The background of the slide features a vibrant, abstract pattern resembling liquid or smoke, with swirling bands of orange, blue, yellow, and green against a dark background.

Ensemble forecasts with fable

Rob J Hyndman

14 August 2020

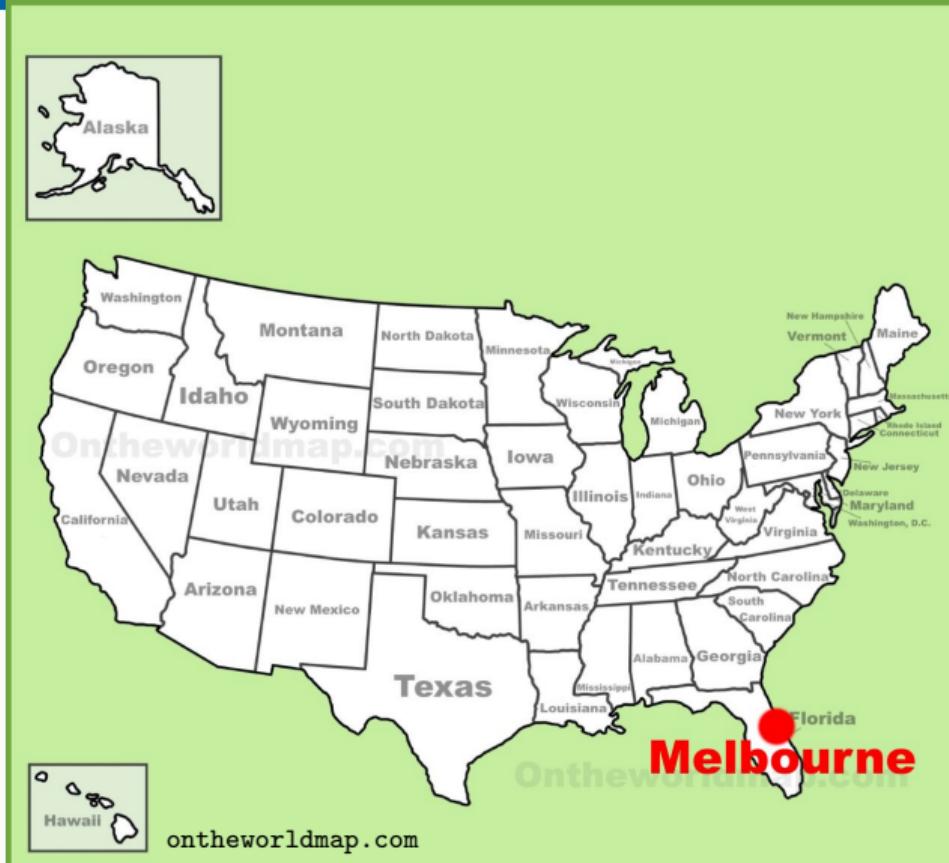
Outline

- 1 Quantile forecasting
- 2 Ensemble forecasting
- 3 Combination forecasting
- 4 Forecasting many series

Outline

- 1 Quantile forecasting
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Where is Melbourne?



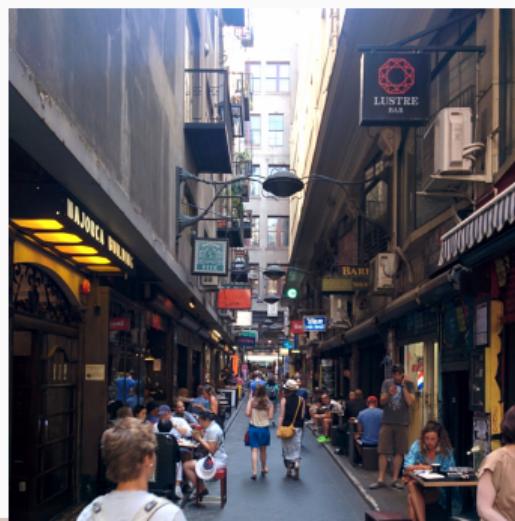
Where is Melbourne?



Where is Melbourne?

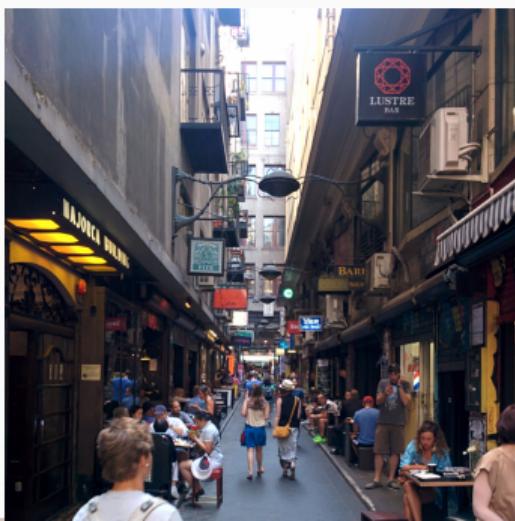
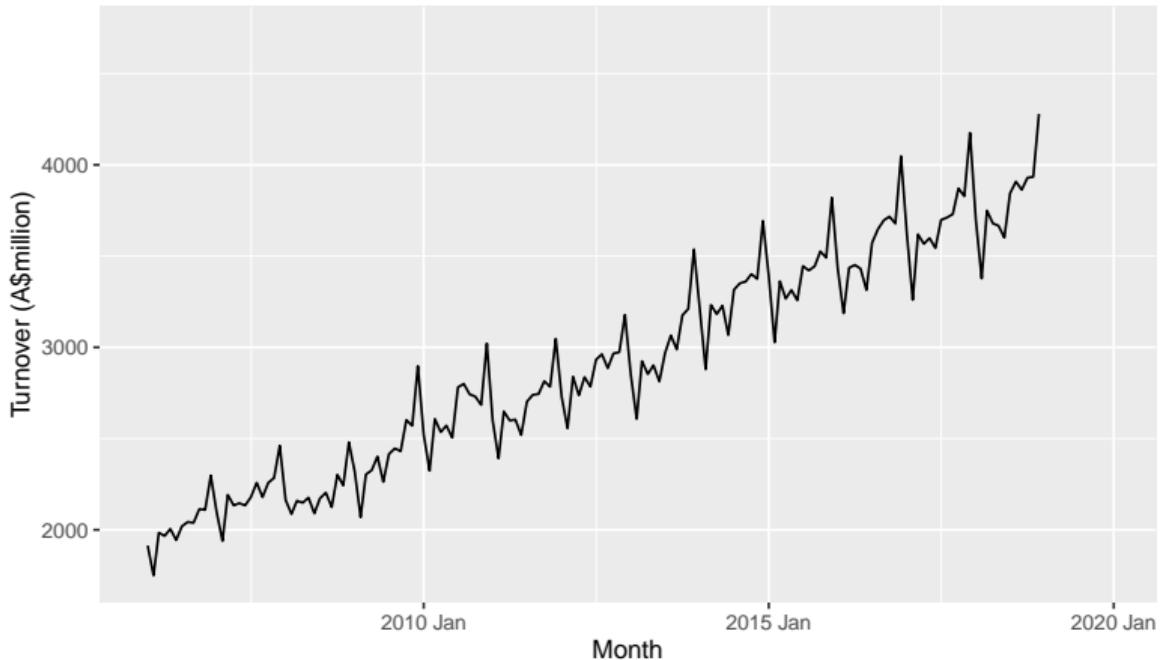


Where is Melbourne?



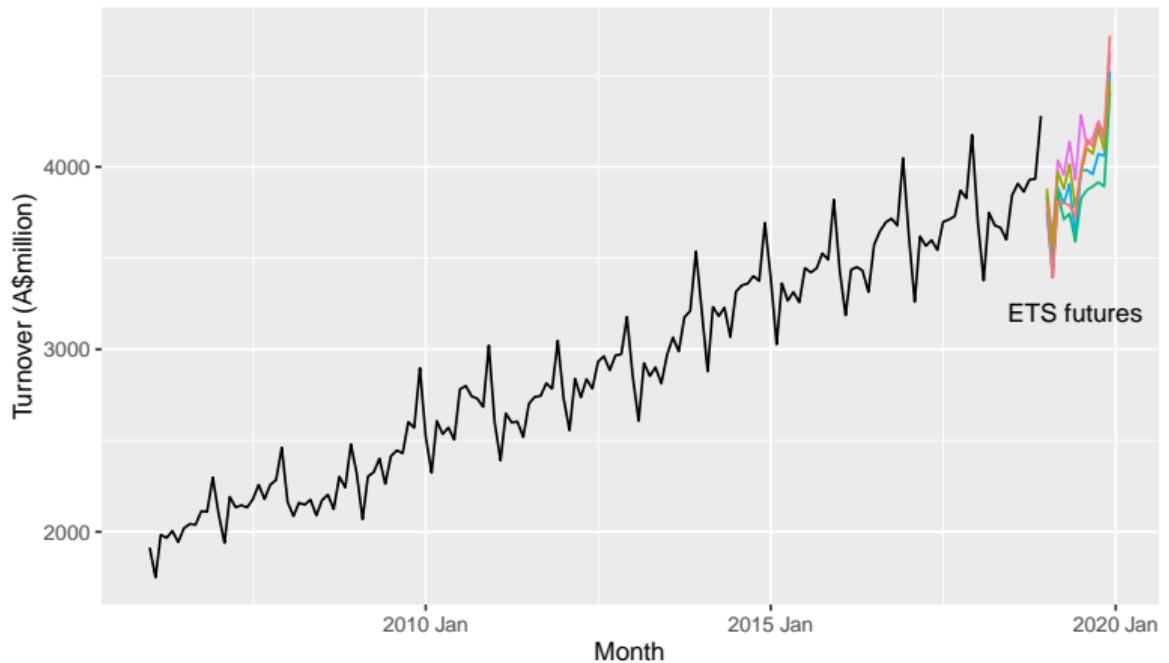
Australian monthly café turnover

Australian monthly café turnover

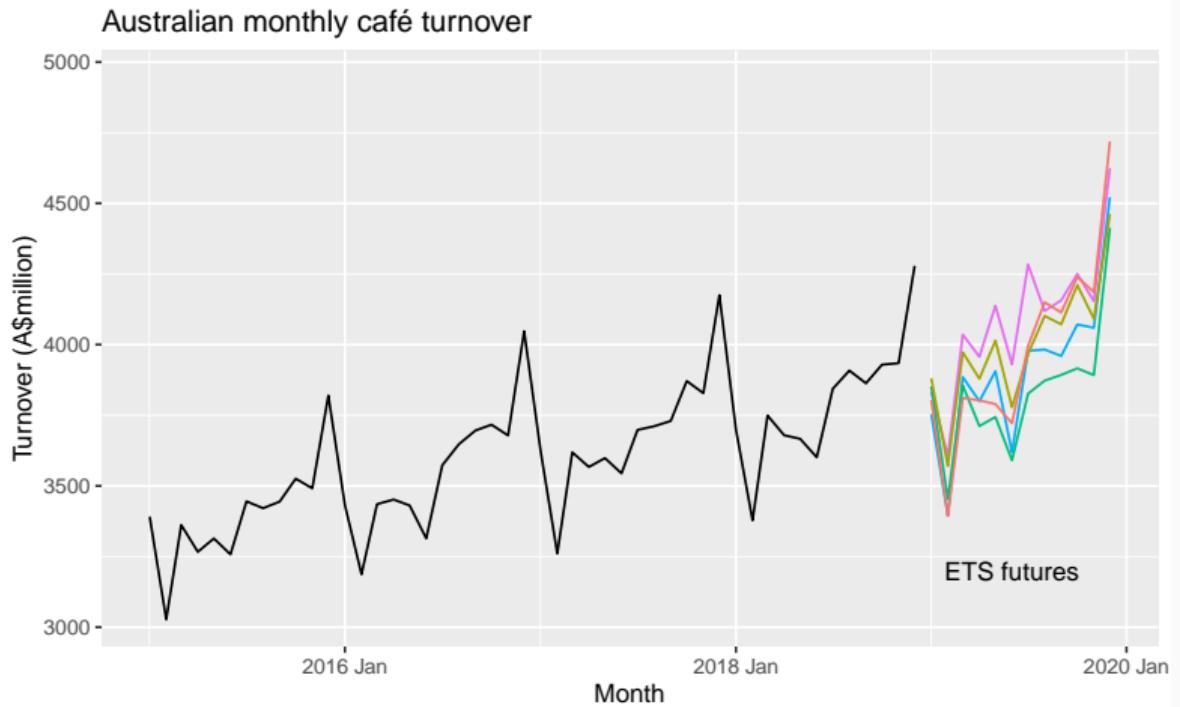


Forecasting using possible futures

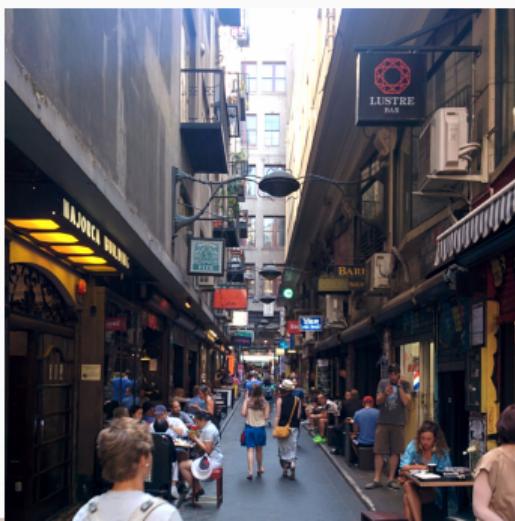
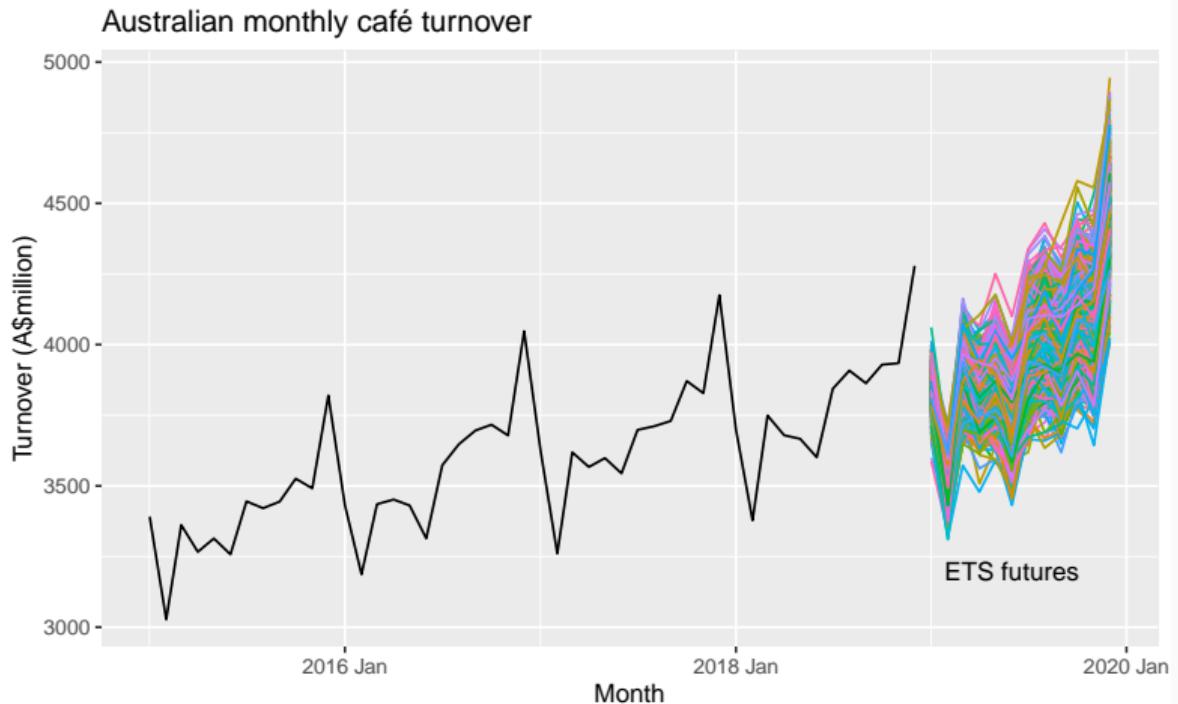
Australian monthly café turnover



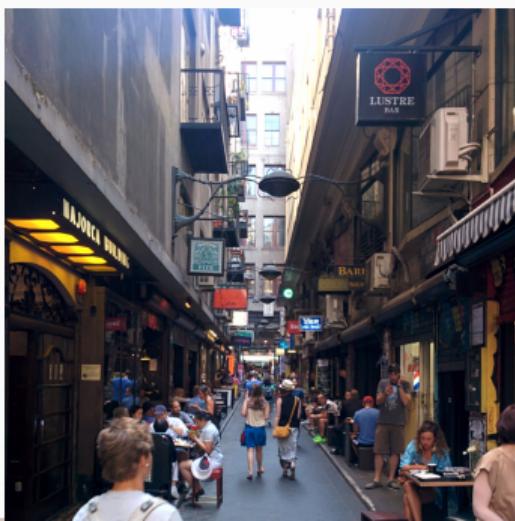
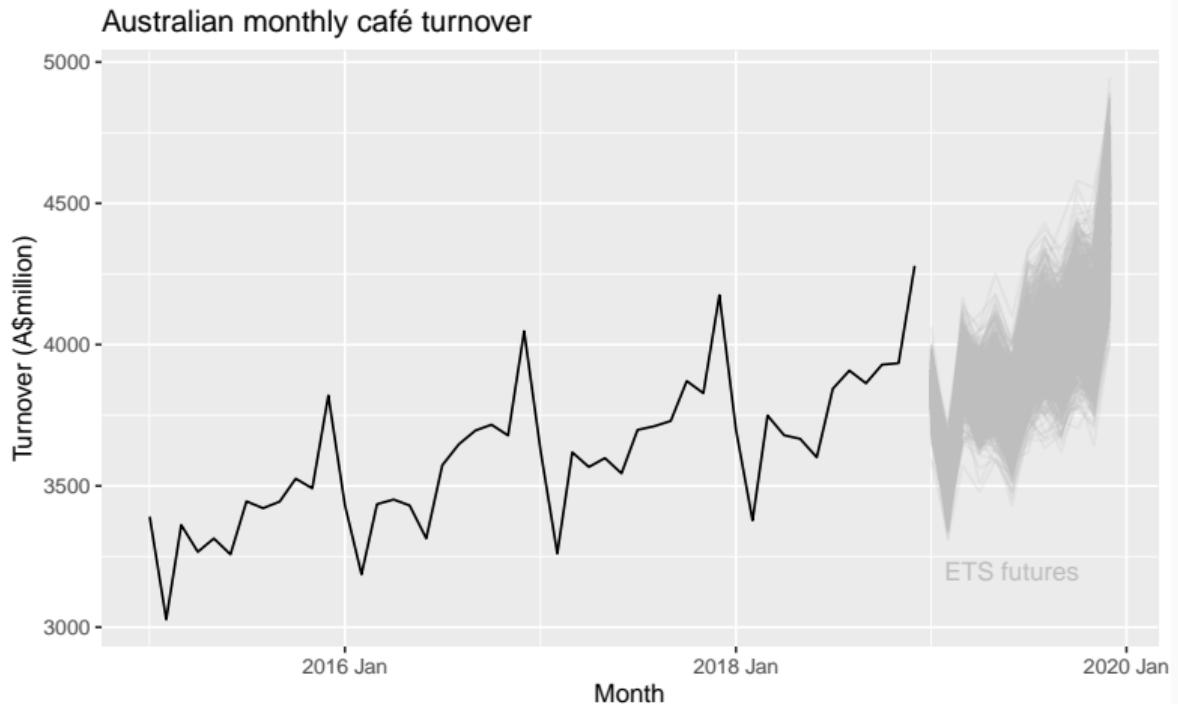
Forecasting using possible futures



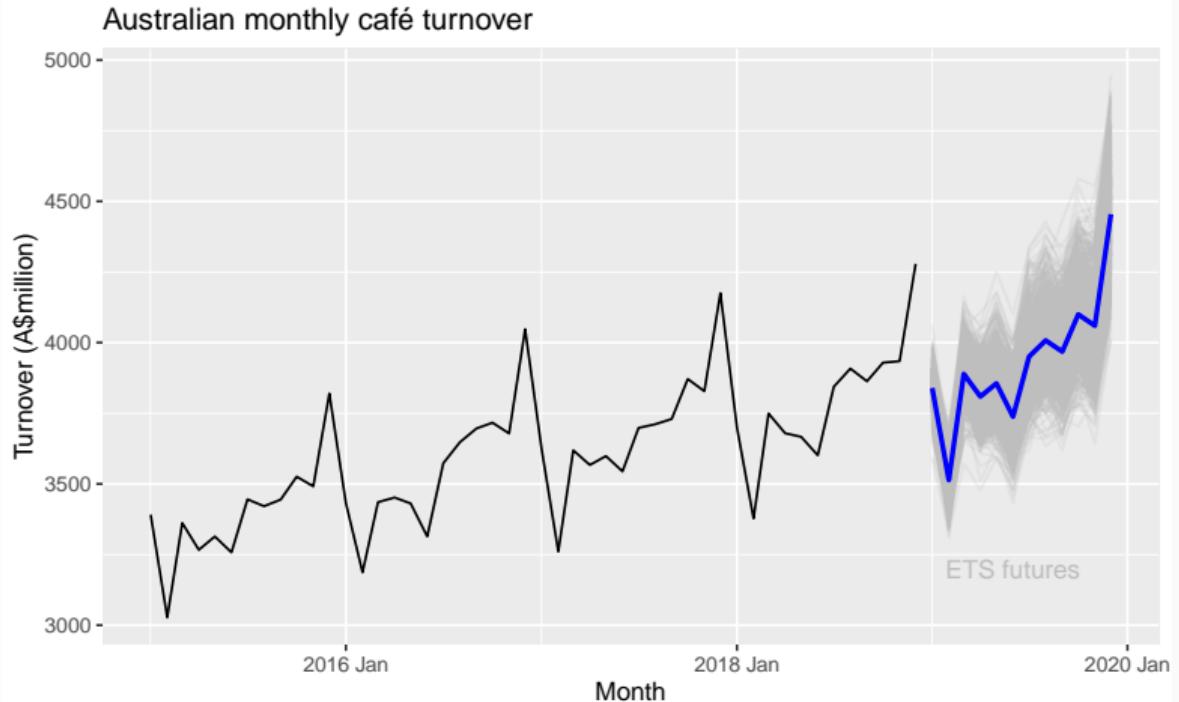
Forecasting using possible futures



Forecasting using possible futures

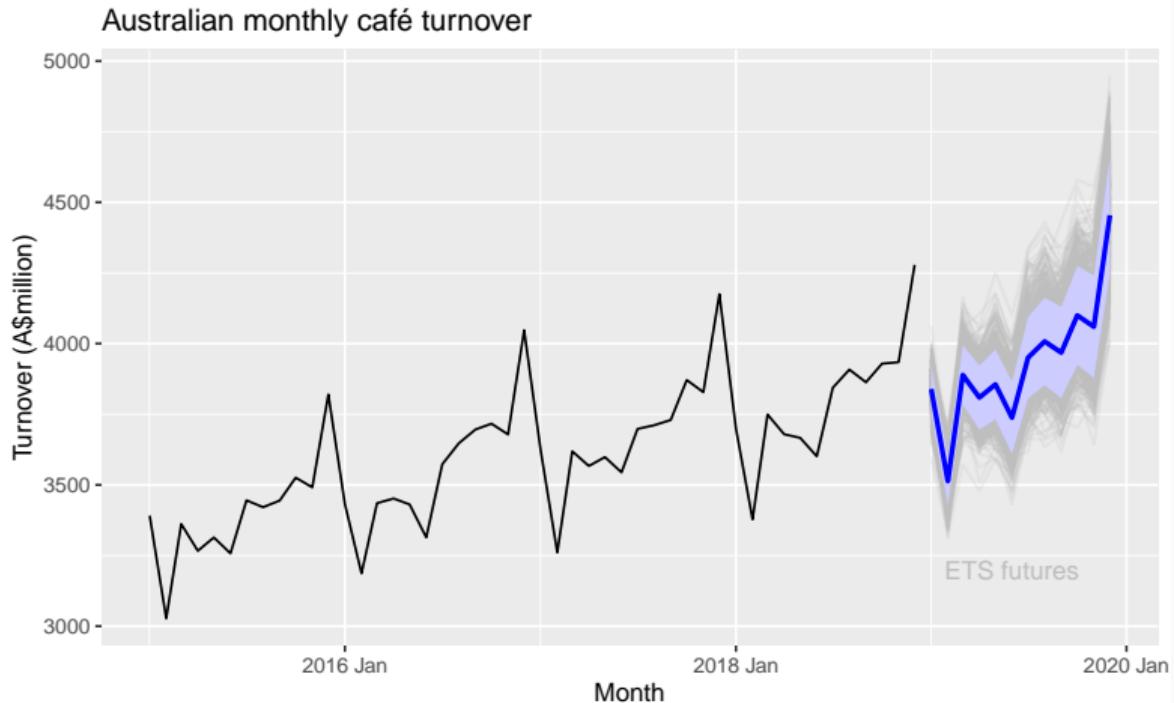


Forecasting using possible futures



Point forecasts: means of the sample paths.

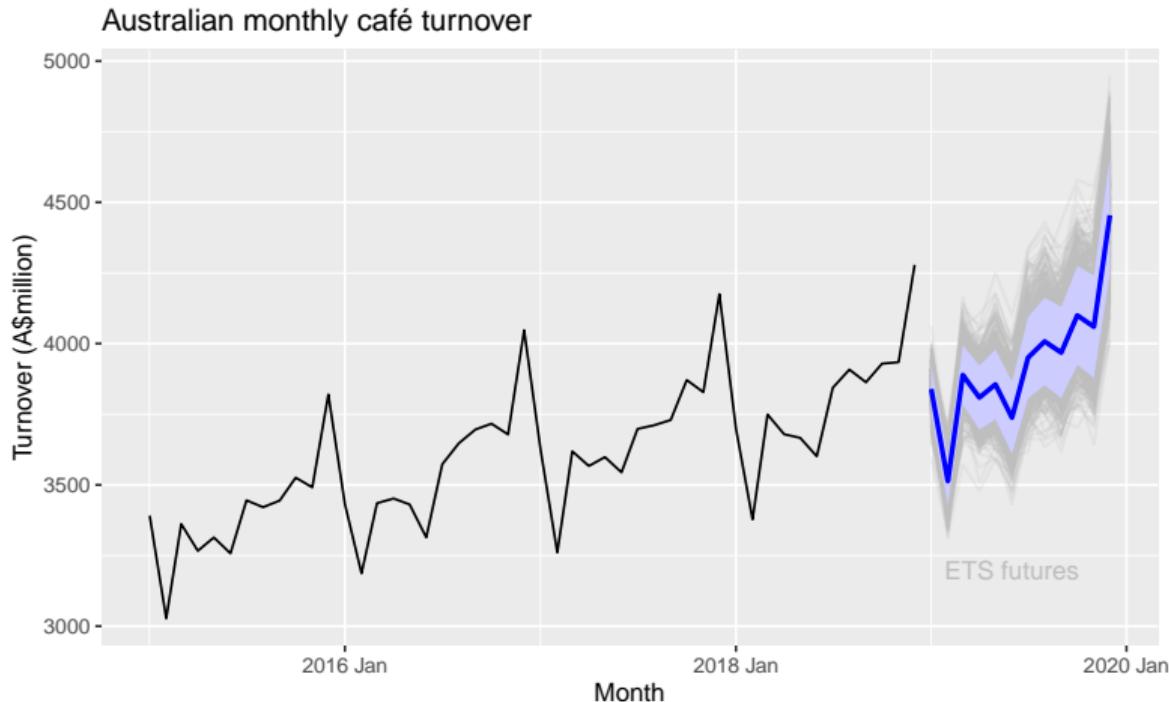
Forecasting using possible futures



Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

Forecasting using possible futures

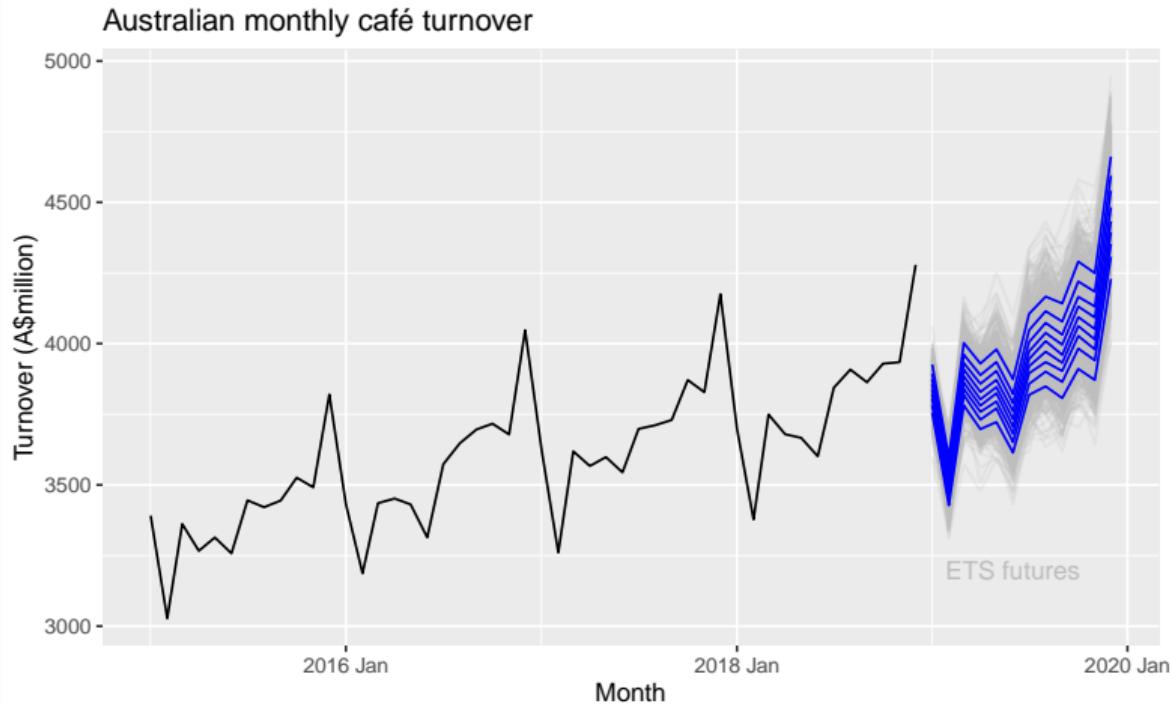


Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

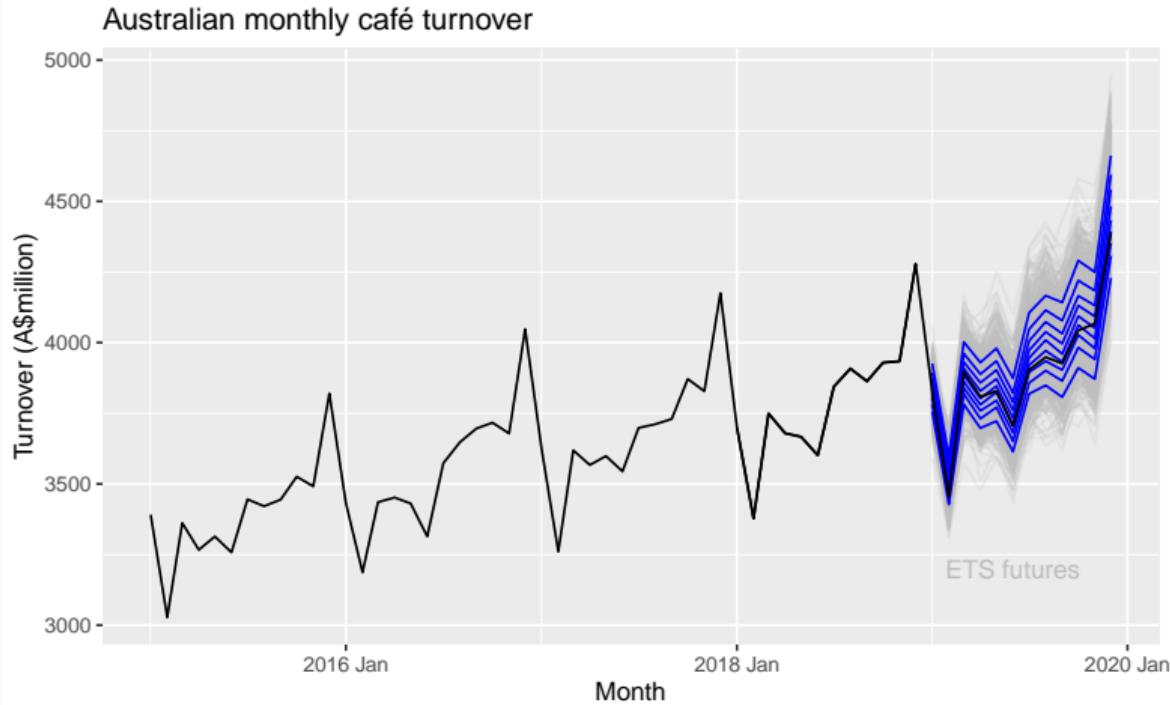
Quantile forecasts: Quantiles of the sample paths at each forecast horizon.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.
Black: Observed values.

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

- Low Q_p is good
- Multiplier of 2 often omitted, but useful for interpretation
- Q_p like absolute error (weighted to account for likely exceedance)
- Average Q_p = CRPS (Continuous Rank Probability Score)

Let's do some coding

tidyverts.org



Earo Wang



Mitchell O'Hara-Wild

Evaluating quantile forecasts

```
auscafe          ## # A tsibble: 168 x 2 [1M]
#>           date turnover
#>           <mth>    <dbl>
#> 1 2006 Jan     1914.
#> 2 2006 Feb     1750
#> 3 2006 Mar     1984.
#> 4 2006 Apr     1967.
#> 5 2006 May     2005.
#> 6 2006 Jun     1944.
#> 7 2006 Jul     2019.
#> 8 2006 Aug     2043.
#> 9 2006 Sep     2039.
#> 10 2006 Oct    2113.
#> # ... with 158 more rows
```

Evaluating quantile forecasts

```
auscafe %>%
  filter(year(date) <= 2018)
## # A tsibble: 156 x 2 [1M]
##       date turnover
##       <mth>    <dbl>
## 1 2006 Jan     1914.
## 2 2006 Feb     1750
## 3 2006 Mar     1984.
## 4 2006 Apr     1967.
## 5 2006 May     2005.
## 6 2006 Jun     1944.
## 7 2006 Jul     2019.
## 8 2006 Aug     2043.
## 9 2006 Sep     2039.
## 10 2006 Oct    2113.
## # ... with 146 more rows
```

Evaluating quantile forecasts

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##           ETS          ARIMA
##           <model>       <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Evaluating quantile forecasts

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year")
```

```
## # A fable: 24 x 4 [1M]
## # Key:     .model [2]
##   .model      date      turnover .mean
##   <chr>      <mth>      <dist> <dbl>
## 1 ETS      2019 Jan   N(3839, 4272) 3839.
## 2 ETS      2019 Feb   N(3514, 4587) 3514.
## 3 ETS      2019 Mar   N(3889, 6892) 3889.
## 4 ETS      2019 Apr   N(3809, 7868) 3809.
## 5 ETS      2019 May   N(3856, 9385) 3856.
## 6 ETS      2019 Jun   N(3738, 10098) 3738.
## 7 ETS      2019 Jul   N(3951, 12748) 3951.
## 8 ETS      2019 Aug   N(4008, 14670) 4008.
## 9 ETS      2019 Sep   N(3968, 15941) 3968.
## 10 ETS     2019 Oct   N(4100, 18726) 4100.
## # ... with 14 more rows
```

Evaluating quantile forecasts

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~ pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = auscafe,
    measures = list(crps=CRPS, rmse=RMSE)
) %>%
  arrange(crps)

## # A tibble: 2 x 4
##   .model .type   crps   rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test    31.3  41.1
## 2 ARIMA  Test    32.9  51.5
```

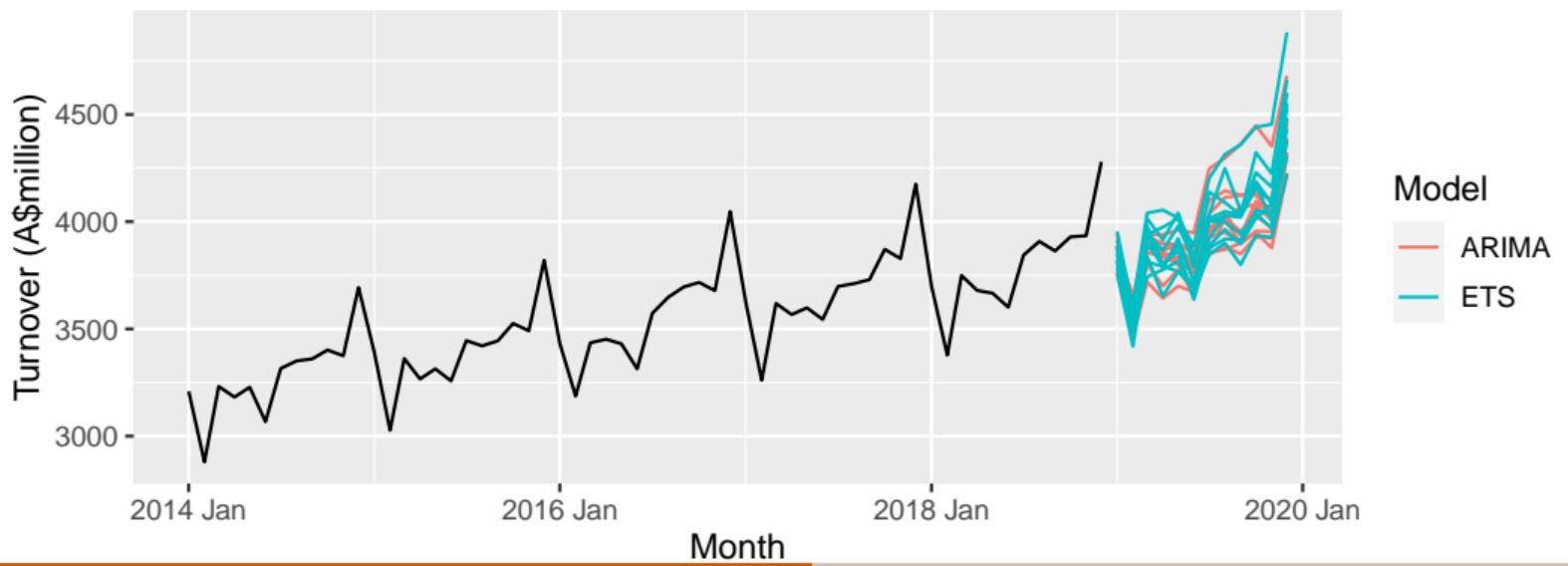
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Ensemble forecasting

Ensemble forecasting: mix the forecast distributions from multiple models.

- “All models are wrong, but some are useful” (George Box, 1976)
- Allows diverse models to be included, while reducing impact of any specific model.
- Allows uncertainty of model selection to be incorporated.



Ensemble forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 years")
```

```
## # A fable: 24 x 4 [1M]
## # Key:   .model [2]
##   .model     date     turnover .mean
##   <chr>     <mth>     <dist> <dbl>
## 1 ETS     2019 Jan   N(3839, 4272) 3839.
## 2 ETS     2019 Feb   N(3514, 4587) 3514.
## 3 ETS     2019 Mar   N(3889, 6892) 3889.
## 4 ETS     2019 Apr   N(3809, 7868) 3809.
## 5 ETS     2019 May   N(3856, 9385) 3856.
## 6 ETS     2019 Jun   N(3738, 10098) 3738.
## 7 ETS     2019 Jul   N(3951, 12748) 3951.
## 8 ETS     2019 Aug   N(4008, 14670) 4008.
## 9 ETS     2019 Sep   N(3968, 15941) 3968.
## 10 ETS    2019 Oct   N(4100, 18726) 4100.
## # ... with 14 more rows
```

Ensemble forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  )
```

```
## # A tsibble: 12 x 2 [1M]
##       date   turnover
##       <mth>   <dist>
## 1 2019 Jan mixture(n=2)
## 2 2019 Feb mixture(n=2)
## 3 2019 Mar mixture(n=2)
## 4 2019 Apr mixture(n=2)
## 5 2019 May mixture(n=2)
## 6 2019 Jun mixture(n=2)
## 7 2019 Jul mixture(n=2)
## 8 2019 Aug mixture(n=2)
## 9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
            distribution = turnover)
```

```
## # A fable: 12 x 2 [1M]
##       date   turnover
##       <mth>   <dist>
## 1 2019 Jan mixture(n=2)
## 2 2019 Feb mixture(n=2)
## 3 2019 Mar mixture(n=2)
## 4 2019 Apr mixture(n=2)
## 5 2019 May mixture(n=2)
## 6 2019 Jun mixture(n=2)
## 7 2019 Jul mixture(n=2)
## 8 2019 Aug mixture(n=2)
## 9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
            distribution = turnover) %>%
  accuracy(
```

A tibble: 1 x 3
.type crps rmse
<chr> <dbl> <dbl>
1 Test 31.7 45.1

Ensemble forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
forecast(h = "1 year") %>%
summarise(
  turnover = dist_mixture(
    turnover[1],
    turnover[2],
    weights = c(0.5, 0.5)
  )
) %>%
as_fable(response = "turnover",
          distribution = turnover) %>%
accuracy(
```

A tibble: 1 x 3
.type crps rmse
<chr> <dbl> <dbl>
1 Test 31.7 45.1

Comparison:

```
## # A tibble: 2 x 4  
##   .model .type   crps   rmse  
##   <chr>  <chr> <dbl> <dbl>  
## 1 ETS    Test    31.3  41.1  
## 2 ARIMA  Test    32.9  51.5
```

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Combination forecasting

Combination forecasting: take weighted average of forecasts from multiple models.

- Often a simple average is used.
- Reduces uncertainty associated with selecting a particular model.
- Combination forecasting usually improves point forecast accuracy.
- Mean forecast identical to that from corresponding weighted ensemble.
- Quantile forecasts need to account for correlations between forecast errors from component models.

Combination forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1)))
)
## # A mable: 1 x 2
##           ETS          ARIMA
##           <model>       <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Combination forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1)))
) %>%
  mutate(COMB = (ETS + ARIMA)/2)

## # A mable: 1 x 3
##           ETS          ARIMA
##           <model>     <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]> <CO
```

Combination forecasting

```
auscafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year")
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [3]
##   .model      date      turnover .mean
##   <chr>      <mth>      <dist> <dbl>
## 1 ETS      2019 Jan    N(3839, 4272) 3839.
## 2 ETS      2019 Feb    N(3514, 4587) 3514.
## 3 ETS      2019 Mar    N(3889, 6892) 3889.
## 4 ETS      2019 Apr    N(3809, 7868) 3809.
## 5 ETS      2019 May    N(3856, 9385) 3856.
## 6 ETS      2019 Jun    N(3738, 10098) 3738.
## 7 ETS      2019 Jul    N(3951, 12748) 3951.
## 8 ETS      2019 Aug    N(4008, 14670) 4008.
## 9 ETS      2019 Sep    N(3968, 15941) 3968.
## 10 ETS     2019 Oct    N(4100, 18726) 4100.
## # ... with 26 more rows
```

Combination forecasting

```
auscafe %>%
  filter(date <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = auscafe,
    measures = list(crps=CRPS, rmse=RMSE)
) %>%
  arrange(crps)

## # A tibble: 3 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 COMB   Test   30.9  45.1
## 2 ETS    Test   31.3  41.1
## 3 ARIMA  Test   32.9  51.5
```

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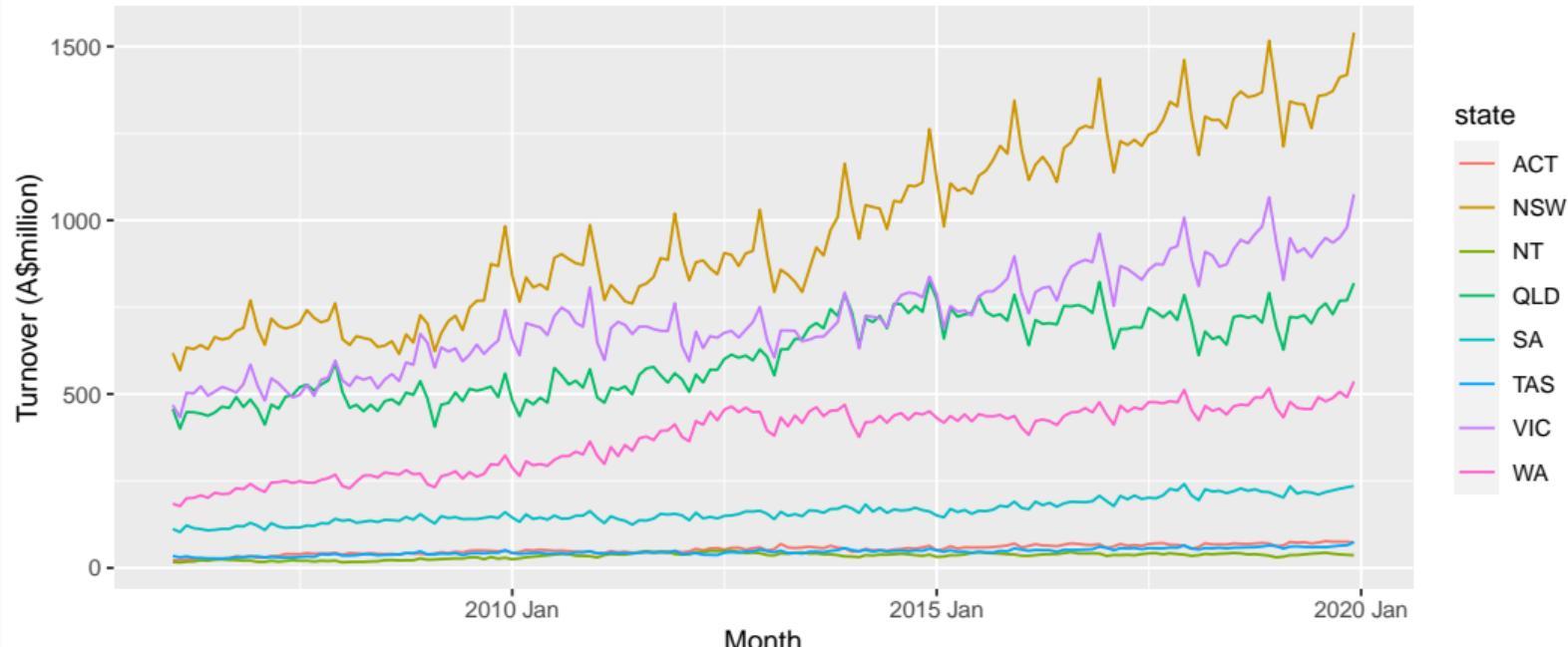
Forecasting many series

```
cafe          ## # A tsibble: 1,344 x 3 [1M]
## # Key:           state [8]
##       date state turnover
##       <mth> <chr>    <dbl>
## 1 2006 Jan ACT     21.7
## 2 2006 Feb ACT     21.9
## 3 2006 Mar ACT     24.9
## 4 2006 Apr ACT     24.8
## 5 2006 May ACT      27
## 6 2006 Jun ACT     24.5
## 7 2006 Jul ACT      24
## 8 2006 Aug ACT     26.1
## 9 2006 Sep ACT     26.2
## 10 2006 Oct ACT    33.7
## # ... with 1,334 more rows
```

Forecasting many series

```
cafe %>% autoplot(turnover)
```

Australian monthly café turnover



Forecasting many series

```
cafe %>%  
  filter(year(date) <= 2018)  
  
## # A tsibble: 1,248 x 3 [1M]  
## # Key:      state [8]  
##       date state turnover  
##       <mth> <chr>    <dbl>  
## 1 2006 Jan ACT     21.7  
## 2 2006 Feb ACT     21.9  
## 3 2006 Mar ACT     24.9  
## 4 2006 Apr ACT     24.8  
## 5 2006 May ACT     27  
## 6 2006 Jun ACT     24.5  
## 7 2006 Jul ACT     24  
## 8 2006 Aug ACT     26.1  
## 9 2006 Sep ACT     26.2  
## 10 2006 Oct ACT    33.7  
## # ... with 1,238 more rows
```

Forecasting many series

```
cafe %>%  
  filter(year(date) <= 2018) %>%  
  model(  
    ETS = ETS(turnover),  
    ARIMA = ARIMA(turnover ~  
                  pdq(d=1) + PDQ(D=1)),  
    SNAIVE = SNAIVE(turnover)  
) %>%  
  mutate(  
    COMB = (ETS+ARIMA)/2  
)
```

```
## # A mable: 8 x 5  
## # Key:      state [8]  
##   state          ETS  
##   <chr>        <model>      <mod  
## 1 ACT <ETS(M,Ad,M)> <ARIMA(2,1,2)(0,1,1)[1]  
## 2 NSW <ETS(M,Ad,M)> <ARIMA(0,1,1)(0,1,2)[1]  
## 3 NT  <ETS(A,N,A)> <ARIMA(1,1,0)(1,1,0)[1]  
## 4 QLD <ETS(M,Ad,M)> <ARIMA(0,1,1)(1,1,1)[1]  
## 5 SA  <ETS(M,Ad,M)> <ARIMA(0,1,1)(0,1,1)[1]  
## 6 TAS <ETS(A,N,A)> <ARIMA(0,1,1)(0,1,1)[1]  
## 7 VIC <ETS(M,Ad,M)> <ARIMA(0,1,2)(0,1,2)[1]  
## 8 WA  <ETS(M,Ad,M)> <ARIMA(0,1,1)(0,1,2)[1]
```

Forecasting many series

```
cafe %>%  
  filter(year(date) <= 2018) %>%  
  model(  
    ETS = ETS(turnover),  
    ARIMA = ARIMA(turnover ~  
                  pdq(d=1) + PDQ(D=1)),  
    SNAIVE = SNAIVE(turnover)  
) %>%  
  mutate(  
    COMB = (ETS+ARIMA)/2  
) %>%  
  forecast(h = "1 year")
```

```
## # A fable: 384 x 5 [1M]  
## # Key:      state, .model [32]  
##      state .model      date   turnover .mean  
##      <chr> <chr>     <mth>     <dist> <dbl>  
## 1 ACT   ETS      2019 Jan N(61, 9.6) 60.7  
## 2 ACT   ETS      2019 Feb N(64, 15) 63.8  
## 3 ACT   ETS      2019 Mar N(72, 26) 71.9  
## 4 ACT   ETS      2019 Apr N(67, 28) 67.0  
## 5 ACT   ETS      2019 May N(70, 36) 69.8  
## 6 ACT   ETS      2019 Jun N(67, 39) 67.3  
## 7 ACT   ETS      2019 Jul N(68, 46) 68.4  
## 8 ACT   ETS      2019 Aug N(70, 53) 69.7  
## 9 ACT   ETS      2019 Sep N(69, 57) 68.5  
## 10 ACT  ETS      2019 Oct N(70, 66) 70.2  
## # ... with 374 more rows
```

Forecasting many series

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)),
    SNAIVE = SNAIVE(turnover)
  ) %>%
  mutate(
    COMB = (ETS+ARIMA)/2
  ) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  )
## # A tibble: 32 x 5
##       .model state .type   crps   rmse
##       <chr>   <chr> <chr>   <dbl>   <dbl>
## 1 ARIMA   NSW   Test    1.64   2.23
## 2 ARIMA   NSW   Test   18.4   28.4
## 3 ARIMA   NT    Test    2.19   3.89
## 4 ARIMA   QLD   Test   15.0   24.9
## 5 ARIMA   SA    Test    4.06   6.70
## 6 ARIMA   TAS   Test    1.52   2.70
## 7 ARIMA   VIC   Test   30.4   48.6
## 8 ARIMA   WA    Test    9.06   14.8
## 9 COMB    ACT   Test    2.02   3.31
## 10 COMB   NSW   Test   17.8   14.8
## # ... with 22 more rows
```

Forecasting many series

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)),
    SNAIVE = SNAIVE(turnover)
  ) %>%
  mutate(
    COMB = (ETS+ARIMA)/2
  ) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  ) ->
  crps
```

```
## # A tibble: 32 x 5
##   .model state .type  crps  rmse
##   <chr>  <chr> <chr> <dbl> <dbl>
## 1 ARIMA  NSW   Test   1.64  2.23
## 2 ARIMA  NT    Test   18.4   28.4
## 3 ARIMA  QLD   Test   2.19  3.89
## 4 ARIMA  SA    Test   15.0   24.9
## 5 ARIMA  TAS   Test   4.06  6.70
## 6 ARIMA  VIC   Test   1.52  2.70
## 7 ARIMA  WA    Test   30.4   48.6
## 8 ARIMA  COMB  ACT   9.06  14.8
## 9 COMB   NSW   Test   2.02  3.31
## 10 COMB   ACT   Test   17.8   14.8
## # ... with 22 more rows
```

Forecasting many series

```
sn_crps <- crps %>%
  filter(.model=="SNAIVE") %>%
  select(state,crps) %>%
  rename(sn_crps = "crps")
sn_crps
```

```
## # A tibble: 8 x 2
##   state    sn_crps
##   <chr>     <dbl>
## 1 ACT        3.06
## 2 NSW       27.5
## 3 NT         1.61
## 4 QLD       25.1
## 5 SA         5.16
## 6 TAS       2.71
## 7 VIC       16.8
## 8 WA        10.3
```

Forecasting many series

```
sn_crps <- crps %>%
  filter(.model=="SNAIVE") %>%
  select(state,crps) %>%
  rename(sn_crps = "crps")
crps %>%
  filter(.model != "SNAIVE") %>%
  left_join(sn_crps, by="state") %>%
  mutate(
    skill = 100*(sn_crps - crps)/sn_crps
  ) %>%
  group_by(.model) %>%
  summarise(
    skill = mean(skill)
  )
```

```
## # A tibble: 3 x 2
##   .model skill
##   <chr>   <dbl>
## 1 ARIMA    9.91
## 2 COMB    14.6 
## 3 ETS     6.23
```

Forecasting many series

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)),
    SNAIVE = SNAIVE(turnover)
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5, 0)
    )
  ) %>%
  as_fable(response = "turnover",
            distribution = turnover) %>%
```

Final words

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robjhyndman.com/nycr2020