Forecast reconciliation

2. Temporal & cross-temporal forecast reconciliation

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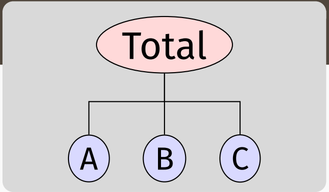
Notation reminder



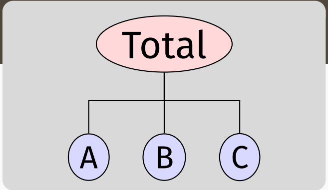
Every collection of time series with linear constraints can be written as

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

- $\boldsymbol{y}_t$ = vector of all series at time t
- $y_{\text{Total},t}$ = aggregate of all series at time t.
- $y_{X,t}$ = value of series X at time t.
- $\boldsymbol{b}_t$ = vector of most disaggregated series at time t
- $\boldsymbol{S}$ = "summing matrix" containing the linear constraints.

$$\boldsymbol{y}_t = \begin{pmatrix} y_{\text{Total},t} \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{S}} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\boldsymbol{b}_t}$$

Notation reminder



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$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

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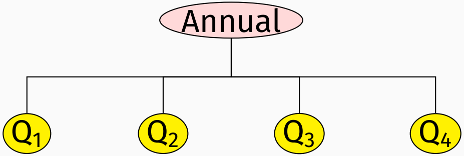
- $\boldsymbol{S}$ = "summing matrix" containing the linear constraints.

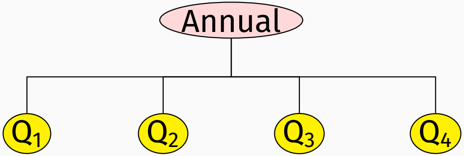
- Base forecasts: $\hat{\boldsymbol{y}}_{T+h|T}$
- Reconciled forecasts: $\tilde{\boldsymbol{y}}_{T+h|T} = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_{T+h|T}$
- MinT: $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$ where $\boldsymbol{W}_h$ is covariance matrix of base forecast errors.

Outline

Temporal reconciliation: quarterly data

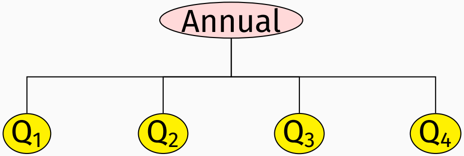


Temporal reconciliation: quarterly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

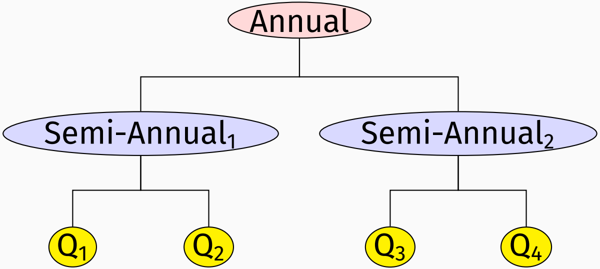
Temporal reconciliation: quarterly data



$$\boldsymbol{y}_\tau = \begin{bmatrix} x_\tau^{[4]} \\ x_{\tau,1}^{[1]} \\ x_{\tau,2}^{[1]} \\ x_{\tau,3}^{[1]} \\ x_{\tau,4}^{[1]} \end{bmatrix} \qquad \boldsymbol{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

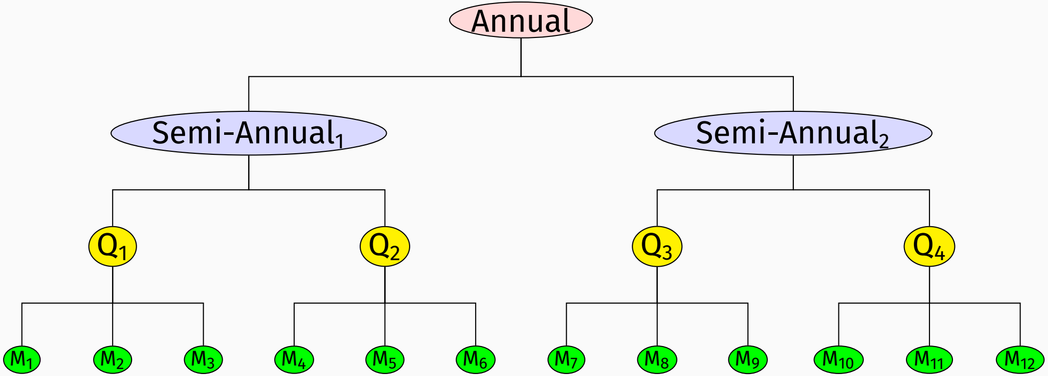
Temporal reconciliation: quarterly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

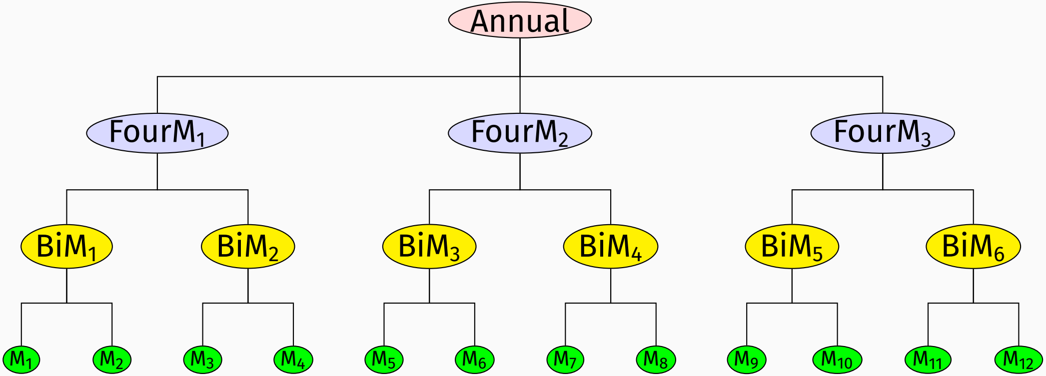
$$\boldsymbol{y}_\tau = \begin{bmatrix} x_\tau^{[4]} \\ x_{\tau,1}^{[2]} \\ x_{\tau,2}^{[2]} \\ x_{\tau,1}^{[1]} \\ x_{\tau,2}^{[1]} \\ x_{\tau,3}^{[1]} \\ x_{\tau,4}^{[1]} \end{bmatrix} \qquad \boldsymbol{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Temporal reconciliation: monthly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

Temporal reconciliation: monthly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

Temporal reconciliation: monthly data

$$\boldsymbol{y}_\tau = \begin{bmatrix} x_\tau^{[12]} \\ \boldsymbol{x}_\tau^{[6]} \\ \boldsymbol{x}_\tau^{[4]} \\ \boldsymbol{x}_\tau^{[3]} \\ \boldsymbol{x}_\tau^{[2]} \\ \boldsymbol{x}_\tau^{[1]} \end{bmatrix} \qquad \boldsymbol{S} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ & & & & & & \boldsymbol{I}_{12} & & & & & \end{bmatrix}$$

Temporal reconciliation

For a time series $y_1, \dots, y_T$, observed at frequency m:

$$x_j^{[k]} = \sum_{t=(j-1)k+1}^{jk} y_t \qquad \text{for } j = 1, \dots, \lfloor T/k \rfloor$$

- $k \in \mathcal{K} = \{k_1, \dots, k_p\}$ denote the p factors of m in ascending order, where $k_1 = 1$ and $k_p = m$
- $x_j^{[1]} = y_t$
- A single unique hierarchy is only possible when there are no coprime pairs in $\mathcal{K}$.
- $M_k = m/k$ is seasonal period of aggregated series.

Temporal reconciliation

$$\boldsymbol{x}_\tau = \boldsymbol{S}\boldsymbol{x}_\tau^{[1]}, \qquad \boldsymbol{S} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{I} \end{bmatrix}$$

where

$$\boldsymbol{x}_\tau = \begin{bmatrix} \boldsymbol{x}_\tau^{[k_p]} \\ \boldsymbol{x}_\tau^{[k_{p-1}]} \\ \vdots \\ \boldsymbol{x}_\tau^{[k_1]} \end{bmatrix} \qquad \boldsymbol{x}_\tau^{[k]} = \begin{bmatrix} x_{M_k(\tau-1)+1}^{[k]} \\ x_{M_k(\tau-1)+2}^{[k]} \\ \vdots \\ x_{M_k\tau}^{[k]} \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} \boldsymbol{1}'_m \\ \boldsymbol{I}_{m/k_{p-1}} \otimes \boldsymbol{1}'_{k_{p-1}} \\ \vdots \\ \boldsymbol{I}_{m/k_2} \otimes \boldsymbol{1}'_{k_2} \end{bmatrix}$$

τ is time index for most aggregated series,

$k \in \mathcal{K} = \{k_1, \dots, k_p\}, \quad k_1 = 1, \quad k_p = m, \quad \tau = 1, \dots, T/m.$

Outline

Temporal reconciliation: thief package

```r
library(thief)
USAccDeaths
```

```
       Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec
1973  9007  8106  8928  9137 10017 10826 11317 10744  9713  9938  9161  8927
1974  7750  6981  8038  8422  8714  9512 10120  9823  8743  9129  8710  8680
1975  8162  7306  8124  7870  9387  9556 10093  9620  8285  8466  8160  8034
1976  7717  7461  7767  7925  8623  8945 10078  9179  8037  8488  7874  8647
1977  7792  6957  7726  8106  8890  9299 10625  9302  8314  8850  8265  8796
1978  7836  6892  7791  8192  9115  9434 10484  9827  9110  9070  8633  9240
```

```r
thief(USAccDeaths, usemodel = "arima", comb = "struc")
```

```
           Jan       Feb       Mar       Apr       May       Jun       Jul
1979  8185.240  7381.008  8131.674  8516.070  9393.247  9764.091 10880.835
1980  8405.850  7601.618  8352.284  8736.680  9613.857  9984.701 11101.445
           Aug       Sep       Oct       Nov       Dec
1979 10059.873  9182.403  9405.069  8869.173  9360.772
1980 10280.483  9403.013  9625.679  9089.783  9581.382
```

Temporal reconciliation: thief package

```r
# Construct aggregates
aggts <- tsaggregates(USAccDeaths)
aggts[[1]]
```

```
       Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec
1973  9007  8106  8928  9137 10017 10826 11317 10744  9713  9938  9161  8927
1974  7750  6981  8038  8422  8714  9512 10120  9823  8743  9129  8710  8680
1975  8162  7306  8124  7870  9387  9556 10093  9620  8285  8466  8160  8034
1976  7717  7461  7767  7925  8623  8945 10078  9179  8037  8488  7874  8647
1977  7792  6957  7726  8106  8890  9299 10625  9302  8314  8850  8265  8796
1978  7836  6892  7791  8192  9115  9434 10484  9827  9110  9070  8633  9240
```

Temporal reconciliation: thief package

```r
aggts[[2]]
```

```
Time Series:
Start = c(1973, 1)
End = c(1978, 6)
Frequency = 6
 [1] 17113 18065 20843 22061 19651 18088 14731 16460 18226 19943 17872 17390
[13] 15468 15994 18943 19713 16751 16194 15178 15692 17568 19257 16525 16521
[25] 14749 15832 18189 19927 17164 17061 14728 15983 18549 20311 18180 17873
```

Temporal reconciliation: thief package

```r
aggts[[3]]
```

```
      Qtr1  Qtr2  Qtr3  Qtr4
1973 26041 29980 31774 28026
1974 22769 26648 28686 26519
1975 23592 26813 27998 24660
1976 22945 25493 27294 25009
1977 22475 26295 28241 25911
1978 22519 26741 29421 26943
```

Temporal reconciliation: thief package

```r
aggts[[4]]
```

```
Time Series:
Start = c(1973, 1)
End = c(1978, 3)
Frequency = 3
 [1] 35178 42904 37739 31191 38169 35262 31462 38656 32945 30870 36825 33046
[13] 30581 38116 34225 30711 38860 36053
```

Temporal reconciliation: thief package

```r
aggts[[5]]
```

```
Time Series:
Start = c(1973, 1)
End = c(1978, 2)
Frequency = 2
 [1] 56021 59800 49417 55205 50405 52658 48438 52303 48770 54152 49260 56364
```

Temporal reconciliation: thief package

```r
aggts[[6]]
```

```
Time Series:
Start = 1973
End = 1978
Frequency = 1
[1] 115821 104622 103063 100741 102922 105624
```

Temporal reconciliation: thief package

```r
# Compute forecasts
fc <- list()
for(i in seq_along(aggts)) {
  fc[[i]] <- forecast(auto.arima(aggts[[i]]), h=2*frequency(aggts[[i]]))
}
reconciled <- reconcilethief(fc)
reconciled[[1]]
```

```
         Point Forecast     Lo 80     Hi 80    Lo 95     Hi 95
Jan 1979       8185.240  7773.891  8596.589 7556.136  8814.344
Feb 1979       7381.008  6907.643  7854.373 6657.059  8104.957
Mar 1979       8131.674  7603.525  8659.822 7323.941  8939.407
Apr 1979       8516.070  7938.310  9093.830 7632.462  9399.678
May 1979       9393.247  8769.810 10016.683 8439.783 10346.710
Jun 1979       9764.091  9098.104 10430.079 8745.552 10782.631
Jul 1979      10880.835 10174.856 11586.813 9801.134 11960.535
Aug 1979      10059.873  9316.050 10803.695 8922.295 11197.451
```

Temporal reconciliation: thief package

```
reconciled[[2]]
```

```
         Point Forecast     Lo 80    Hi 80    Lo 95    Hi 95
1979.000       15566.25 14788.53 16343.97 14376.83 16755.67
1979.167       16647.74 15634.48 17661.00 15098.10 18197.39
1979.333       19157.34 17953.79 20360.89 17316.66 20998.01
1979.500       20940.71 19573.09 22308.33 18849.11 23032.30
1979.667       18587.47 17073.46 20101.48 16271.99 20902.95
1979.833       18229.94 16582.50 19877.39 15710.40 20749.49
1980.000       16007.47 14159.37 17855.57 13181.05 18833.89
1980.167       17088.96 15077.40 19100.53 14012.54 20165.39
1980.333       19598.56 17435.85 21761.27 16290.98 22906.14
1980.500       21381.93 19077.96 23685.89 17858.32 24905.53
1980.667       19028.69 16591.65 21465.73 15301.56 22755.83
1980.833       18671.16 16107.94 21234.39 14751.06 22591.27
```

Temporal reconciliation: thief package

```
reconciled[[3]]
```

```
        Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
1979 Q1       23697.92 22460.67 24935.17 21805.71 25590.13
1979 Q2       27673.41 25923.67 29423.15 24997.41 30349.40
1979 Q3       30123.11 27980.13 32266.09 26845.70 33400.52
1979 Q4       27635.01 25160.51 30109.52 23850.59 31419.44
1980 Q1       24359.75 20860.28 27859.23 19007.77 29711.74
1980 Q2       28335.24 24049.27 32621.20 21780.42 34890.06
1980 Q3       30784.94 25835.93 35733.95 23216.09 38353.79
1980 Q4       28296.84 22763.69 33830.00 19834.61 36759.08
```

Temporal reconciliation: thief package

```r
reconciled[[4]]
```

```
         Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
1979.000       32213.99 30196.88 34231.11 29129.08 35298.90
1979.333       40098.05 37245.41 42950.68 35735.32 44460.77
1979.667       36817.42 33323.67 40311.16 31474.19 42160.64
1980.000       33096.43 27759.64 38433.22 24934.52 41258.34
1980.333       40980.49 34290.47 47670.50 30748.99 51211.98
1980.667       37699.86 29887.60 45512.11 25752.04 49647.67
```

Temporal reconciliation: thief package

```r
reconciled[[5]]
```

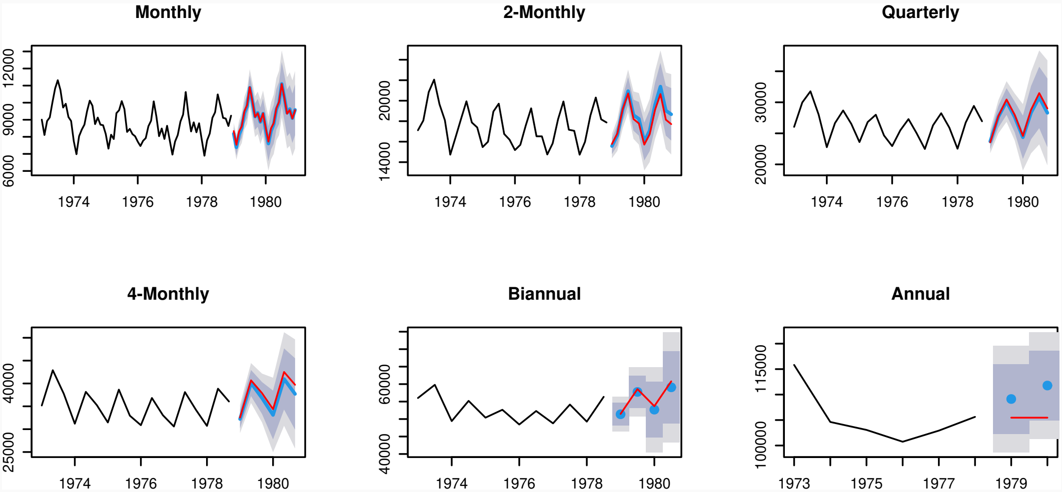
```
        Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
1979.00       51371.33 48111.38 54631.28 46385.66 56357.00
1979.50       57758.12 53147.85 62368.40 50707.32 64808.93
1980.00       52694.99 44709.77 60680.21 40482.64 64907.33
1980.50       59081.78 48772.91 69390.66 43315.72 74847.85
```

Temporal reconciliation: thief package

```r
reconciled[[6]]
```

```
     Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
1979       109129.5 102287.0 115971.9  98664.81 119594.1
1980       111776.8 104934.3 118619.2 101312.13 122241.4
```

Temporal reconciliation: thief package



Outline

Temporal reconciliation: M3 monthly series

- Apply temporal reconciliation to all 1428 monthly series from M3 competition
- Forecast horizon $h = 18$ months
- ETS and ARIMA models
- Measure percentage difference to base forecasts
- Reconciliation methods:
  - WLS_H (diagonal)
  - WLS_V (diagonal with common variances for same frequency)
  - WLS_S (diagonal/structural)

Temporal reconciliation: M3 monthly series

Aggregation level	h	ETS Base	ETS BU	ETS WLS_H	ETS WLS_V	ETS WLS_S	ARIMA Base	ARIMA BU	ARIMA WLS_H	ARIMA WLS_V	ARIMA WLS_S
						RMAE					
Annual	1	1.0	-19.6	-22.0	-22.0	**-25.1**	1.0	-28.6	-33.1	-32.8	**-33.4**
Semi-annual	3	1.0	0.6	-4.0	-3.6	**-5.4**	1.0	-3.4	-8.2	-8.3	**-9.9**
Four-monthly	4	1.0	2.0	-2.4	-2.2	**-3.0**	1.0	-1.7	-5.5	-5.9	**-6.7**
Quarterly	6	1.0	2.4	-1.6	-1.7	**-2.8**	1.0	-3.6	-7.2	-8.1	**-9.1**
Bi-monthly	9	1.0	0.7	-2.9	-3.3	**-4.3**	1.0	-1.5	-4.4	-5.3	**-6.3**
Monthly	18	1.0	0.0	-2.2	-3.2	**-3.9**	1.0	0.0	-0.9	-2.9	**-3.4**
Average			*-2.3*	*-5.9*	*-6.0*	***-7.4***		*-6.5*	*-9.9*	*-10.5*	***-11.5***
						MASE					
Annual	1	1.1	-12.1	-17.9	-17.8	**-18.5**	1.3	-25.4	-29.9	-29.9	**-30.2**
Semi-annual	3	1.0	0.0	-6.3	-6.0	**-6.9**	1.1	-2.9	-8.1	-8.2	**-9.4**
Four-monthly	4	0.9	3.1	-3.2	-3.0	**-3.4**	0.9	-1.8	-6.2	-6.5	**-7.1**
Quarterly	6	0.9	3.2	-2.8	-2.7	**-3.4**	1.0	-2.6	-6.9	-7.4	**-8.1**
Bi-monthly	9	0.9	2.7	-2.9	-3.0	**-3.7**	0.9	-1.3	-5.0	-5.5	**-6.3**
Monthly	18	0.9	0.0	-3.7	-4.6	**-5.0**	0.9	0.0	-1.9	-3.2	**-3.7**
Average			*-0.5*	*-6.1*	*-6.2*	***-6.8***		*-5.7*	*-9.7*	*-10.1*	***-10.8***

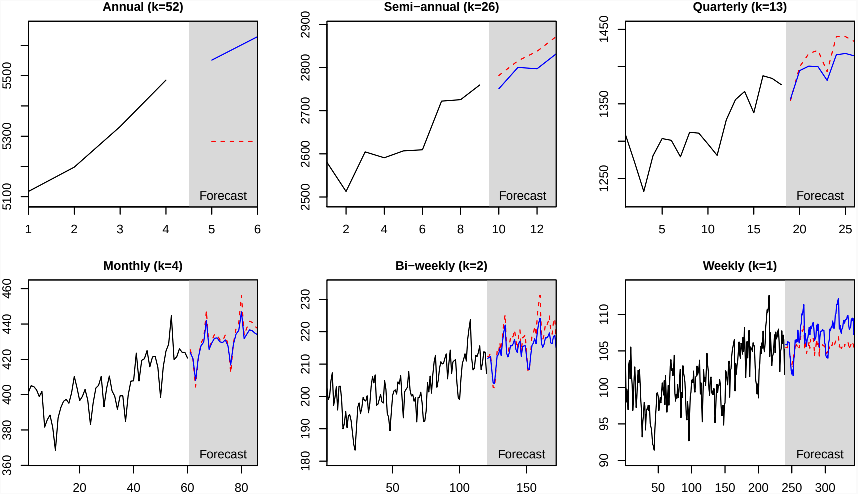
Example: Accident & emergency services demand

Weekly A&E demand data: 7 November 2010 to 7 June 2015.

Type 1 Departments — Major A&E
Type 2 Departments — Single Specialty
Type 3 Departments — Other A&E/Minor Injury Unit
Total Attendances
Type 1 Departments — Major A&E $>$ 2 hours
Type 2 Departments — Single Specialty $>$ 2 hours
Type 3 Departments — Other A&E/Minor Injury Unit $>$ 2 hours
Total Attendances $>$ 2 hours
Emergency Admissions via Type 1 A&E
Total Emergency Admissions via A&E
Other Emergency Admissions (i.e not via A&E)
Total Emergency Admissions
Number of patients spending $>$ 2 hours from decision to admit to admission

Example: Accident & emergency services demand

Total emergency admissions via A&E



Example: Accident & emergency services demand

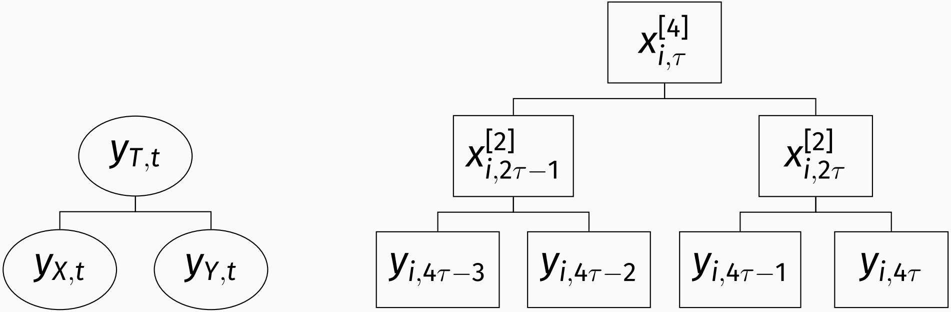
Test set: last 52 weeks

MASE comparison (ARIMA models)

Aggr. Level	h	Base	Reconciled	Change
Annual	1	3.4	1.9	−42.9%
Weekly	1–52	2.0	1.9	−5.0%
Weekly	13	2.3	1.9	−16.2%
Weekly	4	1.9	1.5	−18.6%
Weekly	1	1.6	1.3	−17.2%

Outline

Cross-temporal forecast reconciliation



- $n = 3$, $n_a = 1$, $n_b = 2$
- Quarterly series: $m = 2$, $\mathcal{K} = \{1, 2, 4\}$

Cross-temporal forecast reconciliation

- $\boldsymbol{y}_t$ = series at most temporally disaggregated level, including all cross-sectionally disaggregated and aggregated series.
- $y_{i,t}$ = ith element of $\boldsymbol{y}_t$, $i = 1, \dots, n$.
- For each i, we can expand $y_{i,t}$ to include all temporally aggregated variants:

$$\boldsymbol{x}_{i,\tau} = \begin{bmatrix} x_{i,\tau}^{[k_p]} \\ \vdots \\ \boldsymbol{x}_{i,\tau}^{[k_1]} \end{bmatrix} \qquad \boldsymbol{x}_{\tau} = \begin{bmatrix} \boldsymbol{x}_{1,\tau} \\ \vdots \\ \boldsymbol{x}_{n,\tau} \end{bmatrix}.$$

Cross-temporal forecast reconciliation

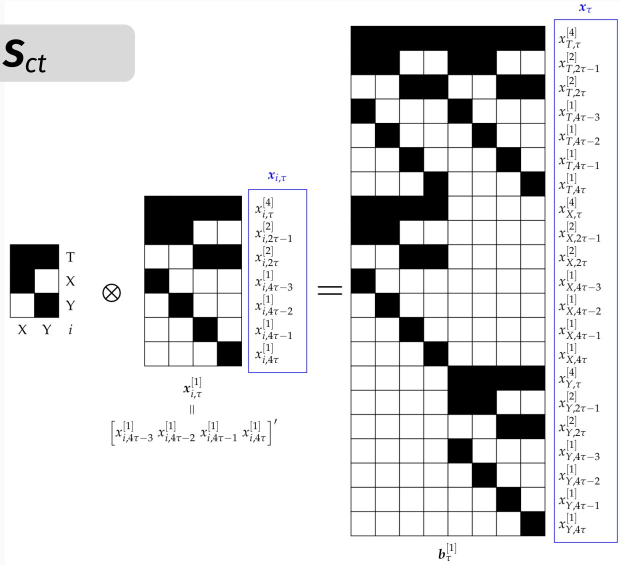
Structural matrix approach

- $\boldsymbol{S}_{cs}$ = structural cross-sectional matrix
- $\boldsymbol{S}_{te}$ = structural temporal matrix
- $\boldsymbol{S}_{ct} = \boldsymbol{S}_{cs} \otimes \boldsymbol{S}_{te}$

$$\boldsymbol{x}_\tau = \boldsymbol{S}_{ct}\boldsymbol{b}_\tau, \qquad \text{where} \qquad \boldsymbol{b}_\tau = \begin{bmatrix} \boldsymbol{x}^{[1]}_{1,\tau} \\ \vdots \\ \boldsymbol{x}^{[1]}_{n,\tau} \end{bmatrix}.$$

Cross-temporal forecast reconciliation

$\boldsymbol{S}_{cs} \otimes \boldsymbol{S}_{te} = \boldsymbol{S}_{ct}$



Cross-temporal forecast reconciliation

Constraint matrix approach

- $\boldsymbol{C}_{cs}$ = cross-sectional constraint matrix
- $\boldsymbol{C}_{te}$ = temporal constraint matrix

$$\boldsymbol{C}_{ct}\boldsymbol{x}_\tau = \boldsymbol{0} \qquad \text{where} \qquad \boldsymbol{C}_{ct} = \begin{bmatrix} (\boldsymbol{0}_{(n_a m \times nk^*)} \quad \boldsymbol{I}_m \otimes \boldsymbol{C}_{cs})\boldsymbol{P}' \\ \boldsymbol{I}_n \otimes \boldsymbol{C}_{te} \end{bmatrix}$$

- $k^* = \displaystyle\sum_{k \in \mathcal{K} \setminus \{k_1\}} \frac{m}{k}$
- $\boldsymbol{P}$ = the commutation matrix such that $\boldsymbol{P}\text{vec}(\boldsymbol{Y}_\tau) = \text{vec}(\boldsymbol{Y}'_\tau)$.

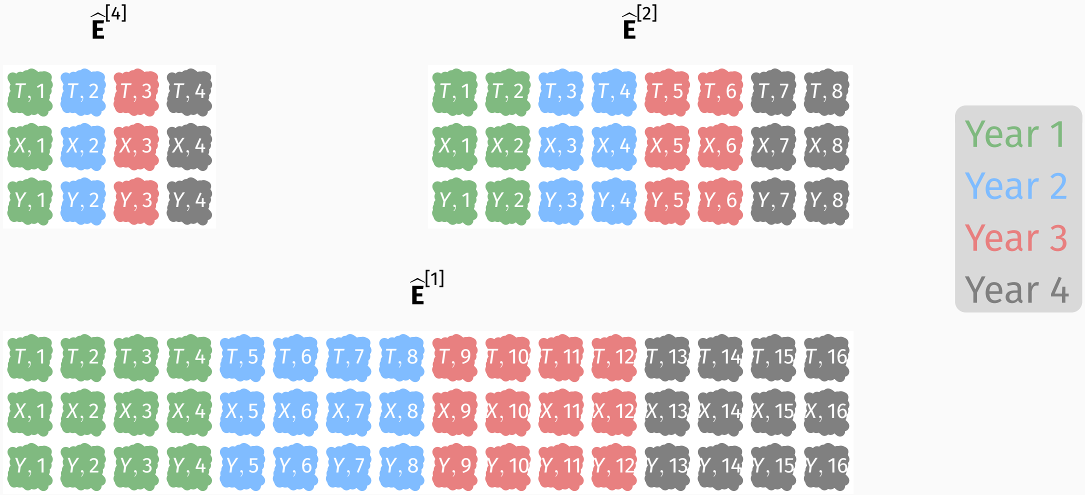
Outline

Cross-temporal probabilistic forecast reconciliation

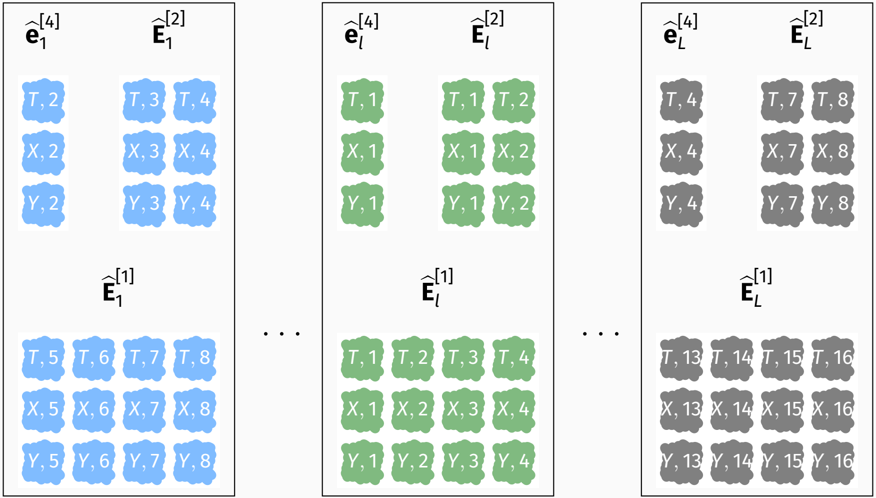
Nonparametric bootstrap

- Simulate future sample paths from all models using bootstrapped residuals, then reconcile them to obtain coherent sample paths.
- Need to generate samples that preserve cross-temporal relationships.
- Draw residual samples of all series at same time from most temporally aggregated level.
- Residuals for other levels obtained using the corresponding time indices.

Cross-temporal probabilistic forecast reconciliation



Cross-temporal probabilistic forecast reconciliation

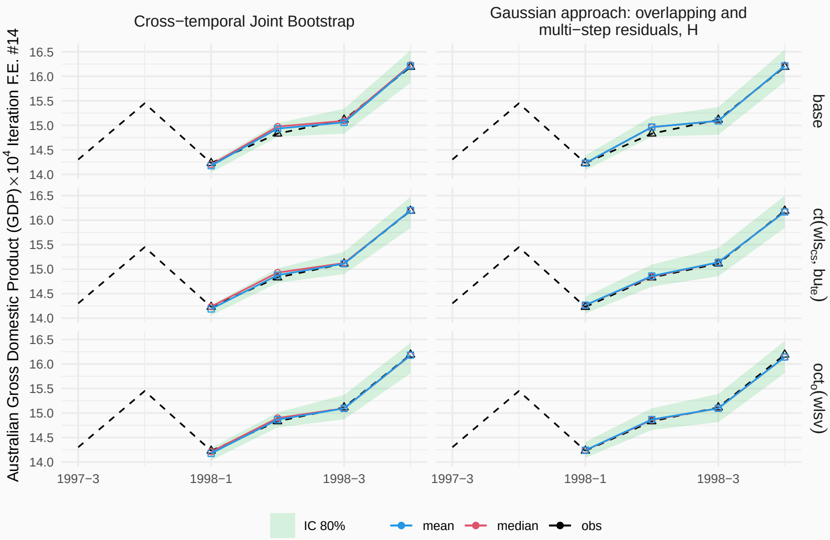


1^{st} bootstrap sample (τ = 2) ℓ^{th} bootstrap sample (τ = 1) L^{th} bootstrap sample (τ = 4)

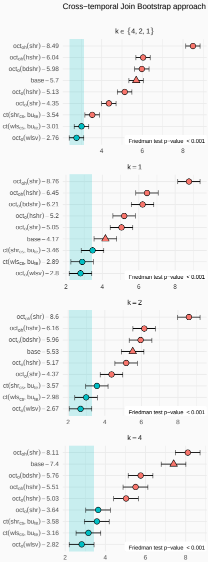
Example: Australian GDP data — CRPS skill scores

Reconciliation approach	**Base forecasts' sample approach**				
	Bootstrap	Gaussian frameworks: sample covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4, 2, 1\}$			
base	1.000	0.979	0.995	0.968	0.976
$\text{oct}_o(wlsv)$	**0.926**	**0.911**	**0.912**	**0.896**	**0.895**
$\text{oct}_o(bdshr)$	0.978	0.964	0.946	0.952	0.930
$\text{oct}_o(shr)$	0.950	0.946	0.922	0.925	0.903
$\text{oct}_o(hshr)$	0.989	0.966	0.984	0.954	0.965
$\text{oct}_{oh}(shr)$	1.102	1.059	1.001	1.094	0.988
$\text{oct}_{oh}(hshr)$	1.006	0.983	1.009	0.974	1.001
		$k = 1$			
base	1.000	0.988	0.988	0.971	0.971

Example: Australian GDP data



Example: Australian GDP data



Example: Australian tourism data

- Monthly data: January 1998 to December 2016
- 7 states, 35 zones, 380 regions. 525 unique series
- Time series cross-validation; initial training set 10 years.

Methods

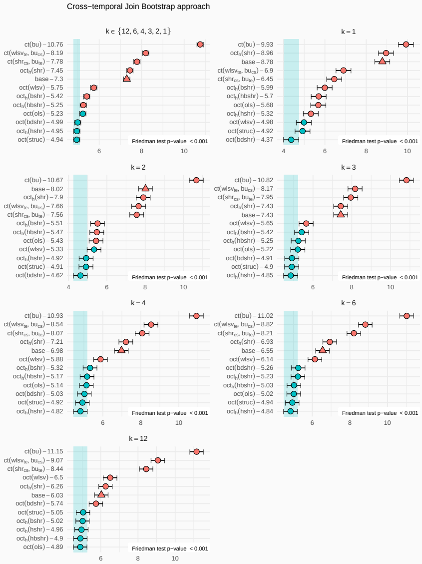
oct$(\,\cdot\,)$ optimal cross-temporal reconciliation for *ols*, *struc*, *wlsv* and *bdshr* approach. When necessary, in-sample residuals were used;

oct$_h(\,\cdot\,)$ optimal cross-temporal reconciliation with multi-step residuals: *shr = Global shrinkage*, *hshr = High frequency shrinkage*, *bshr = bottom time series shrinkage*, *hbshr =*

Example: Australian tourism data – CRPS skill scores

Reconciliation approach	Base forecasts' sample approach									
	ctjb	Gaussian approach*				ctjb	Gaussian approach*			
		G	B	H	HB		G	B	H	HB
	$\forall k \in \{12, 6, 4, 3, 2, 1\}$					$k = 1$				
base	1.000	0.971	0.971	0.973	0.973	1.000	0.972	0.972	0.972	0.972
ct(bu)	1.321	1.011	1.011	1.011	1.011	1.077	0.983	0.982	0.982	0.982
ct(shr_{cs}, bu_{te})	1.057	0.974	0.969	0.974	0.969	0.976	0.963	0.962	0.963	0.962
ct($wlsv_{te}$, bu_{cs})	1.062	0.974	0.974	0.972	0.972	0.976	0.965	0.965	0.966	0.966
oct(ols)	0.989	0.989	0.989	0.987	0.987	0.982	0.986	0.988	0.986	0.989
oct($struc$)	0.982	0.962	0.961	0.961	0.959	0.970	0.963	0.963	0.963	0.963
oct($wlsv$)	0.987	0.959	0.959	0.958	0.957	0.952	0.957	0.957	0.957	0.957
oct($bdshr$)	0.975	**0.956**	**0.953**	**0.952**	**0.951**	**0.949**	**0.955**	**0.953**	**0.954**	**0.954**
oct$_h$($hbshr$)	0.989	1.018	1.020	1.016	1.018	0.982	1.004	1.007	1.004	1.009
oct$_h$($bshr$)	0.994	1.018	1.020	1.016	1.019	0.988	1.007	1.013	1.006	1.012
oct$_{[illegible]}$($hshr$)	**0.969**	0.993	0.993	0.990	0.991	0.953	0.977	0.977	0.979	0.979

Example: Australian tourism data



Thanks!

More information

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References

- Athanasopoulos, G, RJ Hyndman, N Kourentzes, and F Petropoulos (2017). Forecasting with temporal hierarchies. *European J Operational Research* **262**(1), 60–74.
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