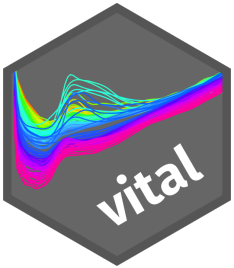

vital: Tidy data analysis for demography

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9 July 2024

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Demographic data structures in R packages

Package	Data class
demography	demogdata
StMoMo	StMoMoData (created by converting a demogdata object)
StanMoMo	Lists of matrices
lifecontingencies	data.frame
BayesMortalityPlus	tibble (that needs to be converted to a matrix for fitting)
MortalityLaws	individual vectors
HMDHFDplus	data.frame

tibble objects

Australian Deaths 1901–2020

```
# A tibble: 145,440 x 7
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>      <dbl>    <dbl>  <dbl>
 1  1901     0 female WA       0.129       2511    325
 2  1901     0 male   WA       0.158       2634    416
 3  1901     1 female WA       0.0275      2219     61
 4  1901     1 male   WA       0.0391      2175     85
 5  1901     2 female WA       0.00688     2180     15
 6  1901     2 male   WA       0.0131      2208     29
 7  1901     3 female WA       0.00584     1884     11
 8  1901     3 male   WA       0.00503     1988     10
 9  1901     4 female WA       0.00290     1722      5
10  1901     4 male   WA       0.00287     1743      5
# i 145,430 more rows
```

tsibble objects

Australian Deaths 1901–2020

```
# A tsibble: 145,440 x 7 [1Y]
# Key:       Age, Sex, State [1,212]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA       0.129      2511    325
 2  1901     0 male   WA       0.158      2634    416
 3  1901     1 female WA       0.0275     2219     61
 4  1901     1 male   WA       0.0391     2175     85
 5  1901     2 female WA       0.00688    2180     15
 6  1901     2 male   WA       0.0131     2208     29
 7  1901     3 female WA       0.00584    1884     11
 8  1901     3 male   WA       0.00503    1988     10
 9  1901     4 female WA       0.00290    1722      5
10  1901     4 male   WA       0.00287    1743      5
# i 145,430 more rows
```

Variables

Index:

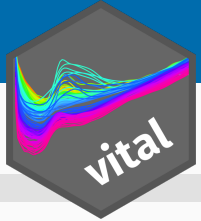
- Year

Keys:

- Age
- Sex
- State

Every row must have a unique combination of Index and Keys

vital objects



Australian Deaths 1901–2020

```
aus
```

```
# A vital: 145,440 x 7 [1Y]
# Key:     Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA       0.129     2511    325
 2  1901     0 male   WA       0.158     2634    416
 3  1901     1 female WA       0.0275    2219     61
 4  1901     1 male   WA       0.0391    2175     85
 5  1901     2 female WA       0.00688   2180     15
 6  1901     2 male   WA       0.0131    2208     29
 7  1901     3 female WA       0.00584   1884     11
 8  1901     3 male   WA       0.00503   1988     10
 9  1901     4 female WA       0.00290   1722      5
10  1901     4 male   WA       0.00287   1743      5
# i 145,430 more rows
```

Variables

Index:

- Year

Keys:

- Age
- Sex
- State

Every row must have a unique combination of Index and Keys

Variables denoting age, sex, deaths, births and population can also be specified as attributes.

vital objects

```
index_var(aus)
```

```
[1] "Year"
```

```
key_vars(aus)
```

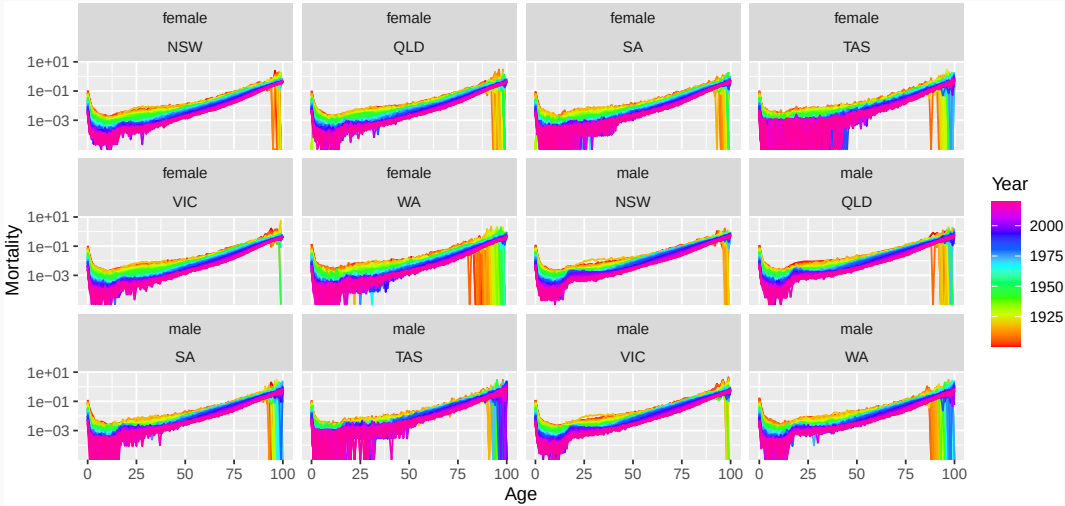
```
[1] "Age"   "Sex"   "State"
```

```
vital_vars(aus)
```

```
       age         sex      deaths population
     "Age"       "Sex"    "Deaths" "Exposure"
```

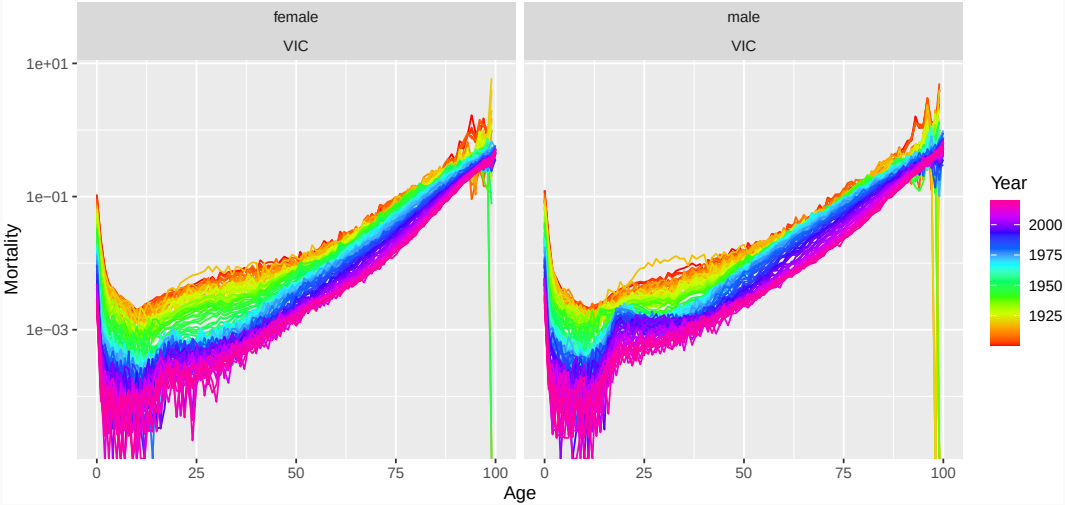
Rainbow plots

```
aus |> autoplot(Mortality) + scale_y_log10()
```



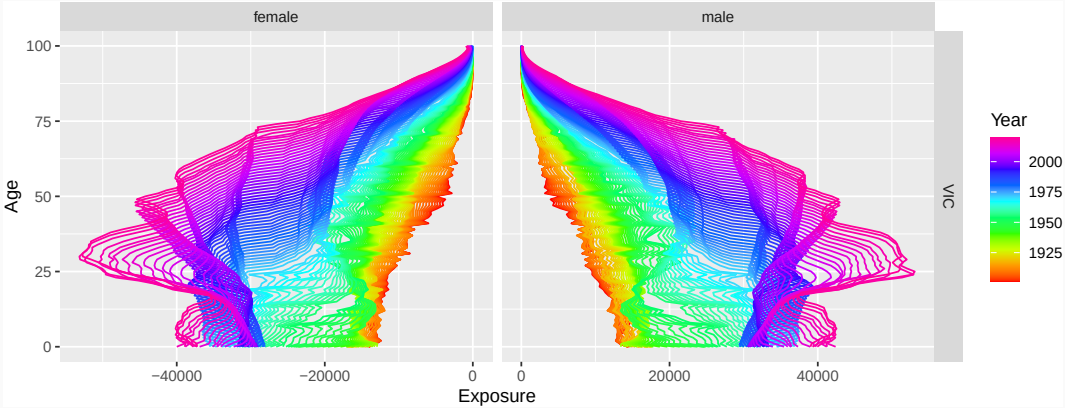
Rainbow plots

```
aus |> filter(State == "VIC") |> autoplot(Mortality) + scale_y_log10()
```



Rainbow plots

```
aus |> filter(State == "VIC") |>
  mutate(Exposure = if_else(Sex == "female", -Exposure, Exposure)) |>
  autoplot(Exposure) +
  facet_grid(State ~ Sex, scales = "free_x") + coord_flip()
```



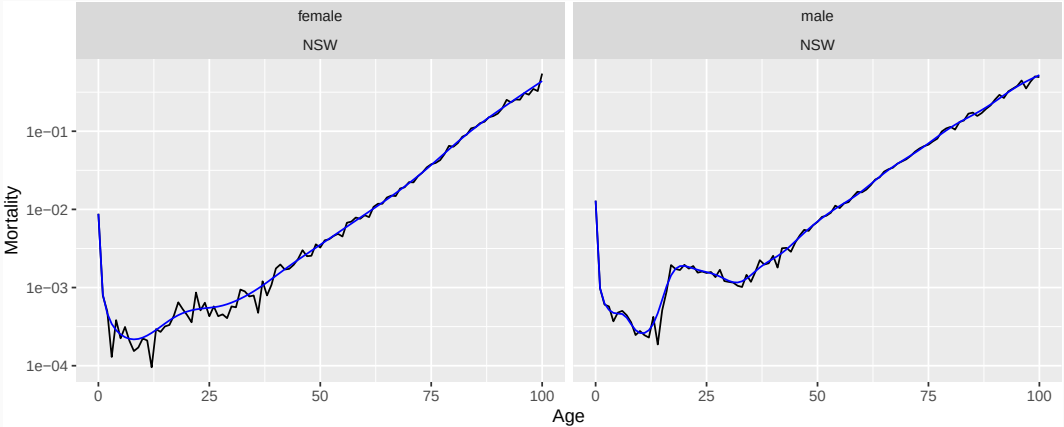
Smoothing

```
sm_aus <- aus |> smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
# Key:     Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths   .smooth .smooth_se
   <int> <dbl> <chr>  <chr>     <dbl>    <dbl>  <dbl> <dbl[1d]>  <dbl[1d]>
 1  1901     0 female NSW     0.107      17143   1833   0.107     0.00295
 2  1901     1 female NSW     0.0247     15071    373   0.0237    0.00141
 3  1901     2 female NSW     0.00686    15461    106   0.00804   0.000670
 4  1901     3 female NSW     0.00441    15629     69   0.00461   0.000405
 5  1901     4 female NSW     0.00374    15762     59   0.00341   0.000305
 6  1901     5 female NSW     0.00274    16030     44   0.00275   0.000251
 7  1901     6 female NSW     0.00252    16289     41   0.00230   0.000215
 8  1901     7 female NSW     0.00216    16639     36   0.00197   0.000189
 9  1901     8 female NSW     0.00169    16554     28   0.00175   0.000173
10  1901     9 female NSW     0.00109    16468     18   0.00162   0.000163
```

Smoothing

```
sm_aus <- aus |> smooth_mortality(Mortality)
sm_aus |> filter(State == "NSW", Year == 1980) |> autoplot(Mortality) +
  geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



Life tables

```
life_table(aus)
```

```
# A vital: 145,440 x 14 [1Y]
# Key:     Age x (Sex, State) [101 x 12]
    Year   Age Sex   State      mx       qx     lx       dx     Lx     Tx     ex     rx
   <int> <int> <chr> <chr>   <dbl>    <dbl> <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 fema~ NSW   0.107   0.100   1      1.00e-1 0.935  56.2  56.2 0.935
 2  1901     1 fema~ NSW   0.0247  0.0244  0.900 2.20e-2 0.889  55.3  61.5 0.951
 3  1901     2 fema~ NSW   0.00686 0.00683 0.878 6.00e-3 0.875  54.4  62.0 0.984
 4  1901     3 fema~ NSW   0.00441 0.00441 0.872 3.84e-3 0.870  53.5  61.4 0.994
 5  1901     4 fema~ NSW   0.00374 0.00374 0.868 3.24e-3 0.867  52.7  60.7 0.996
 6  1901     5 fema~ NSW   0.00274 0.00274 0.865 2.37e-3 0.864  51.8  59.9 0.997
 7  1901     6 fema~ NSW   0.00252 0.00251 0.863 2.17e-3 0.861  50.9  59.1 0.997
 8  1901     7 fema~ NSW   0.00216 0.00216 0.860 1.86e-3 0.859  50.1  58.2 0.998
 9  1901     8 fema~ NSW   0.00169 0.00169 0.859 1.45e-3 0.858  49.2  57.3 0.998
10  1901     9 fema~ NSW   0.00109 0.00109 0.857 9.36e-4 0.857  48.4  56.4 0.999
# i 145,430 more rows
```

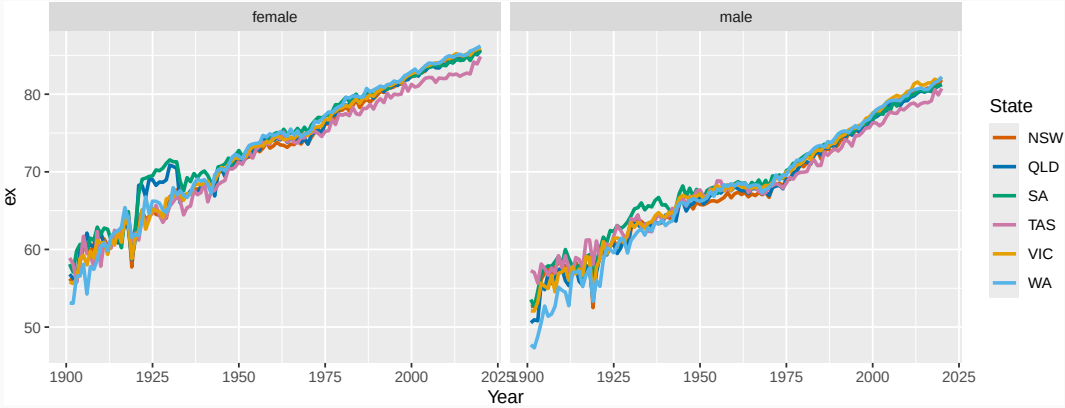
Life expectancy

```
life_expectancy(aus)
```

```
# A vital: 1,440 x 8 [1Y]
# Key:     Age x (Sex, State) [1 x 12]
    Year   Age Sex    State    ex    rx    nx    ax
   <int> <int> <chr>  <chr> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 female NSW    56.2 0.935     1 0.352
 2  1901     0 female QLD    56.8 0.937     1 0.338
 3  1901     0 female SA     58.1 0.939     1 0.324
 4  1901     0 female TAS    58.9 0.946     1 0.275
 5  1901     0 female VIC    55.8 0.937     1 0.334
 6  1901     0 female WA     53.1 0.922     1 0.35
 7  1901     0 male   NSW    52.6 0.925     1 0.33
 8  1901     0 male   QLD    50.6 0.924     1 0.33
 9  1901     0 male   SA     53.5 0.922     1 0.33
10  1901     0 male   TAS    57.3 0.930     1 0.33
# i 1,430 more rows
```

Life expectancy

```
life_expectancy(aus) |>
  ggplot(aes(x = Year, y = ex, colour = State)) +
  geom_line(linewidth = 1) +
  facet_grid(. ~ Sex)
```



Mortality models

Let $m_{x,t}$ be the mortality rate at age x in year t.

Naive model: $m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$

Lee-Carter model: $\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$

where $\varepsilon_{x,t}$ = noise term with variance σ_x^2.

Mortality models

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where $\varepsilon_{x,t}$ = noise term with variance σ_x^2.

```
fit <- aus |>
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
  )
```

Mortality models

```
fit
```

```
# A mable: 12 x 4
# Key:     Sex, State [12]
   Sex    State    naive      lc
   <chr>  <chr>  <model> <model>
 1 female NSW   <FNAIVE>    <LC>
 2 female QLD   <FNAIVE>    <LC>
 3 female SA    <FNAIVE>    <LC>
 4 female TAS   <FNAIVE>    <LC>
 5 female VIC   <FNAIVE>    <LC>
 6 female WA    <FNAIVE>    <LC>
 7 male   NSW   <FNAIVE>    <LC>
 8 male   QLD   <FNAIVE>    <LC>
 9 male   SA    <FNAIVE>    <LC>
10 male   TAS   <FNAIVE>    <LC>
11 male   VIC   <FNAIVE>    <LC>
12 male   WA    <FNAIVE>    <LC>
```

Mortality models

```
fit
```

```
# A mable: 12 x 4
# Key:     Sex, State [12]
   Sex    State    naive      lc
   <chr>  <chr>  <model> <model>
 1 female NSW    <FNAIVE>    <LC>
 2 female QLD    <FNAIVE>    <LC>
 3 female SA     <FNAIVE>    <LC>
 4 female TAS    <FNAIVE>    <LC>
 5 female VIC    <FNAIVE>    <LC>
 6 female WA     <FNAIVE>    <LC>
 7 male   NSW    <FNAIVE>    <LC>
 8 male   QLD    <FNAIVE>    <LC>
 9 male   SA     <FNAIVE>    <LC>
10 male   TAS    <FNAIVE>    <LC>
11 male   VIC    <FNAIVE>    <LC>
12 male   WA     <FNAIVE>    <LC>
```

```
fit |>
  filter(Sex == "female",
         State == "NSW") |>
  select(naive) |>
  report()
```

```
Series: Mortality
Model: FNAIVE

# A tibble: 101 x 2
     Age    sigma
   <int>    <dbl>
 1     0 0.00424
 2     1 0.00180
 3     2 0.000642
 4     3 0.000455
 5     4 0.000382
 6     5 0.000303
 7     6 0.000276
```

Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit |>
  filter(Sex == "female",
         State == "NSW") |>
  select(lc) |>
  report()
```

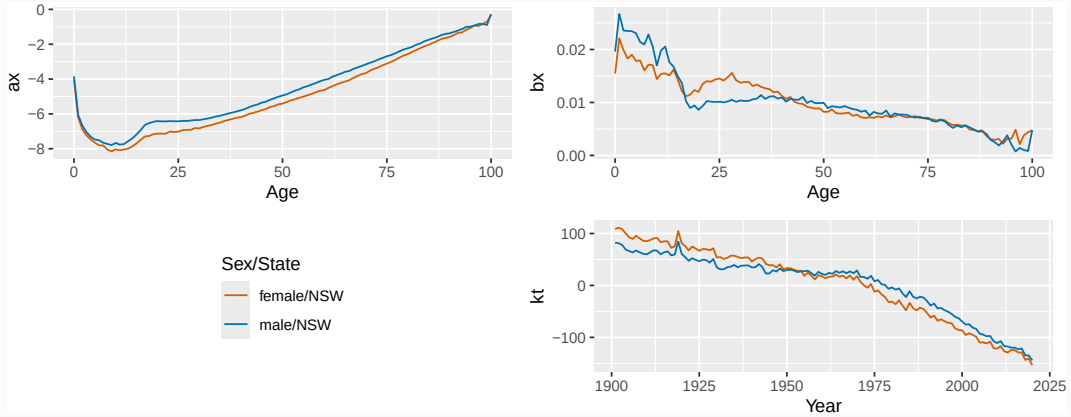
```
Series: Mortality
Model: LC
Transformation: log(Mortality)

Options:
  Adjust method: dt
  Jump choice: fit
```

```
Age functions
# A tibble: 101 × 3
     Age    ax      bx
   <int> <dbl>   <dbl>
1      0 -4.07 0.0155
2      1 -6.20 0.0221
3      2 -6.89 0.0199
# i 98 more rows

Time coefficients
# A tsibble: 120 x 2 [1Y]
    Year    kt
   <int> <dbl>
1  1901 109.
2  1902 111.
3  1903 108.
# i 117 more rows

Time series model: RW w/ drift
Variance explained: 86.61%
```

Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit |>
  filter(State == "NSW") |>
  select(lc) |>
  autoplot()
```



Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit |> select(lc) |> age_components()
```

```
# A tibble: 1,212 x 5
   Sex    State   Age     ax      bx
   <chr>  <chr> <int> <dbl>   <dbl>
 1 female NSW       0 -4.07 0.0155
 2 female NSW       1 -6.20 0.0221
 3 female NSW       2 -6.89 0.0199
 4 female NSW       3 -7.24 0.0183
 5 female NSW       4 -7.47 0.0190
 6 female NSW       5 -7.65 0.0178
 7 female NSW       6 -7.80 0.0179
 8 female NSW       7 -7.81 0.0160
 9 female NSW       8 -8.05 0.0171
10 female NSW       9 -8.15 0.0170
# i 1,202 more rows
```

```
fit |> select(lc) |> time_components()
```

```
# A tsibble: 1,440 x 4 [1Y]
# Key:       Sex, State [12]
   Sex    State  Year    kt
   <chr>  <chr> <int> <dbl>
 1 female NSW    1901 109.
 2 female NSW    1902 111.
 3 female NSW    1903 108.
 4 female NSW    1904 100.
 5 female NSW    1905  92.7
 6 female NSW    1906  89.5
 7 female NSW    1907  95.7
 8 female NSW    1908  90.5
 9 female NSW    1909  85.9
10 female NSW    1910  85.4
# i 1,430 more rows
```

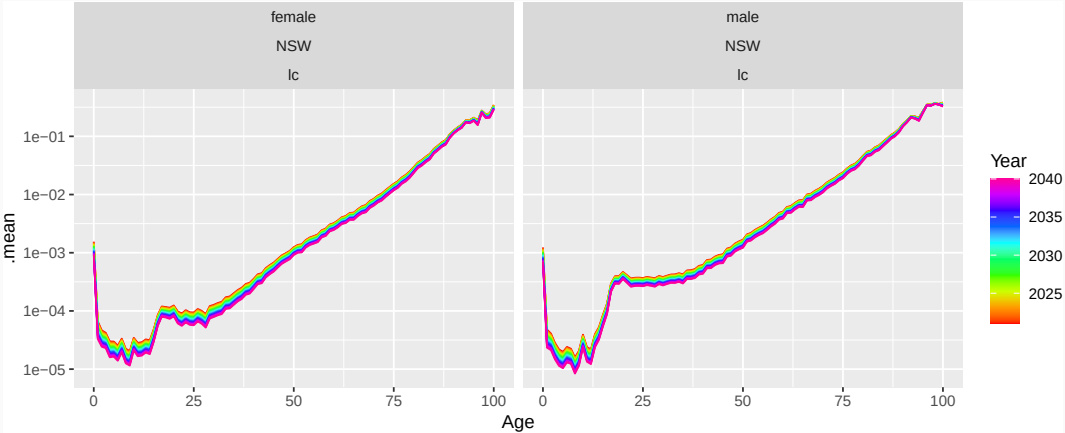
Forecasts

```
fc <- fit |> forecast(h = 20)
fc
```

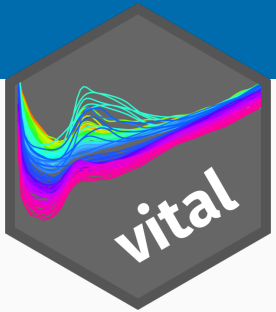
```
# A vital fable: 72,720 x 7 [1Y]
# Key:           Age x (Sex, State, .model) [101 x 36]
   Sex    State .model  Year   Age          Mortality   .mean
   <chr>  <chr> <chr>  <dbl> <int>             <dist>   <dbl>
 1 female NSW   naive   2021     0 N(0.0027, 1.8e-05) 0.00270
 2 female NSW   naive   2022     0 N(0.0027, 3.6e-05) 0.00270
 3 female NSW   naive   2023     0 N(0.0027, 5.4e-05) 0.00270
 4 female NSW   naive   2024     0 N(0.0027, 7.2e-05) 0.00270
 5 female NSW   naive   2025     0   N(0.0027, 9e-05) 0.00270
 6 female NSW   naive   2026     0 N(0.0027, 0.00011) 0.00270
 7 female NSW   naive   2027     0 N(0.0027, 0.00013) 0.00270
 8 female NSW   naive   2028     0 N(0.0027, 0.00014) 0.00270
 9 female NSW   naive   2029     0 N(0.0027, 0.00016) 0.00270
10 female NSW   naive   2030     0 N(0.0027, 0.00018) 0.00270
```

NSW forecasts using Lee-Carter method

```
fc |> filter(State == "NSW", .model == "lc") |>
  autoplot() + scale_y_log10()
```

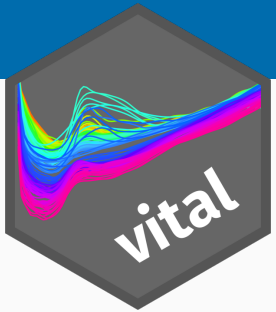


Other functionality



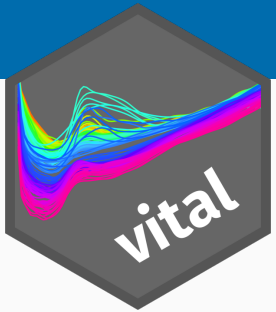
- Import data from Human Mortality Database and Human Fertility Database
- Convert demogdata, tsibble & data.frame objects to vital.
- Compute net migration from population, births and deaths.
- Compute total fertility rates from age-specific fertility rates.
- Various smoothing functions
- Other mortality models including functional data models, and coherent functional data models.

Future plans



- Remaining tools from the `demography` package
- Stochastic population forecasting (as per Hyndman-Booth, IJF, 2008)
- All models handled by `StMoMo` package
- All methods from `MortalityLaws` package
- Suggestions from users

Future plans



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- **Slides**: robjhyndman.com/user2024
- **Package**: pkg.robjhyndman.com/vital/