bit.ly/fable2022

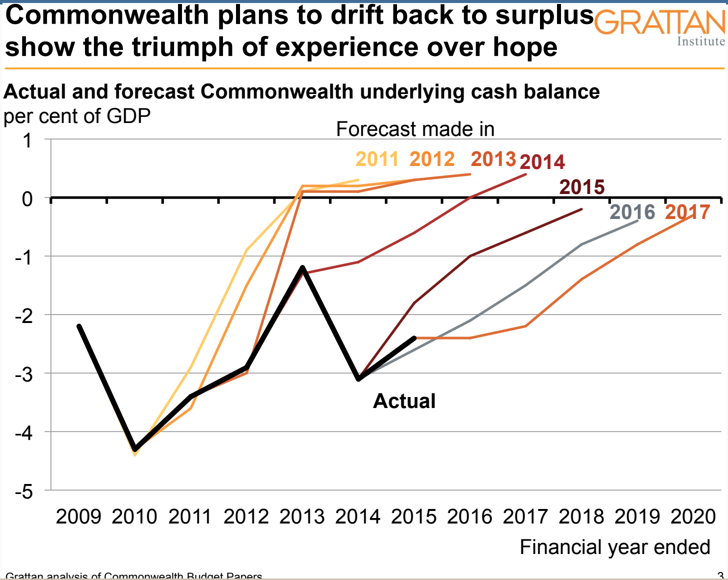
Time Series Analysis & Forecasting Using R

6. Introduction to forecasting

Outline

Outline

Forecasting is difficult



What can we forecast?

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Which is easiest to forecast?

1. daily electricity demand in 3 days time
2. timing of next Halley's comet appearance
3. time of sunrise this day next year
4. Google stock price tomorrow
5. Google stock price in 6 months time
6. maximum temperature tomorrow
7. exchange rate of $US/AUS next week
8. total sales of drugs in Australian pharmacies next month

Which is easiest to forecast?

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6. maximum temperature tomorrow
7. exchange rate of $US/AUS next week
8. total sales of drugs in Australian pharmacies next month

- how do we measure "easiest"?
- what makes something easy/difficult to forecast?

Factors affecting forecastability

Something is easier to forecast if:

- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

Random futures

A forecast is an estimate of the probabilities of possible futures.

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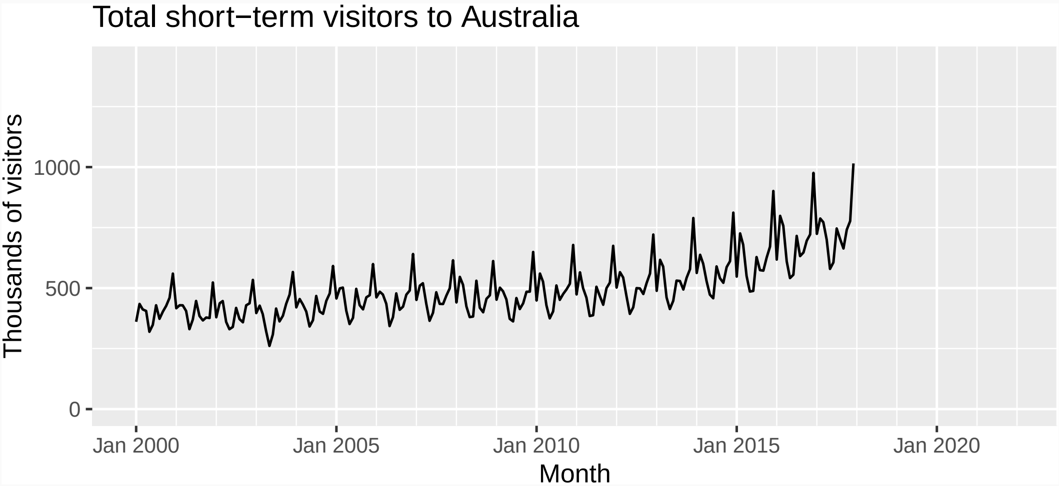

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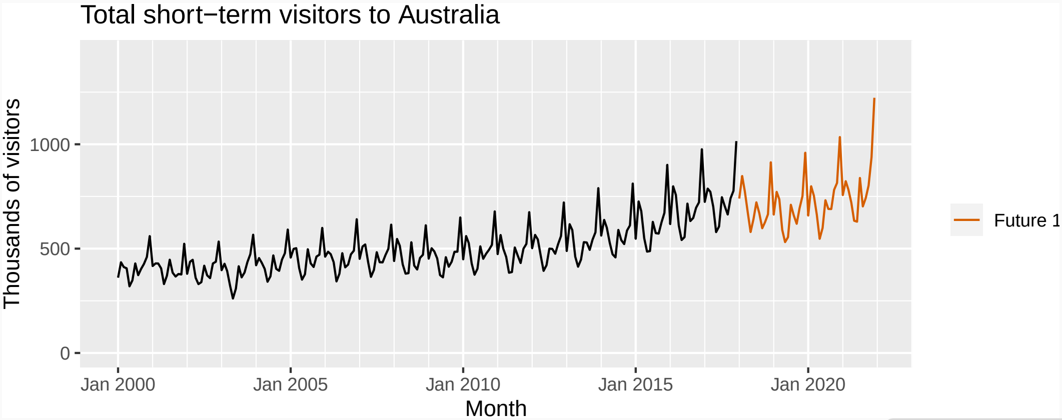

Simulated futures from an ETS model

Random futures

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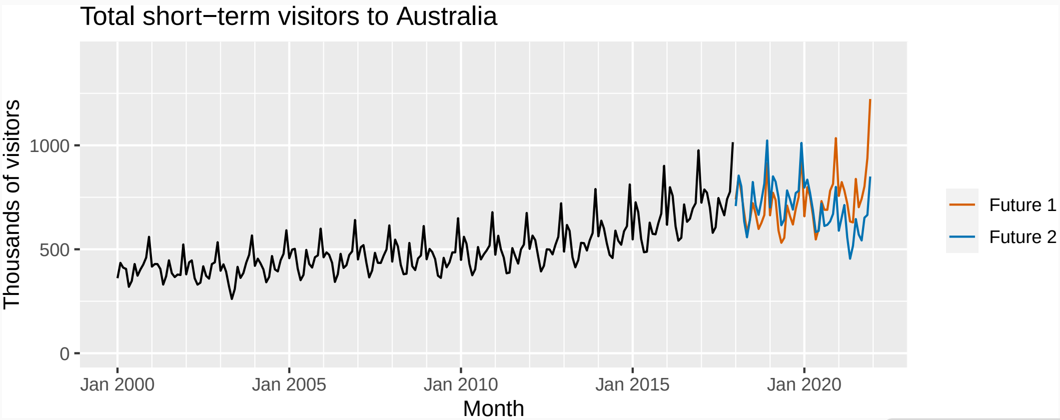

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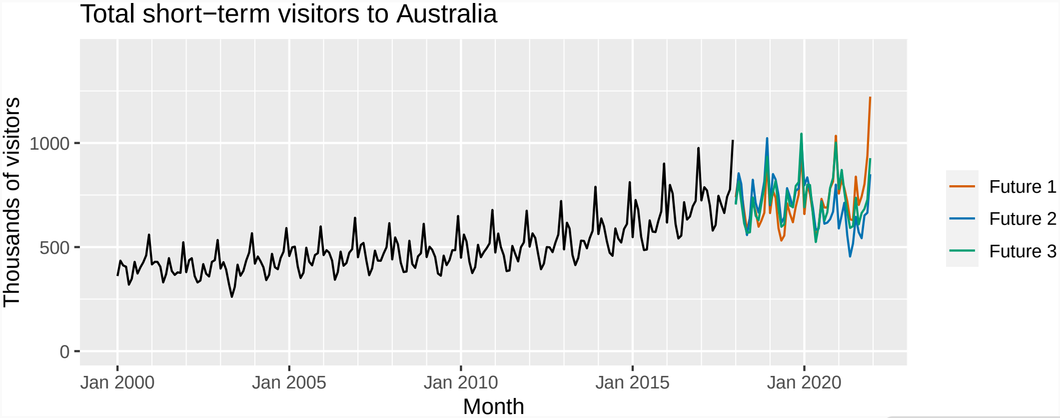

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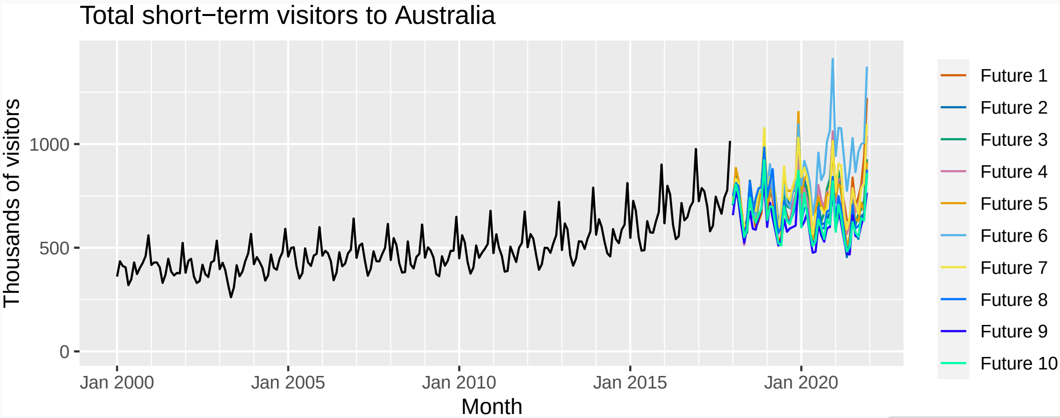

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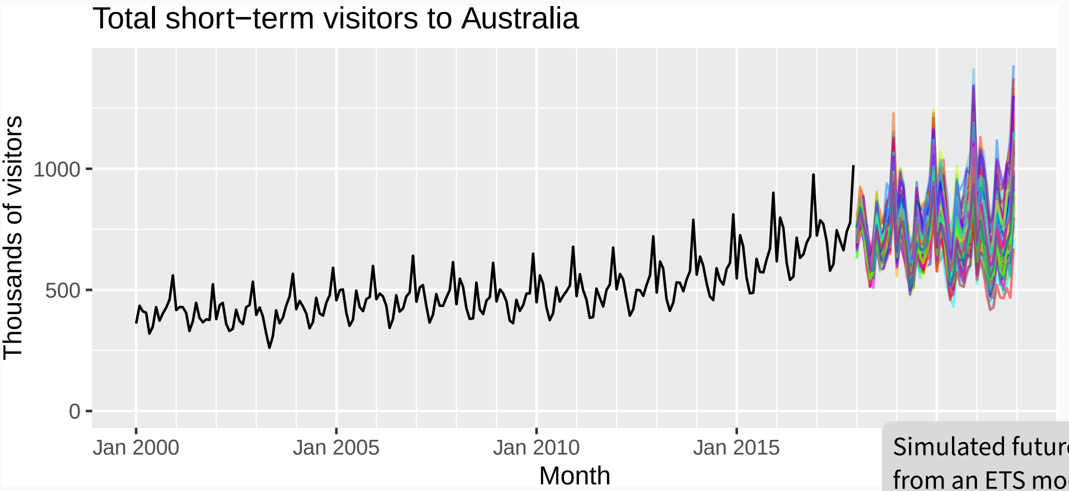
A forecast is an estimate of the probabilities of possible futures.



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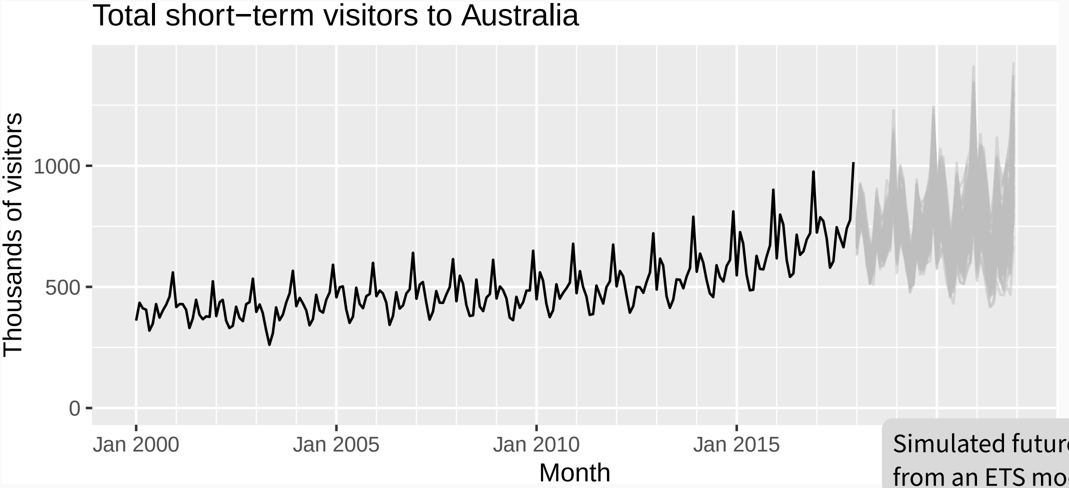

Simulated futures from an ETS model

Random futures

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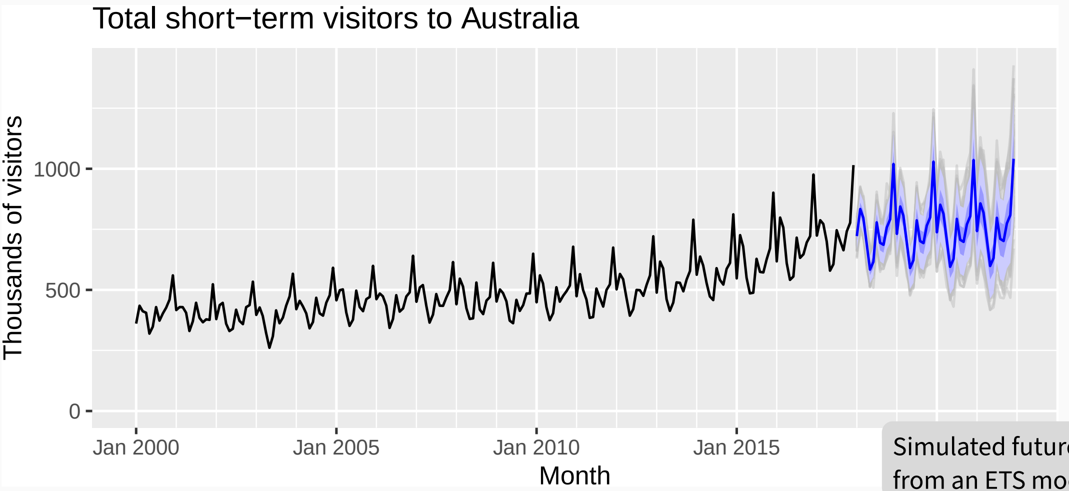

Simulated futures from an ETS model

Random futures

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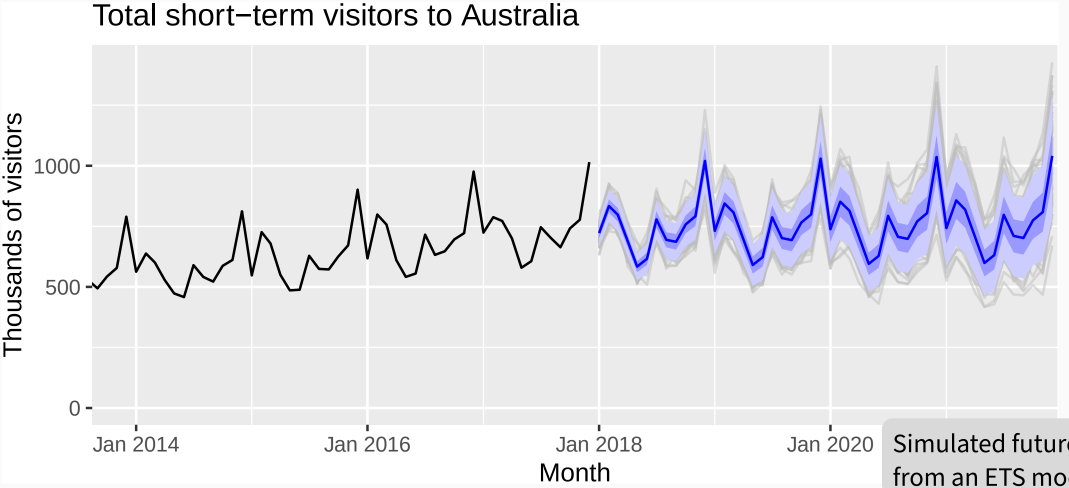

Simulated futures from an ETS model

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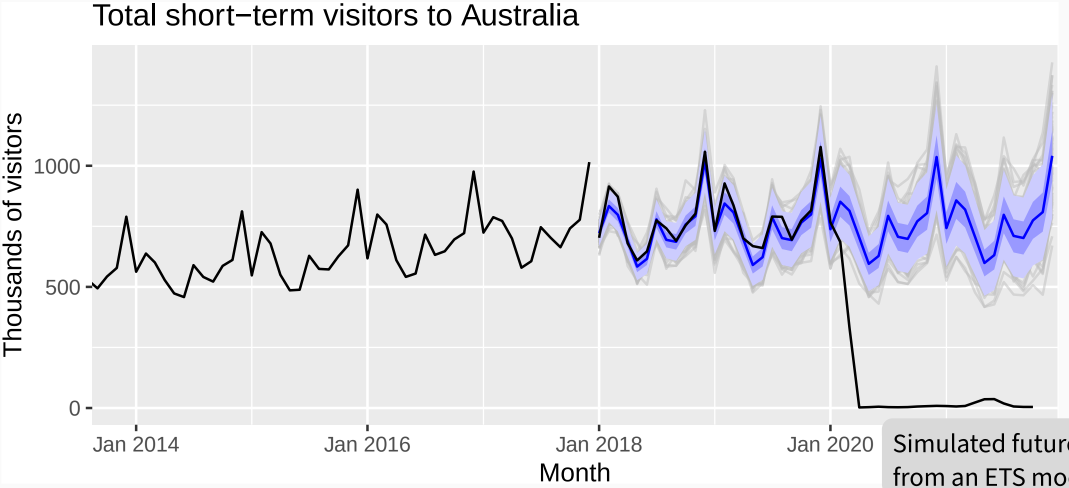
A forecast is an estimate of the probabilities of possible futures.



Simulated futures from an ETS model

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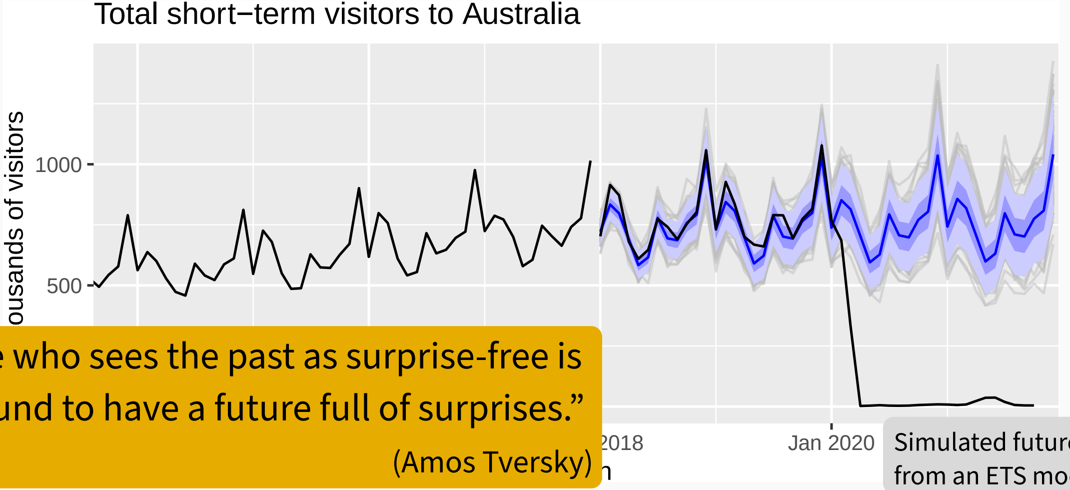


“He who sees the past as surprise-free is bound to have a future full of surprises.”

(Amos Tversky)

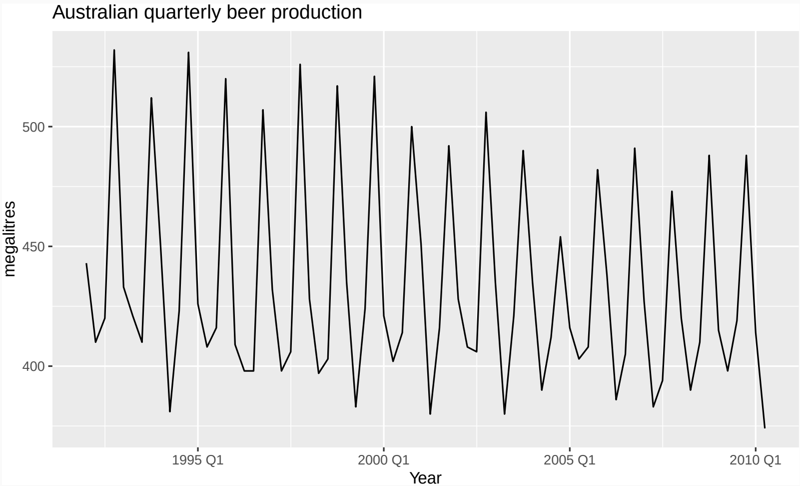
Simulated futures from an ETS model

Statistical forecasting

- Thing to be forecast: y_{T+h}.
- What we know: $y_1, \dots, y_T$.
- Forecast distribution: $y_{T+h|t} = y_{T+h} \mid \{y_1, y_2, \dots, y_T\}$.
- Point forecast: $\hat{y}_{T+h|T} = \text{E}[y_{T+h} \mid y_1, \dots, y_T]$.
- Forecast variance: $\text{Var}[y_t \mid y_1, \dots, y_T]$
- Prediction interval is a range of values of y_{T+h} with high probability.

Outline

Some simple forecasting methods



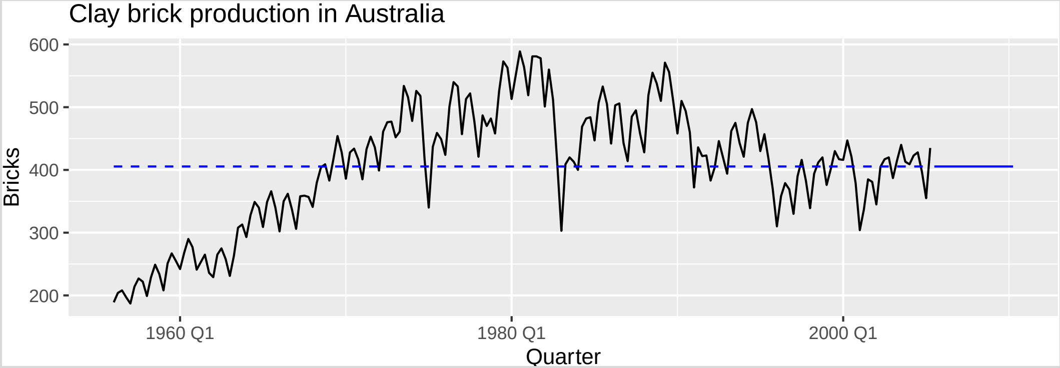
Some simple forecasting methods

Some simple forecasting methods

Some simple forecasting methods

`MEAN(y)`: Average method

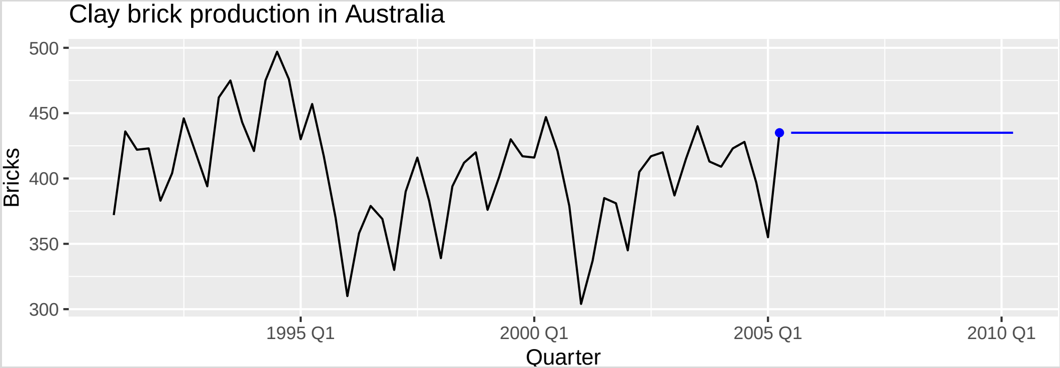
- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$



Some simple forecasting methods

`NAIVE(y)`: Naïve method

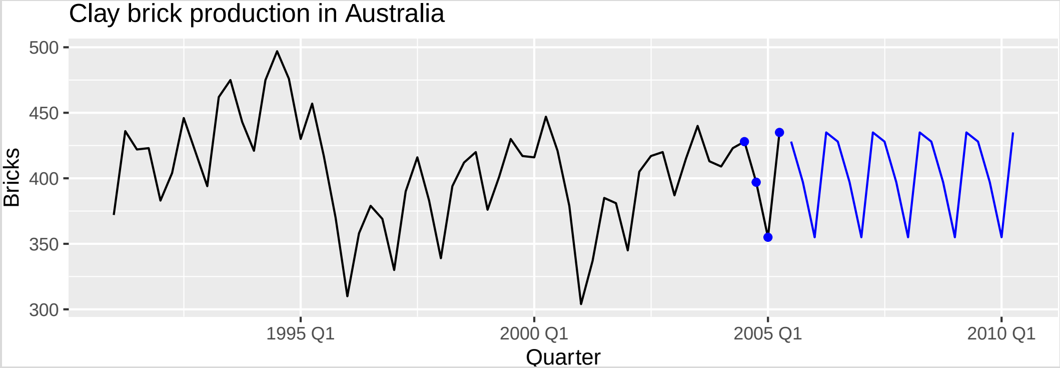
- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.



Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where $m =$ seasonal period and k is the integer part of $(h-1)/m$.



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

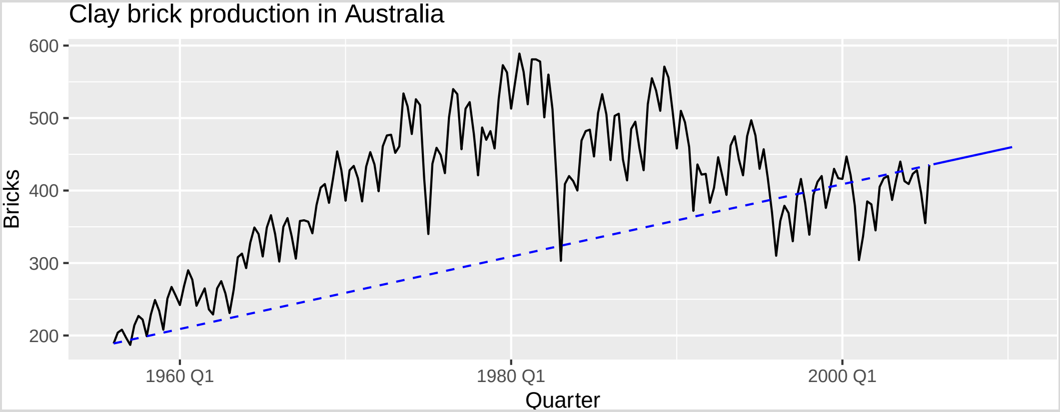
- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method



Model fitting

The `model()` function trains models to data.

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    `Seasonal_naïve` = SNAIVE(Bricks),
    `Naïve` = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

```
## # A mable: 1 x 4
##   Seasonal_naïve   Naïve           Drift    Mean
##          <model> <model>         <model> <model>
## 1       <SNAIVE> <NAIVE> <RW w/ drift>  <MEAN>
```

A mable is a model table, each cell corresponds to a fitted model.

Producing forecasts

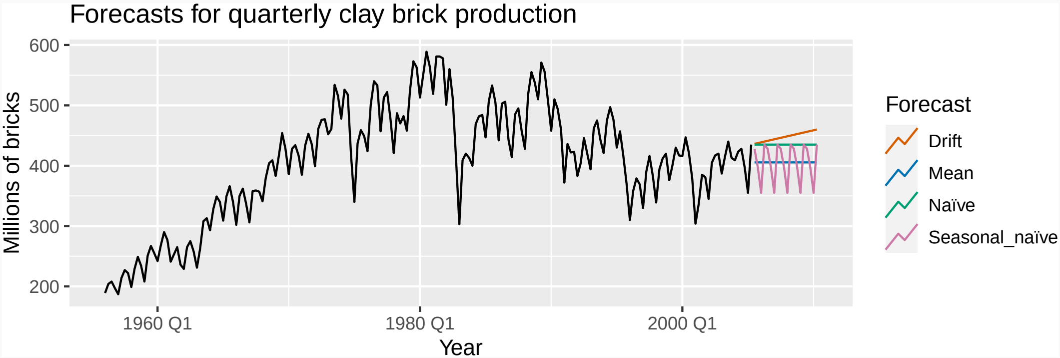
```
brick_fc <- brick_fit |>
  forecast(h = "5 years")
```

```
## # A fable: 80 x 4 [1Q]
## # Key:     .model [4]
##   .model         Quarter       Bricks .mean
##   <chr>            <qtr>       <dist> <dbl>
## 1 Seasonal_naïve 2005 Q3 N(428, 2336)   428
## 2 Seasonal_naïve 2005 Q4 N(397, 2336)   397
## 3 Seasonal_naïve 2006 Q1 N(355, 2336)   355
## 4 Seasonal_naïve 2006 Q2 N(435, 2336)   435
## # ... with 76 more rows
```

A `fable` is a forecast table with point forecasts and distributions.

Visualising forecasts

```
brick_fc |>
  autoplot(aus_production, level = NULL) +
  labs(title = "Forecasts for quarterly clay brick production",
       x = "Year", y = "Millions of bricks") +
  guides(colour = guide_legend(title = "Forecast"))
```



Prediction intervals

```
brick_fc |> hilo(level = c(50, 75))
```

```
## # A tsibble: 80 x 6 [1Q]
## # Key:       .model [4]
##    .model         Quarter        Bricks .mean              `50%`            `75%`
##    <chr>            <qtr>        <dist> <dbl>             <hilo>           <hilo>
##  1 Seasonal_naïve 2005 Q3 N(428, 2336)   428 [395, 461]50 [372, 484]75
##  2 Seasonal_naïve 2005 Q4 N(397, 2336)   397 [364, 430]50 [341, 453]75
##  3 Seasonal_naïve 2006 Q1 N(355, 2336)   355 [322, 388]50 [299, 411]75
##  4 Seasonal_naïve 2006 Q2 N(435, 2336)   435 [402, 468]50 [379, 491]75
##  5 Seasonal_naïve 2006 Q3 N(428, 4672)   428 [382, 474]50 [349, 507]75
##  6 Seasonal_naïve 2006 Q4 N(397, 4672)   397 [351, 443]50 [318, 476]75
##  7 Seasonal_naïve 2007 Q1 N(355, 4672)   355 [309, 401]50 [276, 434]75
##  8 Seasonal_naïve 2007 Q2 N(435, 4672)   435 [389, 481]50 [356, 514]75
##  9 Seasonal_naïve 2007 Q3 N(428, 7008)   428 [372, 484]50 [332, 524]75
```

Prediction intervals

```r
brick_fc |>
  hilo(level = c(50, 75)) |>
  unpack_hilo(c("50%", "75%"))
```

```
## # A tsibble: 80 x 8 [1Q]
## # Key:       .model [4]
##    .model    Quarter        Bricks .mean `50%_lower` `50%_upper` `75%_lower`
##    <chr>       <qtr>        <dist> <dbl>       <dbl>       <dbl>       <dbl>
##  1 Seasonal~ 2005 Q3 N(428, 2336)   428        395.        461.        372.
##  2 Seasonal~ 2005 Q4 N(397, 2336)   397        364.        430.        341.
##  3 Seasonal~ 2006 Q1 N(355, 2336)   355        322.        388.        299.
##  4 Seasonal~ 2006 Q2 N(435, 2336)   435        402.        468.        379.
##  5 Seasonal~ 2006 Q3 N(428, 4672)   428        382.        474.        349.
##  6 Seasonal~ 2006 Q4 N(397, 4672)   397        351.        443.        318.
##  7 Seasonal~ 2007 Q1 N(355, 4672)   355        309.        401.        276.
```

Outline

Lab Session 11

- Produce forecasts using an appropriate benchmark method for household wealth (`hh_budget`). Plot the results using `autoplot()`.
- Produce forecasts using an appropriate benchmark method for Australian takeaway food turnover (`aus_retail`). Plot the results using `autoplot()`.

Outline

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_t$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T-1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

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Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

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Assumptions

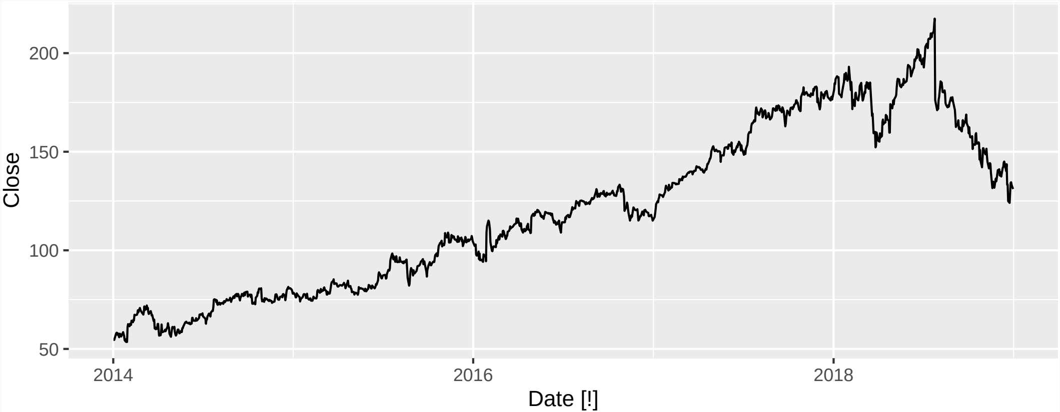
1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock <- gafa_stock |>
  filter(Symbol == "FB")
fb_stock |> autoplot(Close)
```



Facebook closing stock price

```r
fb_stock <- fb_stock |>
  mutate(trading_day = row_number()) |>
  update_tsibble(index = trading_day, regular = TRUE)
fit <- fb_stock |> model(NAIVE(Close))
augment(fit)
```

```
## # A tsibble: 1,258 x 7 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid .innov
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA     NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01  -1.01
##  7 FB     NAIVE(Close)           7  57.9    57.2  0.720  0.720
##  8 FB     NAIVE(Close)           8  55.9    57.9 -2.03  -2.03
```

Facebook closing stock price

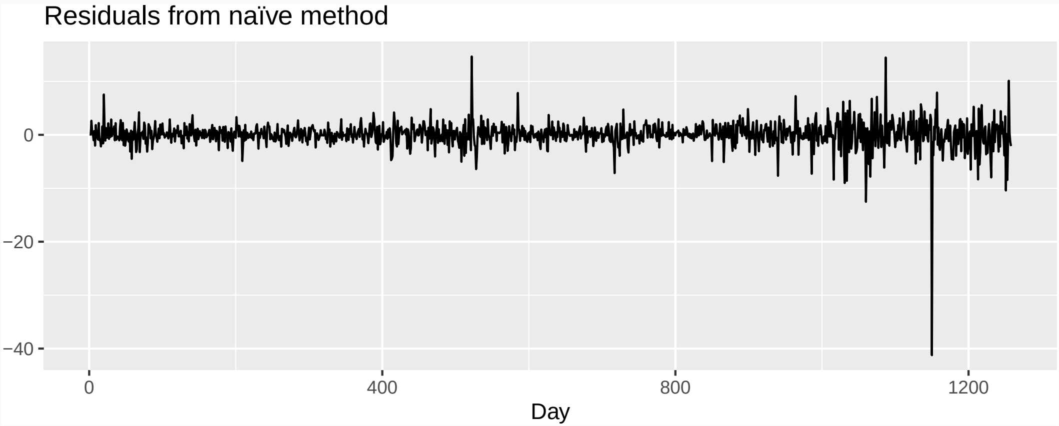
```
augment(fit) |>
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```

Facebook closing stock price

```
augment(fit) ▷
  filter(trading_day > 1100) ▷
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```

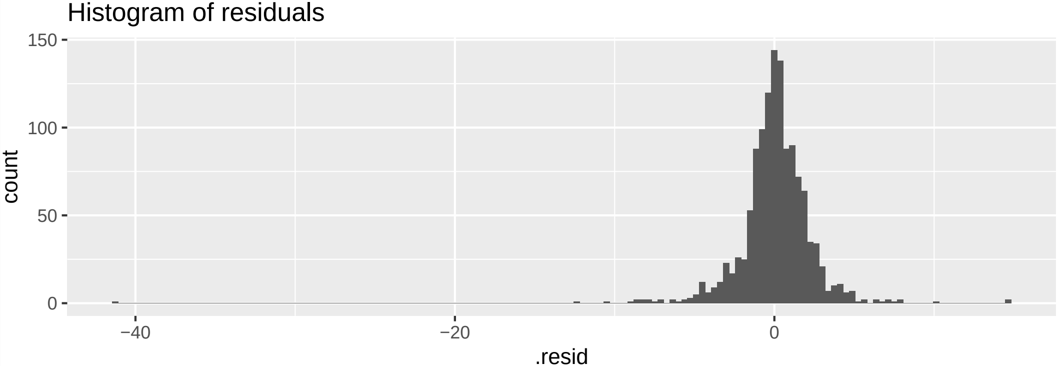
Facebook closing stock price

```
augment(fit) ▷
  autoplot(.resid) +
  labs(x = "Day", y = "", title = "Residuals from naïve method")
```



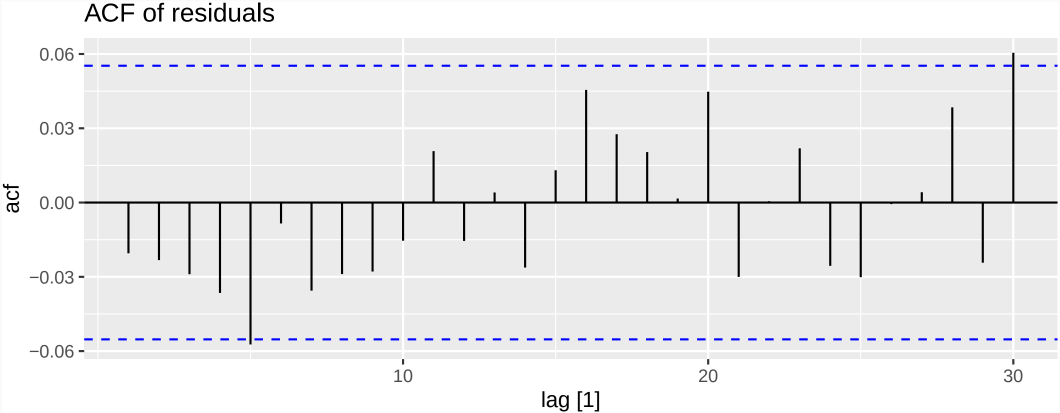
Facebook closing stock price

```
augment(fit) |>
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 150) +
  labs(title = "Histogram of residuals")
```



Facebook closing stock price

```
augment(fit) ▷
  ACF(.resid) ▷
  autoplot() + labs(title = "ACF of residuals")
```

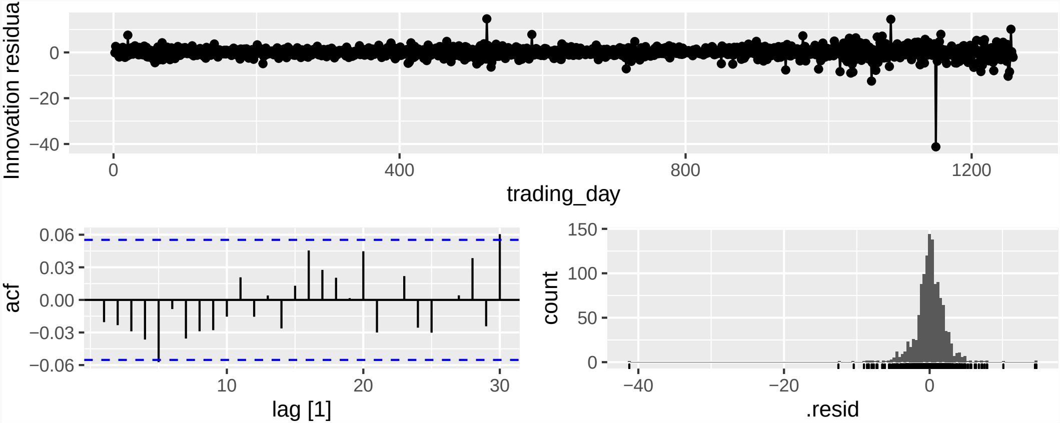


ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Combined diagnostic graph

```
fit ▷ gg_tsresiduals()
```



Ljung-Box test

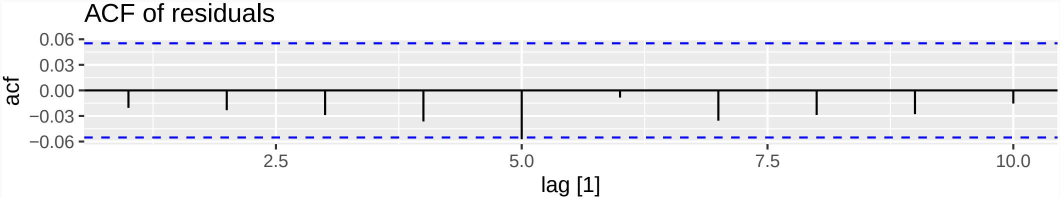
Test whether *whole set* of r_k values is significantly different from zero set.

$$Q = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2 \qquad \text{where } h = \text{max lag and } T = \text{\# observations.}$$

- If each r_k close to zero, Q will be **small**.
- If some r_k values large ($+$ or $-$), Q will be **large**.
- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- If data are WN, $Q \sim \chi^2$ with $(h - K)$ degrees of freedom where $K =$ no. parameters in model.
- When applied to raw data, set $K = 0$.

Ljung-Box test

$$Q = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2 \qquad \text{where } h = \text{max lag and } T = \text{\# observations.}$$




```
# lag=h and dof=K
augment(fit) ▷ features(.resid, ljung_box, dof = 0, lag = 10)
```

```
## # A tibble: 1 x 4
##   Symbol .model        lb_stat lb_pvalue
##   <chr>  <chr>           <dbl>     <dbl>
## 1 FB     NAIVE(Close)     12.1     0.276
```

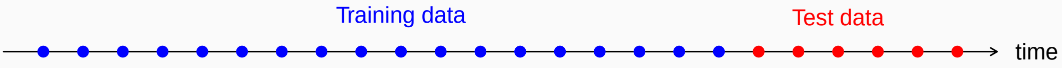
Outline

Lab Session 12

- Compute seasonal naïve forecasts for quarterly Australian beer production.
- Test if the residuals are white noise. What do you conclude?

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- Forecast accuracy is based only on the test set.

Forecast errors

Forecast "error": the difference between an observed value and its forecast.

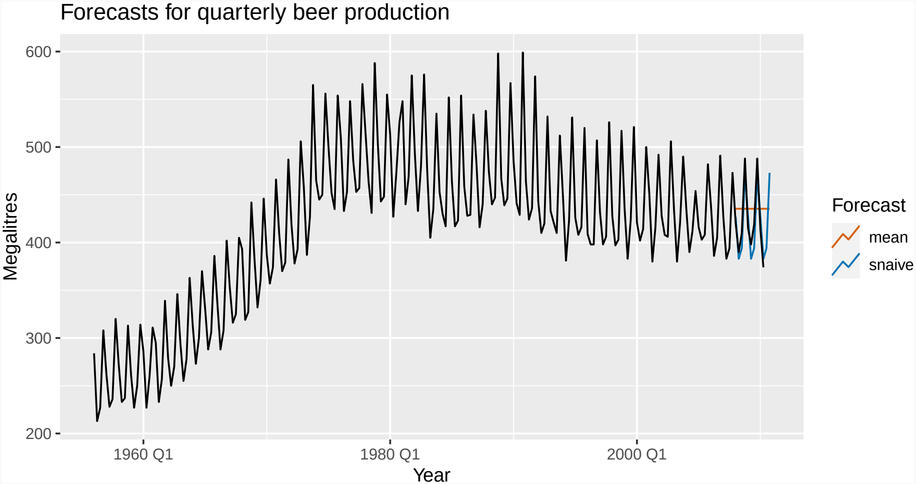
$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

Measures of forecast accuracy

```
beer_fit <- aus_production |>
  filter(between(year(Quarter), 1992, 2007)) |>
  model(
    snaive = SNAIVE(Beer),
    mean = MEAN(Beer)
  )
beer_fit |>
  forecast(h = "3 years") |>
  autoplot(aus_production, level = NULL) +
  labs(title ="Forecasts for quarterly beer production",
       x ="Year", y ="Megalitres") +
  guides(colour = guide_legend(title = "Forecast"))
```

Measures of forecast accuracy



Measures of forecast accuracy

$$\begin{aligned} y_{T+h} &= (T+h)\text{th observation, } h = 1,\dots,H \\ \hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\ e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T} \end{aligned}$$

$$\begin{aligned} \text{MAE} &= \text{mean}(|e_{T+h}|) \\ \text{MSE} &= \text{mean}(e^2_{T+h}) \qquad \text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})} \\ \text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|) \end{aligned}$$

Measures of forecast accuracy

$$\begin{aligned} y_{T+h} &= (T+h)\text{th observation}, h = 1, \dots, H \\ \hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\ e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T} \end{aligned}$$

$$\begin{aligned} \text{MAE} &= \text{mean}(|e_{T+h}|) \\ \text{MSE} &= \text{mean}(e_{T+h}^2) & \qquad \text{RMSE} &= \sqrt{\text{mean}(e_{T+h}^2)} \\ \text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|) \end{aligned}$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy

```r
beer_fc <- forecast(beer_fit, h = "3 years")
accuracy(beer_fc, aus_production)
```

```
## # A tibble: 2 x 10
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE    ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>   <dbl>
## 1 mean   Test  -13.8  38.4  34.8 -3.97  8.28 2.20  1.96  -0.0691
## 2 snaive Test    5.2  14.3  13.4  1.15  3.17 0.847 0.729  0.132
```

Outline

Lab Session 13

- Create a training set for household wealth (`hh_budget`) by witholding the last four years as a test set.
- Fit all the appropriate benchmark methods to the training set and forecast the periods covered by the test set.
- Compute the accuracy of your forecasts. Which method does best?
- Repeat the exercise using the Australian takeaway food turnover data (`aus_retail`) with a test set of four years.