Feasts and fables:
Time series analysis
using R

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Outline

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tidyverts.org


```
library(fpp3)
```

```
-- Attaching packages ---------------- fpp3 0.4.0 --
✓ tibble      3.1.4     ✓ tsibble     1.0.1
✓ dplyr       1.0.7     ✓ tsibbledata 0.3.0
✓ tidyr       1.1.3     ✓ feasts      0.2.2
✓ lubridate   1.7.10    ✓ fable       0.3.1
✓ ggplot2     3.3.5
-- Conflicts --------------------- fpp3_conflicts --
X lubridate::date()     masks base::date()
X dplyr::filter()       masks stats::filter()
X tsibble::intersect()  masks base::intersect()
X tsibble::interval()   masks lubridate::interval()
X dplyr::lag()          masks stats::lag()
```

tsibble objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country              GDP Imports Exports Population
##    <dbl> <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.     7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.     8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.     9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.    16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.    18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.    21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.    18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.    14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.    15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.    15.0    10.1    10854428
```

tsibble objects

global_economy

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country          GDP Imports Exports Population
##    Index <fct>          <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.   7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.   8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.   9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.  16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.  18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.  21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.  18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.  14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.  15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.  15.0    10.1    10854428
```

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tsibble objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:        Country [263]
##      Year Country          GDP Imports Exports Population
##      Index  Key           <dbl>  <dbl>   <dbl>     <dbl>
## 1   1960 Afghanistan  537777811.   7.02    4.13   8996351
## 2   1961 Afghanistan  548888896.   8.10    4.45   9166764
## 3   1962 Afghanistan  546666678.   9.35    4.88   9345868
## 4   1963 Afghanistan  751111191.  16.9     9.17   9533954
## 5   1964 Afghanistan  800000044.  18.1     8.89   9731361
## 6   1965 Afghanistan 1006666638.  21.4    11.3    9938414
## 7   1966 Afghanistan 1399999967.  18.6     8.57  10152331
## 8   1967 Afghanistan 1673333418.  14.2     6.77  10372630
## 9   1968 Afghanistan 1373333367.  15.2     8.90  10604346
## 10  1969 Afghanistan 1408888922.  15.0    10.1   10854428
```

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`tsibble` objects

`global_economy`

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:        Country [263]
##      Year Country         GDP Imports Exports Population
##    1 1960 Afghanistan 537777811.    7.02    4.13    8996351
##    2 1961 Afghanistan 548888896.    8.10    4.45    9166764
##    3 1962 Afghanistan 546666678.    9.35    4.88    9345868
##    4 1963 Afghanistan 751111191.   16.9     9.17    9533954
##    5 1964 Afghanistan 800000044.   18.1     8.89    9731361
##    6 1965 Afghanistan 1006666638.  21.4    11.3     9938414
##    7 1966 Afghanistan 1399999967.  18.6     8.57   10152331
##    8 1967 Afghanistan 1673333418.  14.2     6.77   10372630
##    9 1968 Afghanistan 1373333367.  15.2     8.90   10604346
## 10 1969 Afghanistan 1408888922.   15.0    10.1    10854428
```
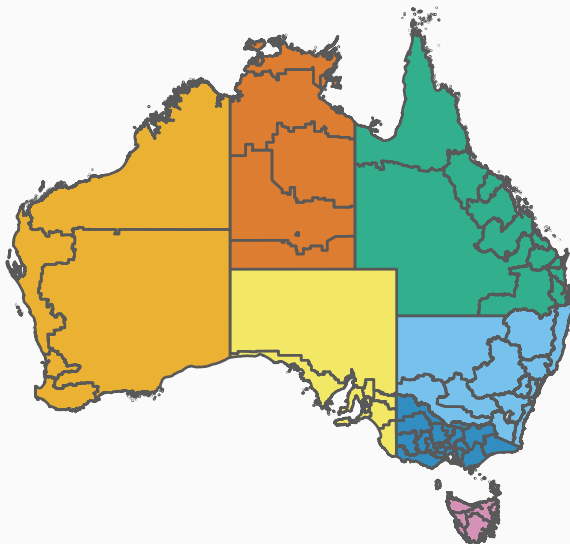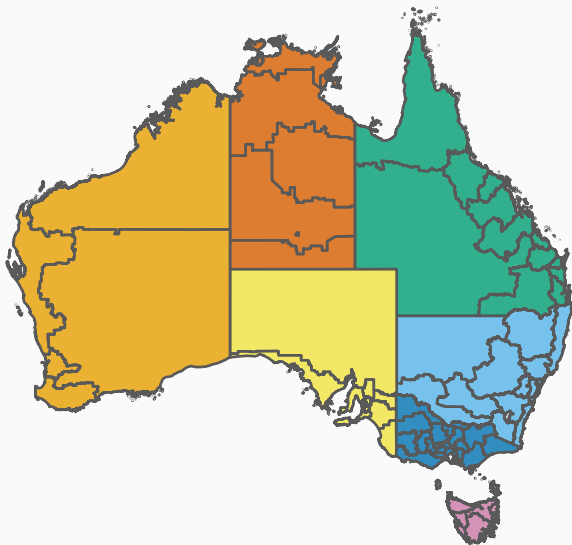
Index | **Key** | **Measured variables**

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Australian tourism regions

State
- New South Wales
- Victoria
- Queensland
- South Australia
- Northern Territory
- Western Australia
- Tasmania
- Australian Capital Territory

Australian tourism regions

- Quarterly data on visitor nights: 1998 – 2017
- From *National Visitor Survey*, interviews of 120,000 Australians aged 15+.
- Geographical hierarchy split by
  - 8 states and territories
  - 76 regions
- Purpose:
  - Holidays
  - Business
  - Visiting friends & relatives
  - Other

tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:        Region, State, Purpose [304]
##     Quarter Region    State Purpose   Trips
##       <qtr> <chr>     <chr> <chr>     <dbl>
##  1 1998 Q1 Adelaide  SA    Business  135.
##  2 1998 Q2 Adelaide  SA    Business  110.
##  3 1998 Q3 Adelaide  SA    Business  166.
##  4 1998 Q4 Adelaide  SA    Business  127.
##  5 1999 Q1 Adelaide  SA    Business  137.
##  6 1999 Q2 Adelaide  SA    Business  200.
##  7 1999 Q3 Adelaide  SA    Business  169.
##  8 1999 Q4 Adelaide  SA    Business  134.
##  9 2000 Q1 Adelaide  SA    Business  154.
## 10 2000 Q2 Adelaide  SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##    Index   <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##     Quarter Region     State Purpose   Trips
##     Index      Keys                    <dbl>
##  1 1998 Q1 Adelaide SA     Business   135.
##  2 1998 Q2 Adelaide SA     Business   110.
##  3 1998 Q3 Adelaide SA     Business   166.
##  4 1998 Q4 Adelaide SA     Business   127.
##  5 1999 Q1 Adelaide SA     Business   137.
##  6 1999 Q2 Adelaide SA     Business   200.
##  7 1999 Q3 Adelaide SA     Business   169.
##  8 1999 Q4 Adelaide SA     Business   134.
##  9 2000 Q1 Adelaide SA     Business   154.
## 10 2000 Q2 Adelaide SA     Business   169.
```

Domestic visitor nights in thousands by state/region and purpose.

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tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:        Region, State, Purpose [304]
##     Quarter Region      State Purpose   Trips
##     [Index] [Keys]                      [Measure]
##  1 1998 Q1 Adelaide    SA    Business   135.
##  2 1998 Q2 Adelaide    SA    Business   110.
##  3 1998 Q3 Adelaide    SA    Business   166.
##  4 1998 Q4 Adelaide    SA    Business   127.
##  5 1999 Q1 Adelaide    SA    Business   137.
##  6 1999 Q2 Adelaide    SA    Business   200.
##  7 1999 Q3 Adelaide    SA    Business   169.
##  8 1999 Q4 Adelaide    SA    Business   134.
##  9 2000 Q1 Adelaide    SA    Business   154.
## 10 2000 Q2 Adelaide    SA    Business   169.
```

Domestic visitor nights in thousands by state/region and purpose.

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`tsibble` objects

- A `tsibble` allows storage and manipulation of multiple time series in R.
- It contains:
  - An index: time information about the observation
  - Measured variable(s): numbers of interest
  - Key variable(s): optional unique identifiers for each series
- It works with tidyverse functions.

Australian prison population

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv")
```

```
## # A tibble: 3,072 x 6
##    date       state gender legal     indigenous count
##    <date>     <chr> <chr>  <chr>     <chr>       <dbl>
##  1 2005-03-01 ACT   Female Remanded  ATSI            0
##  2 2005-03-01 ACT   Female Remanded  Other           2
##  3 2005-03-01 ACT   Female Sentenced ATSI            0
##  4 2005-03-01 ACT   Female Sentenced Other           0
##  5 2005-03-01 ACT   Male   Remanded  ATSI            7
##  6 2005-03-01 ACT   Male   Remanded  Other          58
##  7 2005-03-01 ACT   Male   Sentenced ATSI            0
##  8 2005-03-01 ACT   Male   Sentenced Other           0
##  9 2005-03-01 NSW   Female Remanded  ATSI           51
## 10 2005-03-01 NSW   Female Remanded  Other         131
## # ... with 3,062 more rows
```

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv") %>%
  mutate(Quarter = yearquarter(date))
```

```
## # A tibble: 3,072 x 7
##    date       state gender legal     indigenous count Quarter
##    <date>     <chr> <chr>  <chr>     <chr>      <dbl> <qtr>
##  1 2005-03-01 ACT   Female Remanded  ATSI           0 2005 Q1
##  2 2005-03-01 ACT   Female Remanded  Other          2 2005 Q1
##  3 2005-03-01 ACT   Female Sentenced ATSI           0 2005 Q1
##  4 2005-03-01 ACT   Female Sentenced Other          0 2005 Q1
##  5 2005-03-01 ACT   Male   Remanded  ATSI           7 2005 Q1
##  6 2005-03-01 ACT   Male   Remanded  Other         58 2005 Q1
##  7 2005-03-01 ACT   Male   Sentenced ATSI           0 2005 Q1
##  8 2005-03-01 ACT   Male   Sentenced Other          0 2005 Q1
##  9 2005-03-01 NSW   Female Remanded  ATSI          51 2005 Q1
## 10 2005-03-01 NSW   Female Remanded  Other        131 2005 Q1
```

Read a csv file and convert to a tsibble

```
prison <- readr::read_csv("data/prison_population.csv") %>%
  mutate(Quarter = yearquarter(date)) %>%
  select(-date)
```

```
## # A tibble: 3,072 x 6
##    state gender legal     indigenous count Quarter
##    <chr> <chr>  <chr>     <chr>      <dbl>   <qtr>
## 1 ACT   Female Remanded  ATSI           0 2005 Q1
## 2 ACT   Female Remanded  Other          2 2005 Q1
## 3 ACT   Female Sentenced ATSI           0 2005 Q1
## 4 ACT   Female Sentenced Other          0 2005 Q1
## 5 ACT   Male   Remanded  ATSI           7 2005 Q1
## 6 ACT   Male   Remanded  Other         58 2005 Q1
## 7 ACT   Male   Sentenced ATSI           0 2005 Q1
## 8 ACT   Male   Sentenced Other          0 2005 Q1
## 9 NSW   Female Remanded  ATSI          51 2005 Q1
```

Read a csv file and convert to a tsibble

```r
prison <- readr::read_csv("data/prison_population.csv") %>%
  mutate(Quarter = yearquarter(date)) %>%
  select(-date) %>%
  as_tsibble(
    index = Quarter,
    key = c(state, gender, legal, indigenous)
  )
```

```
## # A tsibble: 3,072 x 6 [1Q]
## # Key:        state, gender, legal, indigenous [64]
##     state gender legal     indigenous count Quarter
##     <chr> <chr>  <chr>     <chr>       <dbl>   <qtr>
## 1 ACT     Female Remanded  ATSI            0 2005 Q1
## 2 ACT     Female Remanded  ATSI            1 2005 Q2
## 3 ACT     Female Remanded  ATSI            0 2005 Q3
## 4 ACT     Female Remanded  ATSI            0 2005 Q4
```

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The `tsibble` index

Common time index variables can be created with these functions:

Frequency	Function
Annual	`start:end`
Quarterly	`yearquarter()`
Monthly	`yearmonth()`
Weekly	`yearweek()`
Daily	`as_date(), ymd()`
Sub-daily	`as_datetime()`

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Australian holidays

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:        Region, State, Purpose [304]
##     Quarter Region    State Purpose   Trips
##       <qtr> <chr>     <chr> <chr>     <dbl>
##  1 1998 Q1 Adelaide   SA    Business  135.
##  2 1998 Q2 Adelaide   SA    Business  110.
##  3 1998 Q3 Adelaide   SA    Business  166.
##  4 1998 Q4 Adelaide   SA    Business  127.
##  5 1999 Q1 Adelaide   SA    Business  137.
##  6 1999 Q2 Adelaide   SA    Business  200.
##  7 1999 Q3 Adelaide   SA    Business  169.
##  8 1999 Q4 Adelaide   SA    Business  134.
##  9 2000 Q1 Adelaide   SA    Business  154.
## 10 2000 Q2 Adelaide   SA    Business  169.
## # ... with 24,310 more rows
```

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Australian holidays

```
holidays <- tourism %>%
  filter(Purpose == "Holiday") %>%
  group_by(State) %>%
  summarise(Trips = sum(Trips))
```

```
## # A tsibble: 640 x 3 [1Q]
## # Key:        State [8]
##    State Quarter Trips
##    <chr>   <qtr> <dbl>
##  1 ACT   1998 Q1  196.
##  2 ACT   1998 Q2  127.
##  3 ACT   1998 Q3  111.
##  4 ACT   1998 Q4  170.
##  5 ACT   1999 Q1  108.
##  6 ACT   1999 Q2  125.
##  7 ACT   1999 Q3  178.
##  8 ACT   1999 Q4  218.
##  9 ACT   2000 Q1  158.
```
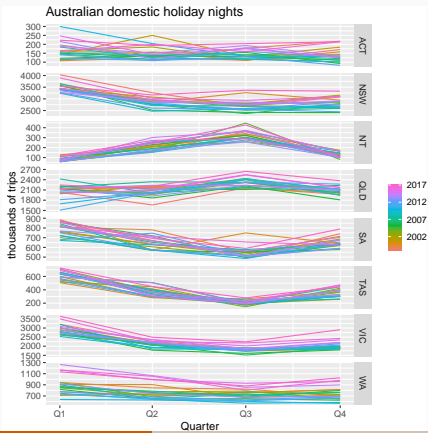
Australian holidays

```
holidays %>% autoplot(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```
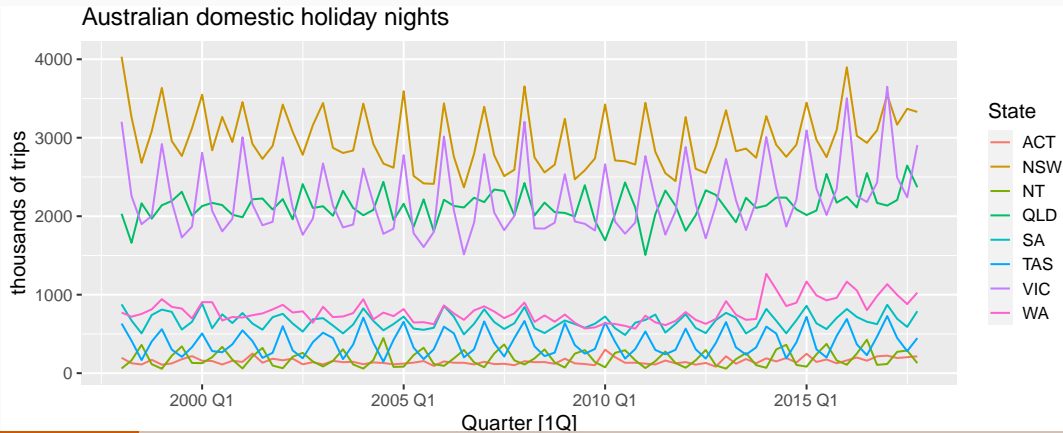
Seasonal plots

```
holidays %>% gg_season(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```


Australian domestic holiday nights

Seasonal subseries plots

```
holidays %>% gg_subseries(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```


Australian domestic holiday nights

Victorian electricity demand

```
vic_elec
```

```
## # A tsibble: 52,608 x 5 [30m] <Australia/Melbourne>
##     Time                Demand Temperature Date       Holiday
##     <dttm>               <dbl>       <dbl> <date>     <lgl>
##  1 2012-01-01 00:00:00  4383.        21.4 2012-01-01 TRUE
##  2 2012-01-01 00:30:00  4263.        21.0 2012-01-01 TRUE
##  3 2012-01-01 01:00:00  4049.        20.7 2012-01-01 TRUE
##  4 2012-01-01 01:30:00  3878.        20.6 2012-01-01 TRUE
##  5 2012-01-01 02:00:00  4036.        20.4 2012-01-01 TRUE
##  6 2012-01-01 02:30:00  3866.        20.2 2012-01-01 TRUE
##  7 2012-01-01 03:00:00  3694.        20.1 2012-01-01 TRUE
##  8 2012-01-01 03:30:00  3562.        19.6 2012-01-01 TRUE
##  9 2012-01-01 04:00:00  3433.        19.1 2012-01-01 TRUE
## 10 2012-01-01 04:30:00  3359.        19.0 2012-01-01 TRUE
## # ... with 52,598 more rows
```

Multiple seasonal periods

```
vic_elec %>% gg_season(Demand)
```
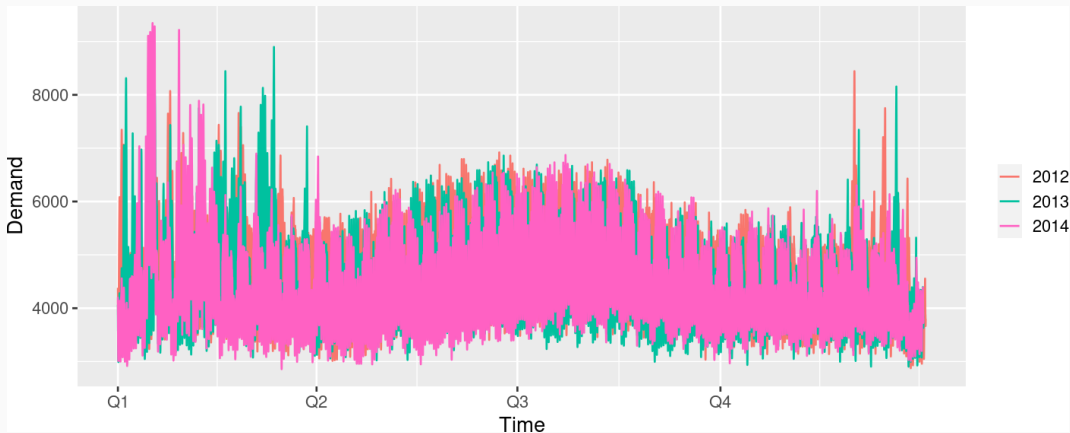
Multiple seasonal periods

```
vic_elec %>% gg_season(Demand, period = "week")
```
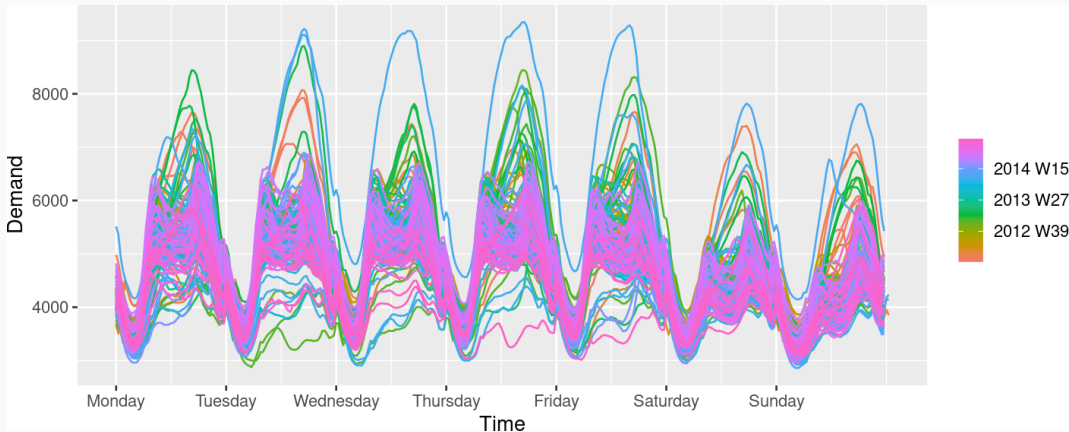
Multiple seasonal periods

```
vic_elec %>% gg_season(Demand, period = "day")
```
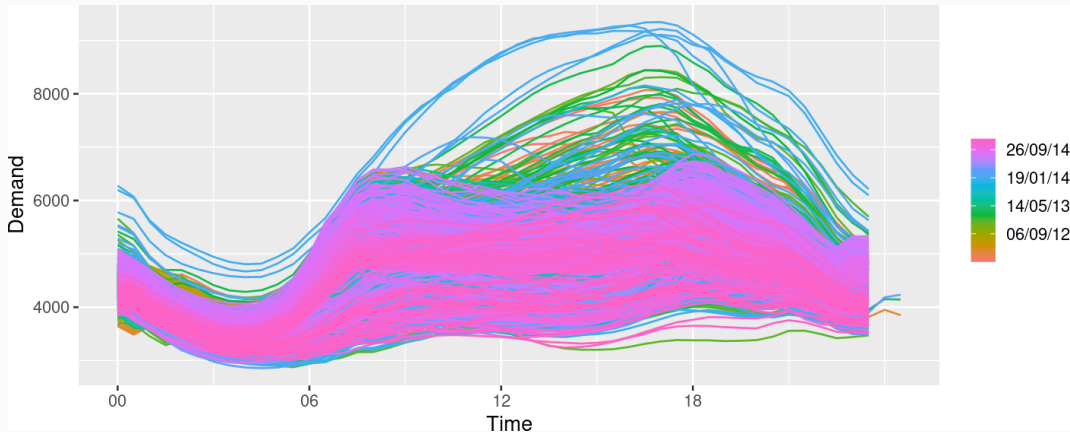
Time plots

- Plotted against time: `autoplot()` (each series overplotted)
- Plotted against season: `gg_season()` (facet by key)
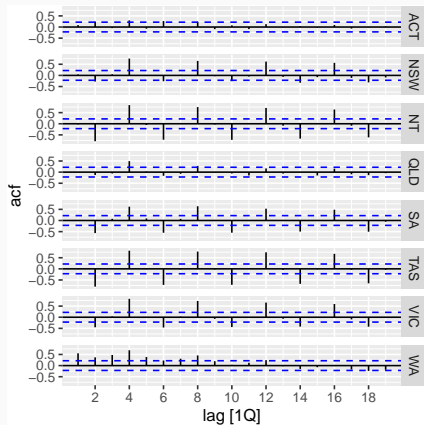- Plotted against time with seasonal facets: `gg_subseries()` (facet by key)

Autocorrelations

```
holidays %>% ACF(Trips)
```

```
## # A tsibble: 152 x 3 [1Q]
## # Key:        State [8]
##    State    lag      acf
##    <chr>  <lag>    <dbl>
##  1 ACT      1Q  0.0877
##  2 ACT      2Q  0.252
##  3 ACT      3Q -0.0496
##  4 ACT      4Q  0.300
##  5 ACT      5Q -0.0741
##  6 ACT      6Q  0.269
##  7 ACT      7Q -0.00504
##  8 ACT      8Q  0.236
##  9 ACT      9Q -0.0953
## 10 ACT     10Q  0.0750
## # ... with 142 more rows
```

ACF plots

```
holidays %>%
  ACF(Trips) %>%
  autoplot()
```

Google stock price

```
gafa_stock
```

```
## # A tsibble: 5,032 x 8 [!]
## # Key:       Symbol [4]
##    Symbol Date        Open  High   Low Close Adj_Close    Volume
##    <chr>  <date>     <dbl> <dbl> <dbl> <dbl>     <dbl>     <dbl>
## 1 AAPL   2014-01-02  79.4  79.6  78.9  79.0      67.0  58671200
## 2 AAPL   2014-01-03  79.0  79.1  77.2  77.3      65.5  98116900
## 3 AAPL   2014-01-06  76.8  78.1  76.2  77.7      65.9 103152700
## 4 AAPL   2014-01-07  77.8  78.0  76.8  77.1      65.4  79302300
## 5 AAPL   2014-01-08  77.0  77.9  77.0  77.6      65.8  64632400
## 6 AAPL   2014-01-09  78.1  78.1  76.5  76.6      65.0  69787200
## 7 AAPL   2014-01-10  77.1  77.3  75.9  76.1      64.5  76244000
```

Google stock price

```
google_2015 <- gafa_stock %>%
  filter(Symbol == "GOOG", year(Date) == 2015) %>%
  select(Date, Close)
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##    Date         Close
##    <date>       <dbl>
##  1 2015-01-02   522.
##  2 2015-01-05   511.
##  3 2015-01-06   499.
##  4 2015-01-07   498.
##  5 2015-01-08   500.
```

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Google stock price

```
google_2015 %>% autoplot(Close)
```
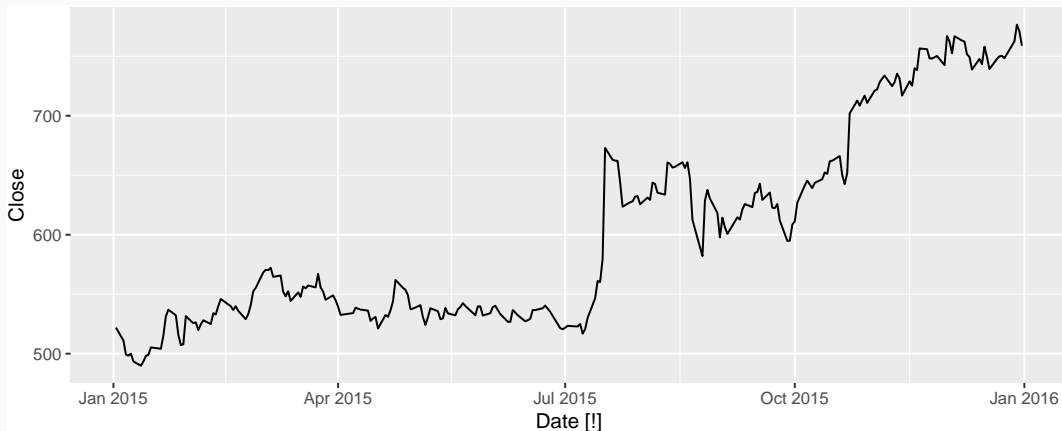
Google stock price

```
google_2015 %>%
  ACF(Close, lag_max = 100)
```

```
## Warning: Provided data has an irregular interval, results should be treated with
## caution. Computing ACF by observation.

## # A tsibble: 100 x 2 [1]
##      lag   acf
##    <lag> <dbl>
## 1     1 0.982
## 2     2 0.959
## 3     3 0.937
## 4     4 0.918
## 5     5 0.901
## 6     6 0.883
## 7     7 0.865
## 8     8 0.849
## 9     9 0.834
## 10   10 0.818
## #    with 90 more rows
```

Google stock price

```
google_2015 %>%
  ACF(Close, lag_max = 100) %>%
  autoplot()
```
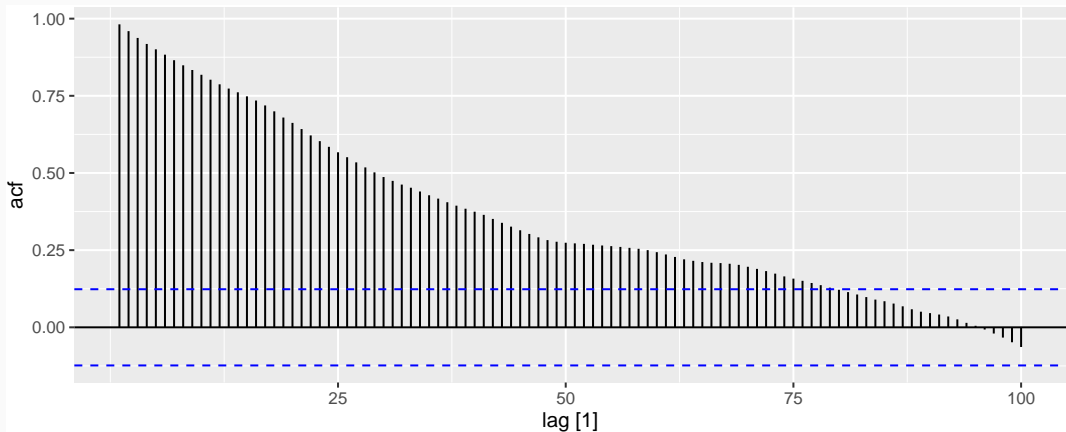
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Strength of seasonality and trend

STL decomposition

$$y_t = T_t + S_t + R_t$$

Seasonal strength

$$\max \left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)} \right)$$

Trend strength

$$\max \left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(T_t + R_t)} \right)$$

Feature extraction and statistics

```
tourism %>% features(Trips, feat_stl)
```

```
## # A tibble: 304 x 12
##    Region        State Purpose  trend_strength seasonal_strength_~ seasonal_peak_ye~
##    <chr>         <chr> <chr>             <dbl>               <dbl>             <dbl>
##  1 Adelaide      SA    Business          0.464               0.407                 3
##  2 Adelaide      SA    Holiday           0.554               0.619                 1
##  3 Adelaide      SA    Other             0.746               0.202                 2
##  4 Adelaide      SA    Visiting          0.435               0.452                 1
##  5 Adelaide Hills SA   Business          0.464               0.179                 3
##  6 Adelaide Hills SA   Holiday           0.528               0.296                 2
##  7 Adelaide Hills SA   Other             0.593               0.404                 2
##  8 Adelaide Hills SA   Visiting          0.488               0.254                 0
##  9 Alice Springs NT    Business          0.534               0.251                 0
## 10 Alice Springs NT    Holiday           0.381               0.832                 3
## # ... with 294 more rows, and 6 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>
```

Feature extraction and statistics

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```

Feature extraction and statistics

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```


- Holidays more seasonal than other travel.
- WA has strongest trends.

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Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

```
tourism %>%
  right_join(most_seasonal, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```
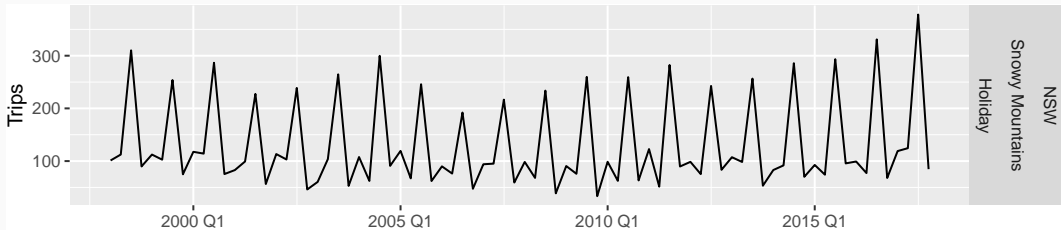
Feature extraction and statistics

Find the most trended time series:

```
most_trended <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength))
```

Feature extraction and statistics

Find the most trended time series:

```
most_trended <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength))
```

```
tourism %>%
  right_join(most_trended, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```


38
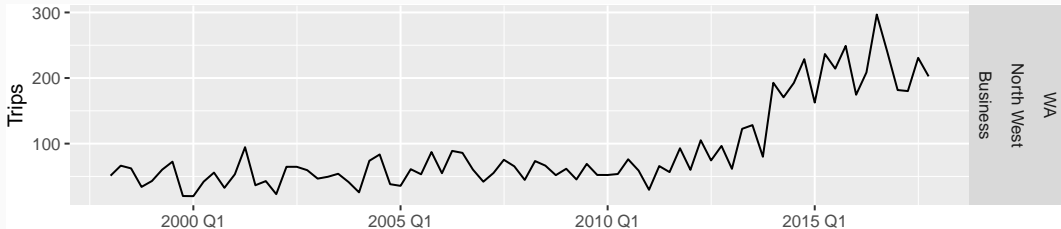
Feature extraction and statistics

```
tourism_features <- tourism %>%
  features(Trips, feature_set(pkgs = "feasts"))
```

All features from the feasts package

```
## # A tibble: 304 x 51
##    Region         State Purpose    trend_strength seasonal_strength_~ seasonal_peak_ye~
##    <chr>          <chr> <chr>              <dbl>               <dbl>             <dbl>
##  1 Adelaide       SA    Business           0.464               0.407                 3
##  2 Adelaide       SA    Holiday            0.554               0.619                 1
##  3 Adelaide       SA    Other              0.746               0.202                 2
##  4 Adelaide       SA    Visiting           0.435               0.452                 1
##  5 Adelaide Hills SA    Business           0.464               0.179                 3
##  6 Adelaide Hills SA    Holiday            0.528               0.296                 2
##  7 Adelaide Hills SA    Other              0.593               0.404                 2
##  8 Adelaide Hills SA    Visiting           0.488               0.254                 0
##  9 Alice Springs  NT    Business           0.534               0.251                 0
## 10 Alice Springs  NT    Holiday            0.381               0.832                 3
## # ... with 294 more rows, and 45 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>, nacf5 <dbl>,
```

Feature extraction and statistics

```
pcs <- tourism_features %>%
  select(-State, -Region, -Purpose) %>%
  prcomp(scale = TRUE) %>%
  broom::augment(tourism_features)
```
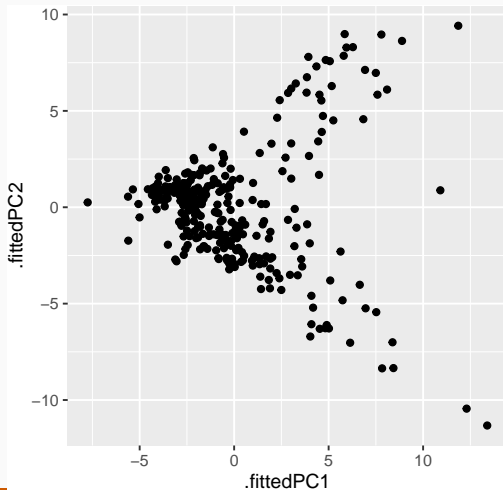
Principal components based on all features from the feasts package

```
## # A tibble: 304 x 100
##    .rownames Region         State Purpose  trend_strength seasonal_streng~ seasonal_peak_y~
##    <chr>     <chr>          <chr> <chr>             <dbl>            <dbl>            <dbl>
##  1 1         Adelaide       SA    Business          0.464            0.407                3
##  2 2         Adelaide       SA    Holiday           0.554            0.619                1
##  3 3         Adelaide       SA    Other             0.746            0.202                2
##  4 4         Adelaide       SA    Visiting          0.435            0.452                1
##  5 5         Adelaide Hills SA    Business          0.464            0.179                3
##  6 6         Adelaide Hills SA    Holiday           0.528            0.296                2
##  7 7         Adelaide Hills SA    Other             0.593            0.404                2
##  8 8         Adelaide Hills SA    Visiting          0.488            0.254                0
##  9 9         Alice Springs  NT    Business          0.534            0.251                0
## 10 10        Alice Springs  NT    Holiday           0.381            0.832                3
## # ... with 294 more rows, and 93 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>, pacf5 <dbl>,
```

Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +
  geom_point() + theme(aspect.ratio=1)
```
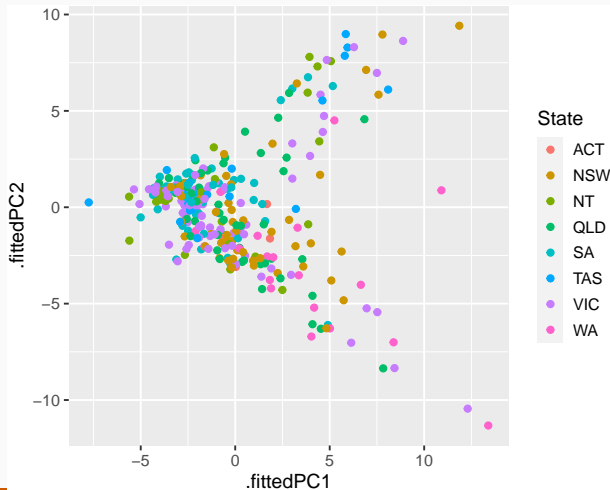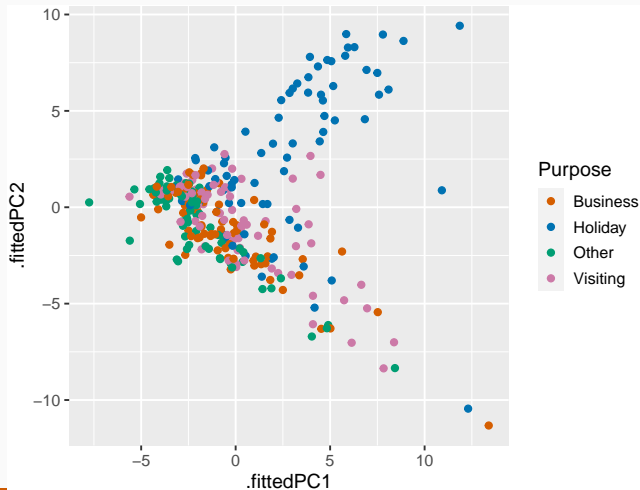
Principal components based on all features from the feasts package

Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package

Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package

Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package
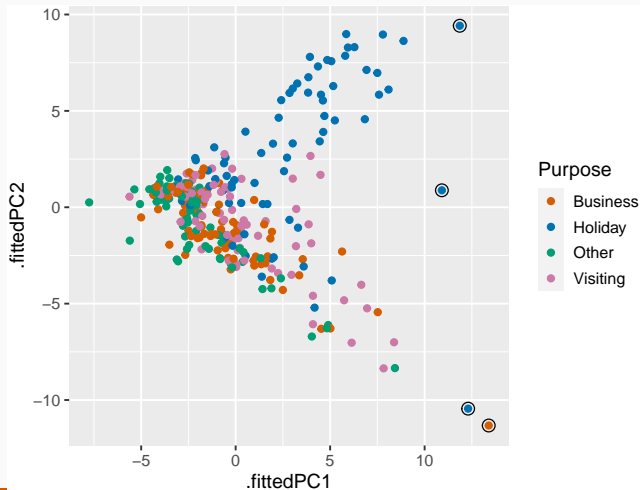
Feature extraction and statistics

```
outliers %>%
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%
  mutate(Series = glue("{State}", "{Region}", "{Purpose}", .sep = "\n\n")) %>%
  ggplot(aes(x = Quarter, y = Trips)) + geom_line() +
  facet_grid(Series ~ ., scales = "free_y") +
  ggtitle("Outlying time series in PC space")
```
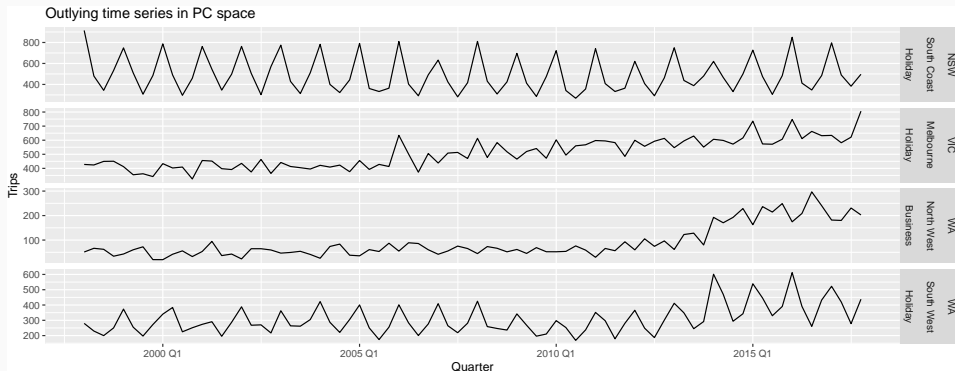

Outlying time series in PC space

Outline

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Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

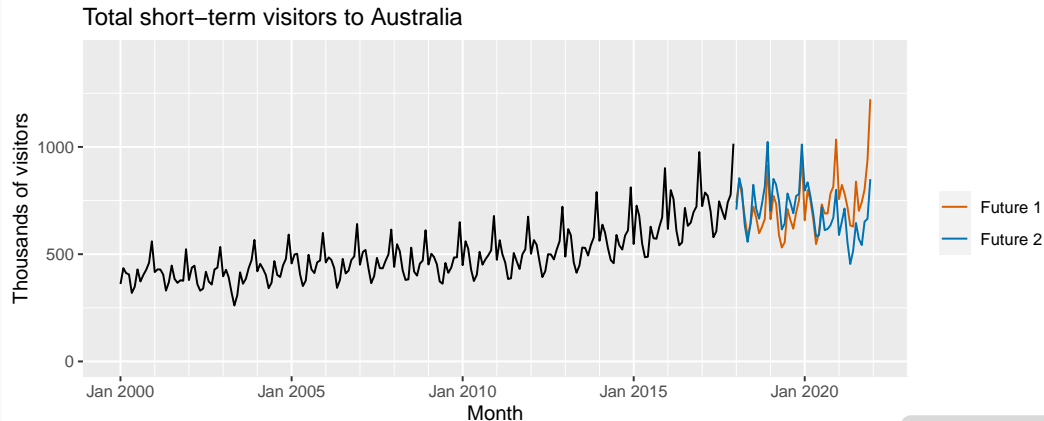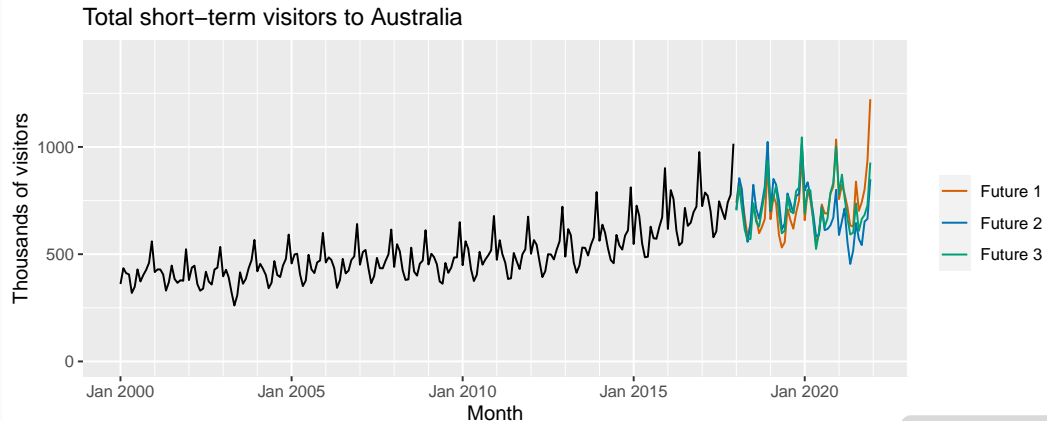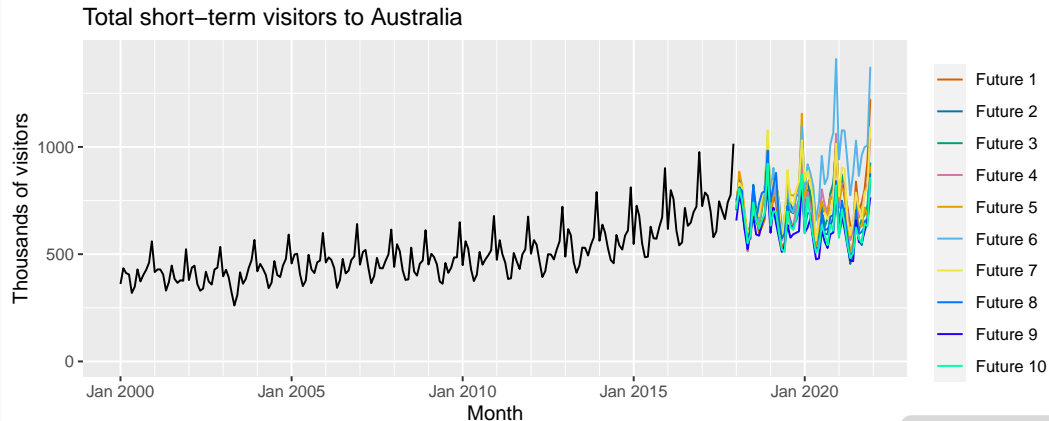Total short−term visitors to Australia

Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

Total short–term visitors to Australia

Simulated futures from an ETS model

Random futures

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Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

Total short–term visitors to Australia

Simulated futures from an ETS model

Model fitting

```
holiday_fit <- holidays %>%
  model(
    snaive = SNAIVE(Trips),
    naive = NAIVE(Trips),
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  )
```

```
## # A mable: 8 x 5
## # Key:     State [8]
##   State  snaive   naive    ets                                    arima
##   <chr>  <model>  <model>  <model>                              <model>
## 1 ACT    <SNAIVE> <NAIVE>  <ETS(M,N,A)>           <ARIMA(0,1,1)(1,0,1)[4]>
## 2 NSW    <SNAIVE> <NAIVE>  <ETS(M,N,A)>           <ARIMA(0,1,1)(0,1,1)[4]>
## 3 NT     <SNAIVE> <NAIVE>  <ETS(M,N,A)>           <ARIMA(0,0,0)(2,1,0)[4]>
## 4 QLD    <SNAIVE> <NAIVE>  <ETS(A,N,A)> <ARIMA(0,0,0)(1,0,0)[4] w/ mean>
```

Model fitting

```
holiday_fit %>%
  filter(State == "VIC") %>%
  select(arima) %>%
  report()

## Series: Trips
## Model: ARIMA(0,1,1)(0,1,1)[4]
##
## Coefficients:
##           ma1     sma1
##       -0.8079  -0.5758
## s.e.   0.0802   0.1640
##
## sigma^2 estimated as 25711:  log likelihood=-487.82
## AIC=981.64   AICc=981.98   BIC=988.6
```

Model fitting

```
glance(holiday_fit)
```

```
## # A tibble: 32 x 12
##    State .model sigma2 log_lik   AIC  AICc   BIC    MSE   AMSE    MAE ar_roots
##    <chr> <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>  <dbl> <list>
##  1 ACT   snaive 2.15e+3     NA    NA    NA    NA     NA     NA     NA <NULL>
##  2 ACT   naive  2.93e+3     NA    NA    NA    NA     NA     NA     NA <NULL>
##  3 ACT   ets    6.80e-2   -463.  940.  941.  956.  1509.  1538.  0.189 <NULL>
##  4 ACT   arima  1.44e+3   -399.  805.  806.  815.    NA     NA     NA <cpl [4~
##  5 NSW   snaive 4.65e+4     NA    NA    NA    NA     NA     NA     NA <NULL>
##  6 NSW   naive  2.54e+5     NA    NA    NA    NA     NA     NA     NA <NULL>
##  7 NSW   ets    3.47e-3   -585. 1184. 1185. 1201. 28271. 29225. 0.0458 <NULL>
##  8 NSW   arima  3.30e+4   -498. 1003. 1003. 1010.    NA     NA     NA <cpl [0~
##  9 NT    snaive 2.45e+3     NA    NA    NA    NA     NA     NA     NA <NULL>
## 10 NT    naive  2.11e+4     NA    NA    NA    NA     NA     NA     NA <NULL>
## # ... with 22 more rows, and 1 more variable: ma_roots <list>
```

Model fitting

```
tidy(holiday_fit)
```

```
## # A tibble: 75 x 7
##    State .model term  estimate std.error statistic  p.value
##    <chr> <chr>  <chr>    <dbl>     <dbl>     <dbl>    <dbl>
##  1 ACT   ets    alpha   0.0513    NA        NA      NA
##  2 ACT   ets    gamma   0.482     NA        NA      NA
##  3 ACT   ets    l[0]  148.        NA        NA      NA
##  4 ACT   ets    s[0]    7.88      NA        NA      NA
##  5 ACT   ets    s[-1]  -2.66      NA        NA      NA
##  6 ACT   ets    s[-2] -20.5       NA        NA      NA
##  7 ACT   ets    s[-3]  15.3       NA        NA      NA
##  8 ACT   arima  ma1    -0.886     0.0668   -13.3    8.33e-22
##  9 ACT   arima  sar1    0.773     0.170      4.55   1.93e- 5
## 10 ACT   arima  sma1   -0.557     0.220     -2.52   1.36e- 2
## # ... with 65 more rows
```

Model fitting

```
augment(holiday_fit)
```

```
## # A tsibble: 2,560 x 7 [1Q]
## # Key:        State, .model [32]
##    State .model Quarter Trips .fitted .resid .innov
##    <chr> <chr>    <qtr> <dbl>   <dbl>  <dbl>  <dbl>
##  1 ACT   snaive 1998 Q1  196.     NA     NA     NA
##  2 ACT   snaive 1998 Q2  127.     NA     NA     NA
##  3 ACT   snaive 1998 Q3  111.     NA     NA     NA
##  4 ACT   snaive 1998 Q4  170.     NA     NA     NA
##  5 ACT   snaive 1999 Q1  108.    196.  -88.4  -88.4
##  6 ACT   snaive 1999 Q2  125.    127.  -2.13  -2.13
##  7 ACT   snaive 1999 Q3  178.    111.   67.3   67.3
##  8 ACT   snaive 1999 Q4  218.    170.   47.2   47.2
##  9 ACT   snaive 2000 Q1  158.    108.   50.6   50.6
## 10 ACT   snaive 2000 Q2  155.    125.   30.2   30.2
```

Ljung-Box test

```
augment(holiday_fit) %>%
  filter(State == "VIC", .model == "arima") %>%
  features(.resid, ljung_box, dof = 2, lag = 8)
```

```
## # A tibble: 1 x 4
##    State .model lb_stat lb_pvalue
##    <chr> <chr>    <dbl>     <dbl>
## 1 VIC    arima     5.42     0.492
```

gg_tsresiduals() function

```
holiday_fit %>%
  filter(State == "VIC") %>%
  select(arima) %>%
  gg_tsresiduals()
```
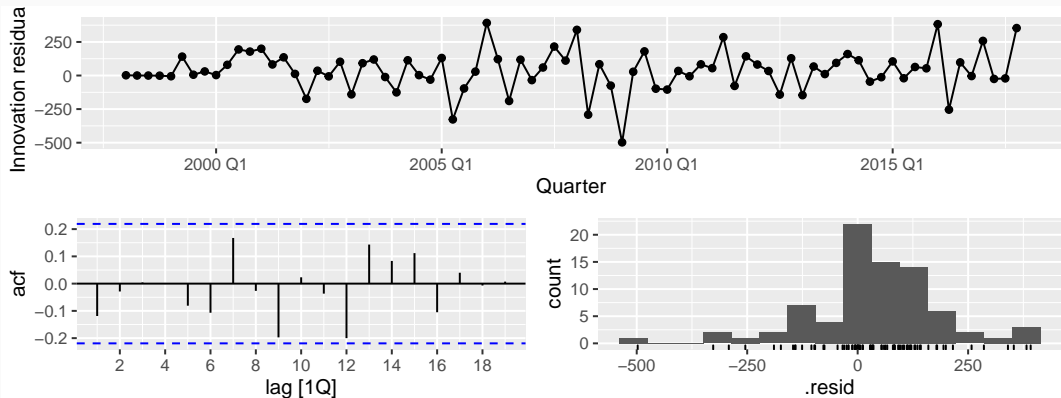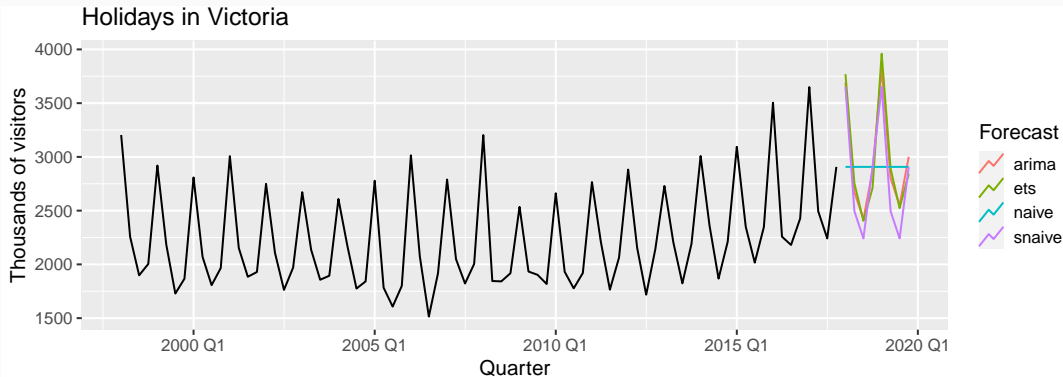
Producing forecasts

```
holiday_fc <- holiday_fit %>%
  forecast(h = "2 years")

## # A fable: 256 x 5 [1Q]
## # Key:      State, .model [32]
##    State .model Quarter        Trips .mean
##    <chr> <chr>    <qtr>        <dist> <dbl>
##  1 ACT   snaive 2018 Q1 N(223, 2128)  223.
##  2 ACT   snaive 2018 Q2 N(193, 2128)  193.
##  3 ACT   snaive 2018 Q3 N(204, 2128)  204.
##  4 ACT   snaive 2018 Q4 N(214, 2128)  214.
##  5 ACT   snaive 2019 Q1 N(223, 4257)  223.
##  6 ACT   snaive 2019 Q2 N(193, 4257)  193.
##  7 ACT   snaive 2019 Q3 N(204, 4257)  204.
##  8 ACT   snaive 2019 Q4 N(214, 4257)  214.
##  9 ACT   naive  2018 Q1 N(214, 2894)  214.
## 10 ACT   naive  2018 Q2 N(214, 5788)  214.
## # ... with 246 more rows
```

Visualising forecasts

```
holiday_fc %>%
  filter(State == "VIC") %>%
  autoplot(holidays, level = NULL) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```

Visualising forecasts

```
holiday_fc %>%
  filter(State == "VIC", .model == "arima") %>%
  autoplot(holidays) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```
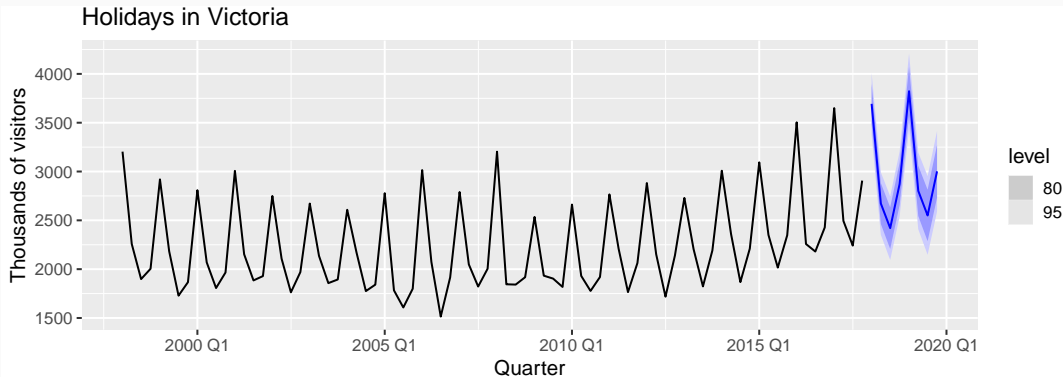
Prediction intervals

```
holiday_fc %>% hilo(level = 95)
```

```
## # A tsibble: 256 x 6 [1Q]
## # Key:       State, .model [32]
##    State .model Quarter        Trips .mean                    `95%`
##    <chr> <chr>    <qtr>        <dist> <dbl>                    <hilo>
##  1 ACT   snaive 2018 Q1 N(223, 2128)  223. [132.71289, 313.5540]95
##  2 ACT   snaive 2018 Q2 N(193, 2128)  193. [103.06575, 283.9069]95
##  3 ACT   snaive 2018 Q3 N(204, 2128)  204. [113.21396, 294.0551]95
##  4 ACT   snaive 2018 Q4 N(214, 2128)  214. [124.04311, 304.8842]95
##  5 ACT   snaive 2019 Q1 N(223, 4257)  223. [ 95.25947, 351.0074]95
##  6 ACT   snaive 2019 Q2 N(193, 4257)  193. [ 65.61234, 321.3603]95
##  7 ACT   snaive 2019 Q3 N(204, 4257)  204. [ 75.76054, 331.5085]95
##  8 ACT   snaive 2019 Q4 N(214, 4257)  214. [ 86.58969, 342.3376]95
##  9 ACT   naive  2018 Q1 N(214, 2894)  214. [109.02634, 319.9010]95
```

Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.

Outline

Training and test sets

Training data · · · · · · · · · · · · · · · · · · Test data → time

- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Measures of forecast accuracy

Holidays in Victoria
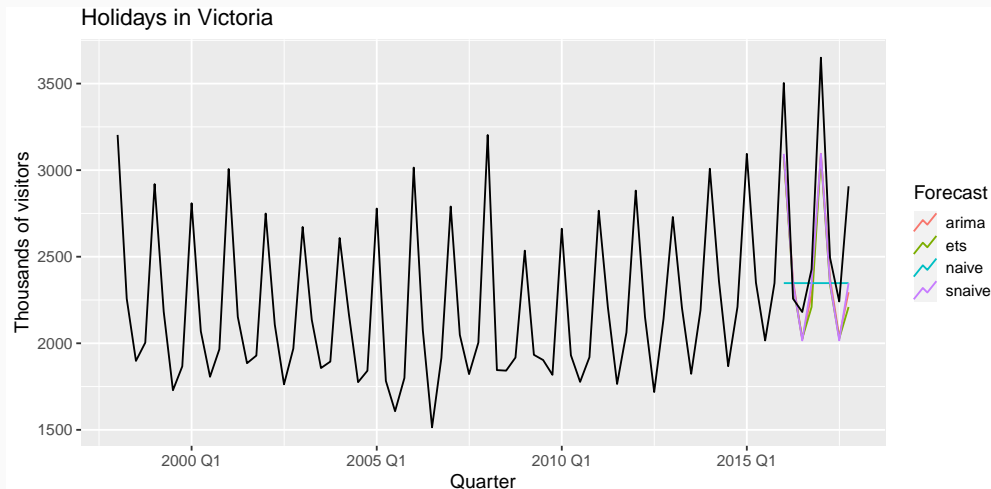
Measures of forecast accuracy

```
accuracy(vic_fc, holidays)
```

```
## # A tibble: 4 x 11
##   .model State .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 arima  VIC   Test   263.  349.  290.  8.83  10.0  2.28  2.07 -0.230
## 2 ets    VIC   Test   295.  378.  315. 10.0   10.9  2.48  2.25 -0.149
## 3 naive  VIC   Test   360.  654.  451. 10.1   14.2  3.54  3.88 -0.181
## 4 snaive VIC   Test   256.  336.  279.  8.59  9.61  2.19  1.99 -0.257
```

Time series cross-validation

Traditional evaluation
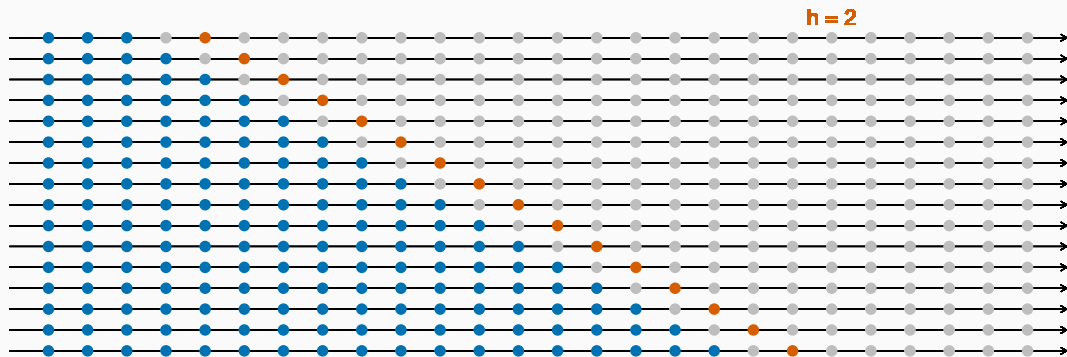

Training data   Test data

time

Time series cross-validation

Traditional evaluation

Time series cross-validation

Time series cross-validation

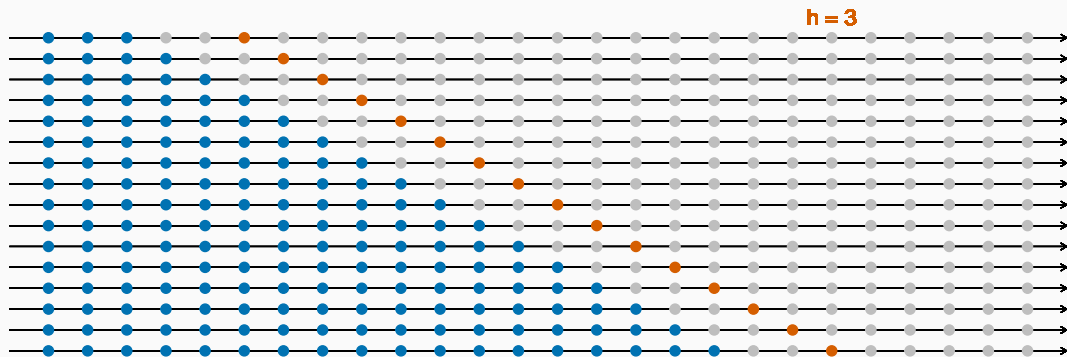## Traditional evaluation

Time series cross-validation

Time series cross-validation

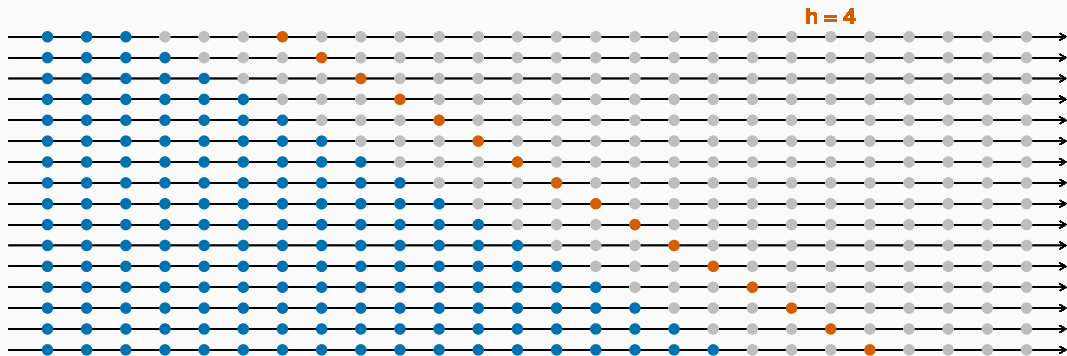## Traditional evaluation

Time series cross-validation

Time series cross-validation

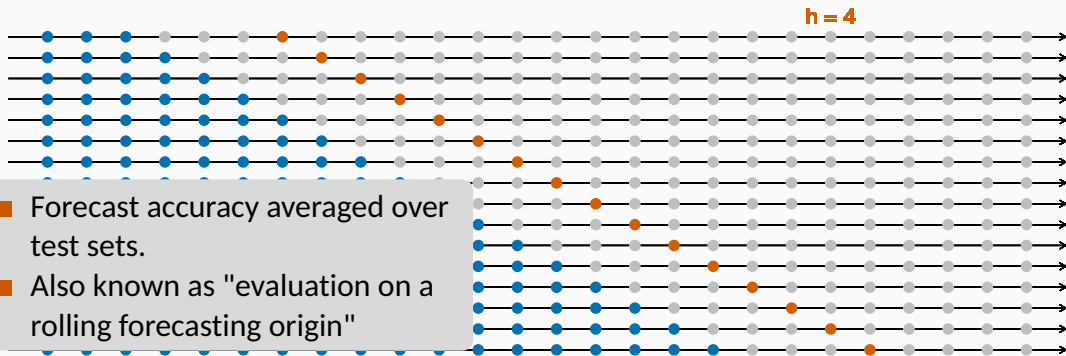## Traditional evaluation

Time series cross-validation

Time series cross-validation

Traditional evaluation

Time series cross-validation

- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"
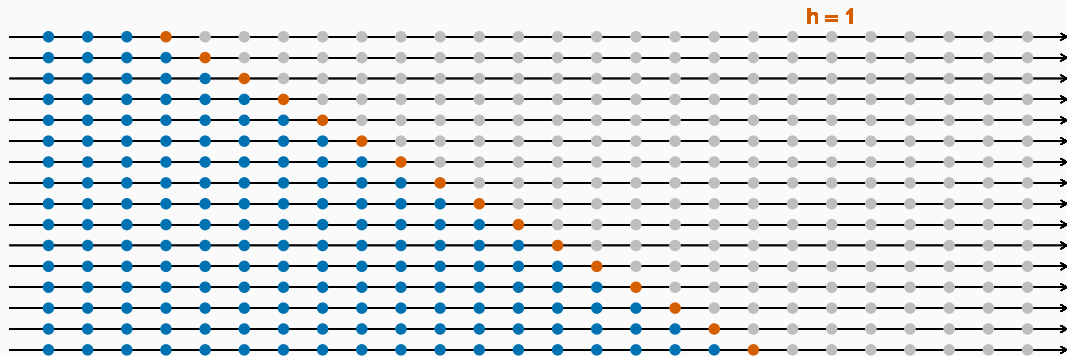
Time series cross-validation

Stretch with a minimum length of 4 years, growing by 1 quarter each step.

```
vic_holiday_stretch <- holidays %>%
  filter(State == "VIC") %>%
  stretch_tsibble(.init = 16, .step = 1)

## # A tsibble: 3,120 x 4 [1Q]
## # Key:        .id, State [65]
##    State Quarter Trips   .id
##    <chr>   <qtr> <dbl> <int>
## 1 VIC    1998 Q1 3204.    1
## 2 VIC    1998 Q2 2258.    1
## 3 VIC    1998 Q3 1898.    1
## 4 VIC    1998 Q4 2004.    1
## 5 VIC    1999 Q1 2919.    1
## 6 VIC    1999 Q2 2183.    1
```

Time series cross-validation

```
fit_cv <- vic_holiday_stretch %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips),
    snaive = SNAIVE(Trips)
  )

## # A mable: 65 x 5
## # Key:     .id, State [65]
##     .id State            ets                       arima   snaive
##   <int> <chr>        <model>                     <model>  <model>
## 1     1 VIC    <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 2     2 VIC    <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 3     3 VIC    <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 4     4 VIC    <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
## 5     5 VIC    <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
```

Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%
  forecast(h = 1)

## # A fable: 195 x 6 [1Q]
## # Key:     .id, State, .model [195]
##     .id State Quarter         Trips .mean .model
##   <int> <chr>   <qtr>        <dist> <dbl> <chr>
## 1     1 VIC    2002 Q1 N(2965, 14682) 2965. ets
## 2     1 VIC    2002 Q1 N(2980, 11447) 2980. arima
## 3     1 VIC    2002 Q1 N(3007, 19035) 3007. snaive
## 4     2 VIC    2002 Q2  N(2024, 8130) 2024. ets
## 5     2 VIC    2002 Q2 N(1961, 14864) 1961. arima
## 6     2 VIC    2002 Q2 N(2153, 22669) 2153. snaive
## # ... with 189 more rows
```

Time series cross-validation

```
# Cross-validated
fc_cv %>% accuracy(holidays)
```

```
## # A tibble: 3 x 11
##    .model State .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE    ACF1
##    <chr>  <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>   <dbl>
## 1 arima  VIC   Test   46.3  183.  147.  1.57  6.41  1.08  1.02  -0.0615
## 2 ets    VIC   Test   43.3  179.  139.  1.51  6.02  1.02  0.997 -0.110
## 3 snaive VIC   Test   36.2  186.  138.  1.14  6.01  1.02  1.04   0.0427
```
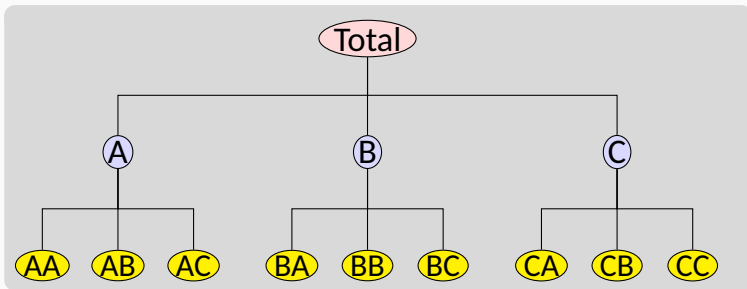
A good way to choose the best forecasting model is to find the model with the smallest
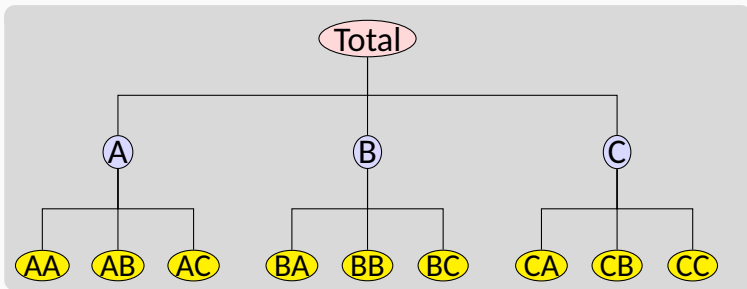RMSE computed using time series cross-validation.

Outline

Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.
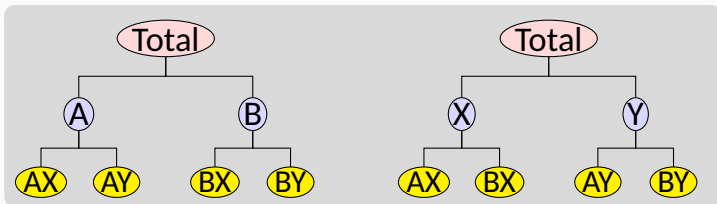
Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.

Examples
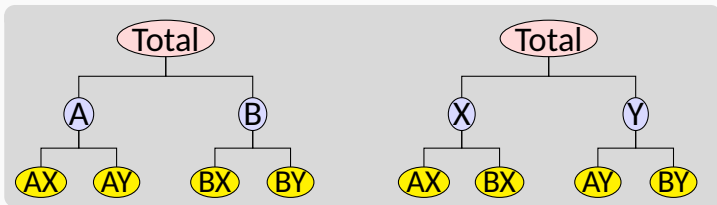- Tourism demand by states, zones, regions

Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.

Grouped time series

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Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

Creating aggregates

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  filter(Quarter == yearquarter("1998 Q1")) %>%
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]
## # Key:       Purpose, State, Region [425]
##    Quarter Purpose      State        Region         Trips
##      <qtr> <chr*>       <chr*>       <chr*>         <dbl>
##  1 1998 Q1 <aggregated> <aggregated> <aggregated>  23182.
##  2 1998 Q1 Business     <aggregated> <aggregated>   3599.
##  3 1998 Q1 Holiday      <aggregated> <aggregated>  11806.
##  4 1998 Q1 Other        <aggregated> <aggregated>    680.
##  5 1998 Q1 Visiting     <aggregated> <aggregated>   7098.
##  6 1998 Q1 <aggregated> ACT          <aggregated>    551.
##  7 1998 Q1 <aggregated> NSW          <aggregated>   8040.
##  8 1998 Q1 <aggregated> NT           <aggregated>    181.
##  9 1998 Q1 <aggregated> QLD          <aggregated>   4041.
## 10 1998 Q1 <aggregated> SA           <aggregated>   1735.
## 11 1998 Q1 <aggregated> TAS          <aggregated>    982.
## 12 1998 Q1 <aggregated> VIC          <aggregated>   6010.
## 13 1998 Q1 <aggregated> WA           <aggregated>   1641.
```

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Creating aggregates

- Similar to `summarise()` but using the key structure
- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```

- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

The problem

1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

The problem

1 How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2 Can we exploit relationships between the series to improve the forecasts?

The solution

1 Forecast all series at all levels of aggregation using an automatic forecasting algorithm.
(e.g., `ETS`, `ARIMA`, ...)
2 Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
3 This is available using `reconcile()`.

Forecast reconciliation

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = 2)

## # A fable: 1,700 x 7 [1Q]
## # Key:     Purpose, State, Region, .model [850]
##    Purpose  State  Region      .model       Quarter         Trips .mean
##    <chr*>   <chr*> <chr*>      <chr>         <qtr>          <dist> <dbl>
##  1 Business ACT    Canberra    ets          2018 Q1  N(144, 1119) 144.
##  2 Business ACT    Canberra    ets          2018 Q2  N(203, 2260) 203.
##  3 Business ACT    Canberra    ets_adjusted 2018 Q1   N(157, 539) 157.
##  4 Business ACT    Canberra    ets_adjusted 2018 Q2   N(214, 951) 214.
##  5 Business ACT    <aggregated> ets         2018 Q1  N(144, 1119) 144.
##  6 Business ACT    <aggregated> ets         2018 Q2  N(203, 2260) 203.
##  7 Business ACT    <aggregated> ets_adjusted 2018 Q1  N(157, 539) 157.
##  8 Business ACT    <aggregated> ets_adjusted 2018 Q2  N(214, 951) 214.
```

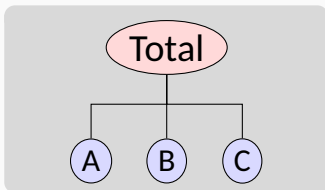# Hierarchical and grouped time series

Every collection of time series with aggregation constraints can be written as

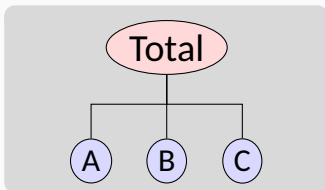$$\mathbf{y}_t = \mathbf{S}\mathbf{b}_t$$

where

- $\mathbf{y}_t$ is a vector of all series at time t
- $\mathbf{b}_t$ is a vector of the most disaggregated series at time t
- $\mathbf{S}$ is a "summing matrix'' containing the aggregation constraints.

Hierarchical time series

y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

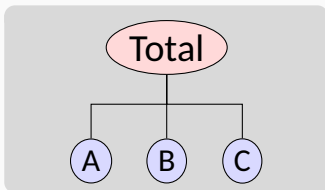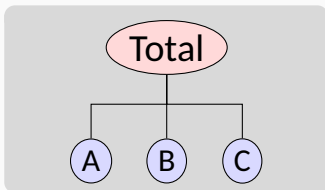# Hierarchical time series

y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

b_t : vector of all series at bottom level in time t.

$$\mathbf{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

Hierarchical time series

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$$\boldsymbol{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{S} \underbrace{\begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}}_{\boldsymbol{b}_t}$$

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

Forecasting notation

Let $\hat{\mathbf{y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\mathbf{y}_t$.

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$$\tilde{\mathbf{y}}_n(h) = \mathbf{SG}\hat{\mathbf{y}}_n(h)$$

for some matrix $\mathbf{G}$.

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Reconciled forecasts must be of the form:
$$\tilde{\mathbf{y}}_n(h) = \mathbf{SG}\hat{\mathbf{y}}_n(h)$$

for some matrix $\mathbf{G}$.

- $\mathbf{G}$ extracts and combines base forecasts $\hat{\mathbf{y}}_n(h)$ to get bottom-level forecasts.
- $\mathbf{S}$ adds them up

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $G = (S'W_h^{-1}S)^{-1}S'W_h^{-1}$, where W_h is the h-step base forecast error covariance matrix.

Optimal combination forecasts

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The best (minimum sum of variances) unbiased forecasts are obtained when $G = (S'W_h^{-1}S)^{-1}S'W_h^{-1}$, where W_h is the h-step base forecast error covariance matrix.

$$\tilde{y}_n(h) = S(S'W_h^{-1}S)^{-1}S'W_h^{-1}\hat{y}_n(h)$$

Problem: W_h hard to estimate, especially for $h > 1$.

Solutions:
- Ignore W_h (OLS) [`min_trace(method='ols')`]
- Assume $W_h = k_h W_1$ is diagonal (WLS) [`min_trace(method='wls')`]
- Assume $W_h = k_h W_1$ and estimate it (GLS)

Features

- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

Example: Australian tourism

```
tourism_agg <- tourism %>%
  aggregate_key(Purpose * (State / Region),
    Trips = sum(Trips)
  )
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = "2 years")
```
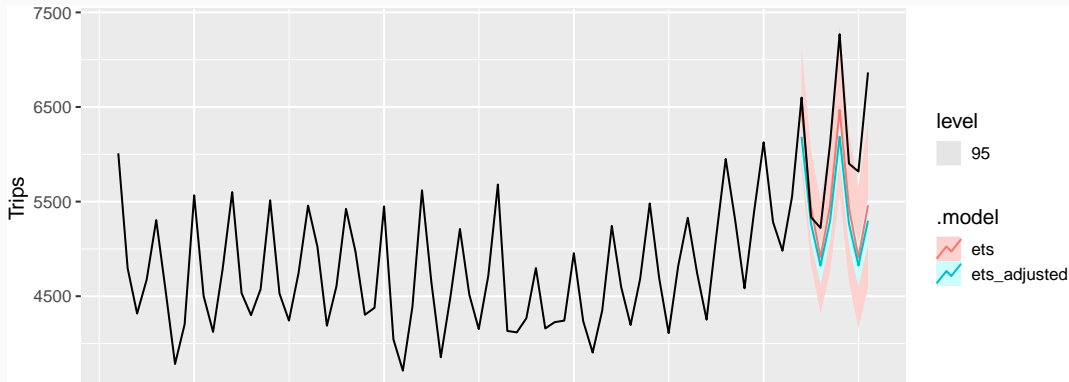
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & is_aggregated(State)) %>%
  autoplot(tourism_agg, level = 95)
```
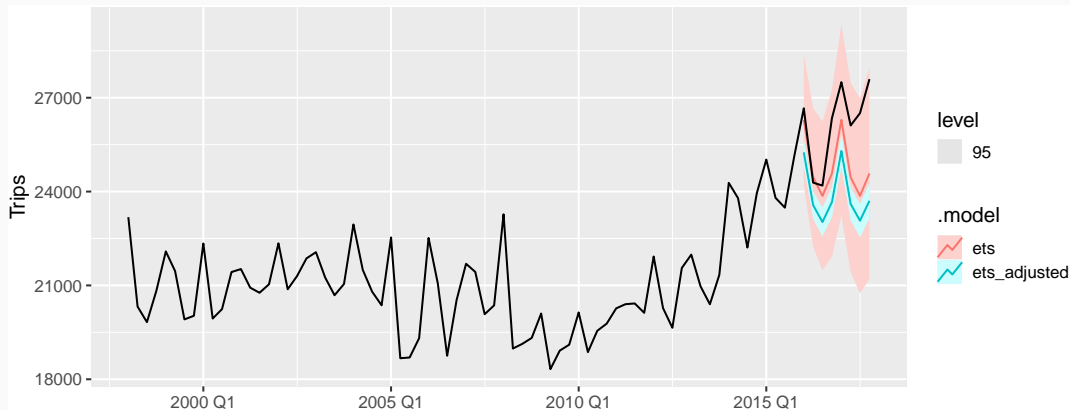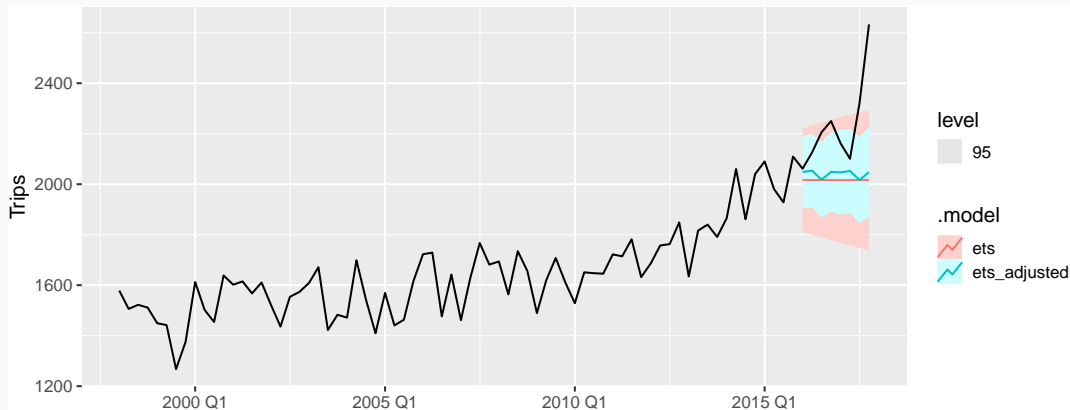
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & State == "VIC" &
    is_aggregated(Region)) %>%
  autoplot(tourism_agg, level = 95)
```
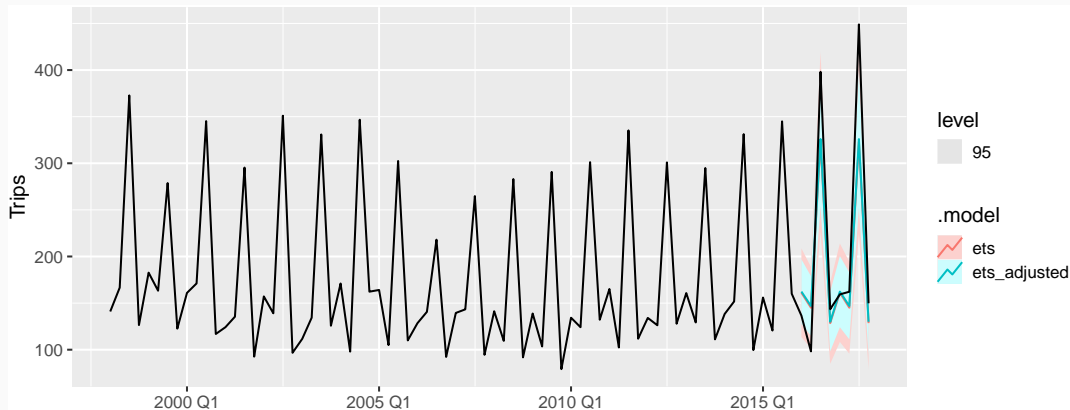
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Melbourne") %>%
  autoplot(tourism_agg, level = 95)
```
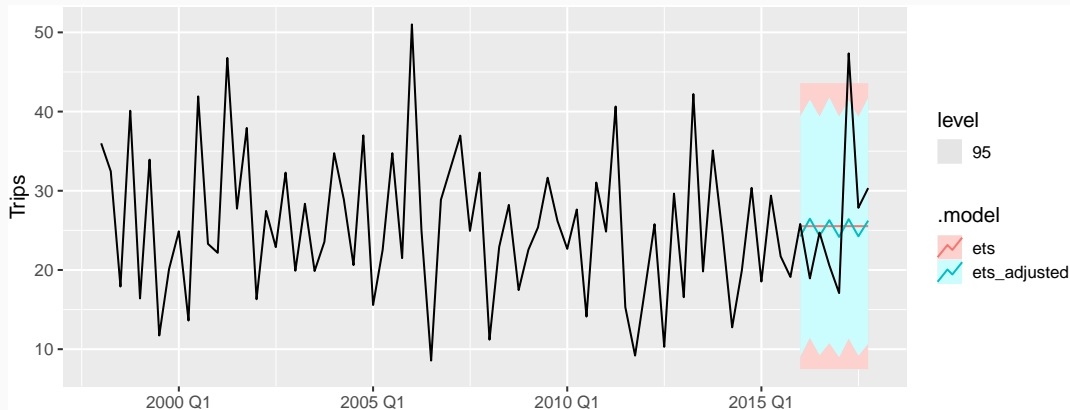
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Snowy Mountains") %>%
  autoplot(tourism_agg, level = 95)
```
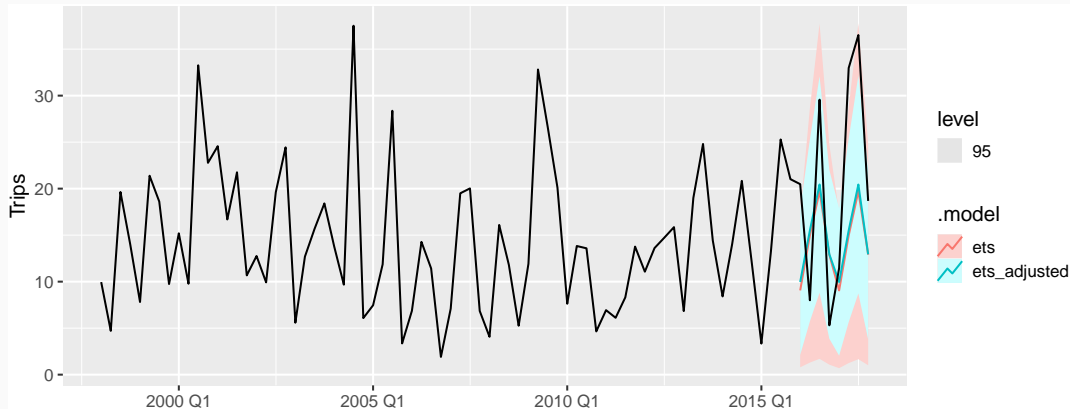
Example: Australian tourism

```
fc %>%
  filter(Purpose == "Holiday" & Region == "Barossa") %>%
  autoplot(tourism_agg, level = 95)
```

Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "MacDonnell") %>%
  autoplot(tourism_agg, level = 95)
```

Example: Australian tourism

```
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  ) %>%
  mutate(
    comb = (ets + arima) / 2
  ) %>%
  reconcile(
    ets_adj = min_trace(ets),
    arima_adj = min_trace(arima),
    comb_adj = min_trace(comb)
  ) %>%
  forecast(h = "2 years")
```

Forecast evaluation

```
fc %>% accuracy(tourism_agg)
```

```
## # A tibble: 2,550 x 13
##    .model Purpose State  Region           .type   ME   RMSE  MAE   MPE  MAPE
##    <chr>  <chr*>  <chr*> <chr*>           <chr> <dbl>  <dbl> <dbl> <dbl> <dbl>
##  1 arima  Business ACT   Canberra        ~ Test  35.9   45.7  35.9  16.9  16.9
##  2 arima  Business ACT   <aggregated>     Test  35.9   45.7  35.9  16.9  16.9
##  3 arima  Business NSW   Blue Mountains  ~ Test   1.93  10.6   8.52 -
## 18.0  48.6
##  4 arima  Business NSW   Capital Country ~ Test   8.08  15.6  10.4  11.8  19.0
##  5 arima  Business NSW   Central Coast   ~ Test  10.0   14.5  10.8  26.9  32.2
##  6 arima  Business NSW   Central NSW     ~ Test  17.7   31.9  28.2  12.0  24.1
##  7 arima  Business NSW   Hunter          ~ Test  35.3   43.9  35.3  24.2  24.2
##  8 arima  Business NSW   New England North~ Test 23.1   31.8  26.8  19.5  28.0
##  9 arima  Business NSW   North Coast NSW ~ Test  24.8   40.1  36.8  11.5  28.5
## 10 arima  Business NSW   Outback NSW     ~ Test   6.87  11.0   7.76 13.7  16.5
```

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Forecast evaluation

```
fc %>%
  accuracy(tourism_agg) %>%
  group_by(.model) %>%
  summarise(MASE = mean(MASE)) %>%
  arrange(MASE)

## # A tibble: 6 x 2
##   .model    MASE
##   <chr>    <dbl>
## 1 ets_adj   1.02
## 2 comb_adj  1.02
## 3 ets       1.04
## 4 comb      1.04
## 5 arima_adj 1.07
## 6 arima     1.09
```

Tidyverts developers

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More information

- Slides and papers: **robjhyndman.com**
- Packages: **tidyverts.org**
- Forecasting textbook using fable package:
  OTexts.com/fpp3

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