# Feasts & fables: Modern tools for time series analysis


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# Outline

1. What does modern time series data look like? (tsibble)
2. EDA for large time series (feasts)
3. Probabilistic forecasting for large time series (fable)
4. fable: Evaluating forecast accuracy
5. fable: Forecast reconciliation

# Tidyverts packages

tidyverts.org




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# What does modern time series data look like? (tsibble)

Show examples of data

Annual Quarterly Monthly Weekly Daily Hourly Half-hourly Minutely data

–> need to deal with high frequency and multiple seasonalities

Show examples with multiple keys

–> need to deal with collections of time series

Hierarchical and grouping structures

# Storing time series as `tsibble` objects

```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##     Year Country              GDP Imports Exports Population
##    <dbl> <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

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## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index <fct>              <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
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##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
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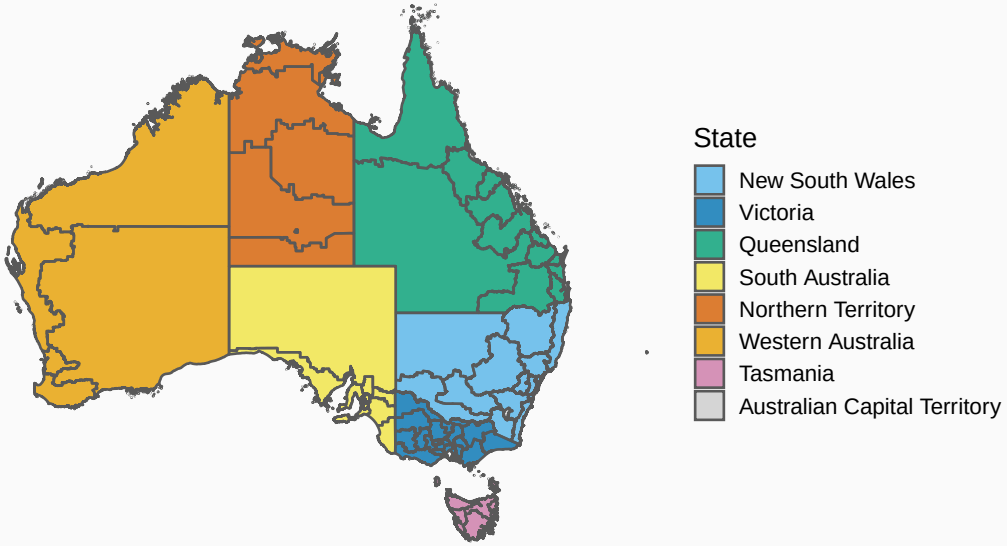
```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index Key                <dbl>   <dbl>   <dbl>      <dbl>
##  1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##  2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##  3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##  4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##  5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##  6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##  7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##  8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##  9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
## 10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

# Storing time series as `tsibble` objects

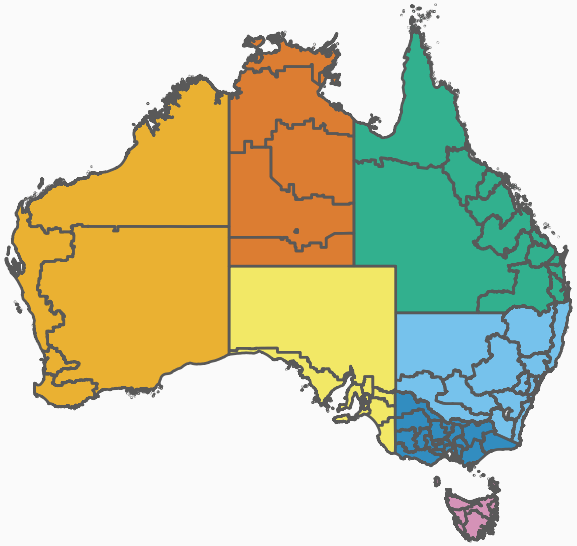
```
global_economy
```

```
## # A tsibble: 15,150 x 6 [1Y]
## # Key:       Country [263]
##      Year Country              GDP Imports Exports Population
##     Index Key                  Measured variables
##   1  1960 Afghanistan  537777811.    7.02    4.13    8996351
##   2  1961 Afghanistan  548888896.    8.10    4.45    9166764
##   3  1962 Afghanistan  546666678.    9.35    4.88    9345868
##   4  1963 Afghanistan  751111191.   16.9     9.17    9533954
##   5  1964 Afghanistan  800000044.   18.1     8.89    9731361
##   6  1965 Afghanistan 1006666638.   21.4    11.3     9938414
##   7  1966 Afghanistan 1399999967.   18.6     8.57   10152331
##   8  1967 Afghanistan 1673333418.   14.2     6.77   10372630
##   9  1968 Afghanistan 1373333367.   15.2     8.90   10604346
##  10  1969 Afghanistan 1408888922.   15.0    10.1    10854428
```

# Australian tourism regions



# Australian tourism regions



- Quarterly data on visitor nights: 1998 – 2017
- From *National Visitor Survey*, interviews of 120,000 Australians aged 15+.
- Geographical hierarchy split by
  - 8 states and territories
  - 76 regions
- Purpose:
  - Holidays
  - Business
  - Visiting friends & relatives
  - Other

# `tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      <qtr> <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

# `tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
```

Index

Domestic visitor nights in thousands by state/region and purpose.

# tsibble objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index Keys                   <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
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```

Domestic visitor nights in thousands by state/region and purpose.

# `tsibble` objects

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      Index Keys                   Measure
##   1 1998 Q1 Adelaide SA    Business  135.
##   2 1998 Q2 Adelaide SA    Business  110.
##   3 1998 Q3 Adelaide SA    Business  166.
##   4 1998 Q4 Adelaide SA    Business  127.
##   5 1999 Q1 Adelaide SA    Business  137.
##   6 1999 Q2 Adelaide SA    Business  200.
##   7 1999 Q3 Adelaide SA    Business  169.
##   8 1999 Q4 Adelaide SA    Business  134.
##   9 2000 Q1 Adelaide SA    Business  154.
##  10 2000 Q2 Adelaide SA    Business  169.
```

Domestic visitor nights in thousands by state/region and purpose.

# `tsibble` objects

- A `tsibble` allows storage and manipulation of multiple time series in R.
- It contains:
  - An index: time information about the observation
  - Measured variable(s): numbers of interest
  - Key variable(s): optional unique identifiers for each series

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# EDA for large time series (feasts)

- PCA
- outliers

# Australian holidays

```
tourism
```

```
## # A tsibble: 24,320 x 5 [1Q]
## # Key:       Region, State, Purpose [304]
##    Quarter Region   State Purpose  Trips
##      <qtr> <chr>    <chr> <chr>    <dbl>
##  1 1998 Q1 Adelaide SA    Business  135.
##  2 1998 Q2 Adelaide SA    Business  110.
##  3 1998 Q3 Adelaide SA    Business  166.
##  4 1998 Q4 Adelaide SA    Business  127.
##  5 1999 Q1 Adelaide SA    Business  137.
##  6 1999 Q2 Adelaide SA    Business  200.
##  7 1999 Q3 Adelaide SA    Business  169.
##  8 1999 Q4 Adelaide SA    Business  134.
##  9 2000 Q1 Adelaide SA    Business  154.
## 10 2000 Q2 Adelaide SA    Business  169.
## # ... with 24,310 more rows
```

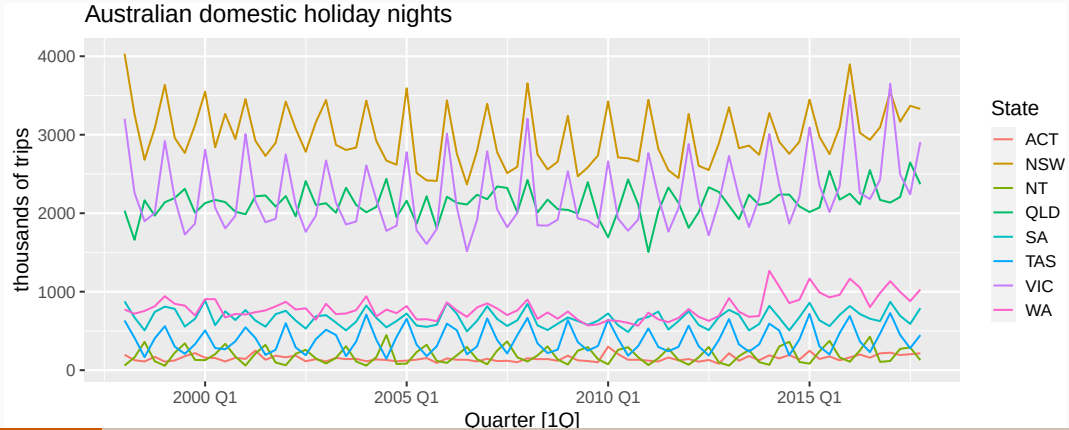
# Australian holidays

```
holidays <- tourism %>%
  filter(Purpose == "Holiday") %>%
  group_by(State) %>%
  summarise(Trips = sum(Trips))
```

```
## # A tsibble: 640 x 3 [1Q]
## # Key:       State [8]
##    State Quarter Trips
##    <chr>   <qtr> <dbl>
##  1 ACT   1998 Q1  196.
##  2 ACT   1998 Q2  127.
##  3 ACT   1998 Q3  111.
##  4 ACT   1998 Q4  170.
##  5 ACT   1999 Q1  108.
##  6 ACT   1999 Q2  125.
##  7 ACT   1999 Q3  178.
##  8 ACT   1999 Q4  218.
##  9 ACT   2000 Q1  158.
```

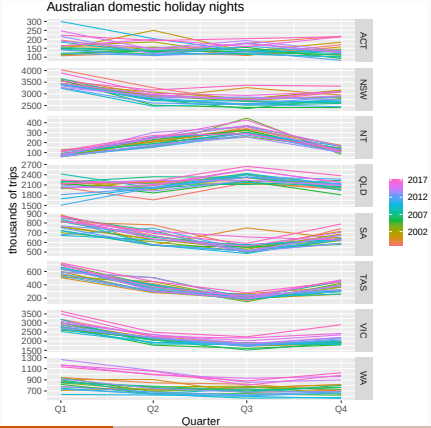
# Australian holidays

```
holidays %>% autoplot(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```



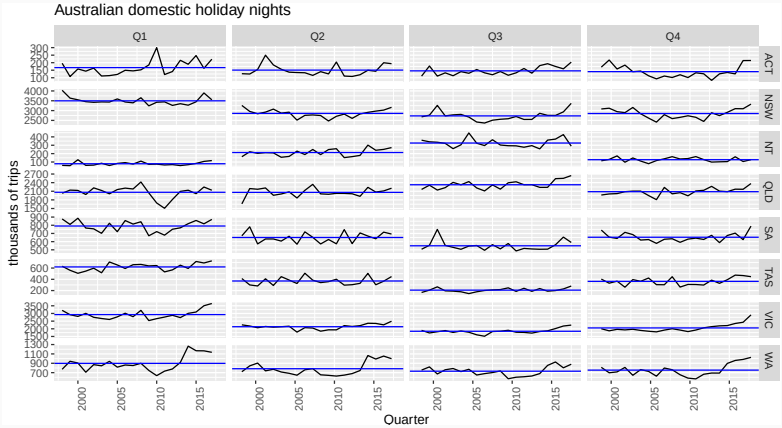
# Seasonal plots

```
holidays %>% gg_season(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```



# Seasonal subseries plots

```
holidays %>% gg_subseries(Trips) +
  ylab("thousands of trips") +
  ggtitle("Australian domestic holiday nights")
```



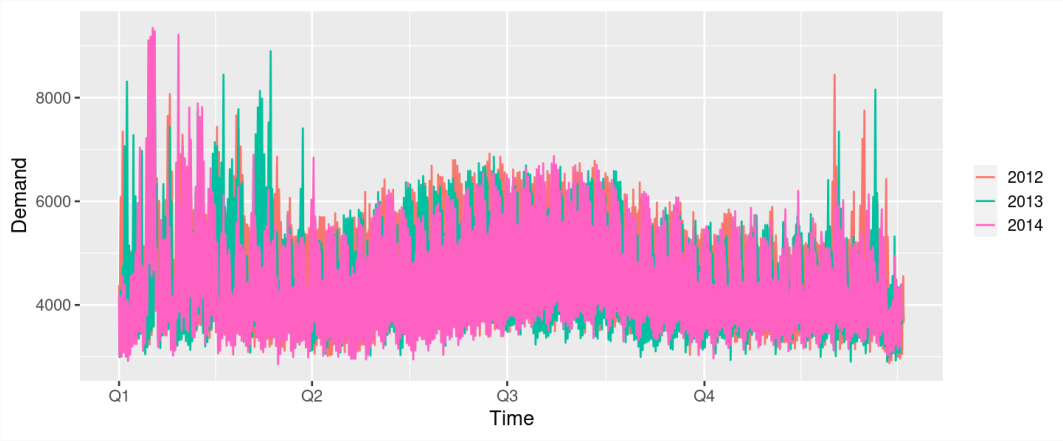
# Victorian electricity demand

```
vic_elec
```

```
## # A tsibble: 52,608 x 5 [30m] <Australia/Melbourne>
##    Time                Demand Temperature Date       Holiday
##    <dttm>               <dbl>       <dbl> <date>     <lgl>
##  1 2012-01-01 00:00:00  4383.        21.4 2012-01-01 TRUE
##  2 2012-01-01 00:30:00  4263.        21.0 2012-01-01 TRUE
##  3 2012-01-01 01:00:00  4049.        20.7 2012-01-01 TRUE
##  4 2012-01-01 01:30:00  3878.        20.6 2012-01-01 TRUE
##  5 2012-01-01 02:00:00  4036.        20.4 2012-01-01 TRUE
##  6 2012-01-01 02:30:00  3866.        20.2 2012-01-01 TRUE
##  7 2012-01-01 03:00:00  3694.        20.1 2012-01-01 TRUE
##  8 2012-01-01 03:30:00  3562.        19.6 2012-01-01 TRUE
##  9 2012-01-01 04:00:00  3433.        19.1 2012-01-01 TRUE
## 10 2012-01-01 04:30:00  3359.        19.0 2012-01-01 TRUE
## # ... with 52,598 more rows
```

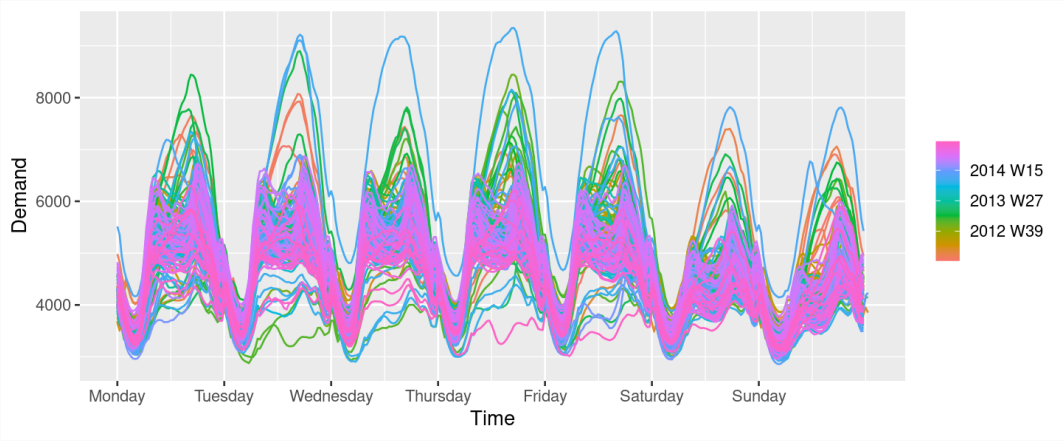
# Multiple seasonal periods

```
vic_elec %>% gg_season(Demand)
```



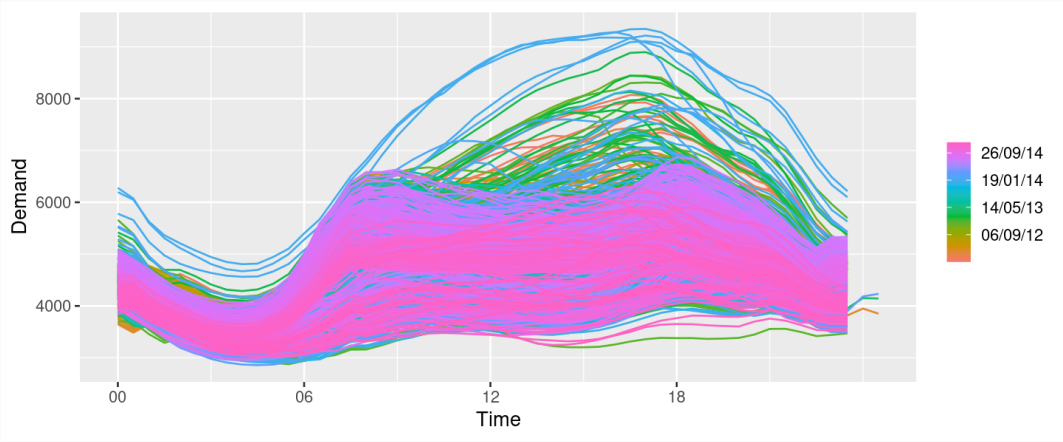
# Multiple seasonal periods

```
vic_elec %>% gg_season(Demand, period = "week")
```



# Multiple seasonal periods

```
vic_elec %>% gg_season(Demand, period = "day")
```



# Time plots

- Plotted against time: `autoplot()` (each series overplotted)
- Plotted against season: `gg_season()` (facet by key)
- Plotted against time with seasonal facets: `gg_subseries()` (facet by key)

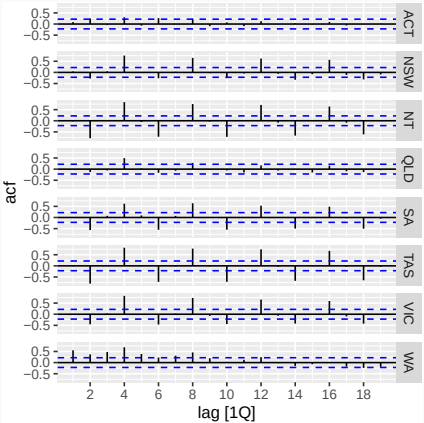
# Autocorrelations

```
holidays %>% ACF(Trips)
```

```
## # A tsibble: 152 x 3 [1Q]
## # Key:       State [8]
##    State   lag      acf
##    <chr> <lag>    <dbl>
##  1 ACT      1Q  0.0877
##  2 ACT      2Q  0.252
##  3 ACT      3Q -0.0496
##  4 ACT      4Q  0.300
##  5 ACT      5Q -0.0741
##  6 ACT      6Q  0.269
##  7 ACT      7Q -0.00504
##  8 ACT      8Q  0.236
##  9 ACT      9Q -0.0953
## 10 ACT     10Q  0.0750
## # ... with 142 more rows
```

# ACF plots

```
holidays %>%
  ACF(Trips) %>%
  autoplot()
```



# Google stock price

```
gafa_stock
```

```
## # A tsibble: 5,032 x 8 [!]
## # Key:       Symbol [4]
##    Symbol Date        Open  High   Low Close Adj_Close    Volume
##    <chr>  <date>     <dbl> <dbl> <dbl> <dbl>     <dbl>     <dbl>
##  1 AAPL   2014-01-02  79.4  79.6  78.9  79.0      67.0  58671200
##  2 AAPL   2014-01-03  79.0  79.1  77.2  77.3      65.5  98116900
##  3 AAPL   2014-01-06  76.8  78.1  76.2  77.7      65.9 103152700
##  4 AAPL   2014-01-07  77.8  78.0  76.8  77.1      65.4  79302300
##  5 AAPL   2014-01-08  77.0  77.9  77.0  77.6      65.8  64632400
##  6 AAPL   2014-01-09  78.1  78.1  76.5  76.6      65.0  69787200
##  7 AAPL   2014-01-10  77.1  77.3  75.9  76.1      64.5  76244000
```

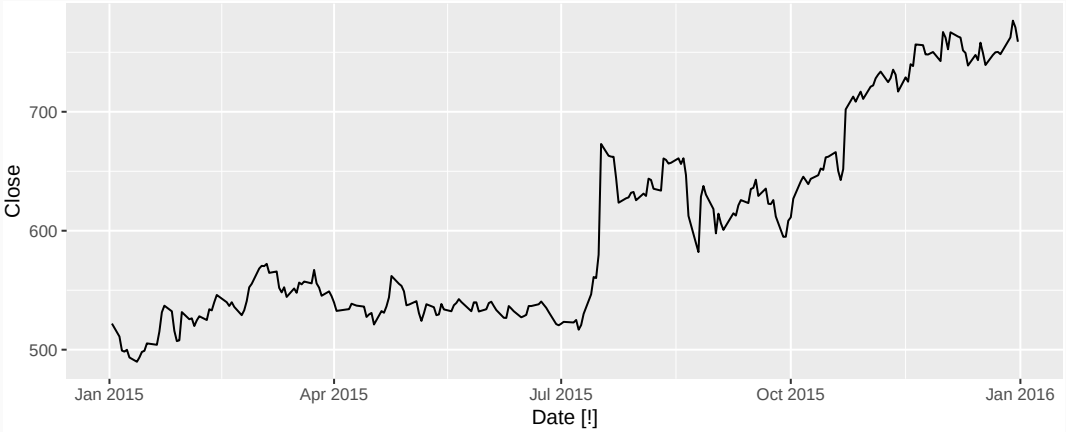
# Google stock price

```
google_2015 <- gafa_stock %>%
  filter(Symbol == "GOOG", year(Date) == 2015) %>%
  select(Date, Close)
google_2015
```

```
## # A tsibble: 252 x 2 [!]
##    Date       Close
##    <date>     <dbl>
##  1 2015-01-02  522.
##  2 2015-01-05  511.
##  3 2015-01-06  499.
##  4 2015-01-07  498.
##  5 2015-01-08  500.
```

# Google stock price

```
google_2015 %>% autoplot(Close)
```



# Google stock price

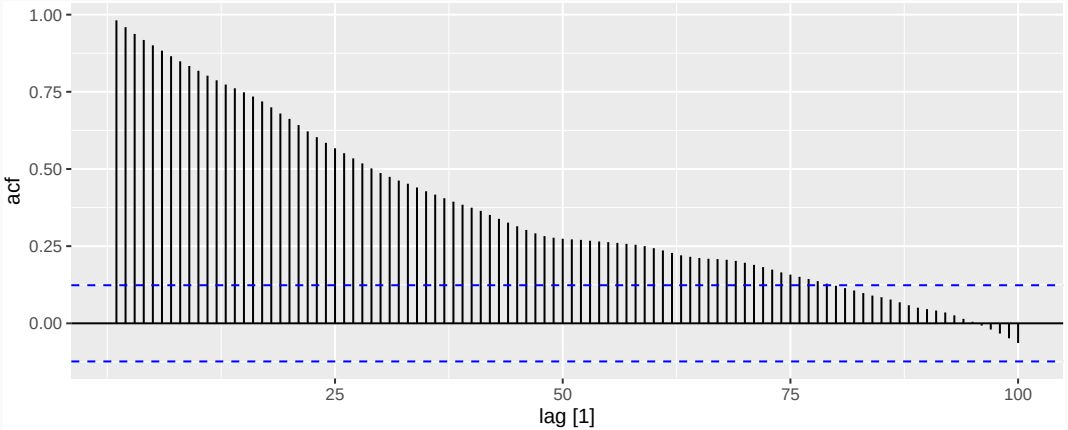
```
google_2015 %>%
  ACF(Close, lag_max = 100)
```

```
## Warning: Provided data has an irregular interval, results should be treated with
## caution. Computing ACF by observation.

## # A tsibble: 100 x 2 [1]
##      lag   acf
##    <lag> <dbl>
##  1     1 0.982
##  2     2 0.959
##  3     3 0.937
##  4     4 0.918
##  5     5 0.901
##  6     6 0.883
##  7     7 0.865
##  8     8 0.849
##  9     9 0.834
## 10    10 0.818
## #     with 90 more rows
```

# Google stock price

```
google_2015 %>%
  ACF(Close, lag_max = 100) %>%
  autoplot()
```



# Strength of seasonality and trend

**STL decomposition**

$$y_t = T_t + S_t + R_t$$

**Seasonal strength**

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right)$$

**Trend strength**

$$\max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(T_t + R_t)}\right)$$

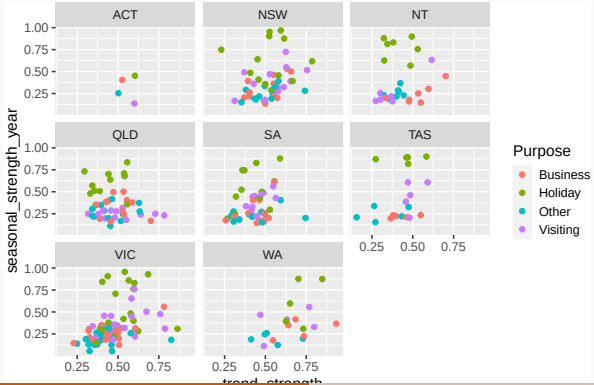
# Feature extraction and statistics

```
tourism %>% features(Trips, feat_stl)
```

```
## # A tibble: 304 x 12
##    Region         State Purpose  trend_strength seasonal_strength_~ seasonal_peak_ye~
##    <chr>          <chr> <chr>             <dbl>               <dbl>             <dbl>
##  1 Adelaide       SA    Business          0.464               0.407                 3
##  2 Adelaide       SA    Holiday           0.554               0.619                 1
##  3 Adelaide       SA    Other             0.746               0.202                 2
##  4 Adelaide       SA    Visiting          0.435               0.452                 1
##  5 Adelaide Hills SA    Business          0.464               0.179                 3
##  6 Adelaide Hills SA    Holiday           0.528               0.296                 2
##  7 Adelaide Hills SA    Other             0.593               0.404                 2
##  8 Adelaide Hills SA    Visiting          0.488               0.254                 0
##  9 Alice Springs  NT    Business          0.534               0.251                 0
## 10 Alice Springs  NT    Holiday           0.381               0.832                 3
## # ... with 294 more rows, and 6 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>
```

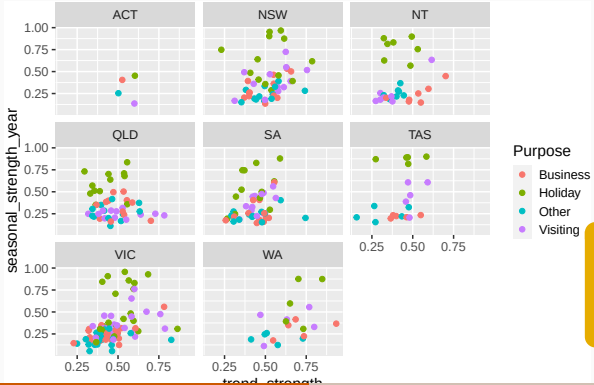
# Feature extraction and statistics

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```



# Feature extraction and statistics

```
tourism %>%
  features(Trips, feat_stl) %>%
  ggplot(aes(x = trend_strength, y = seasonal_strength_year, col = Purpose)) +
  geom_point() +
  facet_wrap(vars(State))
```




- Holidays more seasonal than other travel.
- WA has strongest trends.

# Feature extraction and statistics

Find the most seasonal time series:

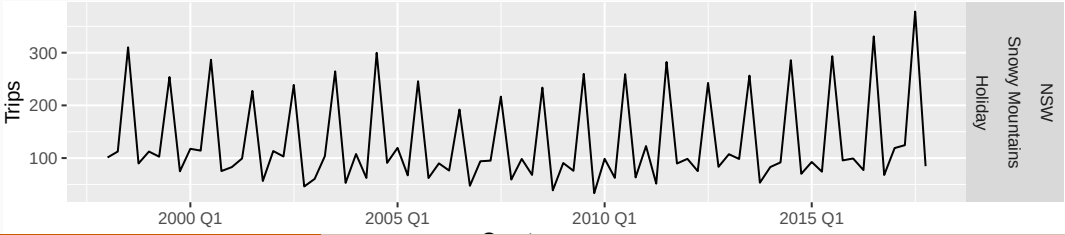
```
most_seasonal <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

# Feature extraction and statistics

Find the most seasonal time series:

```
most_seasonal <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(seasonal_strength_year == max(seasonal_strength_year))
```

```
tourism %>%
  right_join(most_seasonal, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



# Feature extraction and statistics

Find the most trended time series:

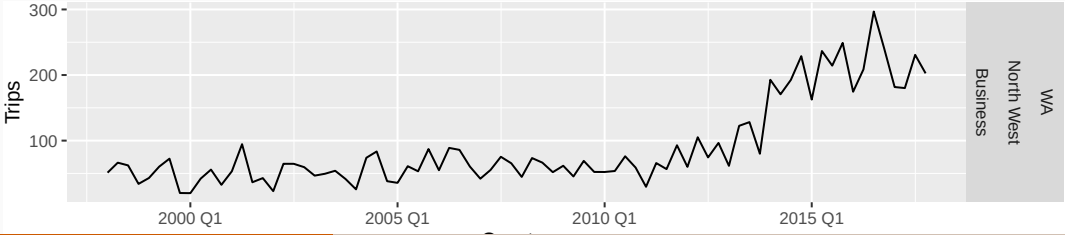
```
most_trended <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength))
```

# Feature extraction and statistics

Find the most trended time series:

```
most_trended <- tourism %>%
  features(Trips, feat_stl) %>%
  filter(trend_strength == max(trend_strength))
```

```
tourism %>%
  right_join(most_trended, by = c("State", "Region", "Purpose")) %>%
  ggplot(aes(x = Quarter, y = Trips)) +
  geom_line() +
  facet_grid(vars(State, Region, Purpose))
```



# Feature extraction and statistics

```
tourism_features <- tourism %>%
  features(Trips, feature_set(pkgs = "feasts"))
```

All features from the feasts package

```
## # A tibble: 304 x 50
##    Region         State Purpose  trend_strength seasonal_strength_~ seasonal_peak_ye~
##    <chr>          <chr> <chr>             <dbl>               <dbl>              <dbl>
##  1 Adelaide       SA    Business          0.464               0.407                  3
##  2 Adelaide       SA    Holiday           0.554               0.619                  1
##  3 Adelaide       SA    Other             0.746               0.202                  2
##  4 Adelaide       SA    Visiting          0.435               0.452                  1
##  5 Adelaide Hills SA    Business          0.464               0.179                  3
##  6 Adelaide Hills SA    Holiday           0.528               0.296                  2
##  7 Adelaide Hills SA    Other             0.593               0.404                  2
##  8 Adelaide Hills SA    Visiting          0.488               0.254                  0
##  9 Alice Springs  NT    Business          0.534               0.251                  0
## 10 Alice Springs  NT    Holiday           0.381               0.832                  3
## # ... with 294 more rows, and 44 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>, pacf5 <dbl>
```

# Feature extraction and statistics

```
pcs <- tourism_features %>%
  select(-State, -Region, -Purpose) %>%
  prcomp(scale = TRUE) %>%
  broom::augment(tourism_features)
```

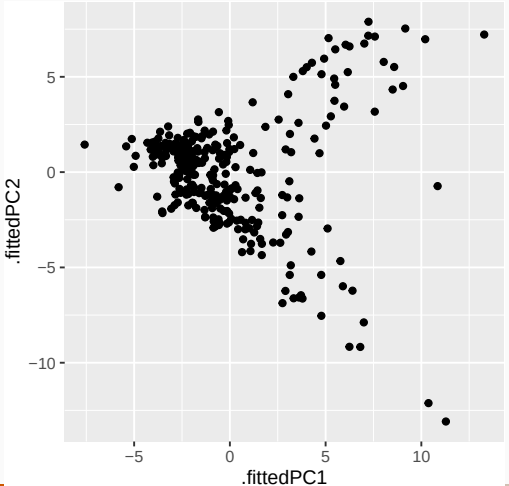
Principal components based on all features from the feasts package

```
## # A tibble: 304 x 98
##    .rownames Region   State Purpose trend_strength seasonal_strengt~ seasonal_peak_y~
##    <chr>     <chr>    <chr> <chr>            <dbl>             <dbl>            <dbl>
##  1 1         Adelaide SA    Busine~          0.464             0.407                3
##  2 2         Adelaide SA    Holiday          0.554             0.619                1
##  3 3         Adelaide SA    Other            0.746             0.202                2
##  4 4         Adelaide SA    Visiti~          0.435             0.452                1
##  5 5         Adelaid~ SA    Busine~          0.464             0.179                3
##  6 6         Adelaid~ SA    Holiday          0.528             0.296                2
##  7 7         Adelaid~ SA    Other            0.593             0.404                2
##  8 8         Adelaid~ SA    Visiti~          0.488             0.254                0
##  9 9         Alice S~ NT    Busine~          0.534             0.251                0
## 10 10        Alice S~ NT    Holiday          0.381             0.832                3
## # ... with 294 more rows, and 91 more variables: seasonal_trough_year <dbl>,
## #   spikiness <dbl>, linearity <dbl>, curvature <dbl>, stl_e_acf1 <dbl>,
## #   stl_e_acf10 <dbl>, acf1 <dbl>, acf10 <dbl>, diff1_acf1 <dbl>, diff1_acf10 <dbl>,
## #   diff2_acf1 <dbl>, diff2_acf10 <dbl>, season_acf1 <dbl>, pacf5 <dbl>,
```

# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2)) +
  geom_point() + theme(aspect.ratio=1)
```

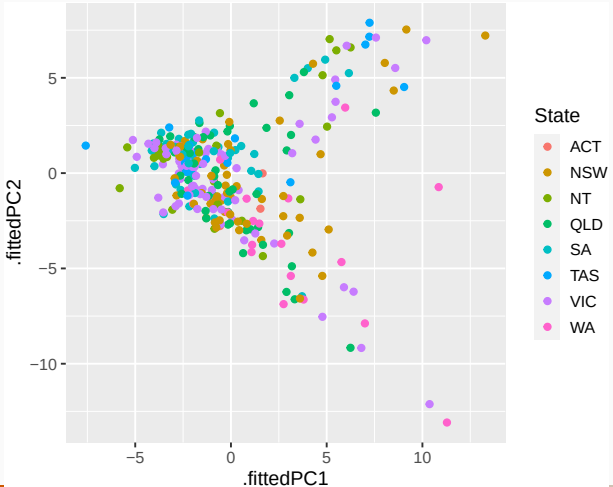
Principal components based on all features from the feasts package



# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=State)) +
  geom_point() + theme(aspect.ratio=1)
```

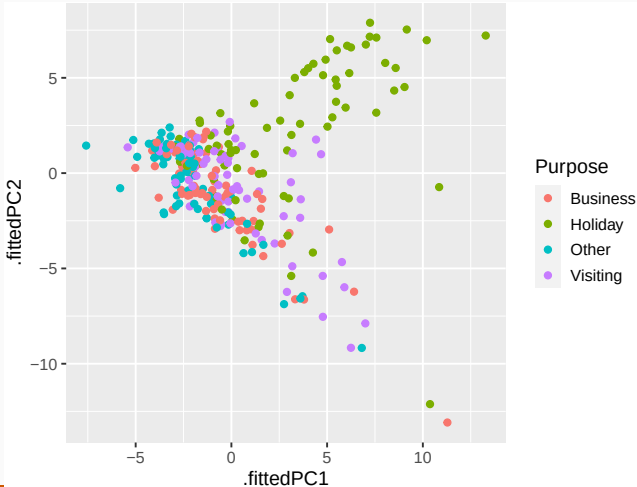
Principal components based on all features from the feasts package



# Feature extraction and statistics

```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

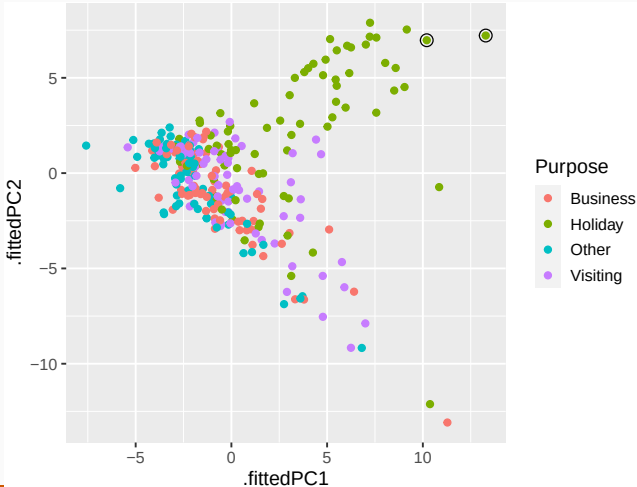
Principal components based on all features from the feasts package



# Feature extraction and statistics

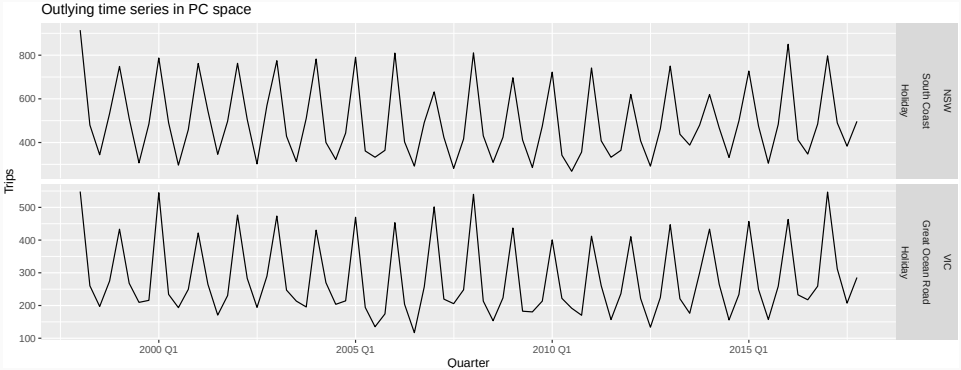
```
pcs %>% ggplot(aes(x=.fittedPC1, y=.fittedPC2, col=Purpose)) +
  geom_point() + theme(aspect.ratio=1)
```

Principal components based on all features from the feasts package



# Feature extraction and statistics

```
outliers %>%
  left_join(tourism, by = c("State", "Region", "Purpose")) %>%
  mutate(Series = glue("{State}", "{Region}", "{Purpose}", .sep = "\n\n")) %>%
  ggplot(aes(x = Quarter, y = Trips)) + geom_line() +
  facet_grid(Series ~ ., scales = "free_y") +
  ggtitle("Outlying time series in PC space")
```



# Outline

1. What does modern time series data look like? (tsibble)
2. EDA for large time series (feasts)
3. Probabilistic forecasting for large time series (fable)
4. fable: Evaluating forecast accuracy
5. fable: Forecast reconciliation

# Probabilistic forecasting for large time series (fable)

- ensemble and hybrid forecasts

# Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.

# Random futures

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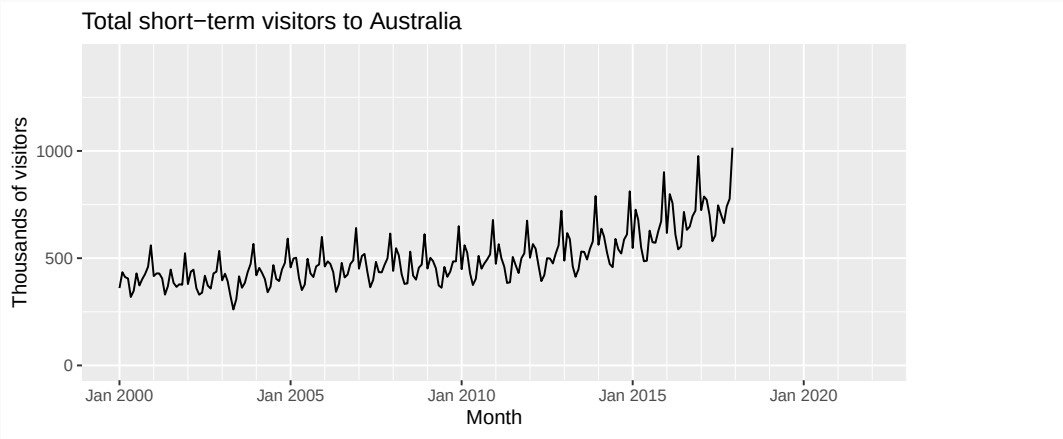
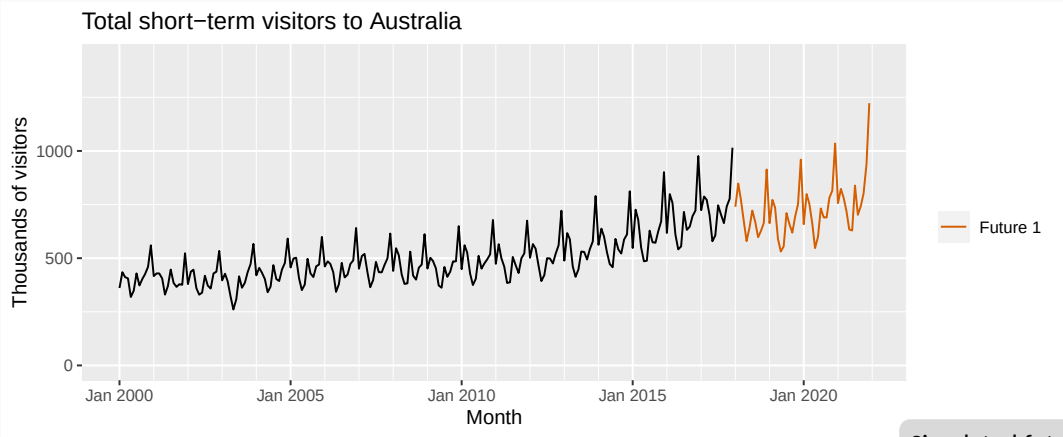


# Random futures

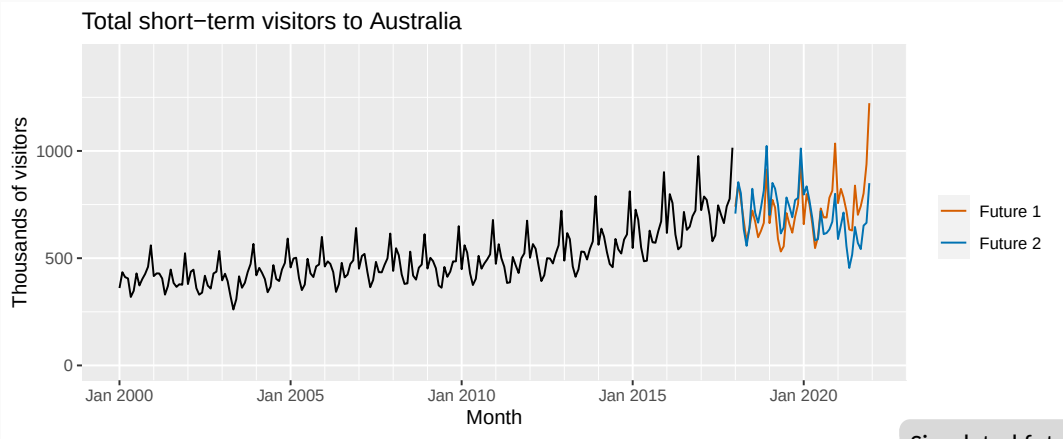
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

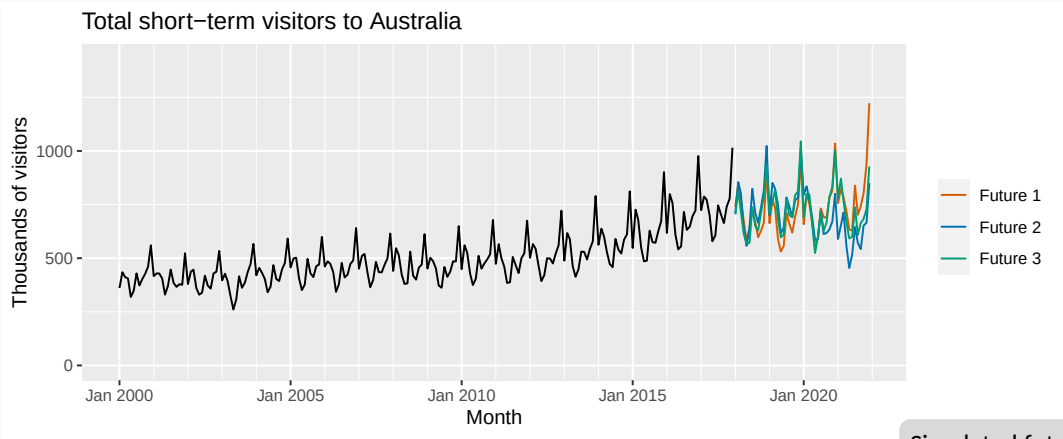
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

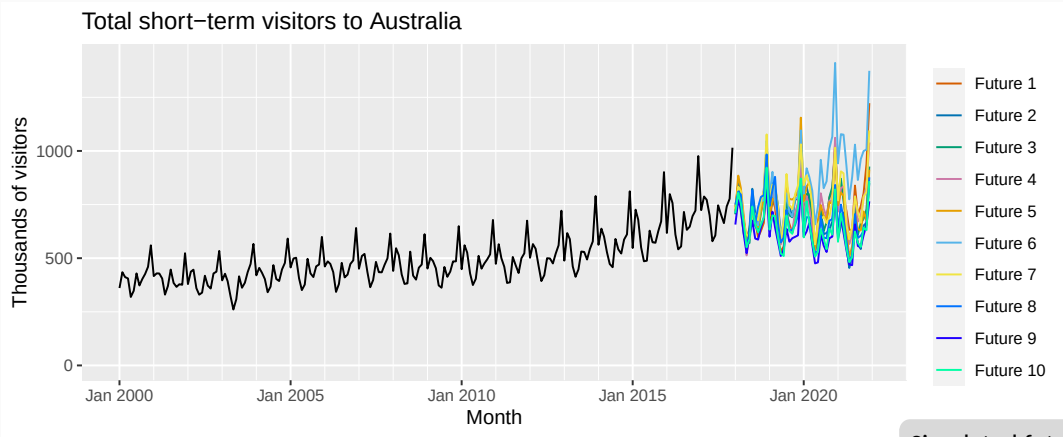
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

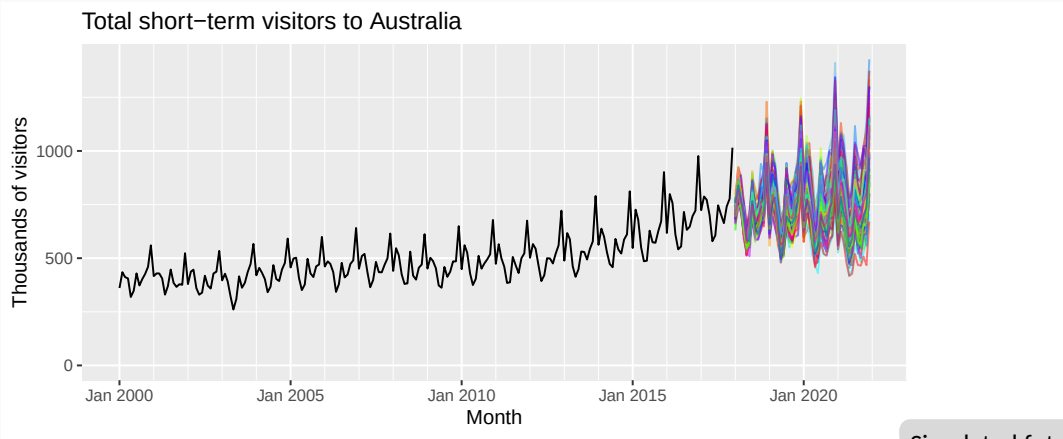
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

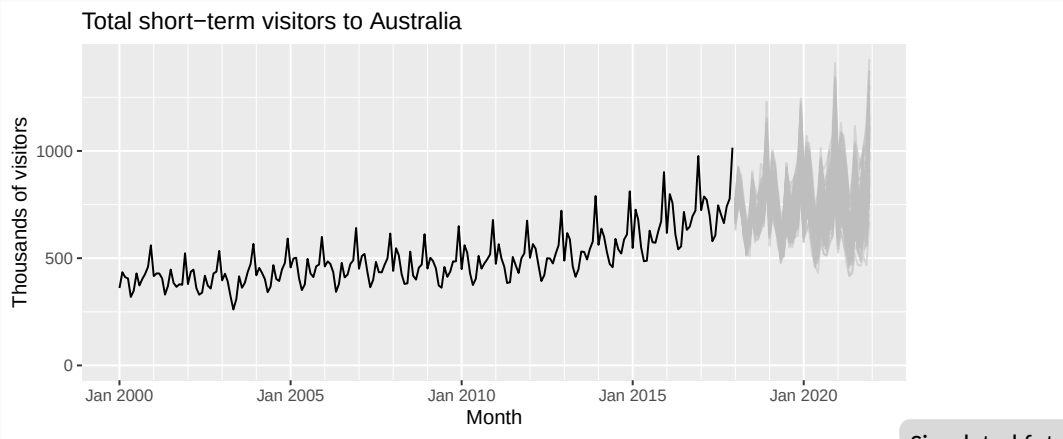
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

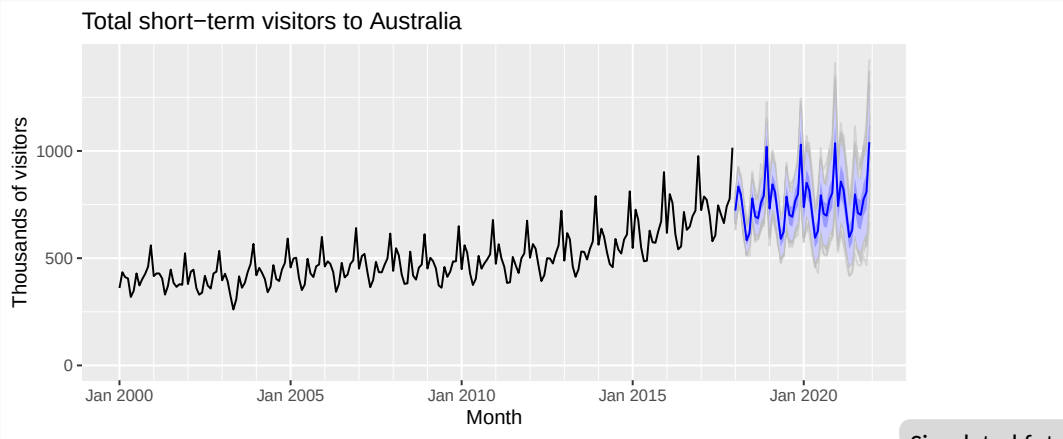
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

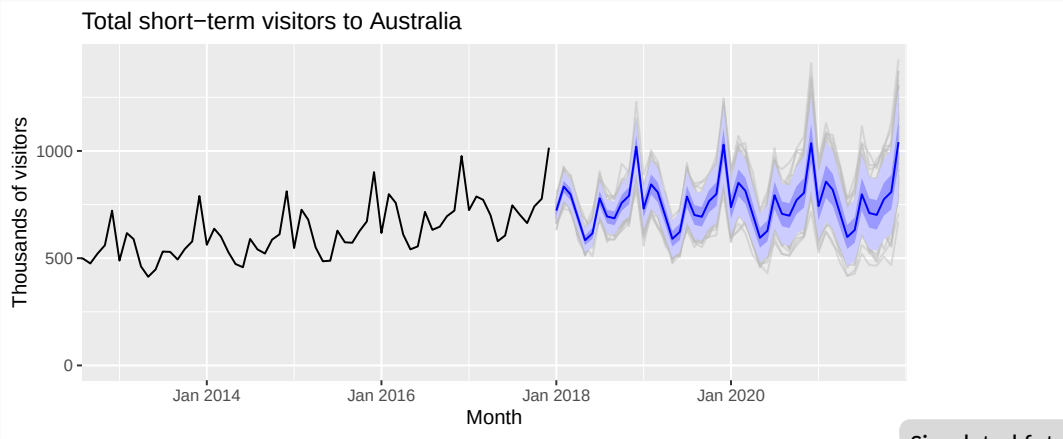
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

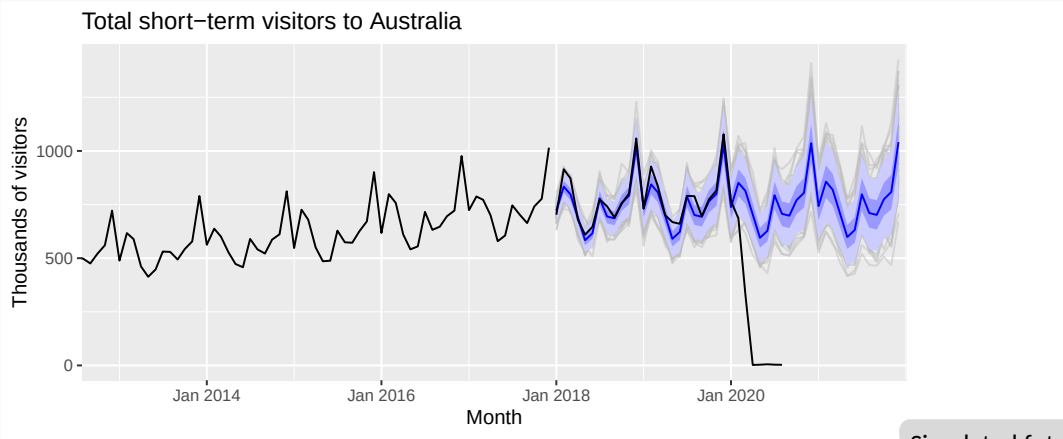
A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Random futures

A forecast is an estimate of the probability distribution of a variable to be observed in the future.




Simulated futures from an ETS model

# Model fitting

```
holiday_fit <- holidays %>%
  model(
    snaive = SNAIVE(Trips),
    naive = NAIVE(Trips),
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  )
```

```
## # A mable: 8 x 5
## # Key:     State [8]
##   State   snaive   naive          ets                                arima
##   <chr>  <model> <model>      <model>                              <model>
## 1 ACT   <SNAIVE> <NAIVE> <ETS(M,N,A)>         <ARIMA(0,1,1)(1,0,1)[4]>
## 2 NSW   <SNAIVE> <NAIVE> <ETS(M,N,A)>         <ARIMA(0,1,1)(0,1,1)[4]>
## 3 NT    <SNAIVE> <NAIVE> <ETS(M,N,A)>         <ARIMA(0,0,0)(2,1,0)[4]>
## 4 QLD   <SNAIVE> <NAIVE> <ETS(A,N,A)> <ARIMA(0,0,0)(1,0,0)[4] w/ mean>
```

# Model fitting

```
holiday_fit %>%
  filter(State == "VIC") %>%
  select(arima) %>%
  report()
```

```
## Series: Trips
## Model: ARIMA(0,1,1)(0,1,1)[4]
##
## Coefficients:
##           ma1    sma1
##       -0.8079  -0.576
## s.e.   0.0802   0.164
##
## sigma^2 estimated as 25711:  log likelihood=-488
## AIC=982   AICc=982   BIC=989
```

# Model fitting

```
glance(holiday_fit)
```

```
## # A tibble: 32 x 12
##    State .model       sigma2 log_lik   AIC  AICc   BIC    MSE   AMSE     MAE ar_r
##    <chr> <chr>         <dbl>   <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>   <dbl> <lis
##  1 ACT   snaive   2147.          NA    NA    NA    NA     NA     NA  NA       <NUL
##  2 ACT   naive    2931.          NA    NA    NA    NA     NA     NA  NA       <NUL
##  3 ACT   ets         0.0680    -463.  940.  941.  956.  1509.  1538.  0.189  <NUL
##  4 ACT   arima    1437.        -399.  805.  806.  815.    NA     NA  NA       <cpl
##  5 NSW   snaive  46496.          NA    NA    NA    NA     NA     NA  NA       <NUL
##  6 NSW   naive  254319.          NA    NA    NA    NA     NA     NA  NA       <NUL
##  7 NSW   ets         0.00347   -585. 1184. 1185. 1201. 28271. 29225.  0.0458 <NUL
##  8 NSW   arima   33036.        -498. 1003. 1003. 1010.    NA     NA  NA       <cpl
##  9 NT    snaive   2445.          NA    NA    NA    NA     NA     NA  NA       <NUL
## 10 NT    naive   21118.          NA    NA    NA    NA     NA     NA  NA       <NUL
## # ... with 22 more rows, and 1 more variable: ma_roots <list>
```

# Model fitting

```
tidy(holiday_fit)
```

```
## # A tibble: 75 x 7
##    State .model term  estimate std.error statistic   p.value
##    <chr> <chr>  <chr>    <dbl>     <dbl>     <dbl>     <dbl>
##  1 ACT   ets    alpha   0.0513   NA          NA    NA
##  2 ACT   ets    gamma   0.482    NA          NA    NA
##  3 ACT   ets    l[0]  148.       NA          NA    NA
##  4 ACT   ets    s[0]    7.88     NA          NA    NA
##  5 ACT   ets    s[-1]  -2.66     NA          NA    NA
##  6 ACT   ets    s[-2] -20.5      NA          NA    NA
##  7 ACT   ets    s[-3]  15.3      NA          NA    NA
##  8 ACT   arima  ma1    -0.886     0.0668    -13.3   8.33e-22
##  9 ACT   arima  sar1    0.773     0.170       4.55  1.93e- 5
## 10 ACT   arima  sma1   -0.557     0.220      -2.52  1.36e- 2
## # ... with 65 more rows
```

# Model fitting

```
augment(holiday_fit)
```

```
## # A tsibble: 2,560 x 7 [1Q]
## # Key:       State, .model [32]
##    State .model Quarter Trips .fitted .resid .innov
##    <chr> <chr>    <qtr> <dbl>   <dbl>  <dbl>  <dbl>
##  1 ACT   snaive 1998 Q1  196.      NA   NA      NA
##  2 ACT   snaive 1998 Q2  127.      NA   NA      NA
##  3 ACT   snaive 1998 Q3  111.      NA   NA      NA
##  4 ACT   snaive 1998 Q4  170.      NA   NA      NA
##  5 ACT   snaive 1999 Q1  108.    196. -88.4  -88.4
##  6 ACT   snaive 1999 Q2  125.    127.  -2.13  -2.13
##  7 ACT   snaive 1999 Q3  178.    111.  67.3   67.3
##  8 ACT   snaive 1999 Q4  218.    170.  47.2   47.2
##  9 ACT   snaive 2000 Q1  158.    108.  50.6   50.6
## 10 ACT   snaive 2000 Q2  155.    125.  30.2   30.2
```

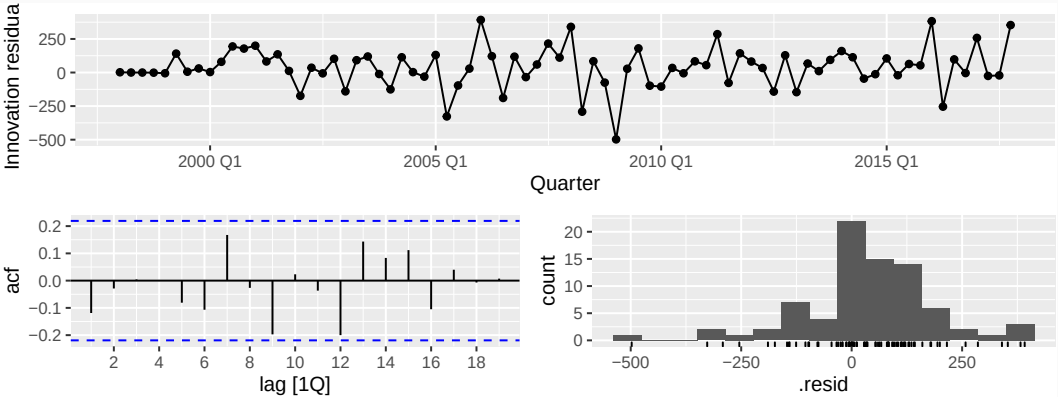
# Ljung-Box test

```
augment(holiday_fit) %>%
  filter(State == "VIC", .model == "arima") %>%
  features(.resid, ljung_box, dof = 2, lag = 8)
```

```
## # A tibble: 1 x 4
##   State .model lb_stat lb_pvalue
##   <chr> <chr>    <dbl>     <dbl>
## 1 VIC   arima     5.42     0.492
```

# gg_tsresiduals() function

```
holiday_fit %>%
  filter(State == "VIC") %>%
  select(arima) %>%
  gg_tsresiduals()
```



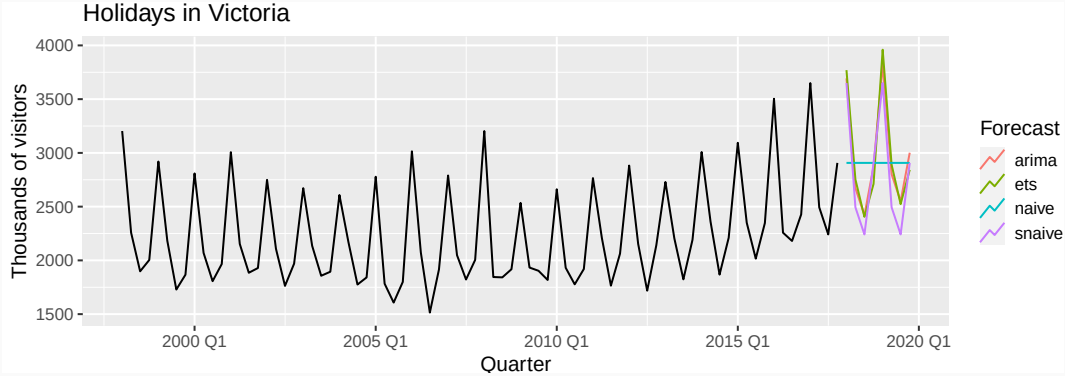
# Producing forecasts

```
holiday_fc <- holiday_fit %>%
  forecast(h = "2 years")
```

```
## # A fable: 256 x 5 [1Q]
## # Key:     State, .model [32]
##    State .model Quarter        Trips .mean
##    <chr> <chr>    <qtr>       <dist> <dbl>
##  1 ACT   snaive 2018 Q1 N(223, 2128)  223.
##  2 ACT   snaive 2018 Q2 N(193, 2128)  193.
##  3 ACT   snaive 2018 Q3 N(204, 2128)  204.
##  4 ACT   snaive 2018 Q4 N(214, 2128)  214.
##  5 ACT   snaive 2019 Q1 N(223, 4257)  223.
##  6 ACT   snaive 2019 Q2 N(193, 4257)  193.
##  7 ACT   snaive 2019 Q3 N(204, 4257)  204.
##  8 ACT   snaive 2019 Q4 N(214, 4257)  214.
##  9 ACT   naive  2018 Q1 N(214, 2894)  214.
## 10 ACT   naive  2018 Q2 N(214, 5788)  214.
## # ... with 246 more rows
```

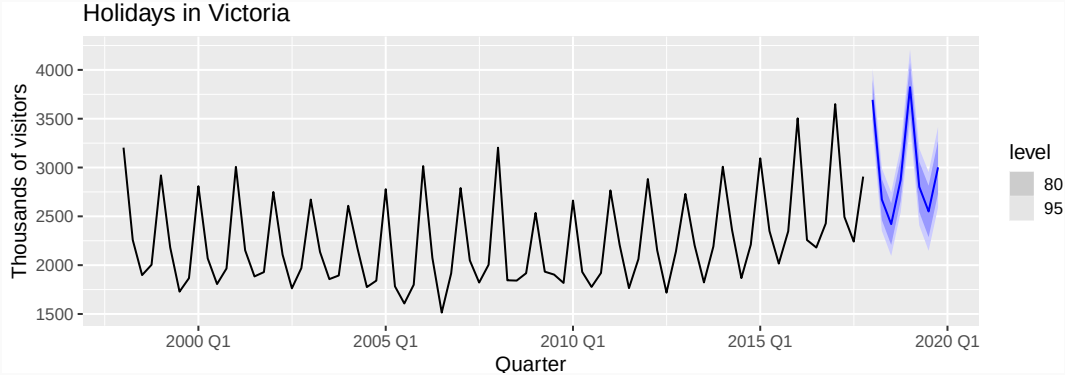
# Visualising forecasts

```
holiday_fc %>%
  filter(State == "VIC") %>%
  autoplot(holidays, level = NULL) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



# Visualising forecasts

```
holiday_fc %>%
  filter(State == "VIC", .model == "arima") %>%
  autoplot(holidays) +
  labs(title = "Holidays in Victoria", y = "Thousands of visitors") +
  guides(color = guide_legend(title = "Forecast"))
```



# Prediction intervals

```
holiday_fc %>% hilo(level = 95)
```

```
## # A tsibble: 256 x 6 [1Q]
## # Key:       State, .model [32]
##    State .model Quarter        Trips .mean          `95%`
##    <chr> <chr>    <qtr>       <dist> <dbl>         <hilo>
##  1 ACT   snaive 2018 Q1 N(223, 2128)  223. [132.7, 314]95
##  2 ACT   snaive 2018 Q2 N(193, 2128)  193. [103.1, 284]95
##  3 ACT   snaive 2018 Q3 N(204, 2128)  204. [113.2, 294]95
##  4 ACT   snaive 2018 Q4 N(214, 2128)  214. [124.0, 305]95
##  5 ACT   snaive 2019 Q1 N(223, 4257)  223. [ 95.3, 351]95
##  6 ACT   snaive 2019 Q2 N(193, 4257)  193. [ 65.6, 321]95
##  7 ACT   snaive 2019 Q3 N(204, 4257)  204. [ 75.8, 332]95
##  8 ACT   snaive 2019 Q4 N(214, 4257)  214. [ 86.6, 342]95
##  9 ACT   naive  2018 Q1 N(214, 2894)  214. [109.0, 320]95
```

# Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.

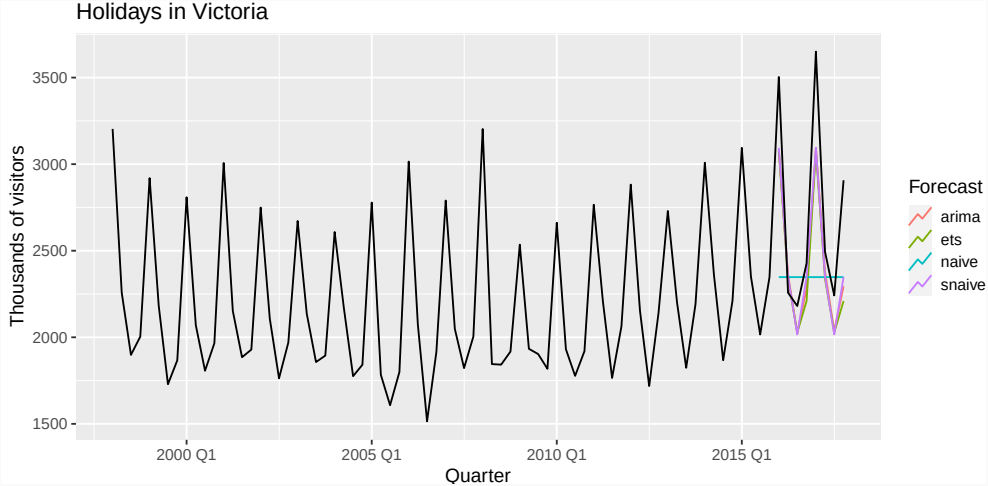
# Outline

1. What does modern time series data look like? (tsibble)
2. EDA for large time series (feasts)
3. Probabilistic forecasting for large time series (fable)
4. fable: Evaluating forecast accuracy
5. fable: Forecast reconciliation

# Training and test sets


- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

# Measures of forecast accuracy



# Measures of forecast accuracy

```
accuracy(vic_fc, holidays)
```

```
## # A tibble: 4 x 11
##   .model State .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 arima  VIC   Test   263.  349.  290.  8.83 10.0   2.28  2.07 -0.230
## 2 ets    VIC   Test   295.  378.  315. 10.0  10.9   2.48  2.25 -0.149
## 3 naive  VIC   Test   360.  654.  451. 10.1  14.2   3.54  3.88 -0.181
## 4 snaive VIC   Test   256.  336.  279.  8.59  9.61  2.19  1.99 -0.257
```

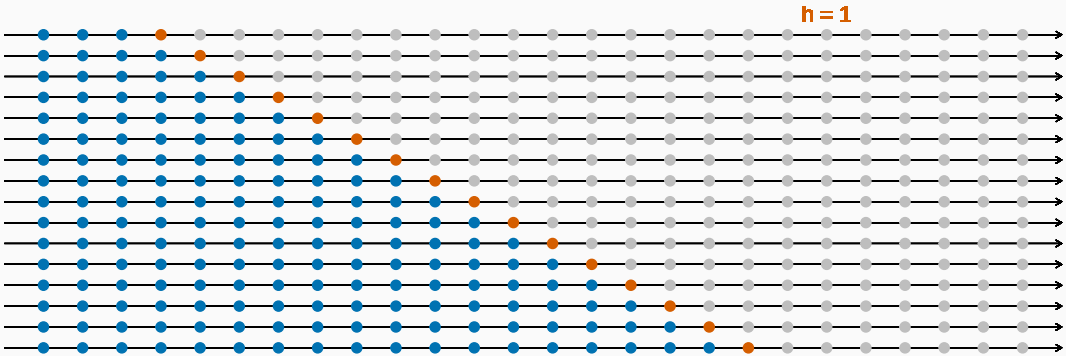
# Time series cross-validation

## Traditional evaluation

# Time series cross-validation

## Traditional evaluation

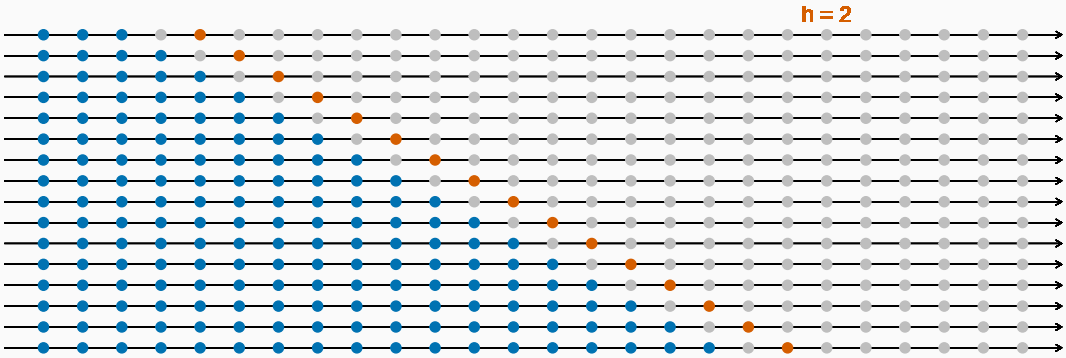

## Time series cross-validation



# Time series cross-validation

## Traditional evaluation


## Time series cross-validation



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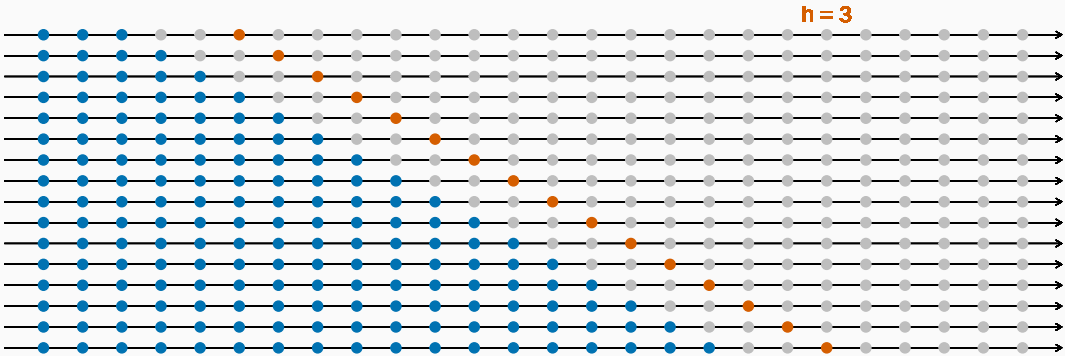

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## Traditional evaluation


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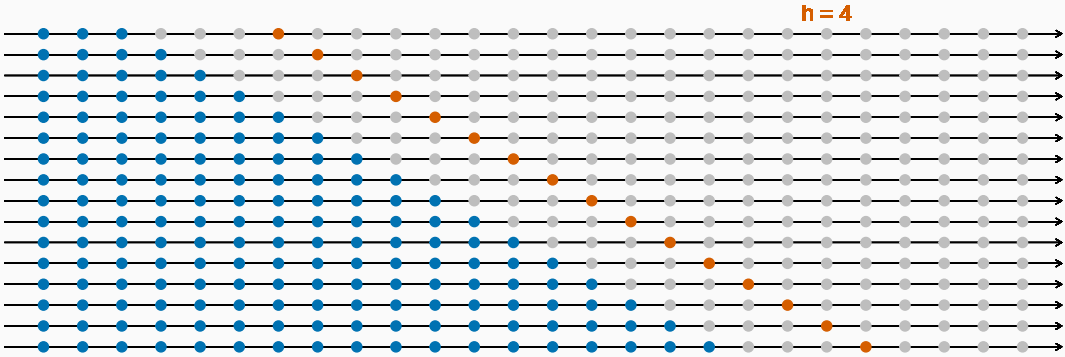
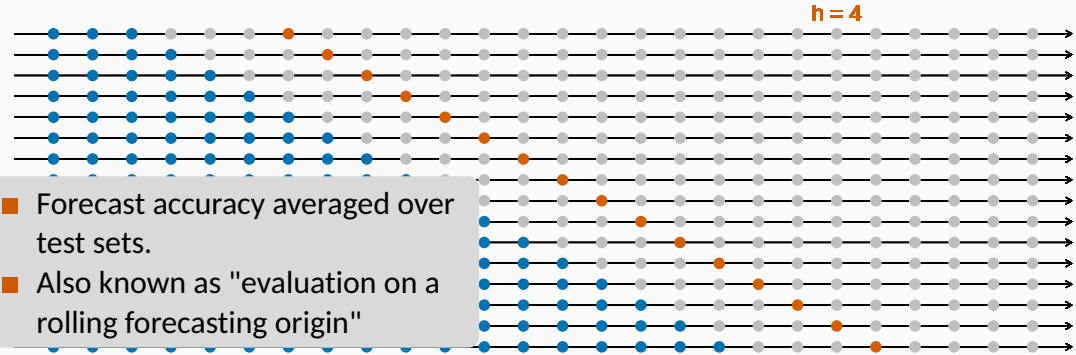


# Time series cross-validation

## Traditional evaluation


## Time series cross-validation




- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"

# Time series cross-validation

Stretch with a minimum length of 4 years, growing by 1 quarter each step.

```
vic_holiday_stretch <- holidays %>%
  filter(State == "VIC") %>%
  stretch_tsibble(.init = 16, .step = 1)
```

```
## # A tsibble: 3,120 x 4 [1Q]
## # Key:       .id, State [65]
##    State Quarter Trips   .id
##    <chr>   <qtr> <dbl> <int>
##  1 VIC   1998 Q1 3204.     1
##  2 VIC   1998 Q2 2258.     1
##  3 VIC   1998 Q3 1898.     1
##  4 VIC   1998 Q4 2004.     1
##  5 VIC   1999 Q1 2919.     1
##  6 VIC   1999 Q2 2183.     1
```

# Time series cross-validation

```
fit_cv <- vic_holiday_stretch %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips),
    snaive = SNAIVE(Trips)
  )
```

```
## # A mable: 65 x 5
## # Key:     .id, State [65]
##     .id State          ets                      arima   snaive
##   <int> <chr>      <model>                    <model>  <model>
## 1     1 VIC   <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 2     2 VIC   <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 3     3 VIC   <ETS(M,N,M)> <ARIMA(1,0,0)(0,1,0)[4]> <SNAIVE>
## 4     4 VIC   <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
## 5     5 VIC   <ETS(M,N,M)> <ARIMA(0,0,2)(0,1,0)[4]> <SNAIVE>
```

# Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv %>%
  forecast(h = 1)
```

```
## # A fable: 195 x 6 [1Q]
## # Key:     .id, State, .model [195]
##     .id State Quarter          Trips .mean .model
##   <int> <chr>   <qtr>         <dist> <dbl> <chr>
## 1     1 VIC   2002 Q1 N(2965, 14682) 2965. ets
## 2     1 VIC   2002 Q1 N(2980, 11447) 2980. arima
## 3     1 VIC   2002 Q1 N(3007, 19035) 3007. snaive
## 4     2 VIC   2002 Q2  N(2024, 8130) 2024. ets
## 5     2 VIC   2002 Q2 N(1961, 14864) 1961. arima
## 6     2 VIC   2002 Q2 N(2153, 22669) 2153. snaive
## # ... with 189 more rows
```

# Time series cross-validation

```
# Cross-validated
fc_cv %>% accuracy(holidays)
```

```
## # A tibble: 3 x 11
##   .model State .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE    ACF1
##   <chr>  <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>   <dbl>
## 1 arima  VIC   Test   46.3  183.  147.  1.57  6.41  1.08 1.02  -0.0615
## 2 ets    VIC   Test   43.3  179.  139.  1.51  6.02  1.02 0.997 -0.110
## 3 snaive VIC   Test   36.2  186.  138.  1.14  6.01  1.02 1.04   0.0427
```

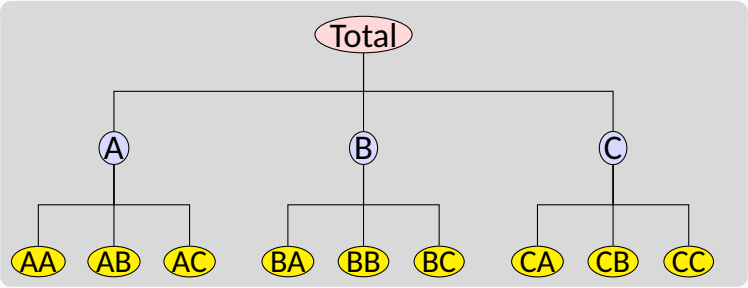
A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.

# Outline

1. What does modern time series data look like? (tsibble)
2. EDA for large time series (feasts)
3. Probabilistic forecasting for large time series (fable)
4. fable: Evaluating forecast accuracy
5. fable: Forecast reconciliation

# Hierarchical time series

A **hierarchical time series** is a collection of several time series that are linked together in a hierarchical structure.



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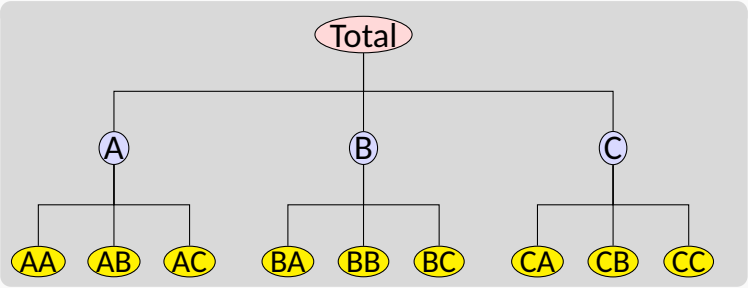
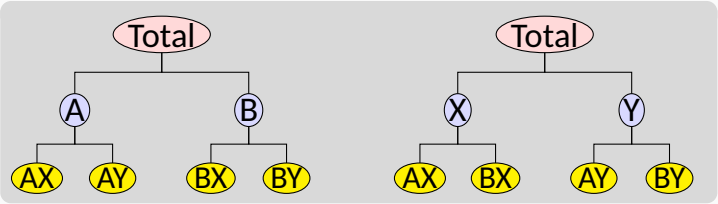



**Examples**

- Tourism demand by states, zones, regions

# Grouped time series

A **grouped time series** is a collection of time series that can be grouped together in a number of non-hierarchical ways.



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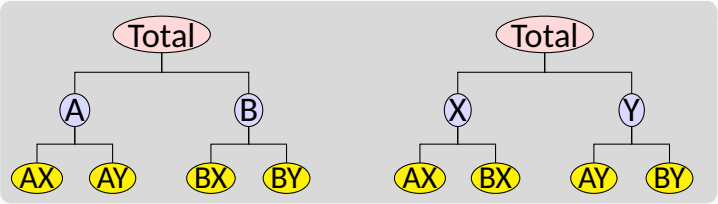


**Examples**

- Tourism by state and purpose of travel
- Retail sales by product groups/sub groups, and by countries/regions

# Creating aggregates

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  filter(Quarter == yearquarter("1998 Q1")) %>%
  print(n = 15)
```

```
## # A tsibble: 425 x 5 [1Q]
## # Key:       Purpose, State, Region [425]
##    Quarter Purpose      State        Region          Trips
##      <qtr> <chr*>       <chr*>       <chr*>          <dbl>
##  1 1998 Q1 <aggregated> <aggregated> <aggregated>   23182.
##  2 1998 Q1 Business     <aggregated> <aggregated>    3599.
##  3 1998 Q1 Holiday      <aggregated> <aggregated>   11806.
##  4 1998 Q1 Other        <aggregated> <aggregated>     680.
##  5 1998 Q1 Visiting     <aggregated> <aggregated>    7098.
##  6 1998 Q1 <aggregated> ACT          <aggregated>     551.
##  7 1998 Q1 <aggregated> NSW          <aggregated>    8040.
##  8 1998 Q1 <aggregated> NT           <aggregated>     181.
##  9 1998 Q1 <aggregated> QLD          <aggregated>    4041.
## 10 1998 Q1 <aggregated> SA           <aggregated>    1735.
## 11 1998 Q1 <aggregated> TAS          <aggregated>     982.
## 12 1998 Q1 <aggregated> VIC          <aggregated>    6010.
## 13 1998 Q1 <aggregated> WA           <aggregated>    1641.
```

# Creating aggregates

- Similar to `summarise()` but using the key structure
- A grouped structure is specified using `grp1 * grp2`
- A nested structure is specified via `parent / child`.
- Groups and nesting can be mixed:

```
(country/region/city) * (brand/product)
```

- All possible aggregates are produced.
- These are useful when forecasting at different levels of aggregation.

# The problem

1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

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1. How to forecast time series at all nodes such that the forecasts add up in the same way as the original data?
2. Can we exploit relationships between the series to improve the forecasts?

## The solution

1. Forecast all series at all levels of aggregation using an automatic forecasting algorithm.
   (e.g., `ETS`, `ARIMA`, `...`)
2. Reconcile the resulting forecasts so they add up correctly using least squares optimization (i.e., find closest reconciled forecasts to the original forecasts).
3. This is available using `reconcile()`.

# Forecast reconciliation

```
tourism %>%
  aggregate_key(Purpose * (State / Region), Trips = sum(Trips)) %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = 2)
```

```
## # A fable: 1,700 x 7 [1Q]
## # Key:     Purpose, State, Region, .model [850]
##    Purpose  State  Region         .model       Quarter        Trips .mean
##    <chr*>   <chr*> <chr*>         <chr>          <qtr>       <dist> <dbl>
##  1 Business ACT    Canberra       ets          2018 Q1 N(144, 1119) 144.
##  2 Business ACT    Canberra       ets          2018 Q2 N(203, 2260) 203.
##  3 Business ACT    Canberra       ets_adjusted 2018 Q1  N(157, 539) 157.
##  4 Business ACT    Canberra       ets_adjusted 2018 Q2  N(214, 951) 214.
##  5 Business ACT    <aggregated>   ets          2018 Q1 N(144, 1119) 144.
##  6 Business ACT    <aggregated>   ets          2018 Q2 N(203, 2260) 203.
##  7 Business ACT    <aggregated>   ets_adjusted 2018 Q1  N(157, 539) 157.
##  8 Business ACT    <aggregated>   ets_adjusted 2018 Q2  N(214, 951) 214.
```

# Hierarchical and grouped time series

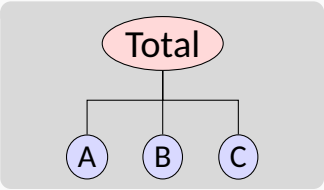
Every collection of time series with aggregation constraints can be written as

$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

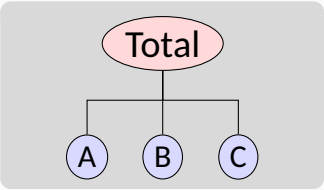
where

- $\boldsymbol{y}_t$ is a vector of all series at time $t$
- $\boldsymbol{b}_t$ is a vector of the most disaggregated series at time $t$
- $\boldsymbol{S}$ is a “summing matrix’ ’ containing the aggregation constraints.

# Hierarchical time series



# Hierarchical time series

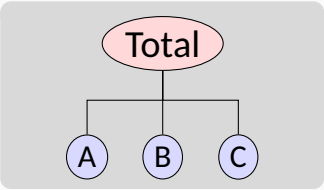



$y_t$ : observed aggregate of all series at time $t$.

$y_{X,t}$ : observation on series $X$ at time $t$.

$\boldsymbol{b}_t$ : vector of all series at bottom level in time $t$.

# Hierarchical time series



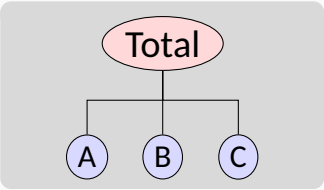

$y_t$ : observed aggregate of all series at time $t$.

$y_{X,t}$ : observation on series $X$ at time $t$.

$\boldsymbol{b}_t$ : vector of all series at bottom level in time $t$.

$$\boldsymbol{y}_t = \begin{pmatrix} y_t \\ y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{A,t} \\ y_{B,t} \\ y_{C,t} \end{pmatrix}$$

# Hierarchical time series

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$$\boldsymbol{y}_t = \boldsymbol{S}\boldsymbol{b}_t$$

# Forecasting notation

Let $\hat{\boldsymbol{y}}_n(h)$ be vector of initial $h$-step forecasts, made at time $n$, stacked in same order as $\boldsymbol{y}_t$.

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Reconciled forecasts must be of the form:

$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_n(h)$$

for some matrix $\boldsymbol{G}$.

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(In general, they will not "add up' '.)

Reconciled forecasts must be of the form:

$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{SG}\hat{\boldsymbol{y}}_n(h)$$

for some matrix $\boldsymbol{G}$.

- $\boldsymbol{G}$ extracts and combines base forecasts $\hat{\boldsymbol{y}}_n(h)$ to get bottom-level forecasts.
- $\boldsymbol{S}$ adds them up

# Optimal combination forecasts

**Main result**

The best (minimum sum of variances) unbiased forecasts are obtained when $\boldsymbol{G} = (\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}$, where $\boldsymbol{W}_h$ is the $h$-step base forecast error covariance matrix.

# Optimal combination forecasts

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$$\tilde{\boldsymbol{y}}_n(h) = \boldsymbol{S}(\boldsymbol{S}'\boldsymbol{W}_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\boldsymbol{W}_h^{-1}\hat{\boldsymbol{y}}_n(h)$$

**Problem:** $\boldsymbol{W}_h$ hard to estimate, especially for $h > 1$.

**Solutions:**

- Ignore $\boldsymbol{W}_h$ (OLS) [`min_trace(method='ols')`]
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ is diagonal (WLS) [`min_trace(method='wls')`]
- Assume $\boldsymbol{W}_h = k_h\boldsymbol{W}_1$ and estimate it (GLS)

# Features

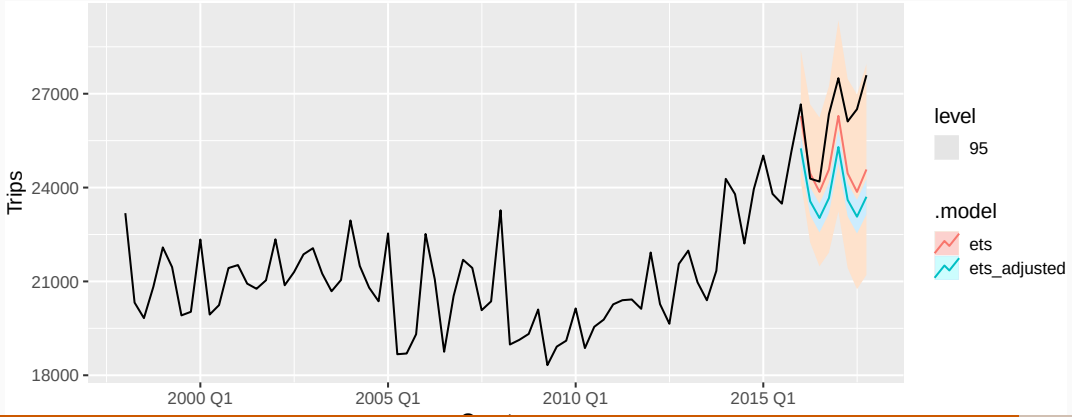
- Covariates can be included in initial forecasts.
- Adjustments can be made to initial forecasts at any level.
- Very simple and flexible method. Can work with *any* hierarchical or grouped time series.
- Conceptually easy to implement: regression of base forecasts on structure matrix.

# Example: Australian tourism

```
tourism_agg <- tourism %>%
  aggregate_key(Purpose * (State / Region),
    Trips = sum(Trips)
  )
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
  model(ets = ETS(Trips)) %>%
  reconcile(ets_adjusted = min_trace(ets)) %>%
  forecast(h = "2 years")
```

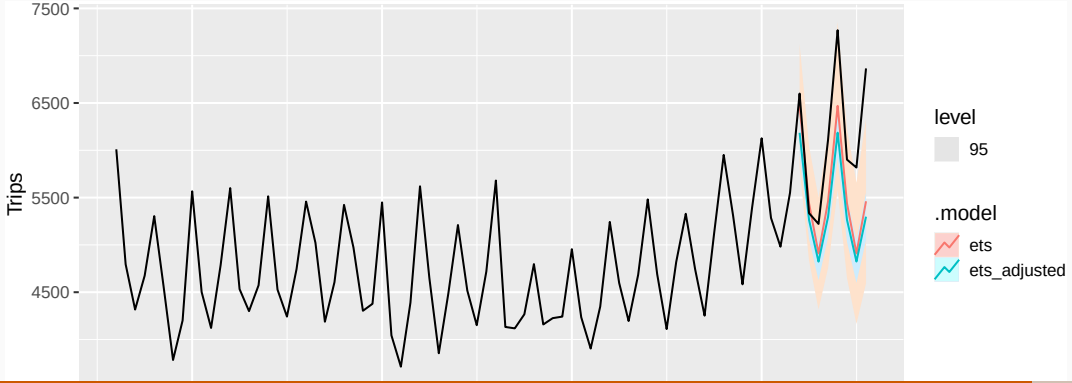
# Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & is_aggregated(State)) %>%
  autoplot(tourism_agg, level = 95)
```



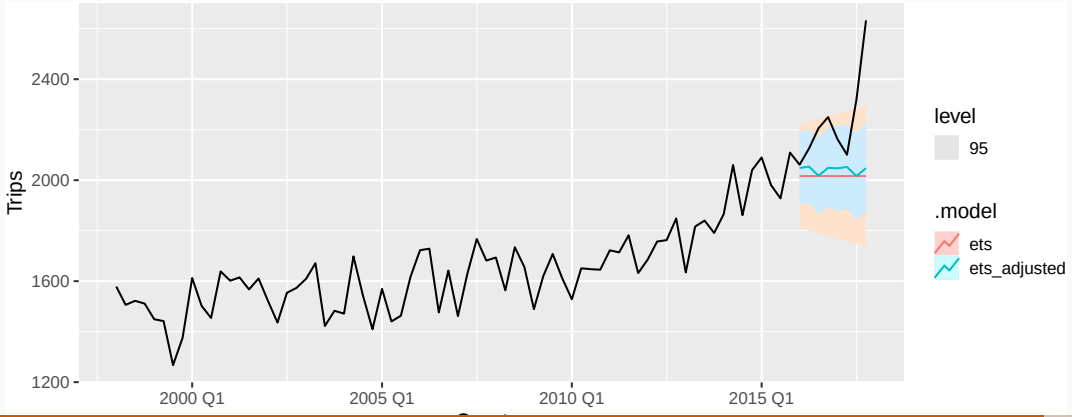
# Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & State == "VIC" &
    is_aggregated(Region)) %>%
  autoplot(tourism_agg, level = 95)
```



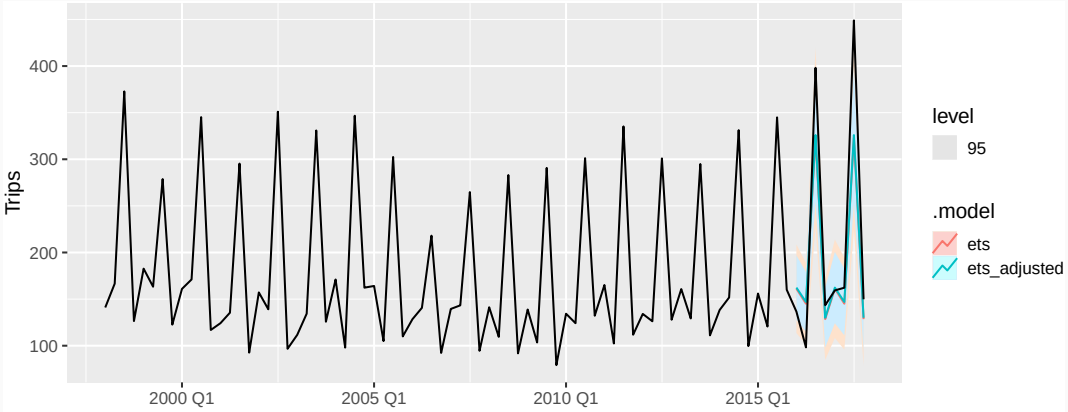
# Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Melbourne") %>%
  autoplot(tourism_agg, level = 95)
```



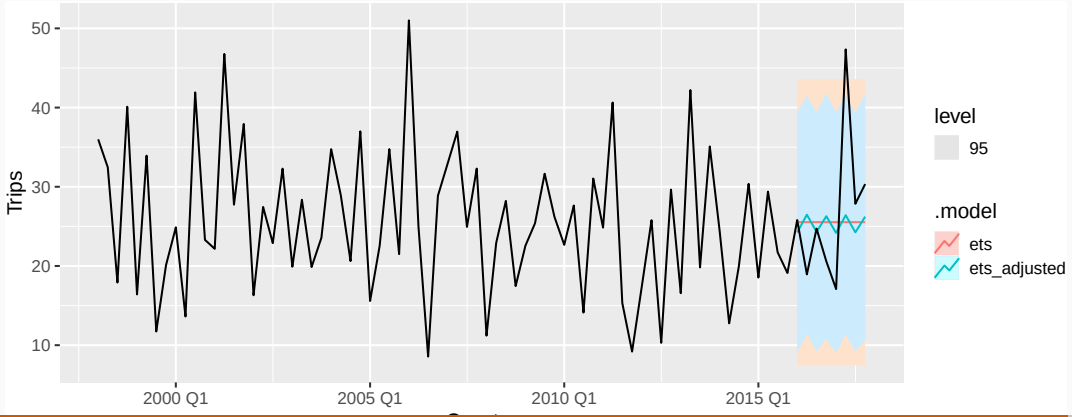
# Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "Snowy Mountains") %>%
  autoplot(tourism_agg, level = 95)
```



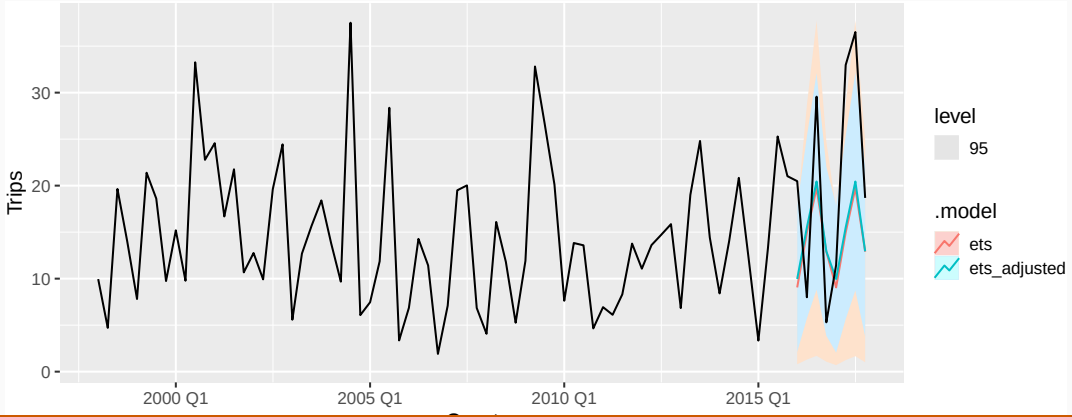
# Example: Australian tourism

```
fc %>%
  filter(Purpose == "Holiday" & Region == "Barossa") %>%
  autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

```
fc %>%
  filter(is_aggregated(Purpose) & Region == "MacDonnell") %>%
  autoplot(tourism_agg, level = 95)
```



# Example: Australian tourism

```
fc <- tourism_agg %>%
  filter_index(. ~ "2015 Q4") %>%
  model(
    ets = ETS(Trips),
    arima = ARIMA(Trips)
  ) %>%
  mutate(
    comb = (ets + arima) / 2
  ) %>%
  reconcile(
    ets_adj = min_trace(ets),
    arima_adj = min_trace(arima),
    comb_adj = min_trace(comb)
  ) %>%
  forecast(h = "2 years")
```

# Forecast evaluation

```
fc %>% accuracy(tourism_agg)
```

```
## # A tibble: 2,550 x 13
##    .model Purpose  State  Region            .type     ME  RMSE   MAE   MPE  MAPE
##    <chr>  <chr*>   <chr*> <chr*>            <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <
##  1 arima  Business ACT    Canberra        ~ Test  35.9   45.7 35.9   16.9  16.9 0
##  2 arima  Business ACT    <aggregated>      Test  35.9   45.7 35.9   16.9  16.9 0
##  3 arima  Business NSW    Blue Mountains  ~ Test   1.93  10.6  8.52 -
18.0  48.6 0.644
##  4 arima  Business NSW    Capital Country ~ Test   8.08  15.6 10.4   11.8  19.0 0
##  5 arima  Business NSW    Central Coast   ~ Test  10.0   14.5 10.8   26.9  32.2 0
##  6 arima  Business NSW    Central NSW     ~ Test  17.7   31.9 28.2   12.0  24.1 1
##  7 arima  Business NSW    Hunter          ~ Test  35.3   43.9 35.3   24.2  24.2 1
##  8 arima  Business NSW    New England Nort~ Test  23.1   31.8 26.8   19.5  28.0 1
##  9 arima  Business NSW    North Coast NSW ~ Test  24.8   40.1 36.8   11.5  28.5 1
## 10 arima  Business NSW    Outback NSW     ~ Test   6.87  11.0  7.76  13.7  16.5 0
```

# Forecast evaluation

```
fc %>%
  accuracy(tourism_agg) %>%
  group_by(.model) %>%
  summarise(MASE = mean(MASE)) %>%
  arrange(MASE)
```

```
## # A tibble: 6 x 2
##   .model     MASE
##   <chr>     <dbl>
## 1 ets_adj    1.02
## 2 comb_adj   1.02
## 3 ets        1.04
## 4 comb       1.04
## 5 arima_adj  1.07
## 6 arima      1.09
```

# Tidyverts developers

## Earo Wang

## Mitchell O'Hara-Wild

# More information

- Slides and papers: **robjhyndman.com**
- Packages: **tidyverts.org**
- Forecasting textbook using fable package: **OTexts.com/fpp3**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