Probabilistic reconciliation: projection vs conditioning

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Point/probabilistic reconciliation

Point forecasts

$\widehat{\mathbf{y}}$: base point forecasts at $T+h$

Point forecasts are incoherent:
$\widehat{\mathbf{y}} \notin \mathcal{S} := \{\mathbf{y} : \ \mathbf{u} = \mathbf{A}\,\mathbf{b}\}$

Probabilistic forecasts

$\widehat{\pi}$: base forecast distribution at $T+h$

Probabilistic forecasts are incoherent:
$supp(\widehat{\pi}) \nsubseteq \mathcal{S}$

Point forecast reconciliation

- $\widehat{\mathbf{y}} \rightsquigarrow \widetilde{\mathbf{b}}$
  E.g. $\widetilde{\mathbf{b}} = \mathbf{P}\,\widehat{\mathbf{y}}$
- $\widetilde{\mathbf{y}} = \mathbf{S}\,\widetilde{\mathbf{b}}$

Probabilistic reconciliation

- $\widehat{\pi} \rightsquigarrow \widetilde{\pi}_B$
- $\widetilde{\pi}(\mathbf{u},\mathbf{b}) = \begin{cases} \widetilde{\pi}_B(\mathbf{b}) & \text{if } \mathbf{u} = \mathbf{A}\mathbf{b} \\ 0 & \text{if } \mathbf{u} \neq \mathbf{A}\mathbf{b} \end{cases}$

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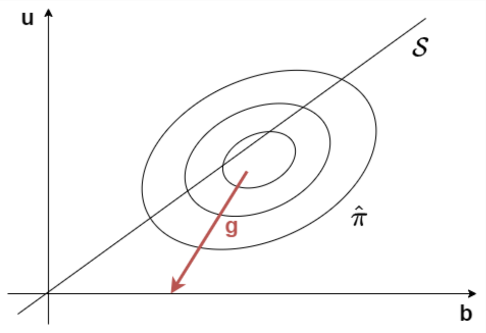
Probabilistic reconciliation via projection

$$\widetilde{\pi}_B := g_{\#}\,\widehat{\pi}$$



- $g : \mathbb{R}^n \to \mathbb{R}^{n_b}$
- $\widetilde{\pi}_B(F) = \widehat{\pi}\left(g^{-1}(F)\right)$
  for any set $F \subseteq \mathbb{R}^{n_b}$
- Sample $\mathbf{y}^{(i)} \sim \widehat{\pi}$
  $\implies g\left(\mathbf{y}^{(i)}\right) \sim \widetilde{\pi}_B$
- g linear: $g(\widehat{\mathbf{y}}) = \mathbf{G}\widehat{\mathbf{y}} + \mathbf{d}$
  $\mathbf{G}, \mathbf{d}$ learned via SGD

Panagiotelis, Gamakumara, Athanasopoulos, Hyndman (EJOR, 2023)

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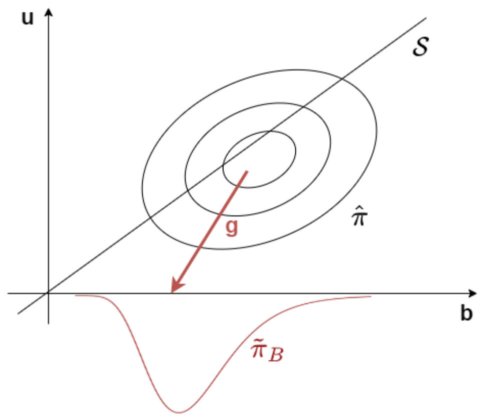

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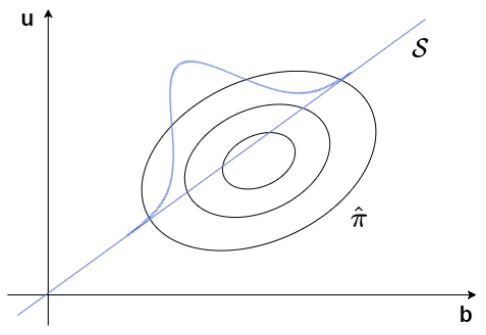
Probabilistic reconciliation via conditioning

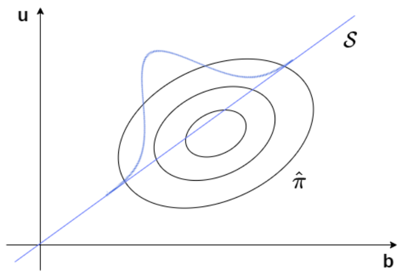
$$\widetilde{\pi}_B(\mathbf{b}) := Prob\big(\widehat{\mathbf{B}} = \mathbf{b} \mid \widehat{\mathbf{U}} = \mathbf{A}\widehat{\mathbf{B}}\big)$$
$$\propto \widehat{\pi}(\mathbf{Ab}, \mathbf{b})$$



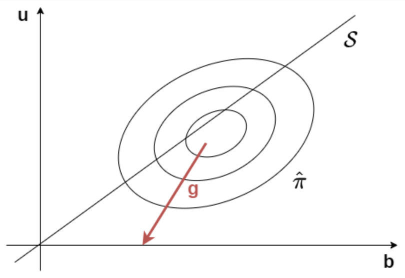
- $\widehat{\mathbf{B}} \sim \widehat{\pi}_B$ and $\widehat{\mathbf{U}} \sim \widehat{\pi}_U$
- $\widehat{\pi}$ Gauss $\implies$ $\widetilde{\pi}_B$ Gauss
  Same formulae of minT!
- $\widehat{\pi}$ non-Gaussian:
  need to sample from $\widetilde{\pi}_B$

Zambon, Azzimonti, Corani (2022); Corani, Azzimonti, Rubattu (2023)

Open problems



Reconciliation via conditioning only uses information about the base distribution on $\mathcal{S}$



Reconciliation via projection combines information coming from coherent and incoherent points

- Reconciliation via conditioning is invariant under modifications of the base forecast probabilities outside $\mathcal{S}$
- Does it make sense to take into account probabilities of points "far away" from $\mathcal{S}$?

Open problems

- $\widehat{\pi}$ Gaussian:
  optimal proj (wrt log score) = conditioning (Wickramasuriya, 2023)
  Results for other distributions/scores?

- Computational load:
  optimization vs sampling

- Empirical comparisons

References

A. Panagiotelis, P. Gamakumara, G. Athanasopoulos, and R. J. Hyndman. Probabilistic forecast reconciliation: Properties, evaluation and score optimisation. *European Journal of Operational Research*, 2023.

Corani, G., Azzimonti, D., Rubattu, N., 2023. Probabilistic reconciliation of count time series. *International Journal of Forecasting*, 2023.

L. Zambon, D. Azzimonti, G. Corani. Efficient probabilistic reconciliation of forecasts for real-valued and count time series. *arXiv:2210.02286*, 2022.

S.L. Wickramasuriya. Probabilistic forecast reconciliation under the Gaussian framework. *Journal of Business & Economic Statistics*, 2023.

Probabilistic reconciliation via importance sampling

$$\widetilde{\pi}_B(\mathbf{b}) := Prob\big(\widehat{\mathbf{B}} = \mathbf{b} \mid \widehat{\mathbf{U}} = \mathbf{A}\widehat{\mathbf{B}}\big)$$
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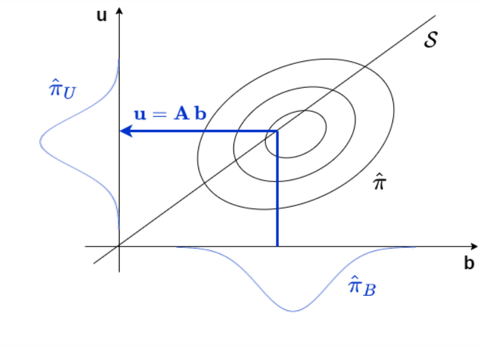

If $\widehat{\mathbf{B}}$, $\widehat{\mathbf{U}}$ condit. independent:
$\widetilde{\pi}_B(\mathbf{b}) \propto \widehat{\pi}_B(\mathbf{b})\,\widehat{\pi}_U(\mathbf{Ab})$

Importance sampling:

- draw samples from $\widehat{\pi}_B$
- compute weights using $\widehat{\pi}_U$

Zambon, Azzimonti, Corani (2022)

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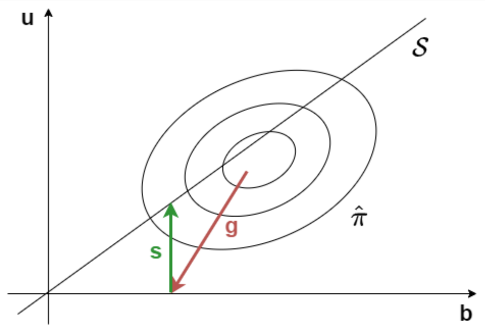

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