Feasts & fables Modern tools for time series analysis feas**ts Rob J Hyndman ts**ibble Sable robjhyndman.com/cornish2021

Outline

- 1 What does modern time series data look like?
- 2 Feature-based time series analysis
- 3 Probabilistic forecasting for large time series
- 4 Evaluating forecast accuracy
- 5 Forecast reconciliation

Tidyverts packages

tidyverts.org



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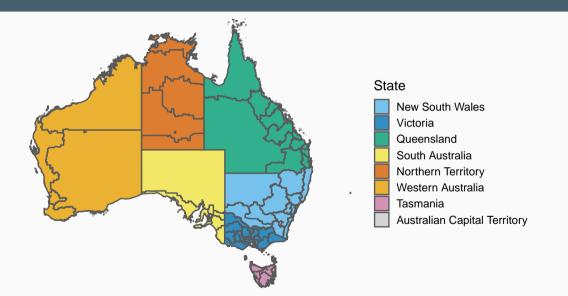
```
## # A tsibble: 15,150 x 6 [1Y]
  # Key:
##
              Country [263]
      Year Country
##
                             GDP Imports Exports Population
##
     <dbl> <fct>
                           <dbl>
                                  <dbl>
                                         <dbl>
                                                    <dbl>
##
      1960 Afghanistan 537777811.
                                   7.02
                                          4.13
                                                  8996351
##
      1961 Afghanistan
                      548888896. 8.10
                                          4.45
                                                  9166764
      1962 Afghanistan 546666678. 9.35
                                          4.88
                                                  9345868
##
                                  16.9
      1963 Afghanistan 751111191.
                                          9.17
                                                  9533954
##
##
      1964 Afghanistan 800000044.
                                  18.1
                                          8.89
                                                  9731361
##
     1965 Afghanistan 1006666638.
                                  21.4
                                         11.3
                                                  9938414
##
      1966 Afghanistan 1399999967.
                                  18.6
                                          8.57
                                                 10152331
##
      1967 Afghanistan 1673333418.
                                  14.2
                                          6.77
                                                 10372630
##
      1968 Afghanistan 1373333367.
                                  15.2
                                          8.90
                                                 10604346
      1969 Afghanistan 1408888922.
                                  15.0
                                         10.1
                                                 10854428
  # ... with 15,140 more rows
```

```
## # A tsibble: 15,150 x 6 [1Y]
##
  # Kev:
              Country [263]
##
      Year Country
                             GDP Imports Exports Population
##
     Index
          <fct>
                            <dbl>
                                   <dbl>
                                           <dbl>
                                                     <dbl>
      1960 Afghanistan 537777811.
                                    7.02
                                           4.13
                                                   8996351
##
##
      1961 Afghanistan
                       548888896. 8.10
                                           4.45
                                                   9166764
      1962 Afghanistan
                       546666678. 9.35
                                           4.88
                                                   9345868
##
      1963 Afghanistan 751111191.
                                   16.9
                                           9.17
                                                   9533954
##
##
      1964 Afghanistan 800000044.
                                   18.1
                                           8.89
                                                   9731361
##
      1965 Afghanistan 1006666638.
                                   21.4
                                           11.3
                                                   9938414
##
      1966 Afghanistan 1399999967.
                                   18.6
                                           8.57
                                                  10152331
##
      1967 Afghanistan 1673333418.
                                   14.2
                                           6.77
                                                  10372630
##
      1968 Afghanistan 1373333367.
                                   15.2
                                           8.90
                                                  10604346
      1969 Afghanistan 1408888922.
                                   15.0
                                           10.1
                                                  10854428
  # ... with 15,140 more rows
```

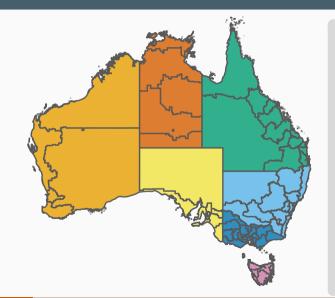
```
## # A tsibble: 15,150 x 6 [1Y]
##
  # Kev:
               Country [263]
      Year Country
##
                              GDP Imports Exports Population
##
      Index
            Kev
                            <dbl>
                                    <dbl>
                                            <dbl>
                                                      <dbl>
      1960 Afghanistan
                        537777811.
                                     7.02
                                             4.13
                                                    8996351
##
##
      1961 Afghanistan
                        548888896. 8.10
                                             4.45
                                                    9166764
      1962 Afghanistan
                        546666678. 9.35
                                             4.88
                                                    9345868
##
      1963 Afghanistan 751111191.
                                    16.9
                                             9.17
                                                    9533954
##
##
      1964 Afghanistan 800000044.
                                    18.1
                                             8.89
                                                    9731361
##
      1965 Afghanistan 1006666638.
                                    21.4
                                            11.3
                                                    9938414
##
      1966 Afghanistan 1399999967.
                                    18.6
                                             8.57
                                                    10152331
##
      1967 Afghanistan 1673333418.
                                    14.2
                                             6.77
                                                    10372630
##
      1968 Afghanistan 1373333367.
                                    15.2
                                             8.90
                                                    10604346
      1969 Afghanistan 1408888922.
                                    15.0
                                            10.1
                                                    10854428
  # ... with 15,140 more rows
```

```
## # A tsibble: 15,150 x 6 [1Y]
##
  # Kev:
               Country [263]
      Year Country
##
                               GDP Imports Exports Population
##
      Index
            Kev
                        Measured variables
      1960 Afghanistan
                        537777811.
                                      7.02
                                             4.13
                                                     8996351
##
##
      1961 Afghanistan
                        548888896.
                                     8.10
                                             4.45
                                                     9166764
      1962 Afghanistan
                        546666678. 9.35
                                             4.88
                                                     9345868
##
      1963 Afghanistan 751111191.
                                     16.9
                                             9.17
                                                     9533954
##
                                             8.89
##
      1964 Afghanistan 800000044.
                                     18.1
                                                     9731361
##
      1965 Afghanistan 1006666638.
                                     21.4
                                             11.3
                                                     9938414
##
      1966 Afghanistan 1399999967.
                                     18.6
                                             8.57
                                                     10152331
##
      1967 Afghanistan 1673333418.
                                     14.2
                                             6.77
                                                     10372630
##
      1968 Afghanistan 1373333367.
                                     15.2
                                             8.90
                                                     10604346
      1969 Afghanistan 1408888922.
                                     15.0
                                             10.1
##
                                                     10854428
  # ... with 15,140 more rows
```

Australian tourism regions



Australian tourism regions



- Quarterly data on visitor nights: 1998 – 2017
- From National Visitor Survey, interviews of 120,000
 Australians aged 15+.
- Geographical hierarchy split by
 - 8 states and territories
 - ► 76 regions
- Purpose:
 - Holidays
 - Business
 - Visiting friends & relatives
 - Other

```
# A tsibble: 24,320 x 5 [10]
##
  # Kev:
                Region, State, Purpose [304]
##
     Quarter Region State Purpose
                                      Trips
        <qtr> <chr> <chr> <chr>
                                      <dbl>
##
##
    1 1998 O1 Adelaide SA
                             Business 135.
   2 1998 Q2 Adelaide SA
                             Business 110.
##
   3 1998 Q3 Adelaide SA
                            Business 166.
##
##
    4 1998 O4 Adelaide SA
                             Business 127.
                                             Domestic visitor
##
    5 1999 O1 Adelaide SA
                             Business 137.
                                             nights in thousands
##
    6 1999 Q2 Adelaide SA
                             Business
                                       200.
                                             by state/region and
                             Business
##
    7 1999 Q3 Adelaide SA
                                       169.
                                             purpose.
##
    8 1999 O4 Adelaide SA
                             Business
                                      134.
##
    9 2000 Q1 Adelaide SA
                             Business 154.
  10 2000 O2 Adelaide SA
                             Business 169.
  # ... with 24,310 more rows
```

```
# A tsibble: 24,320 x 5 [10]
##
   # Kev:
                Region, State, Purpose [304]
##
      Quarter Region
                       State Purpose
                                      Trips
              <chr> <chr> <chr>
                                      <dbl>
##
      Index
   1 1998 O1 Adelaide SA
##
                             Business 135.
   2 1998 Q2 Adelaide SA
                             Business 110.
##
   3 1998 Q3 Adelaide SA
                             Business 166.
##
##
    4 1998 O4 Adelaide SA
                             Business 127.
                                             Domestic visitor
##
    5 1999 O1 Adelaide SA
                             Business
                                       137.
                                             nights in thousands
##
    6 1999 Q2 Adelaide SA
                             Business
                                       200.
                                             by state/region and
                             Business
##
    7 1999 Q3 Adelaide SA
                                       169.
                                             purpose.
##
    8 1999 O4 Adelaide SA
                             Business
                                       134.
   9 2000 Q1 Adelaide SA
##
                             Business
                                       154.
  10 2000 O2 Adelaide SA
                             Business 169.
   # ... with 24,310 more rows
```

```
# A tsibble: 24,320 x 5 [10]
##
   # Kev:
                Region, State, Purpose [304]
##
      Ouarter Region State Purpose
                                       Trips
                                       <dbl>
##
      Index
               Kevs
##
   1 1998 O1 Adelaide SA
                              Business
                                        135.
   2 1998 Q2 Adelaide SA
                              Business 110.
##
   3 1998 Q3 Adelaide SA
                             Business 166.
##
##
    4 1998 O4 Adelaide SA
                              Business 127.
                                              Domestic visitor
##
    5 1999 O1 Adelaide SA
                              Business
                                       137.
                                              nights in thousands
##
    6 1999 Q2 Adelaide SA
                              Business
                                        200.
                                              by state/region and
                              Business
##
    7 1999 Q3 Adelaide SA
                                        169.
                                              purpose.
##
    8 1999 O4 Adelaide SA
                              Business
                                        134.
   9 2000 Q1 Adelaide SA
##
                              Business
                                       154.
   10 2000 02 Adelaide SA
                              Business 169.
   # ... with 24,310 more rows
```

```
# A tsibble: 24,320 x 5 [10]
##
   # Kev:
                Region, State, Purpose [304]
##
      Ouarter Region State Purpose
                                       Trips
##
      Index
               Kevs
                                        Measure
##
   1 1998 O1 Adelaide SA
                              Business
                                        135.
   2 1998 Q2 Adelaide SA
                              Business 110.
##
   3 1998 Q3 Adelaide SA
                             Business 166.
##
##
    4 1998 O4 Adelaide SA
                              Business 127.
                                              Domestic visitor
##
    5 1999 O1 Adelaide SA
                              Business
                                       137.
                                              nights in thousands
##
    6 1999 Q2 Adelaide SA
                              Business
                                        200.
                                              by state/region and
                              Business
##
    7 1999 Q3 Adelaide SA
                                        169.
                                              purpose.
##
    8 1999 O4 Adelaide SA
                              Business
                                        134.
   9 2000 Q1 Adelaide SA
##
                              Business 154.
   10 2000 02 Adelaide SA
                              Business 169.
   # ... with 24,310 more rows
```

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
##
  # Kev:
               State [8]
##
     Time
                         State Date
                                        Holiday Temperature Demand
##
     <dttm>
                         <fct> <date>
                                          <lgl>
                                                       <dbl> <dbl>
##
   1 2012-01-01 00:00:00 VIC 2012-01-01 TRUE
                                                        21.4 4383.
##
   2 2012-01-01 00:30:00 VIC 2012-01-01 TRUE
                                                        21.0 4263.
   3 2012-01-01 01:00:00 VIC 2012-01-01 TRUE
                                                        20.7 4049.
##
                                                        20.6 3878.
##
   4 2012-01-01 01:30:00 VTC 2012-01-01 TRUE
##
   5 2012-01-01 02:00:00 VIC
                               2012-01-01 TRUE
                                                        20.4
                                                              4036.
##
   6 2012-01-01 02:30:00 VIC
                               2012-01-01 TRUE
                                                        20.2
                                                              3866.
##
   7 2012-01-01 03:00:00 VIC
                               2012-01-01 TRUE
                                                        20.1
                                                              3694.
   8 2012-01-01 03:30:00 VIC
                               2012-01-01 TRUE
                                                        19.6 3562.
##
##
   9 2012-01-01 04:00:00 VTC
                               2012-01-01 TRUE
                                                        19.1 3433.
## 10 2012-01-01 04:30:00 VTC
                               2012-01-01 TRUE
                                                        19.0 3359.
  # ... with 420,854 more rows
```

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
##
  # Key:
               State [8]
##
     Time
                         State Date
                                          Holiday Temperature Demand
##
                         <fct> <date>
                                          <lgl>
                                                        <dbl> <dbl>
     Index
   1 2012-01-01 00:00:00 VIC
##
                               2012-01-01 TRUE
                                                         21.4 4383.
##
   2 2012-01-01 00:30:00 VIC 2012-01-01 TRUE
                                                         21.0 4263.
   3 2012-01-01 01:00:00 VIC 2012-01-01 TRUE
                                                         20.7 4049.
##
##
   4 2012-01-01 01:30:00 VTC
                               2012-01-01 TRUE
                                                         20.6 3878.
##
   5 2012-01-01 02:00:00 VIC
                               2012-01-01 TRUE
                                                         20.4
                                                               4036.
##
   6 2012-01-01 02:30:00 VIC
                               2012-01-01 TRUE
                                                         20.2
                                                               3866.
##
   7 2012-01-01 03:00:00 VIC
                               2012-01-01 TRUE
                                                         20.1
                                                               3694.
   8 2012-01-01 03:30:00 VIC
                               2012-01-01 TRUE
                                                              3562.
##
                                                         19.6
##
   9 2012-01-01 04:00:00 VTC
                               2012-01-01 TRUE
                                                         19.1 3433.
  10 2012-01-01 04:30:00 VTC
                               2012-01-01 TRUE
                                                         19.0 3359.
  # ... with 420,854 more rows
```

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
##
  # Kev:
               State [8]
##
     Time
                         State Date
                                          Holiday Temperature Demand
##
                                <date>
                                          <lgl>
                                                        <dbl>
                                                               <dbl>
      Index
                         Kev
   1 2012-01-01 00:00:00 VIC
                               2012-01-01 TRUE
##
                                                         21.4 4383.
##
   2 2012-01-01 00:30:00 VIC 2012-01-01 TRUE
                                                         21.0
                                                               4263.
   3 2012-01-01 01:00:00 VTC 2012-01-01 TRUE
                                                         20.7
                                                               4049.
##
##
   4 2012-01-01 01:30:00 VTC
                               2012-01-01 TRUE
                                                         20.6
                                                               3878.
##
   5 2012-01-01 02:00:00 VIC
                               2012-01-01 TRUE
                                                         20.4
                                                               4036.
##
   6 2012-01-01 02:30:00 VIC
                               2012-01-01 TRUE
                                                          20.2
                                                               3866.
##
   7 2012-01-01 03:00:00 VIC
                               2012-01-01 TRUE
                                                         20.1
                                                               3694.
   8 2012-01-01 03:30:00 VIC
                               2012-01-01 TRUE
                                                               3562.
##
                                                         19.6
##
   9 2012-01-01 04:00:00 VTC
                               2012-01-01 TRUF
                                                         19.1 3433.
  10 2012-01-01 04:30:00 VTC
                               2012-01-01 TRUE
                                                          19.0 3359.
  # ... with 420,854 more rows
```

```
# A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
##
  # Kev:
                State [8]
##
     Time
                          State Date
                                           Holiday Temperature Demand
##
      Index
                          Kev
                                 Measures
   1 2012-01-01 00:00:00 VIC
                                                                4383.
##
                                2012-01-01 TRUE
                                                          21.4
##
    2 2012-01-01 00:30:00 VTC
                                2012-01-01 TRUF
                                                          21.0
                                                                4263.
    3 2012-01-01 01:00:00 VTC 2012-01-01 TRUE
                                                          20.7
                                                                4049.
##
##
    4 2012-01-01 01:30:00 VTC
                                2012-01-01 TRUE
                                                          20.6
                                                                3878.
##
   5 2012-01-01 02:00:00 VIC
                                2012-01-01 TRUE
                                                          20.4
                                                                4036.
##
    6 2012-01-01 02:30:00 VIC
                                2012-01-01 TRUE
                                                          20.2
                                                                3866.
##
   7 2012-01-01 03:00:00 VIC
                                2012-01-01 TRUE
                                                          20.1
                                                                3694.
    8 2012-01-01 03:30:00 VIC
                                2012-01-01 TRUE
                                                                3562.
##
                                                          19.6
##
   9 2012-01-01 04:00:00 VTC
                                2012-01-01 TRUF
                                                          19.1
                                                                3433.
  10 2012-01-01 04:30:00 VTC
                                2012-01-01 TRUE
                                                          19.0 3359.
  # ... with 420,854 more rows
```

Characteristics of modern time series

- Often observed at sub-daily frequency over a long time.
- Multiple keys which may be nested.
- Multiple seasonal patterns.
- Multiple measures for each combination of index and keys.

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- Often observed at sub-daily frequency over a long time.
- Multiple keys which may be nested.
- Multiple seasonal patterns.
- Multiple measures for each combination of index and keys.

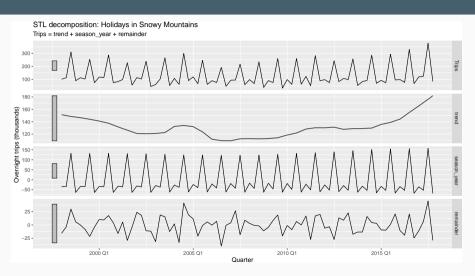
tsibble objects

- A tsibble allows storage and manipulation of multiple time series in R.
- It contains:
 - An index: time information about the observation
 - Key variable(s): optional unique identifiers for each series
 - Measured variable(s): numbers of interest and any other variable

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STL decomposition



Strength of seasonality and trend

STL decomposition

$$y_t = T_t + S_t + R_t$$

Seasonal strength

$$\max\left(0,1-\frac{\mathsf{Var}(R_t)}{\mathsf{Var}(S_t+R_t)}\right)$$

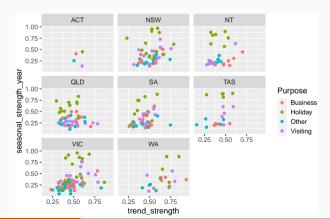
Trend strength

$$\max\left(0,1-\frac{\mathsf{Var}(R_t)}{\mathsf{Var}(T_t+R_t)}\right)$$

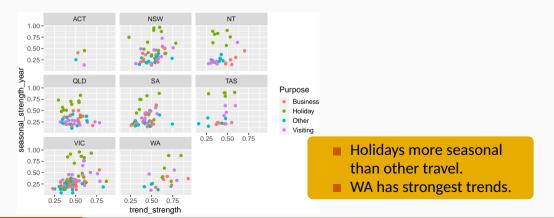
tourism %>% features(Trips, feat_stl)

```
# A tibble: 304 x 12
##
      Region
                     State Purpose trend_strength seasonal_strength_~ seasonal_peak_ye~
##
      <chr>
                     <chr> <chr>
                                             <dbl>
                                                                  <dbl>
                                                                                    <dbl>
##
    1 Adelaide
                     SA
                           Business
                                             0.464
                                                                  0.407
   2 Adelaide
                     SA
                           Holiday
                                             0.554
                                                                  0.619
##
    3 Adelaide
                     SA
                           Other
                                             0.746
                                                                  0.202
##
   4 Adelaide
                     SA
                           Visiting
##
                                             0.435
                                                                  0.452
    5 Adelaide Hills SA
                           Business
                                             0.464
                                                                  0.179
##
##
   6 Adelaide Hills SA
                           Holiday
                                             0.528
                                                                  0.296
   7 Adelaide Hills SA
                           0ther
##
                                             0.593
                                                                  0.404
                           Visiting
   8 Adelaide Hills SA
                                             0.488
                                                                  0.254
##
   9 Alice Springs NT
                           Business
                                             0.534
                                                                  0.251
##
   10 Alice Springs NT
                         Holidav
                                             0.381
                                                                  0.832
   # ... with 294 more rows, and 6 more variables: seasonal_trough_year <dbl>,
## #
       spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
                                                                                          13
       stl e acf10 <dbl>
## #
```

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() + facet_wrap(vars(State))
```



```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() + facet_wrap(vars(State))
```



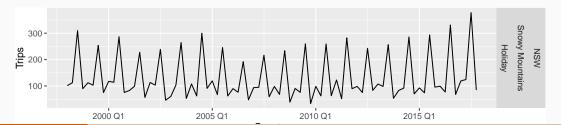
Find the most seasonal time series:

```
most_seasonal <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

Find the most seasonal time series:

```
most_seasonal <- tourism %>%
features(Trips, feat_stl) %>%
filter(seasonal_strength_year == max(seasonal_strength_year))
```

```
tourism %>%
    right_join(most_seasonal, by = c("State", "Region", "Purpose")) %>%
    ggplot(aes(x = Quarter, y = Trips)) +
    geom_line() +
    facet_grid(vars(State, Region, Purpose))
```



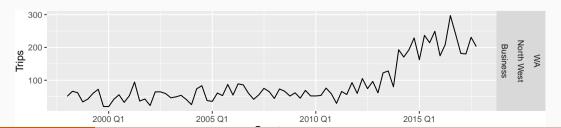
Find the most trended time series:

```
most_trended <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength))
```

Find the most trended time series:

```
most_trended <- tourism %>%
features(Trips, feat_stl) %>%
filter(trend_strength == max(trend_strength))
```

```
tourism %>%
  right_join(most_trended, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



```
tourism_features <- tourism %>%
  features(Trips, feature_set(pkgs = "feasts"))
```

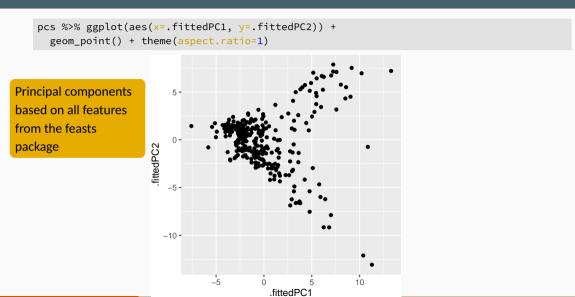
All features from the feasts package

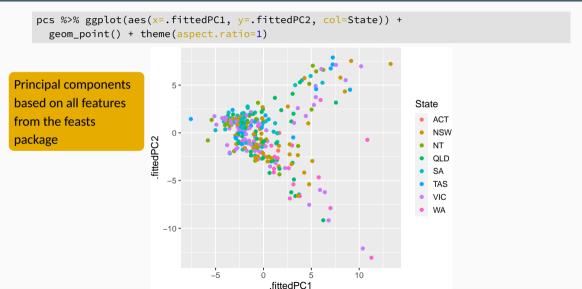
```
# A tibble: 304 x 50
                     State Purpose trend strength seasonal streng~ seasonal peak v~
##
     Region
      <chr>
                     <chr> <chr>
                                            <dbl>
                                                             <dbl>
                                                                              <dbl>
##
   1 Adelaide
                     SA
                           Busine~
                                            0.464
                                                             0.407
##
##
   2 Adelaide
                     SA
                           Holidav
                                            0.554
                                                             0.619
   3 Adelaide
                           Other
##
                     SA
                                            0.746
                                                             0.202
##
   4 Adelaide
                     SA
                           Visiti~
                                            0.435
                                                             0.452
   5 Adelaide Hills SA
                           Busine~
                                            0.464
                                                             0.179
##
   6 Adelaide Hills SA
                           Holiday
                                            0.528
                                                             0.296
##
   7 Adelaide Hills SA
##
                           Other
                                            0.593
                                                             0.404
   8 Adelaide Hills SA
##
                          Visiti~
                                            0.488
                                                             0.254
   9 Alice Springs NT
                           Busine~
                                            0.534
                                                             0.251
##
  10 Alice Springs NT
                           Holiday
                                            0.381
                                                             0.832
  # ... with 294 more rows, and 44 more variables: seasonal_trough_year <dbl>,
##
       spikiness <dbl>. linearity <dbl>. curvature <dbl>. stl e acf1 <dbl>.
## #
       stl e acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1 acf1 <dbl>,
## #
      diff1 acf10 <dhl> diff2 acf1 <dhl> diff2 acf10 <dhl> season acf1 <dhl>
```

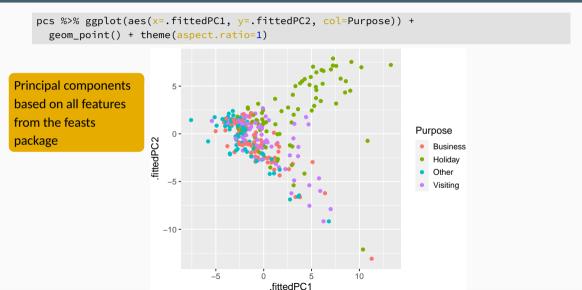
```
pcs <- tourism_features %>%
  select(-State, -Region, -Purpose) %>%
  prcomp(scale = TRUE) %>%
  broom::augment(tourism_features)
```

Principal components based on all features from the feasts package

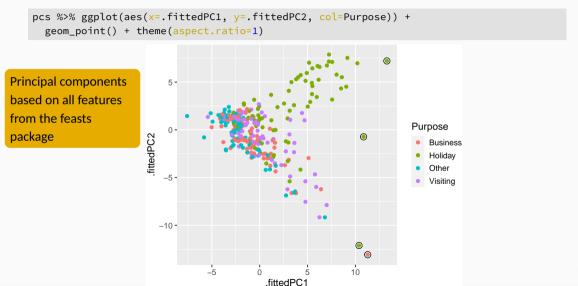
```
## # A tibble: 304 x 98
      .rownames Region
                               State Purpose trend strength seasonal strength year
##
      <chr>>
                <chr>>
                               <chr> <chr>
                                                       <fdb>>
                                                                               <dh1>
##
                Adelaide
##
   1 1
                               SA
                                     Rusiness
                                                       0.464
                                                                               0.407
    2 2
                Adelaide
                               SA Holiday
                                                       0.554
                                                                               0.619
##
    3 3
                Adelaide
                               SA
                                     Other
                                                       0.746
                                                                               0.202
##
    4 4
                Adelaide
                               SA
                                     Visiting
                                                       0.435
                                                                               0.452
    5 5
                Adelaide Hills SA
                                     Business
                                                       0.464
                                                                               0.179
##
   6 6
               Adelaide Hills SA
                                  Holidav
                                                       0.528
                                                                               0.296
##
   7 7
                Adelaide Hills SA
                                     Other
                                                       0.593
                                                                               0.404
   8 8
                Adelaide Hills SA
                                   Visiting
                                                       0.488
                                                                               0.254
                                  Business
    9 9
                Alice Springs NT
                                                       0.534
                                                                               0.251
## 10 10
                Alice Springs NT
                                  Holidav
                                                       0.381
                                                                               0.832
## # ... with 294 more rows, and 92 more variables: seasonal peak year <dbl>...
## #
       seasonal_trough_year <dbl>, spikiness <dbl>, linearity <dbl>,
       curvature <dbl>, stl e acf1 <dbl>, stl e acf10 <dbl>, acf1 <dbl>,
## #
## #
       acf10 <dbl>, diff1 acf1 <dbl>, diff1 acf10 <dbl>, diff2 acf1 <dbl>,
```



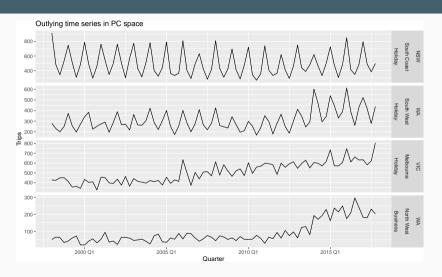




Anomaly detection using time series features



Anomaly detection using time series features



Outline

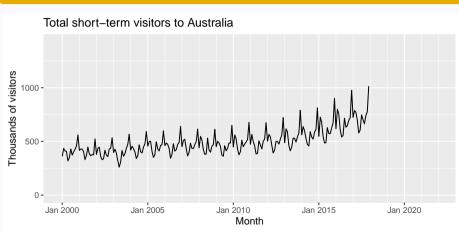
- 1 What does modern time series data look like?
- 2 Feature-based time series analysis
- 3 Probabilistic forecasting for large time series
- 4 Evaluating forecast accuracy
- 5 Forecast reconciliation

Probabilistic forecasting for large time series

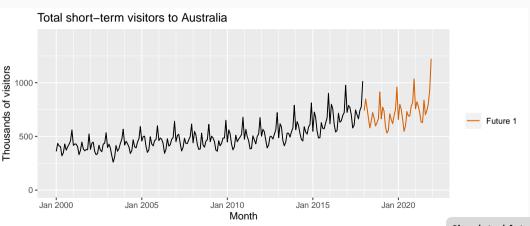
ensemble and hybrid forecasts

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

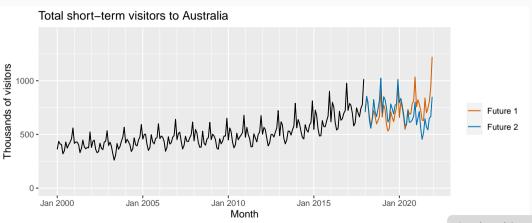
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



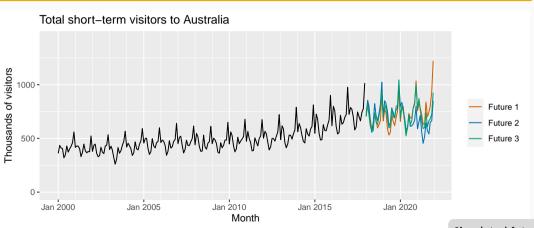
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



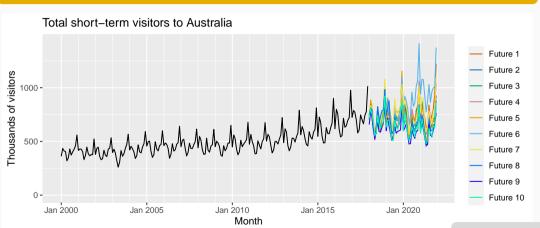
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



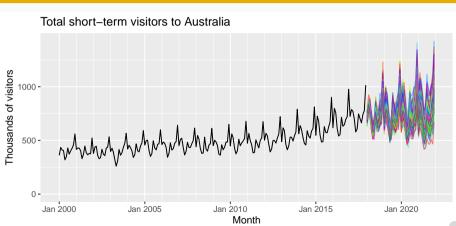
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



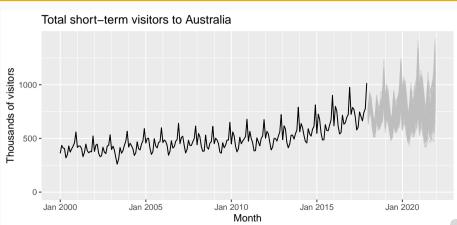
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



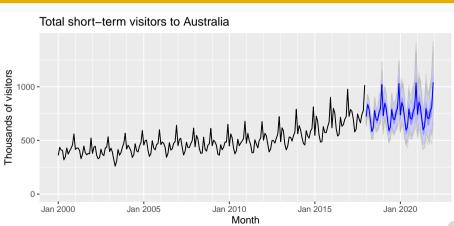
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



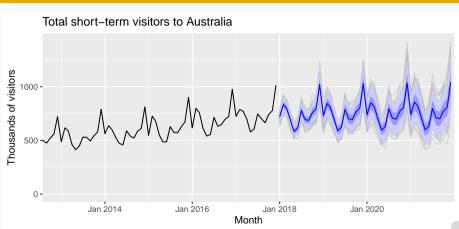
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



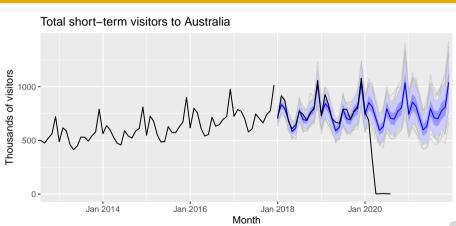
A forecast is an estimate of the probability distribution of a variable to be observed in the future.



A forecast is an estimate of the probability distribution of a variable to be observed in the future.



A forecast is an estimate of the probability distribution of a variable to be observed in the future.



```
holiday_fit <- holidays %>%
model(
    snaive = SNAIVE(Trips),
    naive = NAIVE(Trips),
    ets = ETS(Trips),
    arima = ARIMA(Trips)
)
```

```
## # A mable: 8 x 5
## # Key: State [8]
   State snaive naive
                                                                           arima
##
                                         ets
   <chr> <model> <model> <model>
                                                                         <model>
##
## 1 ACT <SNAIVE> <NAIVE> <ETS(M,N,A)>
                                                      <ARIMA(0,1,1)(1,0,1)[4]>
## 2 NSW
            <SNAIVE> <NAIVE> <ETS(M.N.A)>
                                                      <ARIMA(0,1,1)(0,1,1)[4]>
## 3 NT <SNAIVE> <NAIVE> <ETS(M.N.A)>
                                                      \langle ARIMA(0,0,0)(2,1,0)[4] \rangle
## 4 QLD \langle SNAIVE \rangle \langle NAIVE \rangle \langle ETS(A,N,A) \rangle \langle ARIMA(0,0,0)(1,0,0)[4] w/ mean \rangle
```

```
holiday fit %>%
 filter(State == "VIC") %>%
 select(arima) %>%
 report()
## Series: Trips
## Model: ARIMA(0,1,1)(0,1,1)[4]
##
## Coefficients:
##
            mal smal
## -0.8079 -0.576
## s.e. 0.0802 0.164
##
## sigma^2 estimated as 25711: log likelihood=-488
## ATC=982 ATCc=982 BTC=989
```

##

glance(holiday fit)

A tibble: 32 x 12

State .model

```
<dbl>
                                     <dbl> <dbl> <dbl> <dbl>
                                                                                  <dbl> <
##
      <chr> <chr>
                                                                 <dbl>
                                                                         <dbl>
##
    1 ACT
             snaive
                      2147.
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
##
    2 ACT
             naive
                       2931.
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
    3 ACT
                         0.0680
                                     -463.
                                             940.
                                                    941.
                                                          956.
                                                                 1509.
                                                                         1538.
                                                                                0.189
                                                                                         <NUL
##
             ets
    4 ACT
             arima
                       1437.
##
                                     -399.
                                             805.
                                                    806.
                                                          815.
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <cpl
    5 NSW
             snaive
                     46496.
##
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
##
    6 NSW
             naive
                    254319.
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
##
    7 NSW
             ets
                          0.00347
                                     -585. 1184. 1185. 1201. 28271. 29225.
                                                                                0.0458
                                                                                         <NUL
##
    8 NSW
             arima
                      33036.
                                     -498. 1003. 1003. 1010.
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <cpl
##
    9 NT
             snaive
                      2445.
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
##
   10 NT
             naive
                      21118.
                                       NA
                                              NA
                                                     NA
                                                           NA
                                                                   NA
                                                                           NA
                                                                               NA
                                                                                         <NUL
```

AICc

BIC

MSE

AMSE

MAE ar_r

sigma2 log_lik AIC

... with 22 more rows, and 1 more variable: ma roots <list>

tidy(holiday_fit)

```
# A tibble: 75 \times 7
##
      State .model term estimate std.error statistic
                                                           p.value
                          <dbl>
                                                             <dbl>
##
      <chr> <chr>
                   <chr>
                                        <dbl>
                                                  <dbl>
##
    1 ACT
            ets
                   alpha
                          0.0513
                                     NA
                                                  NA
                                                        NA
##
    2 ACT
            ets
                   gamma
                          0.482
                                     NA
                                                  NA
                                                        NA
    3 ACT
                   1[0]
                          148.
                                     NA
                                                        NA
##
            ets
                                                  NA
    4 ACT
                   s[0]
                            7.88
                                                        NA
##
            ets
                                     NA
                                                  NA
##
    5 ACT
                   s[-1]
                          -2.66
                                     NA
                                                  NA
                                                        NA
            ets
##
    6 ACT
            ets
                   s[-2] -20.5
                                     NA
                                                  NA
                                                        NA
##
    7 ACT
            ets
                   s[-3]
                          15.3
                                      NA
                                                  NA
                                                        NA
    8 ACT
##
            arima
                   ma1
                          -0.886
                                      0.0668
                                                 -13.3
                                                         8.33e-22
    9 ACT
            arima
##
                   sar1
                          0.773
                                      0.170
                                                   4.55
                                                         1.93e- 5
  10 ACT
            arima
                   sma1
                           -0.557
                                      0.220
                                                  -2.52
                                                         1.36e- 2
  # ... with 65 more rows
```

augment(holiday_fit)

```
## # A tsibble: 2,560 x 7 [10]
##
  # Kev:
               State, .model [32]
##
     State .model Quarter Trips .fitted .resid .innov
##
      <chr> <chr> <atr> <dbl>
                                  <dbl>
                                         <dbl>
                                                <dbl>
##
   1 ACT
           snaive 1998 01 196.
                                    NA
                                         NA
                                                NA
   2 ACT
           snaive 1998 Q2 127.
##
                                    NA
                                         NA
                                                NA
   3 ACT
##
           snaive 1998 03 111.
                                    NA
                                         NA
                                                NA
   4 ACT
           snaive 1998 04 170.
##
                                    NA
                                         NA
                                                NA
##
   5 ACT
           snaive 1999 01
                           108.
                                   196. -88.4
                                               -88.4
##
   6 ACT
           snaive 1999 02
                           125.
                                   127.
                                         -2.13
                                                -2.13
##
   7 ACT
           snaive 1999 03
                           178.
                                   111.
                                         67.3
                                                67.3
   8 ACT
##
           snaive 1999 Q4
                           218.
                                   170.
                                         47.2
                                                47.2
##
   9 ACT
           snaive 2000 01
                           158.
                                   108.
                                         50.6
                                                50.6
  10 ACT
           snaive 2000 02
##
                           155.
                                   125.
                                         30.2
                                                30.2
```

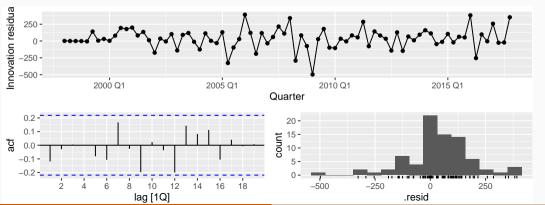
Ljung-Box test

```
augment(holiday_fit) %>%
  filter(State == "VIC", .model == "arima") %>%
  features(.resid, ljung_box, dof = 2, lag = 8)
```

```
## # A tibble: 1 x 4
## State .model lb_stat lb_pvalue
## <chr> <chr> <dbl> <dbl>
## 1 VIC arima 5.42 0.492
```

gg_tsresiduals() function

```
holiday_fit %>%
  filter(State == "VIC") %>%
  select(arima) %>%
  gg_tsresiduals()
```



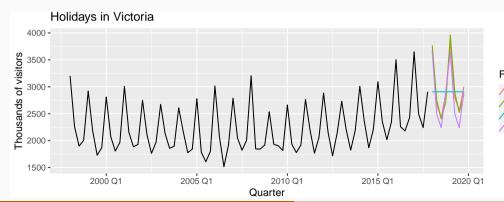
Producing forecasts

```
holiday_fc <- holiday_fit %>%
forecast(h = "2 years")
```

```
# A fable: 256 x 5 [10]
## # Key: State, .model [32]
##
   State .model Quarter Trips .mean
##
  <chr> <chr> <qtr> <dist> <dbl>
##
   1 ACT snaive 2018 01 N(223, 2128) 223.
##
   2 ACT snaive 2018 02 N(193, 2128) 193.
##
   3 ACT snaive 2018 Q3 N(204, 2128)
                                      204.
##
   4 ACT
           snaive 2018 Q4 N(214, 2128)
                                      214.
##
   5 ACT
           snaive 2019 01 N(223, 4257)
                                      223.
##
   6 ACT
           snaive 2019 02 N(193, 4257)
                                      193.
   7 ACT
           snaive 2019 Q3 N(204, 4257)
##
                                      204.
   8 ACT
           snaive 2019 Q4 N(214, 4257)
                                      214.
##
##
   9 ACT naive 2018 01 N(214, 2894)
                                      214.
## 10 ACT naive 2018 02 N(214, 5788)
                                      214.
## # ... with 246 more rows
```

Visualising forecasts

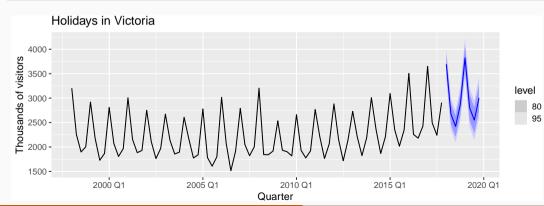
```
holiday_fc %>%
  filter(State == "VIC") %>%
  autoplot(holidays, level = NULL) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



Forecast

Visualising forecasts

```
holiday_fc %>%
  filter(State == "VIC", .model == "arima") %>%
  autoplot(holidays) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



Prediction intervals

##

9 ACT

```
holiday fc %>% hilo(level = 95)
## # A tsibble: 256 x 6 [10]
## # Kev:
               State, .model [32]
##
     State .model Ouarter
                                 Trips .mean
                                                      `95%`
                                <dist> <dbl>
                                                     <hilo>
##
      <chr> <chr>
                    <qtr>
##
   1 ACT
           snaive 2018 01 N(223, 2128) 223. [132.7, 314]95
   2 ACT
           snaive 2018 Q2 N(193, 2128) 193. [103.1, 284]95
##
   3 ACT
           snaive 2018 Q3 N(204, 2128) 204. [113.2, 294]95
##
##
   4 ACT
           snaive 2018 04 N(214, 2128) 214, [124.0, 305]95
   5 ACT
           snaive 2019 Q1 N(223, 4257) 223. [ 95.3, 351]95
##
##
   6 ACT
           snaive 2019 02 N(193, 4257) 193. [ 65.6, 321]95
##
   7 ACT
           snaive 2019 03 N(204, 4257) 204, [ 75.8, 332]95
           snaive 2019 04 N(214, 4257) 214. [ 86.6, 342]95
##
   8 ACT
```

naive 2018 Q1 N(214, 2894) 214. [109.0, 320]95

Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use level argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.

Outline

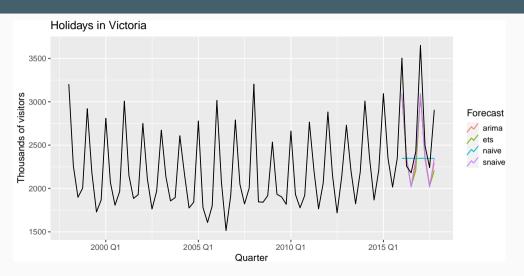
- 1 What does modern time series data look like?
- 2 Feature-based time series analysis
- 3 Probabilistic forecasting for large time series
- 4 Evaluating forecast accuracy
- 5 Forecast reconciliation

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Measures of forecast accuracy



Measures of forecast accuracy

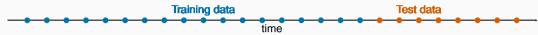
```
accuracy(vic_fc, holidays)
```

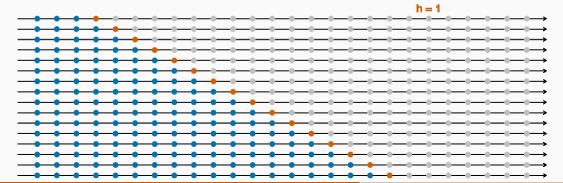
```
# A tibble: 4 x 11
##
     .model State .type
                           ME
                               RMSE
                                      MAE
                                            MPE
                                                 MAPE
                                                       MASE RMSSE
                                                                     ACF1
##
    <chr> <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <</pre>
   1 arima
           VIC
                         263.
                                           8.83 10.0
                  Test
                               349.
                                     290.
                                                       2.28
                                                              2.07 - 0.230
  2 ets
           VIC
                  Test
                         295.
                               378.
                                     315, 10,0 10,9
                                                       2.48
                                                              2.25 - 0.149
  3 naive VIC
               Test
                         360.
                              654.
                                     451. 10.1 14.2
                                                       3.54
                                                              3.88 - 0.181
  4 snaive VIC
                         256.
                               336.
                                     279. 8.59 9.61
                  Test
                                                       2.19
                                                              1.99 - 0.257
```

Traditional evaluation



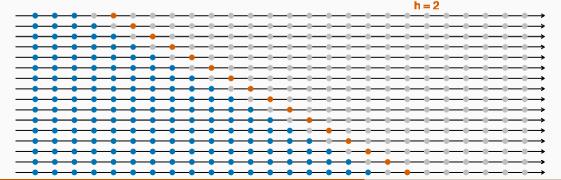
Traditional evaluation



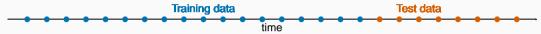


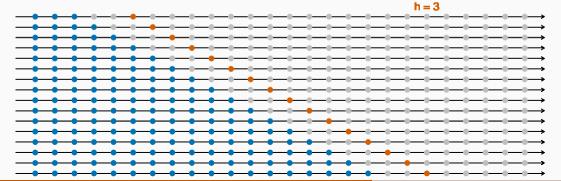
Traditional evaluation





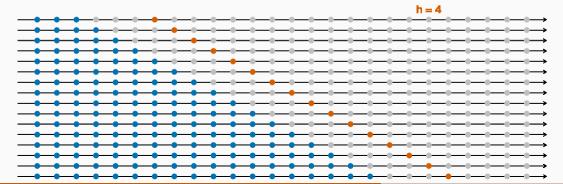
Traditional evaluation

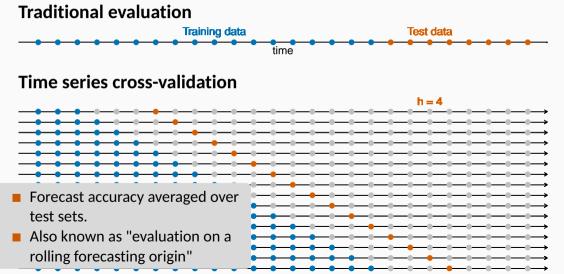




Traditional evaluation







vic holiday stretch <- holidays %>%

Stretch with a minimum length of 4 years, growing by 1 quarter each step.

```
filter(State == "VIC") %>%
 stretch tsibble(.init = 16, .step = 1)
## # A tsibble: 3.120 x 4 [10]
## # Kev: .id. State [65]
## State Quarter Trips .id
## <chr> <qtr> <dbl> <int>
## 1 VIC 1998 Q1 3204. 1
## 2 VIC 1998 Q2 2258. 1
## 3 VIC 1998 03 1898. 1
## 4 VIC 1998 Q4 2004. 1
## 5 VIC 1999 01 2919.
## 6 VIC 1999 Q2 2183.
```

56

```
fit_cv <- vic_holiday_stretch %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips),
    snaive = SNAIVE(Trips)
)
```

```
## # A mable: 65 x 5
## # Kev: .id. State [65]
## .id State
                       ets
                                            arima snaive
## <int> <chr> <model>
                                           <model> <model>
       1 VIC <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 1
## 2 2 VIC
               <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 3 3 VIC
               <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 4 4 VIC
               <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
        5 VIC
## 5
               <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
```

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%
forecast(h = 1)
```

```
## # A fable: 195 x 6 [10]
## # Key: .id, State, .model [195]
## .id State Quarter Trips .mean .model
## <int> <chr> <dist> <dbl> <chr>
## 1 1 VIC 2002 Q1 N(2965, 14682) 2965. ets
## 2 1 VIC 2002 Q1 N(2980, 11447) 2980. arima
## 3 1 VIC 2002 Q1 N(3007, 19035) 3007. snaive
## 4 2 VIC 2002 Q2 N(2024, 8130) 2024. ets
## 5 2 VIC 2002 Q2 N(1961, 14864) 1961. arima
## 6 2 VIC 2002 02 N(2153, 22669) 2153. snaive
## # ... with 189 more rows
```

```
# Cross-validated
fc_cv %>% accuracy(holidays)
```

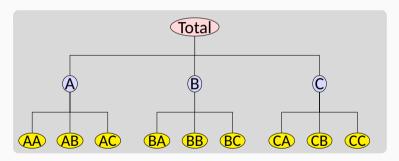
```
## # A tibble: 3 x 11
##
    .model State .type ME RMSE
                                    MAF
                                         MPF
                                              MAPE MASE RMSSE
                                                                 ACF1
##
    <chr> <chr> <chr> <chr> <dhl> <dhl> <dhl> <dhl> <dhl> <dhl> <dhl> <dhl> </hr>
                                                                <fdb>>
  1 arima VIC
               Test 46.3 183. 147. 1.57 6.41 1.08 1.02 -0.0615
## 2 ets VIC Test 43.3 179. 139. 1.51
                                              6.02 1.02 0.997 -0.110
  3 snaive VIC Test 36.2 186, 138, 1.14 6.01 1.02 1.04
                                                               0.0427
```

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.

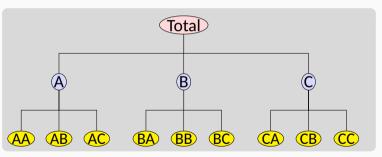
Outline

- 1 What does modern time series data look like?
- 2 Feature-based time series analysis
- 3 Probabilistic forecasting for large time series
- 4 Evaluating forecast accuracy
- 5 Forecast reconciliation

A hierarchical time series is a collection of several time series that are linked together in a hierarchical structure.



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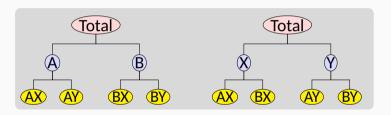


Examples

■ Tourism demand by states, zones, regions

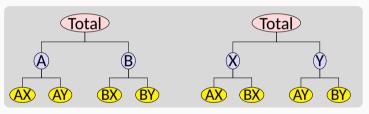
Grouped time series

A grouped time series is a collection of time series that can be grouped together in a number of non-hierarchical ways.



Grouped time series

A grouped time series is a collection of time series that can be grouped together in a number of non-hierarchical ways.



Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

Creating aggregates

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  filter(Quarter == yearquarter("1998 Q1")) %>%
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [10]
## # Kev:
               Purpose, State, Region [425]
##
     Ouarter Purpose
                          State
                                       Region
                                                       Trips
       <atr> <chr*>
                      <chr*>
                                                       <dbl>
##
                                       <chr*>
   1 1998 Q1 <aggregated> <aggregated> <aggregated>
                                                       23182.
##
##
   2 1998 01 Business
                         <aggregated> <aggregated>
                                                       3599.
   3 1998 01 Holiday
                          <aggregated> <aggregated>
##
                                                       11806.
   4 1998 01 Other
                        <aggregated> <aggregated>
                                                        680.
##
   5 1998 01 Visiting <aggregated> <aggregated>
                                                       7098.
##
   6 1998 O1 <aggregated> ACT
                                        <aggregated>
                                                         551.
   7 1998 O1 <aggregated> NSW
##
                                       <aggregated>
                                                       8040.
##
   8 1998 01 <aggregated> NT
                                        <aggregated>
                                                        181.
   9 1998 Q1 <aggregated> OLD
                                        <aggregated>
                                                       4041.
## 10 1998 01 <aggregated> SA
                                       <aggregated>
                                                       1735.
## 11 1998 Q1 <aggregated> TAS
                                                        982.
                                        <aggregated>
## 12 1998 01 <aggregated> VIC
                                        <aggregated>
                                                       6010.
## 13 1998 Q1 <aggregated> WA
                                        <aggregated>
                                                       1641.
```

Creating aggregates

- Similar to summarise() but using the key structure
- A grouped structure is specified using grp1 * grp2
- A nested structure is specified via parent / child.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```

- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

The problem

- How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
- Can we exploit relationships between the series to improve the forecasts?

The problem

- How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
- Can we exploit relationships between the series to improve the forecasts?

The solution

- Forecast all series at all levels of aggregation using an automatic forecasting algorithm.

 (e.g., ETS, ARIMA, ...)
- Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
- This is available using reconcile().

Forecast reconciliation

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = 2)
```

66

```
## # A fable: 1,700 x 7 [10]
## # Key: Purpose, State, Region, .model [850]
##
     Purpose State Region
                                 .model
                                            Quarter Trips .mean
     <chr*> <chr*> <chr*>
                                 <chr>
                                                         <dist> <dbl>
##
                                              <atr>
##
  1 Business ACT Canberra
                                 ets
                                            2018 Q1 N(144, 1119) 144.
##
   2 Business ACT Canberra
                                 ets
                                            2018 Q2 N(203, 2260) 203.
##
   3 Business ACT
                  Canberra
                                 ets_adjusted 2018 Q1 N(157, 539) 157.
                                 ets_adjusted 2018 Q2 N(214, 951) 214.
##
   4 Business ACT
                  Canberra
   5 Business ACT
                   <aggregated>
                                            2018 01 N(144, 1119) 144.
##
                                 ets
##
   6 Business ACT
                   <aggregated>
                                 ets
                                            2018 Q2 N(203, 2260) 203.
##
   7 Business ACT
                   <aggregated>
                                 ets_adjusted 2018 Q1 N(157, 539) 157.
## 8 Business ACT
                   <aggregated>
                                 ets adjusted 2018 02 N(214, 951) 214.
```

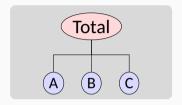
Hierarchical and grouped time series

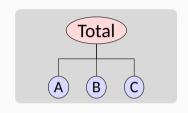
Every collection of time series with aggregation constraints can be written as

$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

where

- \mathbf{y}_t is a vector of all series at time t
- **\mathbf{b}_t** is a vector of the most disaggregated series at time t
- **S** is a "summing matrix' containing the aggregation constraints.

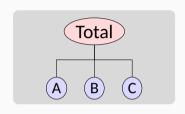




 y_t : observed aggregate of all series at time t.

y_{X,t}: observation on series X at time

b_t: vector of all series at bottom level in time *t*.

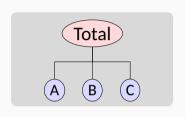


y_t: observed aggregate of all series at time t.

 $y_{X,t}$: observation on series X at time t.

b_t: vector of all series at bottom level in time *t*.

$$\mathbf{y}_{t} = \begin{pmatrix} y_{t} \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$



y_t: observed aggregate of all series at time t.

 $y_{X,t}$: observation on series X at time

b_t: vector of all series at bottom level in time *t*.

$$\mathbf{y}_{t} = \begin{pmatrix} \mathbf{y}_{t} \\ \mathbf{y}_{A,t} \\ \mathbf{y}_{B,t} \\ \mathbf{y}_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{S}} \underbrace{\begin{pmatrix} \mathbf{y}_{A,t} \\ \mathbf{y}_{B,t} \\ \mathbf{y}_{C,t} \end{pmatrix}}_{\mathbf{b}_{t}}$$

Let $\hat{\mathbf{y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as \mathbf{y}_t .

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Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{SG}\hat{\mathbf{y}}_n(h)$$

for some matrix **G**.

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Reconciled forecasts must be of the form:

$$\tilde{\mathbf{y}}_n(h) = \mathbf{S}\mathbf{G}\hat{\mathbf{y}}_n(h)$$

for some matrix G.

- **G** extracts and combines base forecasts $\hat{\mathbf{y}}_n(h)$ to get bottom-level forecasts.
- **S** adds them up

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $G = (S'W_h^{-1}S)^{-1}S'W_h^{-1}$, where W_h is the h-step base forecast error covariance matrix.

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $G = (S'W_h^{-1}S)^{-1}S'W_h^{-1}$, where W_h is the h-step base forecast error covariance matrix.

$$\tilde{\mathbf{y}}_{n}(h) = \mathbf{S}(\mathbf{S}'\mathbf{W}_{h}^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{W}_{h}^{-1}\hat{\mathbf{y}}_{n}(h)$$

Problem: W_h hard to estimate, especially for h > 1.

Solutions:

- Ignore W_h (OLS) [min_trace(method='ols')]
- Assume $W_h = k_h W_1$ is diagonal (WLS) [min_trace(method='wls')]
 - Assume $\mathbf{W}_h = k_h \mathbf{W}_1$ and estimate it (GLS)

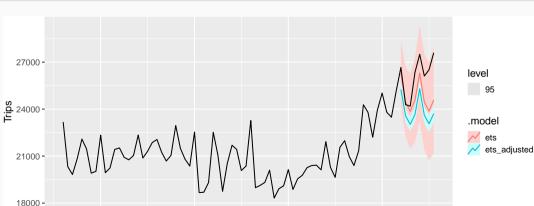
Features

- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with any hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

```
tourism_agg <- tourism %>%
  aggregate key(Purpose * (State / Region),
   Trips = sum(Trips)
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = "2 years")
```

2000 Q1

```
fc %>%
  filter(is_aggregated(Purpose) & is_aggregated(State)) %>%
  autoplot(tourism_agg, level = 95)
```

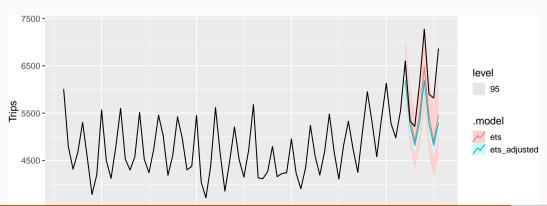


2010 Q1

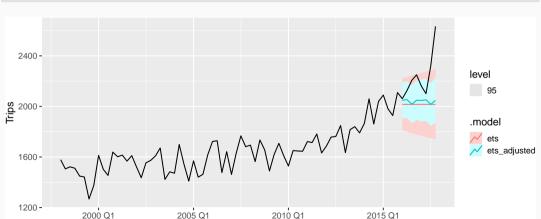
2015 Q1

2005 Q1

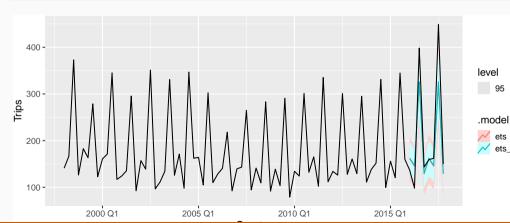
```
fc %>%
  filter(is_aggregated(Purpose) & State == "VIC" &
    is_aggregated(Region)) %>%
  autoplot(tourism_agg, level = 95)
```



```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Melbourne") %>%
  autoplot(tourism_agg, level = 95)
```

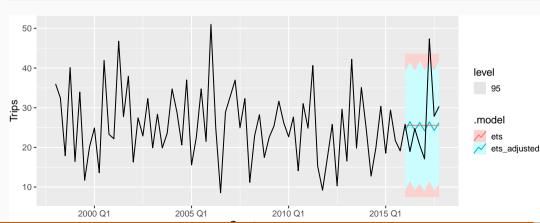


```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Snowy Mountains") %>%
  autoplot(tourism_agg, level = 95)
```

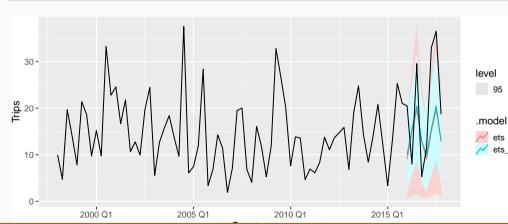


ets_adjusted

```
fc %>%
  filter(Purpose == "Holiday" & Region == "Barossa") %>%
  autoplot(tourism_agg, level = 95)
```



```
fc %>%
  filter(is_aggregated(Purpose) & Region == "MacDonnell") %>%
  autoplot(tourism_agg, level = 95)
```



ets_adjusted

```
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
 model(
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  ) %>%
 mutate(
    comb = (ets + arima) / 2
  ) %>%
  reconcile(
    ets_adj = min_trace(ets),
    arima_adj = min_trace(arima),
    comb adi = min trace(comb)
  ) %>%
  forecast(h = "2 years")
```

Forecast evaluation

```
fc %>% accuracy(tourism_agg)
```

A tibble: 2,550 x 13

Business NSW

10 arima

```
##
      .model Purpose
                       State
                              Region
                                                   .type
                                                            ME
                                                                RMSE
                                                                        MAE
                                                                              MPE
                                                                                    MAPE
                       <chr*> <chr*>
##
      <chr>>
             <chr*>
                                                   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
##
    1 arima
             Business ACT
                              Canberra
                                                ~ Test
                                                         35.9
                                                                45.7 35.9
                                                                             16.9
                                                                                    16.9
##
    2 arima
             Business ACT
                              <aggregated>
                                                  Test
                                                         35.9
                                                                45.7 35.9
                                                                             16.9
                                                                                   16.9
    3 arima
             Business NSW
                              Blue Mountains
                                                ~ Test
                                                          1.93
                                                                 10.6
                                                                      8.52 -
##
18.0
      48.6
    4 arima
             Business NSW
                              Capital Country
                                                                 15.6 10.4
                                                                             11.8
                                                                                   19.0
##
                                                ~ Test
                                                          8.08
    5 arima
##
             Business NSW
                              Central Coast
                                                ~ Test
                                                         10.0
                                                                 14.5 10.8
                                                                             26.9
                                                                                   32.2
##
    6 arima
             Business NSW
                              Central NSW
                                                ~ Test
                                                         17.7
                                                                31.9 28.2
                                                                             12.0
                                                                                    24.1
##
    7 arima
             Business NSW
                              Hunter
                                                ~ Test
                                                         35.3
                                                                43.9 35.3
                                                                             24.2
                                                                                   24.2
##
    8 arima
             Business NSW
                              New England North~ Test
                                                         23.1
                                                                31.8 26.8
                                                                             19.5
                                                                                    28.0
##
    9 arima
             Business NSW
                              North Coast NSW
                                                ~ Test
                                                         24.8
                                                                40.1 36.8
                                                                             11.5
                                                                                    28.5
```

~ Test

6.87

11.0

7.76

Outback NSW

13.7

16.5

Forecast evaluation

```
fc %>%
  accuracy(tourism_agg) %>%
  group_by(.model) %>%
  summarise(MASE = mean(MASE)) %>%
  arrange(MASE)
```

```
## # A tibble: 6 x 2
## .model MASE
## <chr> <dbl>
## 1 ets_adj 1.02
## 2 comb_adj 1.02
## 3 ets 1.04
## 4 comb 1.04
## 5 arima_adj 1.07
## 6 arima 1.09
```

Tidyverts developers

Earo Wang



Mitchell O'Hara-Wild



More information

- Slides and papers: robjhyndman.com
- Packages: tidyverts.org
- Forecasting textbook using fable package:

OTexts.com/fpp3

