residual ACF1 for M3



Distribution of Spectral Entropy for M3



Feature distributions



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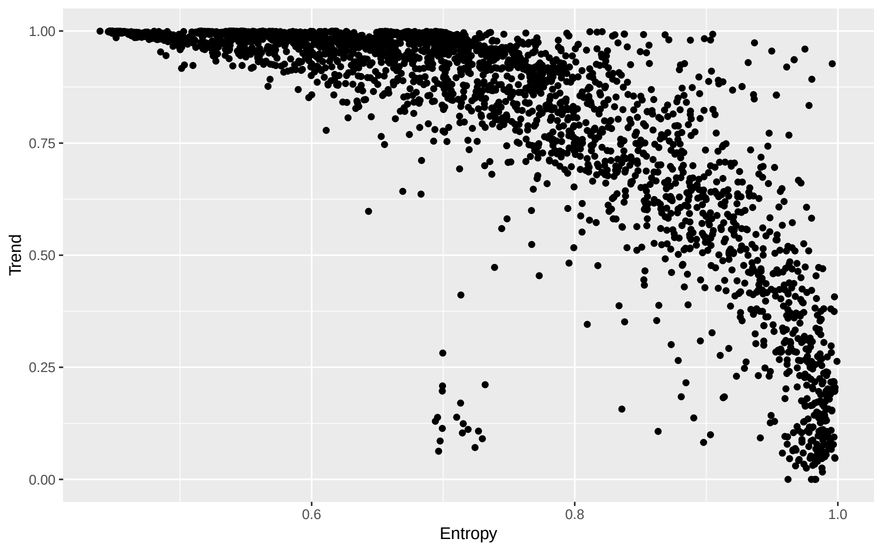

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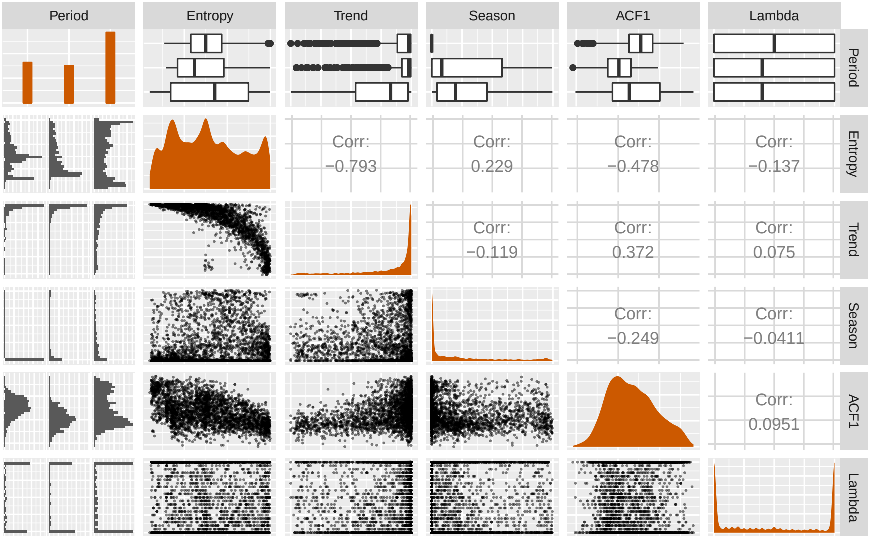

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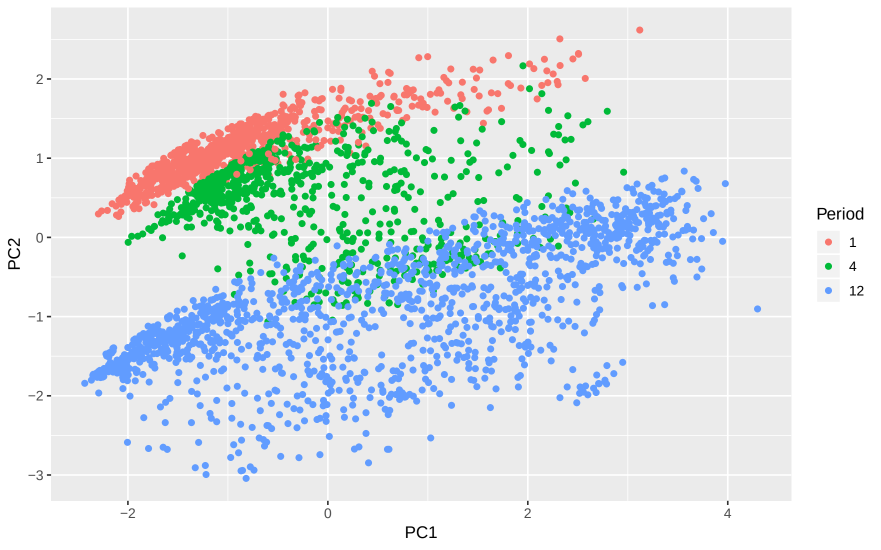
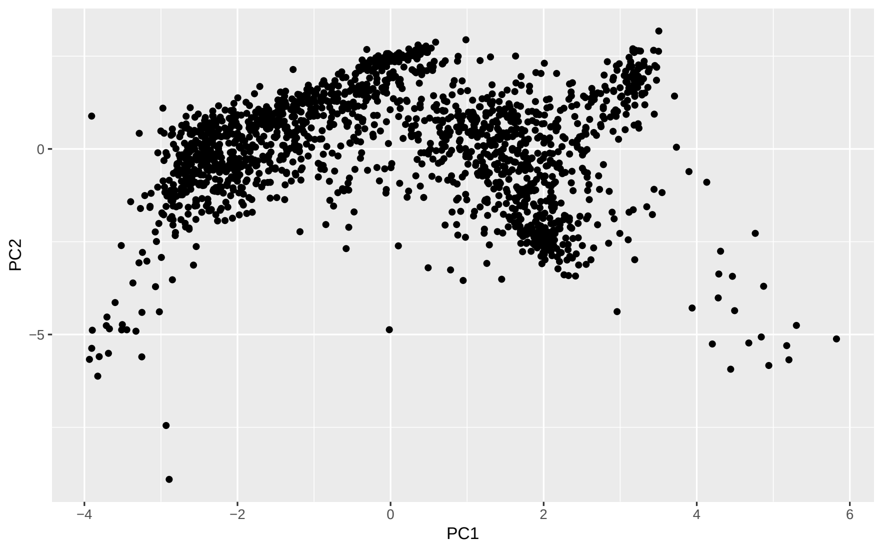


Hyndman, Wang and Laptev (ICDM 2015)



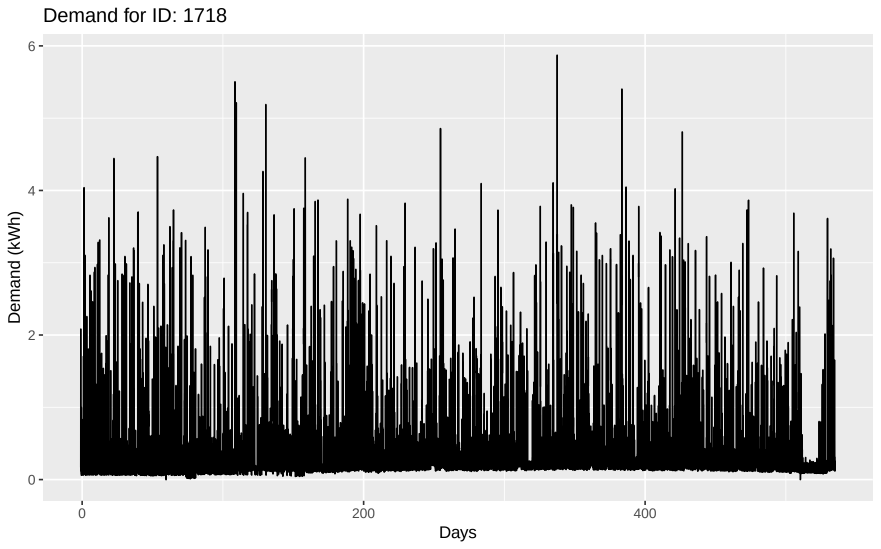
Outline

Irish smart metre data

Figure: http://solutions.3m.com

- 500 households from smart metering trial
- Electricity consumption at 30-minute intervals between 14 July 2009 and 31 December 2010
- Heating/cooling energy usage excluded

Irish smart metre data



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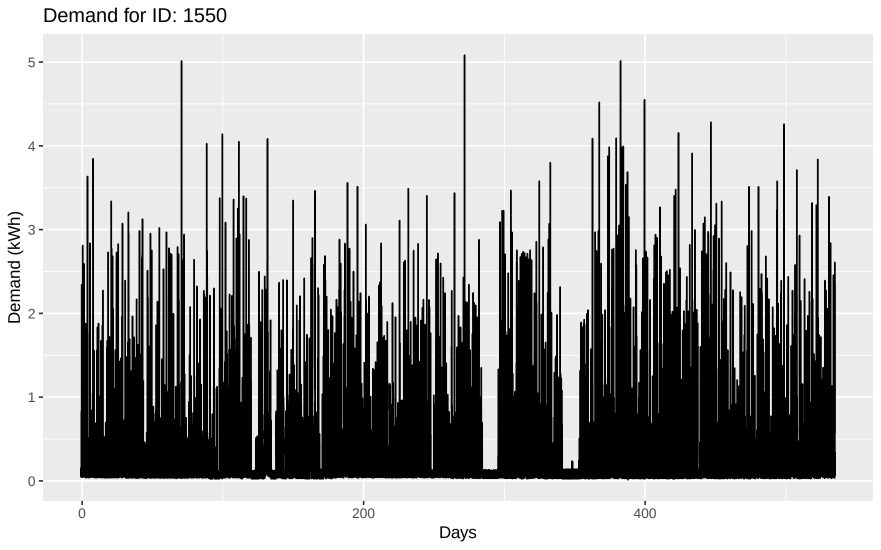

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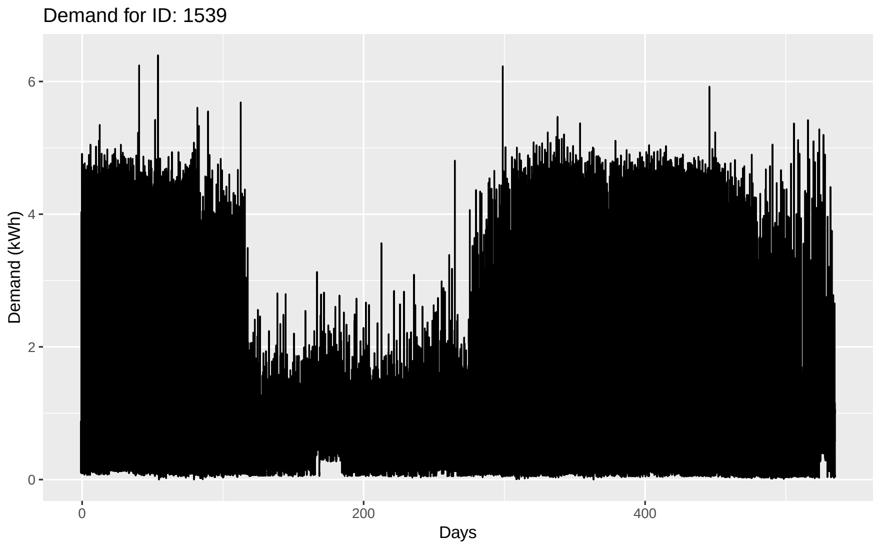
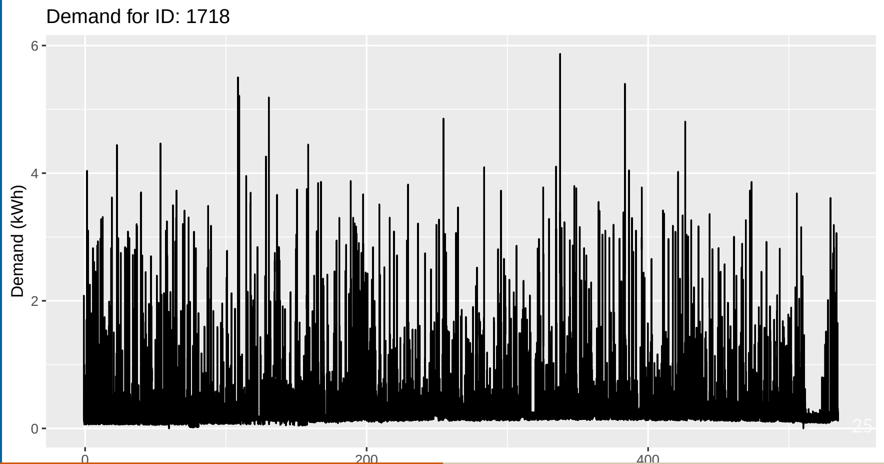


Outline

Quantiles conditional on time of week

- Compute sample quantiles at $p = 0.01, 0.02, \ldots, 0.99$ for each household and each half-hour of the week.
- 336 probability distributions per household.



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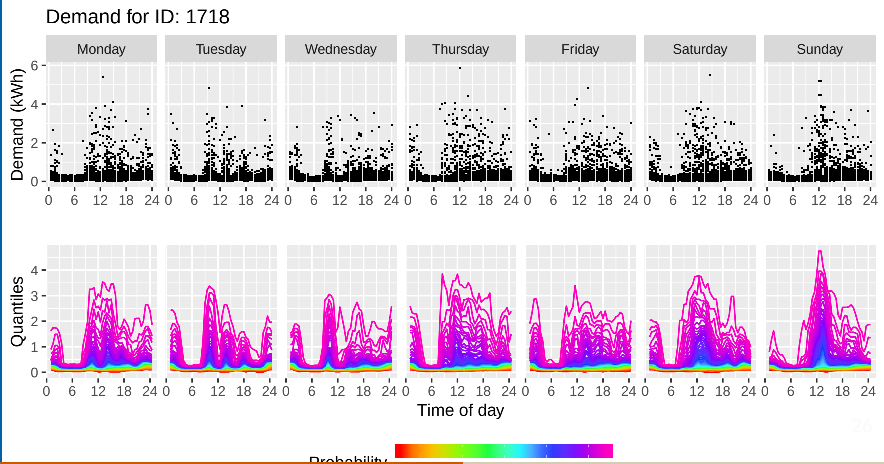

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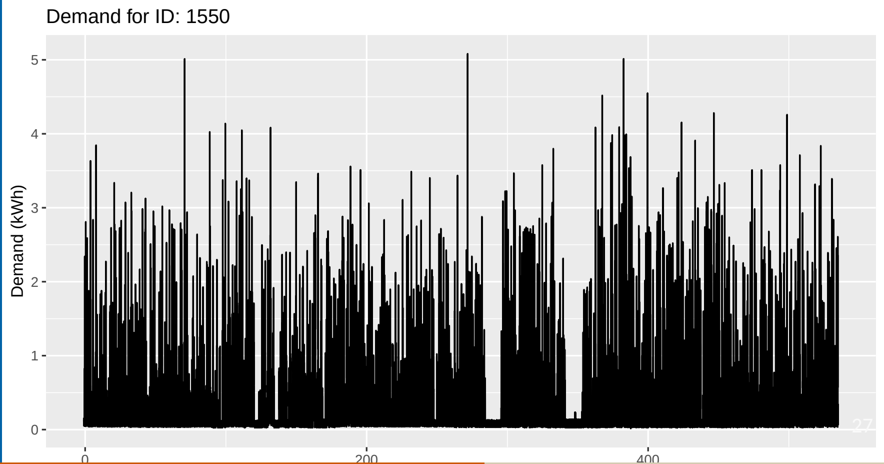

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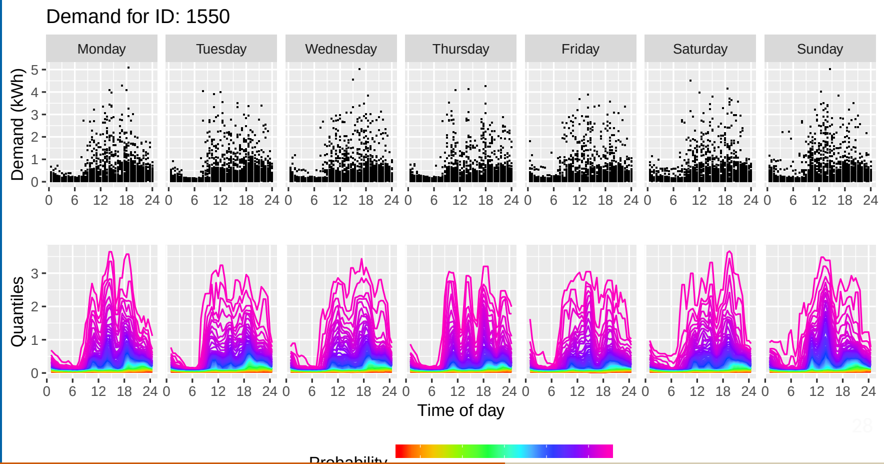

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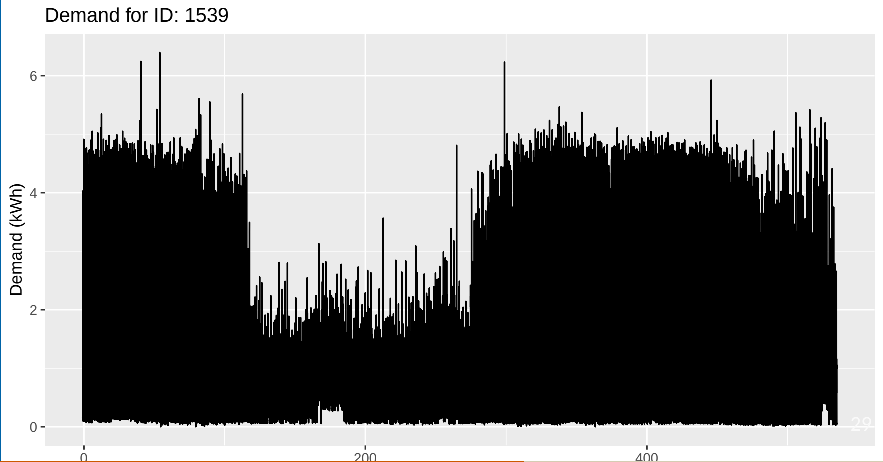

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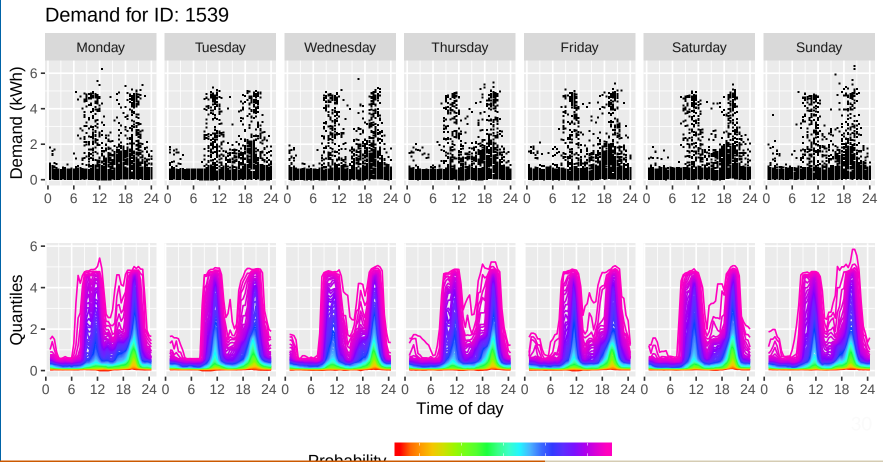

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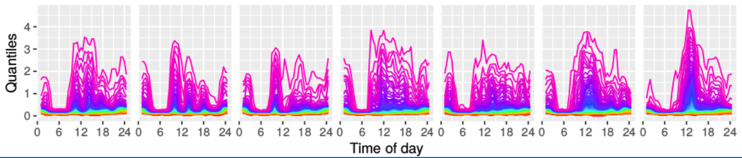

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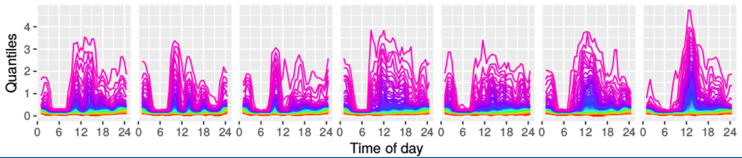

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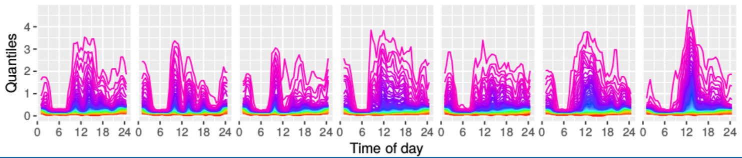

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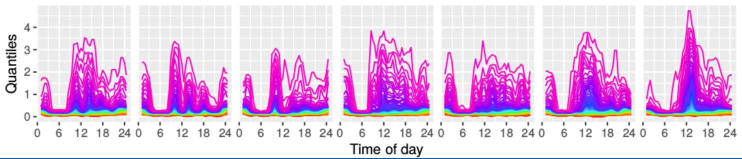

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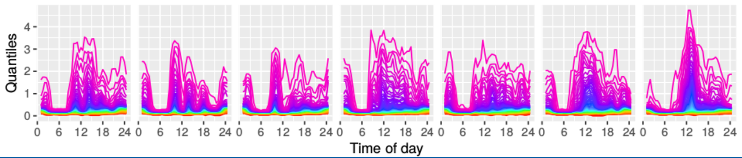

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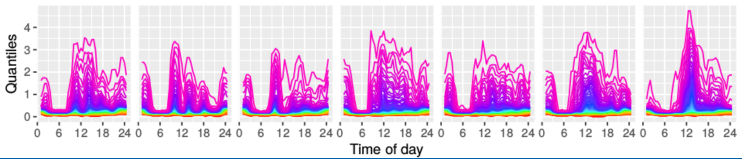

- Sample quantiles better than kernel density estimate:
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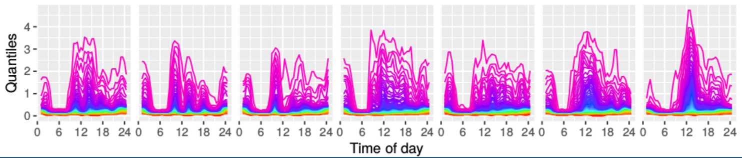

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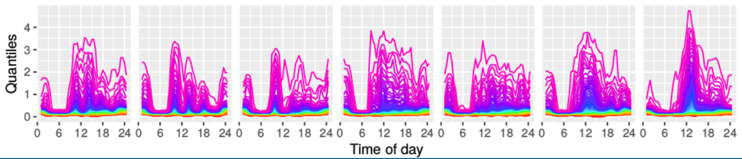

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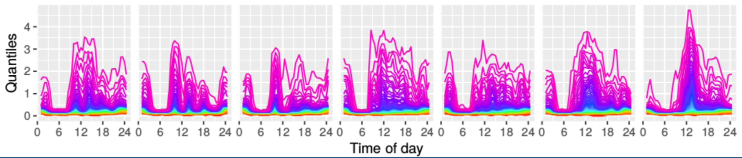

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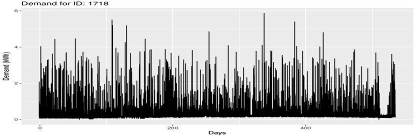
- Allows clustering of households based on probabilistic behaviour rather than coincident behaviour.
- Allows identification of anomalous households.
- Allows estimation of typical household behaviour.

Outline

Pairwise distances



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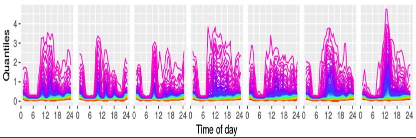
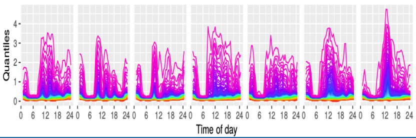
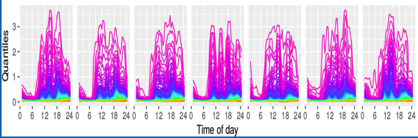


- The time series of 535 $\times$ 48 observations per household is mapped to a set of 7 $\times$ 48 $\times$ 99 quantiles giving a bivariate surface for each household.
- Can we compute pairwise distances between all households?



← ? →
Distance



Jensen-Shannon distances

Kullback-Leibler divergence between two densities

$$D(p, q) = \int_{\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

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Jensen-Shannon distance between two densities

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Distance between two households

$$\Delta_{ij} = \sum_{t=1}^{7\times 48} JS(p_t, q_t)$$

Kernel matrix and density ranking

Similarity between two households

$$w_{ij} = \exp(-\Delta_{ij}^2/h^2).$$

Kernel matrix and density ranking

Similarity between two households

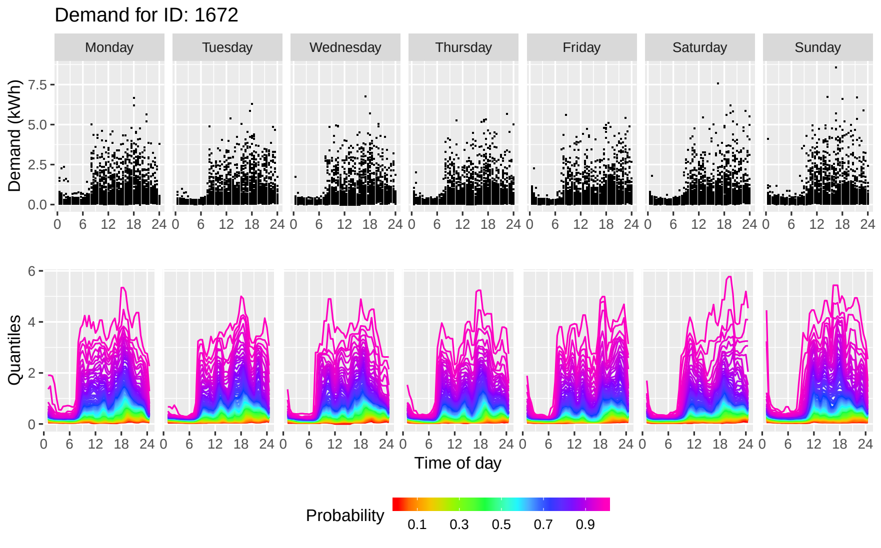
$$w_{ij} = \exp(-\Delta_{ij}^2/h^2).$$

Row sums of the kernel matrix gives a scaled kernel density estimate of households:

$$\hat{f}_i = \sum_{j=1}^{n} w_{ij}$$

- h is bandwidth in Gaussian kernel.
- Households can be ranked by density values.

Typical households



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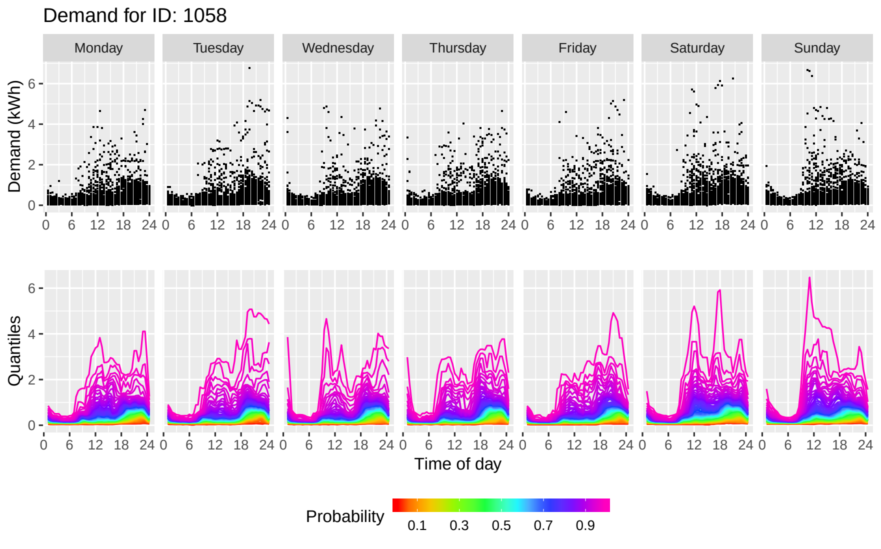

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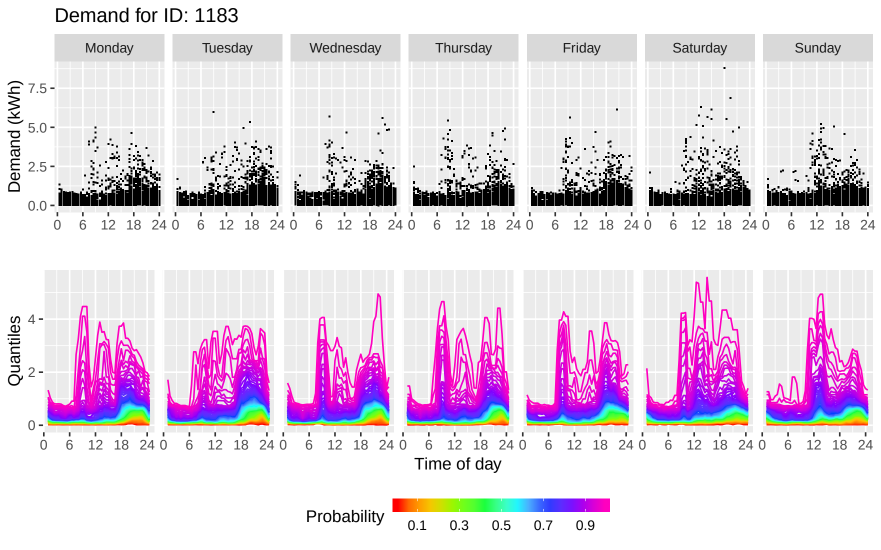
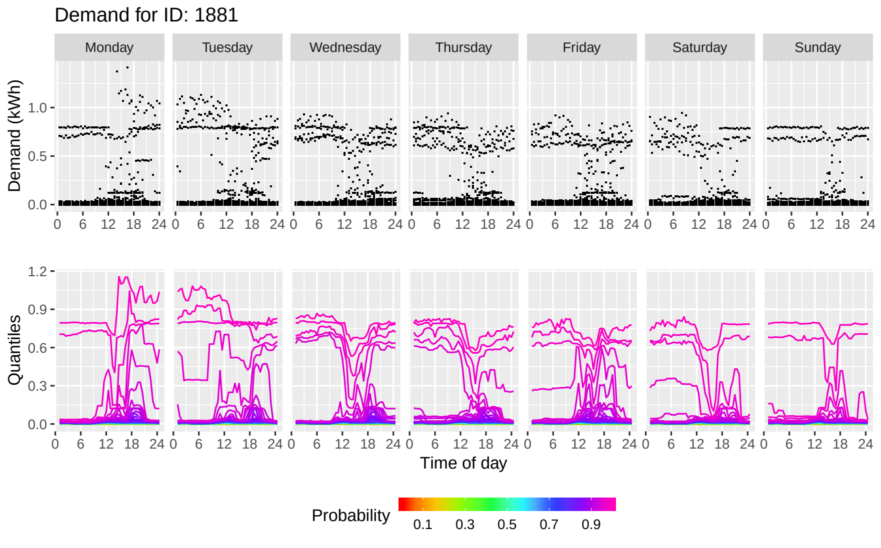


Anomalous households



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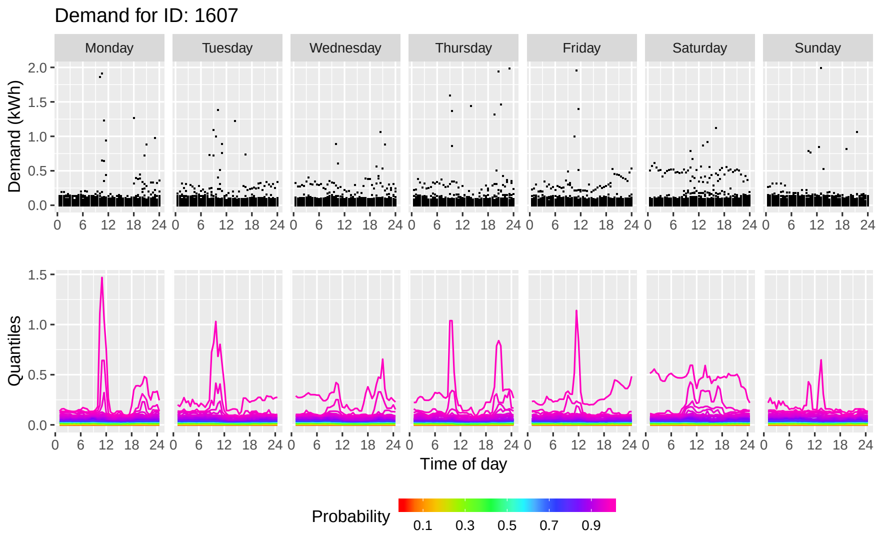

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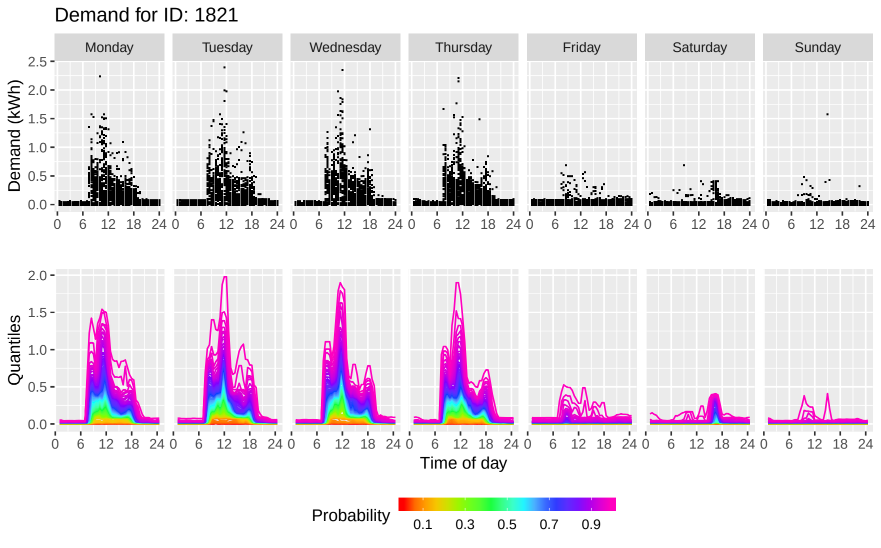

Outline

Laplacian eigenmaps

- **Idea:** Embed conditional densities in a 2d space where the distances are preserved “as far as possible”.

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- Let

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\boldsymbol{W} &= [w_{ij}] && \text{where } w_{ij} = \exp(-\Delta_{ij}^2/h^2). \\
\boldsymbol{D} &= \mathrm{diag}(\hat{f}_i) && \text{where } \hat{f}_i = \sum_{j=1}^{n} w_{ij} \\
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- Let $\boldsymbol{e}_k$ be eigenvector corresponding to kth *smallest* eigenvalue.
- Then $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ create an embedding of households in 2d space.

Key property of Laplacian embedding

Let $y_i = (e_{2,i}, e_{3,i})$ be the embedded point corresponding to household i.

Then the Laplacian eigenmap minimizes

$$\sum_{ij} w_{ij}(y_i - y_j)^2 = \boldsymbol{y}'\boldsymbol{L}\boldsymbol{y} \qquad \text{such that} \quad \boldsymbol{y}'\boldsymbol{D}\boldsymbol{y} = 1.$$

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- First eigenvalue is 0 due to translation invariance.
- Equivalent to optimal embedding using Laplace-Beltrami operator on manifolds.

Outliers computed in embedded space:

Outline

Features and limitations

Features of approach

- Converting time series to quantile surfaces conditional on time of week.
- Using pairwise distances between households
- Using kernel matrices for density ranking, embedding and clustering

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Unresolved issues

- Need to select the bandwidth h in constructing the similarity matrix.
- Two different uses of bandwidth: density-ranking, embedding. Different bandwidth in each case?
- The use of pairwise distances makes it hard to scale this algorithm.