Feasts and fables: Time series analysis using R

tsibble

feasts

Fable

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Outline

Outline

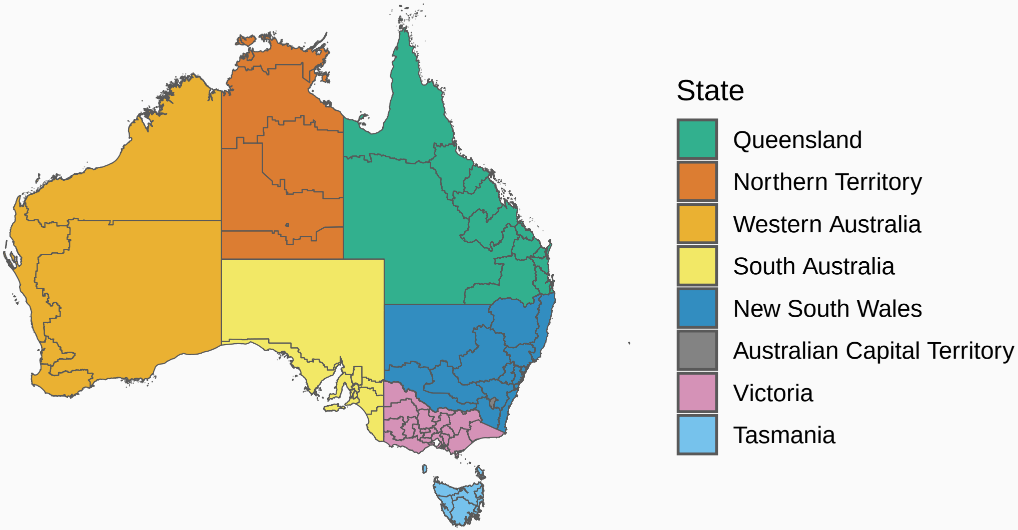
Tidyverts packages

tidyverts.org

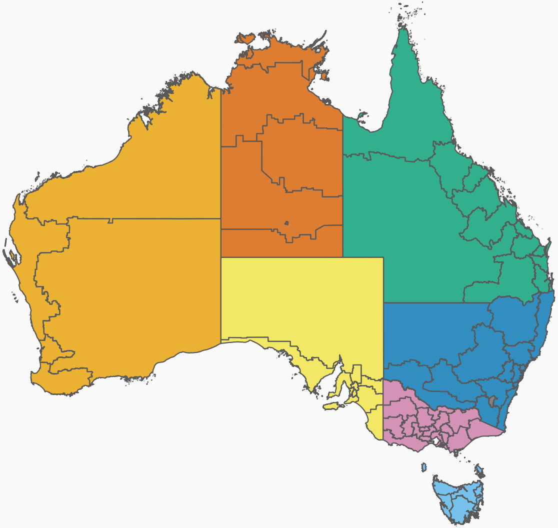
```r
library(fpp3)
```

```
-- Attaching packages -------------- fpp3 0.5.0 --
✓ tibble      3.1.8      ✓ tsibble     1.1.2
✓ dplyr       1.1.0      ✓ tsibbledata 0.4.1
✓ tidyr       1.3.0      ✓ feasts      0.3.0
✓ lubridate   1.9.2      ✓ fable       0.3.2
✓ ggplot2     3.4.1      ✓ fabletools  0.3.2
-- Conflicts ------------------ fpp3_conflicts --
X lubridate::date()     masks base::date()
X dplyr::filter()       masks stats::filter()
X tsibble::intersect() masks base::intersect()
X tsibble::interval()  masks lubridate::interval()
X dplyr::lag()          masks stats::lag()
X tsibble::setdiff()   masks base::setdiff()
X tsibble::union()     masks base::union()
```

Australian tourism regions



Australian tourism regions



- Quarterly data on visitor nights: 1998 – 2017
- From *National Visitor Survey*, interviews of 120,000 Australians aged 15+.
- Geographical hierarchy split by
  - 8 states and territories
  - 76 regions
- Purpose:
  - Holidays
  - Business
  - Visiting friends & relatives
  - Other

tsibble objects

```r
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      <qtr> <chr>    <fct> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

`tsibble` objects

```r
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##            <chr>    <fct> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Index

Domestic visitor nights in thousands by state/region and purpose.

`tsibble` objects

```r
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##    Index   Keys                    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Index

Keys

Measure

Domestic visitor nights in thousands by state/region and purpose.

`tsibble` objects

- A `tsibble` allows storage and manipulation of multiple time series in R.
- It contains:
  - An index: time information about the observation
  - Measured variable(s): numbers of interest
  - Key variable(s): optional unique identifiers for each series
- It works with tidyverse functions.

Outline

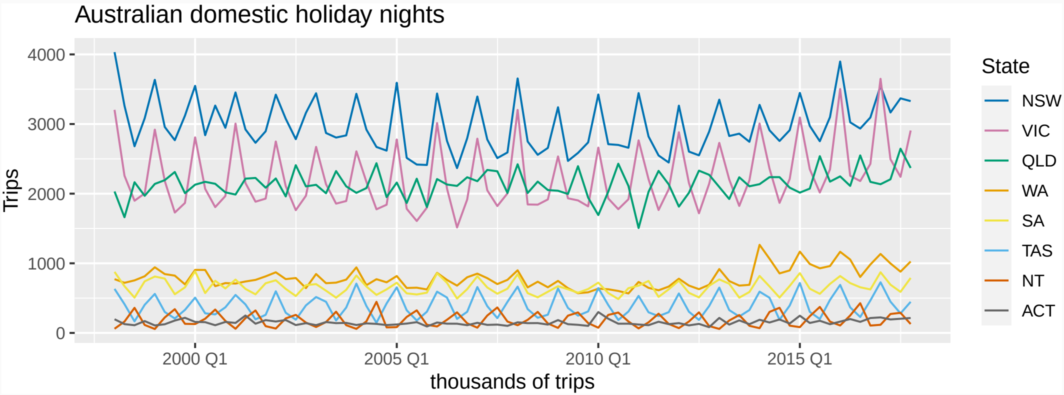
Australian holidays

```r
holidays <- tourism |>
  filter(Purpose == "Holiday") |>
  group_by(State) |>
  summarise(Trips = sum(Trips))
```

```
## # A tsibble: 640 x 3 [1Q]
## # Key:       State [8]
##    State Quarter Trips
##    <fct>   <qtr> <dbl>
##  1 NSW   1998 Q1 4033.
##  2 NSW   1998 Q2 3262.
##  3 NSW   1998 Q3 2681.
##  4 NSW   1998 Q4 3083.
##  5 NSW   1999 Q1 3635.
##  6 NSW   1999 Q2 2958.
##  7 NSW   1999 Q3 2768.
##  8 NSW   1999 Q4 3121.
##  9 NSW   2000 Q1 3548.
```

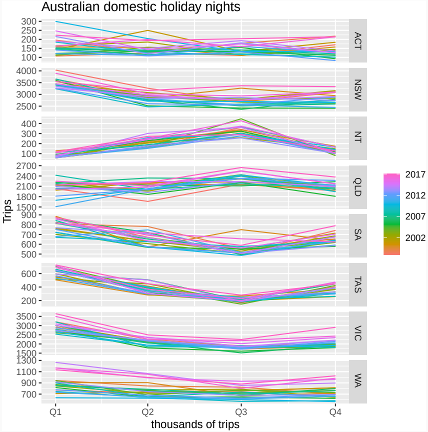
Australian holidays

```r
holidays |> autoplot(Trips) +
  labs(x = "thousands of trips", title = "Australian domestic holiday nights")
```



Seasonal plots

```r
holidays |> gg_season(Trips) +
  labs(x = "thousands of trips", title = "Australian domestic holiday nights")
```



Seasonal subseries plots

```r
holidays |> gg_subseries(Trips) +
  labs(x = "thousands of trips", title = "Australian domestic holiday nights")
```

Outline

Strength of seasonality and trend

STL decomposition

$$y_t = T_t + S_t + R_t$$

Seasonal strength

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right)$$

Trend strength

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(T_t + R_t)}\right)$$

Feature extraction and statistics

```r
tourism |> features(Trips, feat_stl)
```

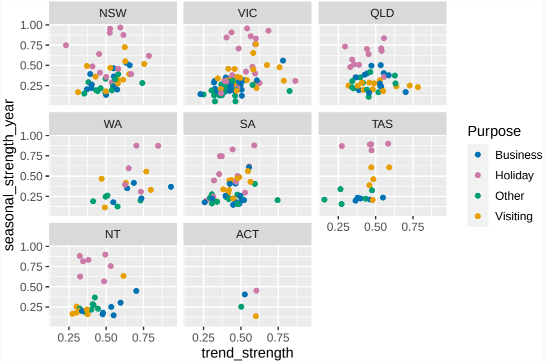
```
## # A tibble: 304 x 12
##    Region  State Purpose trend~1 seaso~2 seaso~3 seaso~4 spiki~5 linea~6 curva~7
##    <chr>   <fct> <chr>     <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
##  1 Adelai~ SA    Busine~   0.464   0.407       3       1 1.58e+2  -5.31   71.6
##  2 Adelai~ SA    Holiday   0.554   0.619       1       2 9.17e+0  49.0    78.7
##  3 Adelai~ SA    Other     0.746   0.202       2       1 2.10e+0  95.1    43.4
##  4 Adelai~ SA    Visiti~   0.435   0.452       1       3 5.61e+1  34.6    71.4
##  5 Adelai~ SA    Busine~   0.464   0.179       3       0 1.03e-1   0.968  -3.22
##  6 Adelai~ SA    Holiday   0.528   0.296       2       1 1.77e-1  10.5    24.0
##  7 Adelai~ SA    Other     0.593   0.404       2       2 4.44e-4   4.28    3.19
##  8 Adelai~ SA    Visiti~   0.488   0.254       0       3 6.50e+0  34.2    -0.529
##  9 Alice ~ NT    Busine~   0.534   0.251       0       1 1.69e-1  23.8    19.5
## 10 Alice ~ NT    Holiday   0.381   0.832       3       1 7.39e-1 -19.6    10.5
## # ... with 294 more rows, 2 more variables: stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, and abbreviated variable names 1: trend_strength,
## #   2: seasonal_strength_year, 3: seasonal_peak_year, 4: seasonal_trough_year,
## #   5: spikiness, 6: linearity, 7: curvature
```

Feature extraction and statistics

```
tourism |>
  features(Trips, feat_stl) |>
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```

Feature extraction and statistics

```
tourism |>
  features(Trips, feat_stl) |>
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```



- Holidays more seasonal than other travel.
- WA has strongest trends.

Feature extraction and statistics

Find the most seasonal time series:

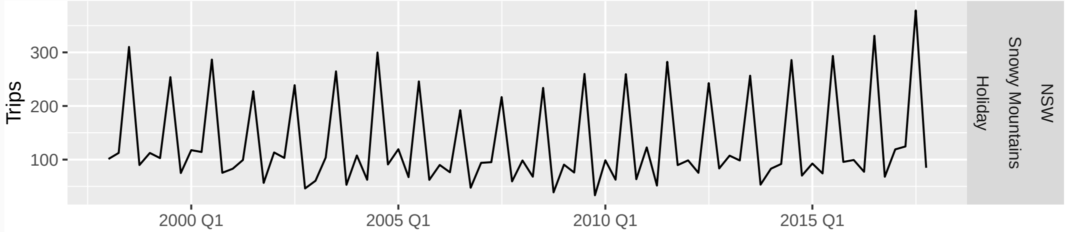
```
most_seasonal <- tourism |>
  features(Trips, feat_stl) |>
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism |>
  features(Trips, feat_stl) |>
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

```
tourism |>
  right_join(most_seasonal, by = c("State", "Region", "Purpose")) |>
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



Feature extraction and statistics

```r
tourism_features <- tourism |>
  features(Trips, feature_set(pkgs = "feasts"))
```

All features from the feasts package

```
## # A tibble: 304 x 51
##    Region  State Purpose trend~1 seaso~2 seaso~3 seaso~4 spiki~5 linea~6 curva~7
##    <chr>   <fct> <chr>     <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
##  1 Adelai~ SA    Busine~   0.464   0.407       3       1 1.58e+2  -5.31   71.6
##  2 Adelai~ SA    Holiday   0.554   0.619       1       2 9.17e+0  49.0    78.7
##  3 Adelai~ SA    Other     0.746   0.202       2       1 2.10e+0  95.1    43.4
##  4 Adelai~ SA    Visiti~   0.435   0.452       1       3 5.61e+1  34.6    71.4
##  5 Adelai~ SA    Busine~   0.464   0.179       3       0 1.03e-1   0.968  -3.22
##  6 Adelai~ SA    Holiday   0.528   0.296       2       1 1.77e-1  10.5    24.0
##  7 Adelai~ SA    Other     0.593   0.404       2       2 4.44e-4   4.28    3.19
##  8 Adelai~ SA    Visiti~   0.488   0.254       0       3 6.50e+0  34.2    -0.529
##  9 Alice ~ NT    Busine~   0.534   0.251       0       1 1.69e-1  23.8    19.5
## 10 Alice ~ NT    Holiday   0.381   0.832       3       1 7.39e-1 -19.6    10.5
## # ... with 294 more rows, 41 more variables: stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>,
## #   diff1_acf10 <dbl>, diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>,
## #   pacf5 <dbl>, diff1_pacf5 <dbl>, diff2_pacf5 <dbl>, season_pacf <dbl>,
## #   zero_run_mean <dbl>, nonzero_squared_cv <dbl>, zero_start_prop <dbl>,
## #   zero_end_prop <dbl>, lambda_guerrero <dbl>, kpss_stat <dbl>,
```

Feature extraction and statistics

```r
pcs <- tourism_features |>
  select(-State, -Region, -Purpose) |>
  prcomp(scale = TRUE) |>
  broom::augment(tourism_features)
```

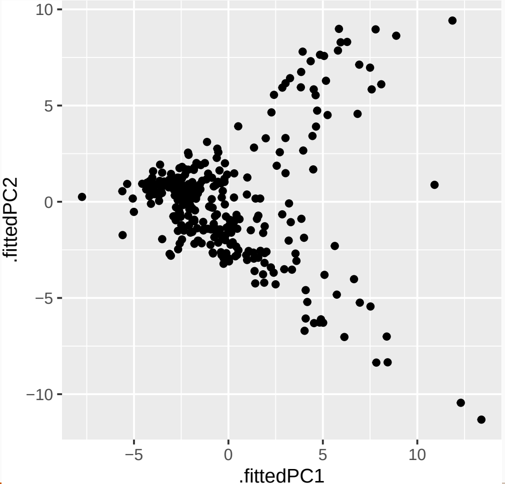
Principal components based on all features from the feasts package

```
## # A tibble: 304 x 100
##    .rowna~1 Region State Purpose trend~2 seaso~3 seaso~4 seaso~5 spiki~6 linea~7
##    <chr>    <chr>  <fct> <chr>     <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
##  1 1        Adela~ SA    Busine~   0.464   0.407       3       1 1.58e+2  -5.31
##  2 2        Adela~ SA    Holiday   0.554   0.619       1       2 9.17e+0  49.0
##  3 3        Adela~ SA    Other     0.746   0.202       2       1 2.10e+0  95.1
##  4 4        Adela~ SA    Visiti~   0.435   0.452       1       3 5.61e+1  34.6
##  5 5        Adela~ SA    Busine~   0.464   0.179       3       0 1.03e-1   0.968
##  6 6        Adela~ SA    Holiday   0.528   0.296       2       1 1.77e-1  10.5
##  7 7        Adela~ SA    Other     0.593   0.404       2       2 4.44e-4   4.28
##  8 8        Adela~ SA    Visiti~   0.488   0.254       0       3 6.50e+0  34.2
##  9 9        Alice~ NT    Busine~   0.534   0.251       0       1 1.69e-1  23.8
## 10 10       Alice~ NT    Holiday   0.381   0.832       3       1 7.39e-1 -19.6
## # ... with 294 more rows, 90 more variables: curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>,
## #   diff1_acf10 <dbl>, diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>,
## #   pacf5 <dbl>, diff1_pacf5 <dbl>, diff2_pacf5 <dbl>, season_pacf <dbl>,
```

Feature extraction and statistics

```
pcs |> ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +
  geom_point() + theme(aspect.ratio=1)
```

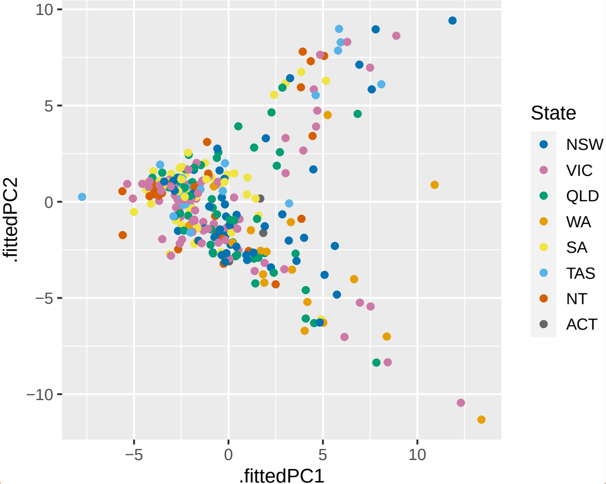
Principal components based on all features from the feasts package



Feature extraction and statistics

```r
pcs |> ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +
  geom_point() + theme(aspect.ratio=1)
```

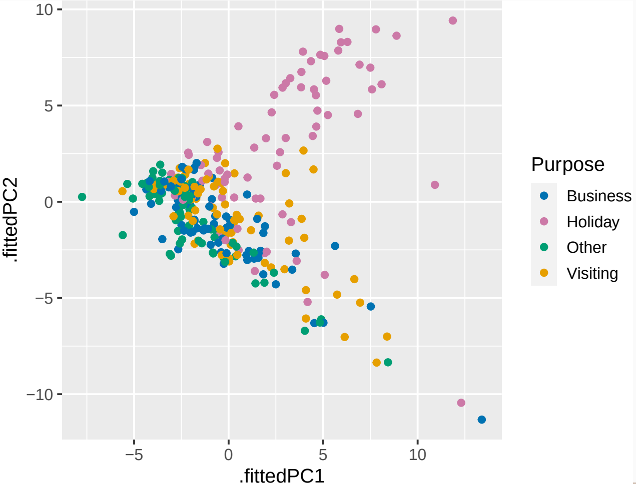
Principal components based on all features from the feasts package



Feature extraction and statistics

```r
pcs |> ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

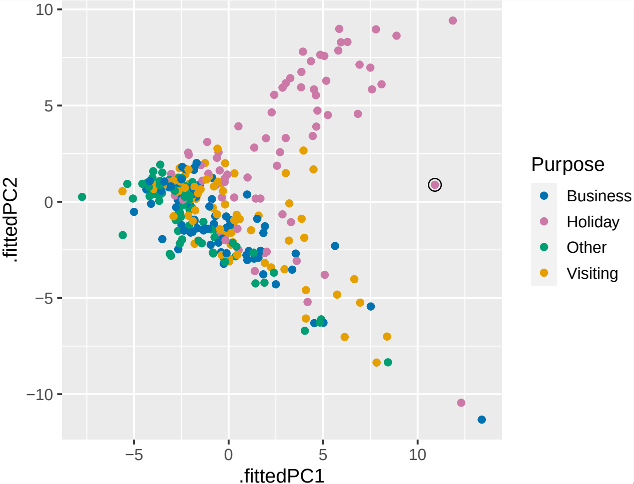
Principal components based on all features from the feasts package



Feature extraction and statistics

```r
pcs |> ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

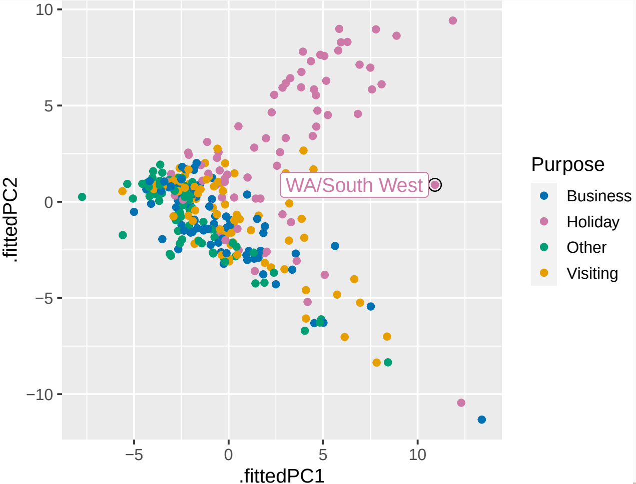
Principal components based on all features from the feasts package



Feature extraction and statistics

```r
pcs |> ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package



Outline

Model fitting

```
training <- tourism |>
  filter(year(Quarter) <= 2015)
fit <- training |>
  model(
    snaive = SNAIVE(Trips),
    naive = NAIVE(Trips),
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  )
```

```
## # A mable: 304 x 7
## # Key:     Region, State, Purpose [304]
##    Region     State Purpose   snaive    naive          ets                                  arima
##    <chr>      <fct> <chr>    <model>  <model>      <model>                                <model>
##  1 Adelaide   SA    Busine~ <SNAIVE> <NAIVE> <ETS(M,N,A)> <ARIMA(0,0,0)(1,0,1)[4] w/ mean>
##  2 Adelaide   SA    Holiday <SNAIVE> <NAIVE> <ETS(M,N,A)> <ARIMA(0,0,0)(2,0,0)[4] w/ mean>
##  3 Adelaide   SA    Other   <SNAIVE> <NAIVE> <ETS(M,A,N)>          <ARIMA(0,1,1) w/ drift>
##  4 Adelaide   SA    Visiti~ <SNAIVE> <NAIVE> <ETS(A,N,A)> <ARIMA(0,0,0)(1,0,1)[4] w/ mean>
##  5 Adelaide ~ SA    Busine~ <SNAIVE> <NAIVE> <ETS(A,N,N)>           <ARIMA(0,0,0) w/ mean>
##  6 Adelaide ~ SA    Holiday <SNAIVE> <NAIVE> <ETS(A,A,N)>           <ARIMA(0,0,0) w/ mean>
##  7 Adelaide ~ SA    Other   <SNAIVE> <NAIVE> <ETS(A,N,N)>         <ARIMA(2,1,1)(2,0,0)[4]>
```

Model fitting

```
fit |>
  filter(Purpose == "Holiday", Region == "Snowy Mountains") |>
  select(arima) |>
  report()
```

```
## Series: Trips
## Model: ARIMA(1,0,0)(0,1,2)[4]
##
## Coefficients:
##          ar1     sma1     sma2
##        0.223   -0.639   -0.288
## s.e.   0.121    0.253    0.163
##
## sigma^2 estimated as 461.3:  log likelihood=-307
## AIC=622   AICc=622   BIC=631
```

Model fitting

```r
augment(fit)
```

```
## # A tsibble: 87,552 x 9 [1Q]
## # Key:       Region, State, Purpose, .model [1,216]
##    Region   State Purpose  .model Quarter Trips .fitted .resid .innov
##    <chr>    <fct> <chr>    <chr>    <qtr> <dbl>   <dbl>  <dbl>  <dbl>
##  1 Adelaide SA    Business snaive 1998 Q1  135.      NA  NA     NA   
##  2 Adelaide SA    Business snaive 1998 Q2  110.      NA  NA     NA   
##  3 Adelaide SA    Business snaive 1998 Q3  166.      NA  NA     NA   
##  4 Adelaide SA    Business snaive 1998 Q4  127.      NA  NA     NA   
##  5 Adelaide SA    Business snaive 1999 Q1  137.     135.  2.37   2.37
##  6 Adelaide SA    Business snaive 1999 Q2  200.     110. 89.9   89.9 
##  7 Adelaide SA    Business snaive 1999 Q3  169.     166.  3.32   3.32
##  8 Adelaide SA    Business snaive 1999 Q4  134.     127.  7.20   7.20
##  9 Adelaide SA    Business snaive 2000 Q1  154.     137. 16.6   16.6 
## 10 Adelaide SA    Business snaive 2000 Q2  169.     200. -31.1  -31.1 
## # ... with 87,542 more rows
```

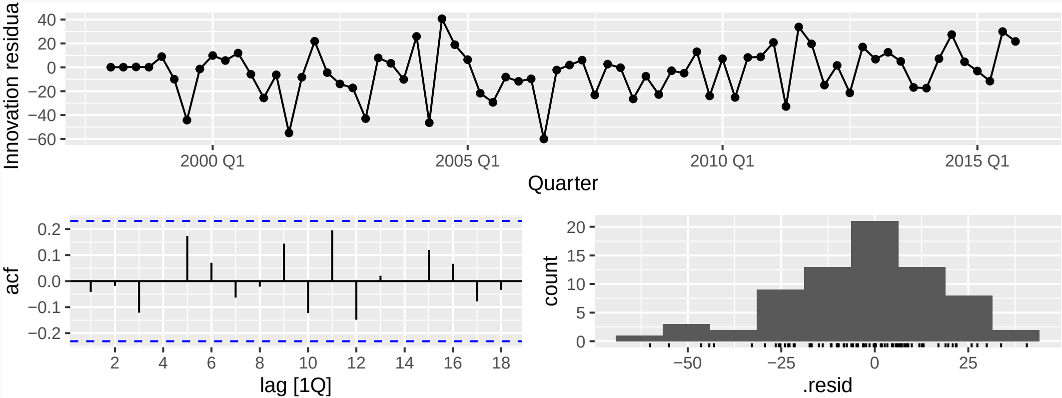
Ljung-Box test

```r
augment(fit) |>
  filter(Purpose == "Holiday", Region == "Snowy Mountains", .model == "arima") |>
  features(.resid, ljung_box, dof = 3, lag = 8)
```

```
## # A tibble: 1 x 6
##   Region          State Purpose .model lb_stat lb_pvalue
##   <chr>           <fct> <chr>   <chr>    <dbl>     <dbl>
## 1 Snowy Mountains NSW   Holiday arima     4.45     0.486
```

gg_tsresiduals() function

```r
fit |>
  filter(Purpose == "Holiday", Region == "Snowy Mountains") |>
  select(arima) |>
  gg_tsresiduals()
```



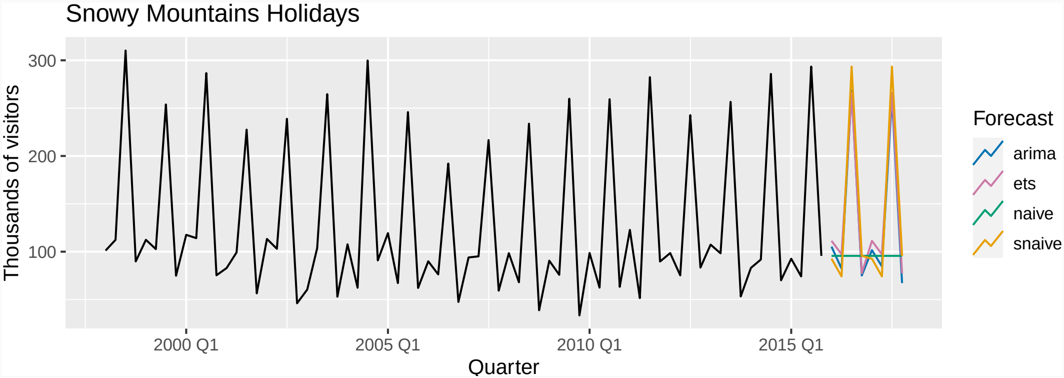
Producing forecasts

```r
fc <- fit |>
  forecast(h = "2 years")
```

```
## # A fable: 9,728 x 7 [1Q]
## # Key:     Region, State, Purpose, .model [1,216]
##    Region   State Purpose  .model Quarter         Trips .mean
##    <chr>    <fct> <chr>    <chr>    <qtr>        <dist> <dbl>
##  1 Adelaide SA    Business snaive 2016 Q1 N(143, 2128)  143.
##  2 Adelaide SA    Business snaive 2016 Q2 N(168, 2128)  168.
##  3 Adelaide SA    Business snaive 2016 Q3 N(176, 2128)  176.
##  4 Adelaide SA    Business snaive 2016 Q4 N(187, 2128)  187.
##  5 Adelaide SA    Business snaive 2017 Q1 N(143, 4257)  143.
##  6 Adelaide SA    Business snaive 2017 Q2 N(168, 4257)  168.
##  7 Adelaide SA    Business snaive 2017 Q3 N(176, 4257)  176.
##  8 Adelaide SA    Business snaive 2017 Q4 N(187, 4257)  187.
##  9 Adelaide SA    Business naive  2016 Q1 N(187, 2635)  187.
## 10 Adelaide SA    Business naive  2016 Q2 N(187, 5270)  187.
## # ... with 9,718 more rows
```

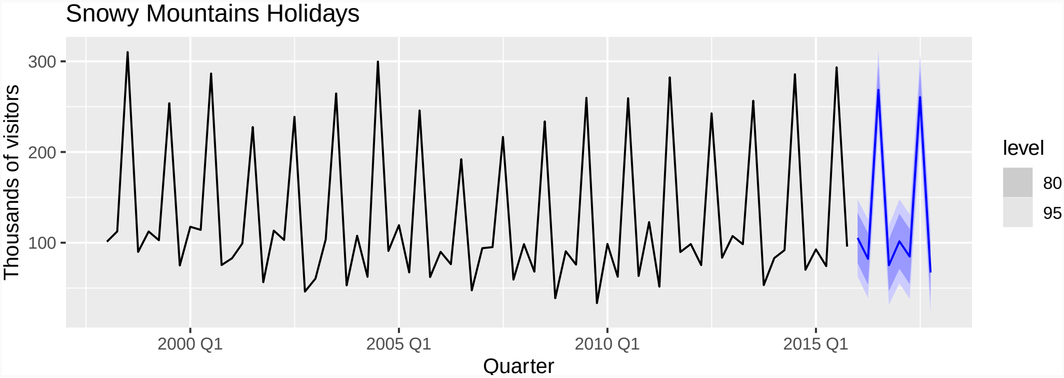
Visualising forecasts

```r
fc |>
  filter(Purpose == "Holiday", Region == "Snowy Mountains") |>
  autoplot(training, level = NULL) +
  labs(title = "Snowy Mountains Holidays", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



Visualising forecasts

```r
fc |>
  filter(Purpose == "Holiday", Region == "Snowy Mountains", .model == "arima") |>
  autoplot(training) +
  labs(title = "Snowy Mountains Holidays", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```

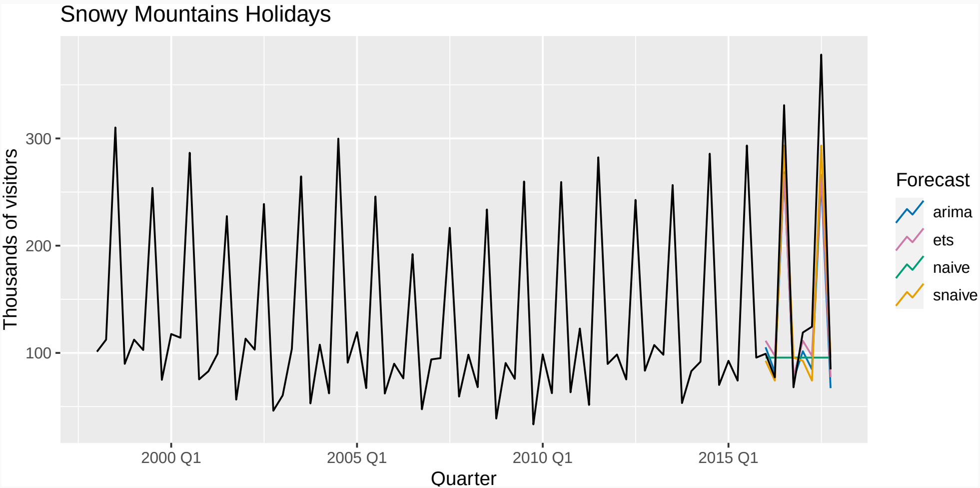


Prediction intervals

```
fc |> hilo(level = 95)
```

```
## # A tsibble: 9,728 x 8 [1Q]
## # Key:       Region, State, Purpose, .model [1,216]
##    Region   State Purpose  .model Quarter        Trips .mean           `95%`
##    <chr>    <fct> <chr>    <chr>    <qtr>       <dist> <dbl>          <hilo>
##  1 Adelaide SA    Business snaive 2016 Q1 N(143, 2128)  143. [52.4, 233]95
##  2 Adelaide SA    Business snaive 2016 Q2 N(168, 2128)  168. [77.5, 258]95
##  3 Adelaide SA    Business snaive 2016 Q3 N(176, 2128)  176. [86.0, 267]95
##  4 Adelaide SA    Business snaive 2016 Q4 N(187, 2128)  187. [96.3, 277]95
##  5 Adelaide SA    Business snaive 2017 Q1 N(143, 4257)  143. [14.9, 271]95
##  6 Adelaide SA    Business snaive 2017 Q2 N(168, 4257)  168. [40.0, 296]95
##  7 Adelaide SA    Business snaive 2017 Q3 N(176, 4257)  176. [48.6, 304]95
##  8 Adelaide SA    Business snaive 2017 Q4 N(187, 4257)  187. [58.9, 315]95
##  9 Adelaide SA    Business naive  2016 Q1 N(187, 2635)  187. [86.1, 287]95
## 10 Adelaide SA    Business naive  2016 Q2 N(187, 5270)  187. [44.4, 329]95
## # ... with 9,718 more rows
```

Measures of forecast accuracy



Measures of forecast accuracy

```r
fc |>
  accuracy(tourism)
```

```
## # A tibble: 1,216 x 13
##    .model Region State Purpose .type     ME  RMSE   MAE    MPE  MAPE  MASE RMSSE
##    <chr>  <chr>  <fct> <chr>   <chr>  <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl>
##  1 arima  Adela~ SA    Busine~ Test  20.8   29.0  27.0    10.5  15.0 0.850 0.628
##  2 arima  Adela~ SA    Holiday Test  21.7   31.1  25.5    10.6  13.1 1.17  1.15
##  3 arima  Adela~ SA    Other   Test   9.79  13.7  12.0    10.6  14.2 0.887 0.772
##  4 arima  Adela~ SA    Visiti~ Test  32.2   36.1  32.2    13.3  13.3 1.04  0.956
##  5 arima  Adela~ SA    Busine~ Test   0.634  4.65  3.13 -Inf   Inf   0.935 0.771
##  6 arima  Adela~ SA    Holiday Test   6.13   7.24  6.13   35.1  35.1 1.07  0.899
##  7 arima  Adela~ SA    Other   Test  -0.923  1.52  1.43 -192.  206.  1.12  0.783
##  8 arima  Adela~ SA    Visiti~ Test   5.67  12.0  10.3   -54.5 107.  1.36  0.903
##  9 arima  Alice~ NT    Busine~ Test   9.11  12.2  10.5    26.6  43.5 1.71  1.56
## 10 arima  Alice~ NT    Holiday Test  -0.536  9.66  8.54  -32.8  56.2 0.883 0.803
## # ... with 1,206 more rows, and 1 more variable: ACF1 <dbl>
```

Measures of forecast accuracy

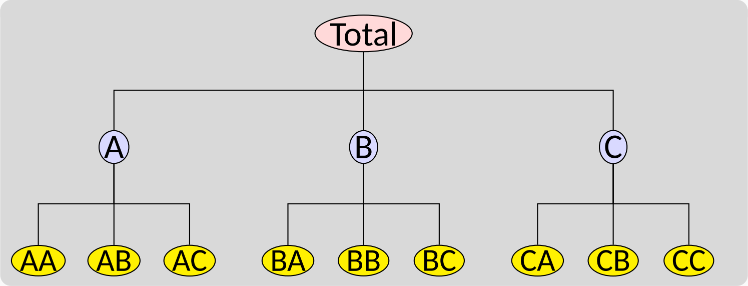
```
fc |>
  accuracy(tourism) |>
  group_by(.model) |>
  summarise(RMSSE = sqrt(mean(RMSSE^2)))
```

```
## # A tibble: 4 x 2
##   .model RMSSE
##   <chr>  <dbl>
## 1 arima  1.04
## 2 ets    0.996
## 3 naive  1.36
## 4 snaive 1.18
```

Outline

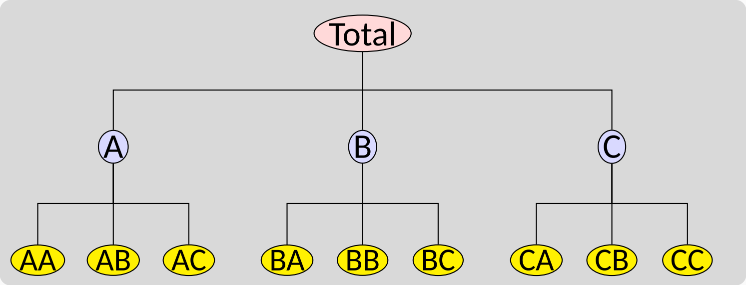
Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.

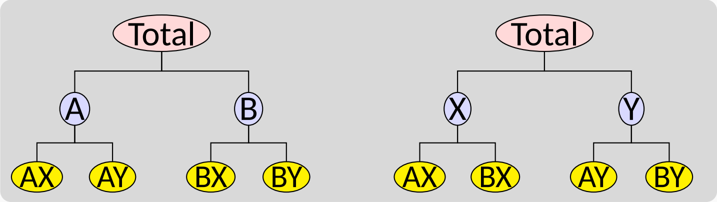


Examples

- Tourism demand by states, zones, regions

Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.



Grouped time series

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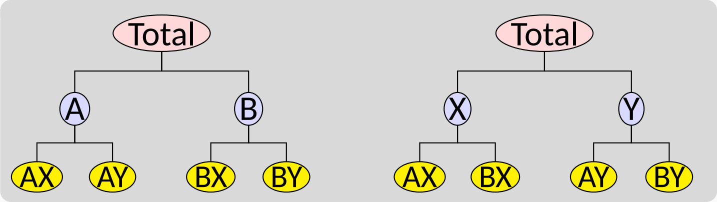


Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

Creating aggregates

```r
tourism |>
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) |>
  filter(Quarter == yearquarter("1998 Q1")) |>
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]
## # Key:       Purpose, State, Region [425]
##    Quarter Purpose      State        Region        Trips
##      <qtr> <chr*>       <fct*>       <chr*>        <dbl>
##  1 1998 Q1 <aggregated> <aggregated> <aggregated> 23182.
##  2 1998 Q1 Business     <aggregated> <aggregated>  3599.
##  3 1998 Q1 Holiday      <aggregated> <aggregated> 11806.
##  4 1998 Q1 Other        <aggregated> <aggregated>   680.
##  5 1998 Q1 Visiting     <aggregated> <aggregated>  7098.
##  6 1998 Q1 <aggregated> NSW          <aggregated>  8040.
##  7 1998 Q1 <aggregated> VIC          <aggregated>  6010.
##  8 1998 Q1 <aggregated> QLD          <aggregated>  4041.
##  9 1998 Q1 <aggregated> WA           <aggregated>  1641.
## 10 1998 Q1 <aggregated> SA           <aggregated>  1735.
## 11 1998 Q1 <aggregated> TAS          <aggregated>   982.
## 12 1998 Q1 <aggregated> NT           <aggregated>   181.
## 13 1998 Q1 <aggregated> ACT          <aggregated>   551.
```

Creating aggregates

- Similar to `summarise()` but using the key structure
- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```

- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

The problem

1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

The problem

1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

The solution

1. Forecast all series at all levels of aggregation using an automatic forecasting algorithm.
   (e.g., `ETS`, `ARIMA`, ...)
2. Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
3. This is available using `reconcile()`.

Forecast reconciliation

```r
tourism |>
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) |>
  model(ets = ETS(Trips)) |>
  reconcile(ets_adjusted = min_trace(ets)) |>
  forecast(h = "2 years")
```

```
## # A fable: 6,800 x 7 [1Q]
## # Key:     Purpose, State, Region, .model [850]
##    Purpose  State  Region          .model       Quarter        Trips .mean
##    <chr*>   <fct*> <chr*>          <chr>          <qtr>       <dist> <dbl>
##  1 Business NSW    Blue Mountains ets          2018 Q1 N(20, 140)   19.7
##  2 Business NSW    Blue Mountains ets          2018 Q2 N(20, 140)   19.7
##  3 Business NSW    Blue Mountains ets          2018 Q3 N(20, 140)   19.7
##  4 Business NSW    Blue Mountains ets          2018 Q4 N(20, 140)   19.7
##  5 Business NSW    Blue Mountains ets          2019 Q1 N(20, 140)   19.7
##  6 Business NSW    Blue Mountains ets          2019 Q2 N(20, 140)   19.7
##  7 Business NSW    Blue Mountains ets          2019 Q3 N(20, 140)   19.7
##  8 Business NSW    Blue Mountains ets          2019 Q4 N(20, 140)   19.7
```

Hierarchical and grouped time series

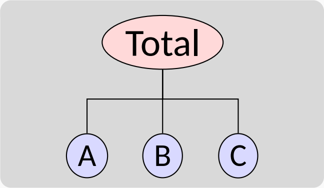
Every collection of time series with aggregation constraints can be written as

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

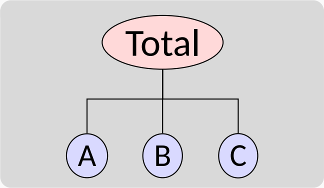
where

- $\boldsymbol{y}_t$ is a vector of all series at time t
- $\boldsymbol{b}_t$ is a vector of the most disaggregated series at time t
- $\boldsymbol{S}$ is a "summing matrix' ' containing the aggregation constraints.

Hierarchical time series



Hierarchical time series

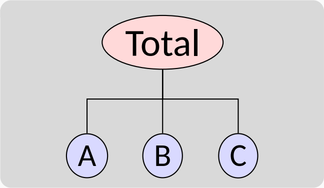


y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

Hierarchical time series



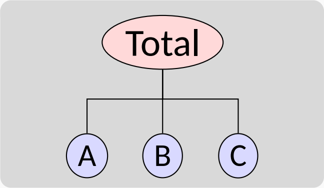
y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

$$\boldsymbol{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

Hierarchical time series



y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

$$\boldsymbol{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{S}} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\boldsymbol{b}_t}$$

$$\boxed{\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t}$$

Forecasting notation

Let $\hat{\boldsymbol{y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\boldsymbol{y}_t$.

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Reconciled forecasts must be of the form:

$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_n(h)$$

for some matrix $\boldsymbol{G}$.

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Let $\hat{\boldsymbol{y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\boldsymbol{y}_t$.
(In general, they will not "add up'".)

Reconciled forecasts must be of the form:

$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_n(h)$$

for some matrix $\boldsymbol{G}$.

- $\boldsymbol{G}$ extracts and combines base forecasts $\hat{\boldsymbol{y}}_n(h)$ to get bottom-level forecasts.
- $\boldsymbol{S}$ adds them up

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$, where $\boldsymbol{W}_h$ is the h-step base forecast error covariance matrix.

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$, where $\boldsymbol{W}_h$ is the h-step base forecast error covariance matrix.

$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}(\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}\hat{\boldsymbol{y}}_n(h)$$

Problem: $\boldsymbol{W}_h$ hard to estimate, especially for $h > 1$.

Solutions:

- Ignore $\boldsymbol{W}_h$ (OLS) [`min_trace(method='ols')`]
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ is diagonal (WLS) [`min_trace(method='wls')`]
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ and estimate it (GLS)

Example: Australian tourism

```
tourism_agg <- tourism |>
  aggregate_key(Purpose * (State / Region),
    Trips = sum(Trips)
  )
fc <- tourism_agg |>
  filter(year(Quarter) <= 2015) |>
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  ) |>
  mutate(
    comb = (ets + arima) / 2
  ) |>
  reconcile(
    ets_adj = min_trace(ets),
    arima_adj = min_trace(arima),
    comb_adj = min_trace(comb)
  ) |>
  forecast(h = "2 years")
```

Forecast evaluation

```r
fc |>
  accuracy(tourism_agg)
```

```
## # A tibble: 2,550 x 13
##    .model Purpose  State  Region               .type    ME  RMSE   MAE    MPE  MAPE
##    <chr>  <chr*>   <fct*> <chr*>               <chr> <dbl> <dbl> <dbl>  <dbl> <dbl>
##  1 arima  Business NSW    Blue Mountains     ~ Test   1.93 10.6   8.52 -18.0   48.6
##  2 arima  Business NSW    Capital Country    ~ Test   8.08 15.6  10.4   11.8   19.0
##  3 arima  Business NSW    Central Coast      ~ Test  10.0  14.5  10.8   26.9   32.2
##  4 arima  Business NSW    Central NSW        ~ Test  17.7  31.9  28.2   12.0   24.1
##  5 arima  Business NSW    Hunter             ~ Test  35.3  43.9  35.3   24.2   24.2
##  6 arima  Business NSW    New England Nort~ Test  23.1  31.8  26.8   19.5   28.0
##  7 arima  Business NSW    North Coast NSW    ~ Test  24.8  40.1  36.8   11.5   28.5
##  8 arima  Business NSW    Outback NSW        ~ Test   6.87 11.0   7.76  13.7   16.5
##  9 arima  Business NSW    Riverina           ~ Test   5.84 20.4  16.5   -2.48  31.5
## 10 arima  Business NSW    Snowy Mountains    ~ Test   5.48  9.54  8.24  12.3   40.4
## # ... with 2,540 more rows, and 3 more variables: MASE <dbl>, RMSSE <dbl>,
## #   ACF1 <dbl>
```

Forecast evaluation

```r
fc |>
  accuracy(tourism_agg) |>
  group_by(.model) |>
  summarise(RMSSE = sqrt(mean(RMSSE^2))) |>
  arrange(RMSSE)
```

```
## # A tibble: 6 x 2
##   .model     RMSSE
##   <chr>      <dbl>
## 1 ets_adj     1.03
## 2 comb_adj    1.04
## 3 ets         1.05
## 4 comb        1.06
## 5 arima_adj   1.08
## 6 arima       1.10
```

More information

- Slides and papers: **robjhyndman.com**
- Packages: **tidyverts.org**
- Forecasting textbook using tsibble, feasts and fable packages: **OTexts.com/fpp3**

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