Feasts & fables

Modern tools for time series analysis

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robjhyndman.com/cornish2021

Outline

1. What does modern time series data look like?
2. Feature-based time series analysis
3. Probabilistic forecasting for large time series
4. Evaluating probabilistic forecasts
5. Forecast reconciliation

E A Cornish (1909–1973)

- Foundation Fellow of the Australian Academy of Science (1954)
- Chief of the CSIRO Mathematical Statistics Division (1954–1973)
- Helped establish CSIRO Division of Computing Research (1963)

E A Cornish (1909–1973)

Rainfall papers

1. Cornish, EA & Coote, GG (1958) *The correlation of monthly rainfall with position and altitude of observing stations in South Australia.* CSIRO Div Math Stats Tech Paper 4.
2. Cornish, EA and Stenhouse, NS (1958) *Inter-station correlations of monthly rainfall in South Australia.* CSIRO Div Math Stats Tech Paper 5.
3. Cornish, EA, Hill, GW, & Evans, MJ (1961) *Inter-station correlations of rainfall in southern Australia.* CSIRO Div Math Stats Tech Paper 10.

- Modelled monthly rainfall at 97 South Australian weather stations based on altitude, longitude and latitude.
- Pairwise correlations of 6-day rainfall totals between weather stations: 90,585 correlation coefficients.

Tidyverts packages

tidyverts.org

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4. Evaluating probabilistic forecasts
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Annual economic data

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country              GDP Imports Exports Population
##    <dbl> <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
## # ... with 15,140 more rows
```

Annual economic data

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
## # ... with 15,140 more rows
```

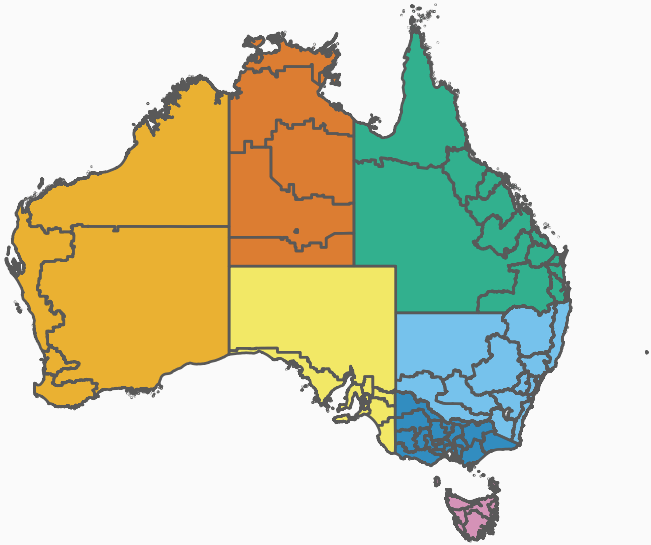
Annual economic data

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country              GDP Imports Exports Population
##    Index Key                <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
## # ... with 15,140 more rows
```

Annual economic data

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index Key                  Measured variables
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
## # ... with 15,140 more rows
```

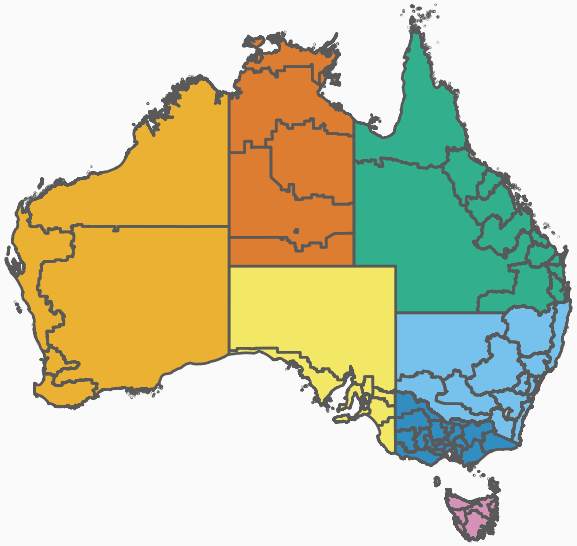
Australian tourism regions



State

- New South Wales
- Victoria
- Queensland
- South Australia
- Northern Territory
- Western Australia
- Tasmania
- Australian Capital Territory

Australian tourism regions



- Quarterly data on visitor nights: 1998 – 2017
- From *National Visitor Survey*, interviews of 120,000 Australians aged 15+.
- Geographical hierarchy split by
  - 8 states and territories
  - 76 regions
- Purpose:
  - Holidays
  - Business
  - Visiting friends & relatives
  - Other

Quarterly tourism data

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      <qtr> <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
## # ... with 24,310 more rows
```

Domestic visitor nights in thousands by state/region and purpose.

Quarterly tourism data

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
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## # ... with 24,310 more rows
```

Domestic visitor nights in thousands by state/region and purpose.

Quarterly tourism data

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index  Keys                  <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
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## 10 2000 Q2 Adelaide SA    Business  169.
## # ... with 24,310 more rows
```

Domestic visitor nights in thousands by state/region and purpose.

Quarterly tourism data

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index  Keys                  Measure
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
## # ... with 24,310 more rows
```

Domestic visitor nights in thousands by state/region and purpose.

Australian electricity demand

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
## # Key:       State [8]
##    Time                State Date       Holiday Temperature Demand
##    <dttm>              <fct> <date>     <lgl>         <dbl>  <dbl>
##  1 2012-01-01 00:00:00 VIC   2012-01-01 TRUE           21.4  4383.
##  2 2012-01-01 00:30:00 VIC   2012-01-01 TRUE           21.0  4263.
##  3 2012-01-01 01:00:00 VIC   2012-01-01 TRUE           20.7  4049.
##  4 2012-01-01 01:30:00 VIC   2012-01-01 TRUE           20.6  3878.
##  5 2012-01-01 02:00:00 VIC   2012-01-01 TRUE           20.4  4036.
##  6 2012-01-01 02:30:00 VIC   2012-01-01 TRUE           20.2  3866.
##  7 2012-01-01 03:00:00 VIC   2012-01-01 TRUE           20.1  3694.
##  8 2012-01-01 03:30:00 VIC   2012-01-01 TRUE           19.6  3562.
##  9 2012-01-01 04:00:00 VIC   2012-01-01 TRUE           19.1  3433.
## 10 2012-01-01 04:30:00 VIC   2012-01-01 TRUE           19.0  3359.
## # ... with 420,854 more rows
```

Australian electricity demand

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
## # Key:       State [8]
##    Time                State Date       Holiday Temperature Demand
##    Index               <fct> <date>     <lgl>          <dbl>  <dbl>
##  1 2012-01-01 00:00:00 VIC   2012-01-01 TRUE            21.4  4383.
##  2 2012-01-01 00:30:00 VIC   2012-01-01 TRUE            21.0  4263.
##  3 2012-01-01 01:00:00 VIC   2012-01-01 TRUE            20.7  4049.
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##  6 2012-01-01 02:30:00 VIC   2012-01-01 TRUE            20.2  3866.
##  7 2012-01-01 03:00:00 VIC   2012-01-01 TRUE            20.1  3694.
##  8 2012-01-01 03:30:00 VIC   2012-01-01 TRUE            19.6  3562.
##  9 2012-01-01 04:00:00 VIC   2012-01-01 TRUE            19.1  3433.
## 10 2012-01-01 04:30:00 VIC   2012-01-01 TRUE            19.0  3359.
## # ... with 420,854 more rows
```

Australian electricity demand

```
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## # Key:       State [8]
##    Time                State Date       Holiday Temperature Demand
##    Index               Key   <date>     <lgl>          <dbl>  <dbl>
##  1 2012-01-01 00:00:00 VIC   2012-01-01 TRUE            21.4  4383.
##  2 2012-01-01 00:30:00 VIC   2012-01-01 TRUE            21.0  4263.
##  3 2012-01-01 01:00:00 VIC   2012-01-01 TRUE            20.7  4049.
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## 10 2012-01-01 04:30:00 VIC   2012-01-01 TRUE            19.0  3359.
## # ... with 420,854 more rows
```

Australian electricity demand

```
## # A tsibble: 420,864 x 6 [30m] <Australia/Melbourne>
## # Key:       State [8]
##    Time                State Date       Holiday Temperature Demand
##    Index               Key   Measures
##  1 2012-01-01 00:00:00 VIC   2012-01-01 TRUE           21.4   4383.
##  2 2012-01-01 00:30:00 VIC   2012-01-01 TRUE           21.0   4263.
##  3 2012-01-01 01:00:00 VIC   2012-01-01 TRUE           20.7   4049.
##  4 2012-01-01 01:30:00 VIC   2012-01-01 TRUE           20.6   3878.
##  5 2012-01-01 02:00:00 VIC   2012-01-01 TRUE           20.4   4036.
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##  8 2012-01-01 03:30:00 VIC   2012-01-01 TRUE           19.6   3562.
##  9 2012-01-01 04:00:00 VIC   2012-01-01 TRUE           19.1   3433.
## 10 2012-01-01 04:30:00 VIC   2012-01-01 TRUE           19.0   3359.
## # ... with 420,854 more rows
```

Characteristics of modern time series

- Often observed at sub-daily frequency over a long time.
- Multiple keys which may be nested.
- Multiple seasonal patterns.
- Multiple measures for each combination of index and keys.

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- Multiple keys which may be nested.
- Multiple seasonal patterns.
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`tsibble` objects

- A `tsibble` allows storage and manipulation of multiple time series in R.
- It contains:
  - An index: time information about the observation
  - Key variable(s): optional unique identifiers for each series
  - Measured variable(s): numbers of interest and any other variable

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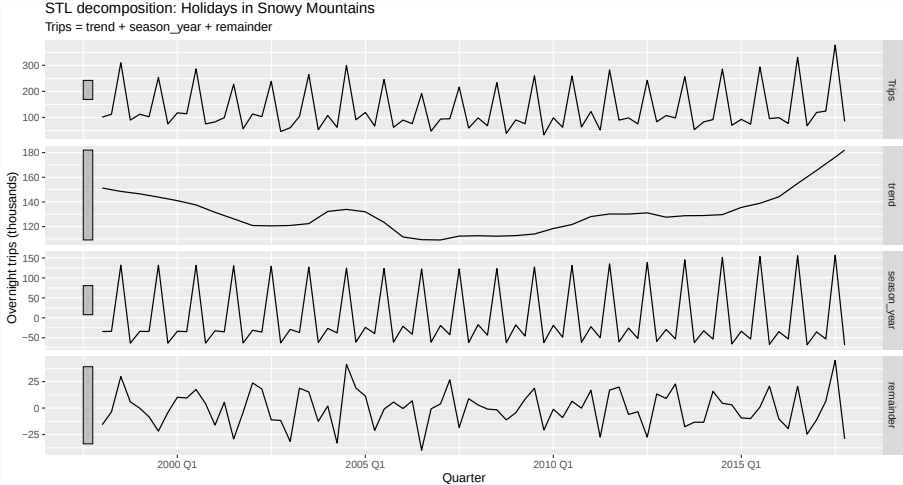
STL decomposition



Strength of seasonality and trend

STL decomposition

$$y_t = T_t + S_t + R_t$$

Seasonal strength

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right)$$

Trend strength

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(T_t + R_t)}\right)$$

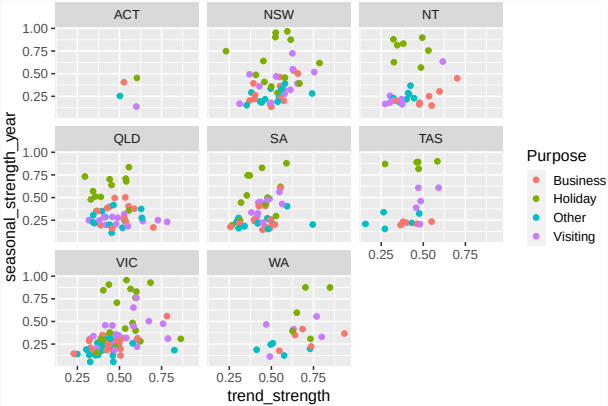
STL-based features

```
tourism %>%
  features(Trips, feat_stl)
```

```
## # A tibble: 304 x 12
##    Region          State Purpose trend_strength seasonal_streng~ seasonal_peak_y~
##    <chr>           <chr> <chr>            <dbl>            <dbl>            <dbl>
##  1 Adelaide        SA    Busine~          0.464            0.407                3
##  2 Adelaide        SA    Holiday          0.554            0.619                1
##  3 Adelaide        SA    Other            0.746            0.202                2
##  4 Adelaide        SA    Visiti~          0.435            0.452                1
##  5 Adelaide Hills SA    Busine~          0.464            0.179                3
##  6 Adelaide Hills SA    Holiday          0.528            0.296                2
##  7 Adelaide Hills SA    Other            0.593            0.404                2
##  8 Adelaide Hills SA    Visiti~          0.488            0.254                0
##  9 Alice Springs  NT    Busine~          0.534            0.251                0
## 10 Alice Springs  NT    Holiday          0.381            0.832                3
## # ... with 294 more rows, and 6 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>
```

STL-based features

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() + facet_wrap(vars(State))
```



STL-based features

```
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  features(Trips, feat_stl) %>%
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```
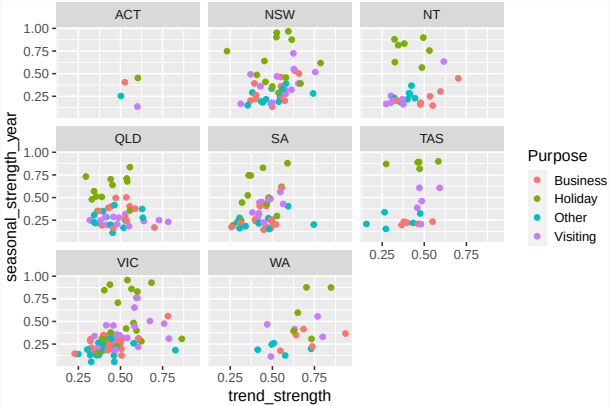
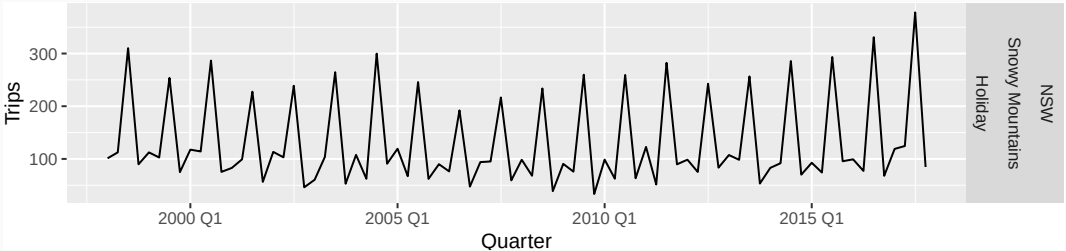



- Holidays more seasonal than other travel.
- WA has strongest trends.

STL-based features

Find the most seasonal time series:

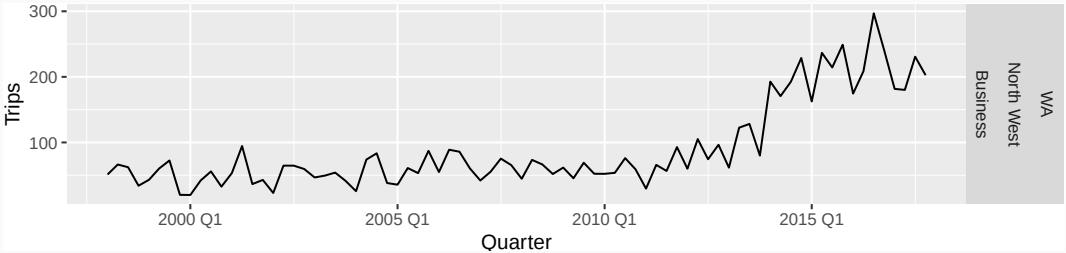
```
tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year)) %>%
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



STL-based features

Find the most trended time series:

```
tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength)) %>%
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



Time series features

```
tourism_features <- tourism %>%
  features(Trips, feature_set(pkgs = "feasts"))
```

All features from the feasts package

```
## # A tibble: 304 x 50
##    Region  State Purpose trend_strength seasonal_streng~ seasonal_peak_y~ seasonal_trough~
##    <chr>   <chr> <chr>            <dbl>            <dbl>            <dbl>            <dbl>
##  1 Adelai~ SA    Busine~          0.464            0.407                3                1
##  2 Adelai~ SA    Holiday          0.554            0.619                1                2
##  3 Adelai~ SA    Other            0.746            0.202                2                1
##  4 Adelai~ SA    Visiti~          0.435            0.452                1                3
##  5 Adelai~ SA    Busine~          0.464            0.179                3                0
##  6 Adelai~ SA    Holiday          0.528            0.296                2                1
##  7 Adelai~ SA    Other            0.593            0.404                2                2
##  8 Adelai~ SA    Visiti~          0.488            0.254                0                3
##  9 Alice ~ NT    Busine~          0.534            0.251                0                1
## 10 Alice ~ NT    Holiday          0.381            0.832                3                1
## # ... with 294 more rows, and 43 more variables: spikiness <dbl>, linearity <dbl>,
## #   curvature <dbl>, stl_e_acf1 <dbl>, stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>,
## #   diff1_acf1 <dbl>, diff1_acf10 <dbl>, diff2_acf1 <dbl>, diff2_acf10 <dbl>,
## #   season_acf1 <dbl>, pacf5 <dbl>, diff1_pacf5 <dbl>, diff2_pacf5 <dbl>,
## #   season_pacf <dbl>, zero_run_mean <dbl>, nonzero_squared_cv <dbl>,
## #   zero_start_prop <dbl>, zero_end_prop <dbl>, lambda_guerrero <dbl>, kpss_stat <dbl>,
## #   kpss_pvalue <dbl>, pp_stat <dbl>, pp_pvalue <dbl>, ndiffs <int>, nsdiffs <int>, ...
```

Reduced feature space

```
pcs <- tourism_features %>%
  select(-State, -Region, -Purpose) %>%
  prcomp(scale = TRUE) %>%
  augment(tourism_features)
```

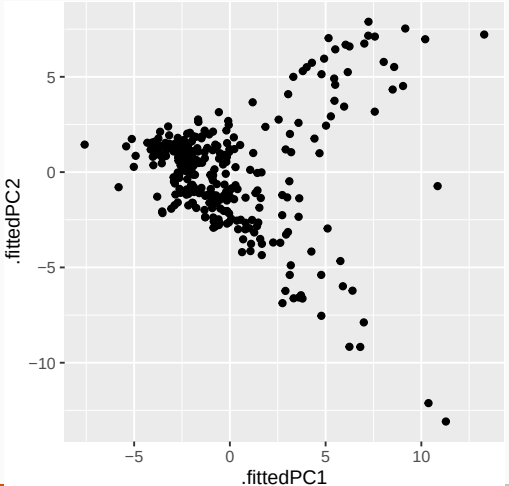
Principal components based on all features from the feasts package

```
## # A tibble: 304 x 98
##    .rownames Region         State Purpose trend_strength seasonal_streng~ seasonal_peak_y~
##    <chr>     <chr>          <chr> <chr>            <dbl>            <dbl>            <dbl>
##  1 1         Adelaide       SA    Busine~          0.464            0.407                3
##  2 2         Adelaide       SA    Holiday          0.554            0.619                1
##  3 3         Adelaide       SA    Other            0.746            0.202                2
##  4 4         Adelaide       SA    Visiti~          0.435            0.452                1
##  5 5         Adelaide Hills SA    Busine~          0.464            0.179                3
##  6 6         Adelaide Hills SA    Holiday          0.528            0.296                2
##  7 7         Adelaide Hills SA    Other            0.593            0.404                2
##  8 8         Adelaide Hills SA    Visiti~          0.488            0.254                0
##  9 9         Alice Springs  NT    Busine~          0.534            0.251                0
## 10 10        Alice Springs  NT    Holiday          0.381            0.832                3
## # ... with 294 more rows, and 91 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>, pacf5 <dbl>,
## #   diff1_pacf5 <dbl>, diff2_pacf5 <dbl>, season_pacf <dbl>, zero_run_mean <dbl>
```

Reduced feature space

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +
  geom_point() + theme(aspect.ratio=1)
```

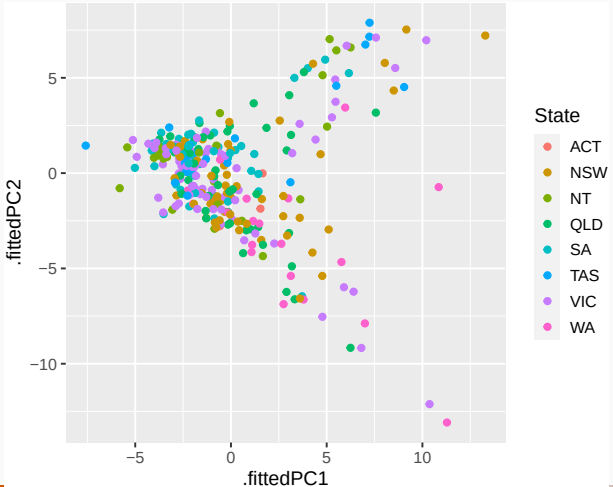
Principal components based on all features from the feasts package



Reduced feature space

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +
  geom_point() + theme(aspect.ratio=1)
```

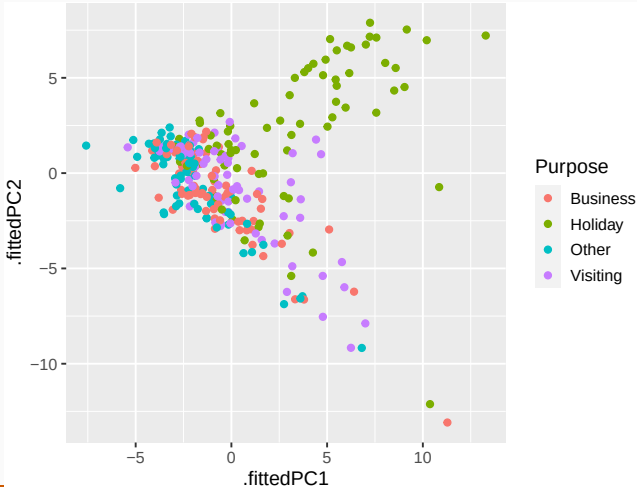
Principal components based on all features from the feasts package



Reduced feature space

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package



Anomaly detection using time series features

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
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```

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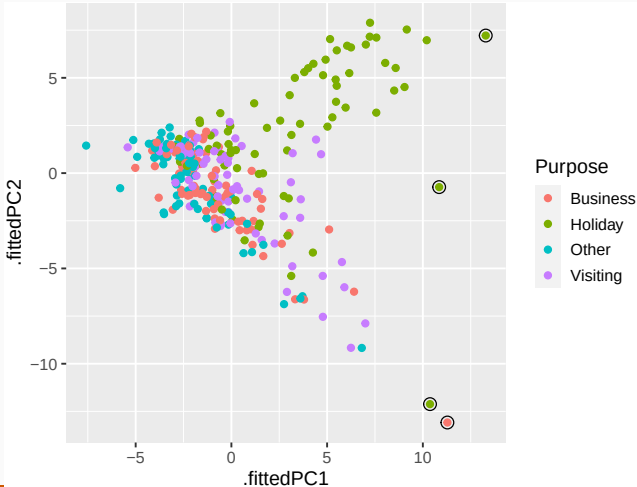

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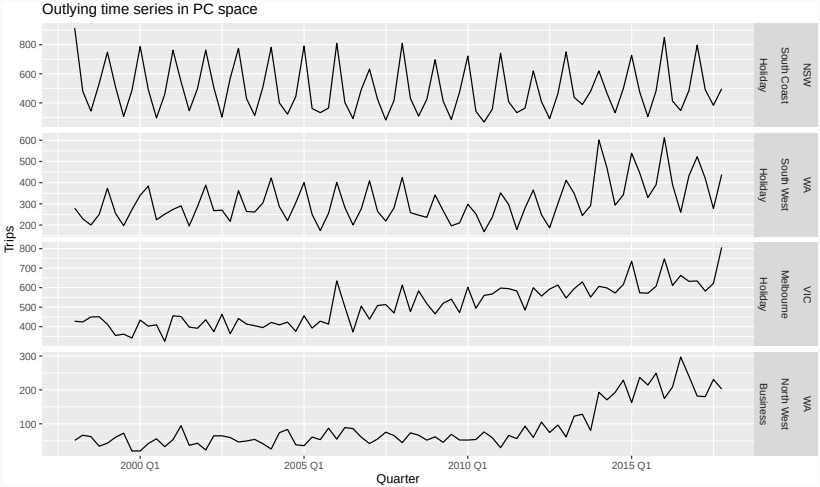

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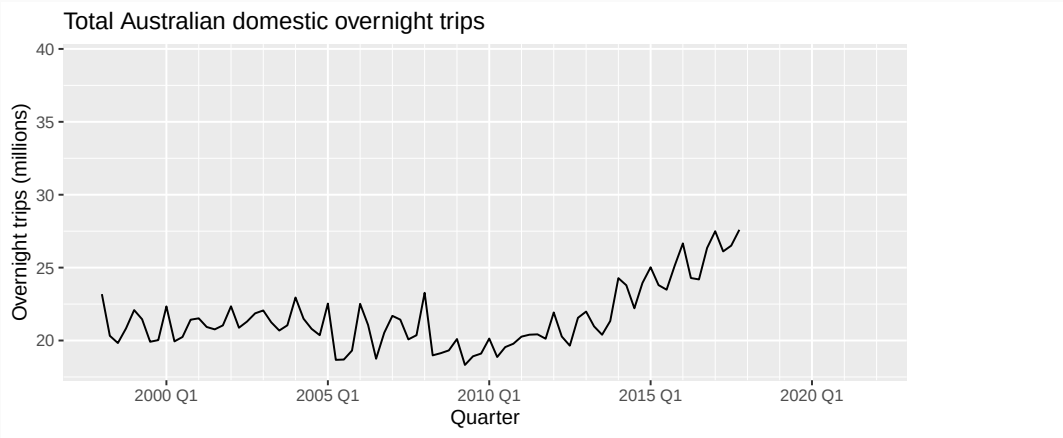

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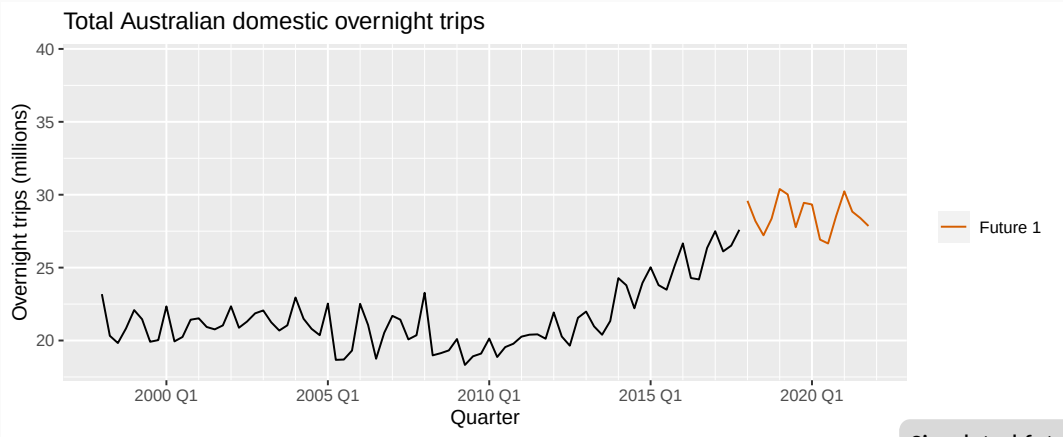

Simulated futures from an ETS model

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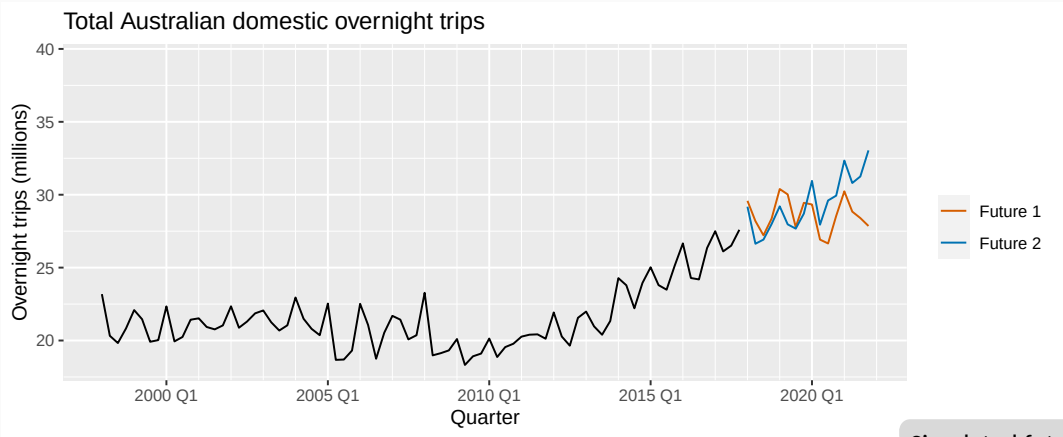

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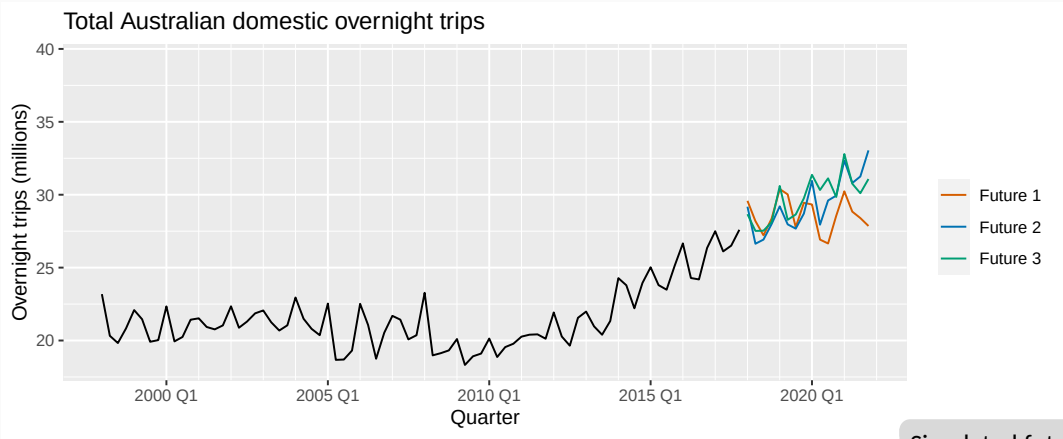

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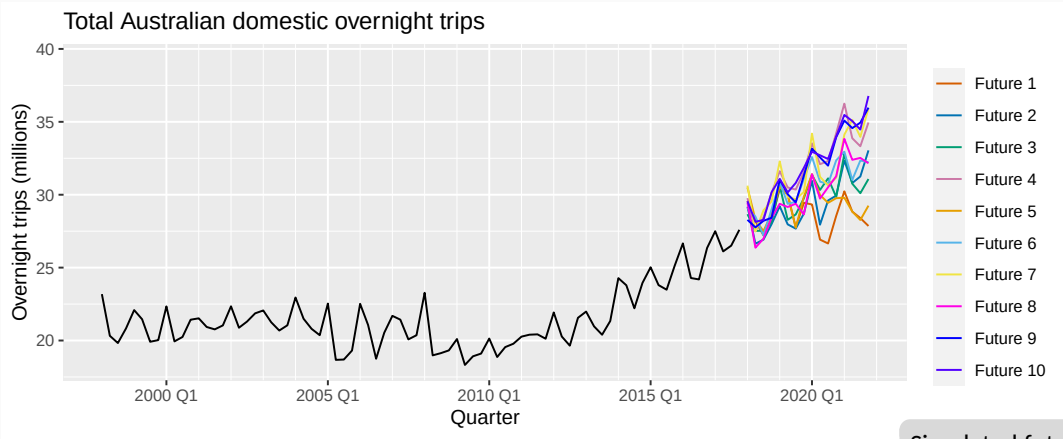

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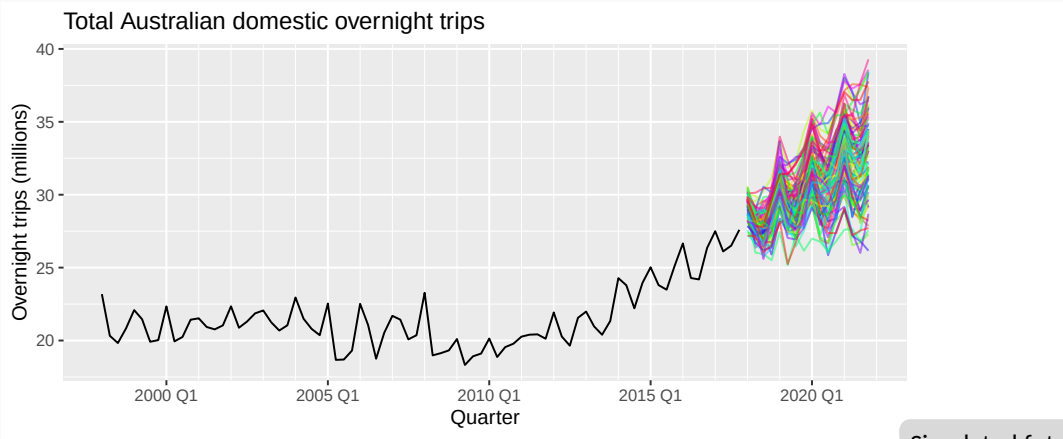

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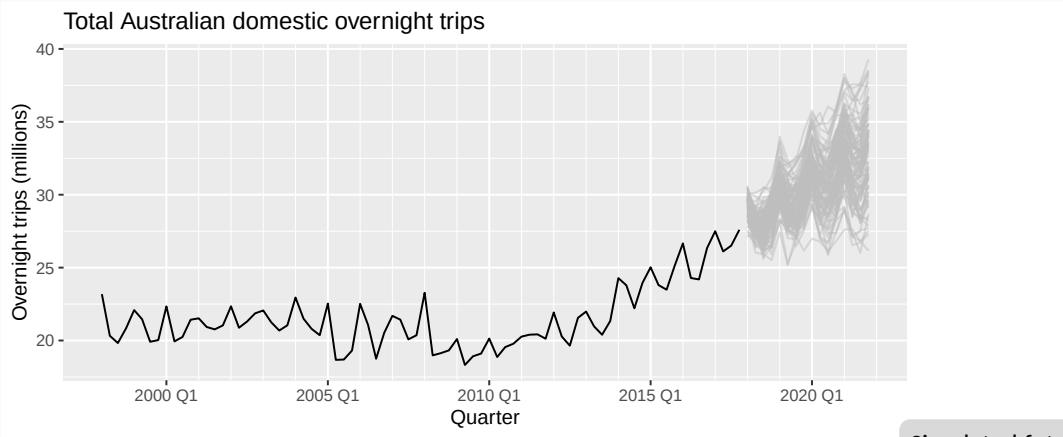

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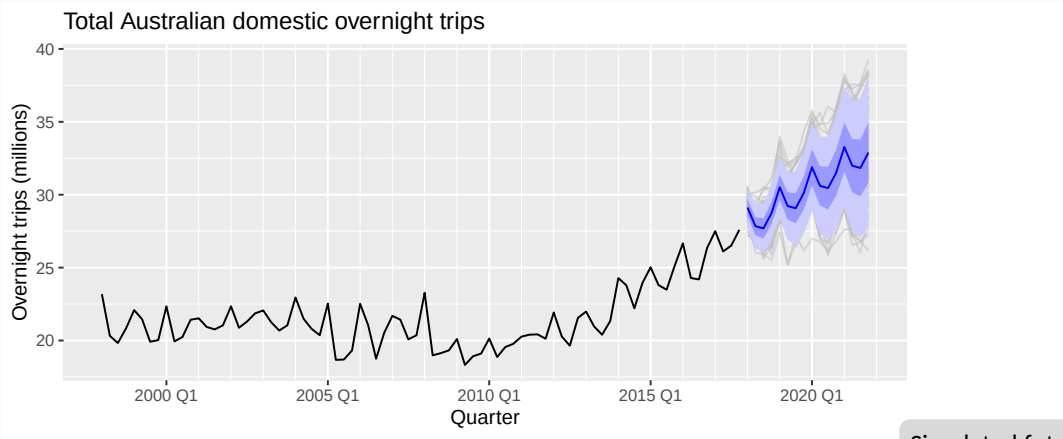

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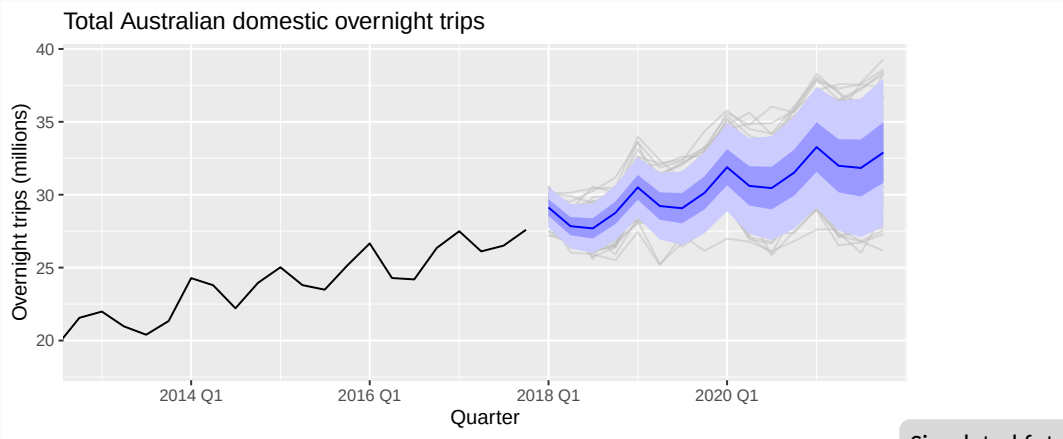


Simulated futures from an ETS model

Model fitting

```
tourism_fit <- tourism %>%
  filter(year(Quarter) < 2015) %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  ) %>%
  mutate(ensemble = (ets + arima)/2)
```

```
## # A mable: 304 x 6
## # Key:     Region, State, Purpose [304]
##    Region          State Purpose          ets                              arima
##    <chr>           <chr> <chr>            <model>                          <model>
##  1 Adelaide        SA    Business <ETS(M,N,M)>  <ARIMA(0,0,0)(1,0,1)[4] w/ mean>
##  2 Adelaide        SA    Holiday  <ETS(M,N,A)>  <ARIMA(0,0,0)(2,0,0)[4] w/ mean>
##  3 Adelaide        SA    Other    <ETS(M,A,N)>           <ARIMA(0,1,1) w/ drift>
##  4 Adelaide        SA    Visiting <ETS(A,N,A)>  <ARIMA(0,0,0)(1,0,1)[4] w/ mean>
##  5 Adelaide Hills SA    Business <ETS(A,N,N)>            <ARIMA(1,0,0) w/ mean>
##  6 Adelaide Hills SA    Holiday  <ETS(A,N,N)>            <ARIMA(0,0,0) w/ mean>
##  7 Adelaide Hills SA    Other    <ETS(A,N,N)>  <ARIMA(0,0,1)(1,0,0)[4] w/ mean>
##  8 Adelaide Hills SA    Visiting <ETS(M,A,M)>            <ARIMA(0,0,0) w/ mean>
```

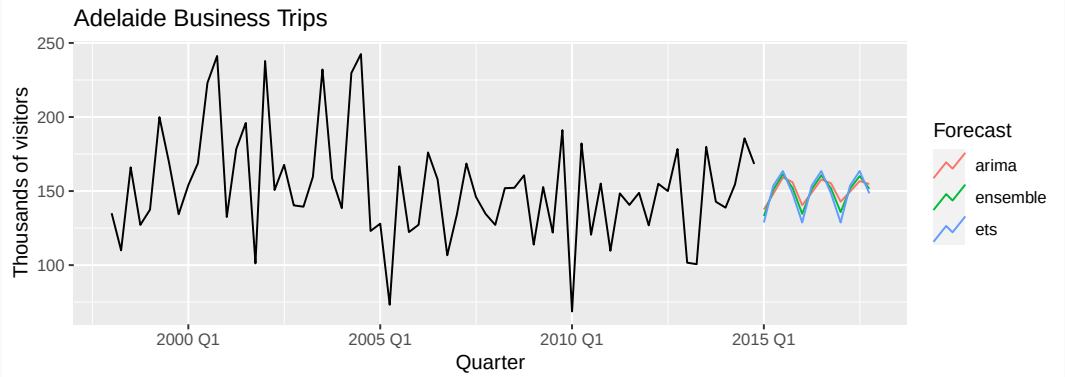
Producing forecasts

```
tourism_fc <- tourism_fit %>%
  forecast(h = "3 years")
```

```
## # A fable: 10,944 x 7 [1Q]
## # Key:     Region, State, Purpose, .model [912]
##    Region   State Purpose  .model Quarter        Trips .mean
##    <chr>    <chr> <chr>    <chr>    <qtr>       <dist> <dbl>
##  1 Adelaide SA    Business ets    2015 Q1  N(129, 842)  129.
##  2 Adelaide SA    Business ets    2015 Q2 N(154, 1214)  154.
##  3 Adelaide SA    Business ets    2015 Q3 N(164, 1391)  164.
##  4 Adelaide SA    Business ets    2015 Q4 N(148, 1159)  148.
##  5 Adelaide SA    Business ets    2016 Q1  N(129, 883)  129.
##  6 Adelaide SA    Business ets    2016 Q2 N(154, 1274)  154.
##  7 Adelaide SA    Business ets    2016 Q3 N(164, 1458)  164.
##  8 Adelaide SA    Business ets    2016 Q4 N(148, 1215)  148.
##  9 Adelaide SA    Business ets    2017 Q1  N(129, 925)  129.
## 10 Adelaide SA    Business ets    2017 Q2 N(154, 1334)  154.
## # ... with 10,934 more rows
```

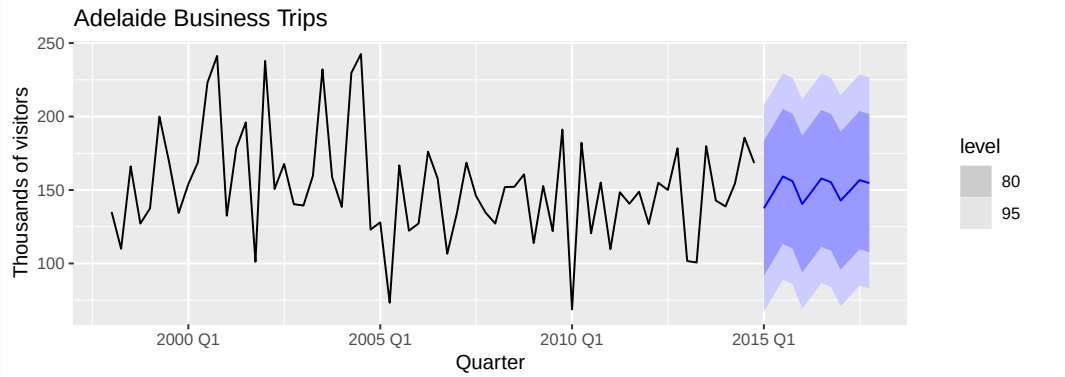
Visualising forecasts

```
tourism_fc %>%
  filter(Region == "Adelaide", Purpose=="Business") %>%
  autoplot(tourism, level = NULL) +
  labs(title = "Adelaide Business Trips", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



Visualising forecasts

```
tourism_fc %>%
  filter(Region == "Adelaide", Purpose=="Business", .model == "arima") %>%
  autoplot(tourism) +
  labs(title = "Adelaide Business Trips", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



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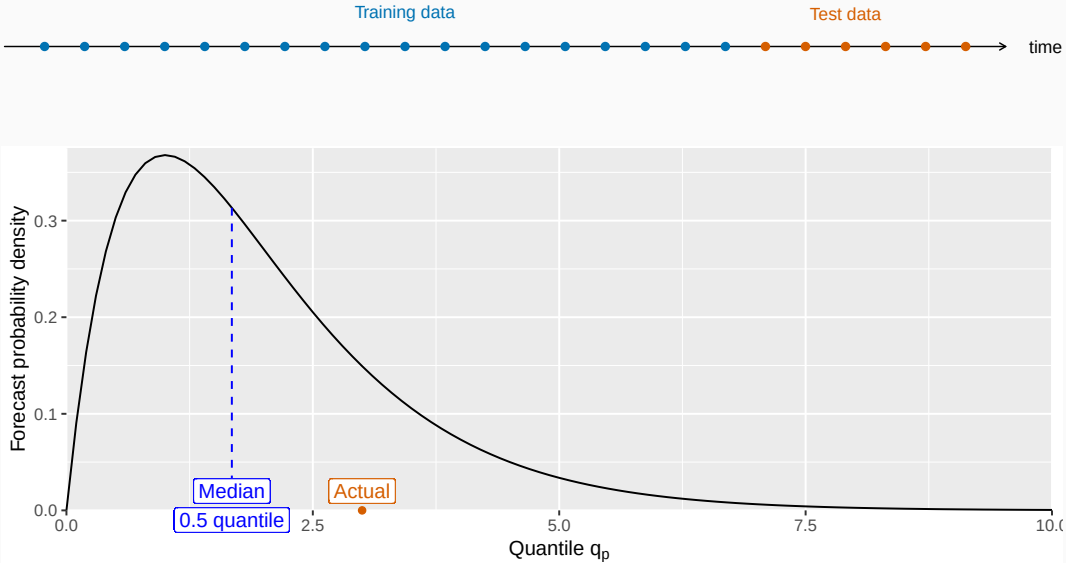

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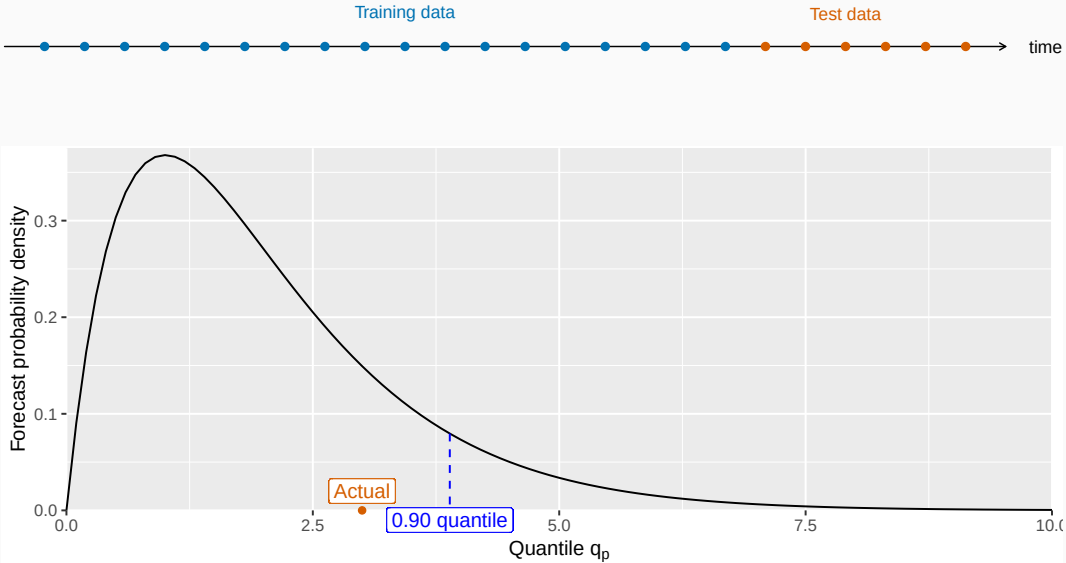

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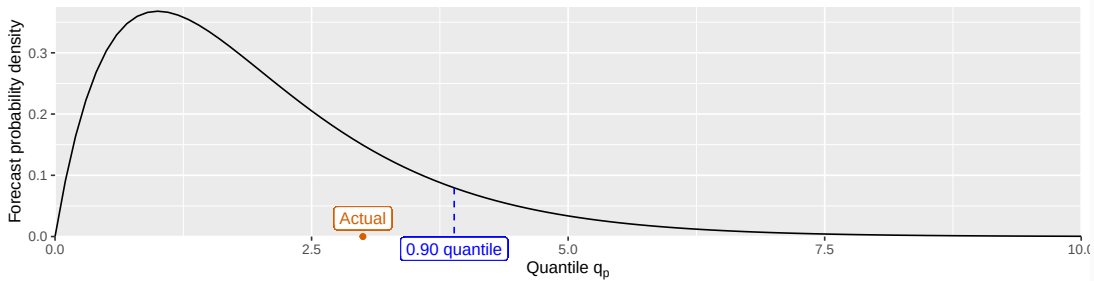


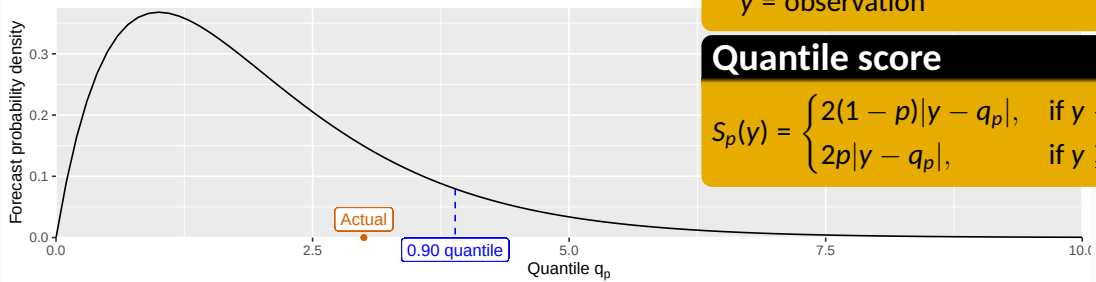
Evaluating probabilistic forecasts

q_p = quantile forecast with prob. p

y = observation

Quantile score

$$S_p(y) = \begin{cases} 2(1-p)|y - q_p|, & \text{if } y < q_p \\ 2p|y - q_p|, & \text{if } y \geq q_p \end{cases}$$



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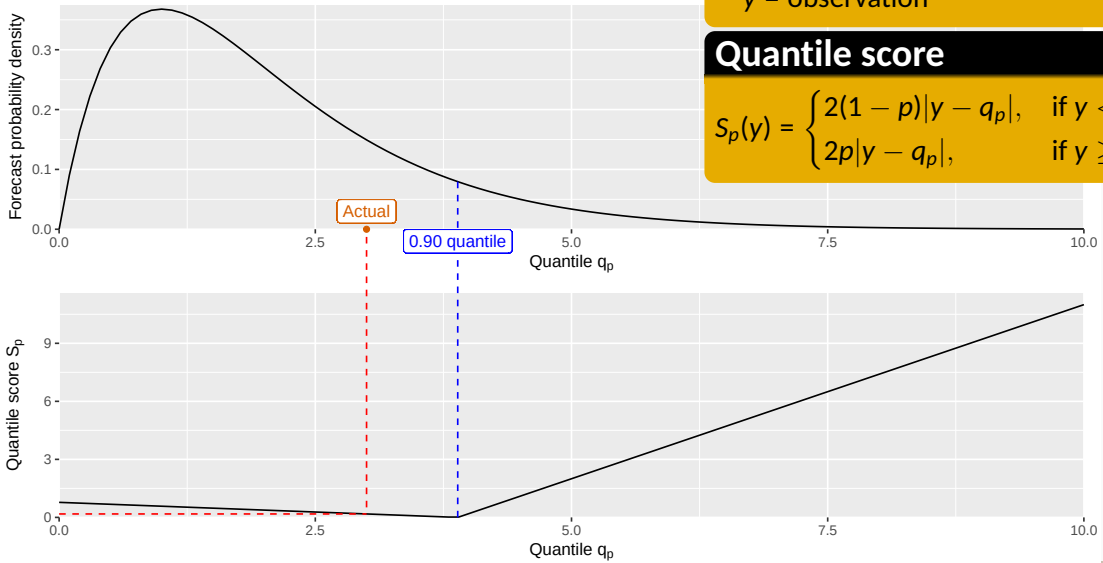

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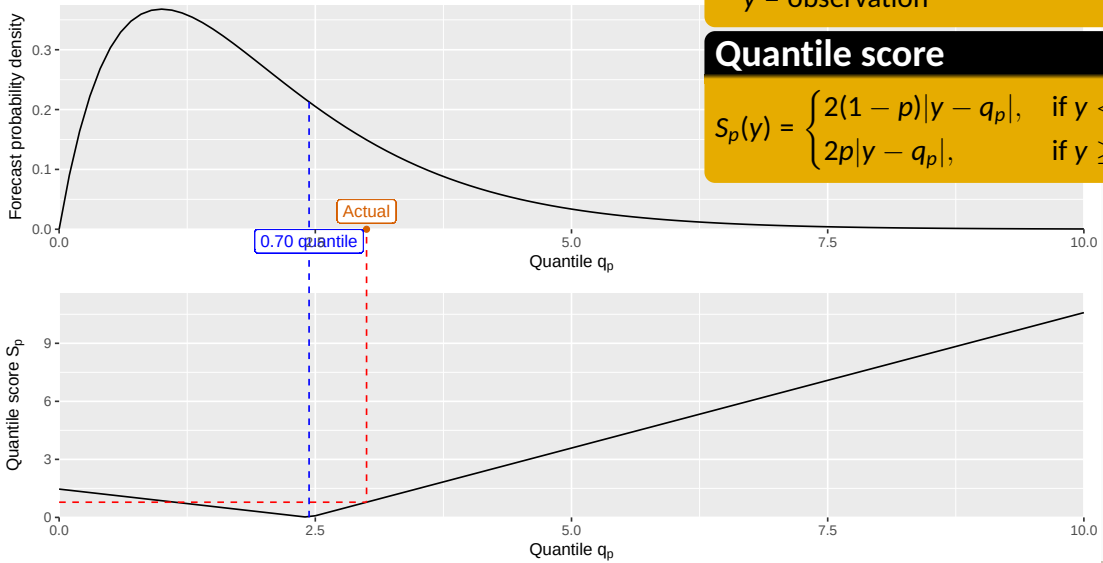

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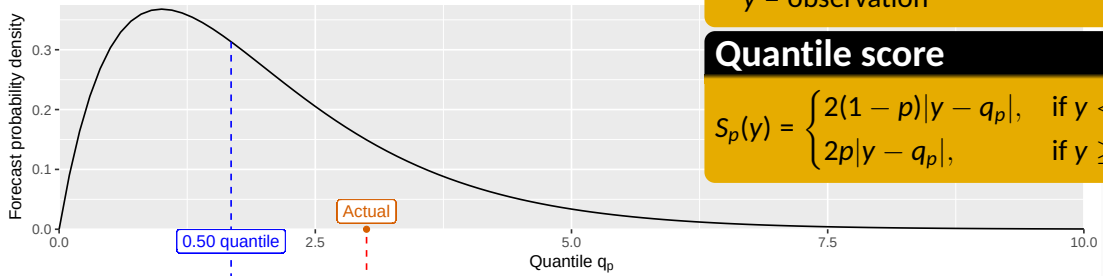

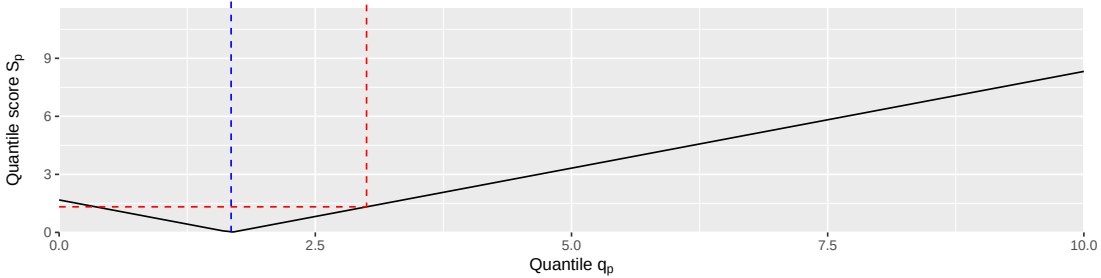

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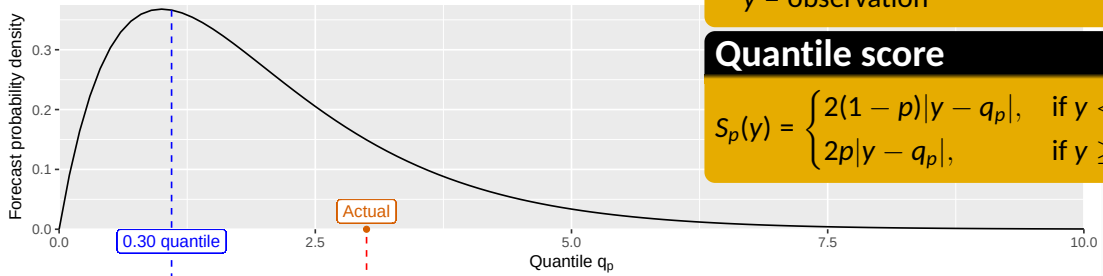

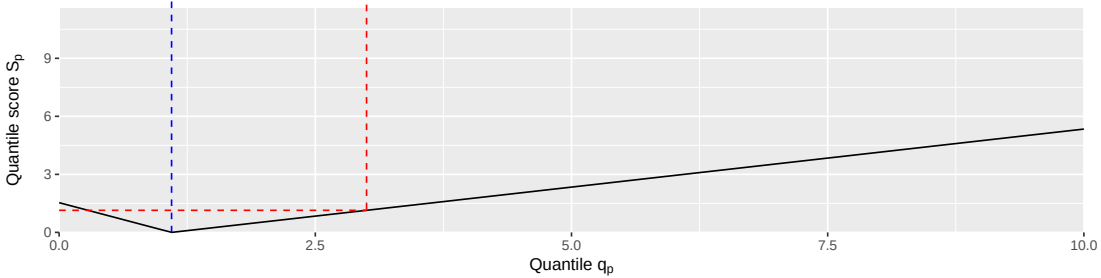

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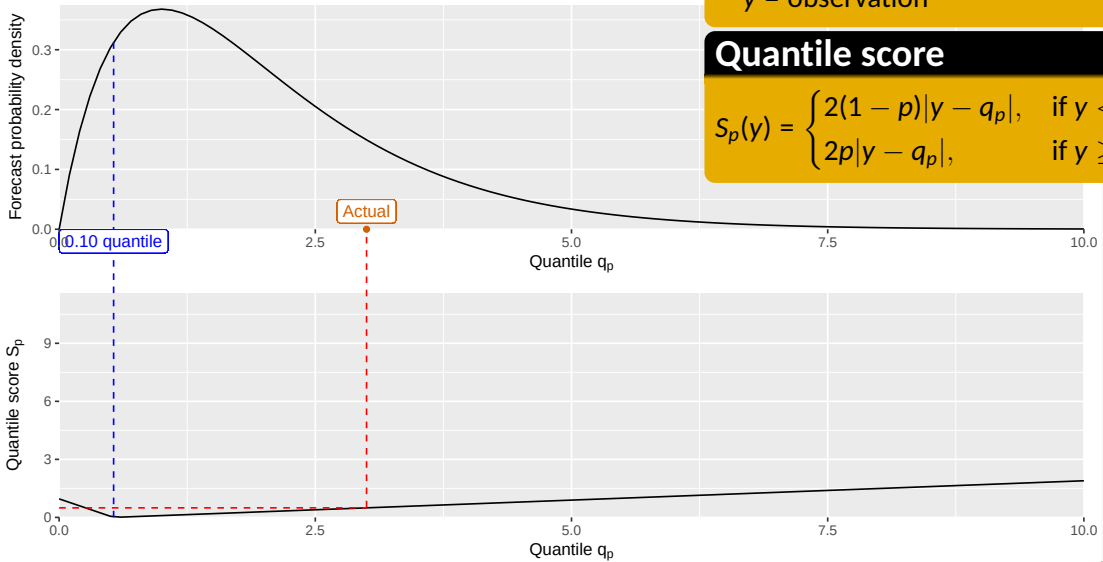
$$S_p(y) = \begin{cases} 2(1-p)|y - q_p|, & \text{if } y < q_p \\ 2p|y - q_p|, & \text{if } y \geq q_p \end{cases}$$



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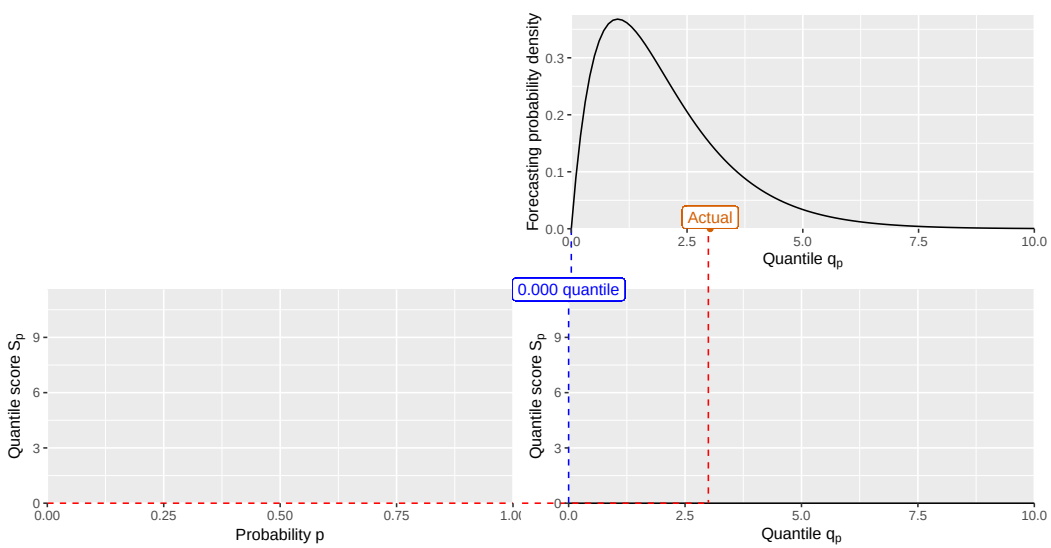

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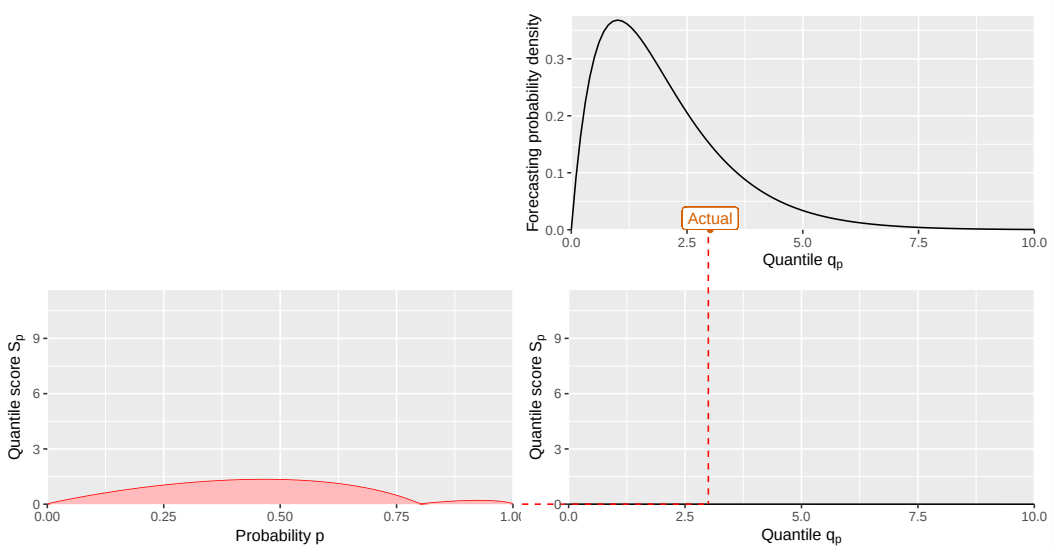

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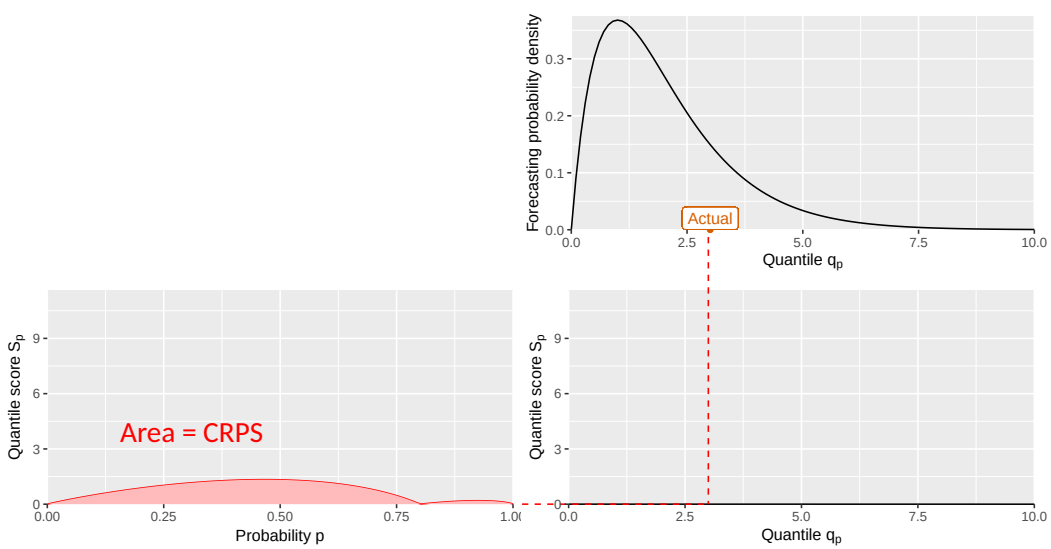

Evaluating probabilistic forecasts

```
tourism_fc %>%
  accuracy(tourism, measures = list(MSE=MSE, CRPS=CRPS))
```

```
## # A tibble: 912 x 7
##    .model Region         State Purpose  .type     MSE  CRPS
##    <chr>  <chr>          <chr> <chr>    <chr>   <dbl> <dbl>
##  1 arima  Adelaide       SA    Business Test   813.   16.7
##  2 arima  Adelaide       SA    Holiday  Test  1212.   20.9
##  3 arima  Adelaide       SA    Other    Test   307.   10.3
##  4 arima  Adelaide       SA    Visiting Test  1379.   22.2
##  5 arima  Adelaide Hills SA    Business Test    31.2   2.85
##  6 arima  Adelaide Hills SA    Holiday  Test    55.1   4.36
##  7 arima  Adelaide Hills SA    Other    Test     7.76  1.51
##  8 arima  Adelaide Hills SA    Visiting Test   158.    7.35
##  9 arima  Alice Springs  NT    Business Test   148.    7.91
## 10 arima  Alice Springs  NT    Holiday  Test   128.    6.48
```

Evaluating probabilistic forecasts

```
tourism_fc %>%
  accuracy(tourism, measures = list(MSE=MSE, CRPS=CRPS)) %>%
  group_by(.model) %>%
  summarise(MSE = mean(MSE), CRPS=mean(CRPS))
```

```
## # A tibble: 3 x 3
##   .model     MSE  CRPS
##   <chr>    <dbl> <dbl>
## 1 arima    1090.  13.0
## 2 ensemble  952.  12.1
## 3 ets       899.  11.9
```

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Hierarchical time series

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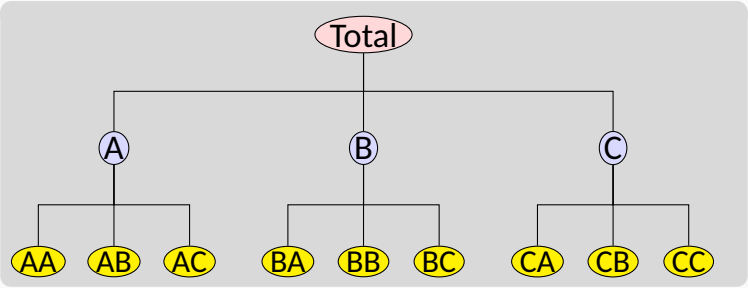

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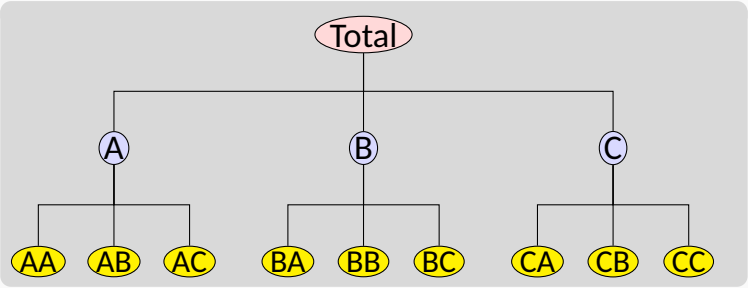
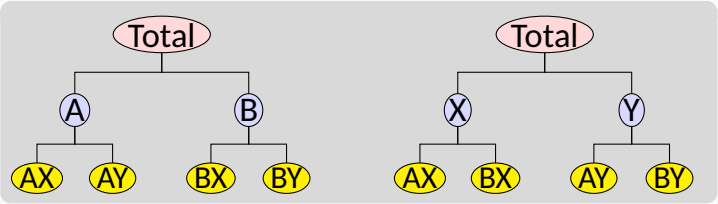


Examples

- Tourism demand by states, zones, regions

Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.



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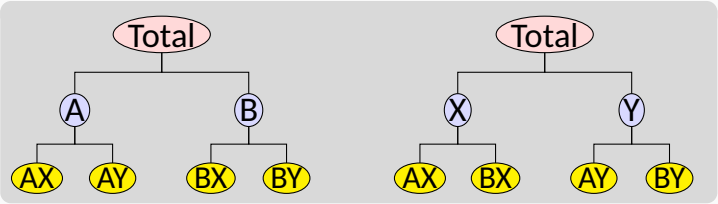

Examples

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

Creating aggregates

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  filter(Quarter == yearquarter("1998 Q1")) %>%
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]
## # Key:       Purpose, State, Region [425]
##    Quarter Purpose      State        Region          Trips
##      <qtr> <chr*>       <chr*>       <chr*>          <dbl>
##  1 1998 Q1 <aggregated> <aggregated> <aggregated>   23182.
##  2 1998 Q1 Business     <aggregated> <aggregated>    3599.
##  3 1998 Q1 Holiday      <aggregated> <aggregated>   11806.
##  4 1998 Q1 Other        <aggregated> <aggregated>     680.
##  5 1998 Q1 Visiting     <aggregated> <aggregated>    7098.
##  6 1998 Q1 <aggregated> ACT          <aggregated>     551.
##  7 1998 Q1 <aggregated> NSW          <aggregated>    8040.
##  8 1998 Q1 <aggregated> NT           <aggregated>     181.
##  9 1998 Q1 <aggregated> QLD          <aggregated>    4041.
## 10 1998 Q1 <aggregated> SA           <aggregated>    1735.
## 11 1998 Q1 <aggregated> TAS          <aggregated>     982.
## 12 1998 Q1 <aggregated> VIC          <aggregated>    6010.
## 13 1998 Q1 <aggregated> WA           <aggregated>    1641.
```

Creating aggregates

- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```

- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

The problem

1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

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2. Can we exploit relationships between the series to improve the forecasts?

The solution

1. Forecast all series at all levels of aggregation using an automatic forecasting algorithm.
   (e.g., `ETS`, `ARIMA`, `...`)
2. Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
3. This is available using `reconcile()`.

Forecast reconciliation

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = 2)
```

```
## # A fable: 1,700 x 7 [1Q]
## # Key:     Purpose, State, Region, .model [850]
##    Purpose  State  Region         .model       Quarter        Trips .mean
##    <chr*>   <chr*> <chr*>         <chr>          <qtr>       <dist> <dbl>
##  1 Business ACT    Canberra       ets          2018 Q1 N(144, 1119) 144.
##  2 Business ACT    Canberra       ets          2018 Q2 N(203, 2260) 203.
##  3 Business ACT    Canberra       ets_adjusted 2018 Q1  N(157, 539) 157.
##  4 Business ACT    Canberra       ets_adjusted 2018 Q2  N(214, 951) 214.
##  5 Business ACT    <aggregated>   ets          2018 Q1 N(144, 1119) 144.
##  6 Business ACT    <aggregated>   ets          2018 Q2 N(203, 2260) 203.
##  7 Business ACT    <aggregated>   ets_adjusted 2018 Q1  N(157, 539) 157.
##  8 Business ACT    <aggregated>   ets_adjusted 2018 Q2  N(214, 951) 214.
```

Hierarchical and grouped time series

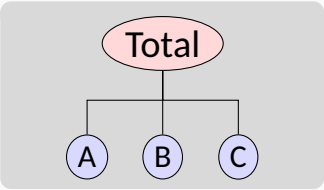
Every collection of time series with aggregation constraints can be written as

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

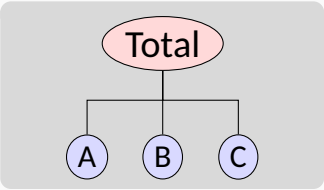
where

- $\boldsymbol{y}_t$ is a vector of all series at time t
- $\boldsymbol{b}_t$ is a vector of the most disaggregated series at time t
- $\boldsymbol{S}$ is a “summing matrix’ ’ containing the aggregation constraints.

Hierarchical time series



Hierarchical time series



y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

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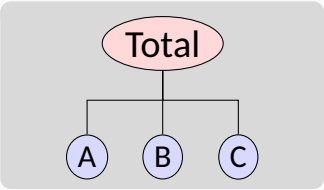


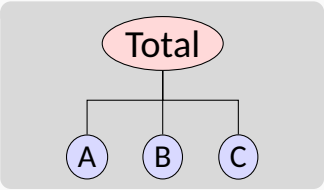
y_t : observed aggregate of all series at time t.

$y_{X,t}$: observation on series X at time t.

$\boldsymbol{b}_t$: vector of all series at bottom level in time t.

$$\boldsymbol{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

Hierarchical time series



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$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

Forecasting notation

Let $\hat{\boldsymbol{y}}_n(h)$ be vector of initial h-step forecasts, made at time n, stacked in same order as $\boldsymbol{y}_t$.

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for some matrix $\boldsymbol{G}$.

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for some matrix $\boldsymbol{G}$.

- $\boldsymbol{G}$ extracts and combines base forecasts $\hat{\boldsymbol{y}}_n(h)$ to get bottom-level forecasts.
- $\boldsymbol{S}$ adds them up

Optimal combination forecasts

Main result

The best (minimum sum of variances) unbiased forecasts are obtained when $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$, where $\boldsymbol{W}_h$ is the h-step base forecast error covariance matrix.

Optimal combination forecasts

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$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}(\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}\hat{\boldsymbol{y}}_n(h)$$

Problem: $\boldsymbol{W}_h$ hard to estimate, especially for $h > 1$.

Solutions:

- Ignore $\boldsymbol{W}_h$ (OLS)
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ is diagonal (WLS)
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ and estimate it (GLS) — the default uses a shrinkage covariance estimator

Features

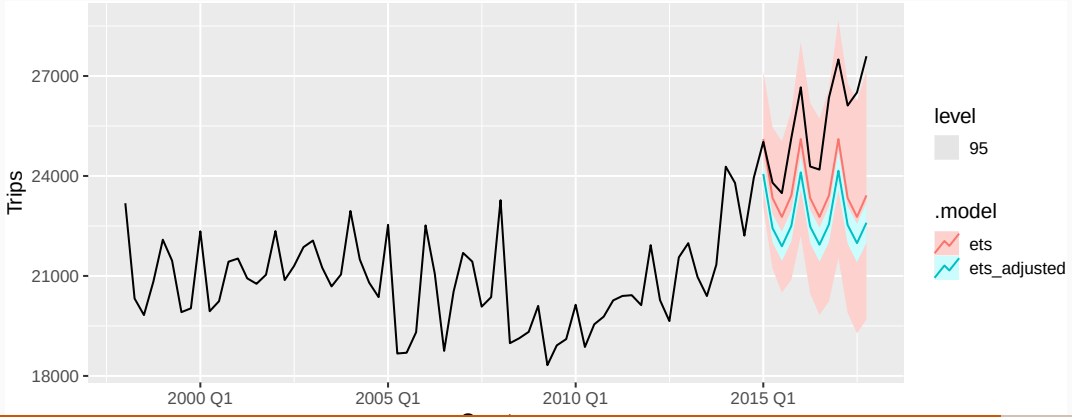
- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

Example: Australian tourism

```
tourism_agg <- tourism %>%
  aggregate_key(Purpose * (State / Region),
    Trips = sum(Trips)
  )
fc <- tourism_agg %>%
  filter(year(Quarter) < 2015) %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = "3 years")
```

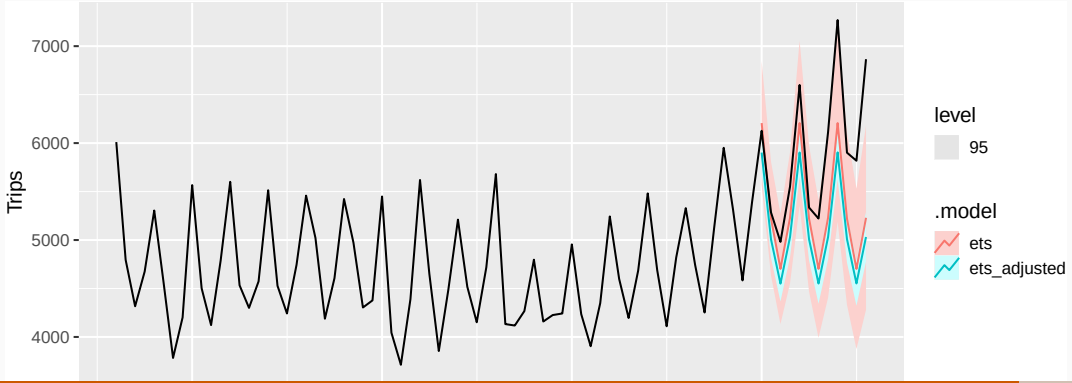
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & is_aggregated(State)) %>%
  autoplot(tourism_agg, level = 95)
```



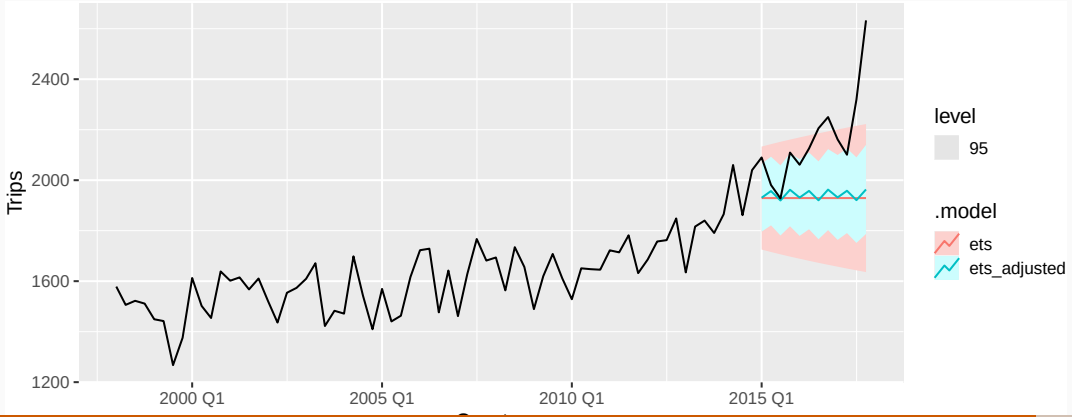
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & State == "VIC" &
    is_aggregated(Region)) %>%
  autoplot(tourism_agg, level = 95)
```



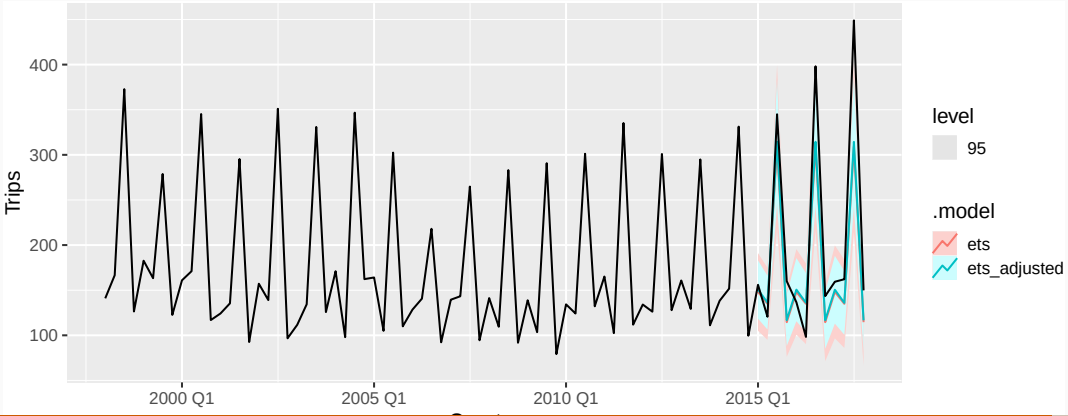
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Melbourne") %>%
  autoplot(tourism_agg, level = 95)
```



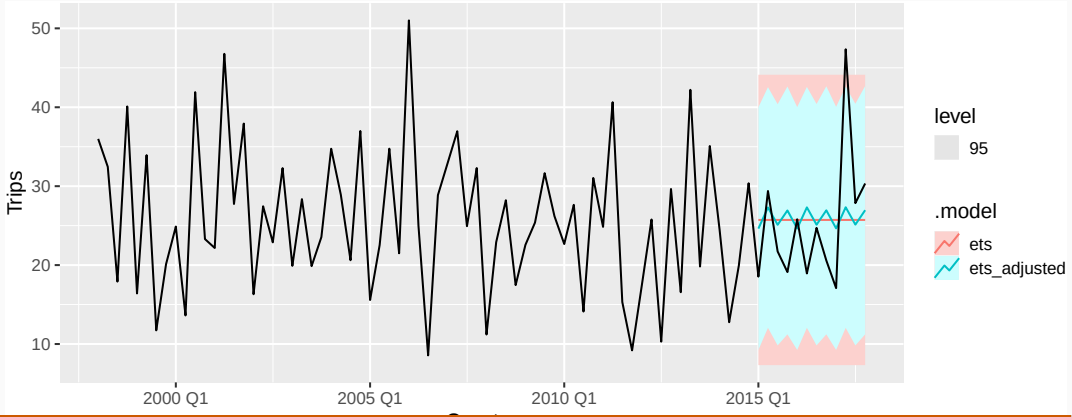
Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Snowy Mountains") %>%
  autoplot(tourism_agg, level = 95)
```



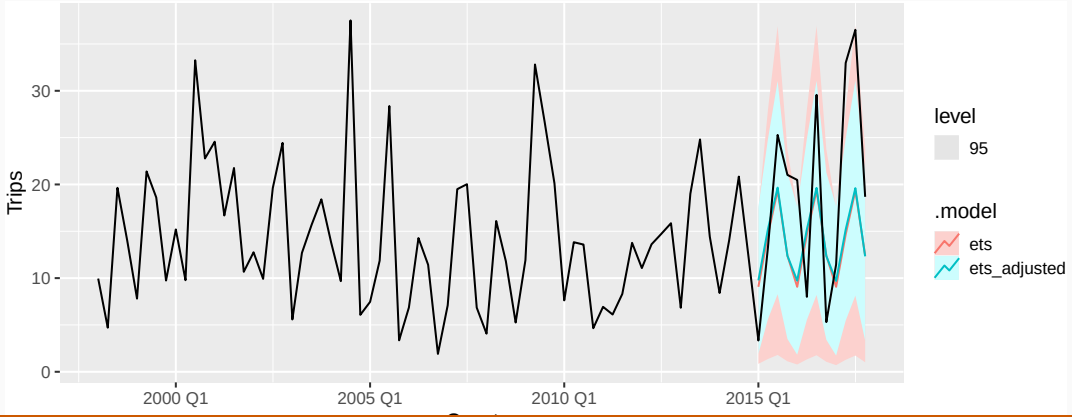
Example: Australian tourism

```
fc %>%
  filter(Purpose == "Holiday" & Region == "Barossa") %>%
  autoplot(tourism_agg, level = 95)
```



Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "MacDonnell") %>%
  autoplot(tourism_agg, level = 95)
```



Forecast evaluation

```
fc %>%
  accuracy(tourism, measures = list(MSE=MSE, CRPS=CRPS))
```

```
## # A tibble: 850 x 7
##    .model Region          State  Purpose      .type     MSE   CRPS
##    <chr>  <chr*>          <chr*> <chr*>       <chr>   <dbl>  <dbl>
##  1 ets    Adelaide        SA     Business     Test    864.    17.1
##  2 ets    Adelaide        SA     Holiday      Test   1188.    21.7
##  3 ets    Adelaide        SA     Other        Test    439.    12.2
##  4 ets    Adelaide        SA     Visiting     Test   1101.    19.9
##  5 ets    Adelaide        SA     <aggregated> Test    NaN     NaN
##  6 ets    Adelaide Hills SA     Business     Test     31.2     2.86
##  7 ets    Adelaide Hills SA     Holiday      Test     44.7     3.84
##  8 ets    Adelaide Hills SA     Other        Test      7.86    1.51
##  9 ets    Adelaide Hills SA     Visiting     Test     67.0    11.9
## 10 ets    Adelaide Hills SA     <aggregated> Test    NaN     NaN
```

Forecast evaluation

```
fc %>%
  accuracy(tourism, measures = list(MSE=MSE, CRPS=CRPS)) %>%
  group_by(.model) %>%
  summarise(MSE = mean(MSE), CRPS=mean(CRPS))
```

```
## # A tibble: 2 x 3
##   .model          MSE  CRPS
##   <chr>         <dbl> <dbl>
## 1 ets             NaN   NaN
## 2 ets_adjusted    NaN   NaN
```

More information

- Slides and papers: **robjhyndman.com**
- Packages: **tidyverts.org**
- Forecasting textbook using tidyverts package: **OTexts.com/fpp3**

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