

Improving forecasts via subspace projections

Rob J Hyndman



Outline

- 1 Improving hierarchical forecasts
- 2 Improving univariate forecasts
- 3 Improving multivariate forecasts
- 4 Final comments

Outline

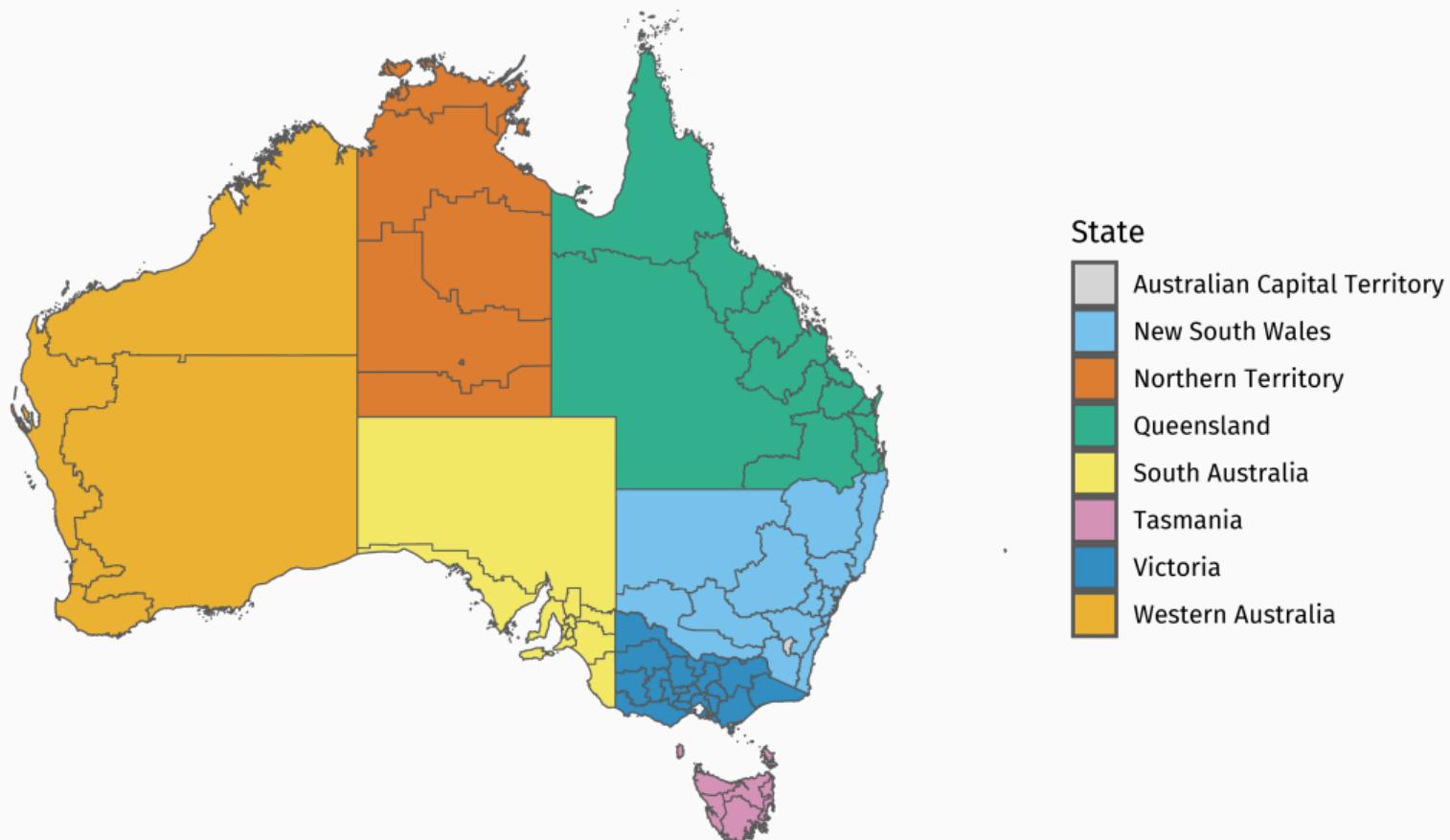
1 Improving hierarchical forecasts

2 Improving univariate forecasts

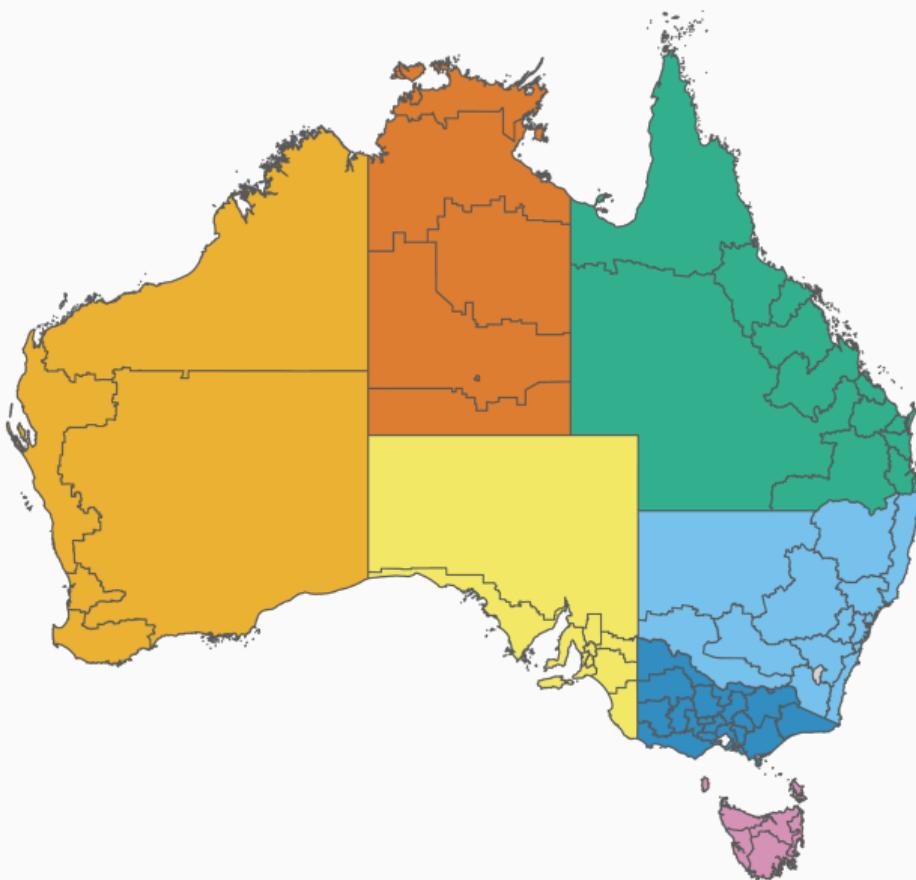
3 Improving multivariate forecasts

4 Final comments

Australian tourism regions



Australian tourism regions



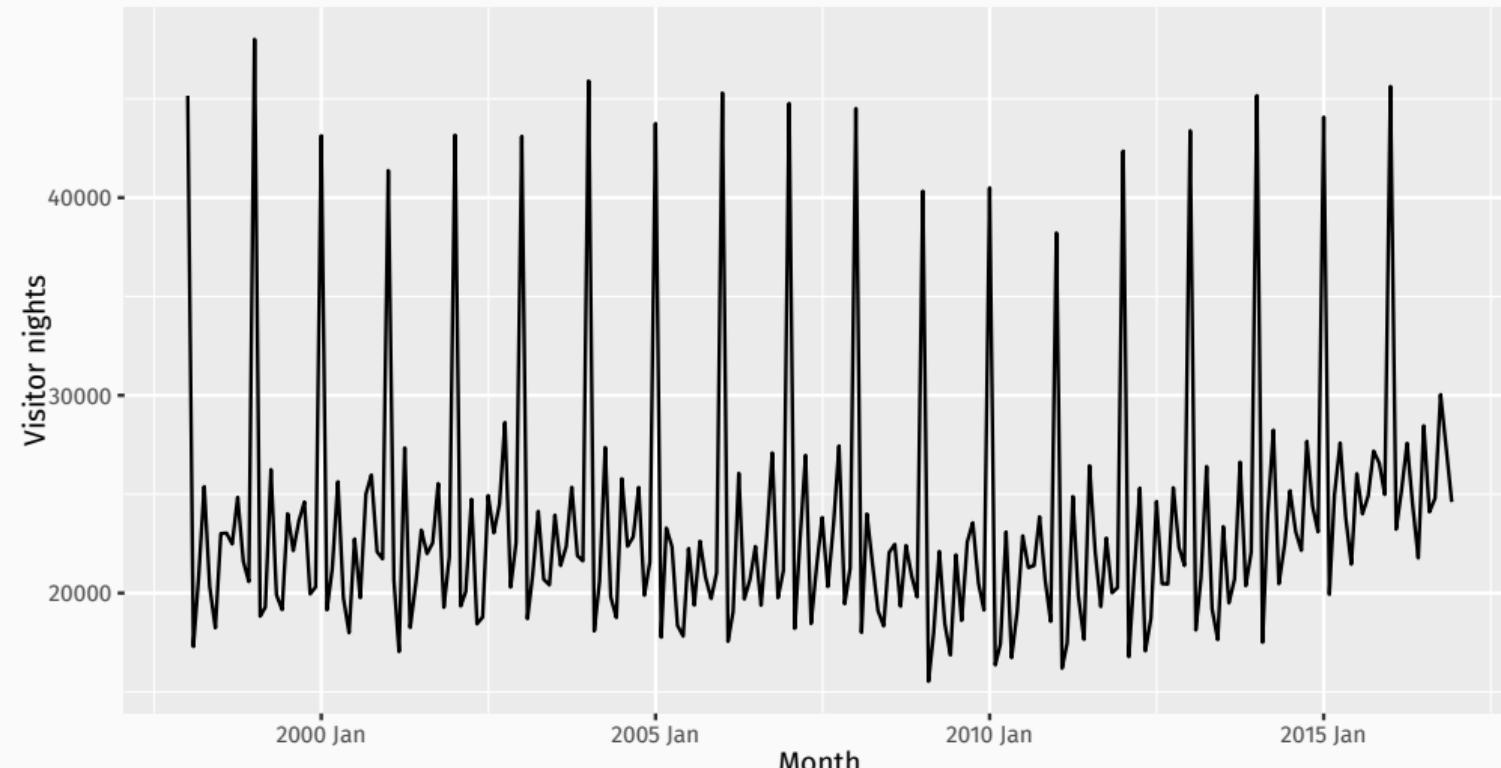
- Monthly data on visitor nights: 1998 – 2016
- 7 states
- 27 zones
- 76 regions

State

Australian Capital Territory
New South Wales
Northern Territory
Queensland
South Australia
Tasmania
Victoria
Western Australia

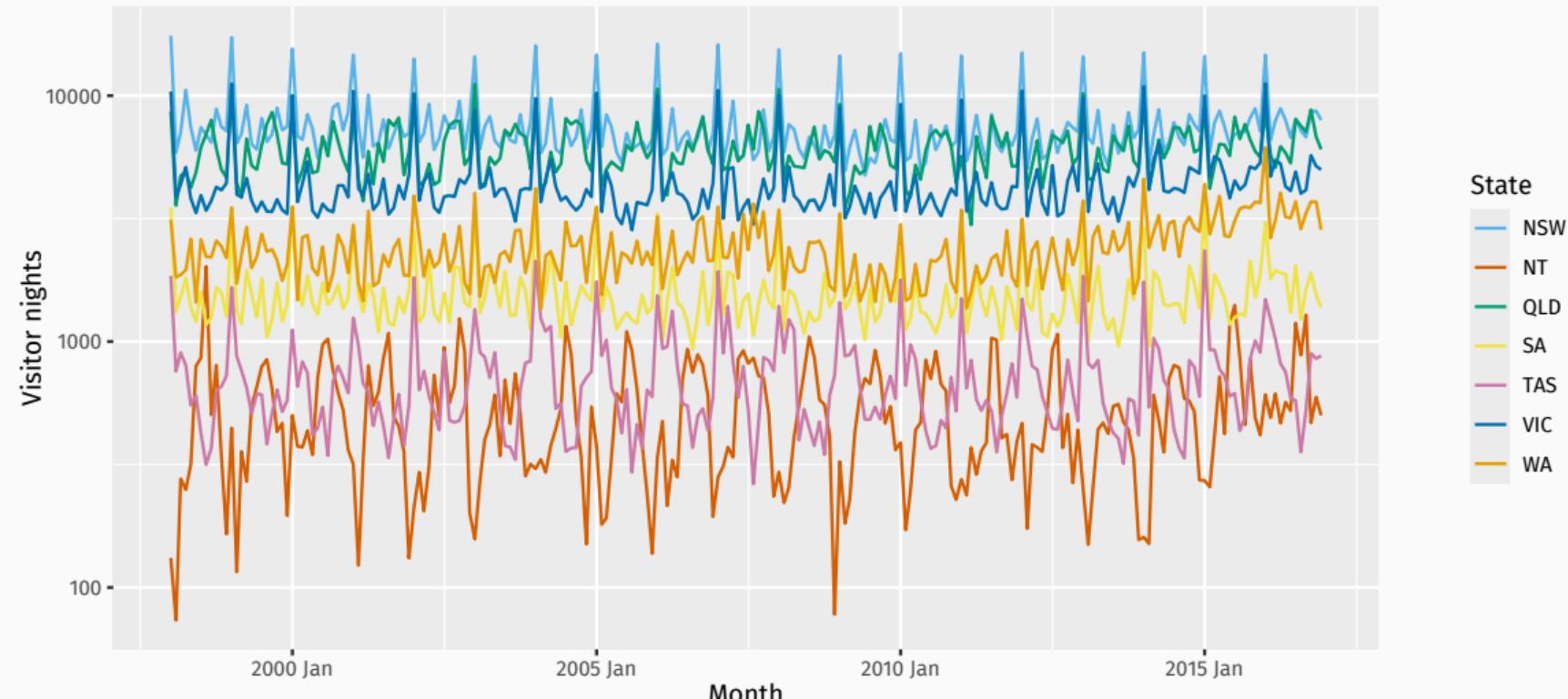
Australian tourism data

Total domestic travel: Australia



Australian tourism data

Total domestic travel: by state



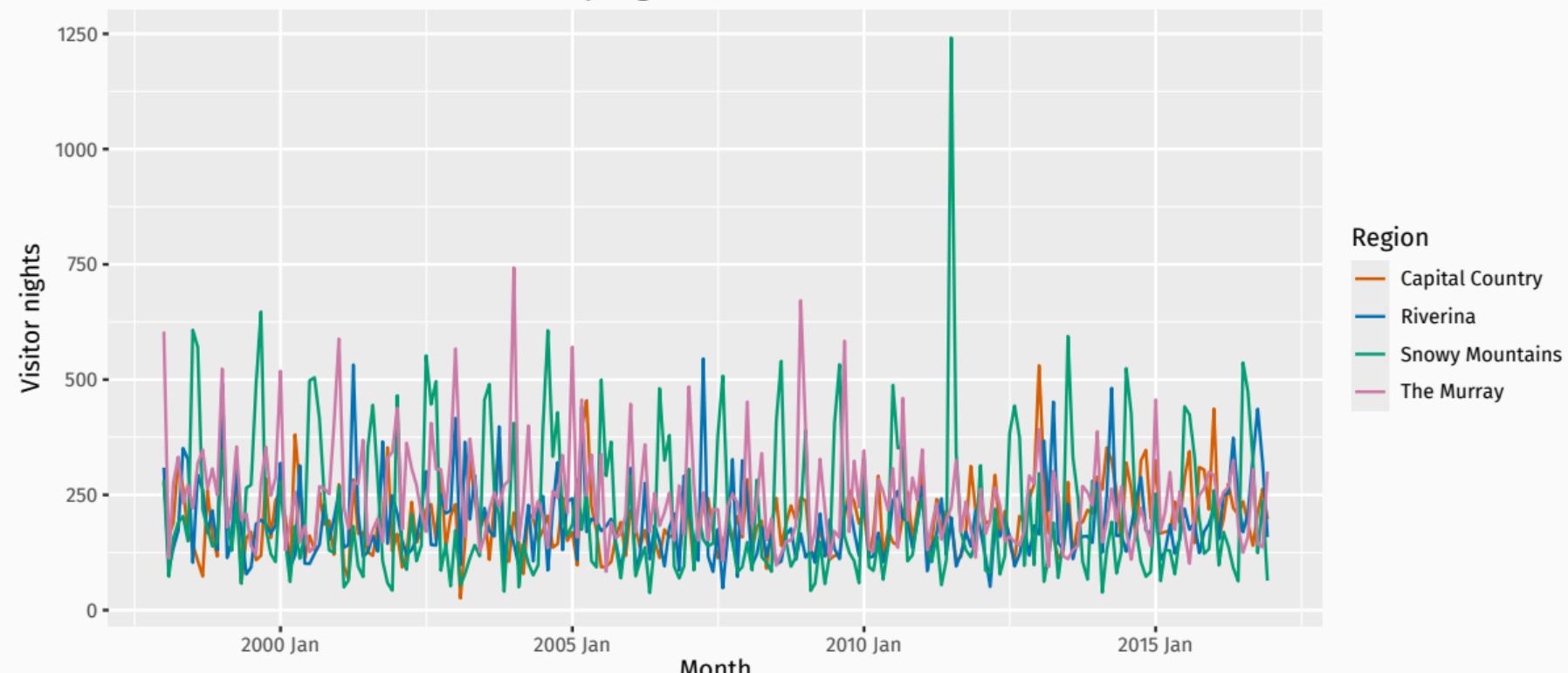
Australian tourism data

Total domestic travel: NSW by zone

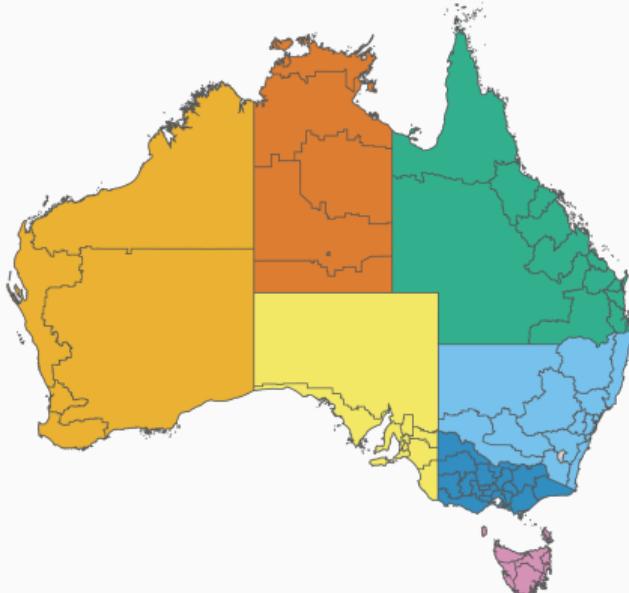


Australian tourism data

Total domestic travel: South NSW by region



Australian tourism data



State
Australian Capital Territory
New South Wales
Northern Territory
Queensland
South Australia
Tasmania
Victoria
Western Australia

Aggregation level	# series
National	1
State	7
Zone	27
Region	76
Total	111

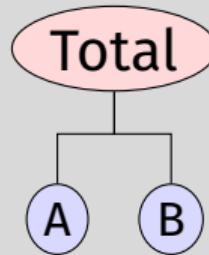
- Need forecasts at all levels of aggregation.
- Compute **base forecasts** using univariate models. These will not add up.
- Adjust base forecasts to ensure they are “coherent” giving **reconciled forecasts**.

Notation

Almost all collections of time series with linear constraints can be written as

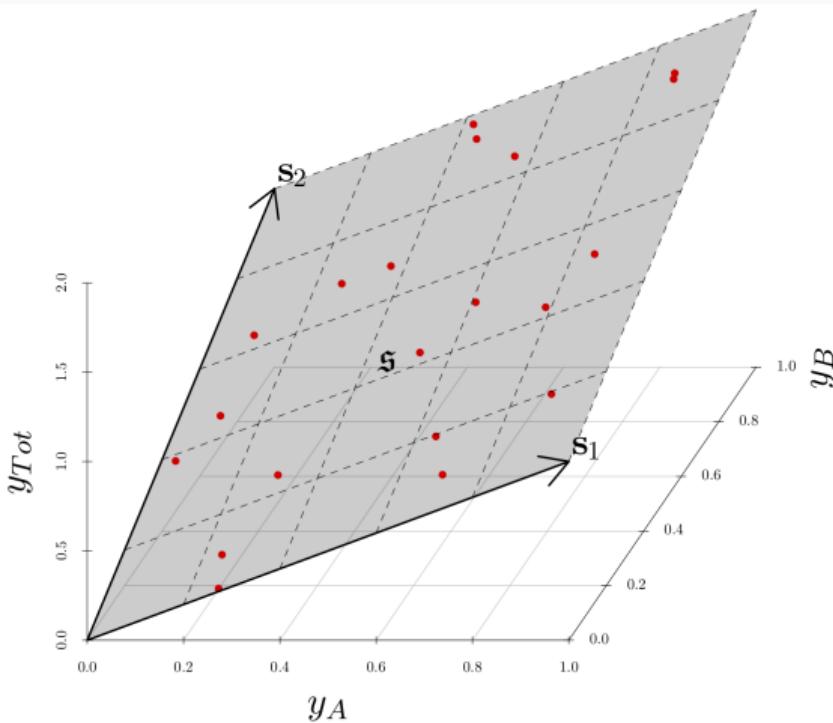
$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

- \mathbf{y}_t = vector of all series at time t
- $y_{\text{Total},t}$ = aggregate of all series at time t .
- $y_{X,t}$ = value of series X at time t .
- \mathbf{b}_t = vector of most disaggregated series at time t
- \mathbf{S} = “summing matrix” containing the linear constraints.



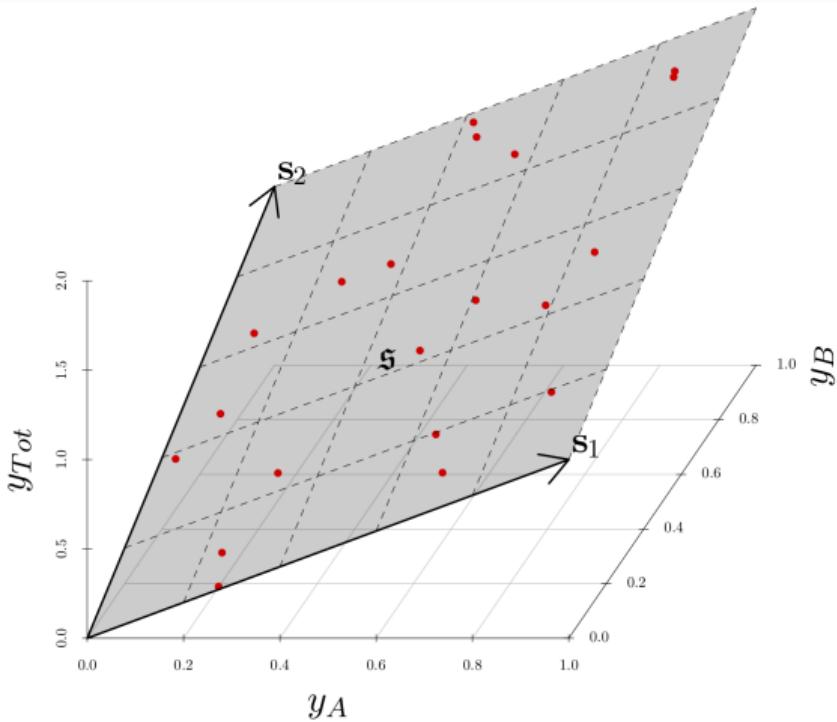
$$\begin{aligned}\mathbf{y}_t &= \begin{pmatrix} y_{\text{Total},t} \\ y_{A,t} \\ y_{B,t} \end{pmatrix} \\ &= \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_S \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \end{pmatrix}}_{\mathbf{b}_t}\end{aligned}$$

Projections onto the coherent subspace



$$y_{\text{Total}} = y_A + y_B$$

Projections onto the coherent subspace

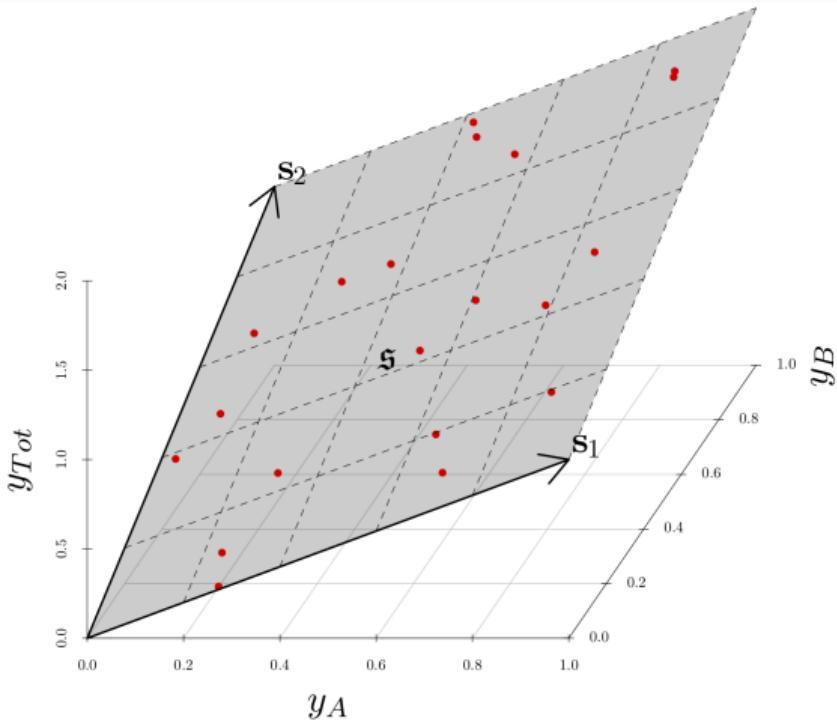


Base forecasts

Let $\hat{y}_{t+h|t}$ be vector of *incoherent* initial h -step forecasts.

$$y_{\text{Total}} = y_A + y_B$$

Projections onto the coherent subspace



$$y_{\text{Total}} = y_A + y_B$$

Base forecasts

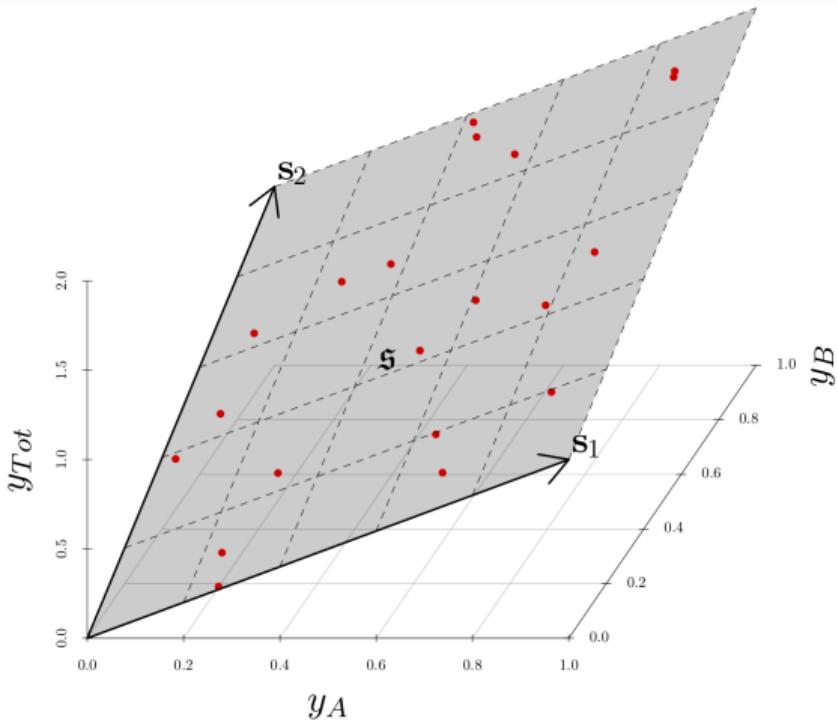
Let $\hat{\mathbf{y}}_{t+h|t}$ be vector of *incoherent* initial h -step forecasts.

Reconciled forecasts

Let \mathbf{M} be a projection matrix.

$\tilde{\mathbf{y}}_{t+h|t} = \mathbf{M}\hat{\mathbf{y}}_{t+h|t}$ “reconciles” $\hat{\mathbf{y}}_{t+h|t}$.

Projections onto the coherent subspace



$$y_{Total} = y_A + y_B$$

Base forecasts

Let $\hat{y}_{t+h|t}$ be vector of *incoherent* initial h -step forecasts.

Reconciled forecasts

Let M be a projection matrix.

$\tilde{y}_{t+h|t} = M\hat{y}_{t+h|t}$ “reconciles” $\hat{y}_{t+h|t}$.

- S forms a basis set for \mathfrak{s}
- All projections are of the form $M = S(S'\Psi S)^{-1}S'\Psi$ where Ψ is a positive definite matrix.
- How to choose the best Ψ ?

$$\tilde{\mathbf{y}}_{t+h|t} = \mathbf{M}\hat{\mathbf{y}}_{t+h|t}$$

Reconciled forecasts

Base forecasts

$$\tilde{\mathbf{y}}_{t+h|t} = \mathbf{M}\hat{\mathbf{y}}_{t+h|t}$$

Reconciled forecasts

Base forecasts

- Base forecast covariance:

$$\mathbf{W}_h = \text{Var}[\mathbf{y}_{T+h} - \hat{\mathbf{y}}_{t+h|t} \mid \mathbf{y}_1, \dots, \mathbf{y}_T]$$

- Reconciled forecast covariance:

$$\mathbf{V}_h = \text{Var}[\mathbf{y}_{T+h} - \tilde{\mathbf{y}}_{t+h|t} \mid \mathbf{y}_1, \dots, \mathbf{y}_T] = \mathbf{M}\mathbf{W}_h\mathbf{M}'$$

$$\tilde{\mathbf{y}}_{t+h|t} = \mathbf{M}\hat{\mathbf{y}}_{t+h|t}$$

Reconciled forecasts

Base forecasts

- Base forecast covariance:

$$\mathbf{W}_h = \text{Var}[\mathbf{y}_{T+h} - \hat{\mathbf{y}}_{t+h|t} \mid \mathbf{y}_1, \dots, \mathbf{y}_T]$$

- Reconciled forecast covariance:

$$\mathbf{V}_h = \text{Var}[\mathbf{y}_{T+h} - \tilde{\mathbf{y}}_{t+h|t} \mid \mathbf{y}_1, \dots, \mathbf{y}_T] = \mathbf{M}\mathbf{W}_h\mathbf{M}'$$

Minimum trace (MinT) reconciliation

If \mathbf{M} is a projection, then trace of \mathbf{V}_h is minimized when $\Psi = \mathbf{W}_h$

$$\mathbf{M} = \mathbf{S}(\mathbf{S}'\mathbf{W}_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{W}_h^{-1}$$

Linear projections

$$\tilde{\mathbf{y}}_{t+h|t} = \mathbf{S}(\mathbf{S}'\mathbf{W}_h^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{W}_h^{-1}\hat{\mathbf{y}}_{t+h|t}$$

Reconciliation method

 \mathbf{W}_h

OLS

 I

WLS(var)

 $\text{diag}(\mathbf{W}_1)$

WLS(struct)

 $\text{diag}(\mathbf{S1})$

MinT(sample)

 $\hat{\mathbf{W}}_{\text{sam}}$

MinT(shrink)

 $\hat{\mathbf{W}}_{\text{shr}}$

- All approximate MinT by assuming $\mathbf{W}_h = k_h \mathbf{W}_1$.
- $\hat{\mathbf{W}}_{\text{sam}}$ is sample estimate of the residual covariance matrix
- $\hat{\mathbf{W}}_{\text{shr}}$ is shrinkage estimator $\tau \text{diag}(\hat{\mathbf{W}}_{\text{sam}}) + (1 - \tau)\hat{\mathbf{W}}_{\text{sam}}$ where τ selected optimally.

Example: Australian tourism

tourism

```
# A tsibble: 17,328 x 5 [1M]
# Key:      state, zone, region [76]
  state zone  region    month visitors
  <chr> <chr> <chr>     <mth>     <dbl>
1 NSW   ACT    Canberra Jan      612.
2 NSW   ACT    Canberra Feb      471.
3 NSW   ACT    Canberra Mar      430.
4 NSW   ACT    Canberra Apr      499.
5 NSW   ACT    Canberra May      338.
6 NSW   ACT    Canberra Jun      236.
7 NSW   ACT    Canberra Jul      371.
8 NSW   ACT    Canberra Aug      375.
9 NSW   ACT    Canberra Sep      449.
10 NSW  ACT    Canberra Oct      517.
# i 17,318 more rows
```

Example: Australian tourism

```
tourism_agg <- tourism |>  
  aggregate_key(state / zone / region, visitors = sum(visitors))
```

```
# A tsibble: 25,308 x 5 [1M]  
# Key:      state, zone, region [111]  
  month state       zone       region     visitors  
  <mth> <chr*>    <chr*>    <chr*>     <dbl>  
1 1998 Jan <aggregated> <aggregated> <aggregated> 45151.  
2 1998 Feb <aggregated> <aggregated> <aggregated> 17295.  
3 1998 Mar <aggregated> <aggregated> <aggregated> 20725.  
4 1998 Apr <aggregated> <aggregated> <aggregated> 25389.  
5 1998 May <aggregated> <aggregated> <aggregated> 20330.  
6 1998 Jun <aggregated> <aggregated> <aggregated> 18238.  
7 1998 Jul <aggregated> <aggregated> <aggregated> 23005.  
8 1998 Aug <aggregated> <aggregated> <aggregated> 23033.  
9 1998 Sep <aggregated> <aggregated> <aggregated> 22483.  
10 1998 Oct <aggregated> <aggregated> <aggregated> 24845.  
# i 25,298 more rows
```

Example: Australian tourism

```
fit <- tourism_agg |>
```

```
  filter(year(month) <= 2015) |>
```

```
  model(ets = ETS(visitors))
```

```
# A mable: 111 x 4
```

```
# Key: state, zone, region [111]
```

	state	zone	region	ets
	<chr*>	<chr*>	<chr*>	<model>
1	NSW	ACT	Canberra	<ETS(M,N,A)>
2	NSW	ACT	<aggregated>	<ETS(M,N,A)>
3	NSW	Metro NSW	Central Coast	<ETS(M,N,A)>
4	NSW	Metro NSW	Sydney	<ETS(M,N,A)>
5	NSW	Metro NSW	<aggregated>	<ETS(M,N,A)>
6	NSW	North Coast NSW	Hunter	<ETS(M,N,M)>
7	NSW	North Coast NSW	North Coast NSW	<ETS(M,N,M)>
8	NSW	North Coast NSW	<aggregated>	<ETS(M,N,M)>
9	NSW	North NSW	Blue Mountains	<ETS(M,N,M)>
10	NSW	North NSW	Central NSW	<ETS(A,N,A)>
# i 101 more rows				

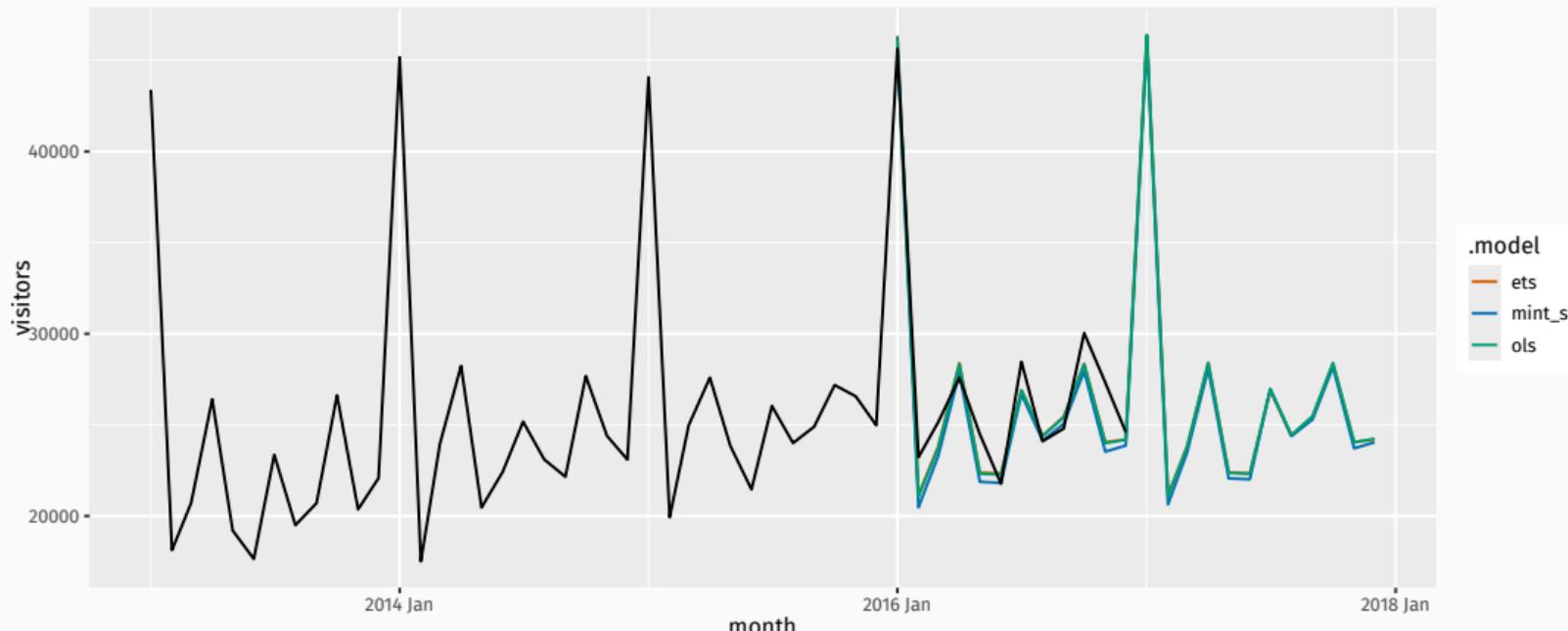
Example: Australian tourism

```
fc <- fit |>
  reconcile(
    ols = min_trace(ets, method = "ols"),
    mint_s = min_trace(ets, method = "mint_shrink"),
  ) |>
  forecast(h = "2 years")
```

```
# A fable: 7,992 x 7 [1M]
# Key:      state, zone, region, .model [333]
  state   zone   region   .model   month
  <chr*> <chr*> <chr*>   <chr>     <mth>
1 NSW     ACT     Canberra  ets     2016 Jan
2 NSW     ACT     Canberra  ets     2016 Feb
3 NSW     ACT     Canberra  ets     2016 Mar
4 NSW     ACT     Canberra  ets     2016 Apr
5 NSW     ACT     Canberra  ets     2016 May
# i 7,987 more rows
# i 2 more variables: visitors <dist>, .mean <dbl>
```

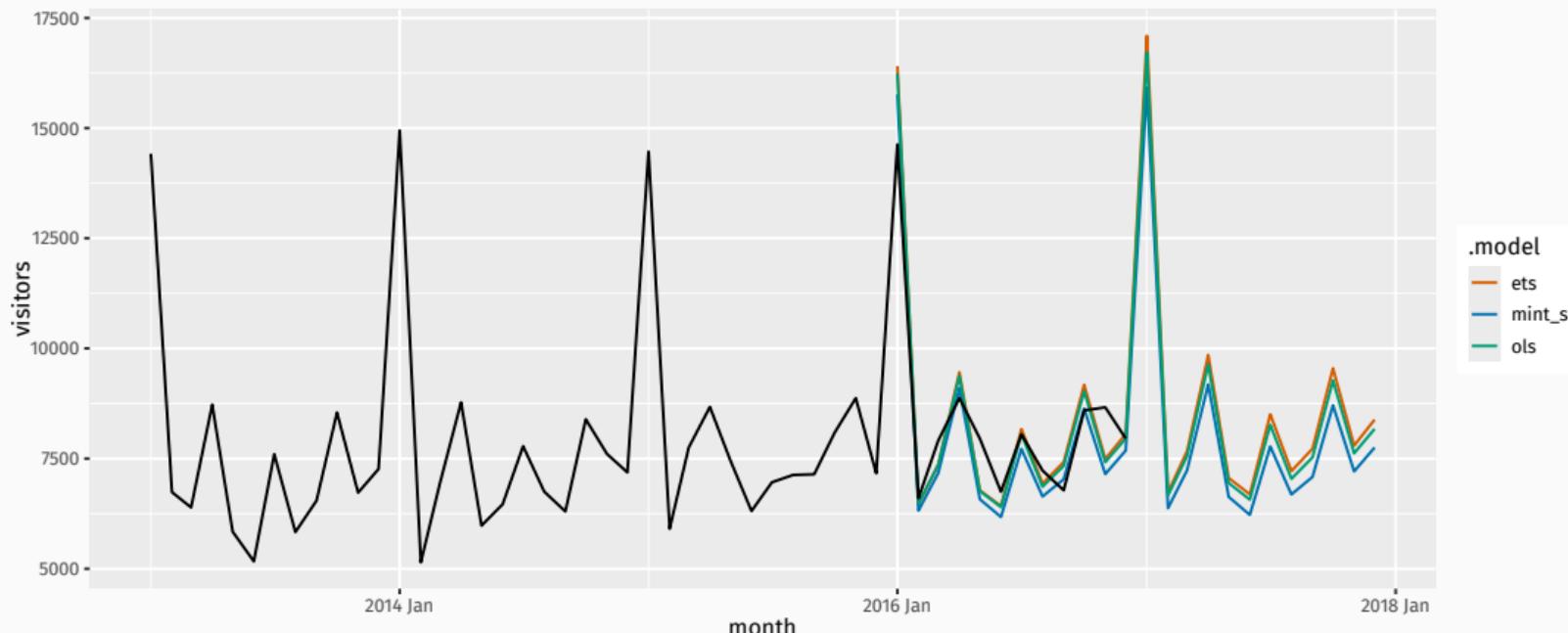
Example: Australian tourism

```
fc |>  
  filter(is_aggregated(state)) |>  
  autoplot(filter(tourism_agg, year(month) > 2012), level = NULL)
```



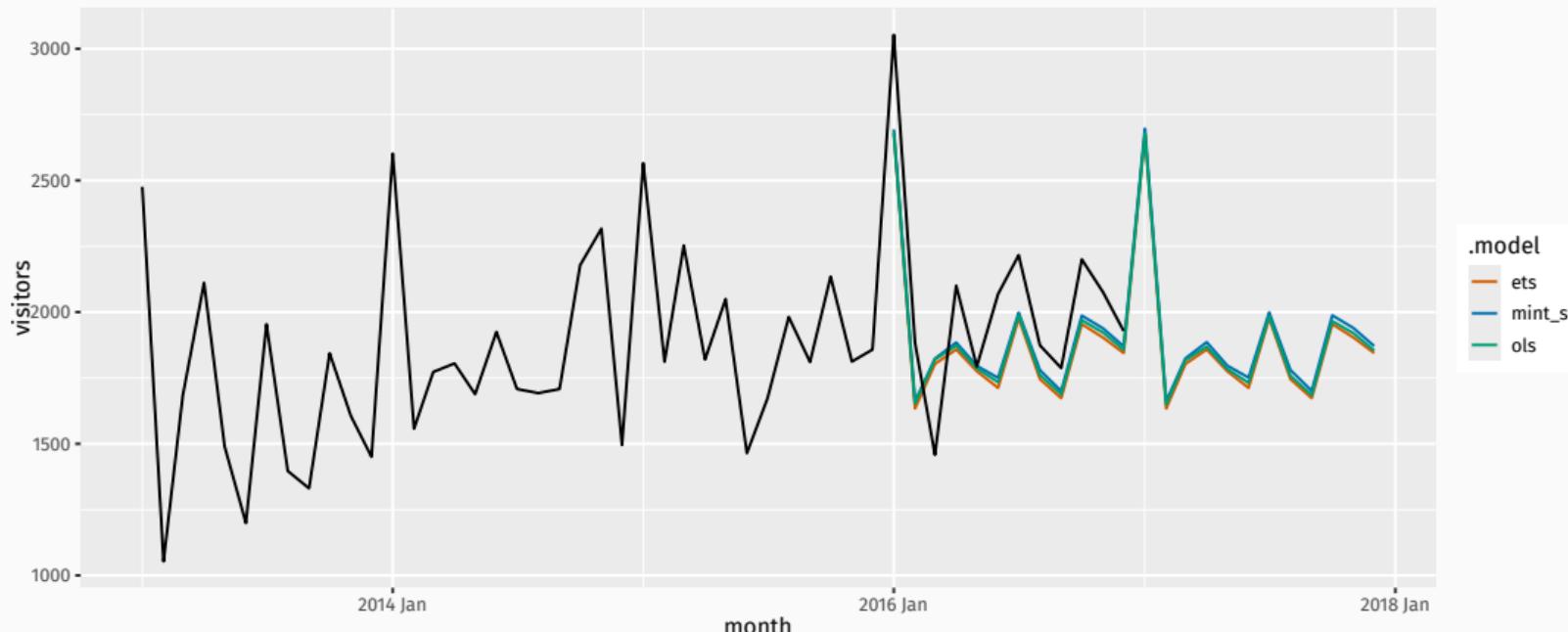
Example: Australian tourism

```
fc |>  
  filter(state == "NSW" & is_aggregated(zone)) |>  
  autoplot(filter(tourism_agg, year(month) > 2012), level = NULL)
```



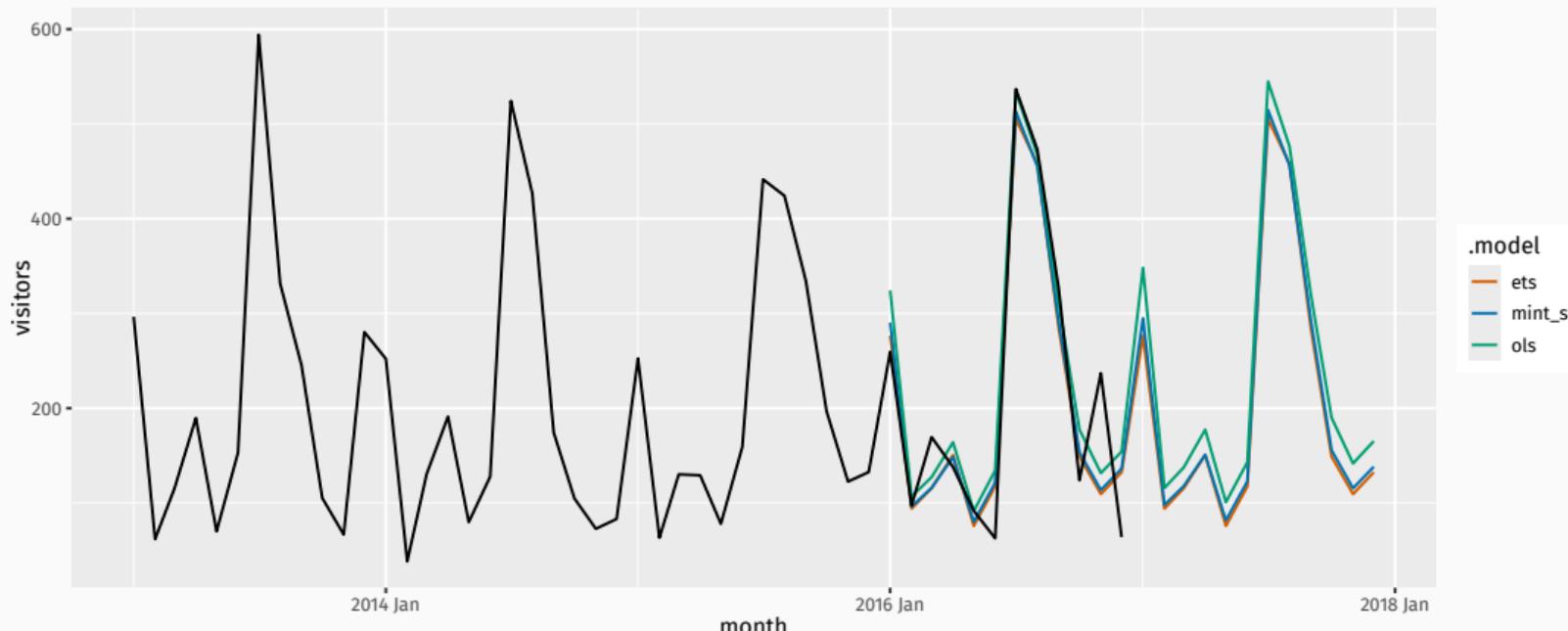
Example: Australian tourism

```
fc |>  
  filter(region == "Melbourne") |>  
  autoplot(filter(tourism_agg, year(month) > 2012), level = NULL)
```



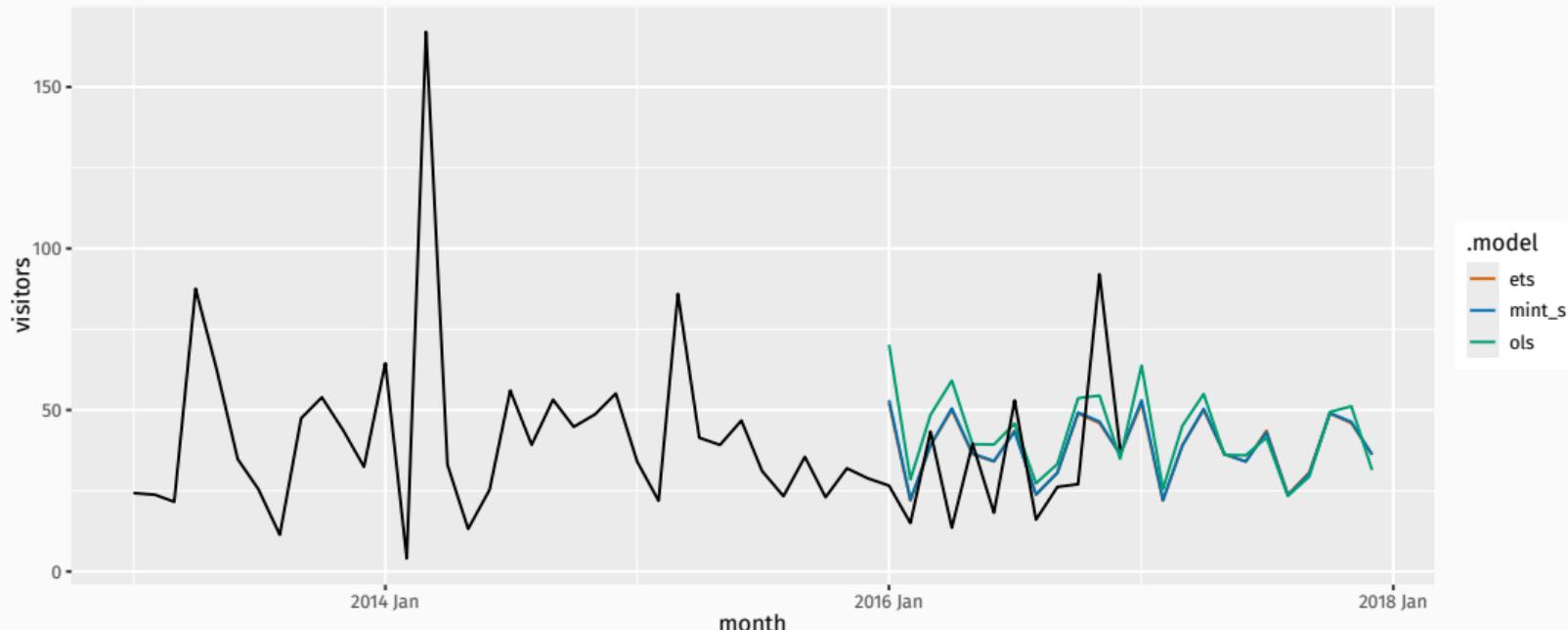
Example: Australian tourism

```
fc |>  
  filter(region == "Snowy Mountains") |>  
  autoplot(filter(tourism_agg, year(month) > 2012), level = NULL)
```



Example: Australian tourism

```
fc |>  
  filter(region == "Barossa") |>  
  autoplot(filter(tourism_agg, year(month) > 2012), level = NULL)
```



Performance evaluation

$$\text{RMSSE} = \sqrt{\text{mean}(q_j^2)}$$

$$q_j^2 = \frac{e_j^2}{\frac{1}{T-m} \sum_{t=m+1}^T (y_t - y_{t-m})^2}$$

- y_t = observation for period t
- e_j = forecast error for forecast horizon j
- T = size of training set
- $m = 12$

Example: Australian tourism

```
accuracy(fc, tourism_agg, measures = list(rmsse = RMSSE))
```

```
# A tibble: 333 x 7
  .model state      zone      region     .type rmsse level
  <chr>   <chr*>    <chr*>    <chr*>    <chr>  <dbl> <fct>
1 ets     <aggregated> <aggregated> <aggregated> Test   0.755 National
2 mint_s  <aggregated> <aggregated> <aggregated> Test   0.896 National
3 ols     <aggregated> <aggregated> <aggregated> Test   0.768 National
4 ets     NSW        <aggregated> <aggregated> Test   0.921 State
5 mint_s NSW        <aggregated> <aggregated> Test   0.893 State
6 ols     NSW        <aggregated> <aggregated> Test   0.881 State
7 ets     NT          <aggregated> <aggregated> Test   1.24  State
8 mint_s NT          <aggregated> <aggregated> Test   1.22  State
9 ols     NT          <aggregated> <aggregated> Test   1.18  State
10 ets    QLD         <aggregated> <aggregated> Test   0.860 State
# i 323 more rows
```

Example: Australian tourism

```
accuracy(fc, tourism_agg, measures = list(rmsse = RMSSE)) |>  
  summarise(rmsse = sqrt(mean(rmsse^2)), .by = .model)
```

```
# A tibble: 3 x 2  
  .model    rmsse  
  <chr>    <dbl>  
1 ols      0.863  
2 mint_s   0.866  
3 ets      0.880
```

Example: Australian tourism

```
accuracy(fc, tourism_agg, measures = list(rmsse = RMSSE)) |>  
  summarise(rmsse = sqrt(mean(rmsse^2)), .by = .model)
```

```
# A tibble: 3 x 2  
  .model    rmsse  
  <chr>    <dbl>  
1 ols      0.863  
2 mint_s   0.866  
3 ets      0.880
```

- Overall, both reconciliation methods are more accurate than the base ETS forecasts.

Example: Australian tourism

```
accuracy(fc, tourism_agg, measures = list(rmsse = RMSSE)) |>
  summarise(rmsse = sqrt(mean(rmsse^2)), .by = c(.model, level)) |>
```

```
# A tibble: 12 x 3
  .model    level   rmsse
  <chr>    <fct>   <dbl>
1 ets      National 0.755
2 ols      National 0.768
3 mint_s   National 0.896
4 ols      State    0.905
5 ets      State    0.919
6 mint_s   State    0.953
7 ols      Zone     0.912
8 mint_s   Zone     0.914
9 ets      Zone     0.935
10 mint_s  Region   0.839
11 ols     Region   0.843
12 ets     Region   0.858
```

Example: Australian tourism

```
accuracy(fc, tourism_agg, measures = list(rmsse = RMSSE)) |>  
  summarise(rmsse = sqrt(mean(rmsse^2)), .by = c(.model, level)) |>
```

```
# A tibble: 12 x 3  
  .model   level    rmsse  
  <chr>   <fct>    <dbl>  
1 ets     National  0.755  
2 ols     National  0.768  
3 mint_s  National  0.896  
4 ols     State     0.905  
5 ets     State     0.919  
6 mint_s  State     0.953  
7 ols     Zone      0.912  
8 mint_s  Zone      0.914  
9 ets     Zone      0.935  
10 mint_s Region    0.839  
11 ols    Region    0.843  
12 ets    Region    0.858
```

- Reconciliation is best for all levels except national.
- Improvements due to reconciliation are greater at lower levels.

Distance reducing property

Let $\|\mathbf{u}\|_{\Psi} = \mathbf{u}'\Psi\mathbf{u}$. Then

$$\|\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|\textcolor{red}{t}}\|_{\Psi} \leq \|\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|\textcolor{blue}{t}}\|_{\Psi}$$

- Ψ -projection is guaranteed to improve forecast accuracy over base forecasts *using this distance measure.*
- Distance reduction holds for any realisation and any forecast.
- OLS reconciliation minimizes Euclidean distance.

$$\begin{aligned}\|\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|t}\|_2^2 &= \|\mathbf{M}(\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|t})\|_2^2 \\ &\leq \|\mathbf{M}\|_2^2 \|\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|t}\|_2^2 \\ &= \sigma_{\max}^2 \|\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|t}\|_2^2\end{aligned}$$

- σ_{\max} is the largest eigenvalue of \mathbf{M}
- $\sigma_{\max} \geq 1$ as \mathbf{M} is a projection matrix.
- Every projection reconciliation is better than base forecasts using Euclidean distance.

$$\begin{aligned} & \text{tr}\left(E[\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|t}^{\text{MinT}}]'[\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|t}^{\text{MinT}}]\right) \\ & \leq \text{tr}\left(E[\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|t}^{\text{OLS}}]'[\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|t}^{\text{OLS}}]\right) \\ & \leq \text{tr}\left(E[\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|t}]'[\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|t}]\right) \end{aligned}$$

Using sums of variances:

- MinT reconciliation is better than OLS reconciliation
- OLS reconciliation is better than base forecasts

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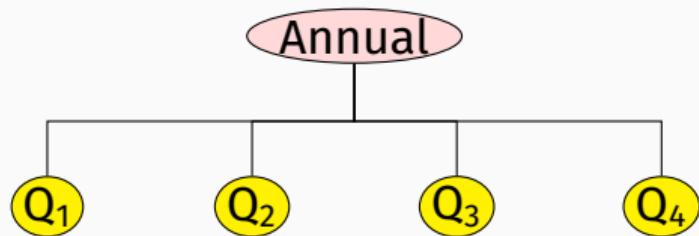
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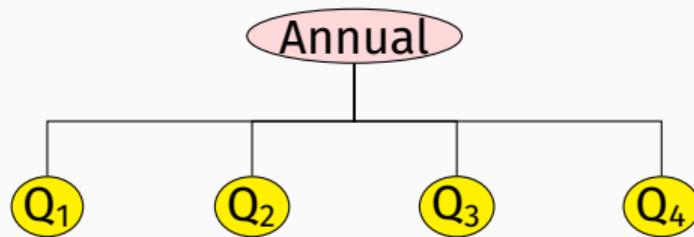
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4 Final comments

Temporal reconciliation: quarterly data

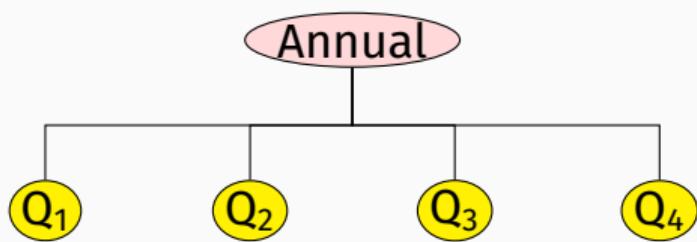


Temporal reconciliation: quarterly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

Temporal reconciliation: quarterly data

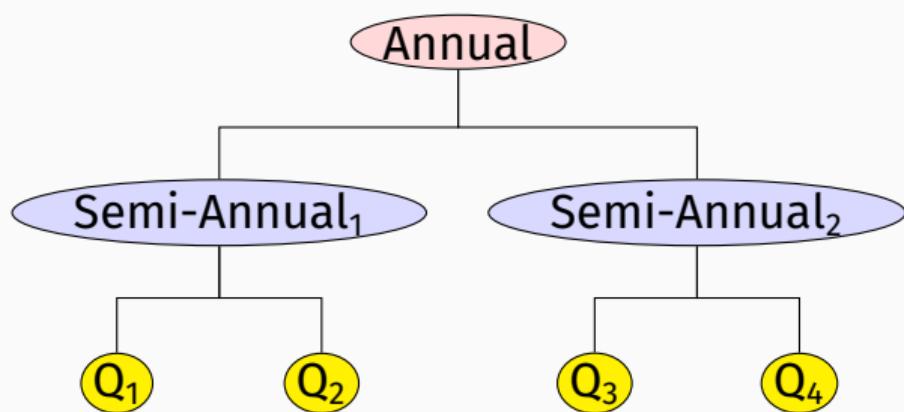


$$\mathbf{y}_\tau = \begin{bmatrix} x_\tau^{[4]} \\ x_{\tau,1}^{[1]} \\ x_{\tau,2}^{[1]} \\ x_{\tau,3}^{[1]} \\ x_{\tau,4}^{[1]} \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

τ = year

Temporal reconciliation: quarterly data

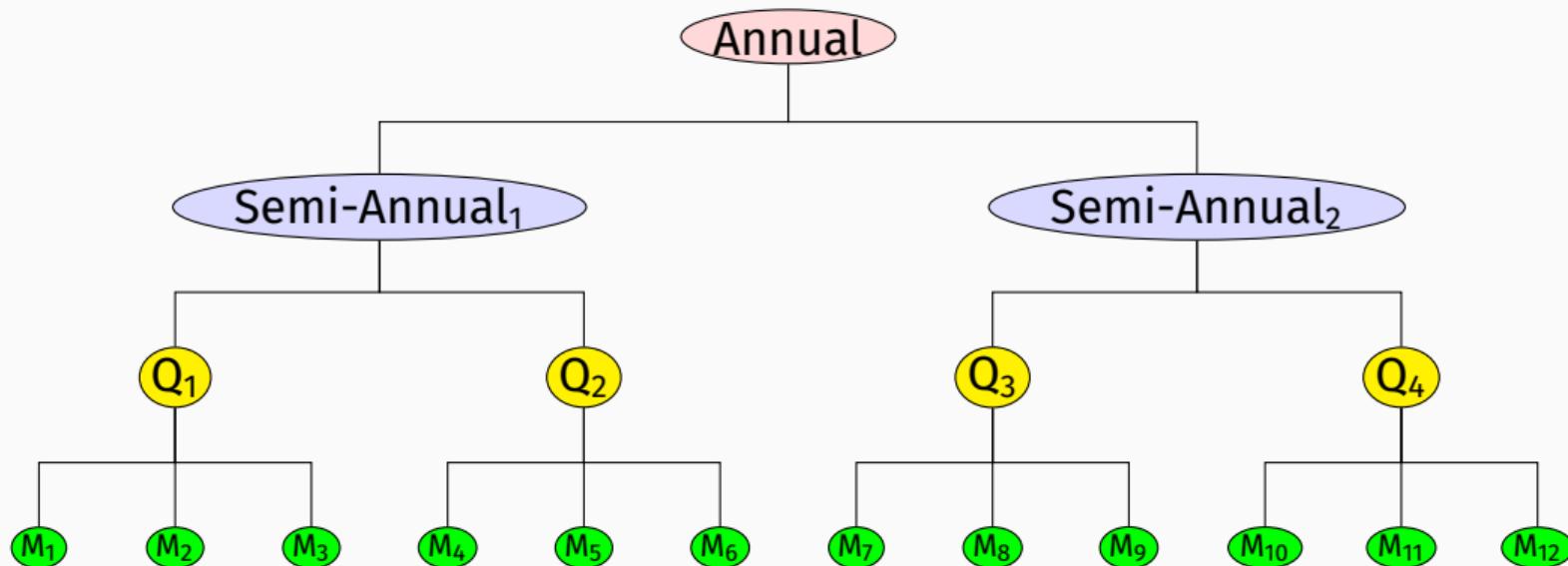


- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

$$\mathbf{y}_\tau = \begin{bmatrix} x_\tau^{[4]} \\ x_{\tau,1}^{[2]} \\ x_{\tau,2}^{[2]} \\ x_{\tau,1}^{[1]} \\ x_{\tau,2}^{[1]} \\ x_{\tau,3}^{[1]} \\ x_{\tau,4}^{[1]} \end{bmatrix}$$
$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

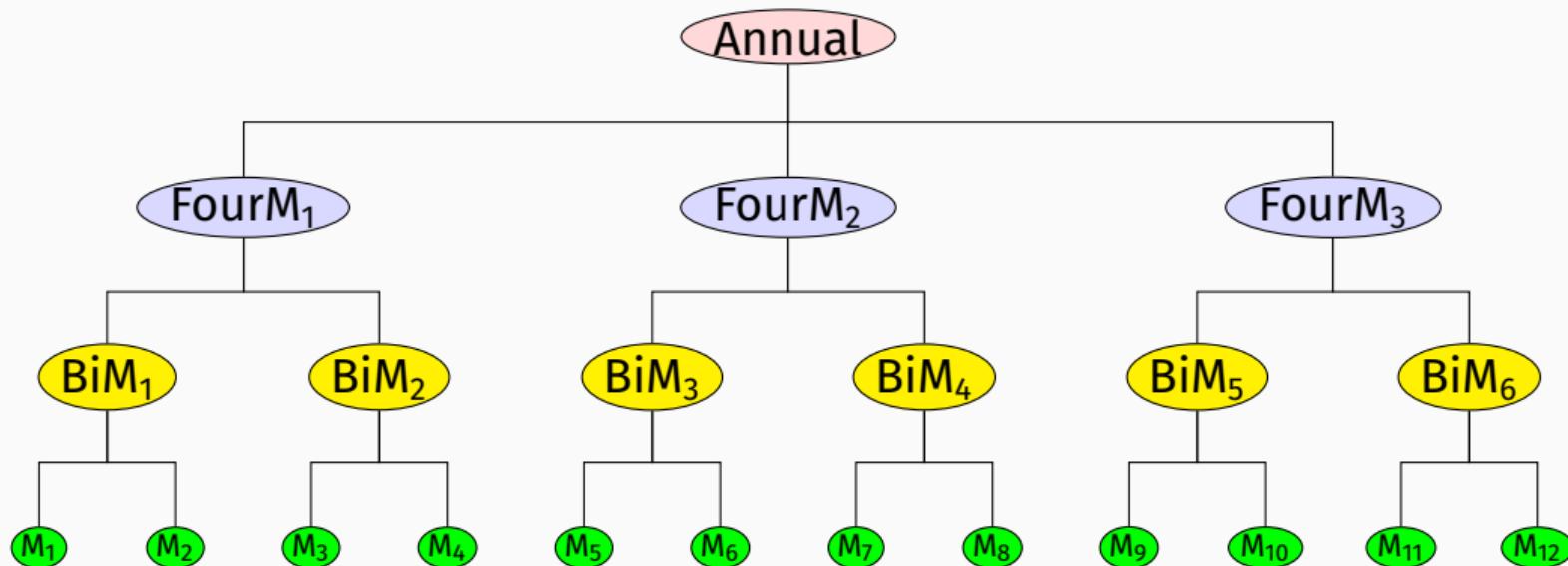
$\tau = \text{year}$

Temporal reconciliation: monthly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

Temporal reconciliation: monthly data



- Forecast series at each available frequency.
- Optimally combine forecasts within the same year.

Temporal reconciliation: monthly data

$$\mathbf{y}_\tau = \begin{bmatrix} \mathbf{x}_\tau^{[12]} \\ \mathbf{x}_\tau^{[6]} \\ \mathbf{x}_\tau^{[4]} \\ \mathbf{x}_\tau^{[3]} \\ \mathbf{x}_\tau^{[2]} \\ \mathbf{x}_\tau^{[1]} \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \mathbf{I}_{12}$$

Temporal reconciliation

For a time series y_1, \dots, y_T , observed at frequency m :

$$x_j^{[k]} = \sum_{t=(j-1)k+1}^{jk} y_t \quad \text{for } j = 1, \dots, [T/k]$$

- $k \in \mathcal{K} = \{k_1, \dots, k_p\}$ denote the p factors of m in ascending order, where $k_1 = 1$ and $k_p = m$
- $x_j^{[1]} = y_t$
- A single unique hierarchy is only possible when there are no coprime pairs in \mathcal{K} .
- $M_k = m/k$ is seasonal period of aggregated series.

Temporal reconciliation

$$\mathbf{x}_\tau = \mathbf{S} \mathbf{x}_\tau^{[1]}, \quad \mathbf{S} = \begin{bmatrix} \mathbf{A} \\ \mathbf{I} \end{bmatrix}$$

where

$$\mathbf{x}_\tau = \begin{bmatrix} \mathbf{x}_\tau^{[k_p]} \\ \mathbf{x}_\tau^{[k_{p-1}]} \\ \vdots \\ \mathbf{x}_\tau^{[k_1]} \end{bmatrix} \quad \mathbf{x}_\tau^{[k]} = \begin{bmatrix} \mathbf{x}_{M_k(\tau-1)+1}^{[k]} \\ \mathbf{x}_{M_k(\tau-1)+2}^{[k]} \\ \vdots \\ \mathbf{x}_{M_k\tau}^{[k]} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} & \mathbf{1}'_m \\ \mathbf{I}_{m/k_{p-1}} \otimes \mathbf{1}'_{k_{p-1}} \\ & \vdots \\ \mathbf{I}_{m/k_2} \otimes \mathbf{1}'_{k_2} \end{bmatrix}$$

τ is time index for most aggregated series,

$$k \in \mathcal{K} = \{k_1, \dots, k_p\}, \quad k_1 = 1, \quad k_p = m, \quad \tau = 1, \dots, T/m.$$

Example: Accident & emergency services demand

Weekly A&E demand data: 7 November 2010 to 7 June 2015.

Type 1 Departments – Major A&E

Type 2 Departments – Single Specialty

Type 3 Departments – Other A&E/Minor Injury Unit

Total Attendances

Type 1 Departments – Major A&E > 2 hours

Type 2 Departments – Single Specialty > 2 hours

Type 3 Departments – Other A&E/Minor Injury Unit > 2 hours

Total Attendances > 2 hours

Emergency Admissions via Type 1 A&E

Total Emergency Admissions via A&E

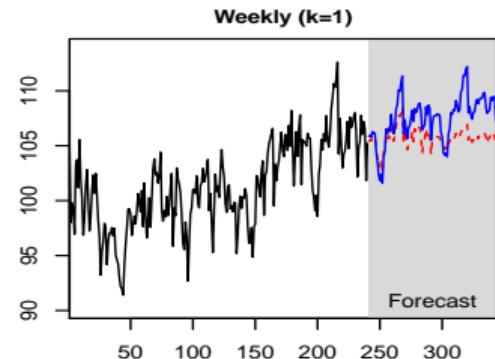
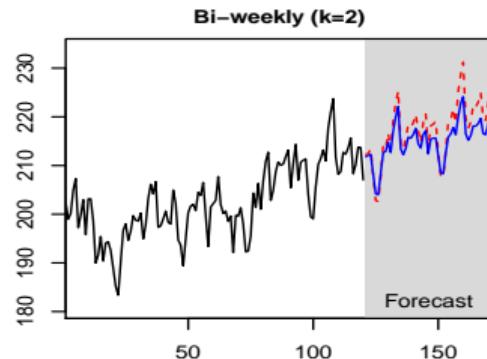
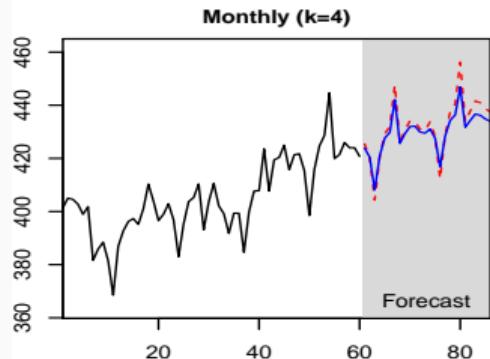
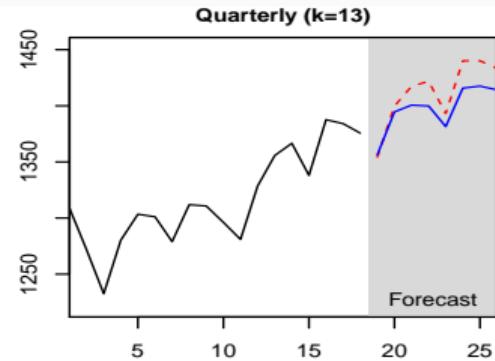
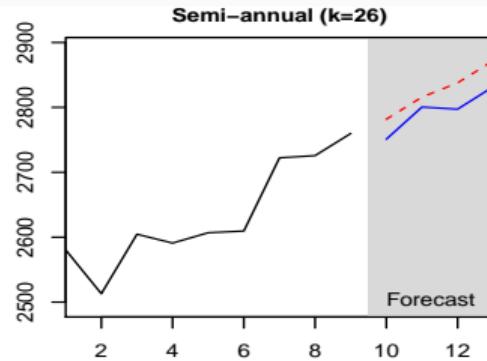
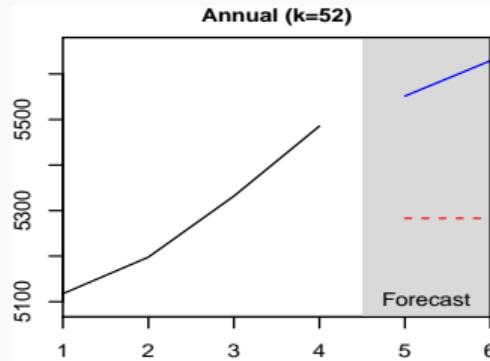
Other Emergency Admissions (i.e not via A&E)

Total Emergency Admissions

Number of patients spending > 2 hours from decision to admit to admission

Example: Accident & emergency services demand

Total emergency admissions via A&E



Example: Accident & emergency services demand

Test set: last 52 weeks

MASE comparison (ARIMA models)

Aggregation Level	h	Base	Reconciled	Change
Annual	1	3.4	1.9	-42.9%
Weekly	1-52	2.0	1.9	-5.0%
Weekly	13	2.3	1.9	-16.2%
Weekly	4	1.9	1.5	-18.6%
Weekly	1	1.6	1.3	-17.2%

Temporal reconciliation: M3 monthly series

- Apply temporal reconciliation to all 1428 monthly series from M3 competition
- Forecast horizon $h = 18$ months
- ETS and ARIMA models
- Measure percentage difference in MASE to base forecasts
- Reconciliation methods:
 - ▶ WLS_H (diagonal)
 - ▶ WLS_V (diagonal with common variances for same frequency)
 - ▶ WLS_S (diagonal/structural)

Temporal reconciliation: M3 monthly series

Improvement in MASE relative to base forecasts

Aggregation level	h	ETS				ARIMA			
		BU	WLS _H	WLS _V	WLS _S	BU	WLS _H	WLS _V	WLS _S
Annual	1	-12.1	-17.9	-17.8	-18.5	-25.4	-29.9	-29.9	-30.2
Semi-annual	3	0.0	-6.3	-6.0	-6.9	-2.9	-8.1	-8.2	-9.4
Four-monthly	4	3.1	-3.2	-3.0	-3.4	-1.8	-6.2	-6.5	-7.1
Quarterly	6	3.2	-2.8	-2.7	-3.4	-2.6	-6.9	-7.4	-8.1
Bi-monthly	9	2.7	-2.9	-3.0	-3.7	-1.3	-5.0	-5.5	-6.3
Monthly	18	0.0	-3.7	-4.6	-5.0	0.0	-1.9	-3.2	-3.7
Average		-0.5	-6.1	-6.2	-6.8	-5.7	-9.7	-10.1	-10.8

Outline

- 1 Improving hierarchical forecasts
- 2 Improving univariate forecasts
- 3 Improving multivariate forecasts
- 4 Final comments

Forecast Linear Augmented Projection (FLAP)

- We want to forecast multivariate series \mathbf{y}_t .
- Linear combinations of \mathbf{y}_t may have better signal-noise ratio
- Construct many linear combinations $\mathbf{c}_t = \Phi \mathbf{y}_t$ (e.g., principal components or random combinations)
- Produce univariate forecasts of all series $\hat{\mathbf{y}}_{t+h|t}$ and all linear combinations $\hat{\mathbf{c}}_{t+h|t}$.
- Reconcile forecasts so they are coherent ($\tilde{\mathbf{c}}_{t+h|t} = \Phi \tilde{\mathbf{y}}_{t+h|t}$)

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$$\mathbf{z}_t = \begin{bmatrix} \mathbf{y}_t \\ \mathbf{c}_t \end{bmatrix} \quad \tilde{\mathbf{z}}_{t+h} = \mathbf{M} \hat{\mathbf{z}}_{t+h|t}$$

Forecast error variance reduction

If we know the covariance matrix $\mathbf{W}_h = \text{Var}(\mathbf{z}_{t+h} - \hat{\mathbf{z}}_{t+h|\mathbf{t}})$, then

- 1 The forecast error variance is reduced:
 - ▶ $\text{Var}(\mathbf{y}_{t+h} - \hat{\mathbf{y}}_{t+h|\mathbf{t}}) - \text{Var}(\mathbf{y}_{t+h} - \tilde{\mathbf{y}}_{t+h|\mathbf{t}})$ is positive semi-definite.
- 2 The forecast error variance of each series monotonically decreases with increasing number of components.

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In practice, we need to:

- Estimate \mathbf{W}_h (use the MinT shrinkage estimator).
- Construct the components, Φ .

Construction of Φ

Principal component analysis (PCA)

Finding the weights matrix Φ so that the resulting components
maximise variance

Simulation

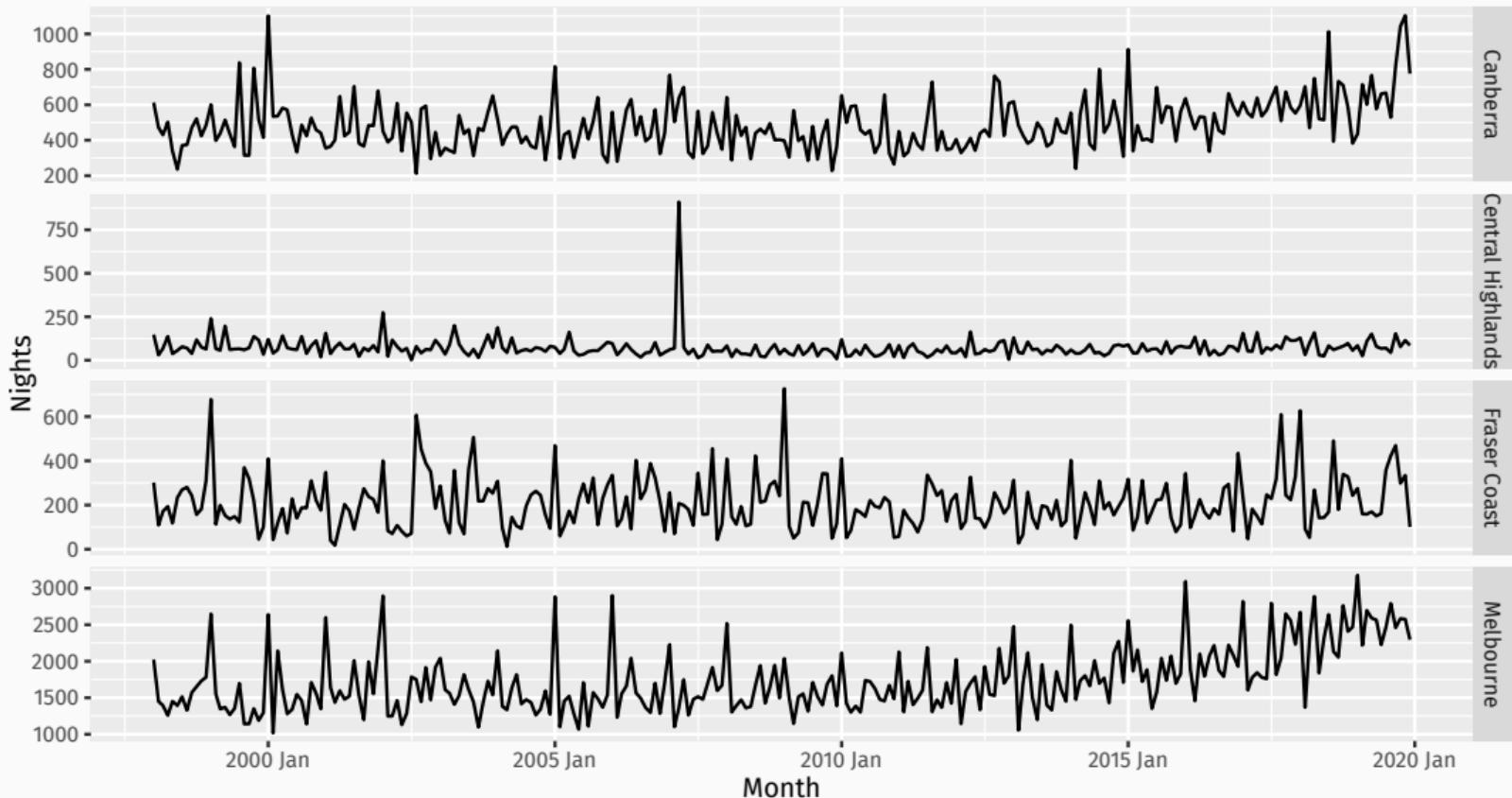
Generating values of Φ from a random distribution and
normalising them to unit vectors

- Normal distribution
- Uniform distribution
- Orthonormal matrix

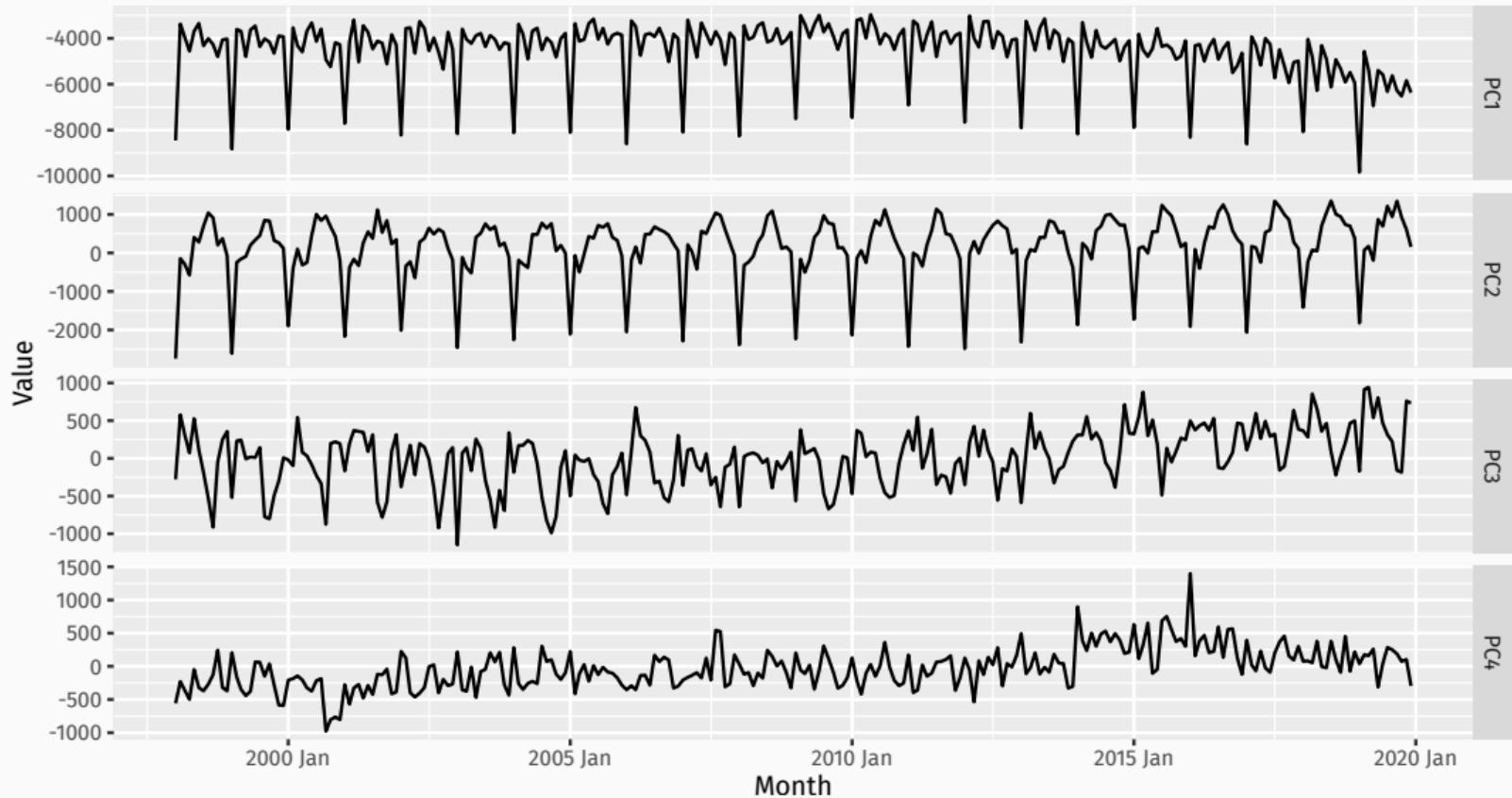
Monthly Australian regional tourism

- Monthly Australian tourism data set aggregated by region giving 77 series, from Jan 1998 to Dec 2019.
- Use expanding window time series cross-validation with $T = 84$ observations in first training set, and forecast horizons $h = 1, 2, \dots, 12$.
- Fit univariate ETS models to each series.

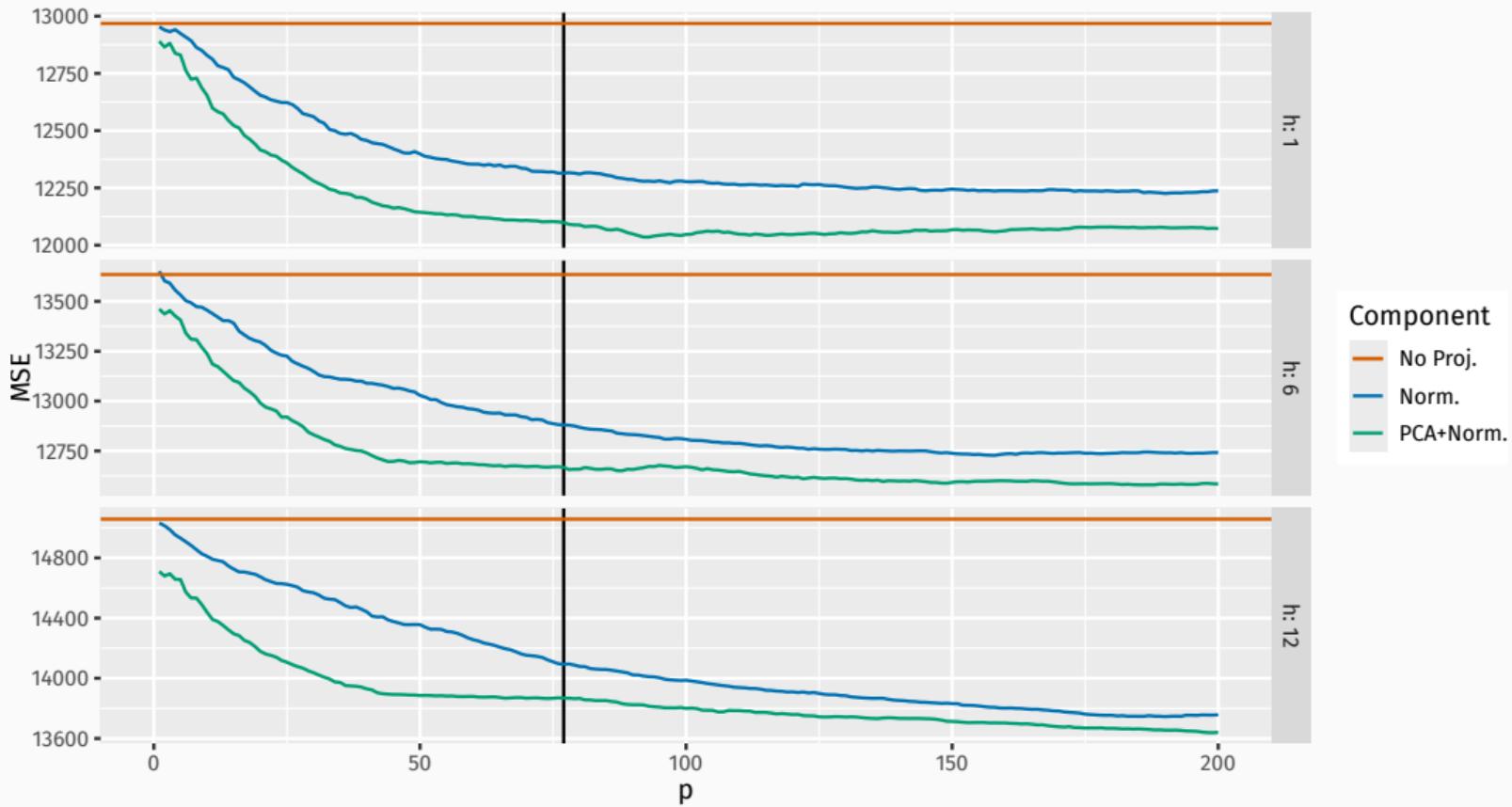
Monthly Australian regional tourism



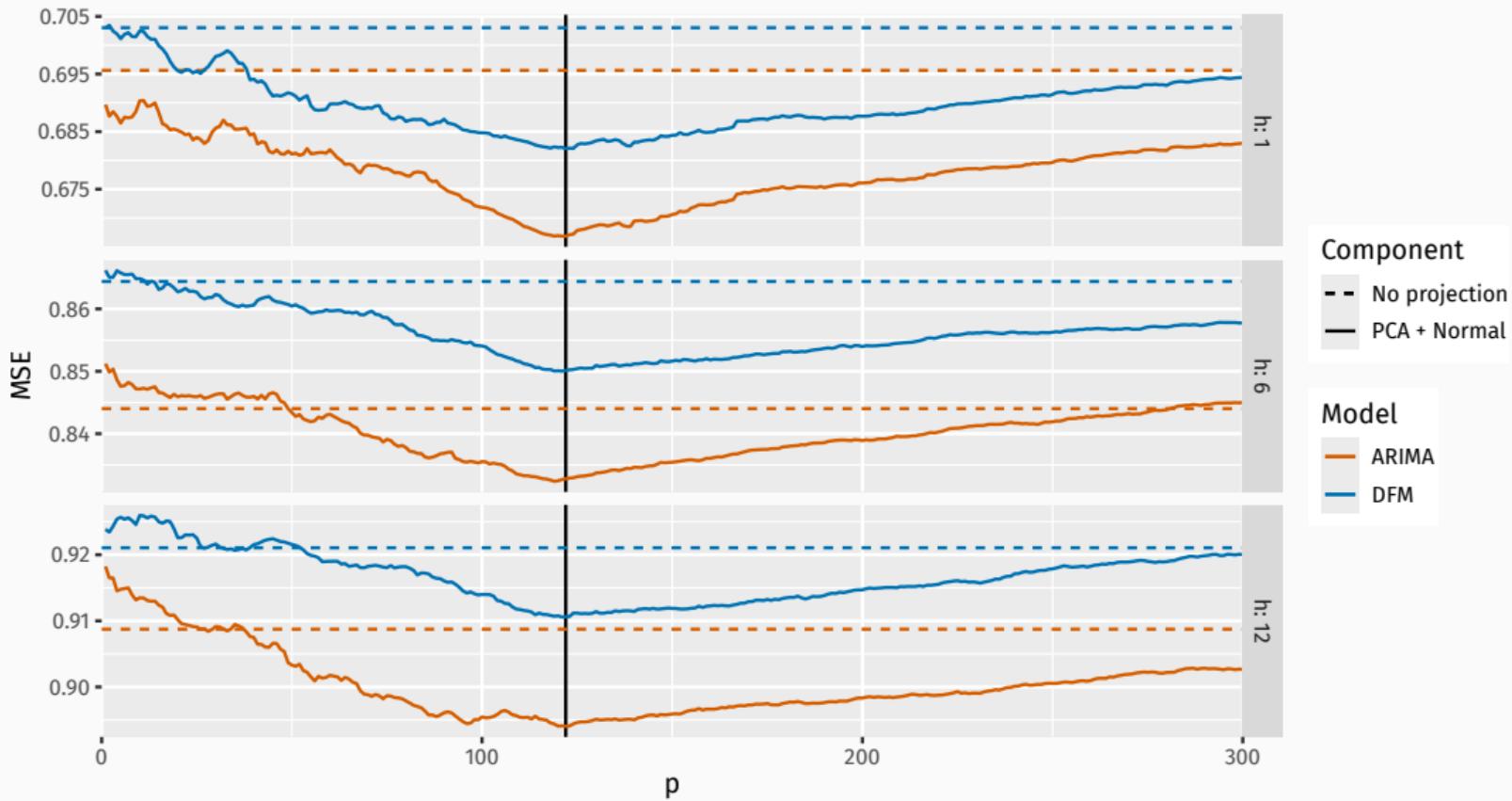
Monthly Australian regional tourism



Monthly Australian regional tourism



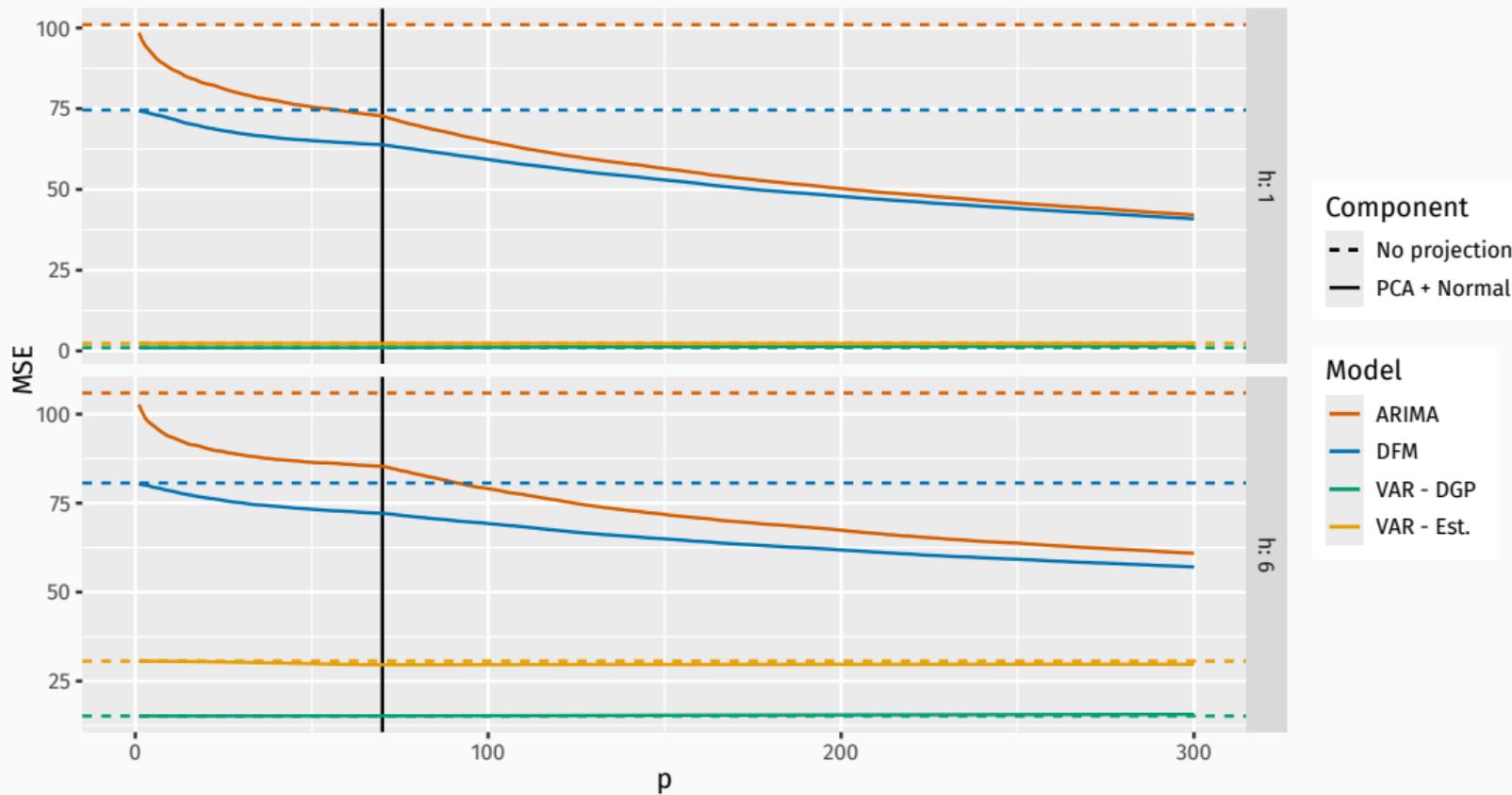
- Monthly data of macroeconomic variables (McCracken and Ng, 2016).
- Data from Jan 1959 – Sep 2023. 777 observations on 122 series.
- Same cleaning process as per McCracken and Ng (2016).
- All series scaled to have mean 0 and variance 1.
- Expanding time series cross-validation with initial size of 25 years and forecast horizon 12 months.



Simulation

- Data generating process: VAR(3) with 70 variables
- Sample size: $T = 400$
- Number of repeated samples: 220
- Base models:
 - ▶ automatic ARIMA (based on AICc)
 - ▶ DFM (structure chosen using BIC, different model for each horizon)

Simulation



Outline

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Software

Package	Language	Cross-sectional	Temporal	Cross-temporal	Probabilistic	Multivariate
hts	R	✓				
thief	R		✓			
fable	R	✓			✓	
FoReco	R	✓	✓	✓	✓	
flap	R					✓
pyhts	Python	✓	✓			
hierarchicalforecast	Python	✓			✓	

- hts, thief, and FoReco use ts objects
- fable uses tsibble objects
- flap uses matrices of base forecasts
- fable has plans to implement temporal and cross-temporal reconciliation

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Thanks!



More information

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-  Athanasopoulos, G, RJ Hyndman, N Kourentzes & A Panagiotelis (2024). Forecast reconciliation: a review. *International J Forecasting* **40**(2), 430–456.
-  Athanasopoulos, G, RJ Hyndman, N Kourentzes & F Petropoulos (2017). Forecasting with temporal hierarchies. *European J Operational Research* **262**(1), 60–74.
-  Di Fonzo, T & D Girolimetto (2023). Cross-temporal forecast reconciliation: Optimal combination method and heuristic alternatives. *International J Forecasting* **39**(1), 39–57.
-  Girolimetto, D, G Athanasopoulos, T Di Fonzo & RJ Hyndman (2024). Cross-temporal probabilistic forecast reconciliation: methodological and practical issues. *International J Forecasting* **40**(3), 1134–1151.
-  Hyndman, RJ, RA Ahmed, G Athanasopoulos & HL Shang (2011). Optimal combination forecasts for hierarchical time series. *Computational Statistics & Data Analysis* **55**(9), 2579–2589.

-  Panagiotelis, A, P Gamakumara, G Athanasopoulos & RJ Hyndman (2021). Forecast reconciliation: A geometric view with new insights on bias correction. *International J Forecasting* **37**(1), 343–359.
-  Panagiotelis, A, P Gamakumara, G Athanasopoulos & RJ Hyndman (2023). Probabilistic forecast reconciliation: properties, evaluation and score optimisation. *European J Operational Research* **306**(2), 693–706.
-  Wickramasuriya, SL (2021). Properties of point forecast reconciliation approaches. *arXiv:2103.11129*.
-  Wickramasuriya, SL, G Athanasopoulos & RJ Hyndman (2019). Optimal forecast reconciliation for hierarchical and grouped time series through trace minimization. *J American Statistical Association* **114**(526), 804–819.
-  Yang, YF, G Athanasopoulos, RJ Hyndman & A Panagiotelis (2024). Forecast Linear Augmented Projection (FLAP): A free lunch to reduce forecast error variance. *arXiv:2407.01868*.