

# Feature-based forecasting algorithms for large collections of time series

Rob J Hyndman

25 January 2019

# Outline

- 1 Time series features
- 2 FFORMS: Feature-based forecast model selection
- 3 FFORMA: Feature-based forecast model averaging

# Outline

- 1 Time series features
- 2 FFORMS: Feature-based forecast model selection
- 3 FFORMA: Feature-based forecast model averaging

# M3 competition



ELSEVIER

International Journal of Forecasting 16 (2000) 451–476

[www.elsevier.com/locate/ijforecast](http://www.elsevier.com/locate/ijforecast)

*international journal  
of forecasting*

## The M3-Competition: results, conclusions and implications

Spyros Makridakis, Michèle Hibon\*

*INSEAD, Boulevard de Constance, 77305 Fontainebleau, France*

---

### Abstract

This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy 4

# M3 competition



ELSEVIER

International Journal of Forecasting 16 (2000) 451–476

*international journal  
of forecasting*

[www.elsevier.com/locate/ijforecast](http://www.elsevier.com/locate/ijforecast)



petition: results, conclusions a

Spyros Makridakis, Michèle Hibon\*

EAD, Boulevard de Constance, 77305 Fontainebleau, Fr

Abst



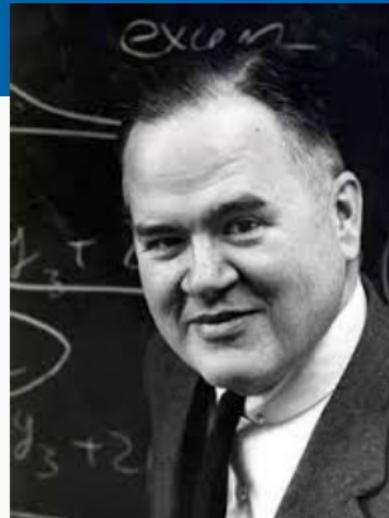
This paper describes the M3-Competition, the latest of the M-Competitions. It explains the reasons for conducting the competition and summarizes its results and conclusions. In addition, the paper compares such results/conclusions with those of the previous two M-Competitions as well as with those of other major empirical studies. Finally, the implications of these results and conclusions are considered, their consequences for both the theory and practice of forecasting are explored and directions for future research are contemplated. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Comparative methods — time series: univariate; Forecasting competitions; M-Competition; Forecasting methods, Forecasting accuracy 4

# Key idea

## Cognostics

Computer-produced diagnostics  
(Tukey and Tukey, 1985).

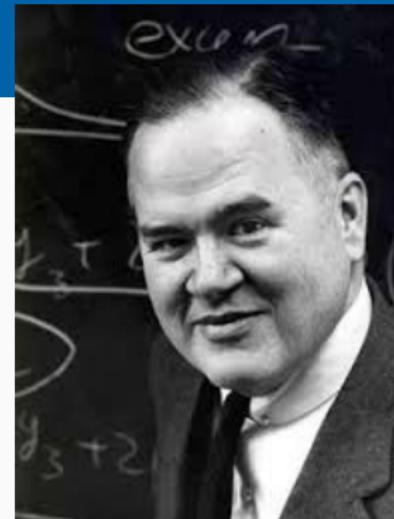


*John W Tukey*

# Key idea

## Cognostics

Computer-produced diagnostics  
(Tukey and Tukey, 1985).



John W Tukey

## Examples for time series

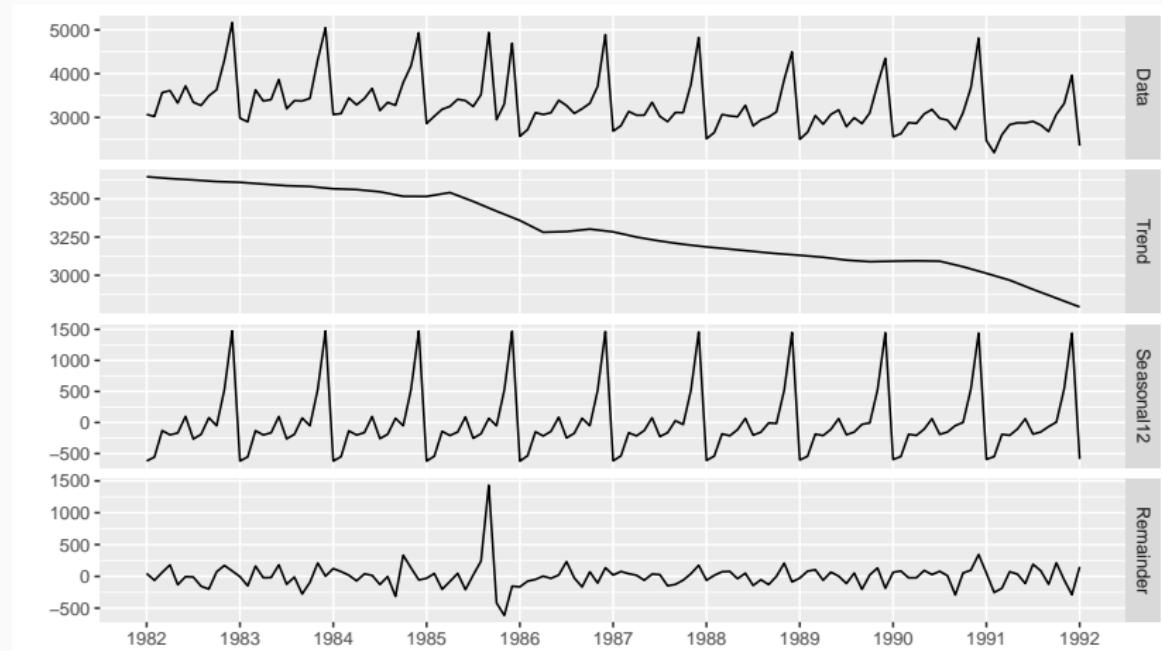
- lag correlation
- size and direction of trend
- strength of seasonality
- timing of peak seasonality
- spectral entropy

Called “features” in the machine learning literature.

# An STL decomposition: N2096

$$Y_t = S_t + T_t + R_t$$

$S_t$  is periodic with mean 0



# Candidate features

## STL decomposition

$$Y_t = S_t + T_t + R_t$$

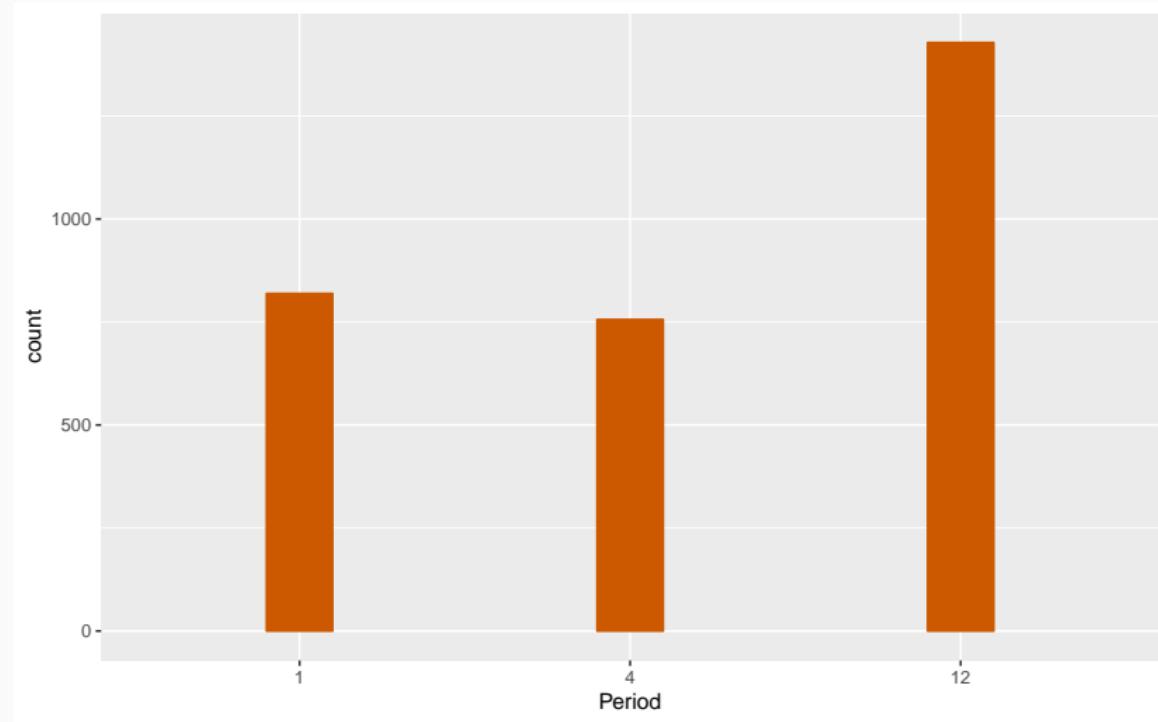
# Candidate features

## STL decomposition

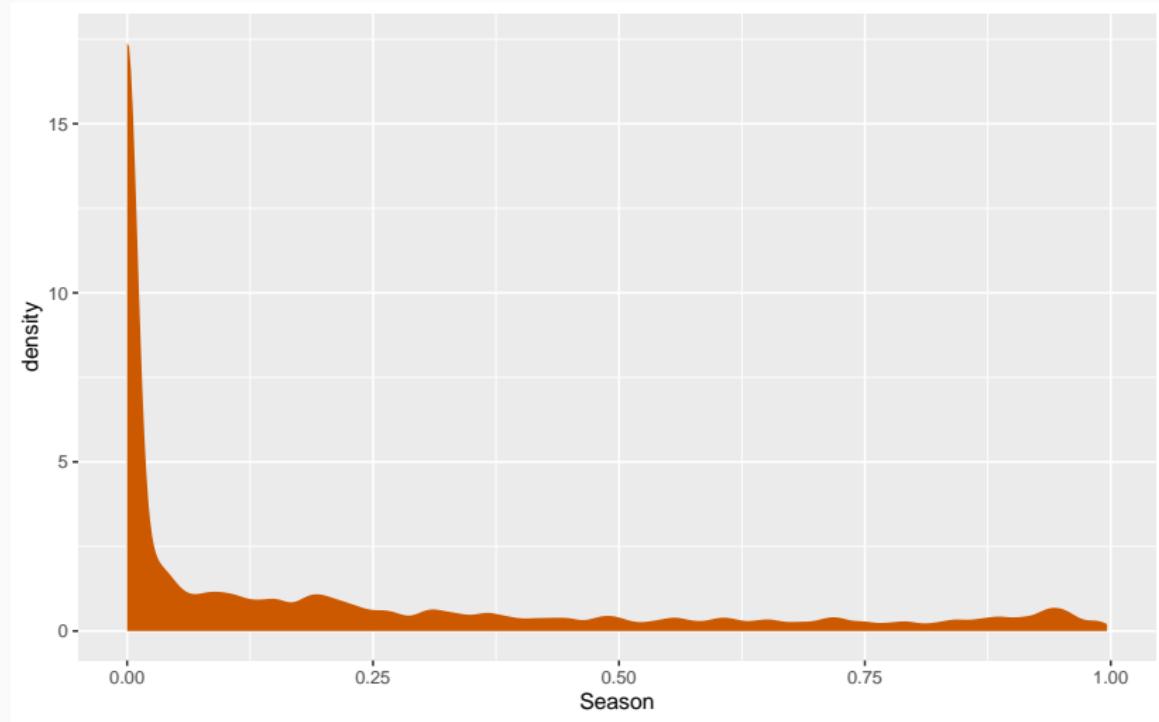
$$Y_t = S_t + T_t + R_t$$

- Seasonal period
- Autocorrelations of data  $(Y_1, \dots, Y_T)$
- Autocorrelations of data  $(R_1, \dots, R_T)$
- Strength of seasonality:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - T_t)} \right)$
- Strength of trend:  $\max \left( 0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(Y_t - S_t)} \right)$
- Spectral entropy:  $H = - \int_{-\pi}^{\pi} f_y(\lambda) \log f_y(\lambda) d\lambda$ ,  
where  $f_y(\lambda)$  is spectral density of  $Y_t$ .  
Low values of  $H$  suggest a time series that is  
easier to forecast (more signal).
- Optimal Box-Cox transformation of data

# Distribution of Period for M3

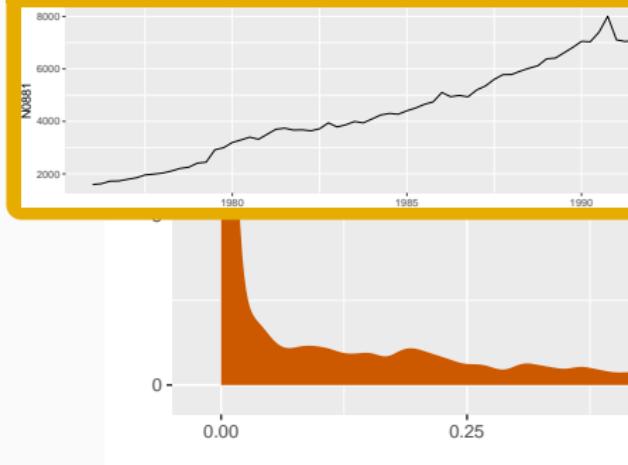


# Distribution of Seasonality for M3



# Distribution of Seasonality for M3

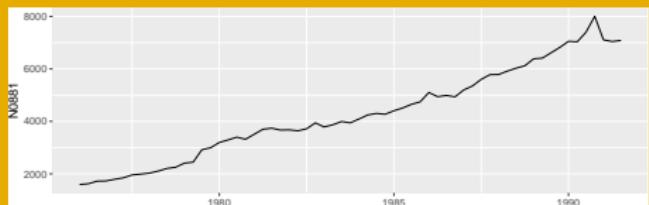
Low Seasonality



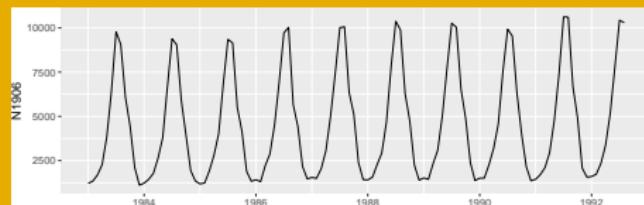
# Distribution of Seasonality for M3



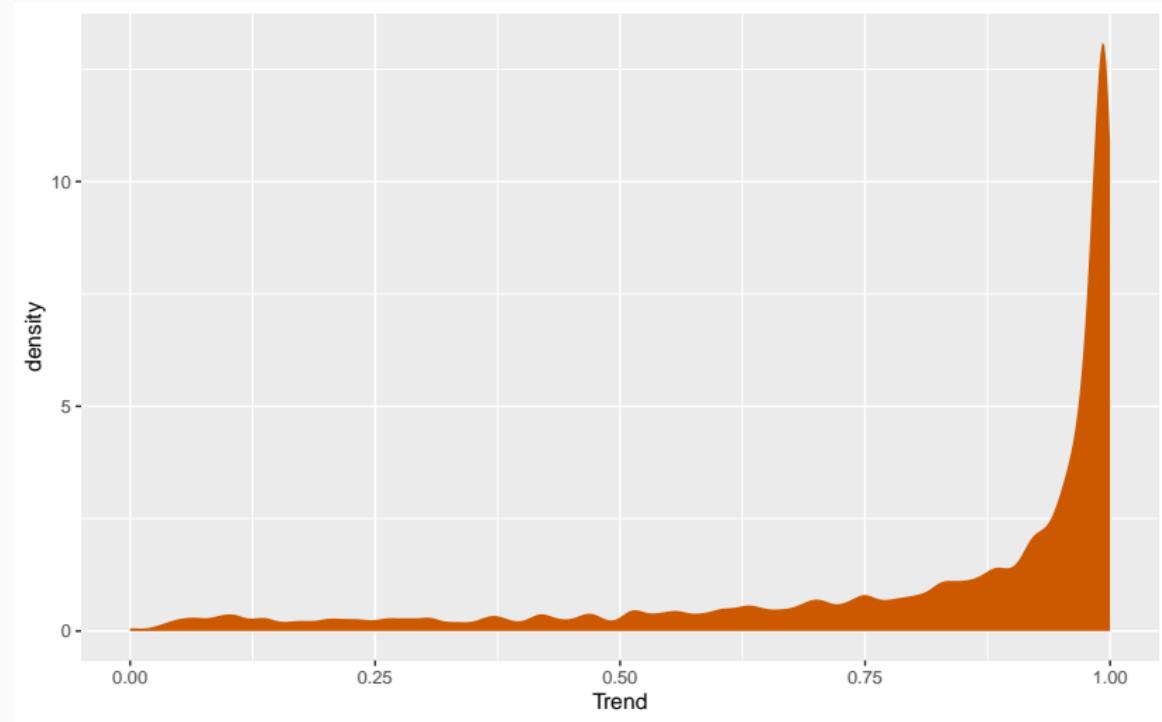
**Low Seasonality**



**High Seasonality**

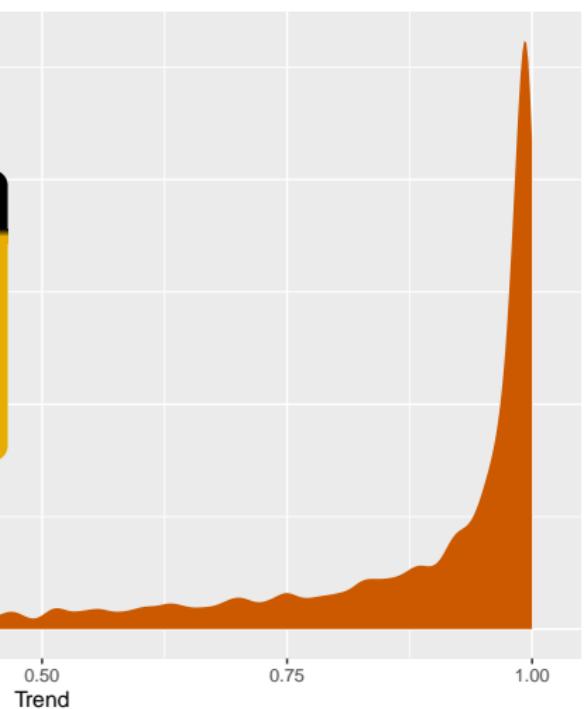
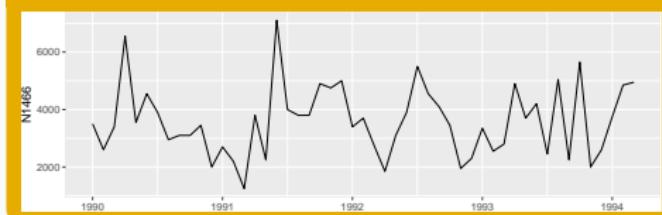


# Distribution of Trend for M3



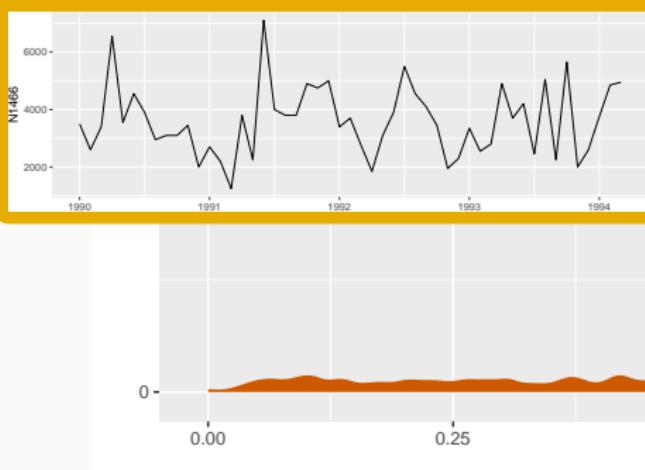
# Distribution of Trend for M3

Low Trend

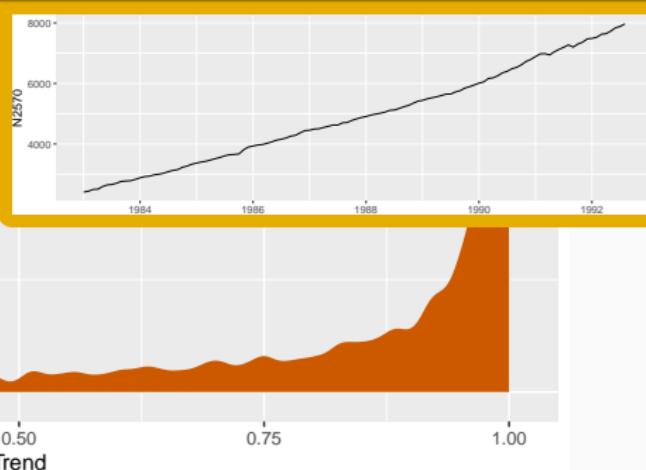


# Distribution of Trend for M3

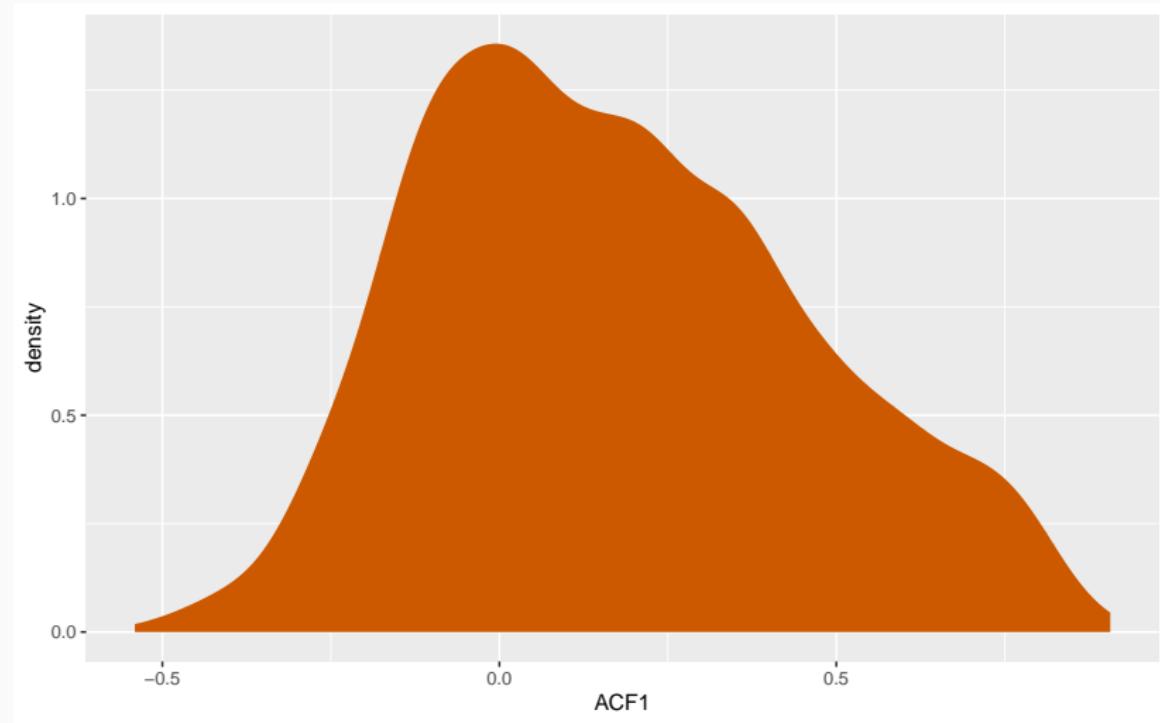
Low Trend



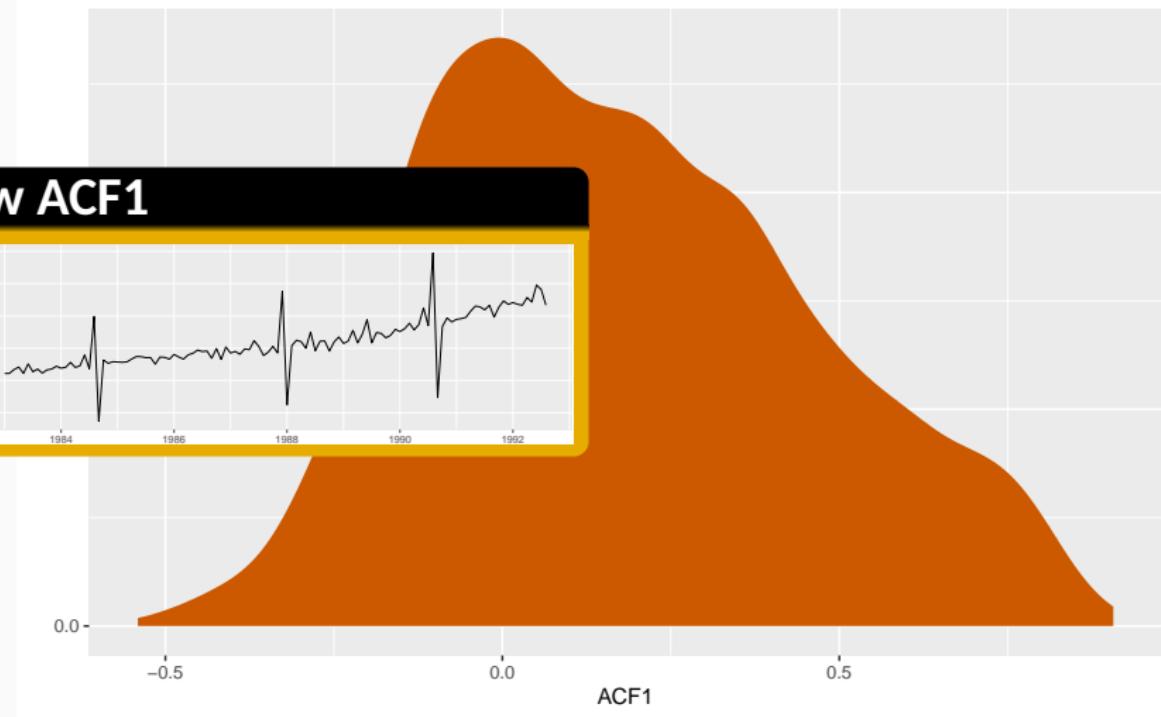
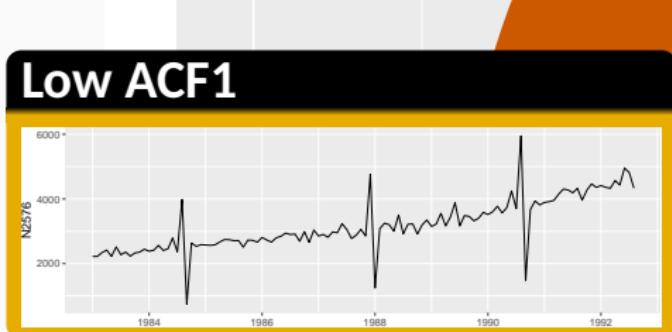
High Trend



# Distribution of Residual ACF1 for M3

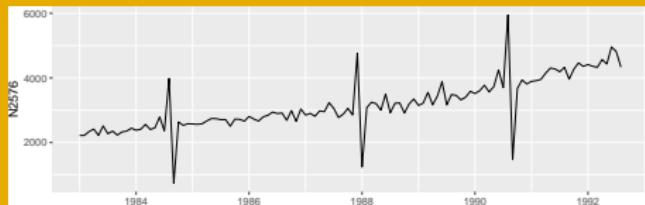


# Distribution of Residual ACF1 for M3

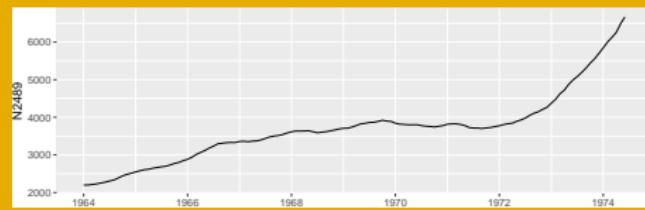


# Distribution of Residual ACF1 for M3

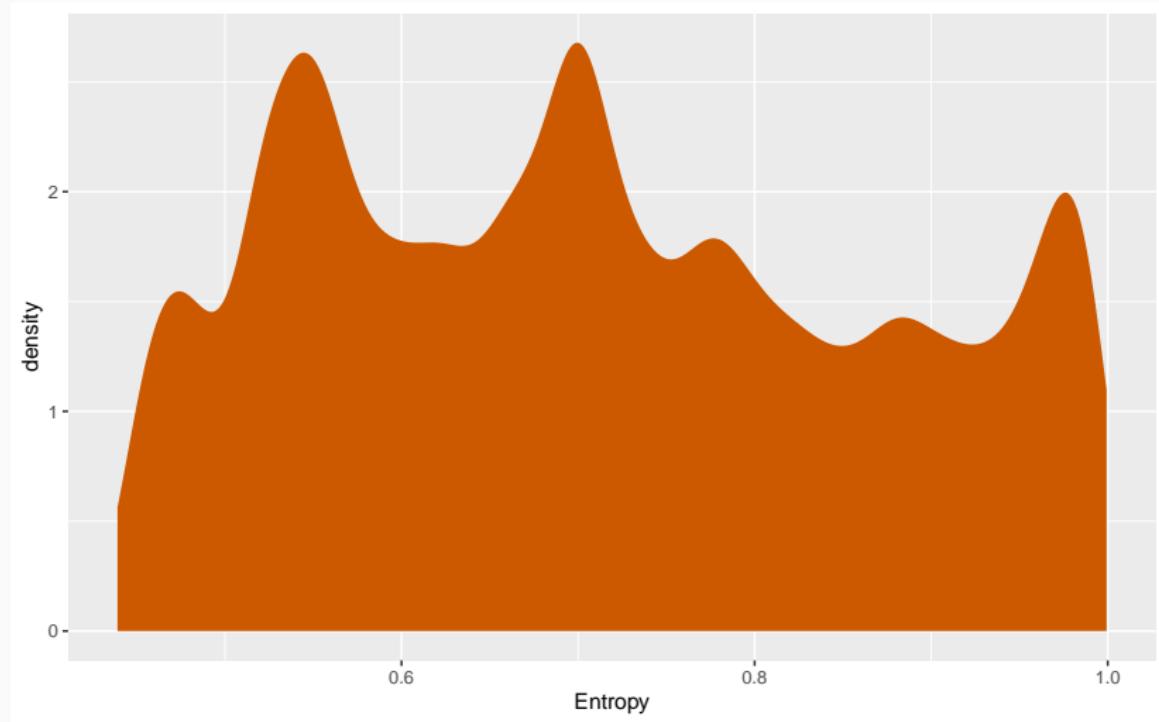
Low ACF1



High ACF1

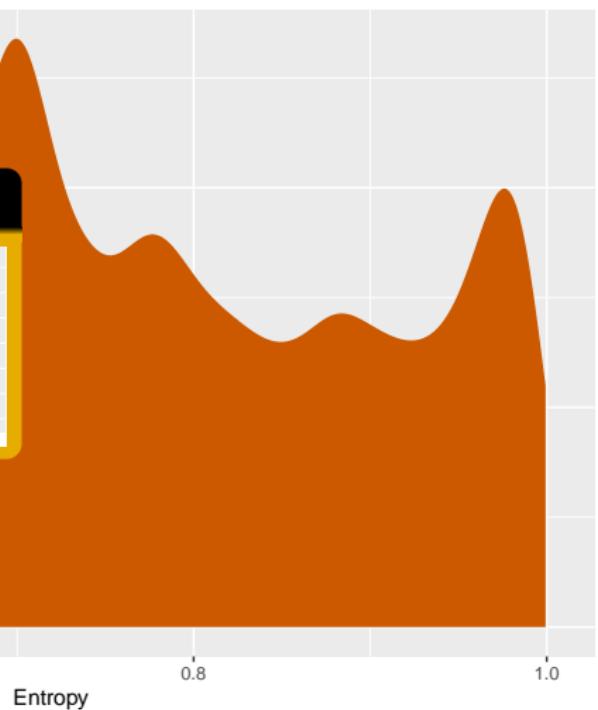
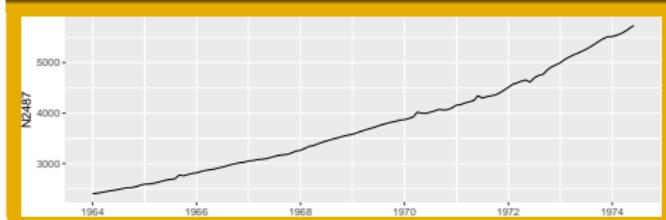


# Distribution of Spectral Entropy for M3



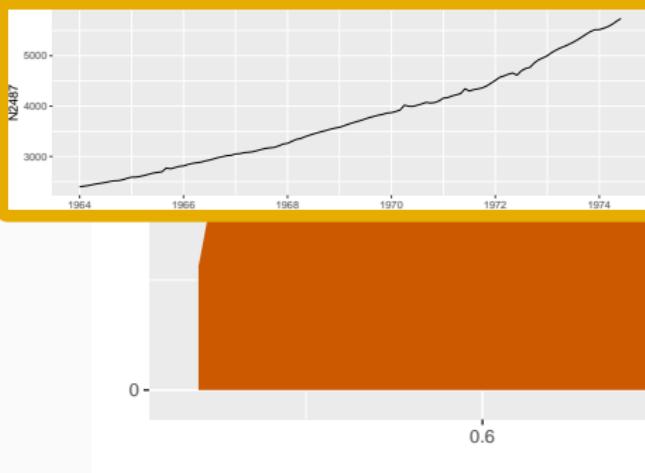
# Distribution of Spectral Entropy for M3

Low Entropy

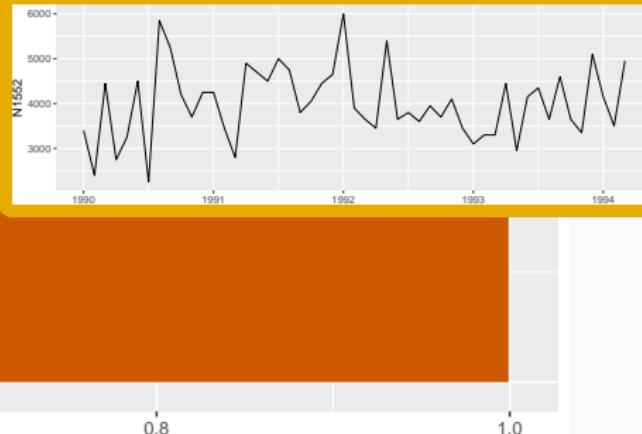


# Distribution of Spectral Entropy for M3

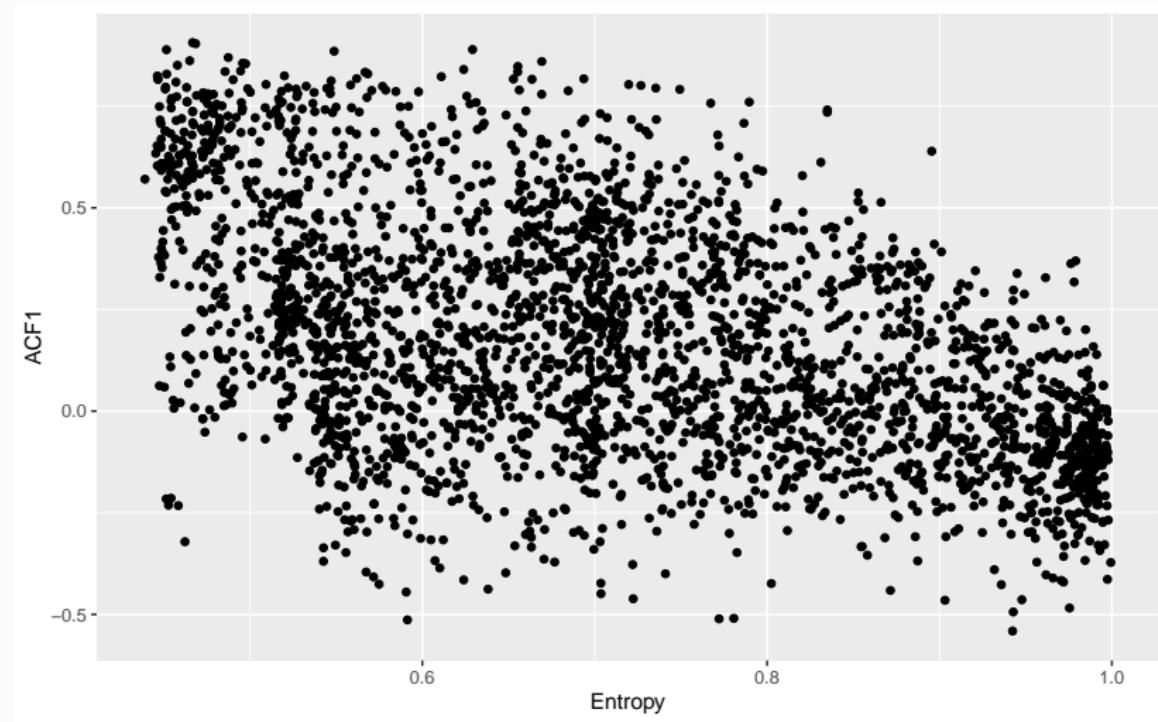
Low Entropy



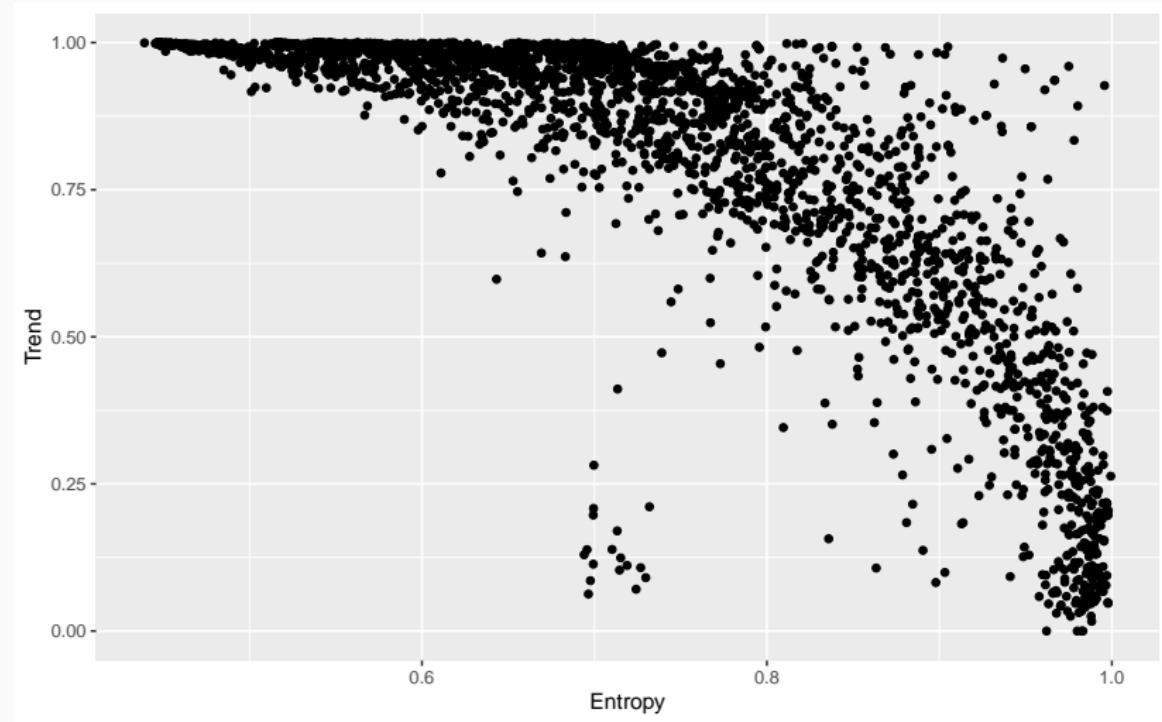
High Entropy



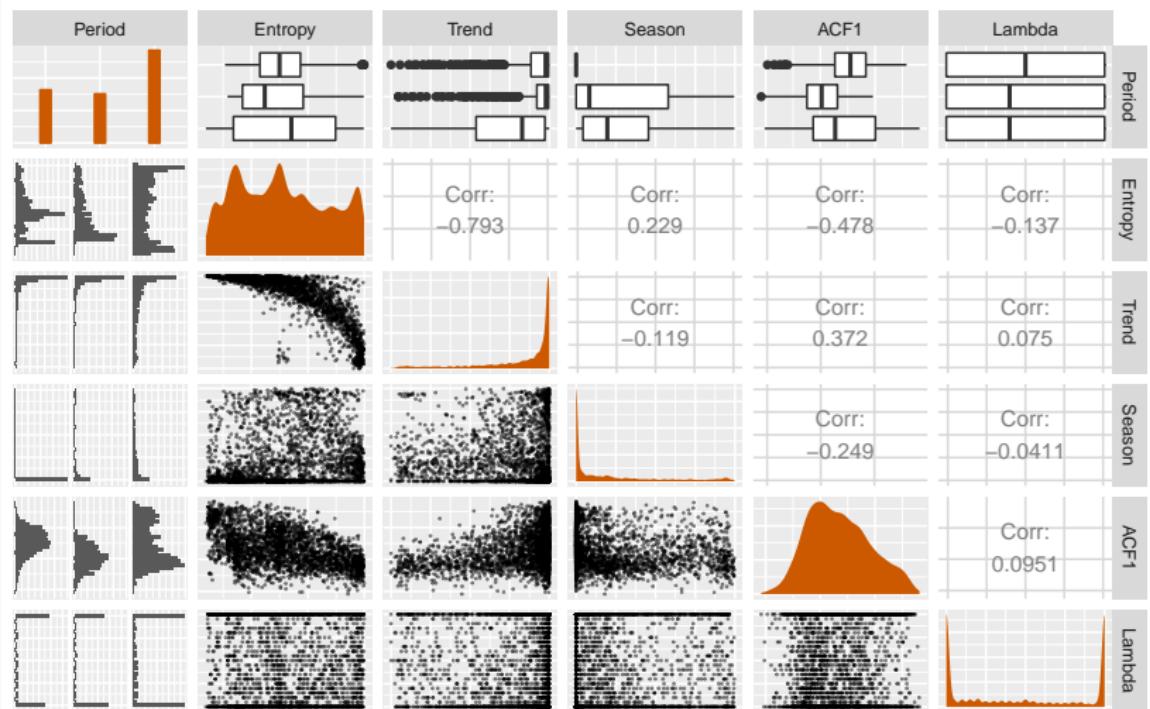
# Feature distributions



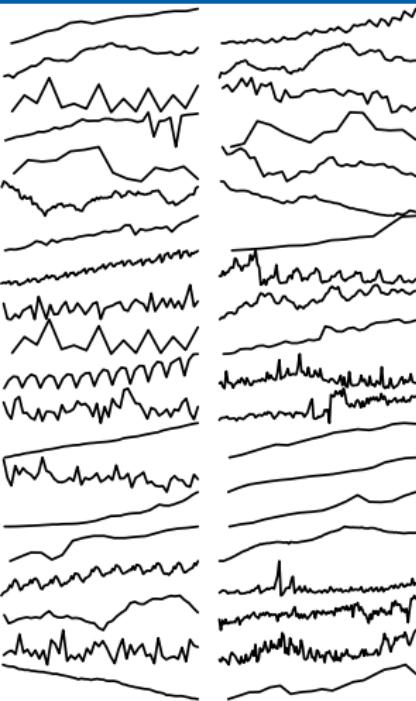
# Feature distributions



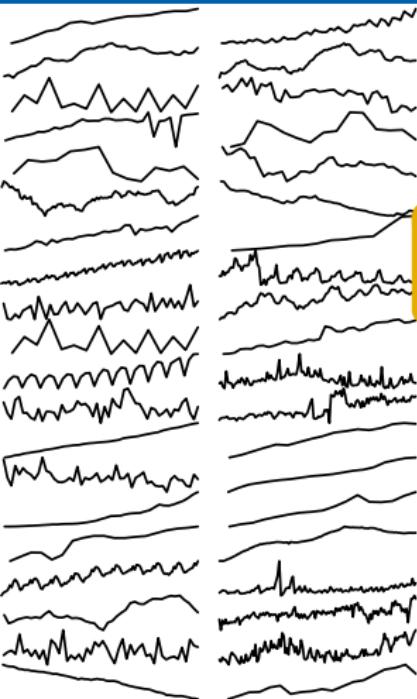
# Feature distributions



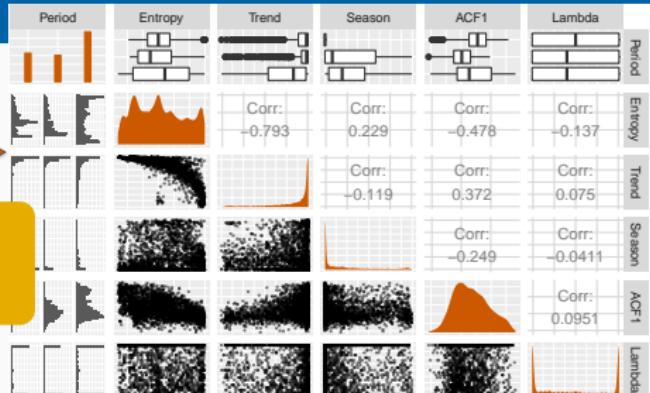
# Dimension reduction for time series



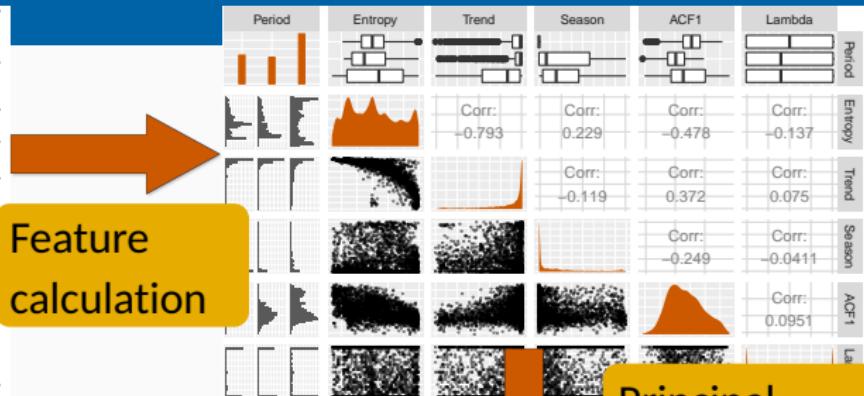
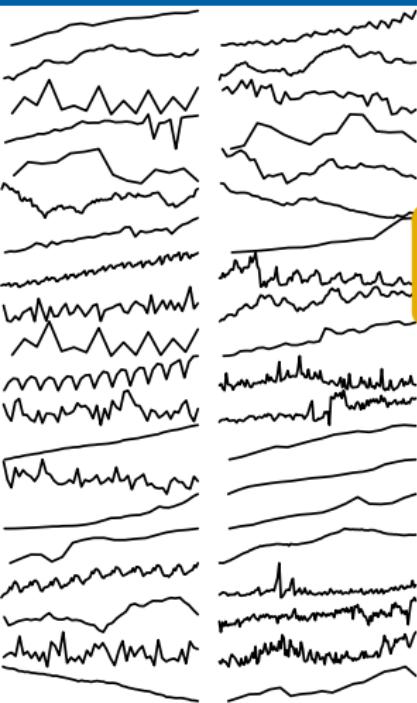
# Dimension reduction for time series



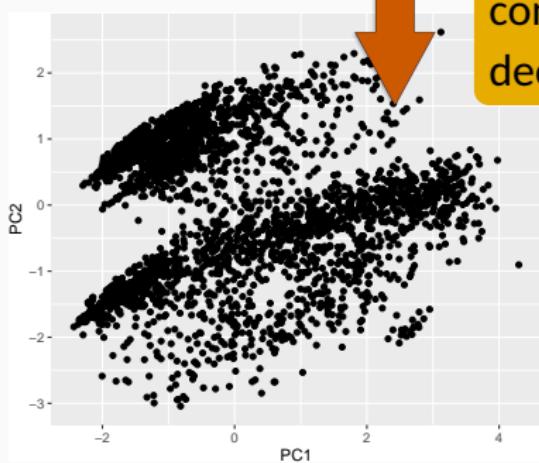
Feature  
calculation



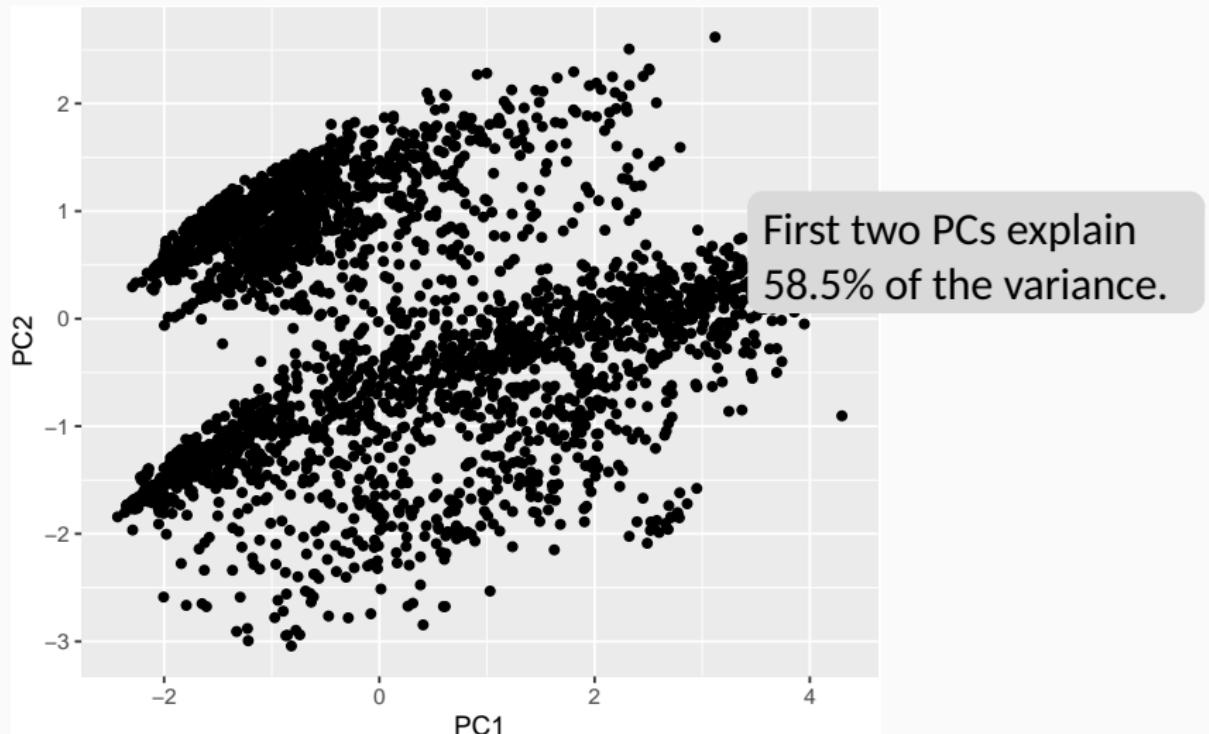
# Dimension reduction for time series



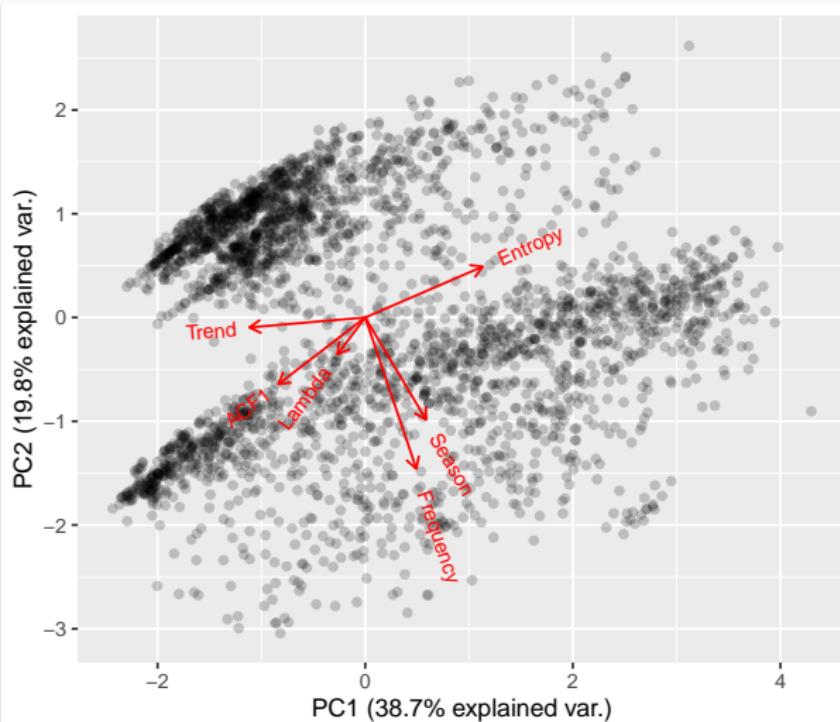
Principal component decomposition



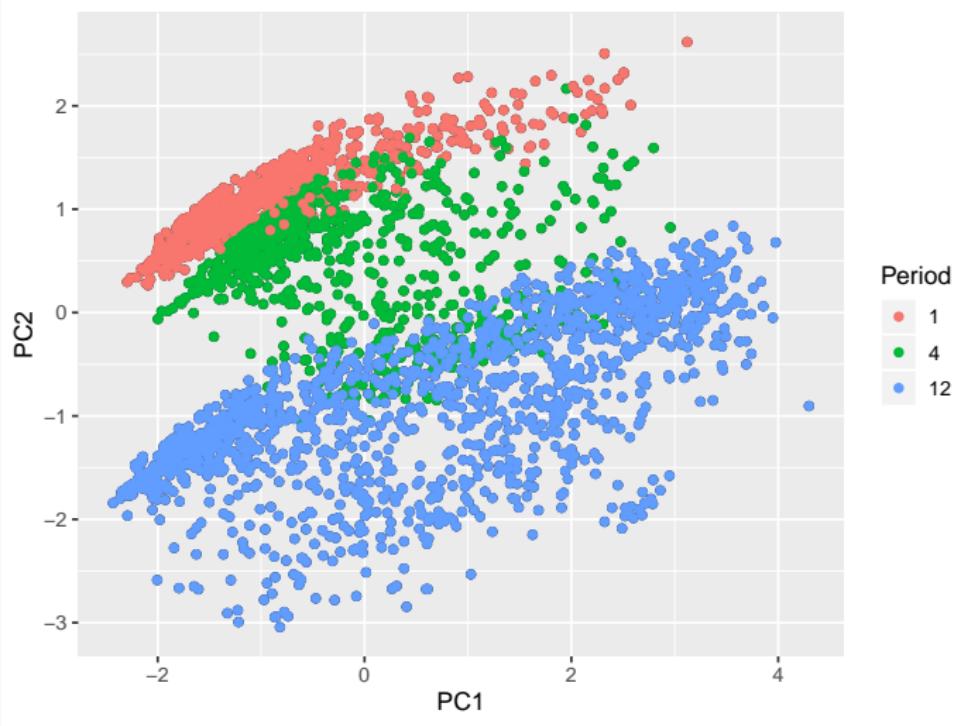
# M3 feature space



# M3 feature space



# M3 feature space



# Feature properties

In this analysis, we have restricted features to be

- ergodic
- scale-independent

For other analyses, it may be appropriate to have different requirements.

# R package: tsfeatures

[github.com/robjhyndman/tsfeatures](https://github.com/robjhyndman/tsfeatures)

```
library(tsfeatures)
library(tidyverse)
library(forecast)

myfeatures <- function(x,...) {
  lambda <- BoxCox.lambda(x, lower=0, upper=1, method='loglik')
  y <- BoxCox(x, lambda)
  c(stl_features(y,s.window='periodic', robust=TRUE, ...),
    lambda=lambda)
}
M3Features <- bind_cols(
  tsfeatures(M3data, c("frequency", "entropy")),
  tsfeatures(M3data, "myfeatures", scale=FALSE))
```

# Outline

- 1 Time series features
- 2 FFORMS: Feature-based forecast model selection
- 3 FFORMA: Feature-based forecast model averaging

# FFORMS: Feature-based FORcast Model Selection

## Features used to select a forecasting model

- length
- strength of seasonality
- strength of trend
- linearity
- curvature
- spikiness
- stability
- lumpiness
- first ACF value of remainder series
- parameter estimates of Holt's linear trend method
- spectral entropy
- Hurst exponent
- nonlinearity
- parameter estimates of Holt-Winters' additive method
- unit root test statistics
- first ACF value of residual series of linear trend model
- ACF and PACF based features
  - calculated on both the raw and differenced series

# FFORMS: Feature-based FORcast Model Selection

## Why these features?

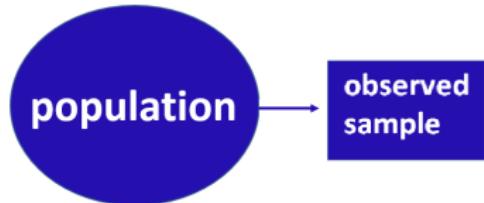
- Hyndman, Wang and Laptev. “Large scale unusual time series detection” (ICDM 2015).
- Kang, Hyndman & Smith-Miles. “Visualising forecasting algorithm performance using time series instance spaces” (IJF 2017).
- Talagala, Hyndman and Athanasopoulos. “Meta-learning how to forecast time series” (2018).
- Implemented in the tsfeatures R package

# FFORMS: Feature-based FOrecast Model Selection

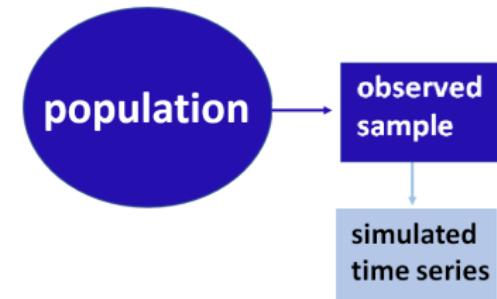


population

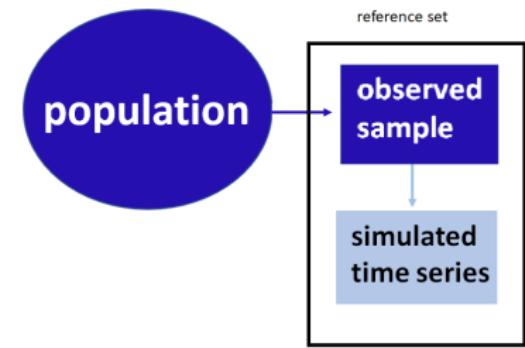
# FFORMS: Feature-based FORcast Model Selection



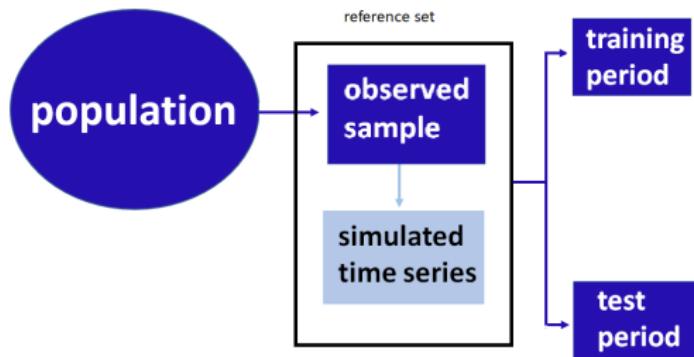
# FFORMS: Feature-based FOrecast Model Selection



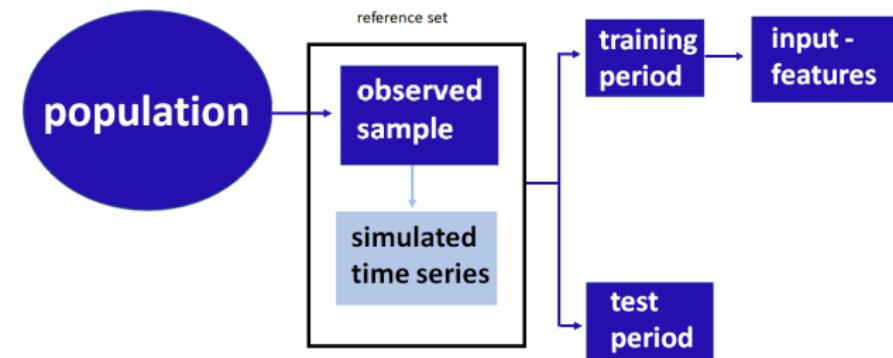
# FFORMS: Feature-based FOrecast Model Selection



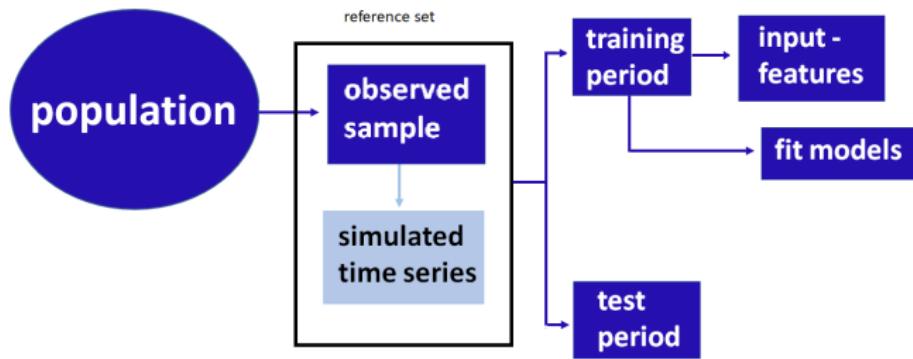
# FFORMS: Feature-based FOrecast Model Selection



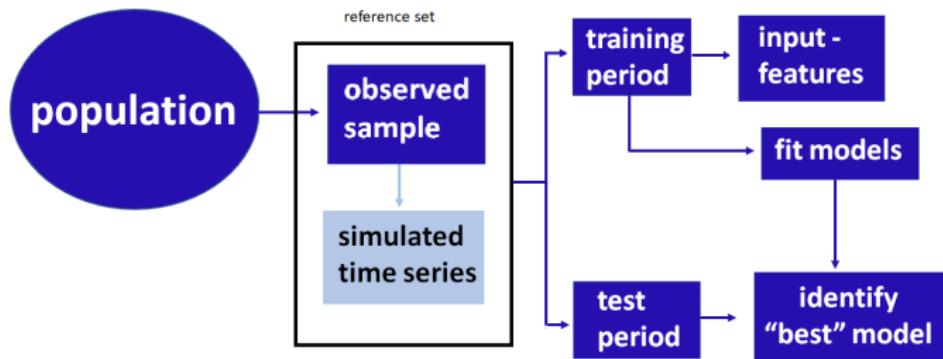
# FFORMS: Feature-based FOrecast Model Selection



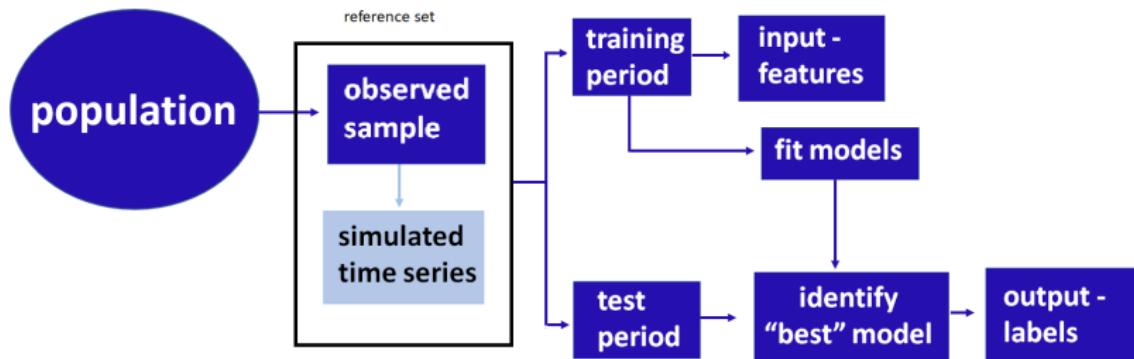
# FFORMS: Feature-based FOrecast Model Selection



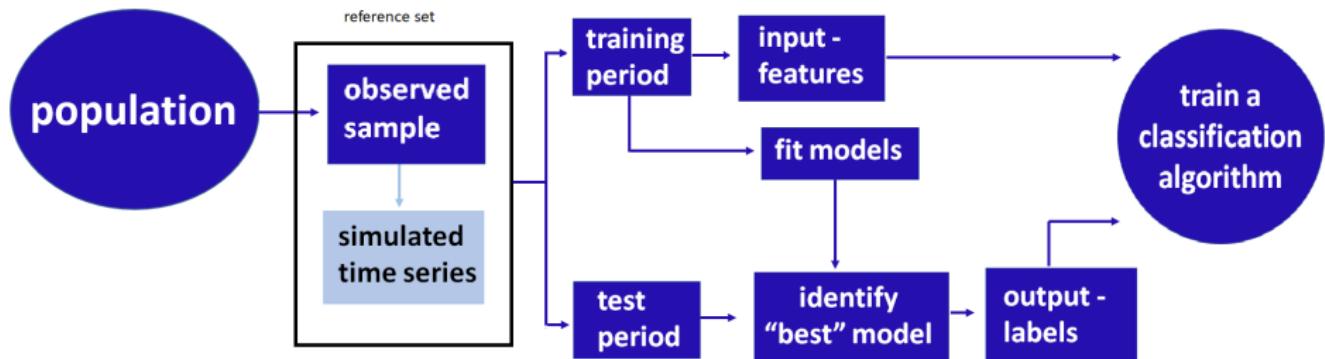
# FFORMS: Feature-based FORecast Model Selection



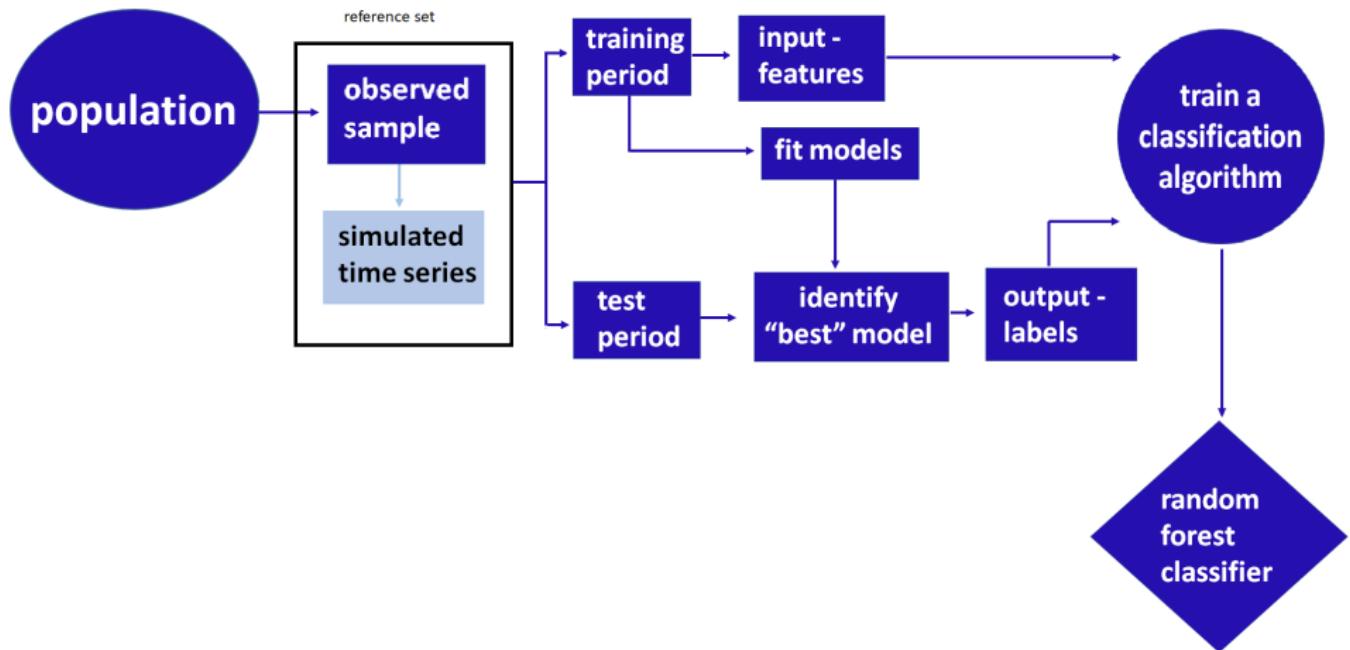
# FFORMS: Feature-based FORecast Model Selection



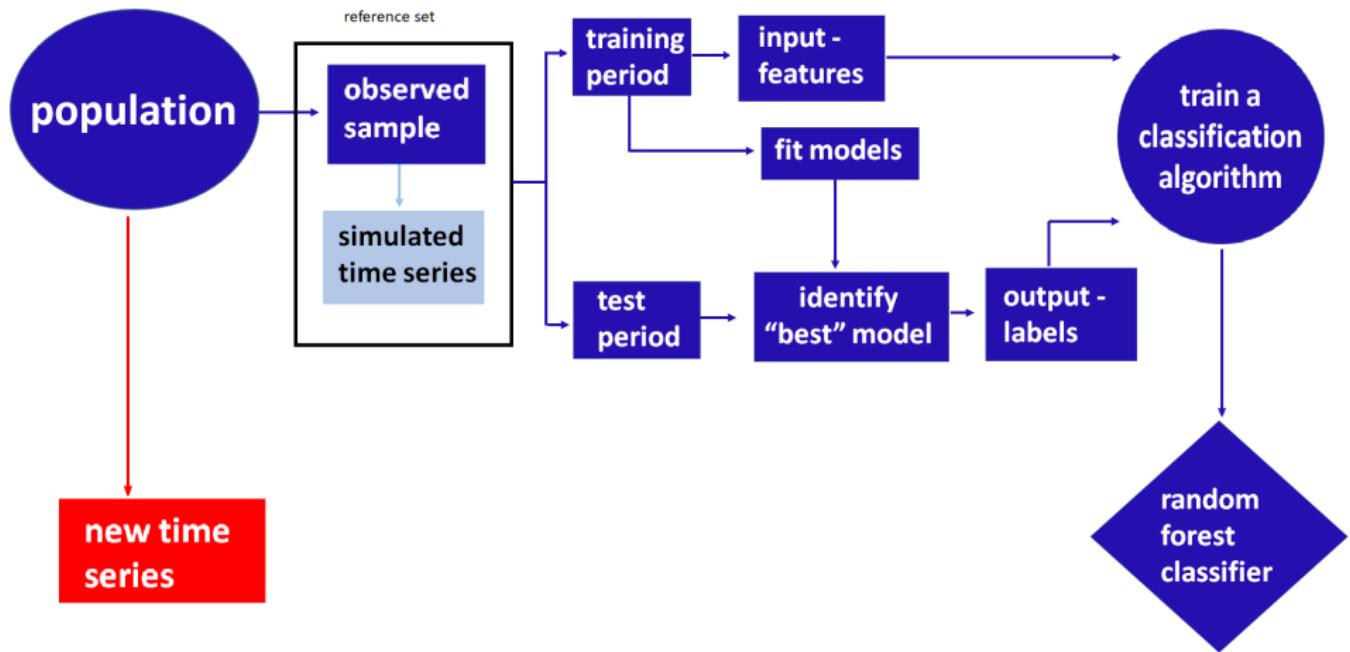
# FFORMS: Feature-based FORecast Model Selection



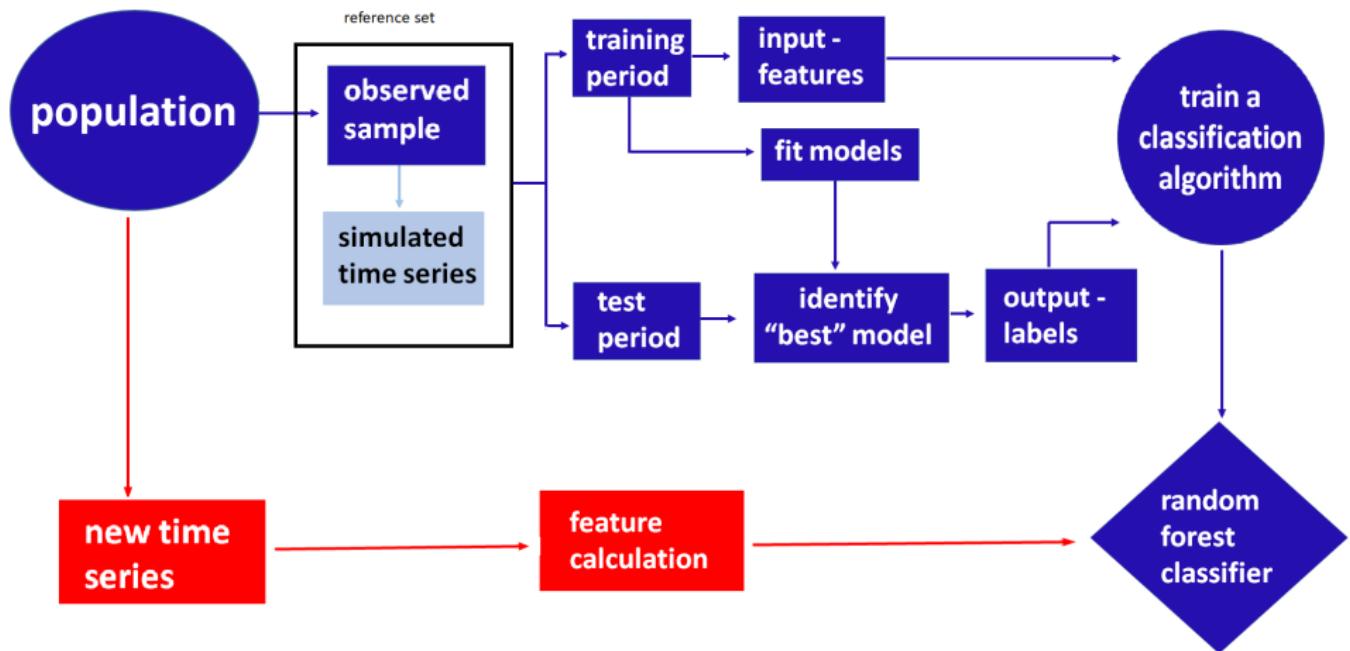
# FFORMS: Feature-based FOREcast Model Selection



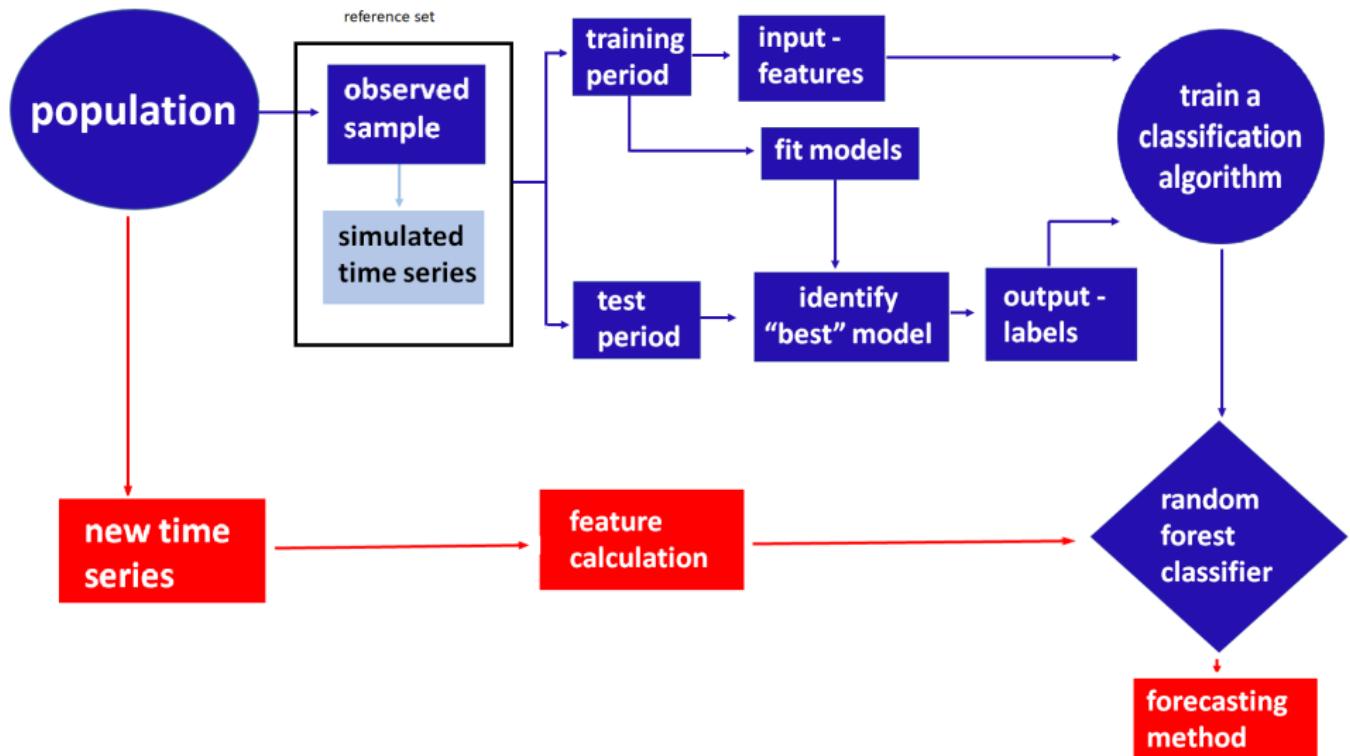
# FFORMS: Feature-based FOREcast Model Selection



# FFORMS: Feature-based FORecast Model Selection



# FFORMS: Feature-based FORecast Model Selection



# Application to M competition data

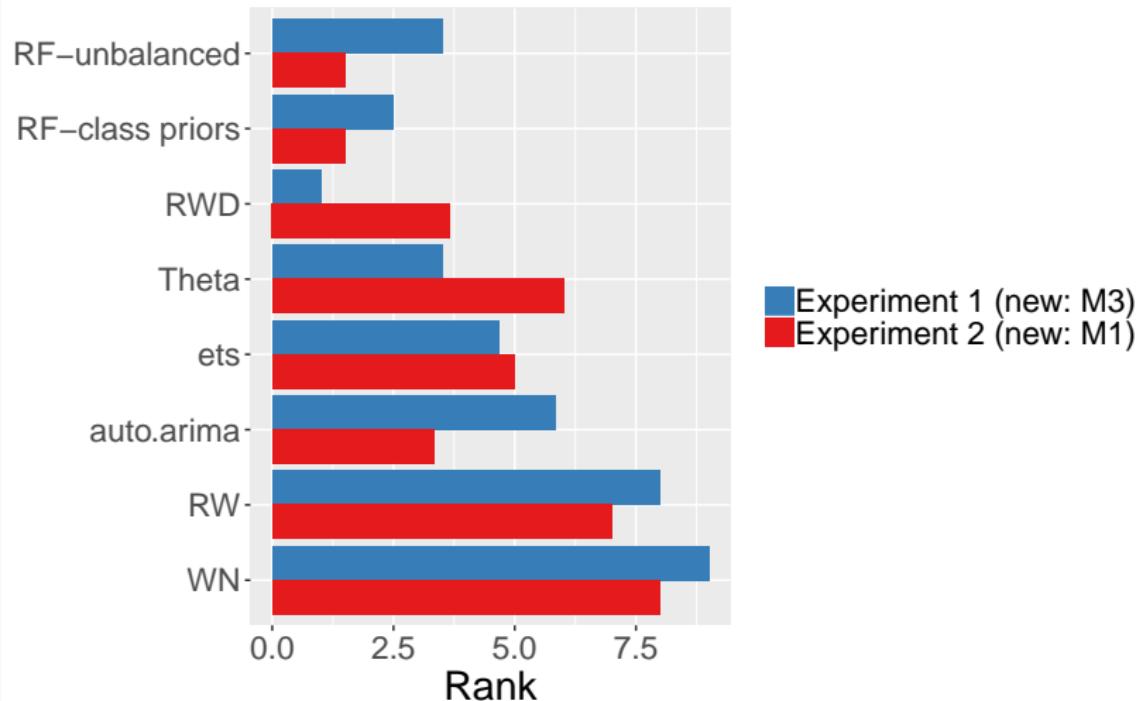
## Experiment 1

	Source	Y	Q	M
Observed series	M1	181	203	617
Simulated series		362000	406000	123400
New series	M3	645	756	1428

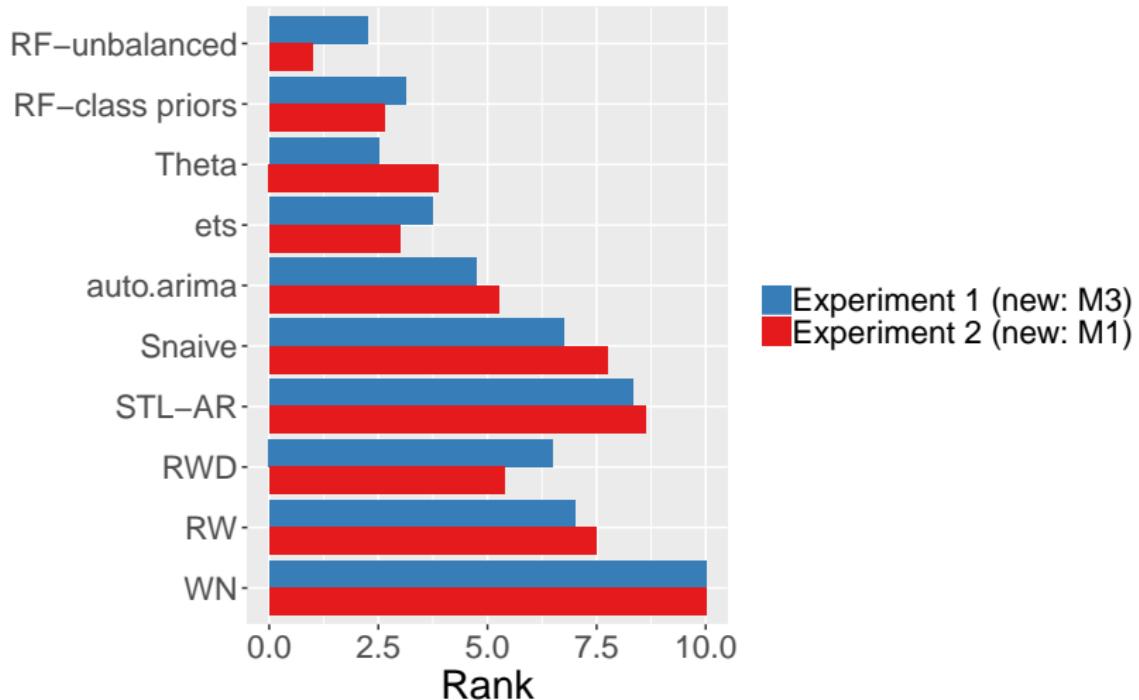
## Experiment 2

	Source	Y	Q	M
Observed series	M3	645	756	1428
Simulated series		1290000	1512000	285600
New series	M1	181	203	617

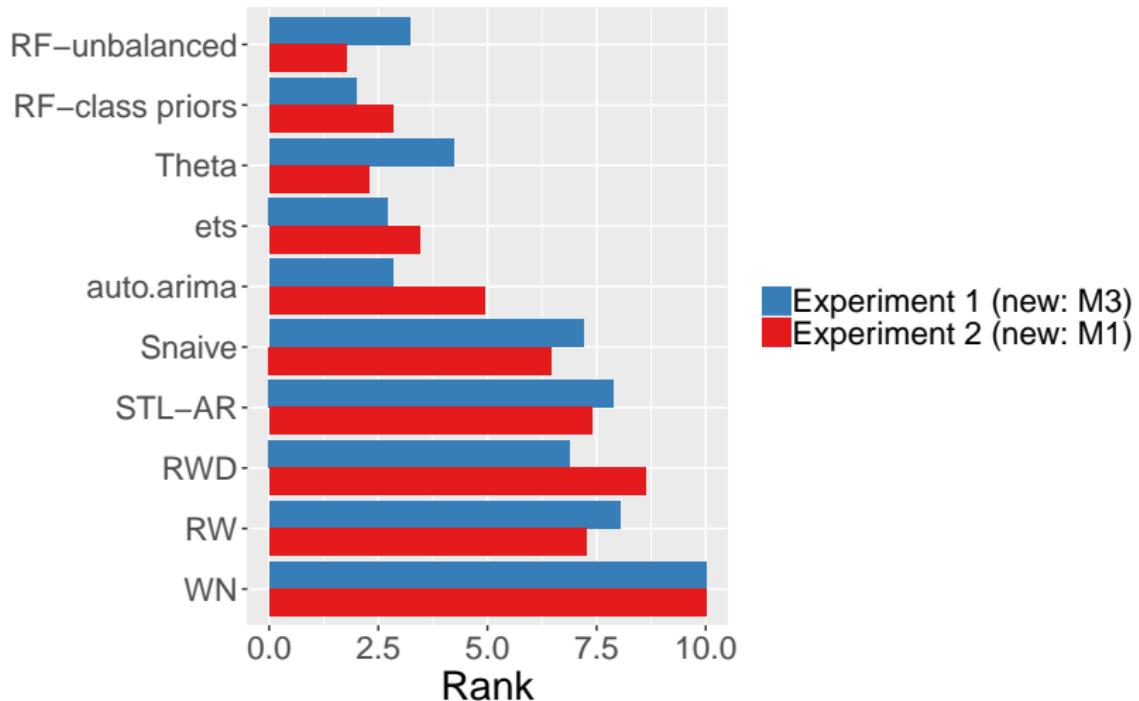
# Results: Yearly



# Results: Quarterly



# Results: Monthly



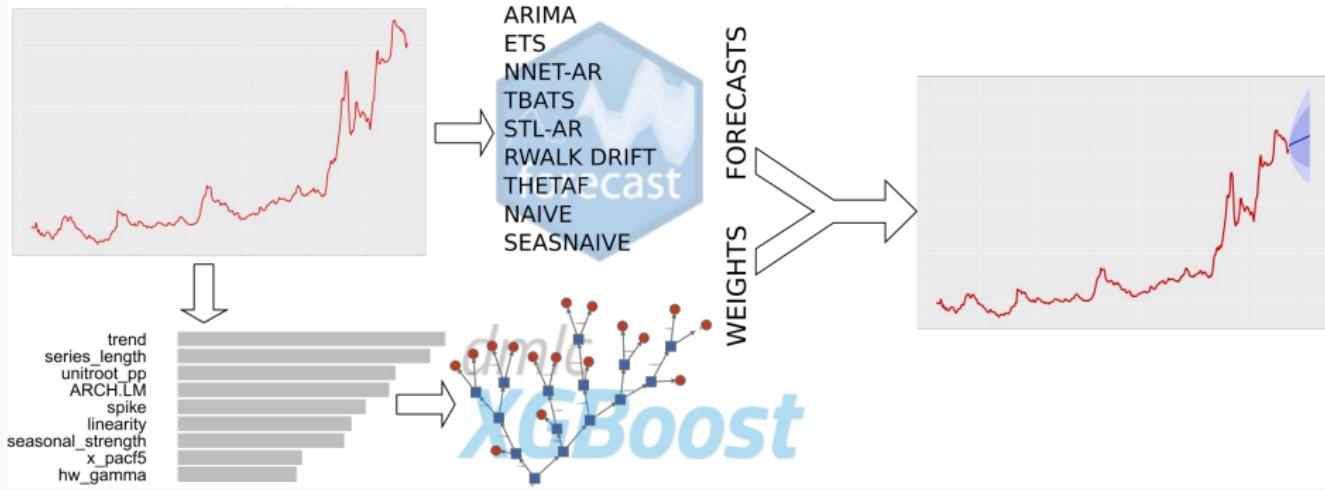
# Outline

- 1 Time series features
- 2 FFORMS: Feature-based forecast model selection
- 3 FFORMA: Feature-based forecast model averaging

## FFORMA: Feature-based FOrecast Model Averaging

- Like FFORMS but we use xgboost rather than a random forest.
- The optimization criterion is forecast accuracy not classification accuracy.
- The probability of each model being best is used to construct a model weight.
- A combination forecast is produced using these weights.
- **Came second in the M4 competition**

# FFORMA: Feature-based FORcast Model Averaging



## Results

2nd according to average OWA: 0.838

1st: 0.821

3rd: 0.841

# FFORMA: Feature-based FOrecast Model Averaging

## Models included

- 1 Naive
- 2 Seasonal naive
- 3 Random walk with drift
- 4 Theta method
- 5 ARIMA
- 6 ETS
- 7 TBATS
- 8 STL decomposition with AR for seasonally  
adjusted series
- 9 Neural network autoregression

# FFORMA: Feature-based FOrecast Model Averaging

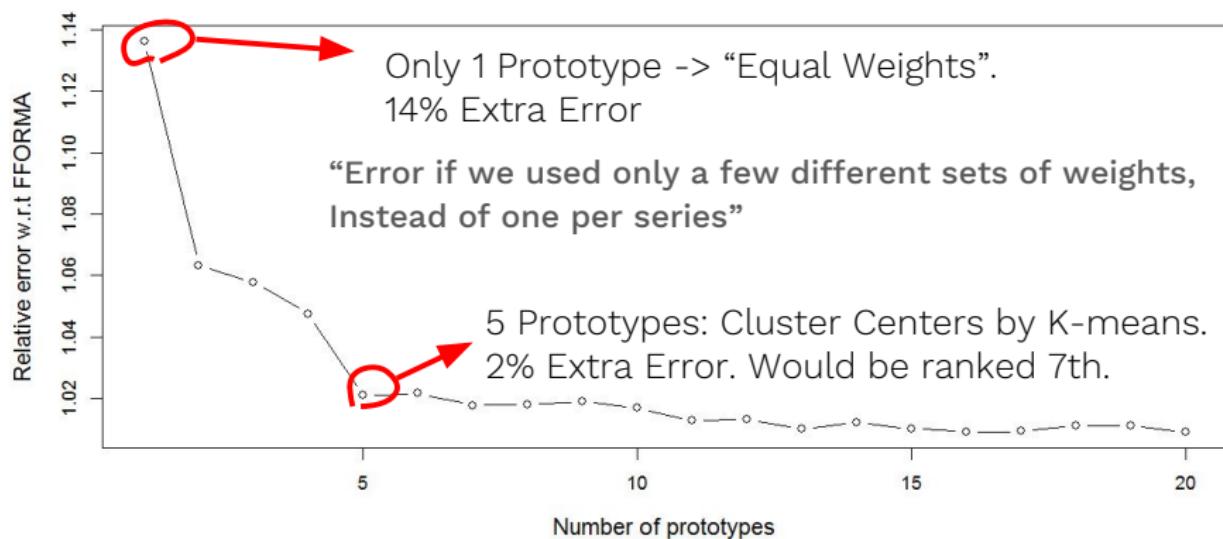
Weights for the combination:

- A **decision tree model** generates the weights using xgboost.
- The model is trained on a temporal holdout version of the M4 dataset, where the test sets are equal to the required forecast horizons.

# FFORMA: Feature-based FOrecast Model Averaging

# FFORMA: Feature-based FOrecast Model Averaging

## Looking for Prototypes in the weights



# FFORMA: Feature-based FOrecast Model Averaging

**“Roughly Equal Weights”. 40000 Series in M4**

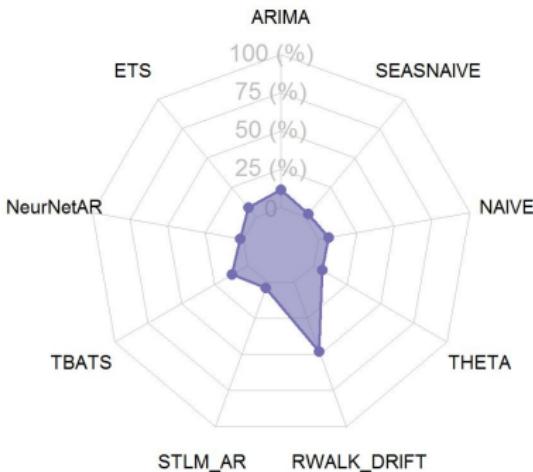
Weights of Prototype I



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly RandomWalk Drift”. 20000 Series in M4**

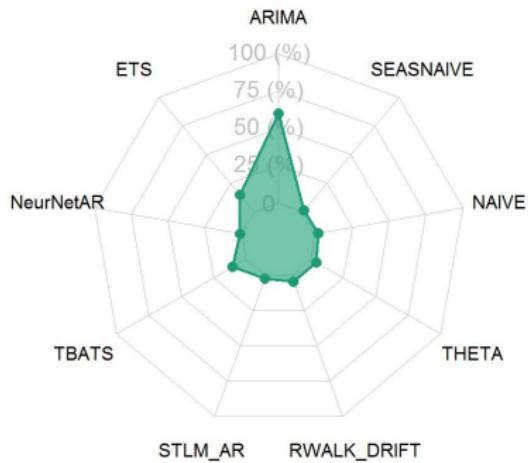
Weights of Prototype II



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly ARIMA”. 16000 Series in M4**

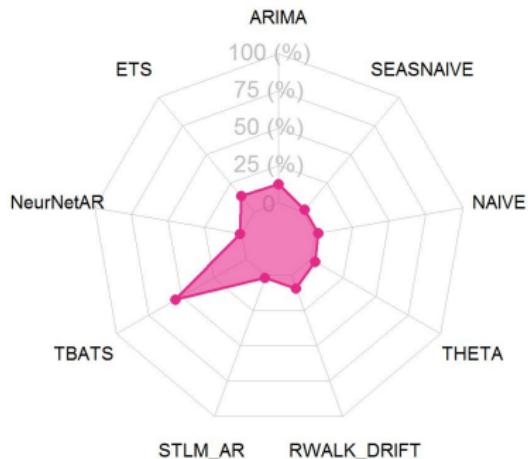
Weights of Prototype III



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly TBATS”. 13000 Series in M4**

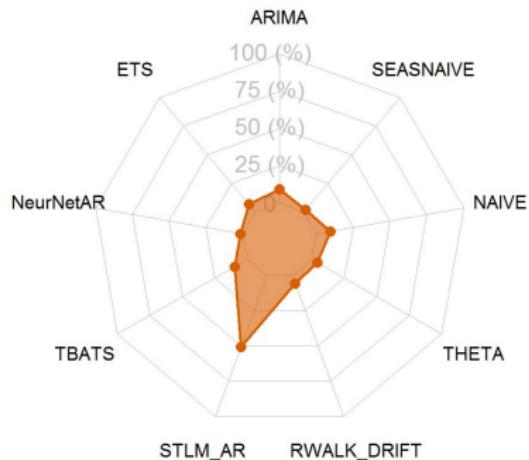
Weights of Prototype IV



# FFORMA: Feature-based FOrecast Model Averaging

**“Mostly STLM-AR”. 8000 Series in M4**

Weights of Prototype V



# R Packages

- **seer**: FFORMS — selecting forecasting model using features.

`github.com/thiyangt/seer`

- **M4metalearning**: FFORMA – forecast combinations using features to choose weights.

`github.com/robjhyndman/M4metalearning`

# Acknowledgements



Kate Smith-Miles



Yanfei Kang



Earo Wang



Thiyanga Talagala



George Athanasopoulos



Pablo Montero-Manso