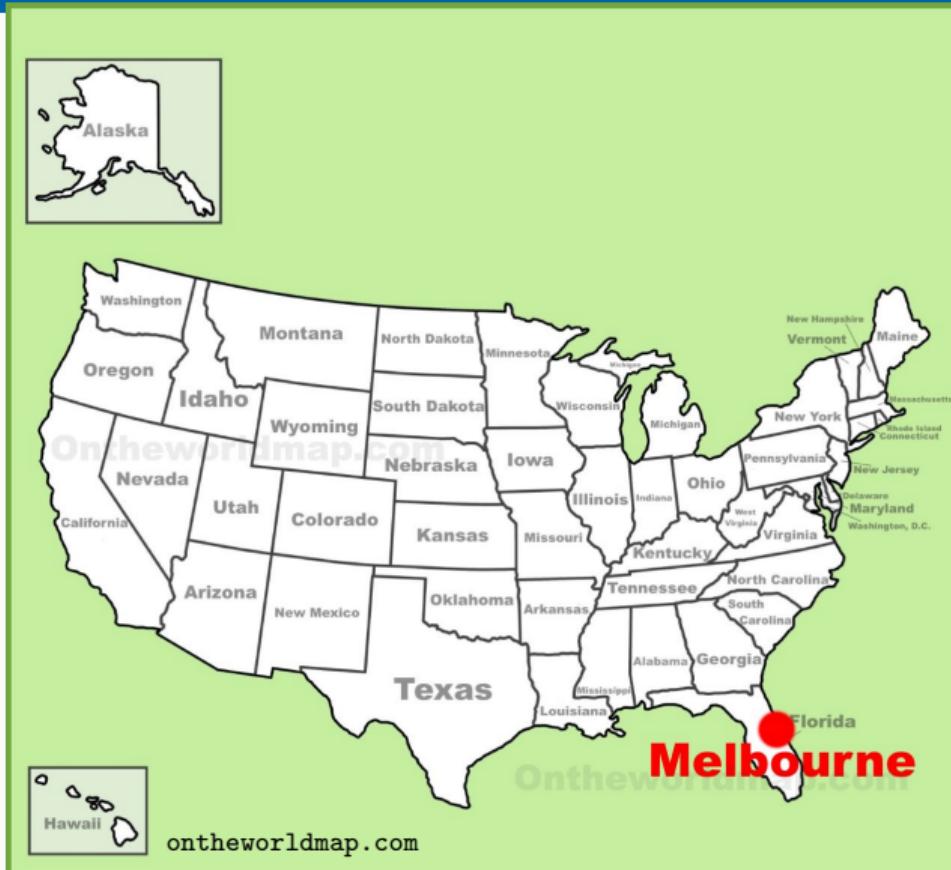
The background of the slide features a vibrant, abstract pattern resembling liquid or smoke, with swirling bands of orange, blue, yellow, and green against a dark background.

Ensemble forecasts with fable

Rob J Hyndman

14 August 2020

Where is Melbourne?



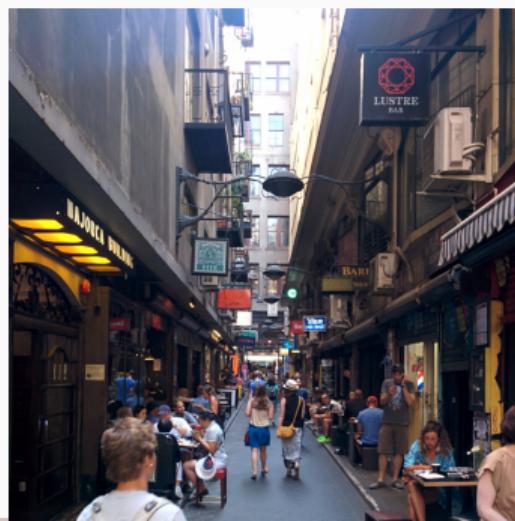
Where is Melbourne?



Where is Melbourne?

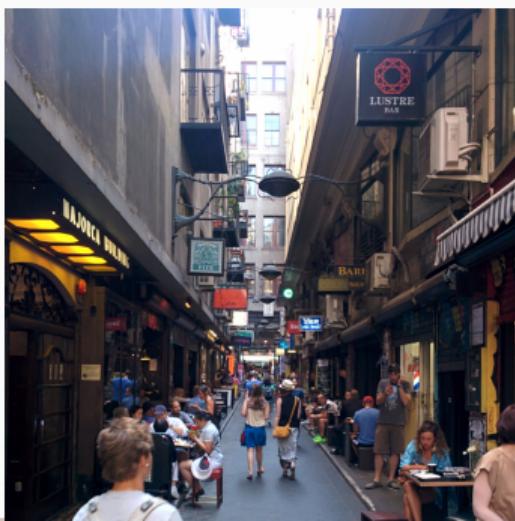
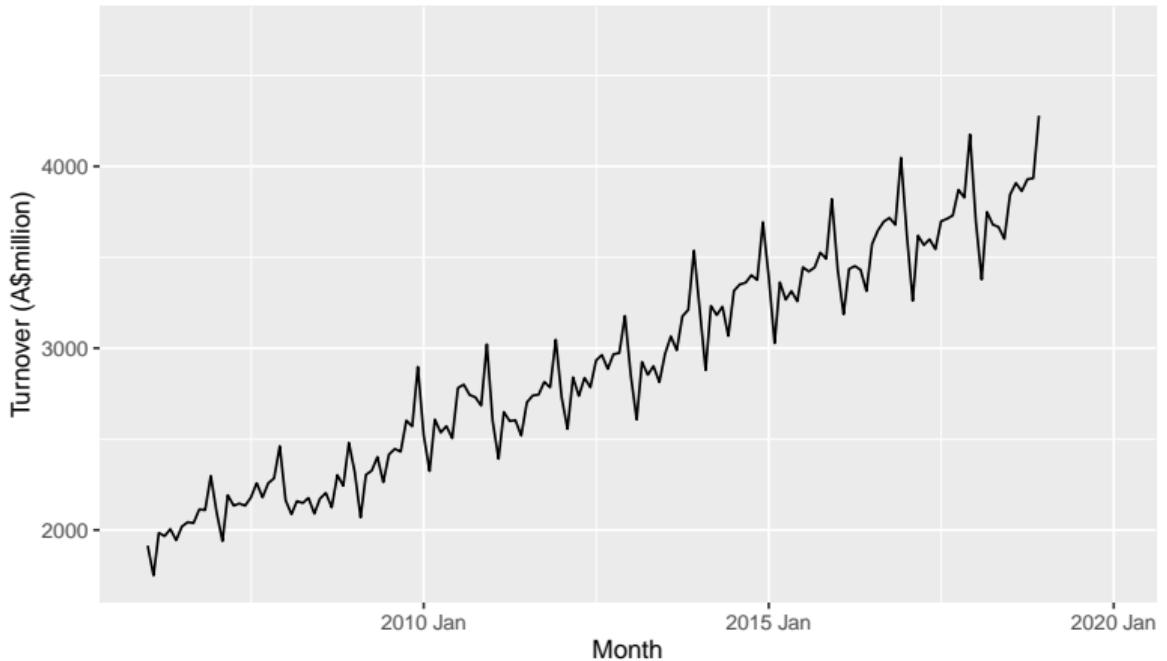


Where is Melbourne?



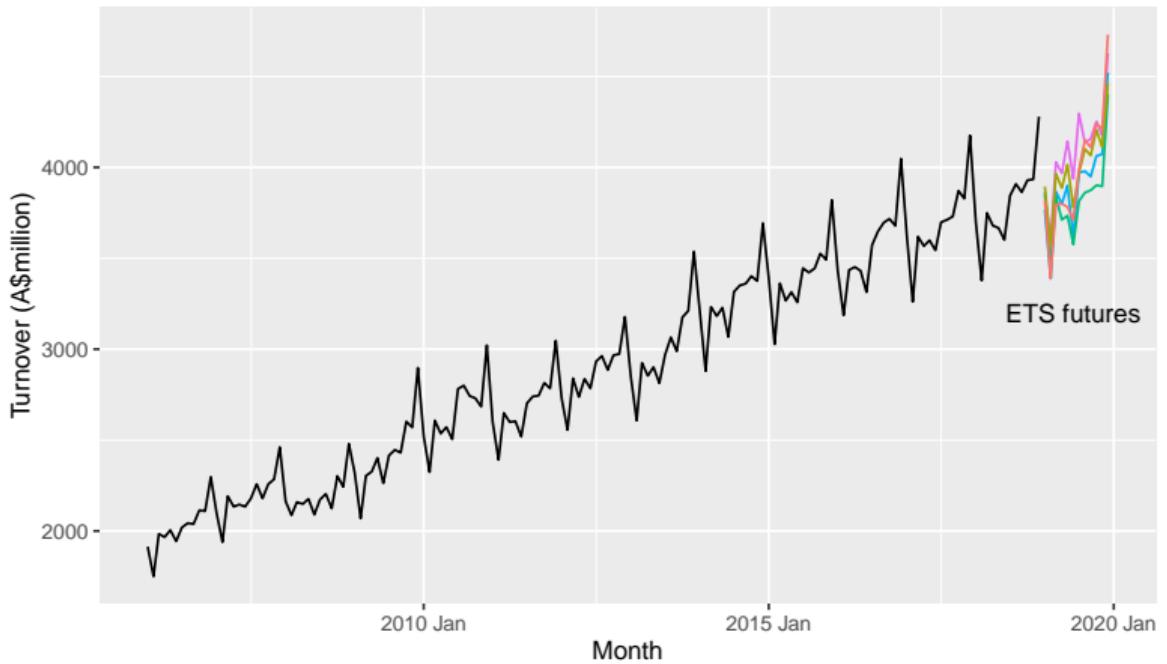
Australian monthly café turnover

Australian monthly café turnover

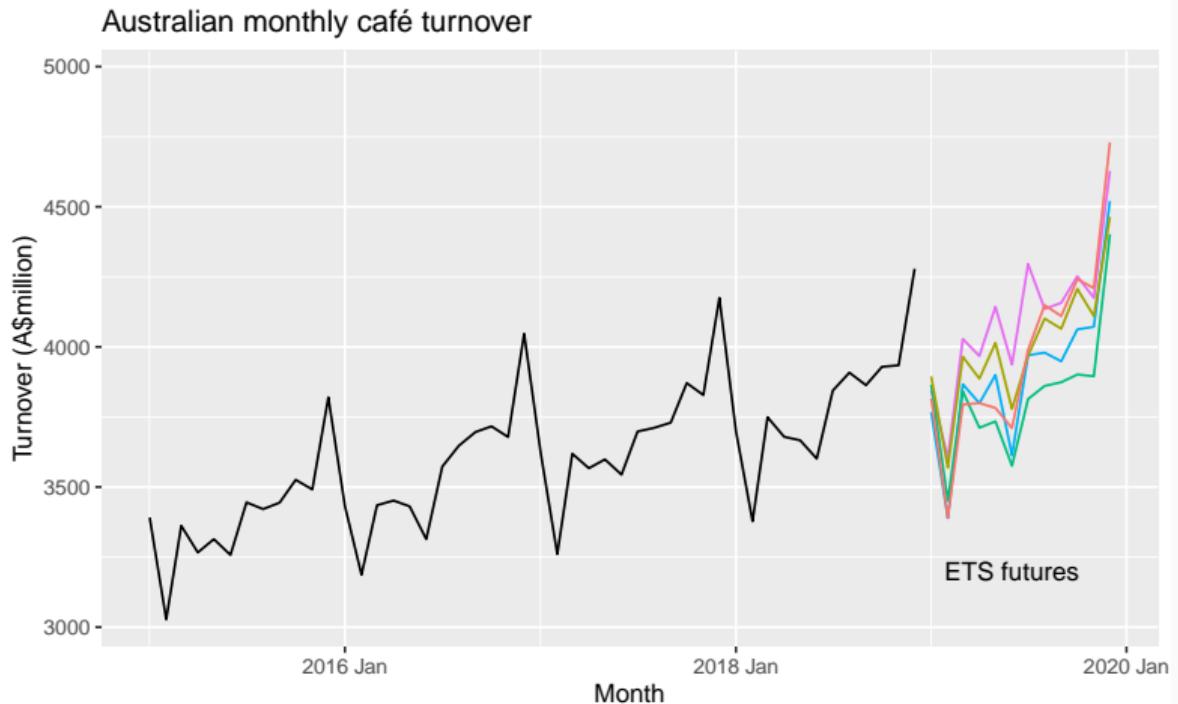


Forecasting using possible futures

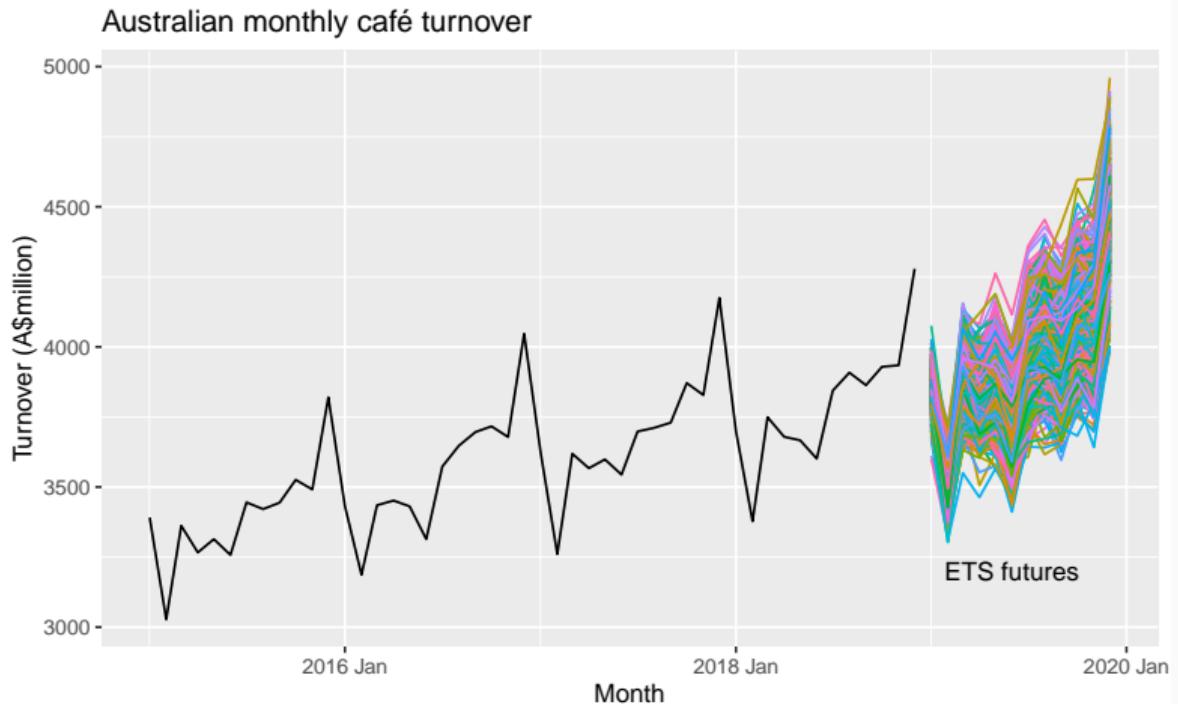
Australian monthly café turnover



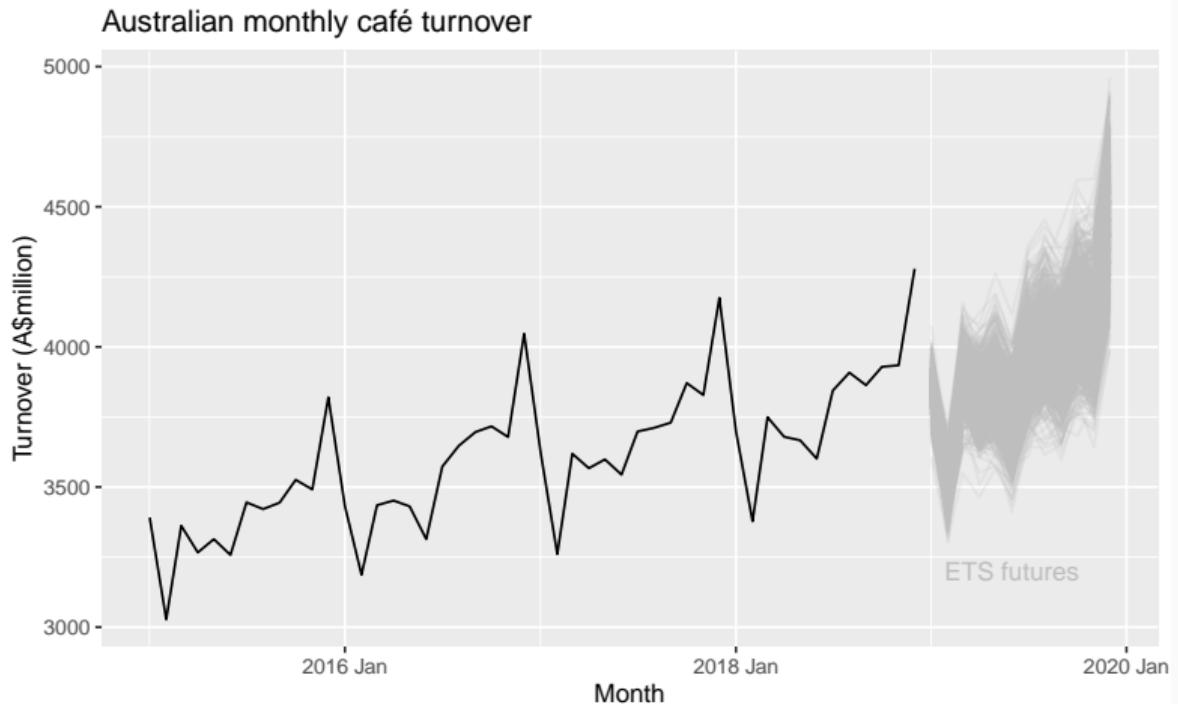
Forecasting using possible futures



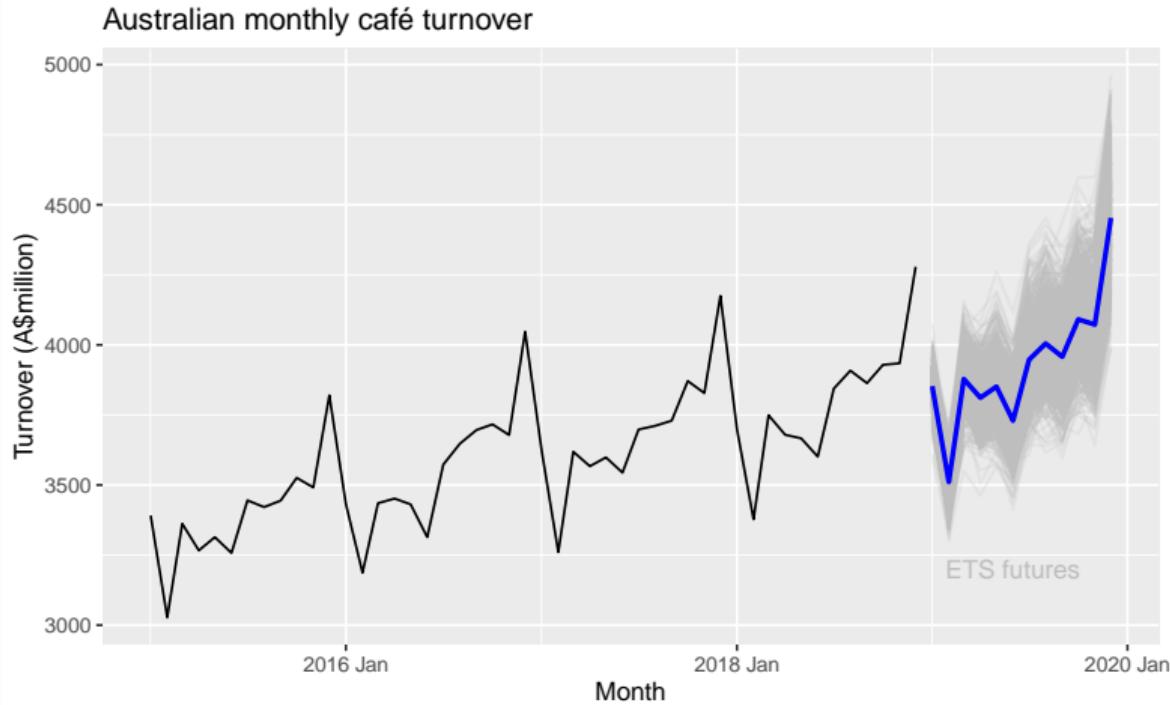
Forecasting using possible futures



Forecasting using possible futures

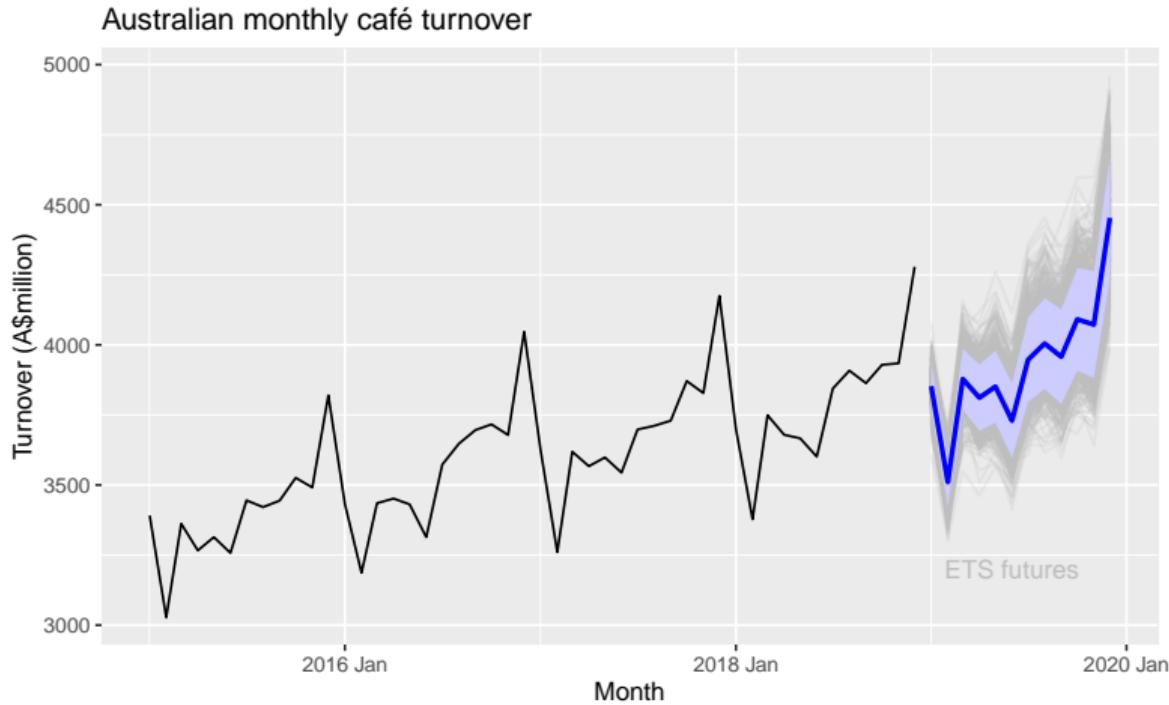


Forecasting using possible futures



Point forecasts: means of the sample paths.

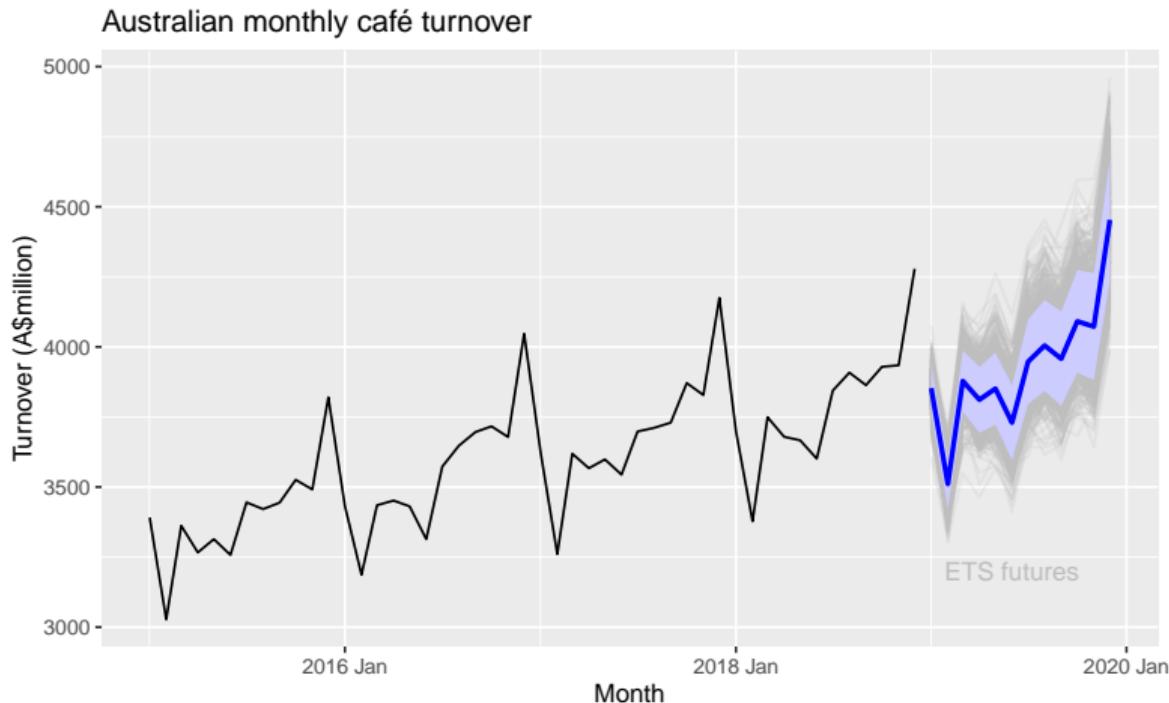
Forecasting using possible futures



Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

Forecasting using possible futures

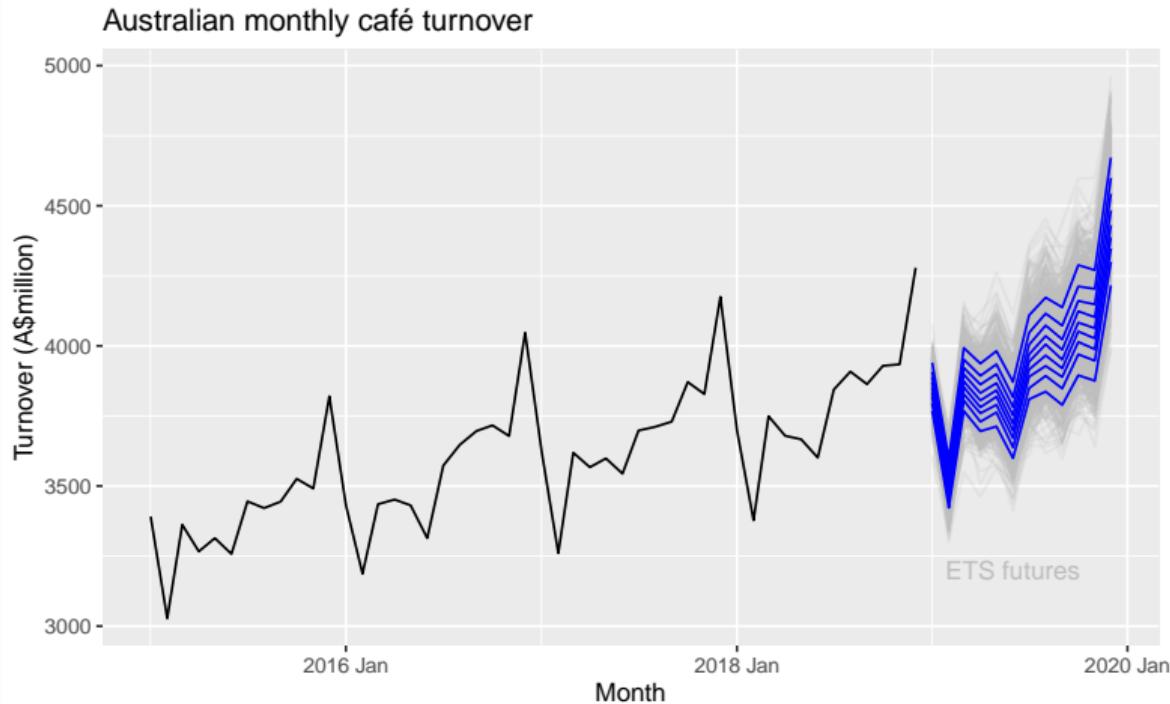


Point forecasts: means of the sample paths.

Prediction intervals: middle 80% of the sample paths at each forecast horizon.

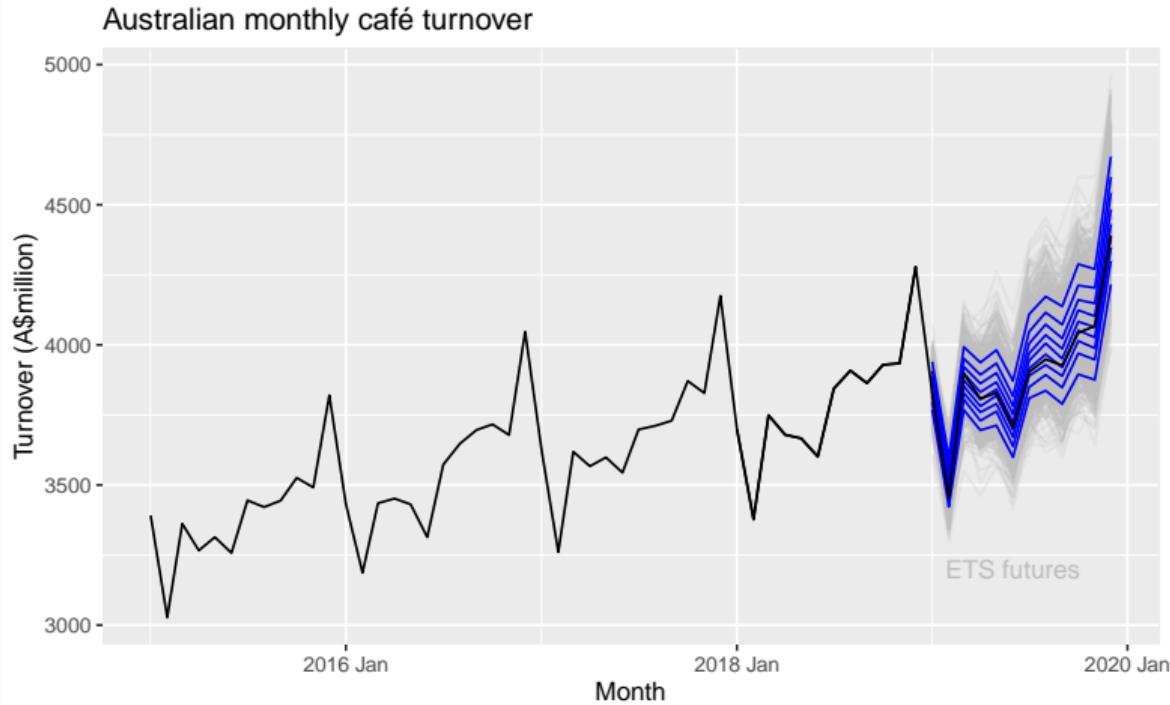
Quantile forecasts: Quantiles of the sample paths at each forecast horizon.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.

Quantile forecasts



Blue: Deciles for the ETS forecasts for the Australian monthly café turnover.
Black: Observed values.

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

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Quantile score

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Evaluating quantile forecasts

$f_{p,t}$ = quantile forecast with prob. p at time t .

y_t = observation at time t

Quantile score

$$Q_{p,t} = \begin{cases} 2(1-p)|y_t - f_{p,t}|, & \text{if } y_t < f_{p,t} \\ 2p|y_t - f_{p,t}|, & \text{if } y_t \geq f_{p,t} \end{cases}$$

- Low Q_p is good
- Multiplier of 2 often omitted, but useful for interpretation
- Q_p like absolute error (weighted to account for likely exceedance)
- Average Q_p = CRPS (Continuous Rank Probability Score)

Evaluating quantile forecasts

```
cafe          ## # A tsibble: 168 x 2 [1M]
              ##       date turnover
              ##     <mth>    <dbl>
              ## 1 2006 Jan    1914.
              ## 2 2006 Feb    1750.
              ## 3 2006 Mar    1984.
              ## 4 2006 Apr    1966.
              ## 5 2006 May    2005.
              ## 6 2006 Jun    1944.
              ## 7 2006 Jul    2019.
              ## 8 2006 Aug    2043.
              ## 9 2006 Sep    2039.
             ## 10 2006 Oct   2113.
## # ... with 158 more rows
```

Evaluating quantile forecasts

```
cafe %>%  
  filter(year(date) <= 2018)  
  
## # A tsibble: 156 x 2 [1M]  
##       date turnover  
##     <mth>    <dbl>  
## 1 2006 Jan    1914.  
## 2 2006 Feb    1750.  
## 3 2006 Mar    1984.  
## 4 2006 Apr    1966.  
## 5 2006 May    2005.  
## 6 2006 Jun    1944.  
## 7 2006 Jul    2019.  
## 8 2006 Aug    2043.  
## 9 2006 Sep    2039.  
## 10 2006 Oct    2113.  
## # ... with 146 more rows
```

Evaluating quantile forecasts

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1))
  )
```

```
## # A mable: 1 x 2
##           ETS          ARIMA
##           <model>       <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Evaluating quantile forecasts

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year")
```

```
## # A fable: 24 x 4 [1M]
## # Key:     .model [2]
##   .model      date      turnover .mean
##   <chr>      <mth>      <dist> <dbl>
## 1 ETS      2019 Jan   N(3853, 4315) 3853.
## 2 ETS      2019 Feb   N(3511, 4760) 3511.
## 3 ETS      2019 Mar   N(3878, 7273) 3878.
## 4 ETS      2019 Apr   N(3812, 8472) 3812.
## 5 ETS      2019 May   N(3851, 10155) 3851.
## 6 ETS      2019 Jun   N(3730, 10968) 3730.
## 7 ETS      2019 Jul   N(3947, 13933) 3947.
## 8 ETS      2019 Aug   N(4005, 16074) 4005.
## 9 ETS      2019 Sep   N(3958, 17421) 3958.
## 10 ETS     2019 Oct   N(4091, 20493) 4091.
## # ... with 14 more rows
```

Evaluating quantile forecasts

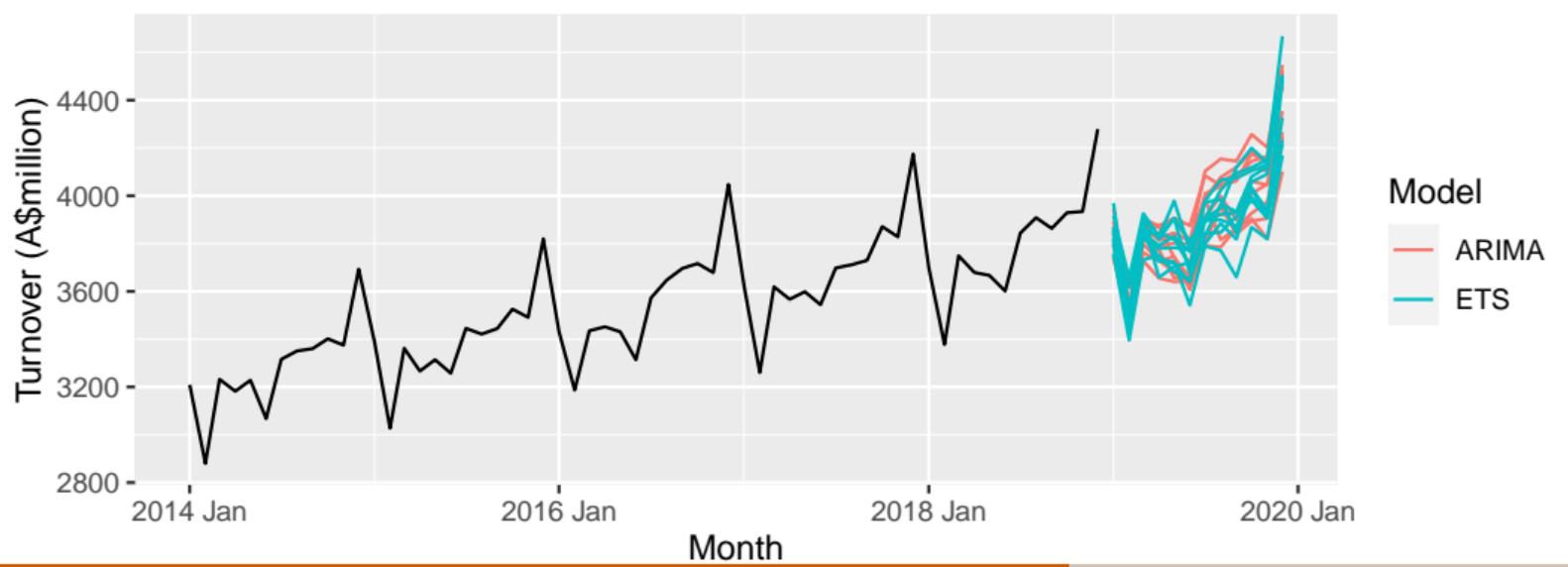
```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~ pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
) %>%
  arrange(crps)

## # A tibble: 2 x 4
##   .model .type   crps   rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS    Test    31.5  38.3
## 2 ARIMA  Test    32.9  51.5
```

Ensemble forecasting

Ensemble forecasting involves combining the forecast distributions from multiple models.

- “All models are wrong, but some are useful” (George Box, 1976)
- Allows diverse models to be included, while reducing impact of any specific model.
- Allows uncertainty of model selection to be incorporated.



Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 years")
```

```
## # A fable: 24 x 4 [1M]
## # Key:     .model [2]
##   .model      date      turnover .mean
##   <chr>      <mth>      <dist> <dbl>
##   1 ETS      2019 Jan   N(3853, 4315) 3853.
##   2 ETS      2019 Feb   N(3511, 4760) 3511.
##   3 ETS      2019 Mar   N(3878, 7273) 3878.
##   4 ETS      2019 Apr   N(3812, 8472) 3812.
##   5 ETS      2019 May   N(3851, 10155) 3851.
##   6 ETS      2019 Jun   N(3730, 10968) 3730.
##   7 ETS      2019 Jul   N(3947, 13933) 3947.
##   8 ETS      2019 Aug   N(4005, 16074) 4005.
##   9 ETS      2019 Sep   N(3958, 17421) 3958.
##  10 ETS     2019 Oct   N(4091, 20493) 4091.
## # ... with 14 more rows
```

Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
)
```

```
## # A tsibble: 12 x 2 [1M]
##       date   turnover
##       <mth>   <dist>
## 1 2019 Jan mixture(n=2)
## 2 2019 Feb mixture(n=2)
## 3 2019 Mar mixture(n=2)
## 4 2019 Apr mixture(n=2)
## 5 2019 May mixture(n=2)
## 6 2019 Jun mixture(n=2)
## 7 2019 Jul mixture(n=2)
## 8 2019 Aug mixture(n=2)
## 9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
            distribution = turnover)
```

```
## # A fable: 12 x 2 [1M]
##       date   turnover
##       <mth>   <dist>
## 1 2019 Jan mixture(n=2)
## 2 2019 Feb mixture(n=2)
## 3 2019 Mar mixture(n=2)
## 4 2019 Apr mixture(n=2)
## 5 2019 May mixture(n=2)
## 6 2019 Jun mixture(n=2)
## 7 2019 Jul mixture(n=2)
## 8 2019 Aug mixture(n=2)
## 9 2019 Sep mixture(n=2)
## 10 2019 Oct mixture(n=2)
## 11 2019 Nov mixture(n=2)
## 12 2019 Dec mixture(n=2)
```

Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  forecast(h = "1 year") %>%
  summarise(
    turnover = dist_mixture(
      turnover[1],
      turnover[2],
      weights = c(0.5, 0.5)
    )
  ) %>%
  as_fable(response = "turnover",
            distribution = turnover) %>%
  accuracy(
```

A tibble: 1 x 3
.type crps rmse
<chr> <dbl> <dbl>
1 Test 31.7 43.6

Ensemble forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
forecast(h = "1 year") %>%
summarise(
  turnover = dist_mixture(
    turnover[1],
    turnover[2],
    weights = c(0.5, 0.5)
  )
) %>%
as_fable(response = "turnover",
          distribution = turnover) %>%
accuracy()
```

```
## # A tibble: 1 x 3
##   .type   crps   rmse
##   <chr> <dbl> <dbl>
## 1 Test    31.7  43.6
```

Comparison:

```
## # A tibble: 2 x 4
##   .model .type   crps   rmse
##   <chr>  <chr> <dbl> <dbl>
## 1 ETS     Test    31.5  38.3
## 2 ARIMA   Test    32.9  51.5
```

Combination forecasting

Combination forecasting: weighted averages of forecasts from multiple models.

- Often a simple average is used.
- Reduces uncertainty associated with selecting a particular model.
- Combination forecasting usually improves point forecast accuracy.
- Mean forecast identical to that from corresponding weighted ensemble.
- Quantile forecasts need to account for correlations between forecast errors from component models.

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1)))
)

## # A mable: 1 x 2
##           ETS          ARIMA
##           <model>       <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]>
```

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
      pdq(d=1) + PDQ(D=1)))
) %>%
  mutate(COMB = (ETS + ARIMA)/2)

## # A mable: 1 x 3
##           ETS          ARIMA
##           <model>     <model>
## 1 <ETS(M,A,M)> <ARIMA(0,1,1)(0,1,1)[12]> <CO
```

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1))
  ) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year")
```

```
## # A fable: 36 x 4 [1M]
## # Key:     .model [3]
##   .model      date      turnover .mean
##   <chr>      <mth>      <dist> <dbl>
## 1 ETS      2019 Jan   N(3853, 4315) 3853.
## 2 ETS      2019 Feb   N(3511, 4760) 3511.
## 3 ETS      2019 Mar   N(3878, 7273) 3878.
## 4 ETS      2019 Apr   N(3812, 8472) 3812.
## 5 ETS      2019 May   N(3851, 10155) 3851.
## 6 ETS      2019 Jun   N(3730, 10968) 3730.
## 7 ETS      2019 Jul   N(3947, 13933) 3947.
## 8 ETS      2019 Aug   N(4005, 16074) 4005.
## 9 ETS      2019 Sep   N(3958, 17421) 3958.
## 10 ETS     2019 Oct   N(4091, 20493) 4091.
## # ... with 26 more rows
```

Combination forecasting

```
cafe %>%
  filter(year(date) <= 2018) %>%
  model(
    ETS = ETS(turnover),
    ARIMA = ARIMA(turnover ~
                  pdq(d=1) + PDQ(D=1)))
) %>%
  mutate(COMB = (ETS + ARIMA)/2) %>%
  forecast(h = "1 year") %>%
  accuracy(
    data = cafe,
    measures = list(crps=CRPS, rmse=RMSE)
) %>%
  arrange(crps)

## # A tibble: 3 x 4
##   .model .type   crps   rmse
##   <chr>  <chr> <dbl>  <dbl>
## 1 COMB   Test    30.9  43.6
## 2 ETS    Test    31.5  38.3
## 3 ARIMA  Test    32.9  51.5
```

fable packages



fable.tidyverts.org



Mitchell O'Hara-Wild

fable packages



fable.tidyverts.org



Mitchell O'Hara-Wild

robjhyndman.com/nycr2020