# Open problem:
# On the uncertainty of reconciliation weights

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# The problem

Uncertainties on forecasts impact forecast combination weights:

- forecast combination puzzle
  (Smith and Wallis, 2009; Graefe et al., 2014; Claeskens et al., 2016)
- hierarchical forecast reconciliation
  (Pritularga et al. 2021; Møller et al. 2023)

- Which uncertainties impact reconciliation weights? How?
- What can we do to better account for those uncertainties?

# Impact of uncertainties on weights

Consider the small example shown in Pritularga et al. 2021.

**Hierarchy:** 4 bottom, 2 intermediate, 1 top.

**Data generating process:** bottom time series generated as

$$b_{q,t} = 0.4 b_{q,t-1} + \epsilon_{q,t}, \qquad q = 1, \ldots, 4$$

and $\boldsymbol{\epsilon_{b,t}} = [\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t}]^T$, $\boldsymbol{\epsilon_{b,t}} \sim N(0, \Sigma_b)$ with known $\Sigma_b$.
Top and intermediate time series generated by aggregating bottom ones.

Even for a well-specified model **a pointwise estimate** of the parameter(s) **does not include the uncertainty!**

# Why are pointwise estimates of parameters problematic?

Let's simplify even further: hierarchy with one upper and two bottoms.

In the Gaussian case, the optimal reconciled forecasts (Corani et al. 2021, Hollyman et al. 2021) are

$$\tilde{u} = \underbrace{\frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_2^2 + \sigma_u^2}}_{w} \hat{\mathbf{u}} + \underbrace{\frac{\sigma_u^2}{\sigma_1^2 + \sigma_2^2 + \sigma_u^2}}_{1-w} \mathbf{A}\hat{\mathbf{b}},$$

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**We would like:** an estimator of the ratio (estimator for $w$)

**We have:** reconciliation weights computed from a <u>ratio of estimators</u>!

# How to improve on pointwise estimates?

**Two possible approaches:**

**Bias corrected estimator:** correct the bias of a ratio of estimators.

- use classical, jackknife methods to correct bias;
- see, e.g. Choquet et al. 1999.

**Bayesian approach:** compute a posterior distribution for the parameters.

- select a prior for the parameters
- compute posterior of the parameters given observations (MC?)

# Bagging to create approximate posterior?

**Possible starting point:**

1. generate boostraped series
   (Bergmeir et al. 2016; Petropoulos et al. 2018);
2. fit one model on each series;
3. gather samples from "posterior distribution" for the parameters ($\sigma$)
4. gerate posterior distribution for $w$

Questions

- does this approximation represents well the uncertainties?
- does this procedure introduce bias?

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