

# Time Series Analysis & Forecasting Using R

[bit.ly/fable2023](https://bit.ly/fable2023)

## 6. Introduction to forecasting



# Outline

- 1 Statistical forecasting
- 2 Benchmark methods
- 3 Lab Session 11
- 4 Residual diagnostics
- 5 Lab Session 12
- 6 Forecast accuracy measures
- 7 Lab Session 13

# Outline

1 Statistical forecasting

2 Benchmark methods

3 Lab Session 11

4 Residual diagnostics

5 Lab Session 12

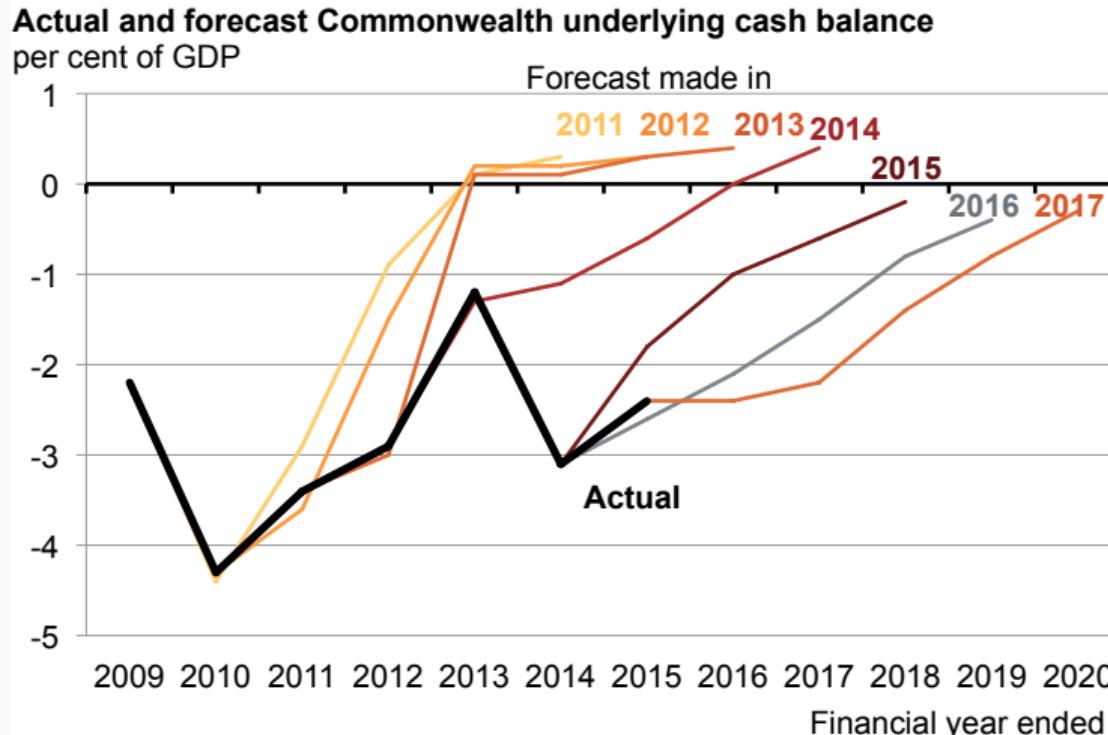
6 Forecast accuracy measures

7 Lab Session 13

# Forecasting is difficult

Commonwealth plans to drift back to surplus  
show the triumph of experience over hope

GRATTAN  
Institute



# What can we forecast?



# What can we forecast?



# What can we forecast?

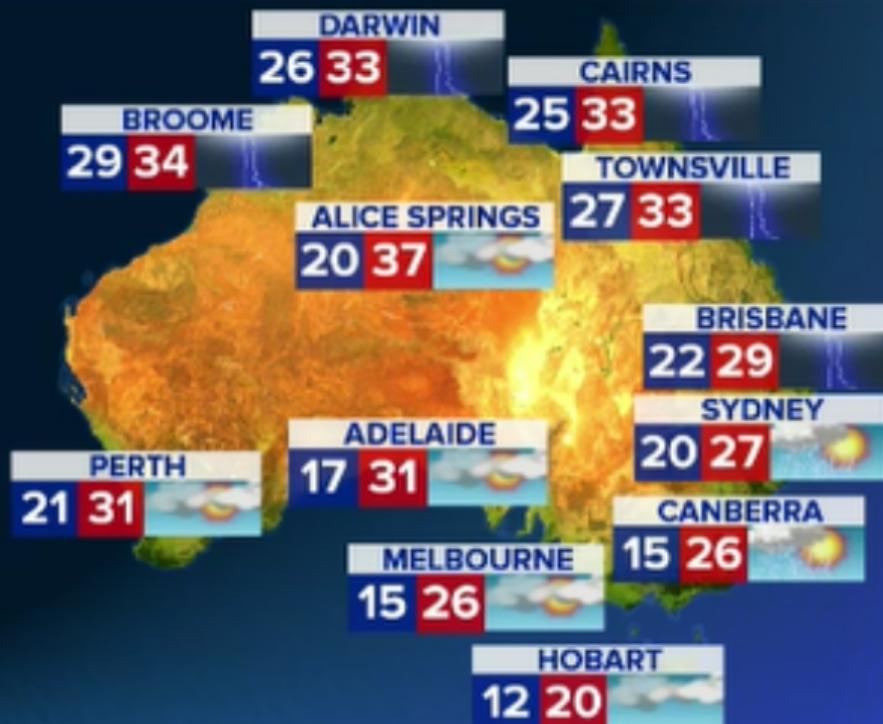


# What can we forecast?



# What can we forecast?

## TOMORROW



# What can we forecast?



# What can we forecast?



# Which is easiest to forecast?

- 1 daily electricity demand in 3 days time
- 2 timing of next Halley's comet appearance
- 3 time of sunrise this day next year
- 4 Google stock price tomorrow
- 5 Google stock price in 6 months time
- 6 maximum temperature tomorrow
- 7 exchange rate of \$US/AUS next week
- 8 total sales of drugs in Australian pharmacies next month

# Which is easiest to forecast?

- 1 daily electricity demand in 3 days time
  - 2 timing of next Halley's comet appearance
  - 3 time of sunrise this day next year
  - 4 Google stock price tomorrow
  - 5 Google stock price in 6 months time
  - 6 maximum temperature tomorrow
  - 7 exchange rate of \$US/AUS next week
  - 8 total sales of drugs in Australian pharmacies next month
- 
- how do we measure “easiest”?
  - what makes something easy/difficult to forecast?

# Factors affecting forecastability

Something is easier to forecast if:

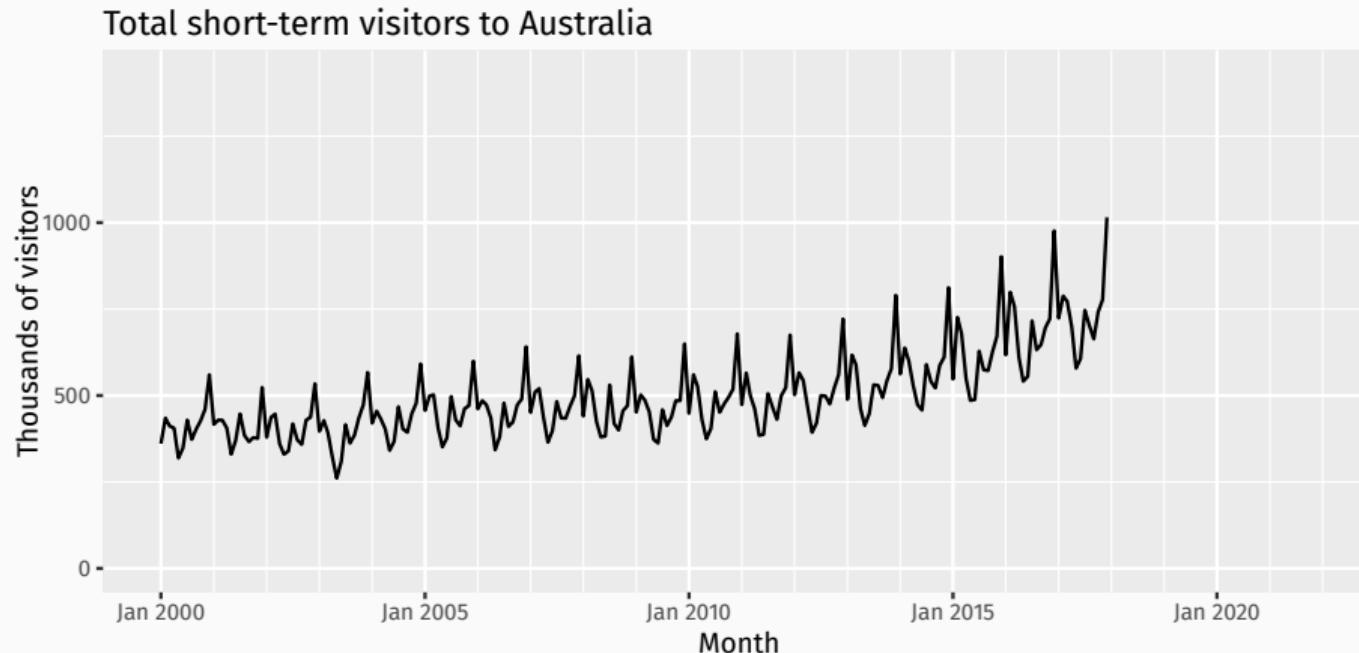
- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

# Random futures

A forecast is an estimate of the probabilities of possible futures.

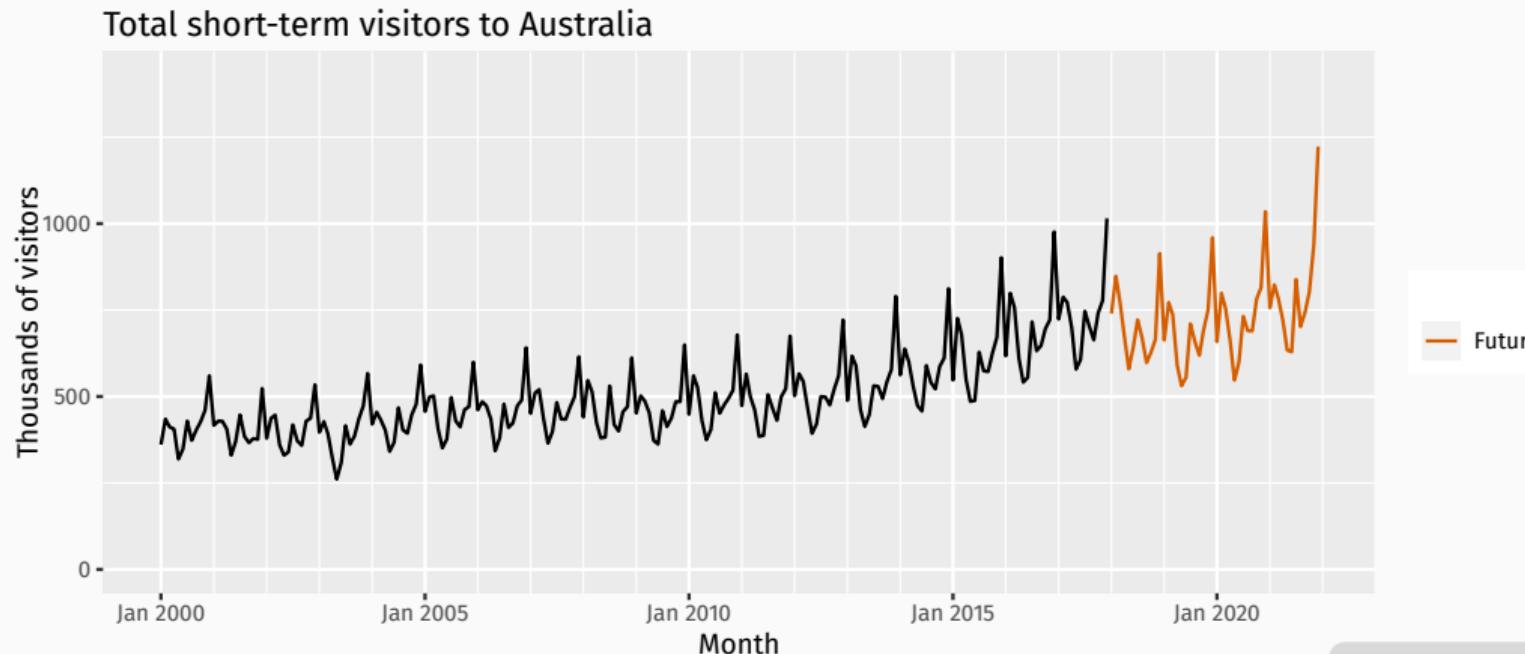
# Random futures

A forecast is an estimate of the probabilities of possible futures.



# Random futures

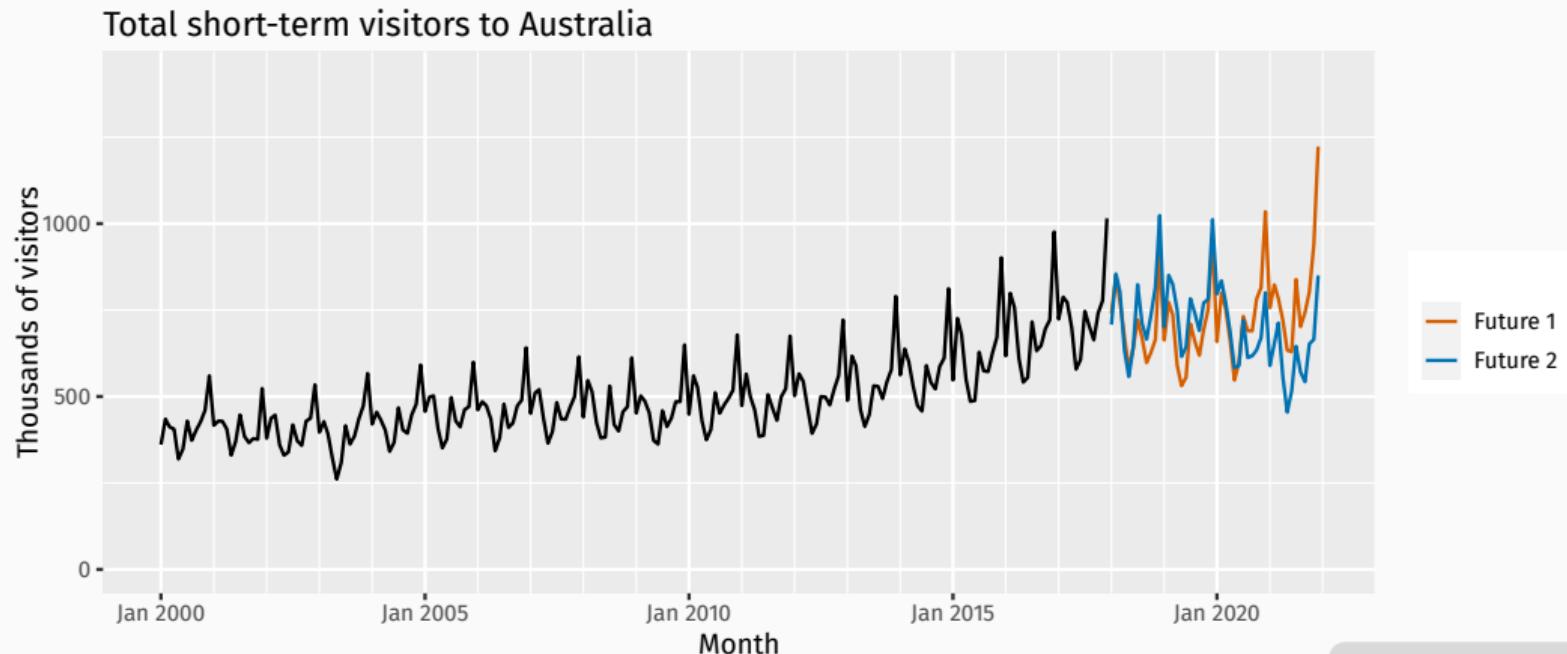
A forecast is an estimate of the probabilities of possible futures.



Simulated futures  
from an ETS model

# Random futures

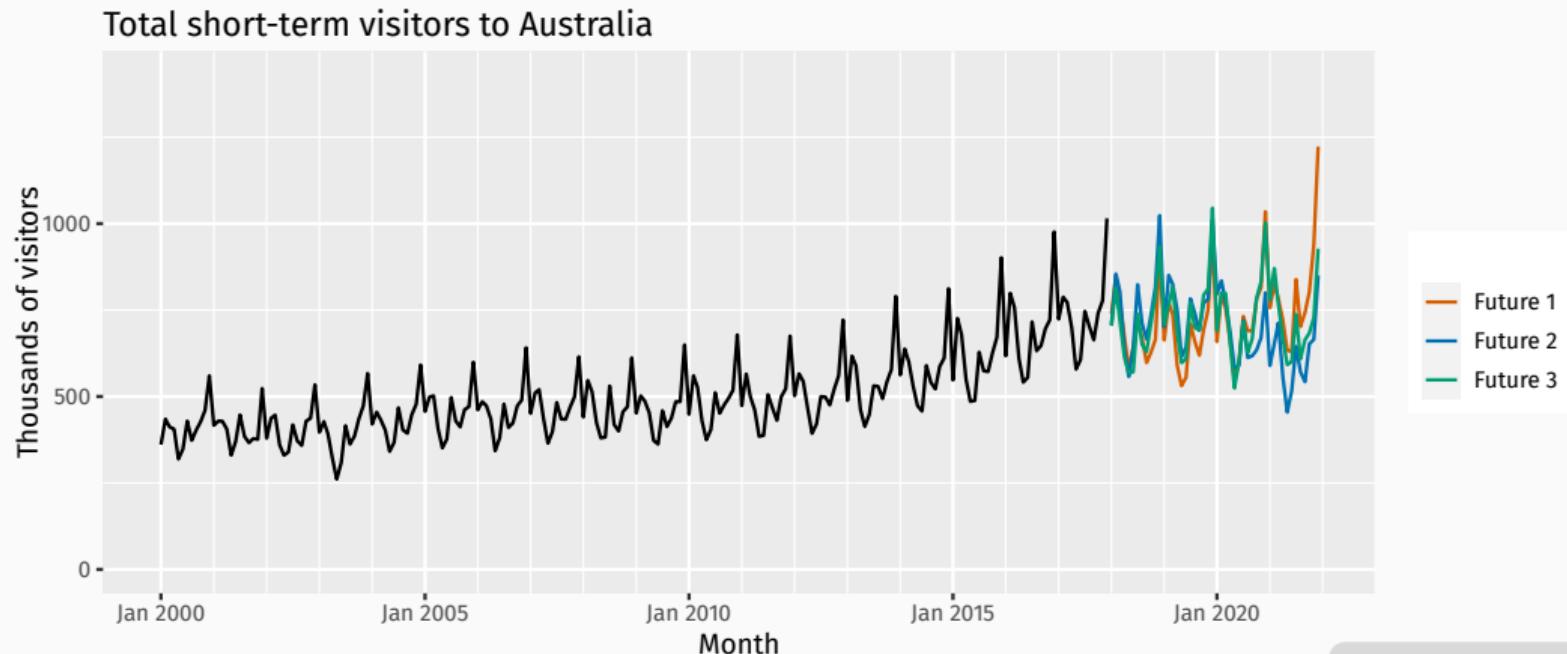
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Simulated futures  
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# Random futures

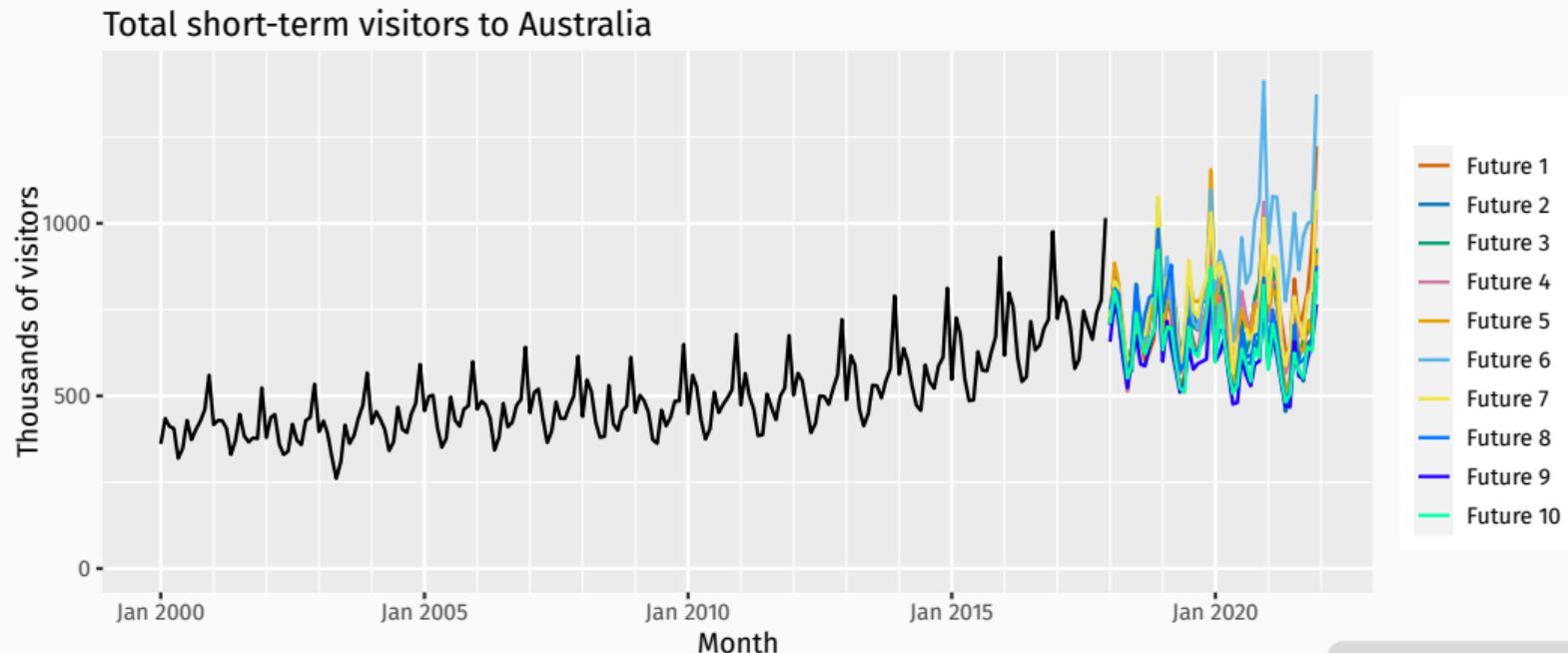
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Simulated futures  
from an ETS model

# Random futures

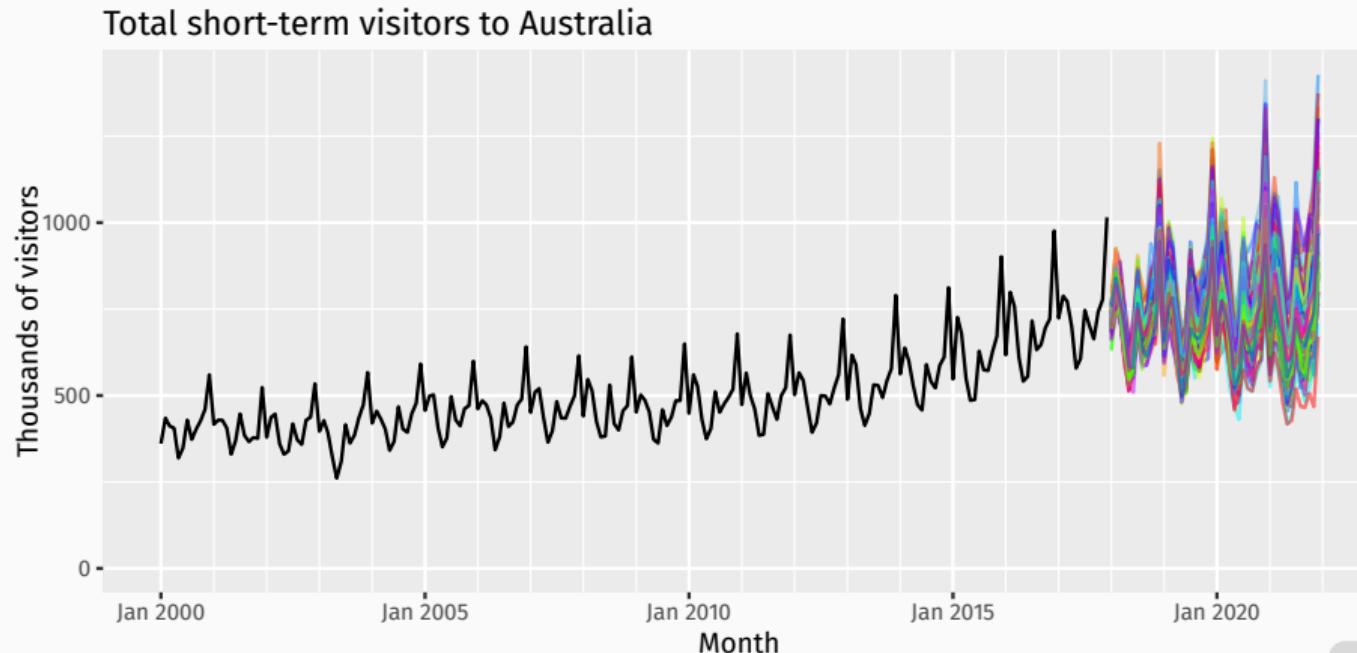
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Simulated futures  
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# Random futures

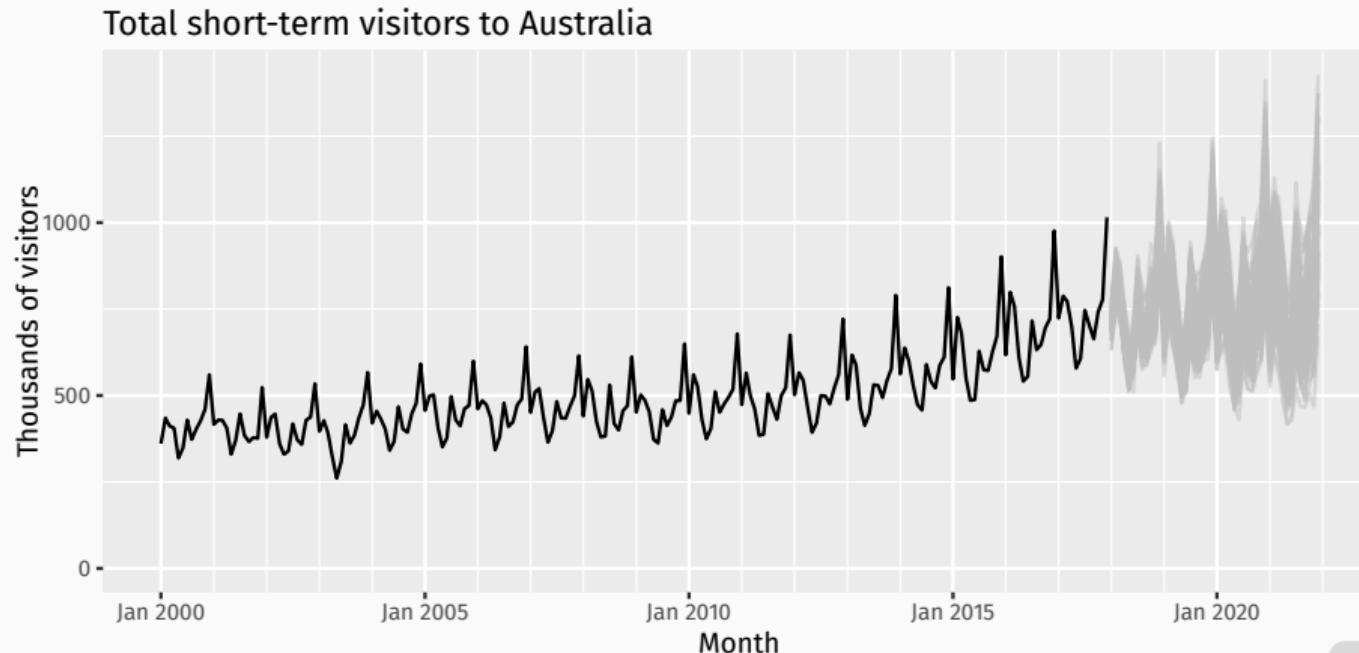
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Simulated futures  
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# Random futures

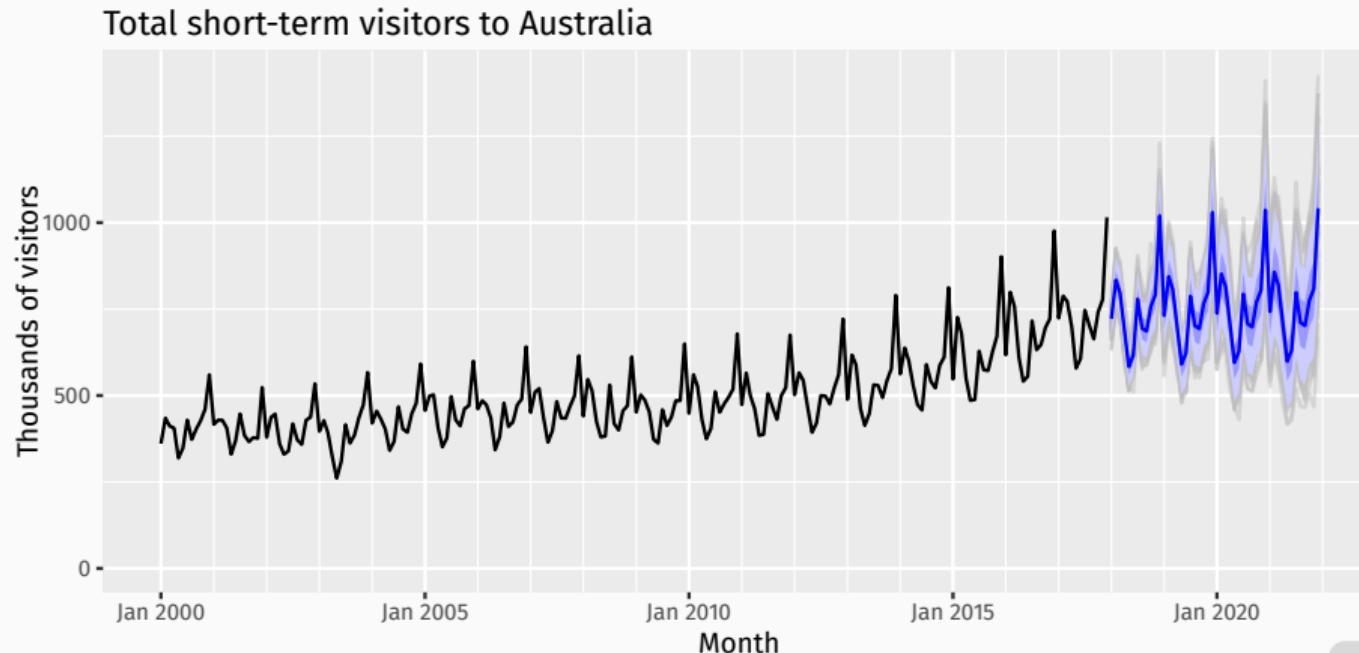
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Simulated futures  
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# Random futures

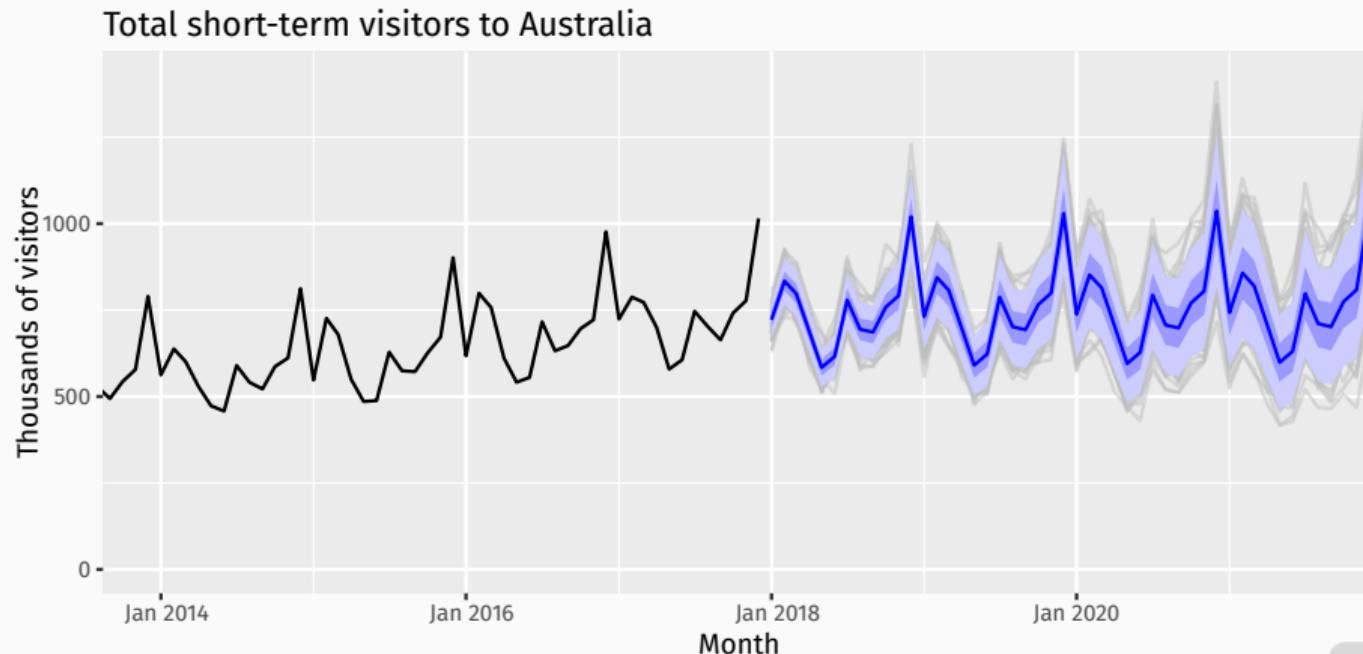
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Simulated futures  
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# Random futures

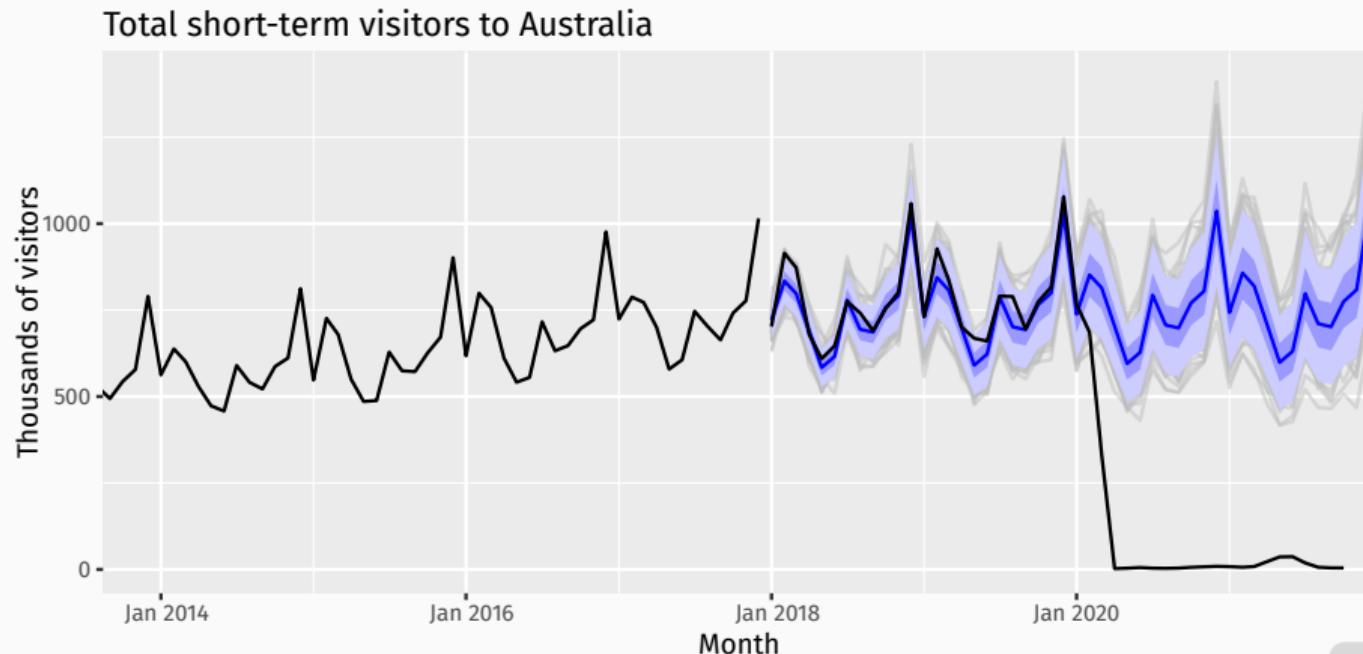
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Simulated futures  
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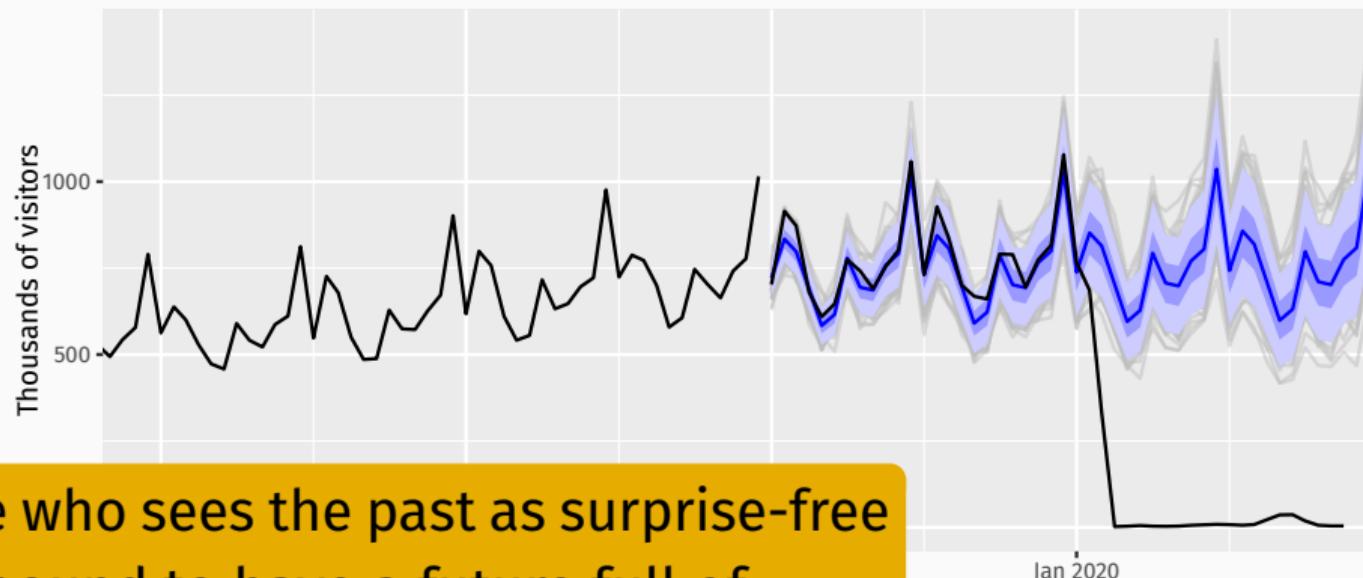


Simulated futures  
from an ETS model

# Random futures

A forecast is an estimate of the probabilities of possible futures.

Total short-term visitors to Australia



“He who sees the past as surprise-free  
is bound to have a future full of  
surprises.”

Jan 2020

Simulated futures  
from an ETS model

# Statistical forecasting

- Thing to be forecast:  $y_{T+h}$ .
- What we know:  $y_1, \dots, y_T$ .
- Forecast distribution:  $y_{T+h|t} = y_{T+h} \mid \{y_1, y_2, \dots, y_T\}$ .
- Point forecast:  $\hat{y}_{T+h|T} = E[y_{T+h} \mid y_1, \dots, y_T]$ .
- Forecast variance:  $\text{Var}[y_t \mid y_1, \dots, y_T]$
- Prediction interval is a range of values of  $y_{T+h}$  with high probability.

# Outline

1 Statistical forecasting

2 Benchmark methods

3 Lab Session 11

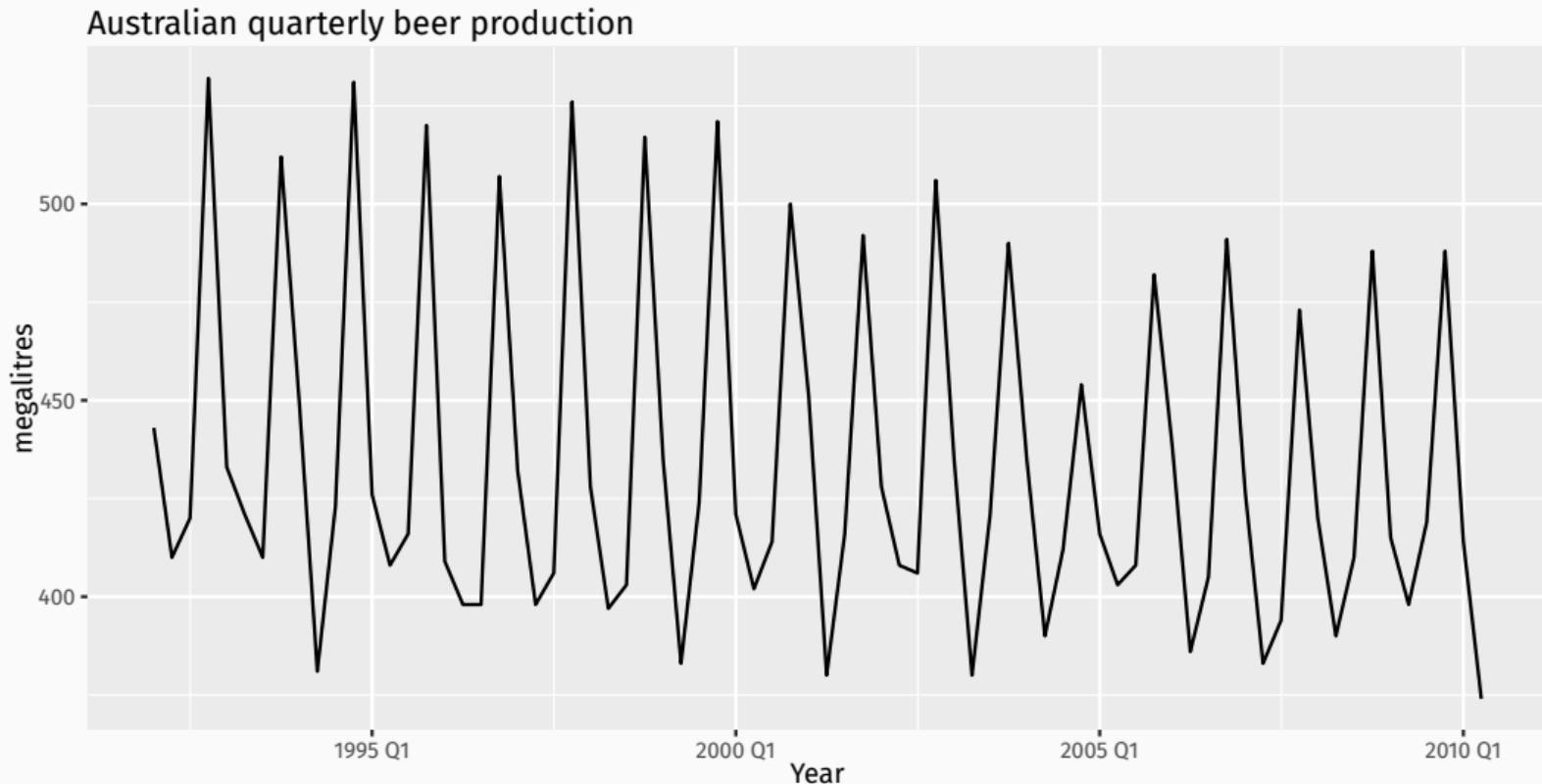
4 Residual diagnostics

5 Lab Session 12

6 Forecast accuracy measures

7 Lab Session 13

# Some simple forecasting methods

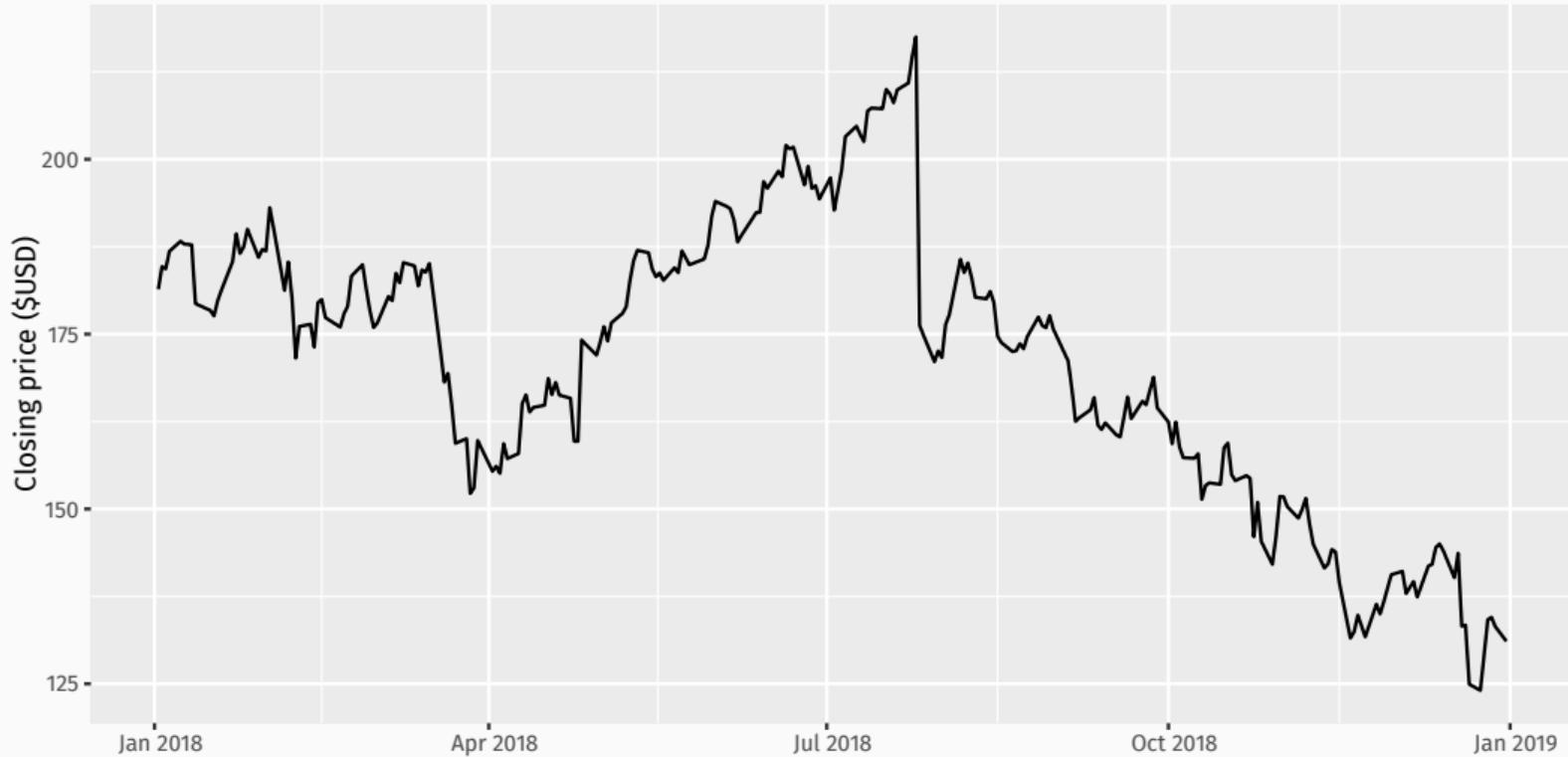


# Some simple forecasting methods



# Some simple forecasting methods

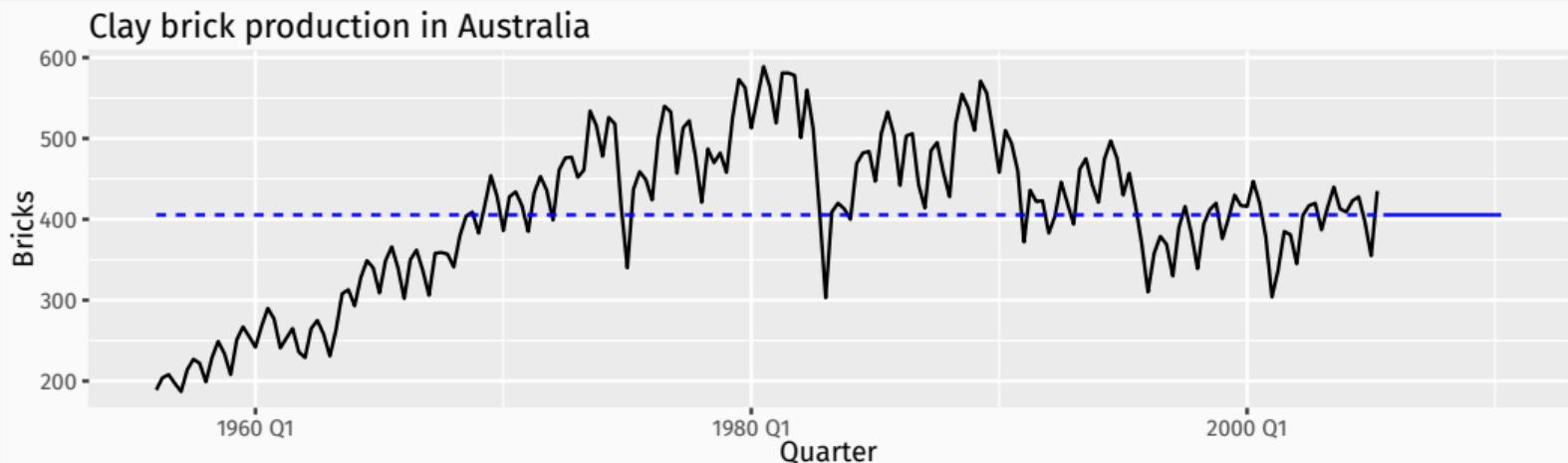
Facebook closing stock price in 2018



# Some simple forecasting methods

## MEAN( $y$ ): Average method

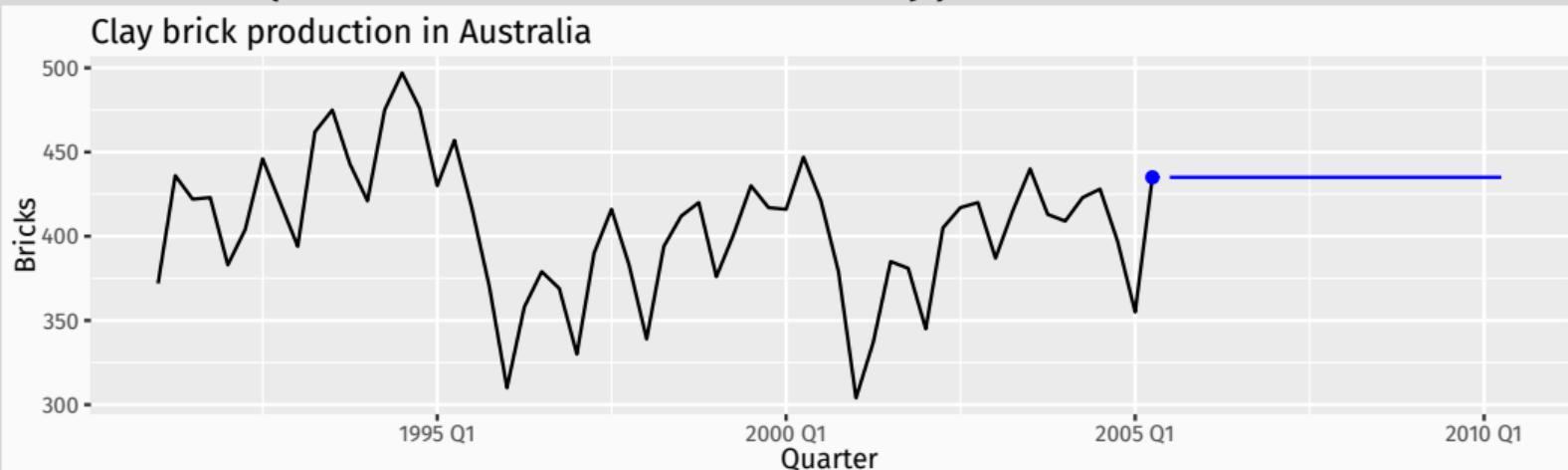
- Forecast of all future values is equal to mean of historical data  $\{y_1, \dots, y_T\}$ .
- Forecasts:  $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$



# Some simple forecasting methods

## NAIVE( $y$ ): Naïve method

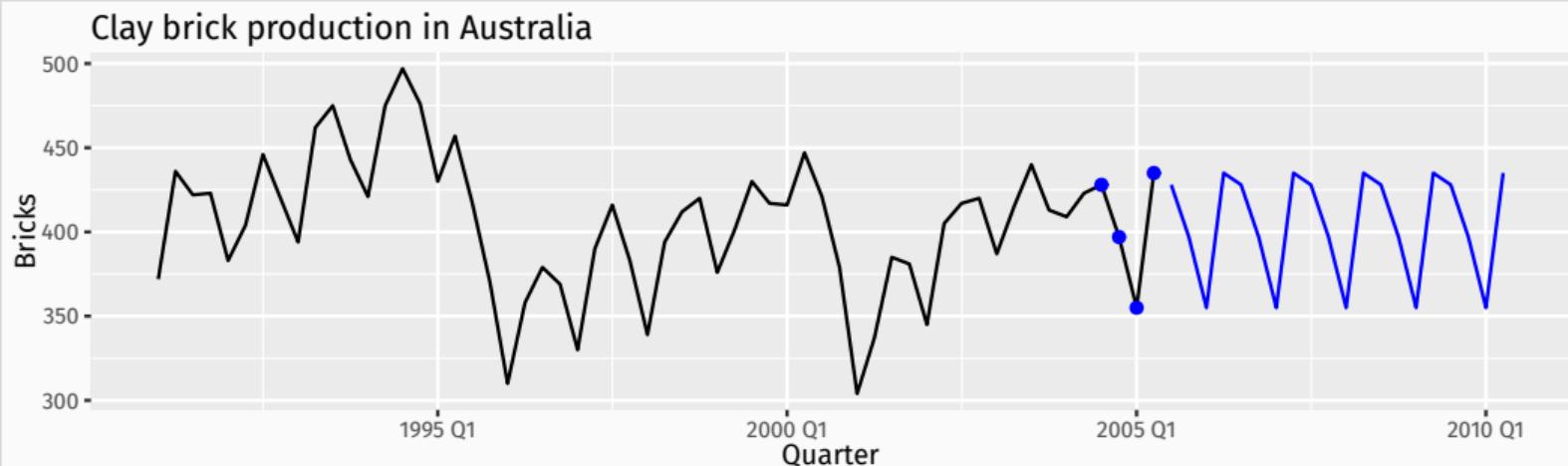
- Forecasts equal to last observed value.
- Forecasts:  $\hat{y}_{T+h|T} = y_T$ .
- Consequence of efficient market hypothesis.



# Some simple forecasting methods

## SNAIVE( $y \sim \text{lag}(m)$ ): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts:  $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$ , where  $m$  = seasonal period and  $k$  is the integer part of  $(h - 1)/m$ .



# Some simple forecasting methods

## RW(y ~ drift()): Drift method

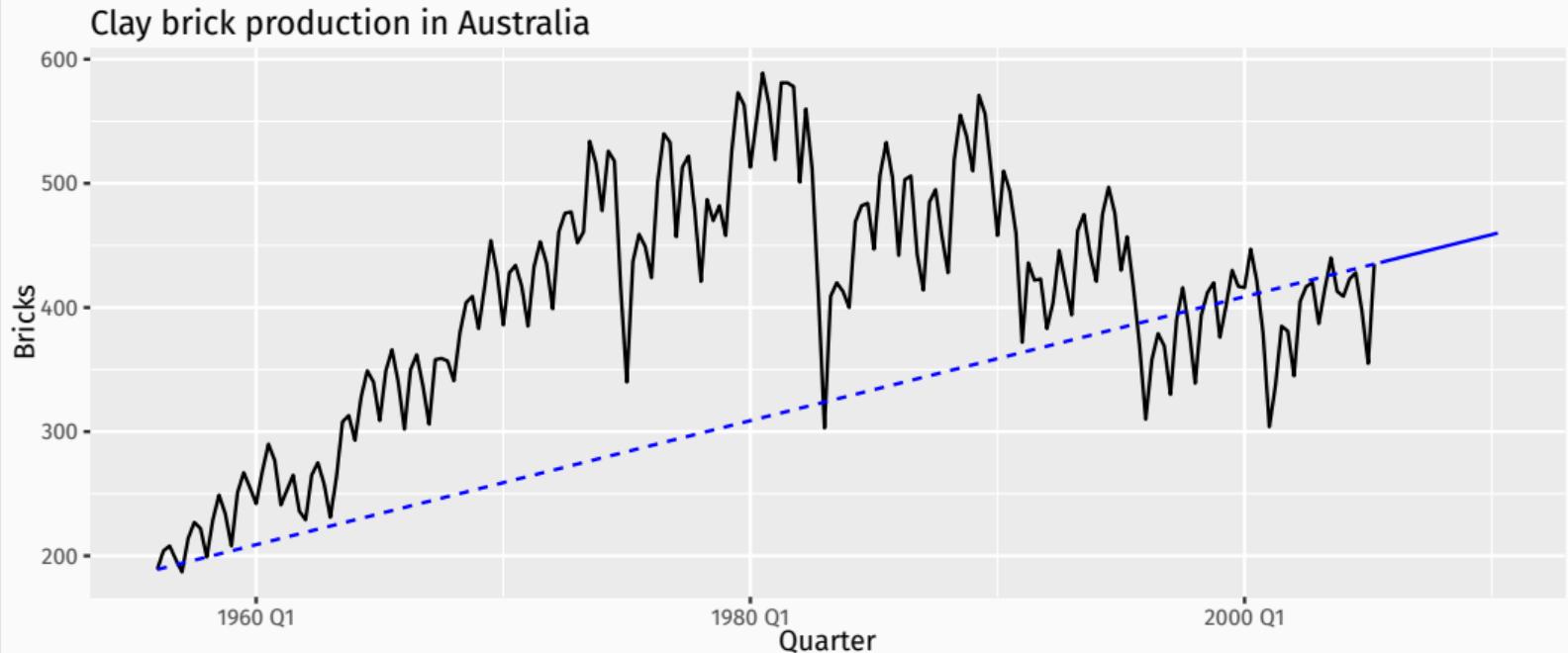
- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

# Some simple forecasting methods

## Drift method



# Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production |>  
  filter(!is.na(Bricks)) |>  
  model(  
    `Seasonal_naïve` = SNAIVE(Bricks),  
    `Naïve` = NAIVE(Bricks),  
    Drift = RW(Bricks ~ drift()),  
    Mean = MEAN(Bricks)  
)
```

```
# A mable: 1 x 4  
  Seasonal_naïve   Naïve          Drift      Mean  
  <model> <model>       <model> <model>  
1     <SNAIVE> <NAIVE> <RW w/ drift> <MEAN>
```

A `mable` is a model table, each cell corresponds to a fitted model.

# Producing forecasts

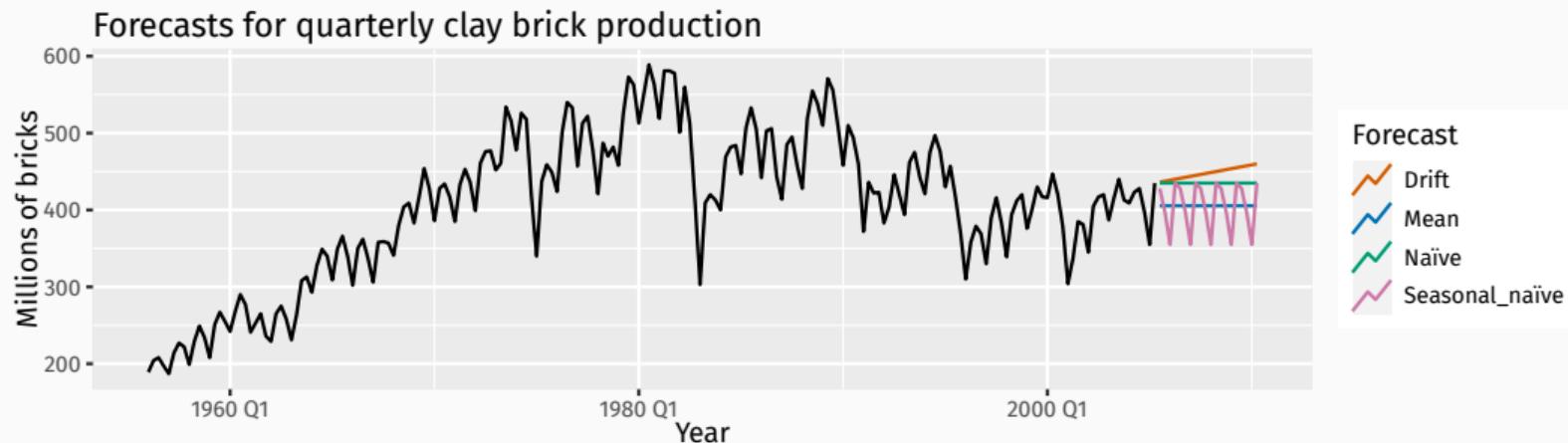
```
brick_fc <- brick_fit |>  
  forecast(h = "5 years")
```

```
# A fable: 80 x 4 [1Q]  
# Key:     .model [4]  
  
.model      Quarter      Bricks .mean  
<chr>       <qtr>       <dist> <dbl>  
1 Seasonal_naïve 2005 Q3 N(428, 2336)    428  
2 Seasonal_naïve 2005 Q4 N(397, 2336)    397  
3 Seasonal_naïve 2006 Q1 N(355, 2336)    355  
4 Seasonal_naïve 2006 Q2 N(435, 2336)    435  
# i 76 more rows
```

A fable is a forecast table with point forecasts and distributions.

# Visualising forecasts

```
brick_fc |>  
  autoplot(aus_production, level = NULL) +  
  labs(title = "Forecasts for quarterly clay brick production",  
       x = "Year", y = "Millions of bricks") +  
  guides(colour = guide_legend(title = "Forecast"))
```



# Prediction intervals

```
brick_fc |> hilo(level = c(50, 75))
```

#	.model	Quarter	Bricks	.mean	`50%`	`75%`
	<chr>	<qtr>	<dist>	<dbl>	<hilo>	<hilo>
1	Seasonal_naïve	2005 Q3	N(428, 2336)	428	[395, 461]	[372, 484]
2	Seasonal_naïve	2005 Q4	N(397, 2336)	397	[364, 430]	[341, 453]
3	Seasonal_naïve	2006 Q1	N(355, 2336)	355	[322, 388]	[299, 411]
4	Seasonal_naïve	2006 Q2	N(435, 2336)	435	[402, 468]	[379, 491]
5	Seasonal_naïve	2006 Q3	N(428, 4672)	428	[382, 474]	[349, 507]
6	Seasonal_naïve	2006 Q4	N(397, 4672)	397	[351, 443]	[318, 476]
7	Seasonal_naïve	2007 Q1	N(355, 4672)	355	[309, 401]	[276, 434]
8	Seasonal_naïve	2007 Q2	N(435, 4672)	435	[389, 481]	[356, 514]
9	Seasonal_naïve	2007 Q3	N(428, 7008)	428	[372, 484]	[332, 524]
10	Seasonal_naïve	2007 Q4	N(397, 7008)	397	[341, 453]	[301, 493]

# Prediction intervals

```
brick_fc |>  
  hilo(level = c(50, 75)) |>  
  unpack_hilo(c("50%", "75%"))
```

#	A tsibble: 80 x 8 [1Q]	# Key:	.model	[4]	.model	Quarter	Bricks	.mean	`50%_lower`	`50%_upper`	`75%_lower`	`75%_upper`
					<chr>	<qtr>	<dist>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	Seasonal ~ 2005 Q3 N(428, 2336)				1	2005 Q3	N(428, 2336)	428	395.	461.	372.	
2	Seasonal ~ 2005 Q4 N(397, 2336)				2	2005 Q4	N(397, 2336)	397	364.	430.	341.	
3	Seasonal ~ 2006 Q1 N(355, 2336)				3	2006 Q1	N(355, 2336)	355	322.	388.	299.	
4	Seasonal ~ 2006 Q2 N(435, 2336)				4	2006 Q2	N(435, 2336)	435	402.	468.	379.	
5	Seasonal ~ 2006 Q3 N(428, 4672)				5	2006 Q3	N(428, 4672)	428	382.	474.	349.	
6	Seasonal ~ 2006 Q4 N(397, 4672)				6	2006 Q4	N(397, 4672)	397	351.	443.	318.	
7	Seasonal ~ 2007 Q1 N(355, 4672)				7	2007 Q1	N(355, 4672)	355	309.	401.	276.	
8	Seasonal ~ 2007 Q2 N(435, 4672)				8	2007 Q2	N(435, 4672)	435	389.	481.	356.	
9	Seasonal ~ 2007 Q3 N(428, 4672)				9	2007 Q3	N(428, 4672)	428	376.	464.	333.	
10	Seasonal ~ 2007 Q4 N(397, 4672)				10	2007 Q4	N(397, 4672)	397	351.	443.	318.	
11	Seasonal ~ 2008 Q1 N(355, 4672)				11	2008 Q1	N(355, 4672)	355	309.	401.	276.	
12	Seasonal ~ 2008 Q2 N(435, 4672)				12	2008 Q2	N(435, 4672)	435	389.	481.	356.	
13	Seasonal ~ 2008 Q3 N(428, 4672)				13	2008 Q3	N(428, 4672)	428	376.	464.	333.	
14	Seasonal ~ 2008 Q4 N(397, 4672)				14	2008 Q4	N(397, 4672)	397	351.	443.	318.	
15	Seasonal ~ 2009 Q1 N(355, 4672)				15	2009 Q1	N(355, 4672)	355	309.	401.	276.	
16	Seasonal ~ 2009 Q2 N(435, 4672)				16	2009 Q2	N(435, 4672)	435	389.	481.	356.	
17	Seasonal ~ 2009 Q3 N(428, 4672)				17	2009 Q3	N(428, 4672)	428	376.	464.	333.	
18	Seasonal ~ 2009 Q4 N(397, 4672)				18	2009 Q4	N(397, 4672)	397	351.	443.	318.	
19	Seasonal ~ 2010 Q1 N(355, 4672)				19	2010 Q1	N(355, 4672)	355	309.	401.	276.	
20	Seasonal ~ 2010 Q2 N(435, 4672)				20	2010 Q2	N(435, 4672)	435	389.	481.	356.	
21	Seasonal ~ 2010 Q3 N(428, 4672)				21	2010 Q3	N(428, 4672)	428	376.	464.	333.	
22	Seasonal ~ 2010 Q4 N(397, 4672)				22	2010 Q4	N(397, 4672)	397	351.	443.	318.	
23	Seasonal ~ 2011 Q1 N(355, 4672)				23	2011 Q1	N(355, 4672)	355	309.	401.	276.	
24	Seasonal ~ 2011 Q2 N(435, 4672)				24	2011 Q2	N(435, 4672)	435	389.	481.	356.	
25	Seasonal ~ 2011 Q3 N(428, 4672)				25	2011 Q3	N(428, 4672)	428	376.	464.	333.	
26	Seasonal ~ 2011 Q4 N(397, 4672)				26	2011 Q4	N(397, 4672)	397	351.	443.	318.	
27	Seasonal ~ 2012 Q1 N(355, 4672)				27	2012 Q1	N(355, 4672)	355	309.	401.	276.	
28	Seasonal ~ 2012 Q2 N(435, 4672)				28	2012 Q2	N(435, 4672)	435	389.	481.	356.	
29	Seasonal ~ 2012 Q3 N(428, 4672)				29	2012 Q3	N(428, 4672)	428	376.	464.	333.	
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31	Seasonal ~ 2013 Q1 N(355, 4672)				31	2013 Q1	N(355, 4672)	355	309.	401.	276.	
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43	Seasonal ~ 2016 Q1 N(355, 4672)				43	2016 Q1	N(355, 4672)	355	309.	401.	276.	
44	Seasonal ~ 2016 Q2 N(435, 4672)				44	2016 Q2	N(435, 4672)	435	389.	481.	356.	
45	Seasonal ~ 2016 Q3 N(428, 4672)				45	2016 Q3	N(428, 4672)	428	376.	464.	333.	
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50	Seasonal ~ 2017 Q4 N(397, 4672)				50	2017 Q4	N(397, 4672)	397	351.	443.	318.	
51	Seasonal ~ 2018 Q1 N(355, 4672)				51	2018 Q1	N(355, 4672)	355	309.	401.	276.	
52	Seasonal ~ 2018 Q2 N(435, 4672)				52	2018 Q2	N(435, 4672)	435	389.	481.	356.	
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58	Seasonal ~ 2019 Q4 N(397, 4672)				58	2019 Q4	N(397, 4672)	397	351.	443.	318.	
59	Seasonal ~ 2020 Q1 N(355, 4672)				59	2020 Q1	N(355, 4672)	355	309.	401.	276.	
60	Seasonal ~ 2020 Q2 N(435, 4672)				60	2020 Q2	N(435, 4672)	435	389.	481.	356.	
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66	Seasonal ~ 2021 Q4 N(397, 4672)				66	2021 Q4	N(397, 4672)	397	351.	443.	318.	
67	Seasonal ~ 2022 Q1 N(355, 4672)				67	2022 Q1	N(355, 4672)	355	309.	401.	276.	
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83	Seasonal ~ 2026 Q1 N(355, 4672)				83	2026 Q1	N(355, 4672)	355	309.	401.	276.	
84	Seasonal ~ 2026 Q2 N(435, 4672)				84	2026 Q2	N(435, 4672)	435	389.	481.	356.	
85	Seasonal ~ 2026 Q3 N(428, 4672)				85	2026 Q3	N(428, 4672)	428	376.	464.	333.	
86	Seasonal ~ 2026 Q4 N(397, 4672)				86	2026 Q4	N(397, 4672)	397	351.	443.	318.	
87	Seasonal ~ 2027 Q1 N(355, 4672)				87	2027 Q1	N(355, 4672)	355	309.	401.	276.	
88	Seasonal ~ 2027 Q2 N(435, 4672)				88	2027 Q2	N(435, 4672)	435	389.	481.	356.	
89	Seasonal ~ 2027 Q3 N(428, 4672)				89	2027 Q3	N(428, 4672)	428	376.	464.	333.	
90	Seasonal ~ 2027 Q4 N(397, 4672)				90	2027 Q4	N(397, 4672)	397	351.	443.	318.	
91	Seasonal ~ 2028 Q1 N(355, 4672)				91	2028 Q1	N(355, 4672)	355	309.	401.	276.	
92	Seasonal ~ 2028 Q2 N(435, 4672)				92	2028 Q2	N(435, 4672)	435	389.	481.	356.	
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94	Seasonal ~ 2028 Q4 N(397, 4672)				94	2028 Q4	N(397, 4672)	397	351.	443.	318.	
95	Seasonal ~ 2029 Q1 N(355, 4672)				95	2029 Q1	N(355, 4672)	355	309.	401.	276.	
96	Seasonal ~ 2029 Q2 N(435, 4672)				96	2029 Q2	N(435, 4672)	435	389.	481.	356.	
97	Seasonal ~ 2029 Q3 N(428, 4672)				97	2029 Q3	N(428, 4672)	428	376.	464.	333.	
98	Seasonal ~ 2029 Q4 N(397, 4672)				98	2029 Q4	N(397, 4672)	397	351.	443.	318.	
99	Seasonal ~ 2030 Q1 N(355, 4672)				99	2030 Q1	N(355, 4672)	35				

# Outline

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4 Residual diagnostics

5 Lab Session 12

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7 Lab Session 13

# Lab Session 11

- Produce forecasts using an appropriate benchmark method for household wealth (`hh_budget`). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for Australian takeaway food turnover (`aus_retail`). Plot the results using `autoplot()`.

# Outline

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# Fitted values

- $\hat{y}_{t|t-1}$  is the forecast of  $y_t$  based on observations  $y_1, \dots, y_{t-1}$ .
- We call these “fitted values”.
- Sometimes drop the subscript:  $\hat{y}_t \equiv \hat{y}_{t|t-1}$ .
- Often not true forecasts since parameters are estimated on all data.

## For example:

- $\hat{y}_t = \bar{y}$  for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$  for drift method.

# Forecasting residuals

**Residuals in forecasting:** difference between observed value and its fitted value:  $e_t = y_t - \hat{y}_{t|t-1}$ .

# Forecasting residuals

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## Assumptions

- 1  $\{e_t\}$  uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2  $\{e_t\}$  have mean zero. If they don't, then forecasts are biased.

# Forecasting residuals

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## Assumptions

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- 2  $\{e_t\}$  have mean zero. If they don't, then forecasts are biased.

## Useful properties (for prediction intervals)

- 3  $\{e_t\}$  have constant variance.
- 4  $\{e_t\}$  are normally distributed.

# Facebook closing stock price

```
fb_stock <- gafa_stock |>  
  filter(Symbol == "FB")  
fb_stock |> autoplot(Close)
```



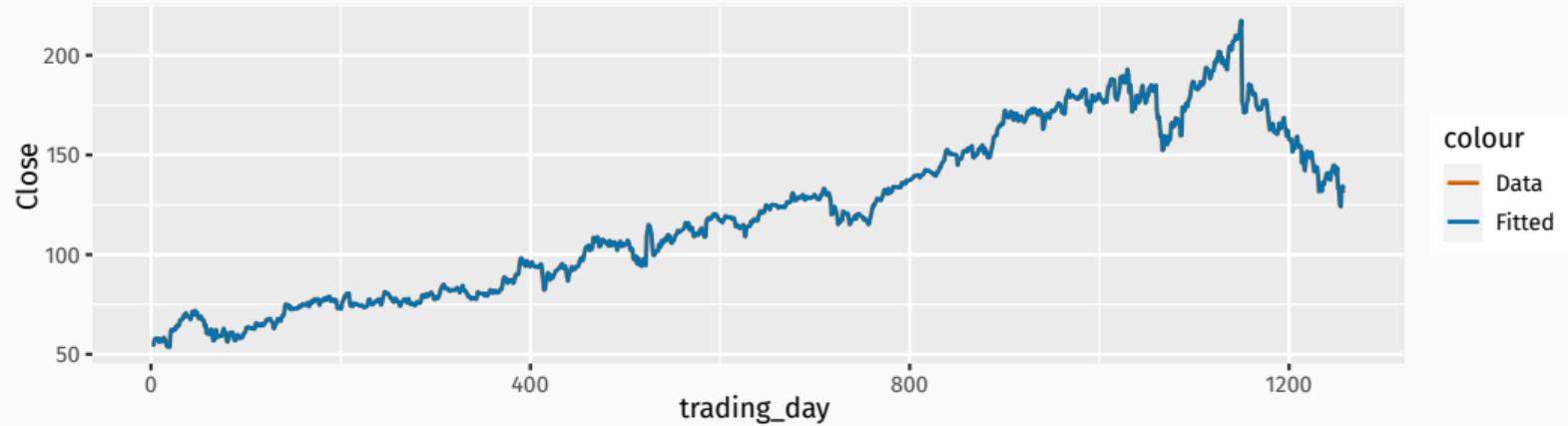
# Facebook closing stock price

```
fb_stock <- fb_stock |>  
  mutate(trading_day = row_number()) |>  
  update_tsibble(index = trading_day, regular = TRUE)  
fit <- fb_stock |> model(NAIVE(Close))  
augment(fit)
```

```
# A tsibble: 1,258 x 7 [1]  
# Key:     Symbol, .model [1]  
  Symbol .model      trading_day Close .fitted .resid .innov  
  <chr>  <chr>        <int>  <dbl>   <dbl>   <dbl>   <dbl>  
1 FB     NAIVE(Close)       1  54.7    NA    NA    NA  
2 FB     NAIVE(Close)       2  54.6  54.7 -0.150 -0.150  
3 FB     NAIVE(Close)       3  57.2  54.6  2.64  2.64  
4 FB     NAIVE(Close)       4  57.9  57.2  0.720 0.720  
5 FB     NAIVE(Close)       5  58.2  57.9  0.310 0.310  
6 FB     NAIVE(Close)       6  57.2  58.2 -1.01 -1.01  
7 FB     NAIVE(Close)       7  57.9  57.2  0.720 0.720  
8 FB     NAIVE(Close)       8  55.9  57.9 -2.03 -2.03  
9 FB     NAIVE(Close)       9  57.7  55.9  1.83  1.83
```

# Facebook closing stock price

```
augment(fit) |>  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



# Facebook closing stock price

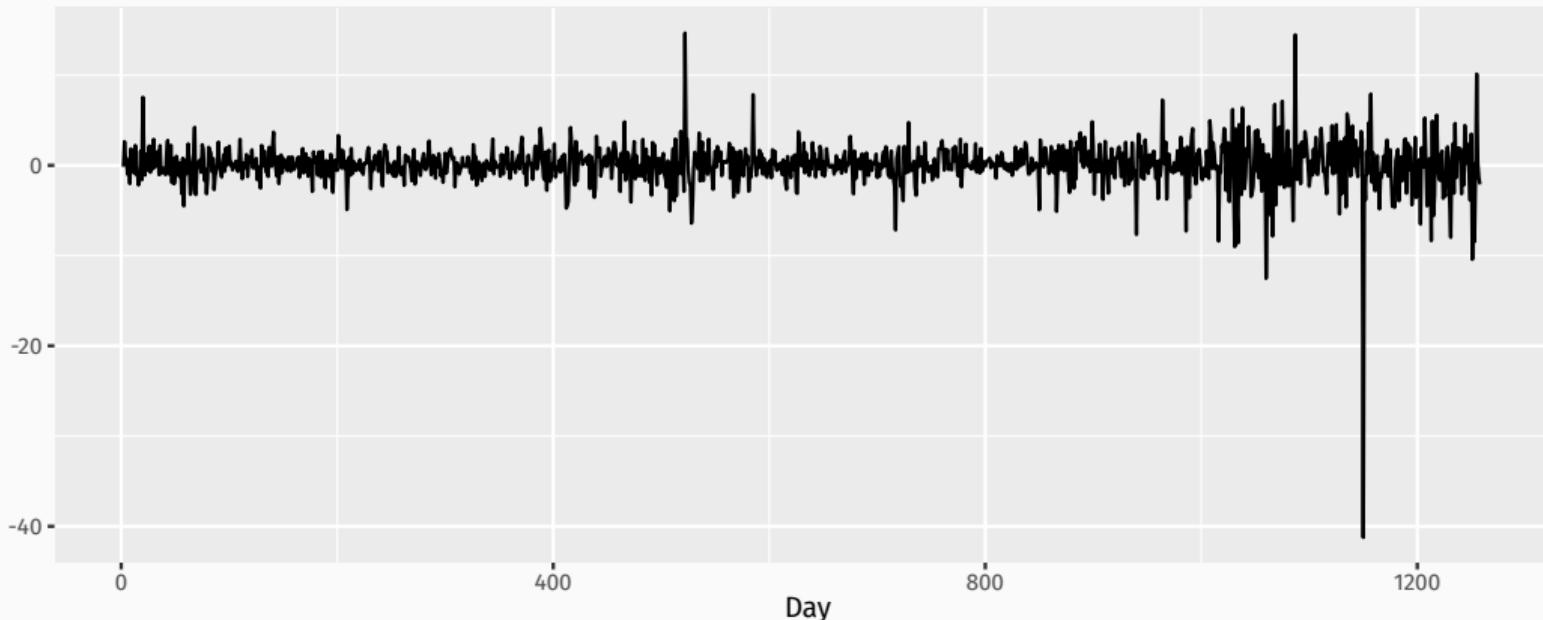
```
augment(fit) |>  
  filter(trading_day > 1100) |>  
  ggplot(aes(x = trading_day)) +  
  geom_line(aes(y = Close, colour = "Data")) +  
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



# Facebook closing stock price

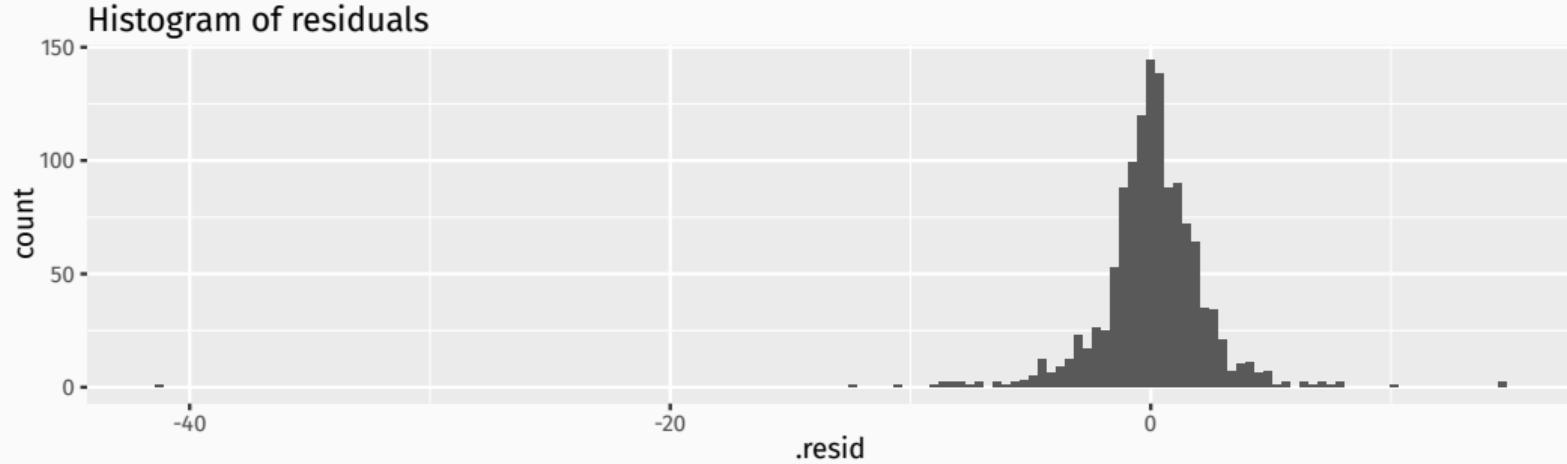
```
augment(fit) |>  
  autoplot(.resid) +  
  labs(x = "Day", y = "", title = "Residuals from naïve method")
```

Residuals from naïve method



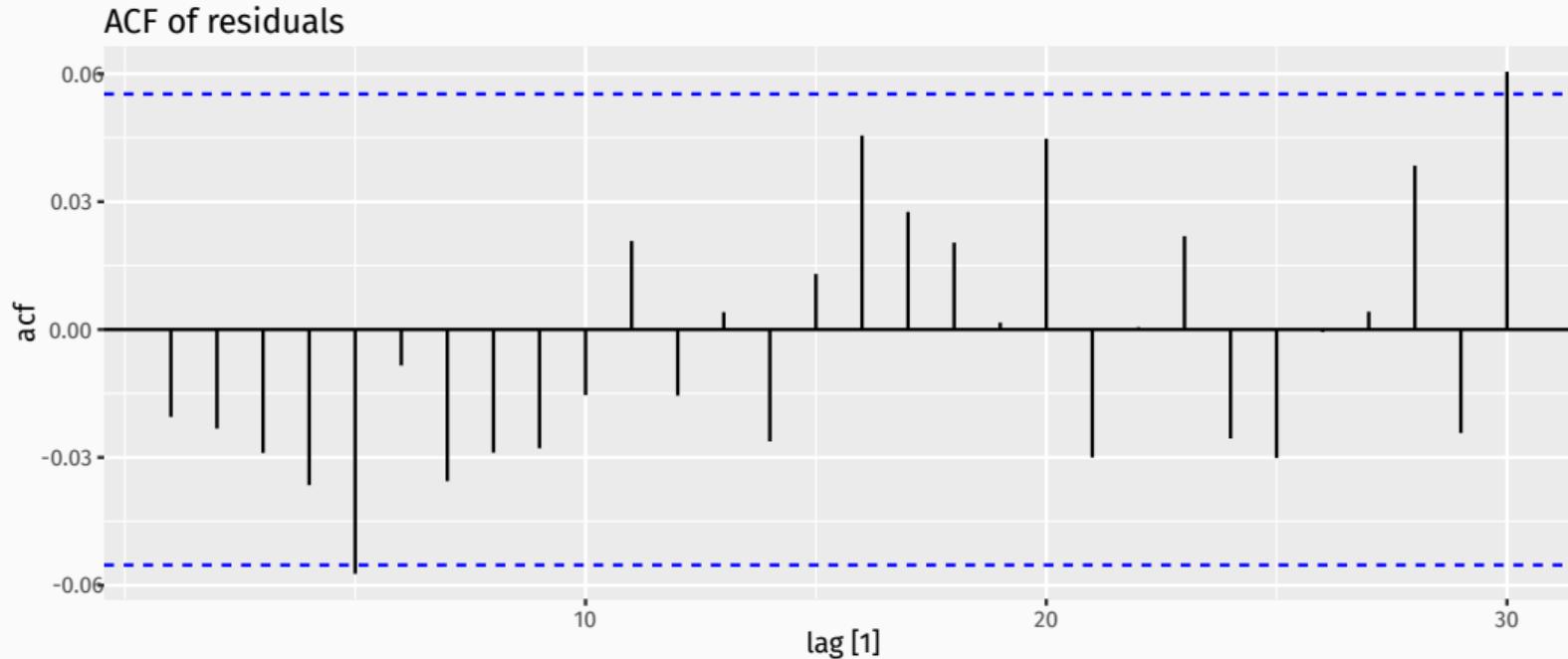
# Facebook closing stock price

```
augment(fit) |>  
  ggplot(aes(x = .resid)) +  
  geom_histogram(bins = 150) +  
  labs(title = "Histogram of residuals")
```



# Facebook closing stock price

```
augment(fit) |>  
ACF(.resid) |>  
autoplot() + labs(title = "ACF of residuals")
```

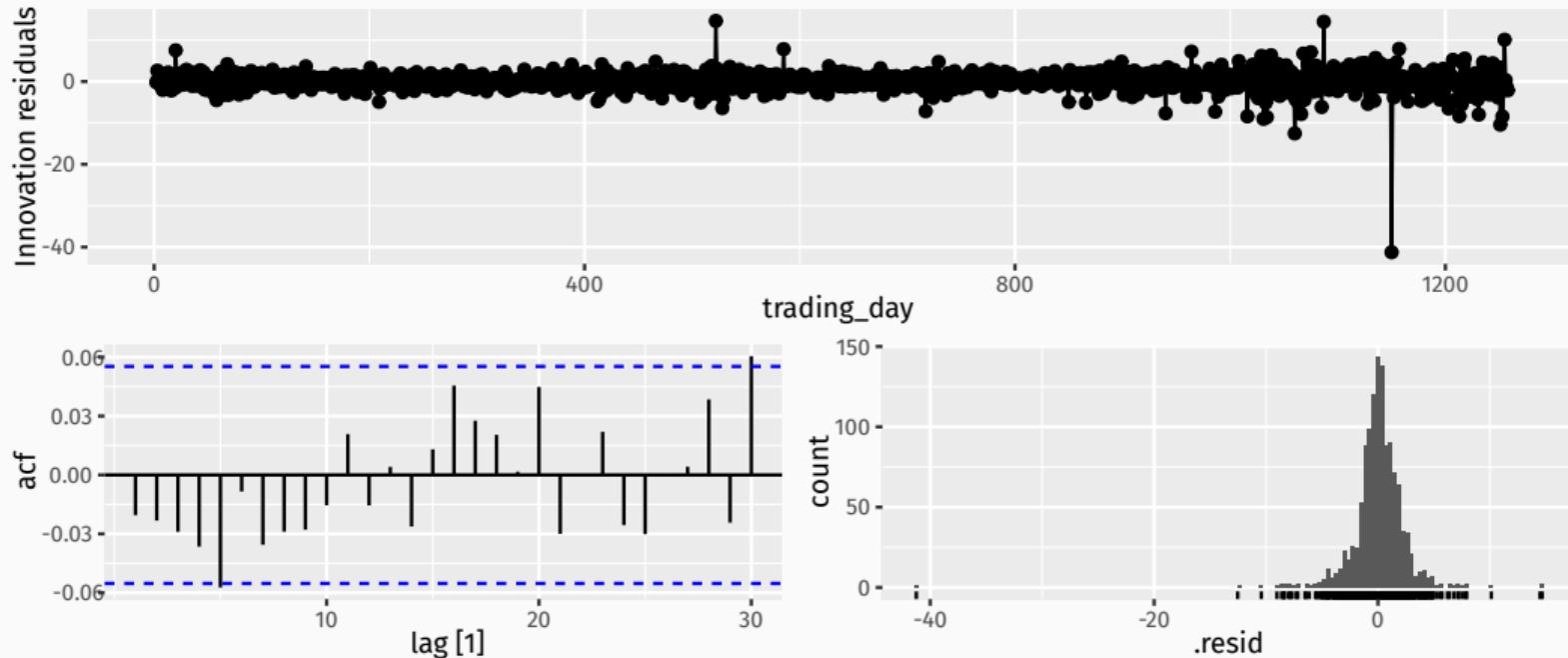


# ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

# Combined diagnostic graph

```
fit |> gg_tsresiduals()
```



# Ljung-Box test

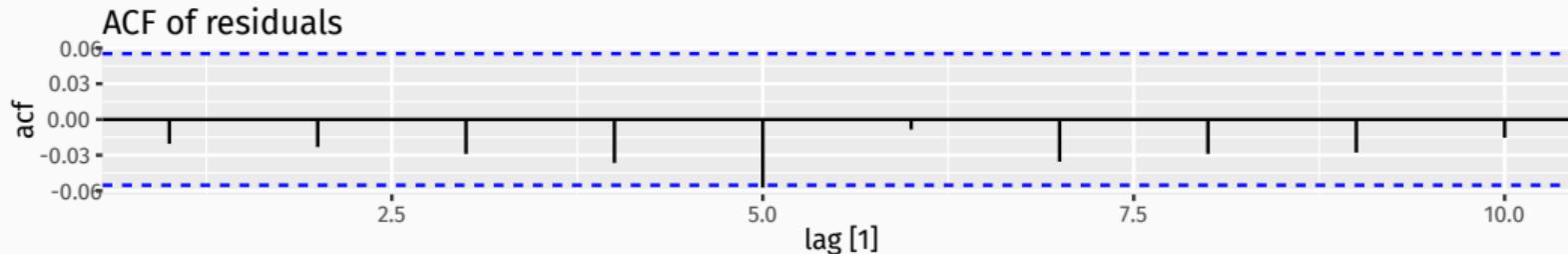
Test whether *whole set* of  $r_k$  values is significantly different from zero set.

$$Q = T(T + 2) \sum_{k=1}^h (T - k)^{-1} r_k^2 \quad \text{where } h = \max \text{ lag and } T = \# \text{ observations.}$$

- If each  $r_k$  close to zero,  $Q$  will be **small**.
- If some  $r_k$  values large (+ or -),  $Q$  will be **large**.
- My preferences:  $h = 10$  for non-seasonal data,  $h = 2m$  for seasonal data.
- If data are WN,  $Q \sim \chi^2$  with  $(h - K)$  degrees of freedom where  $K$  = no. parameters in model.
- When applied to raw data, set  $K = 0$ .

# Ljung-Box test

$$Q = T(T + 2) \sum_{k=1}^h (T - k)^{-1} r_k^2 \quad \text{where } h = \max \text{ lag and } T = \# \text{ observations.}$$



```
# lag=h and dof=K
augment(fit) |> features(.resid, ljung_box, dof = 0, lag = 10)
```

```
# A tibble: 1 x 4
  Symbol .model      lb_stat lb_pvalue
  <chr>  <chr>       <dbl>     <dbl>
1 FB     NAIVE(Close) 12.1      0.276
```

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## Lab Session 12

- Compute seasonal naïve forecasts for quarterly Australian beer production.
- Test if the residuals are white noise. What do you conclude?

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# Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- Forecast accuracy is based only on the test set.

## Forecast errors

Forecast “error”: the difference between an observed value and its forecast.

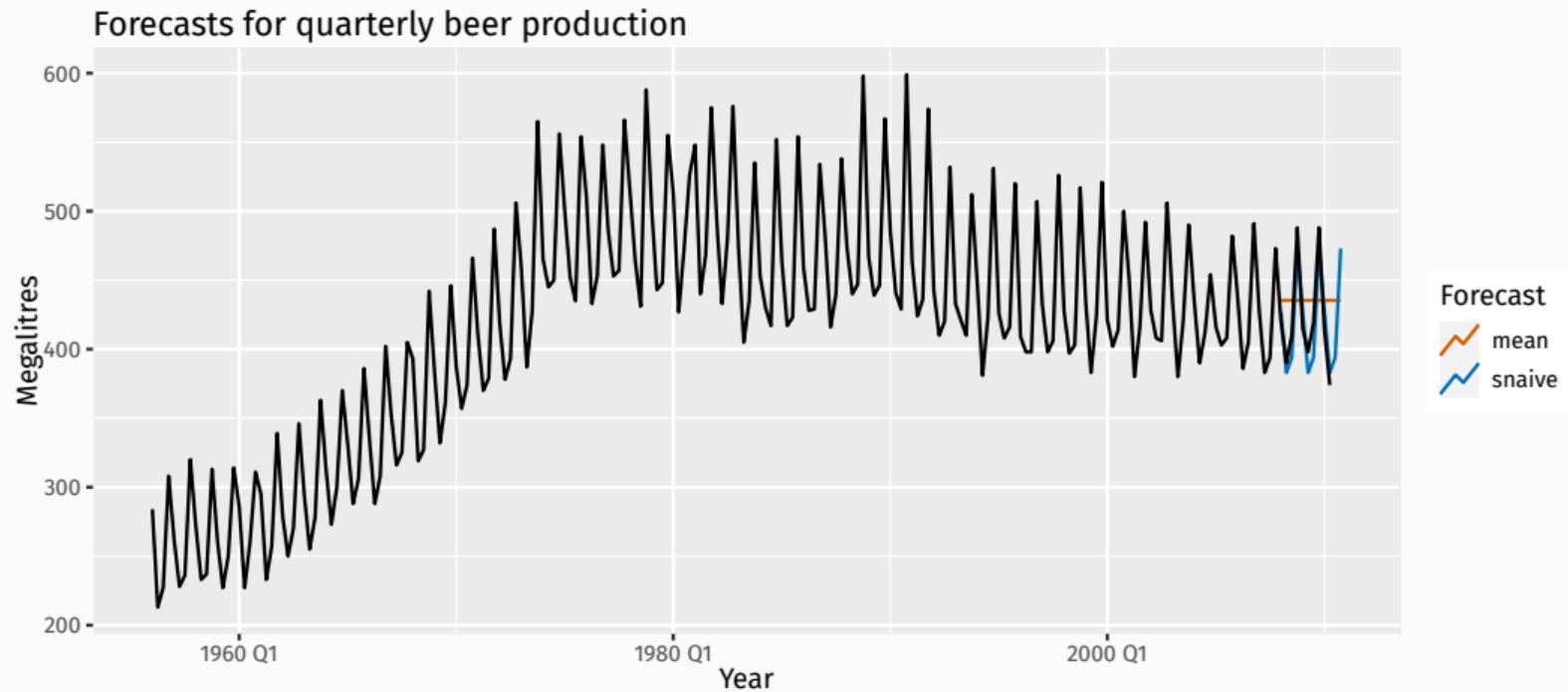
$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by  $\{y_1, \dots, y_T\}$

# Measures of forecast accuracy

```
beer_fit <- aus_production |>
  filter(between(year(Quarter), 1992, 2007)) |>
  model(
    snaive = SNAIVE(Beer),
    mean = MEAN(Beer)
  )
beer_fit |>
  forecast(h = "3 years") |>
  autoplot(aus_production, level = NULL) +
  labs(title ="Forecasts for quarterly beer production",
       x ="Year", y ="Megalitres") +
  guides(colour = guide_legend(title = "Forecast"))
```

# Measures of forecast accuracy



# Measures of forecast accuracy

$y_{T+h}$  =  $(T + h)$ th observation,  $h = 1, \dots, H$

$\hat{y}_{T+h|T}$  = its forecast based on data up to time  $T$ .

$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$

MAE =  $\text{mean}(|e_{T+h}|)$

MSE =  $\text{mean}(e_{T+h}^2)$

RMSE =  $\sqrt{\text{mean}(e_{T+h}^2)}$

MAPE =  $100\text{mean}(|e_{T+h}|/|y_{T+h}|)$

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MSE =  $\text{mean}(e_{T+h}^2)$

RMSE =  $\sqrt{\text{mean}(e_{T+h}^2)}$

MAPE =  $100\text{mean}(|e_{T+h}|/|y_{T+h}|)$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if  $y_t \gg 0$

# Measures of forecast accuracy

## Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where  $Q$  is a stable measure of the scale of the time series  $\{y_t\}$ .

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

# Measures of forecast accuracy

## Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where  $Q$  is a stable measure of the scale of the time series  $\{y_t\}$ .

Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

# Measures of forecast accuracy

```
beer_fc <- forecast(beer_fit, h = "3 years")
accuracy(beer_fc, aus_production)
```

```
# A tibble: 2 x 10
  .model .type     ME   RMSE    MAE    MPE   MAPE   MASE RMSSE     ACF1
  <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
1 mean    Test   -13.8  38.4  34.8 -3.97  8.28  2.20  1.96 -0.0691
2 snaive   Test    5.2  14.3  13.4  1.15  3.17  0.847 0.729  0.132
```

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# Lab Session 13

- Create a training set for household wealth (`hh_budget`) by withholding the last four years as a test set.
- Fit all the appropriate benchmark methods to the training set and forecast the periods covered by the test set.
- Compute the accuracy of your forecasts. Which method does best?
- Repeat the exercise using the Australian takeaway food turnover data (`aus_retail`) with a test set of four years.