Hierarchical Time Series Forecasting in Emergency Medical Services

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Outline

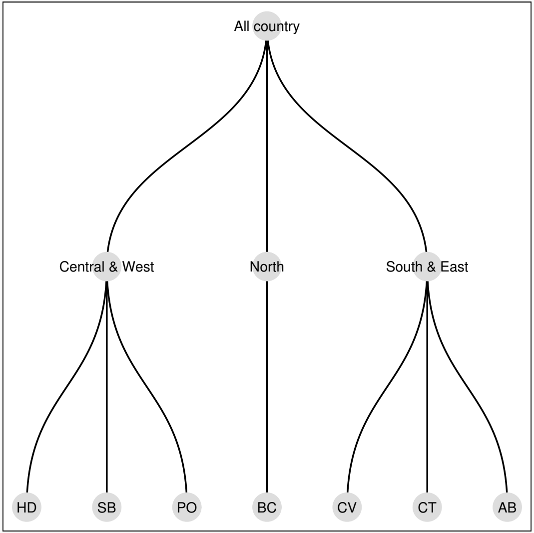
1. Data
2. Forecast methods and evaluation
3. Results

Outline

Data

- Daily number of attended incidents:
  1 October 2015 – 31 July 2019
- Disaggregated by nature of incidents, priority, the health board managing the service and the control area (or region).
- 2,142,000 rows observations from 1,530 time series.

Data structure



*

Priority
Red
Amber
Green

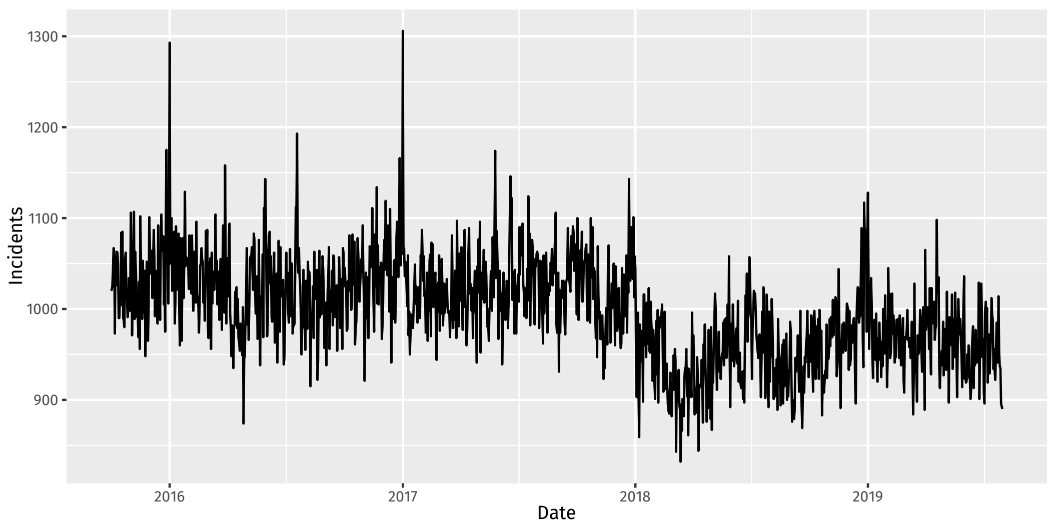
*

Nature of incident
Chest pain
Stroke
Breathing problem
...
Abdominal pain

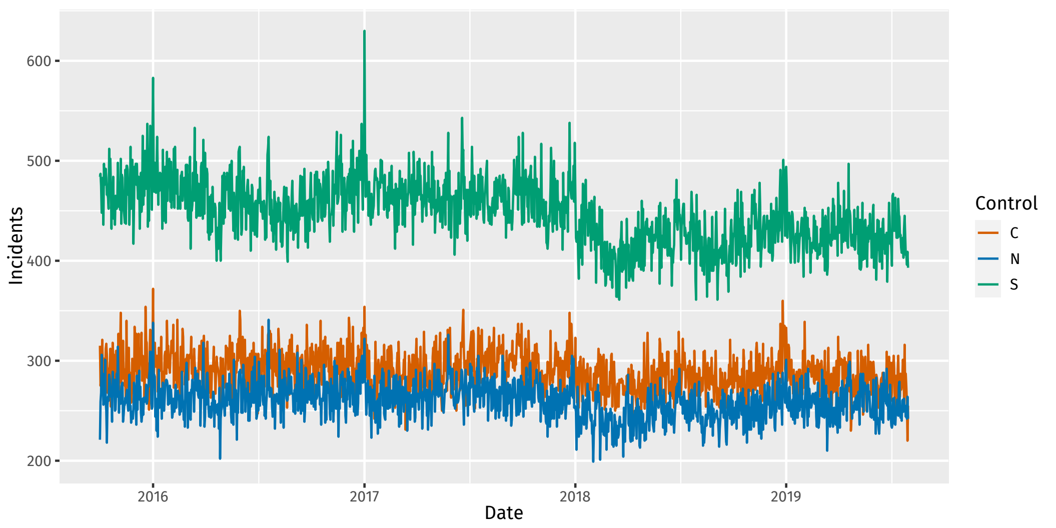
Data structure

Level	Number of series
All country	1
Control	3
Health board	7
Priority	3
Priority * Control	9
Priority * Health board	21
Nature of incident	35
Nature of incident * Control	105
Nature of incident * Health board	245
Priority * Nature of incident	104
Control * Priority * Nature of incident	306
Control * Health board * Priority * Nature of incident (Bottom level)	691
Total	1530

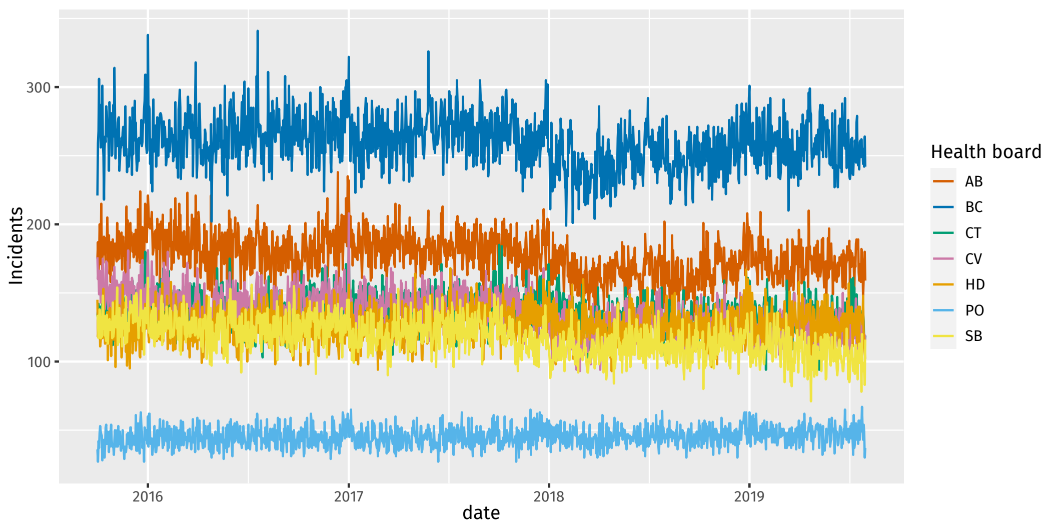
Aggregated daily incidents



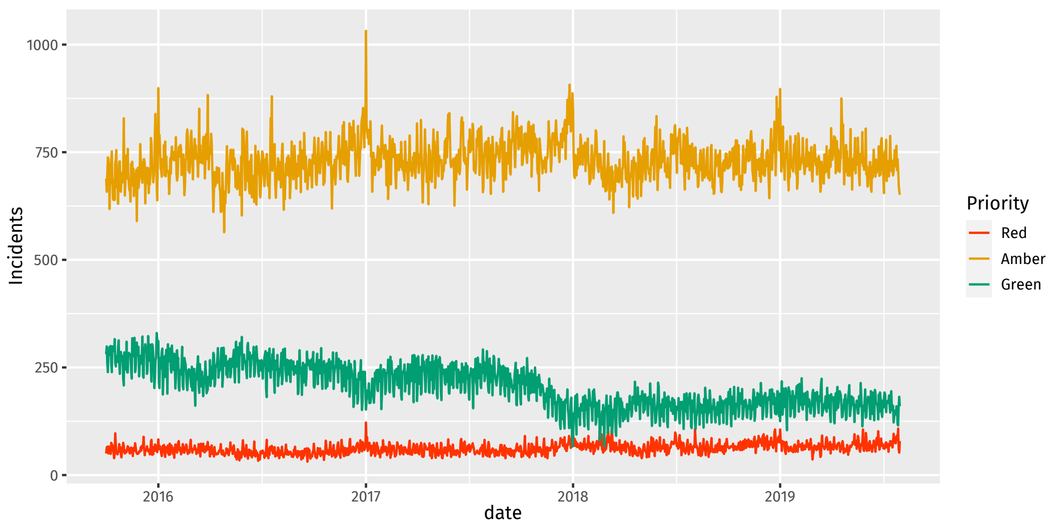
Daily incidents by control area



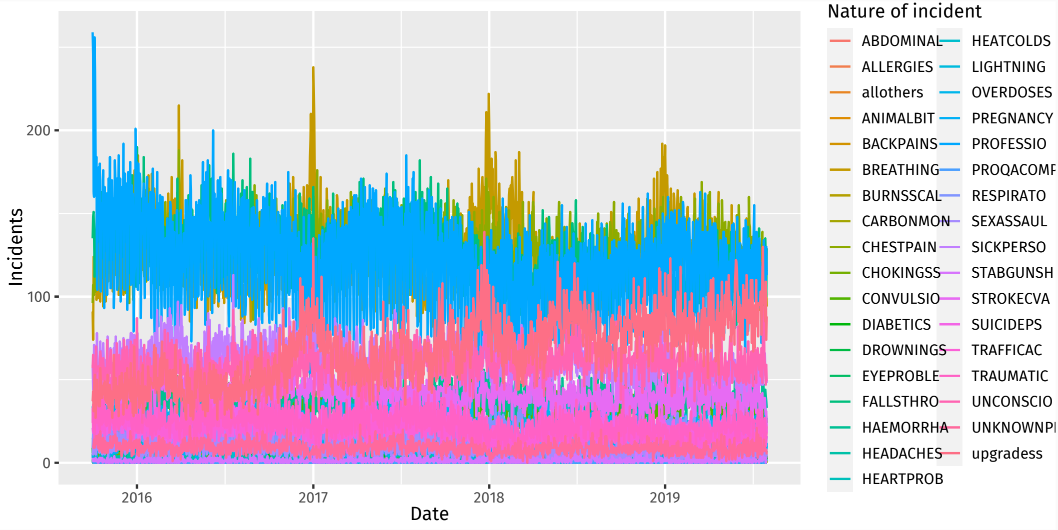
Data incidents by health board



Data incidents by priority



Data incidents by nature of incident



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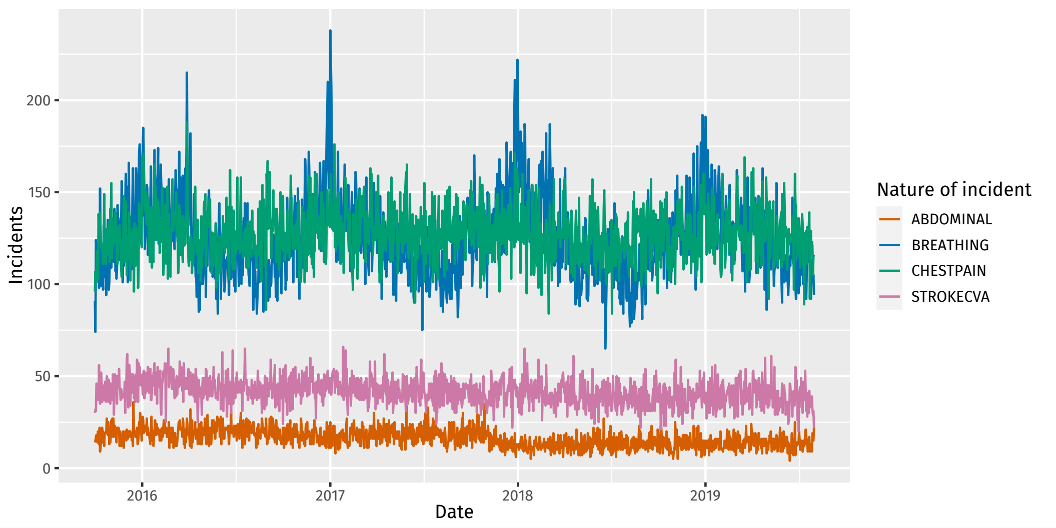
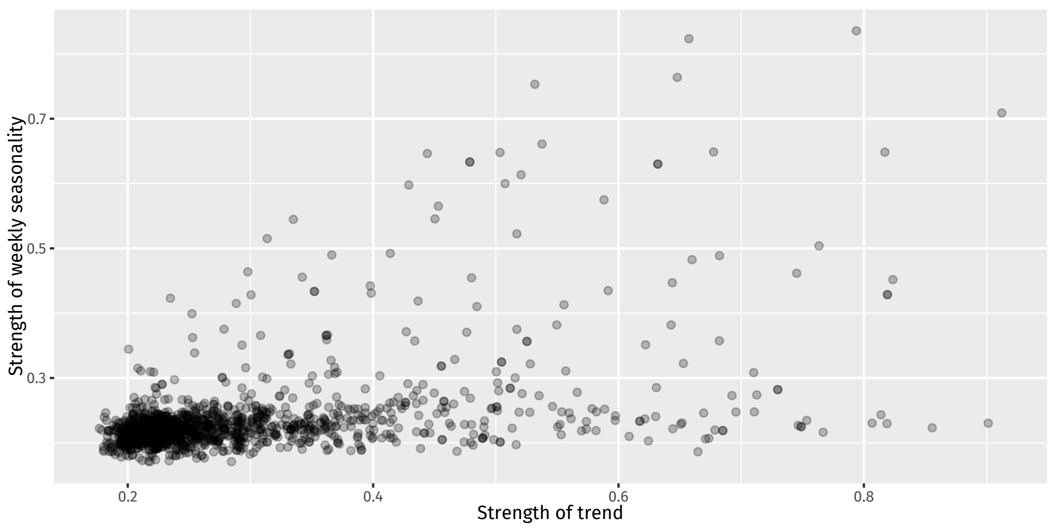


Data features

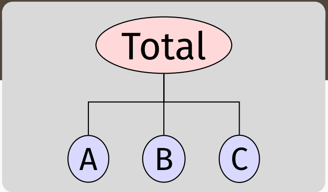


Outline

Forecasting methods

1. **Naïve**: Empirical distribution of past daily attended incidents.
2. **ETS**: Exponential Smoothing State Space models.
3. **GLM**: Poission Regression with spline trend, day of the week, annual Fourier seasonality, public holidays, school holidays, Christmas Day, New Year’s Day.
4. **TSGLM**: Poisson Regression with same covariates plus three autoregressive terms.
5. **Ensemble**: Mixture distribution of 1–4.

Notation reminder



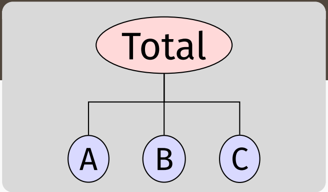
Every collection of time series with linear constraints can be written as

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

- $\boldsymbol{y}_t$ = vector of all series at time t
- $y_{\text{Total},t}$ = aggregate of all series at time t.
- $y_{X,t}$ = value of series X at time t.
- $\boldsymbol{b}_t$ = vector of most disaggregated series at time t
- $\boldsymbol{S}$ = "summing matrix" containing the linear constraints.

$$\boldsymbol{y}_t = \begin{pmatrix} y_{\text{Total},t} \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{S}} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\boldsymbol{b}_t}$$

Notation reminder



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- $\boldsymbol{S}$ = "summing matrix" containing the linear constraints.

- Base forecasts: $\hat{\boldsymbol{y}}_{T+h|T}$
- Reconciled forecasts: $\tilde{\boldsymbol{y}}_{T+h|T} = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_{T+h|T}$
- MinT: $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$ where $\boldsymbol{W}_h$ is covariance matrix of base forecast errors.

Nonparametric bootstrap reconciliation

Performance evaluation

- Time series cross-validation: forecast horizon of 1–84 days.
- Forecasts at 43–84 days correspond to planning horizon.

1. Training set: all data to 2018-04-25.
2. Training set: all data to 2018-06-06.

⋮

10. Training set: all data to 2019-05-09.

Performance evaluation

$$\text{MASE} = \text{mean}(|q_j|)$$

where

$$q_j = \frac{e_j}{\frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|},$$

- e_j = forecast error for forecast horizon j,
- $m = 7$
- y_t = observation for period t
- T = size of training set

Performance evaluation

$$\text{MSSE} = \text{mean}(q_j^2)$$

where

$$q_j^2 = \frac{e_j^2}{\displaystyle\frac{1}{T-m}\sum_{t=m+1}^{T}(y_t - y_{t-m})^2},$$

- e_j = forecast error for forecast horizon j,
- $m = 7$
- y_t = observation for period t
- T = size of training set

Performance evaluation

$$\text{CRPS} = \text{mean}(p_j)$$

where

$$p_j = \int_{-\infty}^{\infty} \left(G_j(x) - F_j(x)\right)^2 dx,$$

- $G_j(x)$ = forecasted probability distribution function for forecast horizon j
- $F_j(x)$ = true probability distribution function for the same period.

Outline

Results

Total
Control areas
Health boards
MASE
MSSE
CRPS
Average accuracy
Week ahead
1.3
1.1
0.9
0.7
1.5
1.2
0.9
0.6
0.3
1.0
0.8
0.6
0.4
1 2 3 4 5 6 7 8 9 10 11 12
model
Naïve
ETS
TSGLM
GLM
Ensemble

Results

		MSSE			
Method	Model	Total	Control areas	Health boards	Bottom
Base	Naïve	1.169	1.056	1.062	1.031
Base	ETS	0.979	0.875	0.816	**0.975**
Base	GLM	0.813	0.897	0.875	1.009
Base	TSGLM	0.822	0.901	0.875	1.050
Base	Ensemble	0.599	0.729	0.774	0.993
MinT	Naïve	1.168	1.057	1.062	2.095
MinT	ETS	0.785	0.852	0.845	0.994
MinT	GLM	0.720	0.827	0.837	1.803
MinT	TSGLM	0.722	0.833	0.839	1.851
MinT	Ensemble	**0.560**	**0.706**	**0.765**	1.557

Results

		MASE			
Method	Model	Total	Control areas	Health boards	Bottom
Base	Naïve	1.139	1.059	1.047	1.019
Base	ETS	0.963	0.930	0.899	1.038
Base	GLM	0.910	0.940	0.923	**1.002**
Base	TSGLM	0.911	0.939	0.924	1.005
Base	Ensemble	0.782	0.856	0.876	1.008
MinT	Naïve	1.138	1.059	1.047	2.651
MinT	ETS	0.877	0.916	0.915	1.289
MinT	GLM	0.848	0.901	0.902	2.493
MinT	TSGLM	0.852	0.903	0.903	2.513
MinT	Ensemble	**0.753**	**0.844**	**0.872**	2.260

Results

		CRPS			
Method	Model	Total	Control areas	Health boards	Bottom
Base	Naïve	30.387	10.882	5.500	0.302
Base	ETS	14.309	6.074	3.476	0.244
Base	GLM	15.396	6.253	3.576	0.244
Base	TSGLM	15.316	6.227	3.575	0.245
Base	Ensemble	12.978	**5.727**	3.430	0.243
MinT	Naïve	30.368	10.902	5.498	0.313
MinT	ETS	13.515	5.967	3.547	**0.243**
MinT	GLM	13.839	5.917	3.453	0.246
MinT	TSGLM	14.000	5.947	3.455	0.248
MinT	Ensemble	**12.585**	5.728	**3.426**	0.247

Conclusions