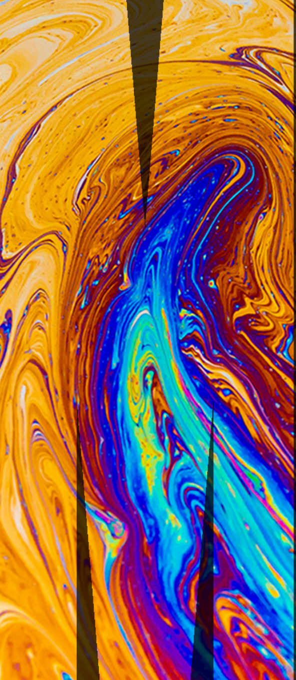
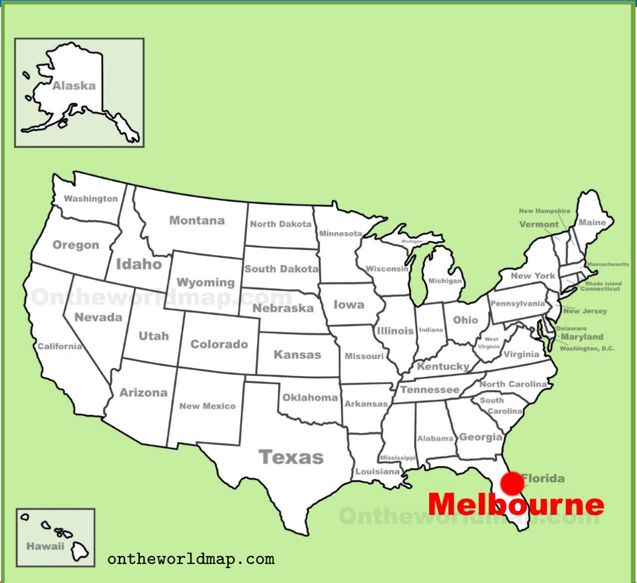


Ensemble forecasts with fable

Rob J Hyndman

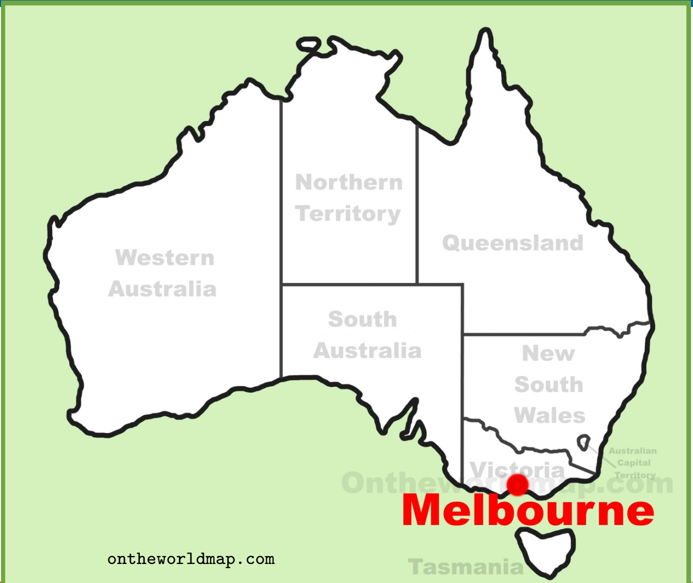
14 August 2020

Where is Melbourne?



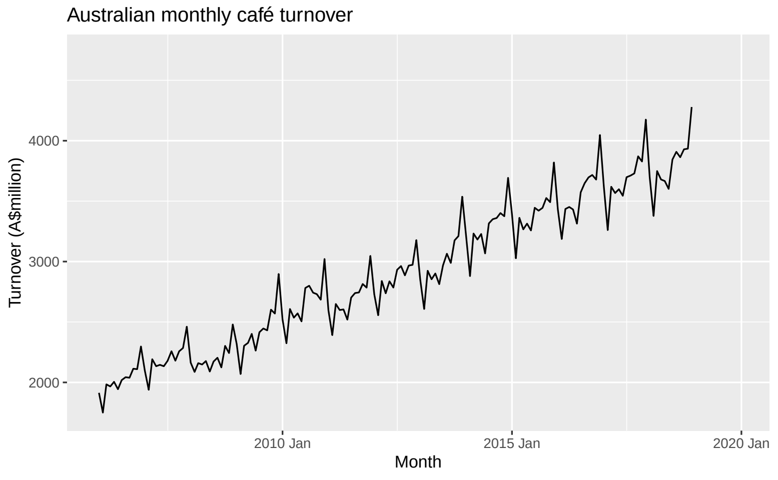
Where is Melbourne?

Where is Melbourne?

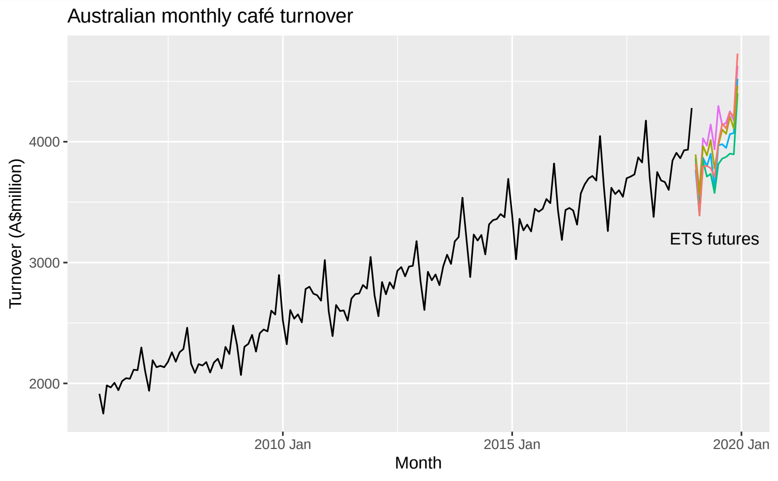


Where is Melbourne?

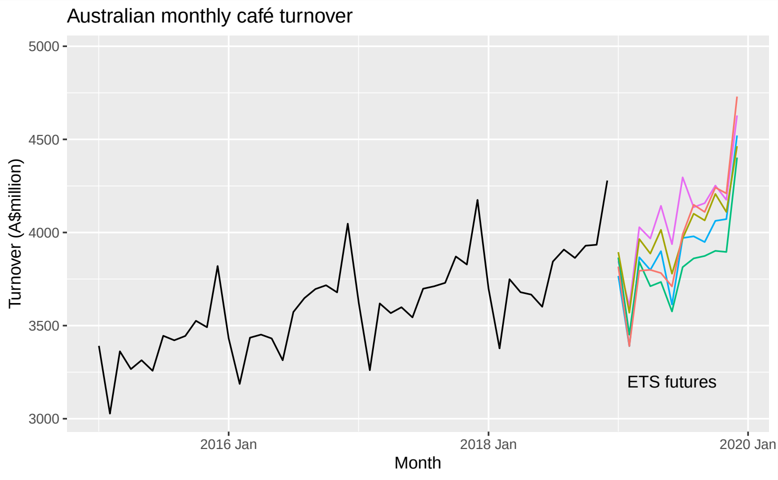
Australian monthly café turnover



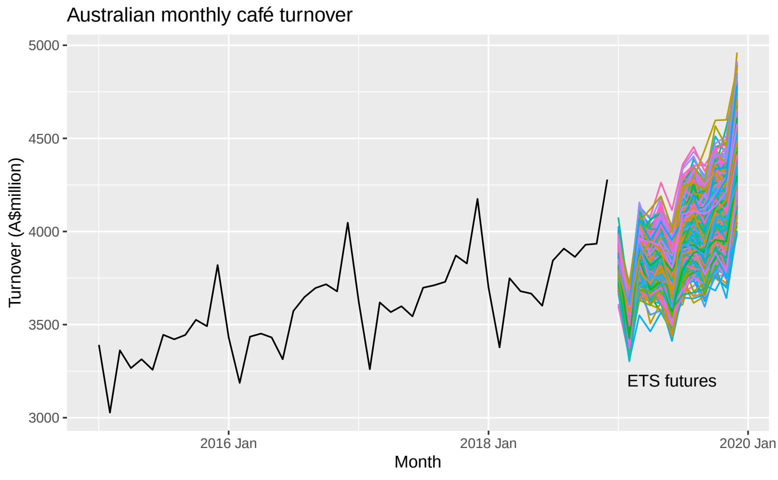
Forecasting using possible futures



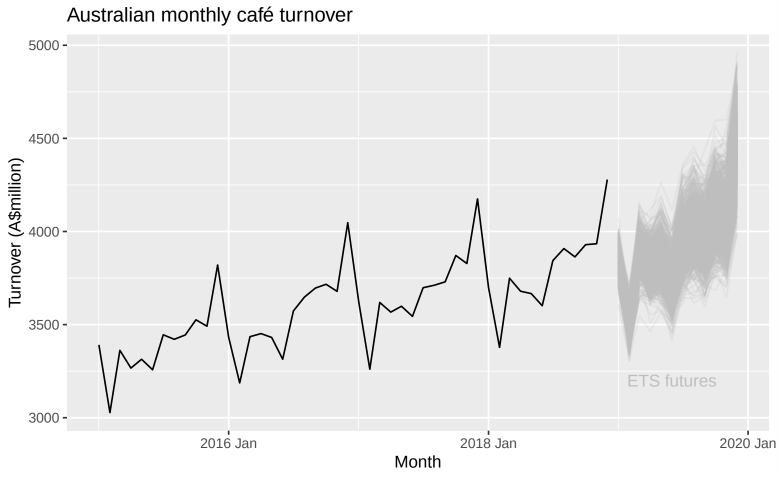
Forecasting using possible futures



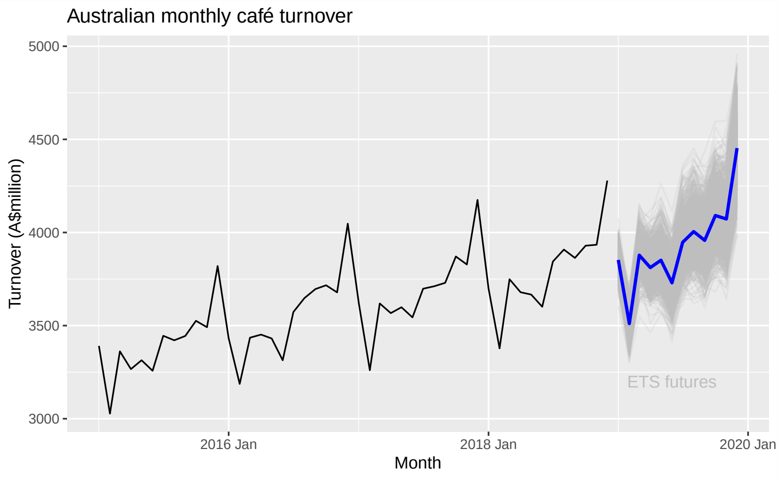
Forecasting using possible futures



Forecasting using possible futures

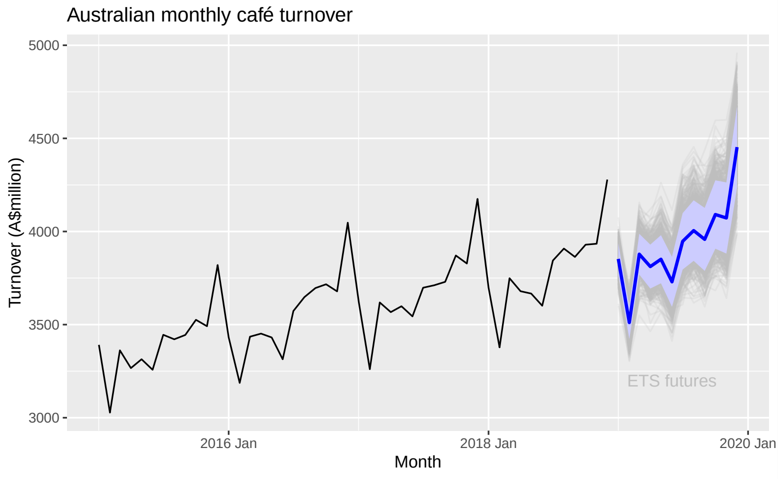


Forecasting using possible futures



Point forecasts: means of the sample paths.

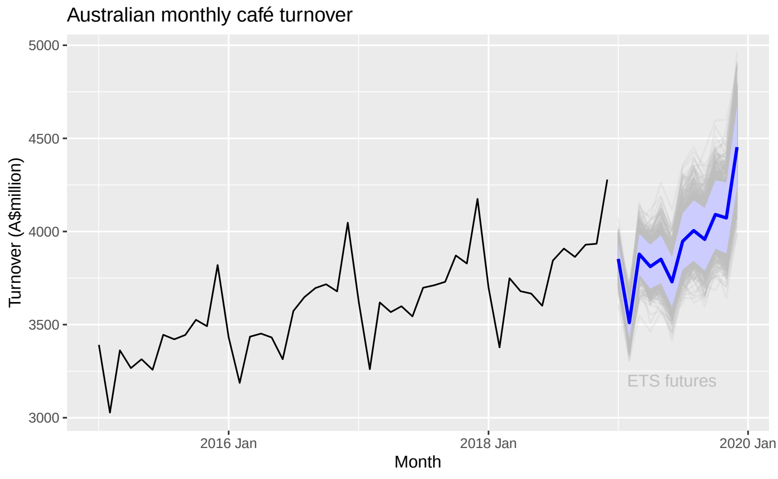
Forecasting using possible futures



Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

Forecasting using possible futures

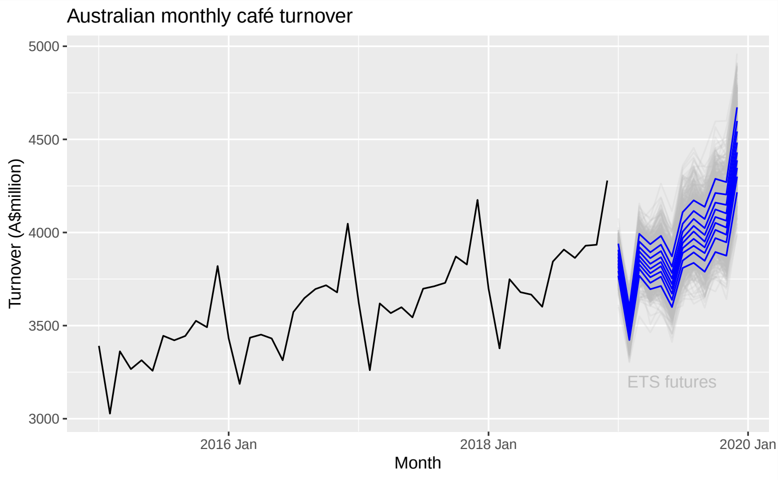


Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

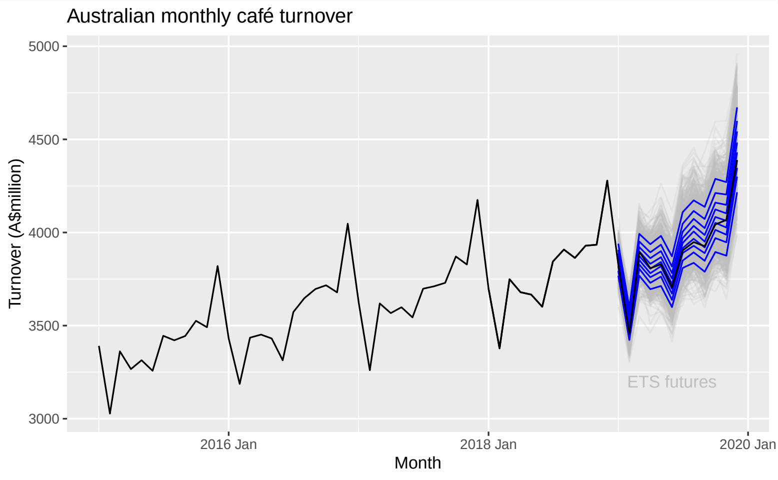
Quantile forecasts: Quantiles of the sample paths at each forecast horizon.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.
Black: Observed values.

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)\big|y_t - f_{p,t}\big|, & \text{if } y_t < f_{p,t} \\ 2p\big|y_t - f_{p,t}\big|, & \text{if } y_t \ge f_{p,t} \end{cases}$$

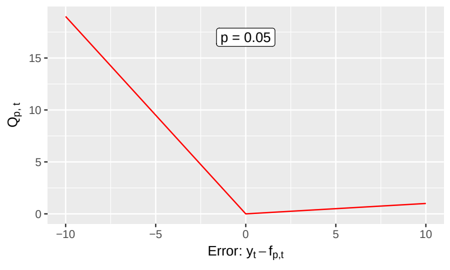
Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$



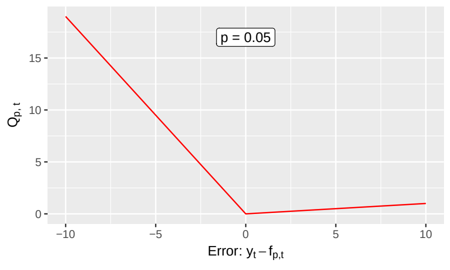
Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t.

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)\big|y_t - f_{p,t}\big|, & \text{if } y_t < f_{p,t} \\ 2p\big|y_t - f_{p,t}\big|, & \text{if } y_t \ge f_{p,t} \end{cases}$$



- Low Q_p is good
- Multiplier of 2 often omitted, but useful for interpretation
- Q_p like absolute error (weighted to account for likely exceedance)
- Average Q_p = CRPS (Continuous Rank Probability Score)

Evaluating quantile forecasts

```r
cafe
```

```
## # A tsibble: 168 x 2 [1M]
##         date turnover
##        <mth>    <dbl>
##  1 2006 Jan    1914.
##  2 2006 Feb    1750.
##  3 2006 Mar    1984.
##  4 2006 Apr    1966.
##  5 2006 May    2005.
##  6 2006 Jun    1944.
##  7 2006 Jul    2019.
##  8 2006 Aug    2043.
##  9 2006 Sep    2039.
## 10 2006 Oct    2113.
## # ... with 158 more rows
```

Evaluating quantile forecasts

```r
cafe %>%
  filter(year(date) <= 2018)
```

```
## # A tsibble: 156 x 2 [1M]
##        date turnover
##       <mth>    <dbl>
##  1 2006 Jan    1914.
##  2 2006 Feb    1750.
##  3 2006 Mar    1984.
##  4 2006 Apr    1966.
##  5 2006 May    2005.
##  6 2006 Jun    1944.
##  7 2006 Jul    2019.
##  8 2006 Aug    2043.
##  9 2006 Sep    2039.
## 10 2006 Oct    2113.
## # ... with 146 more rows
```

Evaluating quantile forecasts

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##              ETS                          ARIMA
##          <model>                        <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Evaluating quantile forecasts

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year")
```

```
## # A fable: 24 x 4 [1M]
## # Key:     .model [2]
##    .model     date        turnover .mean
##    <chr>     <mth>          <dist> <dbl>
##  1 ETS    2019 Jan  N(3853, 4315) 3853.
##  2 ETS    2019 Feb  N(3511, 4760) 3511.
##  3 ETS    2019 Mar  N(3878, 7273) 3878.
##  4 ETS    2019 Apr  N(3812, 8472) 3812.
##  5 ETS    2019 May N(3851, 10155) 3851.
##  6 ETS    2019 Jun N(3730, 10968) 3730.
##  7 ETS    2019 Jul N(3947, 13933) 3947.
##  8 ETS    2019 Aug N(4005, 16074) 4005.
##  9 ETS    2019 Sep N(3958, 17421) 3958.
## 10 ETS    2019 Oct N(4091, 20493) 4091.
## # ... with 14 more rows
```

Evaluating quantile forecasts

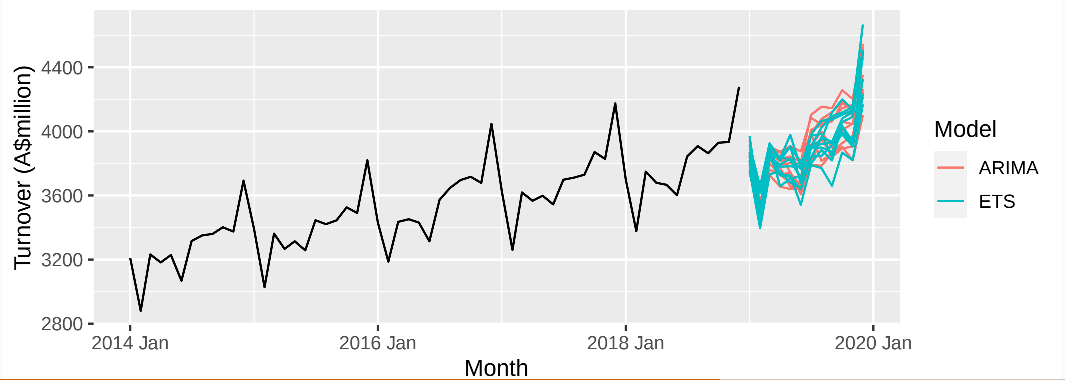
```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  ) %>%
  arrange(crps)
```

```
## # A tibble: 2 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test   31.5  38.3
## 2 ARIMA  Test   32.9  51.5
```

Ensemble forecasting

Ensemble forecasting involves combining the forecast distributions from multiple models.

- "All models are wrong, but some are useful" (George Box, 1976)
- Allows diverse models to be included, while reducing impact of any specific model.
- Allows uncertainty of model selection to be incorporated.



Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 years")
```

```
## # A fable: 24 x 4 [1M]
## # Key:      .model [2]
##     .model     date         turnover .mean
##      <chr>     <mth>          <dist> <dbl>
##   1 ETS     2019 Jan  N(3853, 4315) 3853.
##   2 ETS     2019 Feb  N(3511, 4760) 3511.
##   3 ETS     2019 Mar  N(3878, 7273) 3878.
##   4 ETS     2019 Apr  N(3812, 8472) 3812.
##   5 ETS     2019 May N(3851, 10155) 3851.
##   6 ETS     2019 Jun N(3730, 10968) 3730.
##   7 ETS     2019 Jul N(3947, 13933) 3947.
##   8 ETS     2019 Aug N(4005, 16074) 4005.
##   9 ETS     2019 Sep N(3958, 17421) 3958.
##  10 ETS     2019 Oct N(4091, 20493) 4091.
## # ... with 14 more rows
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  )
```

```
## # A tsibble: 12 x 2 [1M]
##          date       turnover
##         <mth>         <dist>
##   1 2019 Jan mixture(n=2)
##   2 2019 Feb mixture(n=2)
##   3 2019 Mar mixture(n=2)
##   4 2019 Apr mixture(n=2)
##   5 2019 May mixture(n=2)
##   6 2019 Jun mixture(n=2)
##   7 2019 Jul mixture(n=2)
##   8 2019 Aug mixture(n=2)
##   9 2019 Sep mixture(n=2)
##  10 2019 Oct mixture(n=2)
##  11 2019 Nov mixture(n=2)
##  12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover)
```

```
## # A fable: 12 x 2 [1M]
##          date      turnover
##         <mth>        <dist>
##  1 2019 Jan mixture(n=2)
##  2 2019 Feb mixture(n=2)
##  3 2019 Mar mixture(n=2)
##  4 2019 Apr mixture(n=2)
##  5 2019 May mixture(n=2)
##  6 2019 Jun mixture(n=2)
##  7 2019 Jul mixture(n=2)
##  8 2019 Aug mixture(n=2)
##  9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover) %>%
  accuracy(
```

```
## # A tibble: 1 x 3
##   .type  crps  rmse
##   <chr> <dbl> <dbl>
## 1 Test   31.7  43.6
```

Ensemble forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                    pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
           distribution = turnover) %>%
  accuracy(
```

```
## # A tibble: 1 x 3
##   .type  crps  rmse
##   <chr> <dbl> <dbl>
## 1 Test   31.7  43.6
```

Comparison:

```
## # A tibble: 2 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test   31.5  38.3
## 2 ARIMA  Test   32.9  51.5
```

Combination forecasting

Combination forecasting: weighted averages of forecasts from multiple models.

- Often a simple average is used.
- Reduces uncertainty associated with selecting a particular model.
- Combination forecasting usually improves point forecast accuracy.
- Mean forecast identical to that from corresponding weighted ensemble.
- Quantile forecasts need to account for correlations between forecast errors from component models.

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                      pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##             ETS                       ARIMA
##         <model>                     <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Combination forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2)
```

```
## # A mable: 1 x 3
##             ETS                     ARIMA
##         <model>                   <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]> <CO
```

Combination forecasting

```r
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year")
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [3]
##    .model     date        turnover .mean
##    <chr>     <mth>          <dist> <dbl>
##  1 ETS    2019 Jan  N(3853, 4315) 3853.
##  2 ETS    2019 Feb  N(3511, 4760) 3511.
##  3 ETS    2019 Mar  N(3878, 7273) 3878.
##  4 ETS    2019 Apr  N(3812, 8472) 3812.
##  5 ETS    2019 May N(3851, 10155) 3851.
##  6 ETS    2019 Jun N(3730, 10968) 3730.
##  7 ETS    2019 Jul N(3947, 13933) 3947.
##  8 ETS    2019 Aug N(4005, 16074) 4005.
##  9 ETS    2019 Sep N(3958, 17421) 3958.
## 10 ETS    2019 Oct N(4091, 20493) 4091.
## # ... with 26 more rows
```

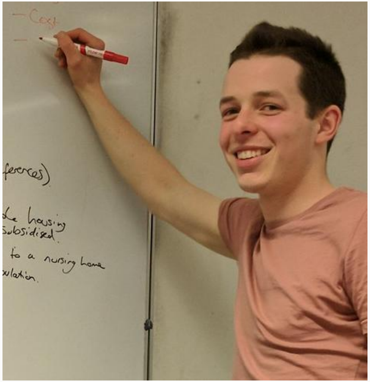
Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
  ) %>%
  arrange(crps)
```

```
## # A tibble: 3 x 4
##   .model .type  crps  rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 COMB   Test   30.9  43.6
## 2 ETS    Test   31.5  38.3
## 3 ARIMA  Test   32.9  51.5
```

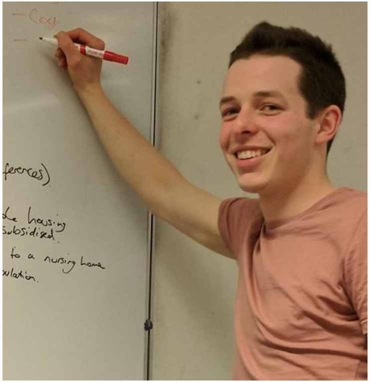
fable packages

fable.tidyverts.org



Mitchell O'Hara-Wild

fable packages



fable.tidyverts.org

Mitchell O’Hara-Wild

robjhyndman.com/nycr2020