

vital: Tidy data analysis for demography

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Demographic data structures in R packages

Package	Data class
demography	demogdata
StMoMo	StMoMoData (created by converting a
	demogdata object)
StanMoMo	Lists of matrices
lifecontingencies	data.frame
BayesMortalityPlus	tibble (that needs to be converted to a matrix for fitting)
MortalityLaws	individual vectors
HMDHFDplus	data.frame

tibble objects

Australian Deaths 1901–2020

A tibble: 145,440 x 7

			-				
	Year	Age	Sex	State	Mortality	Exposure	Deaths
	<int></int>	<int></int>	<chr></chr>	<chr></chr>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
1	1901	0	female	WA	0.129	2511	325
2	1901	0	male	WA	0.158	2634	416
3	1901	1	female	WA	0.0275	2219	61
4	1901	1	male	WA	0.0391	2175	85
5	1901	2	female	WA	0.00688	2180	15
6	1901	2	male	WA	0.0131	2208	29
7	1901	3	female	WA	0.00584	1884	11
8	1901	3	male	WA	0.00503	1988	10
9	1901	4	female	WA	0.00290	1722	5
10	1901	4	male	WA	0.00287	1743	5
# i	145,4	130 moi	re rows				



7133LE

3

tsibble objects

1901

1901

i 145,430 more rows

Australian Deaths 1901–2020

4 female WA

WΔ

4 male

```
tsibble
# A tsibble: 145,440 x 7 [1Y]
             Age, Sex, State [1,212]
# Kev:
    Year
           Age Sex
                      State Mortality Exposure Deaths
   <int> <int> <chr> <chr>
                                 <fdb>>
                                          <fdb1>
                                                 <dbl>
    1901
             0 female WA
                               0.129
                                           2511
                                                    325
                                                   416
    1901
             0 male
                      WA
                               0.158
                                           2634
    1901
             1 female WA
                               0.0275
                                           2219
                                                    61
    1901
             1 male
                      WΑ
                               0.0391
                                           2175
                                                     85
    1901
             2 female WA
                               0.00688
                                           2180
                                                     15
    1901
             2 male
                      WΑ
                               0.0131
                                           2208
                                                     29
    1901
             3 female WA
                               0.00584
                                           1884
                                                     11
                                                     10
    1901
             3 male
                      WA
                               0.00503
                                           1988
```

0.00290

0.00287

Variables

Index:

Year

Kevs:

5

5

1722

1743

- Age
- Sex
- State

Every row must have a unique combination of Index and Kevs

vital objects

i 145.430 more rows

Australian Deaths 1901–2020

aus

```
# A vital: 145,440 x 7 [1Y]
# Key:
           Age x (Sex, State) [101 \times 12]
    Year
           Age Sex
                       State Mortality Exposure Deaths
   <int> <int> <chr> <chr>
                                  <dbl>
                                            <dbl>
                                                   <dbl>
    1901
              0 female WA
                                0.129
                                             2511
                                                     325
    1901
              0 male
                       WΑ
                                0.158
                                             2634
                                                     416
    1901
              1 female WA
                                0.0275
                                             2219
                                                      61
                                                      85
    1901
              1 male
                       WA
                                0.0391
                                             2175
    1901
              2 female WA
                                0.00688
                                             2180
                                                      15
    1901
              2 male
                       WΑ
                                0.0131
                                             2208
                                                      29
                                                      11
    1901
              3 female WA
                                0.00584
                                             1884
    1901
              3 male
                       WA
                                0.00503
                                             1988
                                                      10
              4 female WA
                                0.00290
                                             1722
    1901
    1901
              4 male
                       WA
                                0.00287
                                             1743
```

Variables

Index:

Year

Keys:

■ Age

Sex

State

Every row must have a unique combination of Index and Keys

Variables denoting age, sex, deaths, births and population can also be specified as attributes.

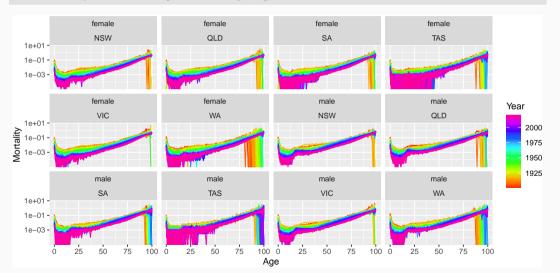
5

vital objects

```
index_var(aus)
[1] "Year"
key_vars(aus)
[1] "Age" "Sex" "State"
vital_vars(aus)
                 sex
                         deaths population
       age
               "Sex" "Deaths" "Exposure"
     "Age"
```

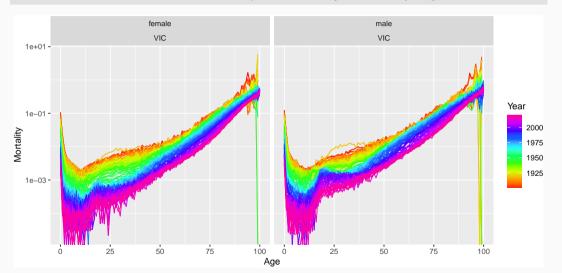
Rainbow plots

aus ▷ autoplot(Mortality) + scale_y_log10()



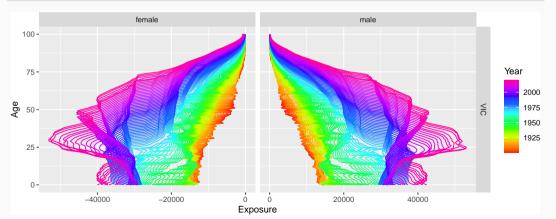
Rainbow plots

aus ▷ filter(State == "VIC") ▷ autoplot(Mortality) + scale_y_log10()



Rainbow plots

```
aus ▷ filter(State == "VIC") ▷
mutate(Exposure = if_else(Sex == "female", -Exposure, Exposure)) ▷
autoplot(Exposure) +
facet_grid(State ~ Sex, scales = "free_x") + coord_flip()
```



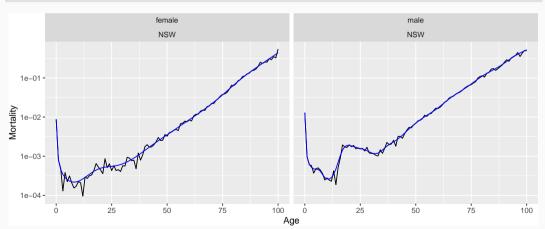
Smoothing

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
           Age x (Sex, State) [101 \times 12]
# Key:
    Year
           Age Sex
                      State Mortality Exposure Deaths
                                                          .smooth .smooth_se
                                 <dbl>
                                          <dbl>
                                                  <dbl> <dbl[1d]>
                                                                    <dbl[1d]>
   <int> <dbl> <chr> <chr>
    1901
             0 female NSW
                               0.107
                                          17143
                                                   1833
                                                          0.107
                                                                     0.00295
    1901
             1 female NSW
                               0.0247
                                          15071
                                                    373
                                                          0.0237
                                                                     0.00141
    1901
             2 female NSW
                               0.00686
                                          15461
                                                    106
                                                          0.00804
                                                                     0.000670
    1901
             3 female NSW
                               0.00441
                                          15629
                                                     69
                                                          0.00461
                                                                     0.000405
             4 female NSW
                                          15762
                                                     59
                                                          0.00341
    1901
                               0.00374
                                                                     0.000305
             5 female NSW
                                          16030
                                                     44
                                                          0.00275
                                                                     0.000251
 6
    1901
                               0.00274
    1901
             6 female NSW
                               0.00252
                                          16289
                                                     41
                                                          0.00230
                                                                     0.000215
    1901
             7 female NSW
                               0.00216
                                          16639
                                                     36
                                                          0.00197
                                                                     0.000189
    1901
             8 female NSW
                               0.00169
                                          16554
                                                     28
                                                          0.00175
                                                                     0.000173
10
    1901
             9 female NSW
                               0.00109
                                           16468
                                                     18
                                                          0.00162
                                                                     0.000163
```

Smoothing

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus ▷ filter(State == "NSW", Year == 1980) ▷ autoplot(Mortality) +
geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



Life tables

life_table(aus)

```
# A vital: 145,440 x 14 [1Y]
# Kev:
          Age x (Sex, State) [101 \times 12]
    Year
          Age Sex
                    State
                               mx
                                            lχ
                                                    dx
                                                          Lx
                                                                Tx
                                       qx
                                                                      ex
                                                                            rx
   <int> <int> <chr> <chr> <dbl>
                                    1 1901
            0 fema~ NSW
                          0.107
                                  0.100
                                         1
                                               1.00e-1 0.935
                                                              56.2
                                                                    56.2 0.935
   1901
            1 fema~ NSW
                          0.0247
                                  0.0244 0.900 2.20e-2 0.889
                                                              55.3
                                                                    61.5 0.951
   1901
            2 fema~ NSW
                          0.00686 0.00683 0.878 6.00e-3 0.875
                                                                    62.0 0.984
                                                              54.4
   1901
            3 fema~ NSW
                          0.00441 0.00441 0.872 3.84e-3 0.870
                                                              53.5
                                                                    61.4 0.994
   1901
            4 fema~ NSW
                          0.00374 0.00374 0.868 3.24e-3 0.867
                                                              52.7
                                                                    60.7 0.996
   1901
            5 fema~ NSW
                          0.00274 0.00274 0.865 2.37e-3 0.864
                                                              51.8
                                                                    59.9 0.997
   1901
            6 fema~ NSW
                          0.00252 0.00251 0.863 2.17e-3 0.861
                                                              50.9
                                                                    59.1 0.997
                                                                    58.2 0.998
   1901
            7 fema~ NSW
                          0.00216 0.00216 0.860 1.86e-3 0.859
                                                              50.1
                          0.00169 0.00169 0.859 1.45e-3 0.858
   1901
            8 fema~ NSW
                                                              49.2
                                                                    57.3 0.998
10
   1901
            9 fema~ NSW
                          0.00109 0.00109 0.857 9.36e-4 0.857
                                                              48.4
                                                                    56.4 0.999
# i 145,430 more rows
```

12

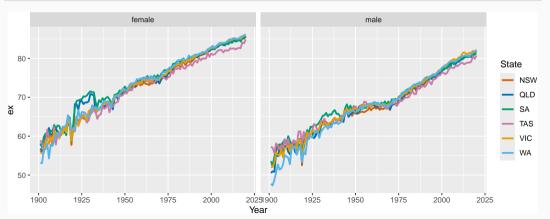
Life expectancy

life_expectancy(aus)

```
# A vital: 1,440 x 8 [1Y]
# Kev:
         Age x (Sex, State) [1 \times 12]
   Year
         Age Sex
                   State
                            ex
                                 rx
                                       nx
                                            ax
  1 1901
           0 female NSW
                          56.2 0.935
                                        1 0.352
2 1901
           0 female QLD
                          56.8 0.937
                                        1 0.338
   1901
           0 female SA
                          58.1 0.939
                                        1 0.324
   1901
           0 female TAS
                          58.9 0.946
                                        1 0.275
   1901
           0 female VIC
                          55.8 0.937
                                        1 0.334
           O female WA
                          53.1 0.922
   1901
                                        1 0.35
   1901
           0 male
                   NSW
                          52.6 0.925
                                        1 0.33
   1901
           0 male
                   QLD
                          50.6 0.924
                                        1 0.33
   1901
           0 male
                    SA
                          53.5 0.922
                                        1 0.33
10
   1901
           0 male
                    TAS
                          57.3 0.930
                                        1 0.33
# i 1,430 more rows
```

Life expectancy

```
life_expectancy(aus) >
    ggplot(aes(x = Year, y = ex, colour = State)) +
    geom_line(linewidth = 1) +
    facet_grid(. ~ Sex)
```



Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

Naive:
$$m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$$

Lee-Carter:
$$log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

$$\varepsilon_{x,t}$$
 = noise term with variance σ_x^2 .

Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

```
Naive: m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}

Lee-Carter: \log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}

\varepsilon_{x,t} = noise term with variance \sigma_x^2.
```

```
fit ← aus ▷
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
)
```

Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

```
Naive: m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}

Lee-Carter: \log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}

\varepsilon_{x,t} = noise term with variance \sigma_x^2.
```

```
fit ← aus ▷
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
)
```

```
fit
# A mable: 12 x 4
# Key: Sex, State [12]
   Sex
         State
                  naive
                             1 c
   <chr> <chr>
                <model> <model>
 1 female NSW
               <FNAIVE>
                           <LC>
 2 female OLD <FNAIVE>
                           <LC>
 3 female SA <FNAIVE>
                           <LC>
 4 female TAS <FNAIVE>
                           <I C>
 5 female VIC
                           <LC>
               <FNAIVE>
                           <LC>
 6 female WA
               <FNAIVE>
 7 male
         NSW
               <FNATVF>
                           <LC>
                           <LC>
8 male
         0LD
               <FNAIVE>
                           <LC>
9 male
         SA
               <FNATVE>
10 male
         TAS
               <FNAIVE>
                           <1 (>
11 male
         VIC
                           <LC>
               <FNAIVE>
                           <LC>
12 male
         WA
               <FNATVF>
```

Lee-Carter models

```
\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}
```

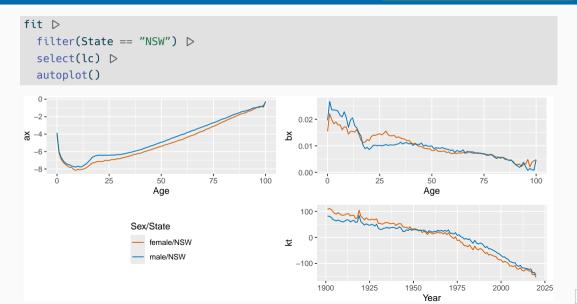
```
fit ▷
  filter(Sex == "female",
         State == "NSW") ▷
  select(lc) ▷
  report()
Series: Mortality
Model: LC
Transformation: log(Mortality)
Options:
  Adjust method: dt
  Jump choice: fit
```

```
Age functions
# A tibble: 101 \times 3
    Age ax
                 bx
  <int> <dbl> <dbl>
     0 -4.07 0.0155
     1 -6.20 0.0221
     2 -6.89 0.0199
# i 98 more rows
Time coefficients
# A tsibble: 120 x 2 [1Y]
  Year kt
  <int> <dbl>
1 1901 109
2 1902 111.
3 1903 108.
# i 117 more rows
```

Time series model: RW w/ drift Variance explained: 86.61%

Lee-Carter models

 $\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$



Lee-Carter models

```
fit ▷ select(lc) ▷ age_components()
# A tibble: 1,212 x 5
  Sex State Age ax
                            bx
  <chr> <chr> <int> <dbl> <dbl>
 1 female NSW
                  0 -4.07 0.0155
2 female NSW 1 -6.20 0.0221
3 female NSW 2 -6.89 0.0199
4 female NSW 3 -7.24 0.0183
5 female NSW
            4 -7.47 0.0190
6 female NSW
             5 -7.65 0.0178
7 female NSW
             6 -7.80 0.0179
8 female NSW
                 7 -7.81 0.0160
9 female NSW
                  8 -8.05 0.0171
10 female NSW
                  9 -8.15 0.0170
# i 1.202 more rows
```

```
fit ▷ select(lc) ▷ time_components()
```

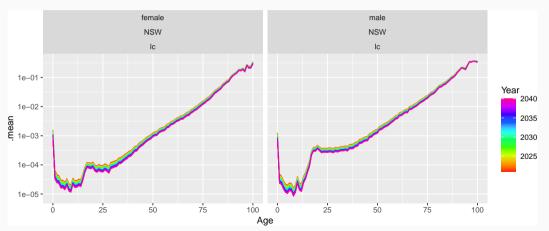
```
# A tsibble: 1,440 x 4 [1Y]
# Key: Sex, State [12]
  Sex State Year kt
  <chr> <chr> <int> <dhl>
1 female NSW
              1901 109.
2 female NSW
              1902 111.
3 female NSW
              1903 108.
4 female NSW
              1904 100.
5 female NSW
               1905 92.7
6 female NSW
               1906 89.5
7 female NSW
               1907
                     95.7
8 female NSW
               1908
                     90.5
9 female NSW
               1909 85.9
10 female NSW
               1910 85.4
# i 1,430 more rows
```

Forecasts

```
fc \leftarrow fit \triangleright forecast(h = 20)
fc
# A vital fable: 48,480 x 7 [1Y]
# Key:
                 Age x (Sex, State, .model) [101 \times 24]
   Sex
          State .model Year Age
                                            Mortality
                                                         .mean
   <chr> <chr> <chr> <dbl> <int>
                                                <dist>
                                                         <dbl>
 1 female NSW
                naive 2021
                             0 N(0.0027, 1.8e-05) 0.00270
 2 female NSW
               naive 2022
                                 0 N(0.0027, 3.6e-05) 0.00270
 3 female NSW
               naive
                        2023
                                 0 N(0.0027, 5.4e-05) 0.00270
 4 female NSW
               naive
                        2024
                                 0 N(0.0027, 7.2e-05) 0.00270
                                     N(0.0027, 9e-05) 0.00270
 5 female NSW
                naive
                        2025
                                 0 N(0.0027, 0.00011) 0.00270
 6 female NSW
                naive
                        2026
 7 female NSW
                naive
                        2027
                                 0 N(0.0027, 0.00013) 0.00270
                                 0 N(0.0027, 0.00014) 0.00270
 8 female NSW
                        2028
                naive
 9 female NSW
                naive
                        2029
                                 0 N(0.0027, 0.00016) 0.00270
10 female NSW
                naive
                        2030
                                 0 N(0.0027, 0.00018) 0.00270
```

NSW forecasts using Lee-Carter method

```
fc > filter(State == "NSW", .model == "lc") >
  autoplot() + scale_y_log10()
```



Let $m_{x,t}$ be the mortality rate at age x in year t.

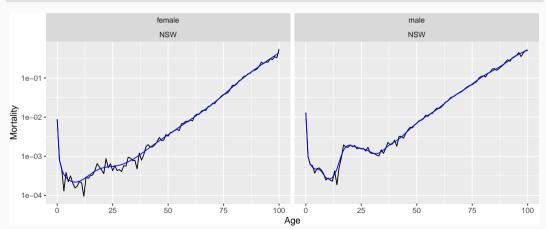
$$\log(m_{t,x}) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$
$$s_t(x) = \mu(x) + \sum_{i=1}^{J} \beta_{t,i}\phi_j(x) + e_t(x)$$

- $s_{\star}(x)$ = smoothed version of $y_{\star}(x)$
- $\mu(x) = \text{mean } s_t(x) \text{ across years.}$
- $\phi_j(x)$ and $\beta_{t,j}$ estimated using principal component analysis.
- $\beta_{1,i}, \dots, \beta_{T,i}$ modelled with ARIMA or ARFIMA processes.

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145.440 x 9 [1Y]
           Age x (Sex, State) [101 \times 12]
# Key:
    Year
           Age Sex
                      State Mortality Exposure Deaths
                                                          .smooth .smooth_se
                                 <dbl>
                                           <dbl>
                                                  <dbl> <dbl[1d]>
                                                                    <dbl[1d]>
   <int> <dbl> <chr> <chr>
    1901
             0 female NSW
                               0.107
                                           17143
                                                   1833
                                                          0.107
                                                                     0.00295
    1901
             1 female NSW
                               0.0247
                                           15071
                                                    373
                                                          0.0237
                                                                     0.00141
    1901
             2 female NSW
                               0.00686
                                           15461
                                                    106
                                                          0.00804
                                                                     0.000670
    1901
             3 female NSW
                               0.00441
                                           15629
                                                     69
                                                          0.00461
                                                                     0.000405
             4 female NSW
                                           15762
                                                     59
                                                          0.00341
    1901
                               0.00374
                                                                     0.000305
             5 female NSW
                                           16030
                                                     44
                                                          0.00275
                                                                     0.000251
    1901
                               0.00274
    1901
             6 female NSW
                               0.00252
                                           16289
                                                          0.00230
                                                                     0.000215
                                                     41
    1901
             7 female NSW
                               0.00216
                                           16639
                                                     36
                                                          0.00197
                                                                     0.000189
    1901
             8 female NSW
                               0.00169
                                           16554
                                                     28
                                                          0.00175
                                                                     0.000173
10
    1901
             9 female NSW
                               0.00109
                                           16468
                                                     18
                                                          0.00162
                                                                     0.000163
```

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus ▷ filter(State == "NSW", Year == 1980) ▷ autoplot(Mortality) +
geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



```
fit ← sm_aus ▷ model(hu = FDM(log(.smooth)))
fit
# A mable: 12 x 3
# Key: Sex, State [12]
  Sex State
                    hu
   <chr> <chr> <model>
 1 female NSW
                 <FDM>
 2 female QLD
              <FDM>
 3 female SA
                 <FDM>
 4 female TAS
                 <FDM>
 5 female VIC
                 <FDM>
 6 female WA
                 <FDM>
 7 male
         NSW
                 <FDM>
 8 male
        QLD
                 <FDM>
 9 male
         SA
                 <FDM>
10 male
          TAS
                  <FDM>
```

```
fit ▷
  filter(Sex == "female", State == "NSW") >
  select(hu) ▷
  report()
Series: .smooth
Model: FDM
Transformation: log(.smooth)
Basis functions
# A tibble: 101 x 8
   Age mean phi1 phi2 phi3 phi4 phi5 phi6
  <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
                                        <dbl>
                                                   <dbl>
     0 -4.07 0.147 0.0625 -0.0270
                                  0.0986
                                         0.0112 -0.0624
     1 -6.16 0.200 -0.0609 -0.194
                                  0.116
                                         0.0383
                                                  -0.238
     2 -6.82 0.182 -0.0483 -0.157
                                  0.0924
                                         0.0443
                                                  -0.264
     3 -7.17 0.170 -0.0368 -0.130
                                  0.0362
                                         0.000338 - 0.321
     4 -7.40 0.164 -0.0165 -0.114 -0.0154 -0.0303
                                                  -0.374
# i 96 more rows
```

```
Coefficients
# A tsibble: 120 x 8 [1Y]
  Year mean betal beta2
                        heta3
                                   beta4
                                           beta5
  <int> <dbl> <dbl> <dbl> <dbl>
                                   <dbl> <dbl> <dbl>
1 1901
          1 11.1 -0.522 -0.0553
                                0.207
                                        0.358
2 1902 1 11.8 -0.649 0.399
                                0.856
                                        0.0319
3 1903 1 11.5 -0.930 -0.485 0.398
                                       0.399
4 1904 1 11.1 -0.827 -0.214
                                -0.000305 0.00125 -0.0783
5 1905 1 10.2 -0.563 -0.105 0.324
                                       0.122
# i 115 more rows
# i Use 'print(n = ...)' to see more rows
Time series models
  beta1 : ARIMA(0.1.1) w/ drift
  beta2 : ARIMA(0,2,2)
  beta3 : ARIMA(1,0,1)
  beta4 : ARIMA(0,0,2)
  beta5 : ARIMA(0,0,0)
  beta6 : ARIMA(2,0,2)
Variance explained
  91.38 + 1.81 + 0.58 + 0.49 + 0.42 + 0.39 = 95.06%
```

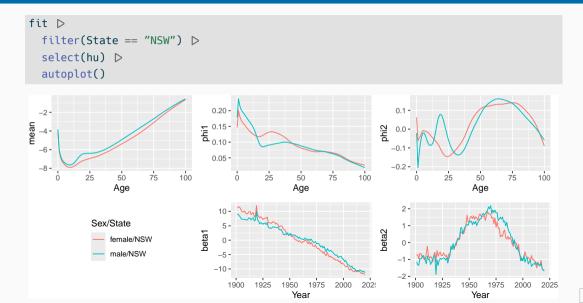
beta6

0.0305

0.422

-0.376

0.0478



```
fit ▷ select(hu) ▷ age_components()
```

```
# A tibble: 1,212 x 10
  Sex
         State
                 Age mean phi1
                                 phi2
                                            phi3
                                                    phi4
                                                              phi5
                                                                     phi6
  <chr> <chr> <dbl> <dbl> <dbl>
                                   <dbl>
                                           <dbl>
                                                   <dbl>
                                                             <dbl>
                                                                     <dbl>
1 female NSW
                   0 -4.07 0.147
                                 0.0625 -0.0270
                                                  0.0986
                                                          0.0112
                                                                   -0.0624
2 female NSW
                   1 -6.16 0.200 -0.0609 -0.194
                                                  0.116
                                                          0.0383
                                                                   -0.238
3 female NSW
                   2 -6.82 0.182 -0.0483 -0.157
                                                  0.0924
                                                          0.0443
                                                                   -0.264
4 female NSW
                   3 -7.17 0.170 -0.0368 -0.130
                                                  0.0362
                                                          0.000338 -0.321
 5 female NSW
                   4 -7.40 0.164 -0.0165 -0.114
                                                 -0.0154 -0.0303
                                                                   -0.374
6 female NSW
                   5 -7.57 0.158 -0.00759 -0.121
                                                 -0.0564
                                                          0.0247
                                                                   -0.315
 7 female NSW
                   6 -7.71 0.153 -0.00942 -0.133
                                                 -0.0976
                                                          0.112
                                                                   -0.197
8 female NSW
                   7 -7.81 0.149 -0.0121 -0.143
                                                 -0.143
                                                          0.175
                                                                   -0.0863
9 female NSW
                   8 -7.88 0.143 -0.0141 -0.148 -0.181
                                                          0.211
                                                                    0.0131
10 female NSW
                   9 -7.92 0.138 -0.0185 -0.142 -0.196
                                                          0.236
                                                                    0.101
# i 1,202 more rows
```

28

fit ▷ select(hu) ▷ time_components()

```
# A tsibble: 1,440 x 10 [1Y]
# Key:
           Sex, State [12]
         State Year mean beta1 beta2
                                                  beta4
                                                           beta5
   Sex
                                          beta3
                                                                  beta6
   <chr> <chr> <int> <dbl> <dbl> <dbl>
                                                           <dbl>
                                          <dbl> <dbl>
                                                                  <dbl>
 1 female NSW
                         1 11.2 -0.756 -0.0301
                                                 0.269
                                                                 0.409
                1901
                                                        -0.155
 2 female NSW
                1902
                         1 11.6
                                 -0.708
                                         0.0899
                                                 0.207
                                                         0.0282
                                                                 0.507
 3 female NSW
                1903
                         1 11.5 -0.962
                                         0.169
                                                -0.103
                                                         0.366
                                                                 0.323
 4 female NSW
                 1904
                         1 11.1 -0.648
                                         0.0985 -0.433
                                                         0.131
                                                                 0.270
 5 female NSW
                 1905
                         1 10.1
                                 -0.660
                                         0.342
                                                -0.0910
                                                         0.0862
                                                                 0.612
                                                -0.147
 6 female NSW
                 1906
                         1 9.78 -0.865
                                         0.496
                                                        -0.101
                                                                 0.306
 7 female NSW
                 1907
                            9.90 - 0.861
                                         0.0530
                                                 1.33
                                                         0.278
                                                                 0.181
 8 female NSW
                 1908
                         1 10.1 -1.01
                                         0.554
                                                -0.0198 -0.00428 0.578
 9 female NSW
                 1909
                         1 9.42 -1.02
                                         0.293
                                                -0.365
                                                        -0.149
                                                                 0.353
10 female NSW
                            9.08 -0.650
                                         0.172 - 0.559
                 1910
                                                        -0.253
                                                                 0.0110
# i 1,430 more rows
```

$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

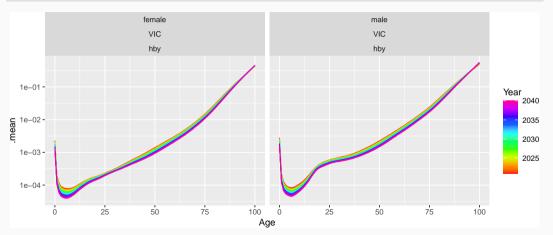
$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{t,j}\phi_j(x) + e_t(x)$$

- $y_t(x) = \log(m_{x,t}^M m_{x,t}^F)$ and $\log(m_{x,t}^M / m_{x,t}^F)$
- $s_t(x)$ = smoothed version of $y_t(x)$
- $\mu(x) = \text{mean } s_t(x) \text{ across years.}$
- $\phi_i(x)$ and $\beta_{t,i}$ estimated using principal component analysis.
- $\beta_{1,j},...,\beta_{T,j}$ modelled with ARIMA for products and ARMA for ratios (to ensure stationary sex-ratios)

```
pr ← sm_aus ▷ make_pr(.smooth)
pr
# A vital: 218,160 x 9 [1Y]
# Key:
           Age x (Sex, State) [101 \times 18]
    Year
           Age Sex
                      State Mortality Exposure Deaths .smooth .smooth_se
   <int> <dbl> <chr> <chr>
                                 <dbl>
                                          <dbl> <dbl> <dbl[1d]>
                                                                   <dbl[1d1>
 1 1901
             0 female NSW
                               0.107
                                                   1833
                                                            0.939
                                                                    0.00295
                                          17143
    1901
             1 female NSW
                               0.0247
                                          15071
                                                    373
                                                            1.03
                                                                    0.00141
    1901
             2 female NSW
                               0.00686
                                          15461
                                                    106
                                                            0.965
                                                                    0.000670
    1901
             3 female NSW
                               0.00441
                                          15629
                                                    69
                                                            0.982
                                                                    0.000405
             4 female NSW
                                                     59
    1901
                               0.00374
                                          15762
                                                            1.02
                                                                    0.000305
    1901
             5 female NSW
                               0.00274
                                          16030
                                                     44
                                                            1.04
                                                                    0.000251
    1901
             6 female NSW
                               0.00252
                                          16289
                                                     41
                                                            1.04
                                                                    0.000215
    1901
             7 female NSW
                               0.00216
                                          16639
                                                     36
                                                            1.01
                                                                    0.000189
             8 female NSW
                                          16554
                                                     28
                                                            0.972
    1901
                               0.00169
                                                                    0.000173
10
    1901
             9 female NSW
                               0.00109
                                          16468
                                                     18
                                                            0.938
                                                                    0.000163
```

```
pr ← sm_aus ▷ make_pr(.smooth)
fit \leftarrow pr \triangleright model(hby = FDM(log(.smooth), coherent = TRUE))
fit
# A mable: 18 x 3
# Key: Sex, State [18]
   Sex
                   State
                             hby
   <chr>
                   <chr> <model>
 1 female
                   NSW
                           <FDM>
 2 female
                   0LD
                           <FDM>
 3 female
                   SA
                           <FDM>
 4 female
                   TAS
                           <FDM>
 5 female
                   VIC
                           <FDM>
 6 female
                   WA
                           <FDM>
 7 geometric_mean NSW
                            <FDM>
 8 geometric_mean QLD
                            <FDM>
 9 geometric mean SA
                            <FDM>
```

```
fc ← fit ▷ forecast(h = 20) ▷ undo_pr(.smooth)
fc ▷ filter(State == "VIC") ▷ autoplot() + scale_y_log10()
```



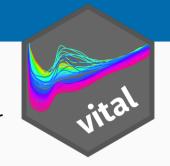
Other functionality

- Import data from Human Mortality Database and Human Fertility Database
- Convert demogdata, tsibble & data.frame objects to vital.
- Compute net migration from population, births and deaths.
- Compute total fertility rates from age-specific fertility rates.
- Various smoothing functions
- Other mortality models including functional data models, and coherent functional data models.



Future plans

- Remaining tools from the demography package
- Stochastic population forecasting (as per Hyndman-Booth, IJF, 2008)
- All models handled by StMoMo package
- All methods from MortalityLaws package
- Suggestions from users



Future plans

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- All methods from MortalityLaws package
- Suggestions from users
- **Slides**: robjhyndman.com/user2024
- **Package**: pkg.robjhyndman.com/vital/

