Discussion

Forecast reconciliation with clustering structure: application to stock prices

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IIF Workshop on Forecast Reconciliation

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- The authors correctly stress that this can be seen as an **original use of forecast combination** (Di Fonzo and Girolimetto, 2022) for financial time series forecasting
- **Very promising conclusion**:

*If the hierarchical structure of the stock market is properly included in the forecasting problem, we can obtain
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Main issues considered in the discussion

Not exhaustive, only topics close to the issues I usually deal with

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- The industry hierarchy is **unbalanced** (in 15 cases out of 30, only one stock per industry). Possible **impacts** on point reconciled forecasts and their variances

- **The literature on the merits and failings of the DJIA is almost as old as the index itself**
  Comer (1952), Milne (1966), and Rudd (1979) discuss the role of stock splits, the weighting methods, and the industry composition of the DJIA. The importance of dividend payments for the DJIA has been discussed by Clarke and Statman (2000) and Shoven and Sialm (2000)

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The importance of each company in the index does not depend on the total market capitalization (a measure of the size) of the company Instead, a highly priced stock has a higher weight than a lower priced stock. Each time a company in the DJIA splits, the weight of this company decreases because the stock price falls by the ratio of the split

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- **The constituents of the DJIA are not representative of the market as a whole**
  The 30 firms are chosen among large-capitalization firms to represent different industries, but they are not chosen according to fixed pre-determined rules. In particular, the DJIA is not an index of the 30 largest companies in the US. Furthermore, a more representative index would include a much larger number of companies

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- As of November 4, 2021, the current divisor for DJIA is 0.1517.
- It would be interesting (useful for forecasting purposes?) to know how the **Dow Divisor changed over the time**

Balanced and unbalanced hierarchy

Hierarchy of the DJIA components (30) according to Industrial-based clusters (20): IND

Balanced hierarchy

$\mathcal{L}_0$: Top node $\mathcal{L}_1$: Intermediate nodes $\mathcal{L}_2$: Bottom nodes   Duplicated nodes

Unbalanced hierarchy

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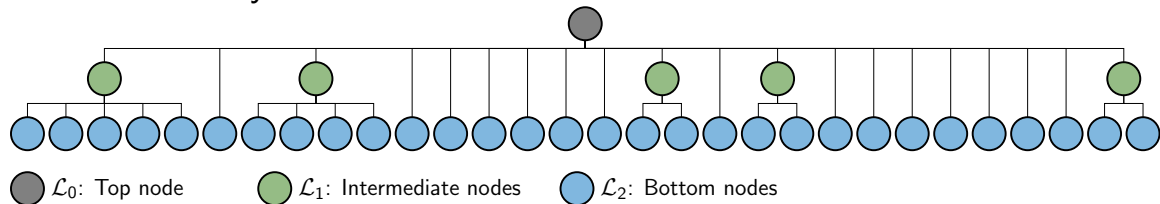|	AAPL	CRM	CSCO	IBM	MSFT	AMGN	AXP	GS	JPM	V	BA	CAT	CVX	DIS	DOW	HD	HON	MMM	INTC	JNJ	MRK	KO	MCD	NKE	PG	TRV	UNH	VZ	WBA	WMT
Information technology	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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Aerospace and defense	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Construction and Mining	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Petroleum industry	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Broadcasting and entertainment	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Chemical industry	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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Drink industry	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.
Food industry	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
Clothing industry	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.
Fast-moving consumer goods	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.
Insurance	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.
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Information technology	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Financial services	.	.	.	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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- Thus, if the balanced framework is used in cases where it is not justified by the data structure, the forecast evaluation/comparison might be **not well grounded** (e.g., confidence intervals will shrink)

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- When comparing the Sharpe ratios between two forecast-based investment strategies (Table 10), **RW is not considered**. Why?

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Conclusions

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  Rather, it seems to be confirmed that the Efficient Market Hypothesis is **hard to beat**!
- A little **take-home message** to practitioners: when facing unbalanced hierarchies, please **do not balance** them

References I

[1] Clarke, R. G., Statman, M. (2000), The DJIA crossed 652,230, *The Journal of Portfolio Management* 26(2), 89–93.

[2] Comer, H.D. (1952), The market-looking backward, *The Financial Analysts Journal* 8(4), 143–144.

[3] Di Fonzo, T., Girolimetto, D. (2022), Forecast combination based forecast reconciliation, *International Journal of Forecasting*, in press.

[4] Ledoit, O., Wolf, M. (2008), Robust performance hypothesis testing with the Sharpe ratio, *Journal of Empirical Finance* 15, 850–859.

[5] Lin, J., Selden, G.C., Shoven, J.B., Sialm, C. (2021), *Replicating the Dow Jones Industrial Average*, National Bureau of Economic Research Working Paper.

[6] Milne, R. D. (1966), The Dow-Jones Industrial Average re-examined, *The Financial Analysts Journal*, 22(6), 83–88.

[7] Rudd, A.T. (1979), The revised Dow Jones Industrial Average: New wine in old bottles?, *The Financial Analysts Journal* 35(6), 57–63.

[8] Shoven, J.B., Sialm, C. (2000), The Dow Jones Industrial Average: The impact of fixing its flaws, *The Journal of Wealth Management* 3(3), 9–18.

Thank you!

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