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Monitoring processes with changing variances

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Abstract

Statistical process control (SPC) has evolved beyond its classical applications in manufacturing to monitoring economic and social phenomena. This extension has required the consideration of autocorrelated and possibly non-stationary time series. Less attention has been paid to the possibility that the variance of the process may also change over time. In this paper we use the innovations state space modeling framework to develop conditionally heteroscedastic models. We provide examples to show that the incorrect use of homoscedastic models may lead to erroneous decisions about the nature of the process.

Keywords: Control charts; GARCH; Heteroscedasticity; Innovations; State space; Statistical process control

1. Introduction

The seminal work of Shewhart (1931) on statistical control charts focused upon manufacturing processes, where production conditions might be expected to be stable over time if and when the process was in control. Typically, a small sample (e.g. $n = 5$) was taken and the sample mean plotted on an $\bar{X}$ chart. The process was deemed to be out of control if the sample mean was more than three standard deviations from the underlying mean. The time between successive sets of readings was typically sufficiently large that the samples might be viewed as independent. Further, if the process was deemed to be out of control, it was reasonable to assume that the equipment could be reset so that the process was in control again by the time the next sample was taken. Thus, we are led to the standard formulation that the sample means are independent and identically distributed.

The Shewhart chart is designed to identify single extreme values, but it is also important to identify shifts in the mean level; this requirement has led to the use of CUSUM and EWMA (exponentially weighted moving average) charts; see, for example, Montgomery (2004) for further details.

These methods still rely upon the assumption of independent and identically distributed observations.

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However, when we come to consider monitoring social and economic processes over time, we usually have only one observation per time period (which we assume henceforth); normality then becomes questionable, and independence over time seems an overly strong assumption. Alwan and Roberts (1988) recognized the need to allow for time-dependence, and developed a monitoring process based upon ARIMA processes. We describe their approach in Section 2, but it is worth providing an outline here. As in any application of statistical process control (SPC), we seek to partition the variability of the process into that part which is explainable in terms of the past history of the process, and that part inherently due to random factors. Alwan and Roberts refer to the charts generated from these two components as:

- *Common Cause Control Charts (CCC):* the fitted values generated by the time series model; and
- *Special (or Assignable) Cause Charts (SCC):* the plots of residuals, which may be treated as Shewhart charts given independent, identically distributed errors.

Alwan and Roberts (1988) urged the consideration of both charts. SCC charts enable the investigator to identify outliers in the usual way, whereas CCC charts allow the recognition of an unacceptable trend, *even when the process is in a state of statistical control*. That is, we seek to understand the common causes leading to autocorrelated behavior, with a view to improving the process by some form of intervention. Alwan and Roberts (1995) review a number of published studies and show how the failure to allow for temporal dependence can lead to misleading conclusions.

In all of these methods, a basic assumption is that the variance is constant over time. This assumption is usually checked by means of an R chart or an S chart (based upon measures of range and standard deviation, respectively; see Montgomery, 2004). For recent developments, see Stoumbos, Reynolds, and Woodall (2003). If the variance is not constant, efforts are made to stabilize the variance before applying the control chart methodology. However, in social and economic processes, such interventions are usually not possible, and we must take changes in the variance into account. Thus, our purpose in this paper is to extend the Alwan–Roberts methodology to include changes over time in both the mean and the variance.

A further point needs to be made when we consider monitoring social and economic processes. In order to calibrate the charts, we must either assume that the process is in statistical control during the calibration period, or that outliers can be successfully identified and adjusted. This outlier modification step must be approached with care; the parameter estimates typically may not change dramatically, but the residual variance may reduce considerably, thereby narrowing the control limits. If we are overly zealous in outlier removal, we may induce a “Chicken Little” effect whereby excessive numbers of out-of-control signals are generated in later periods. We do not pursue that question here, but observe that Ord and Young (2004) provide one possible framework for dealing with outliers in such situations.

The paper is structured as follows. In Section 2 we briefly review the work of Alwan and Roberts, and subsequent developments. Their approach is formulated in a state space framework in Section 3, and then extended to allow for changing variances in Section 4. In Section 5 we then provide two examples of processes that are time dependent in both the mean and the variance, and illustrate how the proposed approach enables the monitoring of each process in an effective manner. Section 6 presents the conclusions.

2. Time-dependent control charts

Alwan and Roberts (1988) introduced an extension to the Shewhart chart by using an ARIMA(0,1,1) model to describe the underlying process; the extension to more general ARIMA processes introduces no new points of principle. The control limits for the SCC are readily determined using the residual variance, and the residuals may also be used to generate EWMA charts to identify shifts; see Koehler, Marks, and O’Connell (2001).

Alwan and Roberts did not provide limits for the CCC, but Wardell, Moskowitz, and Plante (1992) did so for stationary processes; in particular, they considered the ARMA(1,1) scheme. If we denote the process of interest by $\{y_t, t = 1, 2, \ldots\}$ and a random error term by $\{\varepsilon_t, t = 1, 2, \ldots\}$, we may write this model as:

$$y_t = (1 - \varphi)\mu + \varphi y_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1}. \quad (1)$$

When the errors have zero means and a common variance σ^2, the steady-state mean of y_t is μ and the variance is $V_y = \sigma^2(1 - 2\varphi\theta + \theta^2)/(1 - \varphi^2)$. When $\varphi = 1$, the process reduces to the ARIMA(0,1,1) scheme considered by Alwan and Roberts, but the process becomes non-stationary, so that the mean and variance are undefined.

Wardell et al. (1992) then create control limits for the CCC using the steady-state values; the variance of the fitted values is $V_F = \sigma^2(1 - 2\varphi\theta + \theta^2)/(1 - \varphi^2) - \sigma^2 = \sigma^2(\varphi - \theta)^2/(1 - \varphi^2)$. An obvious drawback of this is that the limits become very wide as the autoregressive parameter approaches the non-stationarity boundary. We explore an alternative to the original solution in Section 3. Overall, there is some question about the value of limits for the CCC, as its original purpose was to examine changes in the level of the process qualitatively, rather than to determine whether the process was in statistical control. Used in this way, the CCC might usefully employ target values as the center line, rather than the process mean. Potential users need to ask whether and how such limits might serve a useful purpose in their analysis.

3. A state–space formulation

In place of the ARIMA scheme considered in the previous section, we may alternatively formulate a statistical model using the innovations state space approach (Hyndman, Koehler, Ord, & Snyder, 2008; Ord, Koehler, & Snyder, 1997; Snyder, 1985). We denote the process of interest by $\{y_t, t = 1, 2, \ldots\}$, an unobserved state variable by $\{x_t, t = 0, 1, \ldots\}$, and a random error term by $\{\varepsilon_t, t = 1, 2, \ldots\}$. For the present, we assume that the errors are independent and identically distributed, with zero means and a common variance σ^2, or $\varepsilon_t \sim \text{IID}(0, \sigma^2)$. Akin to the ARMA(1,1) scheme, we may formulate the *damped local level (DLL)* model in terms of a *measurement* (or observation) equation and a *transition* (or state) equation, written respectively as:

$$\begin{aligned} y_t &= \mu + \phi x_{t-1} + \varepsilon_t \\ x_t &= \phi x_{t-1} + \alpha\varepsilon_t. \end{aligned} \tag{2}$$

The measurement equation describes the variations about the underlying mean level (the unobservable state), whereas the transition equation updates the state in light of the latest error term; the parameter lies in the range $0 \le \alpha < 2$. The usual range for stationarity is $0 < \alpha < 2$, but the DLL includes $\alpha = 0$, corresponding to a constant conditional mean, since we start the process at a finite time origin.

Eliminating the state variable between the two equations reduces to the ARMA(1,1) form given in Eq. (1), with $\theta = \varphi(1 - \alpha)$. When $\varphi = 1$, we are back at the ARIMA(0,1,1) scheme:

$$y_t - y_{t-1} = \varepsilon_t - \theta\varepsilon_{t-1}, \quad \text{where } \theta = 1 - \alpha.$$

ARIMA modeling typically assumes that the process generating the series extends back into the infinite past in order to arrive at unconditional steady-state results, as noted above. However, the state space models may be viewed as starting at time $t = 1$, with initial starting values $\{x_0, \varepsilon_0\}$. Then, rather than work with unconditional values, we may condition on the value of the process at a given time t. If we make the same partition into fitted values and residuals, the conditional mean and variance for the fitted values are:

$$E[y_{t+h} \mid x_t] = \mu + \varphi^h x_t \tag{3}$$

$$V[y_{t+h} \mid x_t] = \frac{\sigma^2\alpha^2\varphi^2(1 - \varphi^{2h-2})}{1 - \varphi^2}, \quad h \ge 2. \tag{4}$$

Provided $|\varphi| < 1$, these values converge to those used by Wardell et al. (1992), who also quote Eqs. (3)–(4). When $|\varphi|$ is close to one, the limits may be very wide, as noted earlier. Therefore, instead of using unconditional and possibly very wide limits, we may use conditional limits, using limits for a fixed number of periods ahead (the planning horizon). In this way we can forecast the values for the duration of the planning horizon and plot them on the CCC.

We have restricted the discussion to the simplest models, since these versions often suffice for short-term monitoring; the extensions to more complex schemes are conceptually straightforward. For details of such models, see Hyndman et al. (2008, Chapter 11).

4. GARCH models

The focus of this paper is on how to monitor processes whose variances change over time. To do so, we must extend the models described in the previous section to accommodate such structural movements. The first formulation of this type was the Autoregressive Conditional Heteroscedastic (ARCH) model proposed by Engle (1982). Intuitively speaking, the ARCH models represent the conditional variance in a purely autoregressive way, which may be extravagant in terms of the number of parameters to be estimated. For this reason, the generalized ARCH (or GARCH) model proposed by Bollerslev (1986) is now generally preferred. The GARCH version may be thought of as an ARMA formulation, although the details are more involved. As with models for the mean, the question of stationarity is important for ARMA models. Since the state space model may assume a finite start-up, stationarity is not necessary.

The original ARCH and GARCH models formulated changes in the variance directly in terms of the variance itself, so that conditions are required on the parameters to ensure that the estimated variance remains positive. Nelson (1991) introduced the exponential or EGARCH model, which considers the logarithm of the variance, thereby avoiding the need for such conditions. In the present discussion, we will restrict attention to the EGARCH form, but both approaches are viable. Tsay (2005, Chapter 3) provides an excellent guide to recent extensions of these models. It will be evident from the ensuing discussion that more complex models are readily incorporated into the proposed framework.

Our discussion leads to a modification of model (2) to allow the error terms to be independent but not identically distributed, with zero means and the variance at time t being dependent on previous observations, denoted by v_{t-1}. The variance is then updated according to the relationship:

$$\ln v_t = u_0 + u_1 \ln v_{t-1} + u_2(\varepsilon_t). \tag{5}$$

The function $u_2(\cdot)$ is open to choice, and the selection may well be application-specific. However, a reasonable choice is $u_2(\varepsilon) = u_2 \ln(|\varepsilon|)$. To avoid complications when the error is (close to) zero, we could add a small positive constant, say u_3, and use the function $u_2(\varepsilon_t) = u_2 \ln(|\varepsilon_t| + u_3)$. The constant variance case corresponds to $u_2 = 0$, given the appropriate initial conditions.

In addition to the variance specification, there are many variations on the basic form. For example, we may include slope and seasonal state equations in the usual way (cf. Hyndman et al., 2008, Chapter 2), or allow the variance to depend upon the state variables used to describe the mean.

The benefit of the innovations state space approach is that we have considerable freedom in the specification of expression (5), and yet parameter estimation is still straightforward and may be performed by maximum likelihood, by minimizing the sum of squared or absolute errors, or by using any other appropriate objective function.

5. Monitoring heteroscedastic processes

Following from the discussion in the previous section, we extend model (2) to include a variance function:

$$\begin{aligned} y_t &= \mu + \varphi x_{t-1} + \varepsilon_t \\ x_t &= \varphi x_{t-1} + \alpha \varepsilon_t \\ \ln v_t &= u_0 + u_1 \ln v_{t-1} + u_2 \ln(|\varepsilon_t|) \\ \varepsilon_t &\sim \mathrm{NID}(0, v_{t-1}). \end{aligned} \tag{6}$$

Initial data analysis indicated that $u_0 \approx 0$, so we imposed that constraint. Thus, the parameters may be estimated by maximum likelihood using the likelihood function:

$$\ell(x_0, v_0, \mu, \varphi, \alpha, u_1, u_2) = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi v_{t-1}}} \times \exp\left(-\frac{\varepsilon_t^2}{2v_{t-1}}\right), \tag{7}$$

where x_0 and v_0 denote the start-up values for the state variables. Recall that this approach differs from the standard ARIMA version, where it is assumed that the series has an infinite past. For a detailed discussion of the estimation issues, see Hyndman et al. (2008, Chapter 5). We now consider two examples to illustrate the importance of allowing for possibly heteroscedastic processes. Note that the purpose of these examples is to illustrate the impact of a changing variance, so we have not attempted to either remove outliers prior to estimation, or "reset" the control charts after an out-of-control condition has been observed.

5.1. Running mileage

The first example uses a time series of the number of miles run per month by a middle-aged forecaster

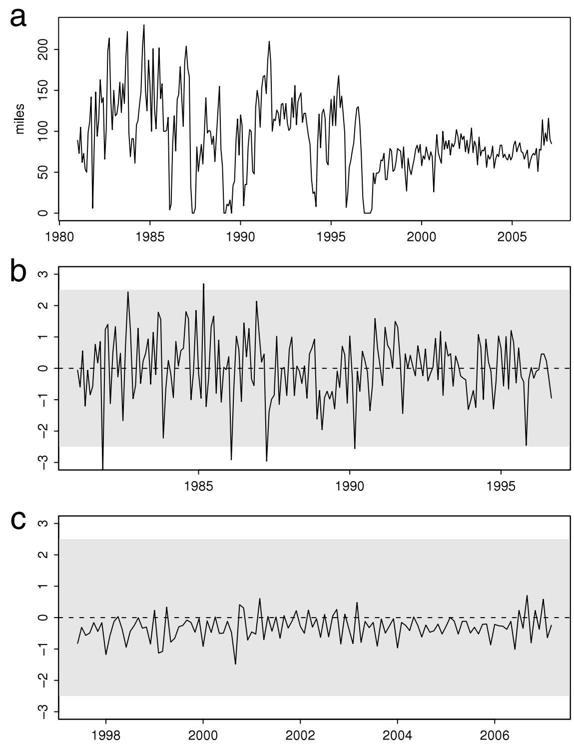

Fig. 1. Plots of (a) monthly mileage, 1/81–3/07; (b) standardized residuals for an AR (1) model fitted over 1/81–9/96; (c) standardized one-step-ahead forecast errors from the same model for 6/97–3/07. The shaded regions show the limits on the standardized charts at ±2.5 standard deviations.

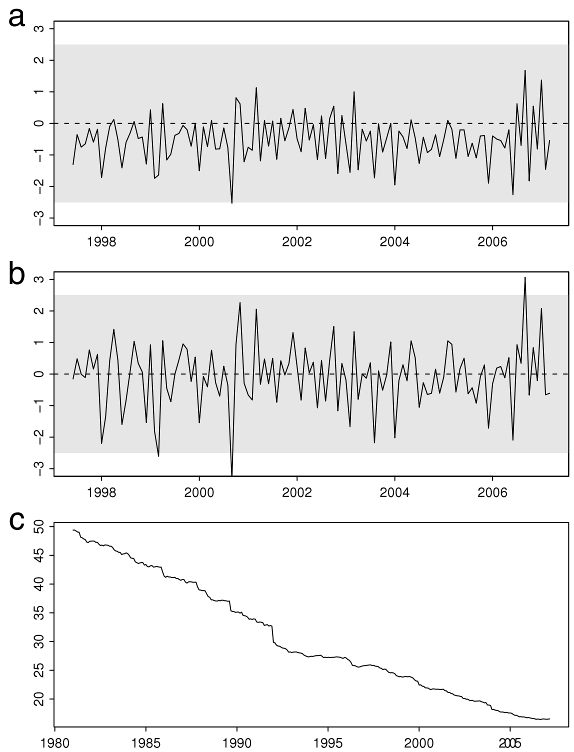

Fig. 2. Plots of (a) standardized one-step-ahead forecast errors for the GARCH model for the monthly mileage model for the period 6/97–3/07; (b) forecast errors for the same period after recalibrating the model; (c) standard deviations from the original model for the period 1/81–3/07. The shaded regions show the limits on the standardized charts at ±2.5 standard deviations.

over the period January 1981–March 2007. The series is plotted in Fig. 1, panel (a). Model (2) was fitted over the period 1/81 to 9/96 ($n = 189$) and yielded the estimates $\hat{\phi} = 0.61$, $\hat{\alpha} = 1.15$, $\hat{\mu} = 106$ and standard error $= 37.8$. The standardized residuals are plotted in Fig. 1, panel (b). The Box–Ljung test gave a p-value of 0.643 for the first 12 lags, so that there is no indication of a seasonal pattern. This model was then used to generate one-step-ahead forecasts for the period 6/97 to 3/07, and the one-step-ahead standardized forecast errors are plotted in Fig. 1, panel (c). The eight-month hiatus was when the runner was injured, and in most of those months he recorded zero mileage; the error was set to zero at 5/97 to restart the series for the one-step-ahead predictions.

It is clear from Fig. 1 panel (c) that the runner's training regime changed post-injury; the average mileage is lower and the variability in the series is greatly reduced. Because the variance has declined so much, the monitoring process is completely ineffective, with virtually all of the residuals being within one standard deviation of the center line.

The results for model (6) show interesting differences. The likelihood estimates derived from (7) for the first part of the series are: $\hat{\varphi} = 0.66$, $\hat{\alpha} = 1.10$, $\hat{u}_0 \simeq 0$, $\hat{u}_1 = 0.992$, $\hat{u}_2 = 0.008$ and $\hat{\mu} = 106$. On inspection, we observe that the estimates relating to the mean level of the process are very similar to those for the constant variance model. By contrast, Fig. 2 panel (a) shows the standardized one-step-ahead forecast errors based upon model (6), which presents a more reasonable picture than Fig. 1 panel (c). Even so, it appears that the standard deviations may still be on the high side. In practice, the model would be periodically recalibrated when used for monitoring, so that adjustments would be incorporated more rapidly. When we refit the model over the period 6/97 to

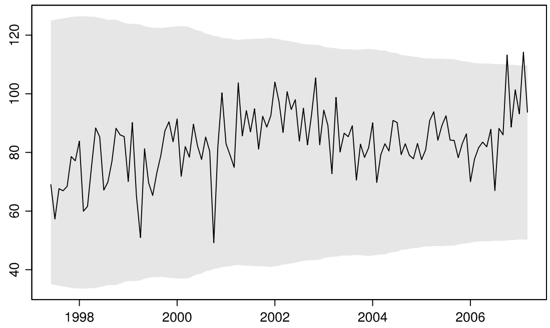

Fig. 3. Common Cause Chart with changing variances for the monthly mileage data, over the period 6/97–3/07.

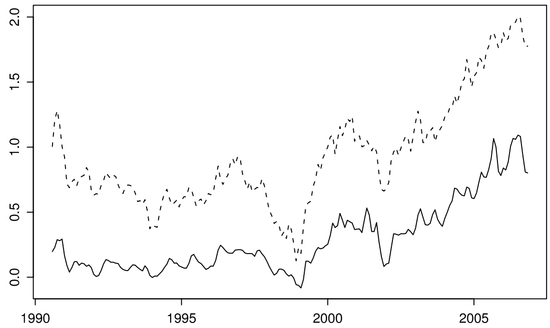

Fig. 4. Plots for the log gas price (solid line) and log spot price/10 (dotted line) series over the period 8/90–11/06. The spot price is scaled simply to place both series on the same diagram.

3/07, we obtain the modified plot shown in Fig. 2 panel (b), which is much more reasonable, although it clearly benefits from the wisdom of hindsight in the estimation process. Finally, Fig. 2 panel (c) shows how the standard deviation of the process has declined over time, by roughly a factor of three.

The Common Cause Chart (CCC) for the period 6/97 to 3/07 is shown in Fig. 3, using a center target line of 80 and the EGARCH variances. The plot indicates a departure from the target level at the end of the series.

5.2. *Gasoline prices*

The first example shows reduced volatility, which is usually the result of process improvements (e.g. new laws) or structural changes. It is important that the monitoring system adjust to such changes so as to avoid missing shifts under the new regime. More common, perhaps, are processes whose volatility increases over time. Gasoline prices provide a recent painful example of such a process. We consider a simple regression model, for which the dependent variable is:

$Y=$ logarithm of US retail gas prices[1] (the average price per gallon, in dollars),

and the predictor variable (lagged one month) is:

$Z=$ logarithm of the spot price of a barrel of West Texas Intermediate (WTI) oil, as traded at Cushing, Oklahoma (in dollars).

[1] These series are available from the US Energy Information Administration website www.eia.doe.gov.

The Cushing spot price is widely used in the industry as a “marker” for pricing a number of other crude oil supplies traded in the domestic spot market at Cushing, Oklahoma. The data are monthly and cover the period January 1991 to November 2006. The data are plotted in Fig. 4.

We use model (6) as a local level model with $\phi = 1$ to reflect the random walk-like nature of the series, and extend the measurement equation to the form:

$$y_t = \beta_0 + x_{t-1} + \beta z_{t-1} + \varepsilon_t.$$

Different versions of the model were fitted to data for the period January 1991 to December 2001, but little improvement was found over $u_3 = 0$. The estimated form of the final model is:

$$\hat{y}_t = -0.137 + 0.134 z_{t-1} + x_{t-1}$$
$$x_t = x_{t-1} + 1.47 e_t$$
$$\ln v_t = 0.931 \ln v_{t-1} + 0.114 \ln(|\varepsilon_t|).$$

Fig. 5, panels (a) and (b), show the control charts based upon the constant variance and EGARCH models respectively. As is to be expected, the general shapes of the two plots are similar. What is very different is the scale of the y-axis. The constant variance model ranges from roughly $+3$ to -4 standard deviations, with a number of warning signals. By contrast, the EGARCH values fluctuate within the warning limits. The reason for the difference can readily be deduced from Fig. 5 panel (c), which shows the increase in the standard deviation over time. The estimated standard deviation from the earlier part of the series is only 0.030, whereas the most recent values are roughly double that. We may conclude that the series

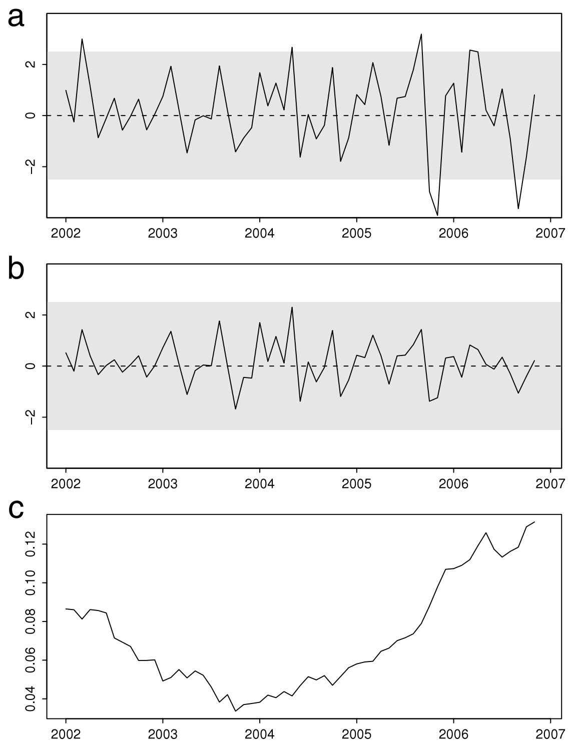

Fig. 5. Plots for the log gas price series over the period 1/02–11/06. The limits on the standardized residuals charts are set at ±2.5 standard deviations. (a) Standardized residuals for the constant variance model. (b) Standardized residuals for the GARCH model. (c) Standard deviations for the GARCH model.

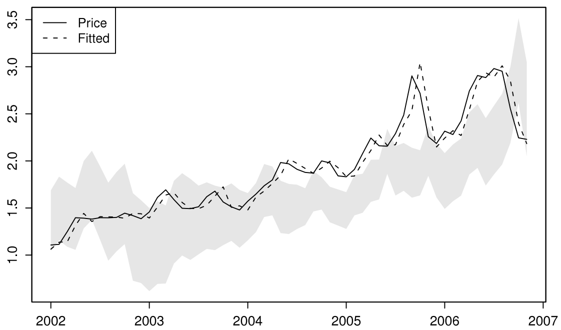

Fig. 6. Plot of the fitted values and 3-step-ahead control limits for the gas price series over the period 8/90–11/06. The shaded regions show the limits on the standardized charts at ±2 standard deviations.

has indeed become more volatile, but that the basic model continues to describe its general movements. By contrast, as we saw in Fig. 4, the average price is increasing rapidly. This effect is captured by a local CCC chart. Since the process is non-stationary, we cannot use the steady-state results of Wardell et al. (1992). Instead, we may use an h-step-ahead warning scheme for which the variance of the h-step-ahead fitted value at time t is approximately $(h-1)\alpha^2 v_t$. Fig. 6 shows this plot for $h = 3$.

In combination, the two plots indicate that the price is clearly increasing, but the process is in (EGARCH-adjusted) statistical control. The impacts of potential policies should be evaluated in terms of their ability to affect both the price level and the magnitude of the fluctuations.

6. Conclusions

When we use control charts to monitor social or economic processes, temporal dependence is often a given. Further, both the mean and the variance may evolve over time in a recognizable fashion. The objective is then to model such anticipated changes so that unexpected shifts in the level of the process can be identified. We have used innovations state space models to describe such evolving processes, and shown that constant variance models may be quite inadequate for the monitoring task; by contrast, models that allow for conditional heteroscedasticity are much more effective. Such models enable us to separate out issues of statistical control from those of underlying trends, thereby providing a decision maker with a clearer view of the underlying process.

The paper focuses upon extensions to Shewhart charts for monitoring univariate time series. The ideas in this paper clearly extend to the use of EWMA charts for the residuals, which will be more effective in detecting level shifts. Likewise, conditionally heteroscedastic R or S charts may be developed for monitoring variability. A systematic analysis of the performance of this approach also needs to be considered, using data coded for exceptions, as in Cohen, Garman, and Gorr (2009).

Finally, we note that our analysis focuses exclusively upon the univariate case, although the monitoring of social and economic processes will often require the evaluation of multiple series. Extensions of standard control charts to the multivariate case has been examined in a number of papers; see for example Lowry, Woodall, Champ, and Rigdon (1992), Pan and Jarrett (2004) and Runger, Barton, Del Castillo, and

Woodall (2007). Clearly this is an area where considerable further research is needed.

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