Ensemble forecasts with fable

Rob J Hyndman

14 August 2020

Where is Melbourne?

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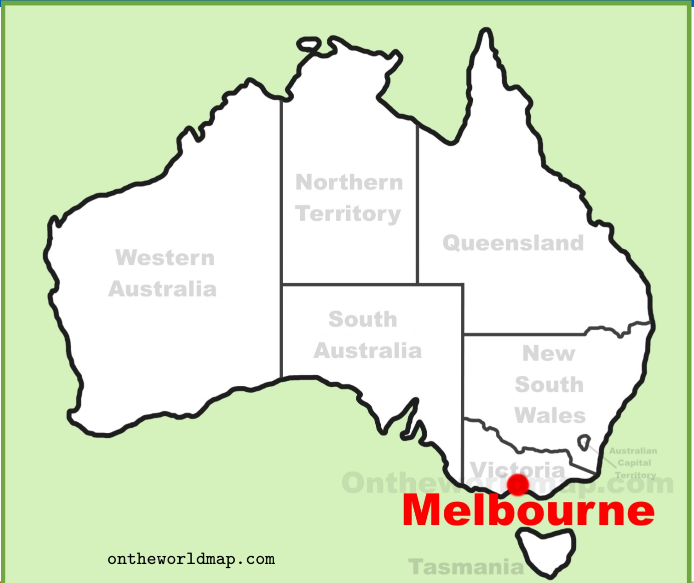
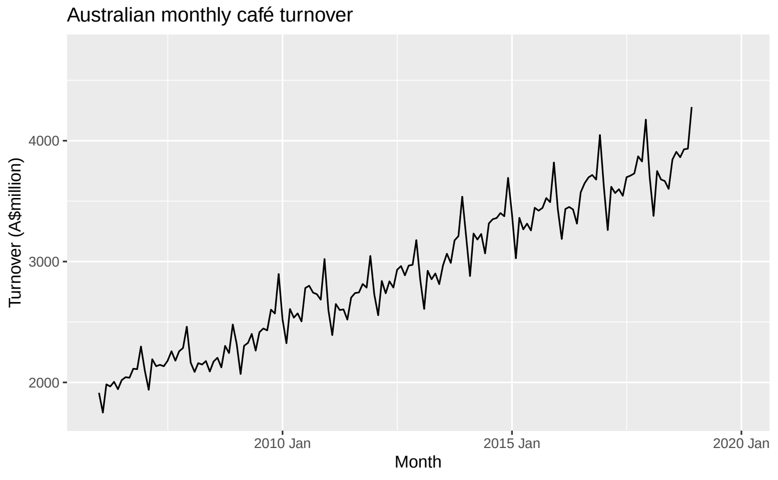
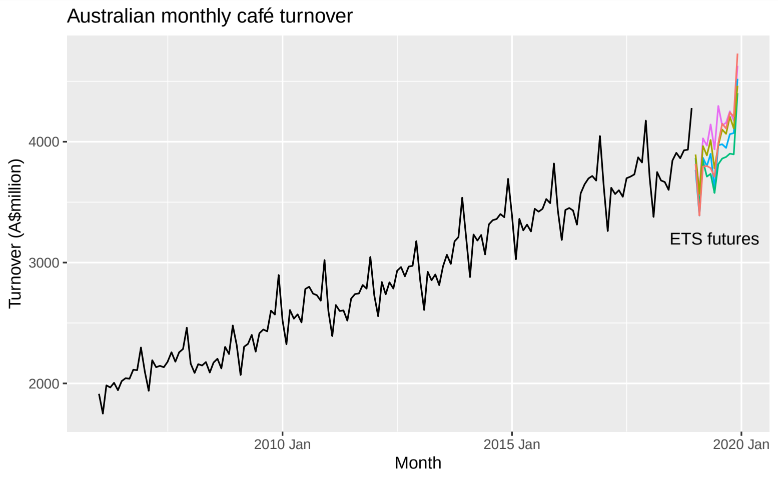


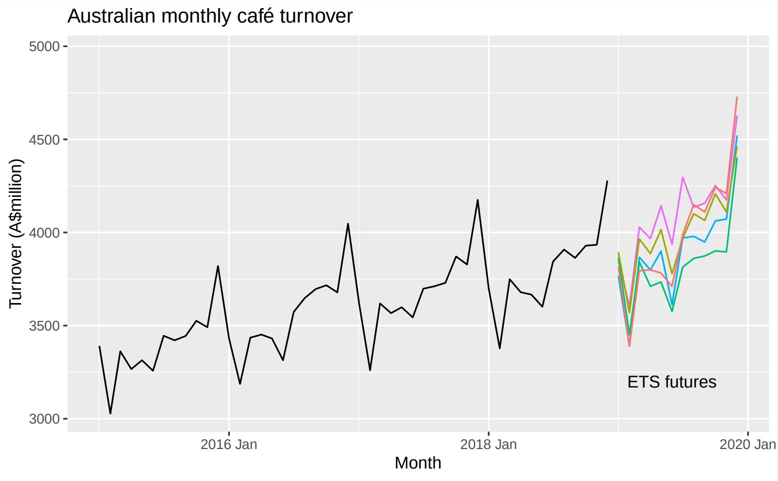
Australian monthly café turnover



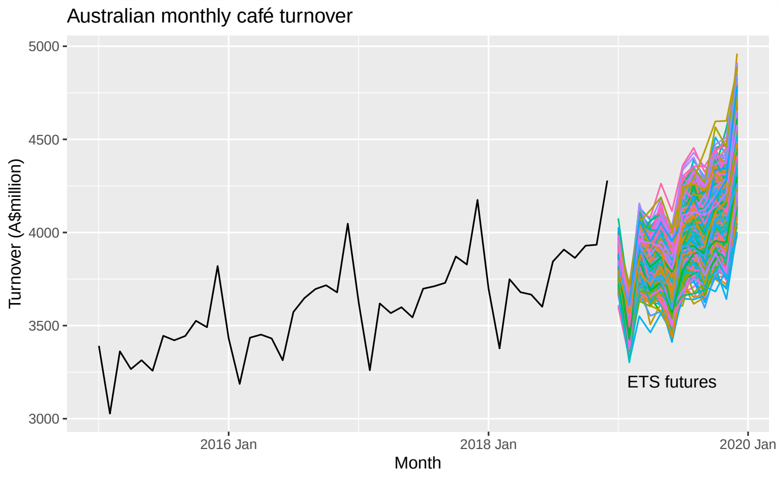
Forecasting using possible futures



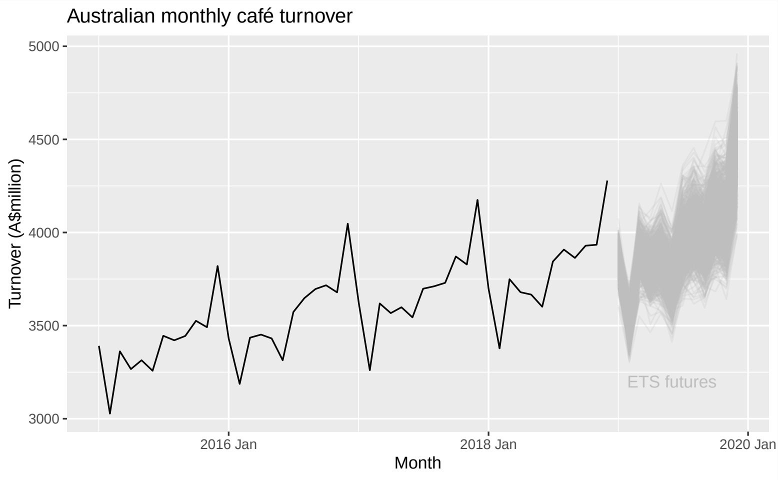
Forecasting using possible futures



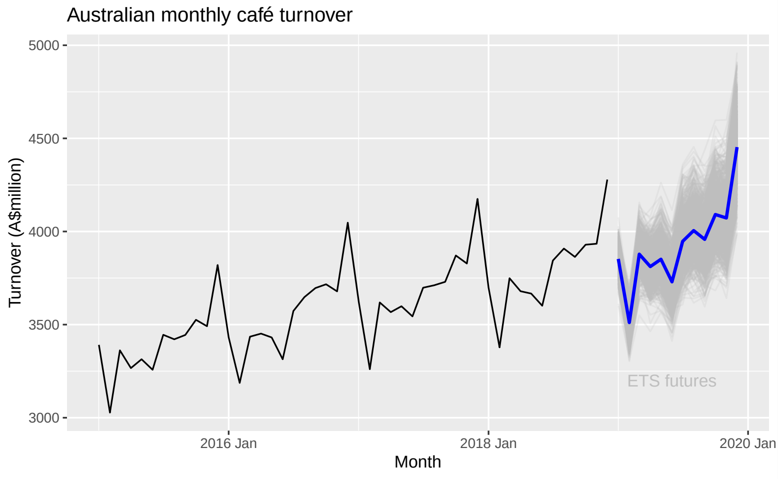
Forecasting using possible futures



Forecasting using possible futures

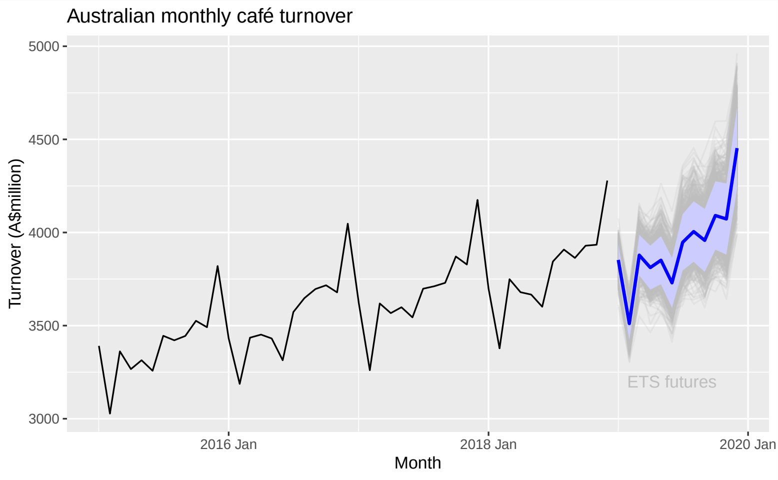


Forecasting using possible futures



Point forecasts: means of the sample paths.

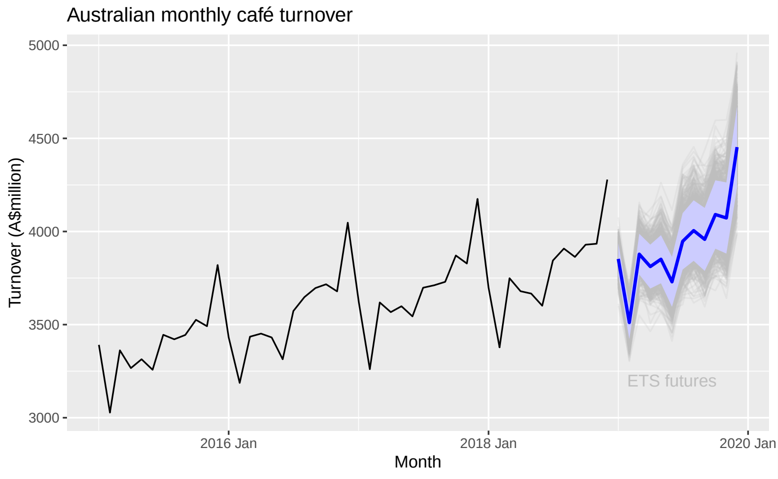
Forecasting using possible futures



Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

Forecasting using possible futures

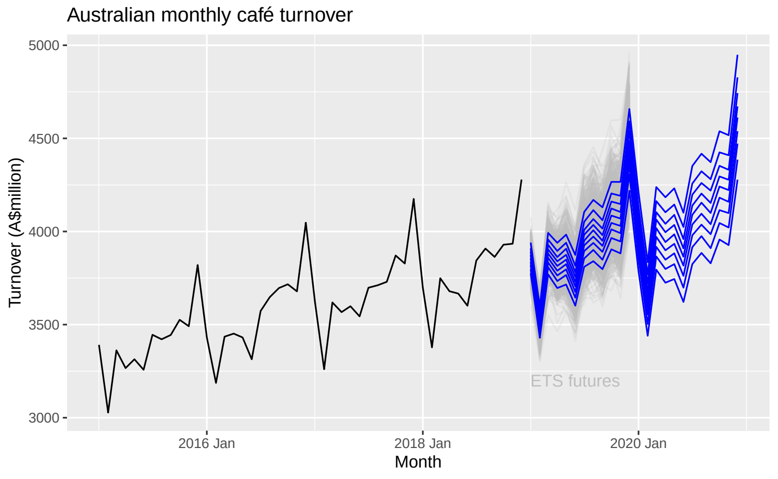


Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

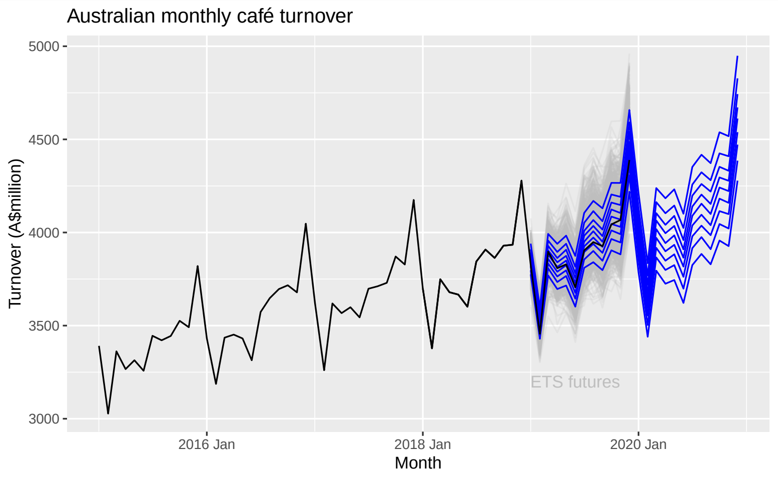
Quantile forecasts: Quantiles of the sample paths at each forecast horizon.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.
Black: Observed values.

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)\big|y_t - f_{p,t}\big|, & \text{if } y_t < f_{p,t} \\ 2p\big|y_t - f_{p,t}\big|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

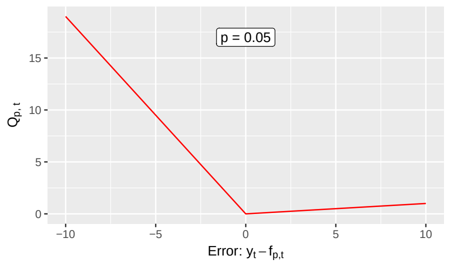
Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)\big|y_t - f_{p,t}\big|, & \text{if } y_t < f_{p,t} \\ 2p\big|y_t - f_{p,t}\big|, & \text{if } y_t \ge f_{p,t} \end{cases}$$



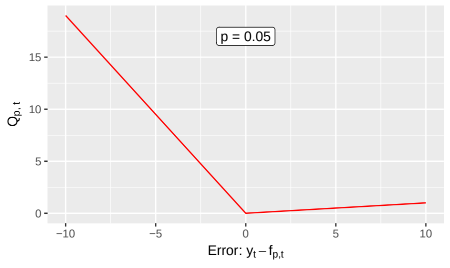
Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \ge f_{p,t} \end{cases}$$



- Low Q_p is good
- Multiplier of 2 often omitted, but useful for interpretation
- Q_p like absolute error (weighted to account for likely exceedance)
- Average Q_p = CRPS (Continuous Rank Probability Score)

Evaluating quantile forecasts

```r
cafe
```

```
## # A tsibble: 168 x 2 [1M]
##        date turnover
##       <mth>    <dbl>
##  1 2006 Jan    1914.
##  2 2006 Feb    1750.
##  3 2006 Mar    1984.
##  4 2006 Apr    1966.
##  5 2006 May    2005.
##  6 2006 Jun    1944.
##  7 2006 Jul    2019.
##  8 2006 Aug    2043.
##  9 2006 Sep    2039.
## 10 2006 Oct    2113.
## # ... with 158 more rows
```

Evaluating quantile forecasts

```r
cafe %>%
  filter(year(date) <= 2018)
```

```
## # A tsibble: 156 x 2 [1M]
##        date turnover
##       <mth>    <dbl>
##  1 2006 Jan    1914.
##  2 2006 Feb    1750.
##  3 2006 Mar    1984.
##  4 2006 Apr    1966.
##  5 2006 May    2005.
##  6 2006 Jun    1944.
##  7 2006 Jul    2019.
##  8 2006 Aug    2043.
##  9 2006 Sep    2039.
## 10 2006 Oct    2113.
## # ... with 146 more rows
```

Evaluating quantile forecasts

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##             ETS                        ARIMA
##         <model>                      <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Evaluating quantile forecasts

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year")
```

```
## # A fable: 24 x 4 [1M]
## # Key:     .model [2]
##    .model     date        turnover .mean
##    <chr>     <mth>          <dist> <dbl>
##  1 ETS    2019 Jan  N(3853, 4315) 3853.
##  2 ETS    2019 Feb  N(3511, 4760) 3511.
##  3 ETS    2019 Mar  N(3878, 7273) 3878.
##  4 ETS    2019 Apr  N(3812, 8472) 3812.
##  5 ETS    2019 May N(3851, 10155) 3851.
##  6 ETS    2019 Jun N(3730, 10968) 3730.
##  7 ETS    2019 Jul N(3947, 13933) 3947.
##  8 ETS    2019 Aug N(4005, 16074) 4005.
##  9 ETS    2019 Sep N(3958, 17421) 3958.
## 10 ETS    2019 Oct N(4091, 20493) 4091.
## # ... with 14 more rows
```

Evaluating quantile forecasts

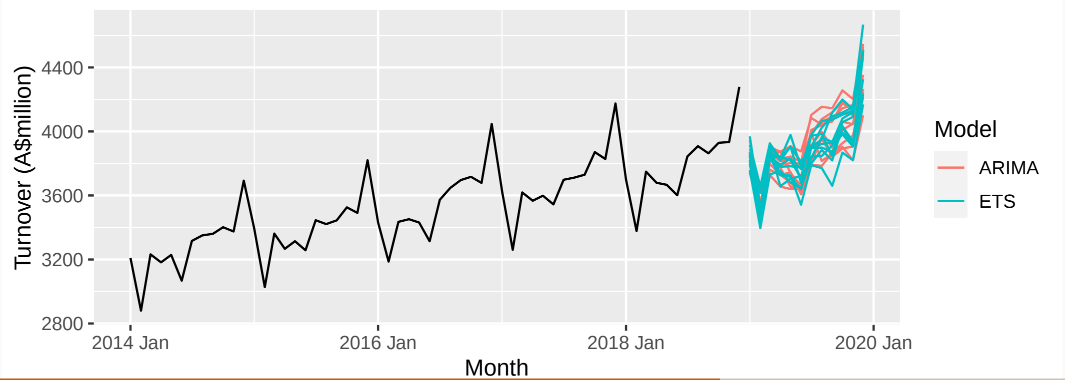
```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  ) %>%
  arrange(crps)
```

```
## # A tibble: 2 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test   31.5  38.3
## 2 ARIMA  Test   32.9  51.5
```

Ensemble forecasting

Ensemble forecasting involves combining the forecast distributions from multiple models.

- “All models are wrong, but some are useful” (George Box, 1976)
- Allows diverse models to be included, while reducing impact of any specific model.
- Allows uncertainty of model selection to be incorporated.



Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 years")
```

```
## # A fable: 24 x 4 [1M]
## # Key:      .model [2]
##    .model     date        turnover .mean
##     <chr>     <mth>         <dist> <dbl>
##  1 ETS     2019 Jan  N(3853, 4315) 3853.
##  2 ETS     2019 Feb  N(3511, 4760) 3511.
##  3 ETS     2019 Mar  N(3878, 7273) 3878.
##  4 ETS     2019 Apr  N(3812, 8472) 3812.
##  5 ETS     2019 May N(3851, 10155) 3851.
##  6 ETS     2019 Jun N(3730, 10968) 3730.
##  7 ETS     2019 Jul N(3947, 13933) 3947.
##  8 ETS     2019 Aug N(4005, 16074) 4005.
##  9 ETS     2019 Sep N(3958, 17421) 3958.
## 10 ETS     2019 Oct N(4091, 20493) 4091.
## # ... with 14 more rows
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  )
```

```
## # A tsibble: 12 x 2 [1M]
##          date      turnover
##         <mth>        <dist>
##  1 2019 Jan mixture(n=2)
##  2 2019 Feb mixture(n=2)
##  3 2019 Mar mixture(n=2)
##  4 2019 Apr mixture(n=2)
##  5 2019 May mixture(n=2)
##  6 2019 Jun mixture(n=2)
##  7 2019 Jul mixture(n=2)
##  8 2019 Aug mixture(n=2)
##  9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover)
```

```
## # A fable: 12 x 2 [1M]
##        date     turnover
##       <mth>       <dist>
##  1 2019 Jan mixture(n=2)
##  2 2019 Feb mixture(n=2)
##  3 2019 Mar mixture(n=2)
##  4 2019 Apr mixture(n=2)
##  5 2019 May mixture(n=2)
##  6 2019 Jun mixture(n=2)
##  7 2019 Jul mixture(n=2)
##  8 2019 Aug mixture(n=2)
##  9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover) %>%
  accuracy(
```

```
## # A tibble: 1 x 3
##   .type  crps  rmse
##   <chr> <dbl> <dbl>
## 1 Test   31.7  43.6
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover) %>%
  accuracy(
```

```
## # A tibble: 1 x 3
##   .type  crps  rmse
##   <chr> <dbl> <dbl>
## 1 Test   31.7  43.6
```

Comparison:

```
## # A tibble: 2 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test   31.5  38.3
## 2 ARIMA  Test   32.9  51.5
```

Combination forecasting

Combination forecasting: weighted averages of forecasts from multiple models.

- Often a simple average is used.
- Reduces uncertainty associated with selecting a particular model.
- Combination forecasting usually improves point forecast accuracy.
- Mean forecast identical to that from corresponding weighted ensemble.
- Quantile forecasts need to account for correlations between forecast errors from component models.

Combination forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##              ETS                         ARIMA
##          <model>                       <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2)
```

```
## # A mable: 1 x 3
##               ETS                          ARIMA
##           <model>                        <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]> <COI
```

Combination forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
              pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year")
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [3]
##    .model     date         turnover .mean
##    <chr>     <mth>           <dist> <dbl>
##  1 ETS    2019 Jan  N(3853, 4315) 3853.
##  2 ETS    2019 Feb  N(3511, 4760) 3511.
##  3 ETS    2019 Mar  N(3878, 7273) 3878.
##  4 ETS    2019 Apr  N(3812, 8472) 3812.
##  5 ETS    2019 May N(3851, 10155) 3851.
##  6 ETS    2019 Jun N(3730, 10968) 3730.
##  7 ETS    2019 Jul N(3947, 13933) 3947.
##  8 ETS    2019 Aug N(4005, 16074) 4005.
##  9 ETS    2019 Sep N(3958, 17421) 3958.
## 10 ETS    2019 Oct N(4091, 20493) 4091.
## # ... with 26 more rows
```

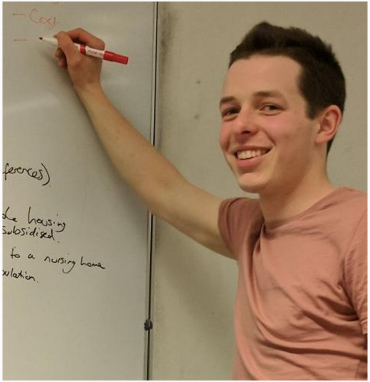
Combination forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  ) %>%
  arrange(crps)
```

```
## # A tibble: 3 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 COMB   Test   30.9  43.6
## 2 ETS    Test   31.5  38.3
## 3 ARIMA  Test   32.9  51.5
```

fable packages

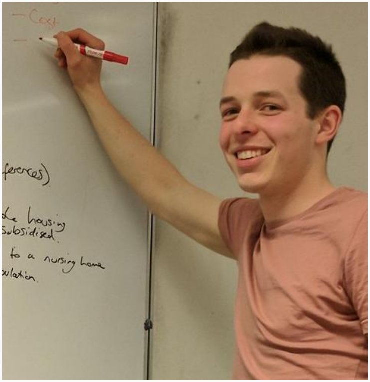
fable.tidyverts.org



Mitchell O’Hara-Wild

fable packages

fable.tidyverts.org



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