

vital: Tidy data analysis for demography

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Demographic data structures in R packages

Package	Data class
demography	demogdata
StMoMo	StMoMoData (created by converting a
	demogdata object)
StanMoMo	Lists of matrices
lifecontingencies	data.frame
BayesMortalityPlus	tibble (that needs to be converted to a matrix for fitting)
MortalityLaws	individual vectors
HMDHFDplus	data.frame

tibble objects

Australian Deaths 1901–2020

A tibble: $145,440 \times 7$

, w copper, 113,110 x ,							
	Year	Age	Sex	State	Mortality	Exposure	Deaths
	<int></int>	<int></int>	<chr></chr>	<chr></chr>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
1	1901	0	female	WA	0.129	2511	325
2	1901	0	male	WA	0.158	2634	416
3	1901	1	female	WA	0.0275	2219	61
4	1901	1	male	WA	0.0391	2175	85
5	1901	2	female	WA	0.00688	2180	15
6	1901	2	male	WA	0.0131	2208	29
7	1901	3	female	WA	0.00584	1884	11
8	1901	3	male	WA	0.00503	1988	10
9	1901	4	female	WA	0.00290	1722	5
10	1901	4	male	WA	0.00287	1743	5
# i	145,4	30 mor	e rows				



7133LE

tsibble objects

Australian Deaths 1901–2020

```
# A tsibble: 145,440 x 7 [1Y]
# Key:
             Age, Sex, State [1,212]
           Age Sex State Mortality Exposure Deaths
    Year
                                           <fd>1
   <int> <int> <chr> <chr>
                                 <1db>>
                                                  <dhl>
    1901
             0 female WA
                               0.129
                                            2511
                                                     325
    1901
             0 male
                       WA
                               0.158
                                            2634
                                                    416
             1 female WA
                                            2219
                                                      61
    1901
                               0.0275
    1901
             1 male
                       WΑ
                               0.0391
                                            2175
                                                      85
    1901
             2 female WA
                               0.00688
                                            2180
                                                      15
    1901
             2 male
                       WΑ
                               0.0131
                                            2208
                                                      29
    1901
             3 female WA
                               0.00584
                                            1884
                                                      11
    1901
             3 male
                               0.00503
                                            1988
                                                      10
                       WΔ
             4 female WA
                                            1722
    1901
                               0.00290
10
    1901
             4 male
                       WΑ
                               0.00287
                                            1743
    145,430
            more rows
```



Index:

Year

Kevs:

- Age
- Sex
- State

Every row must have a unique combination of **Index and Kevs**

vital objects

i 145,430

Australian Deaths 1901–2020

more rows

aus

```
# A vital: 145,440 x 7 [1Y]
# Kev:
           Age x (Sex, State) [101 \times 12]
           Age Sex State Mortality Exposure Deaths
    Year
   <int> <int> <chr> <chr>
                                  <dhl>
                                            <dhl>
                                                   <dbl>
    1901
             0 female WA
                                0.129
                                             2511
                                                     325
    1901
             0 male
                       WA
                                0.158
                                             2634
                                                     416
              1 female WA
    1901
                                0.0275
                                             2219
                                                      61
    1901
              1 male
                       WΑ
                                0.0391
                                             2175
                                                      85
    1901
              2 female WA
                                0.00688
                                             2180
                                                       15
    1901
             2 male
                       WA
                                0.0131
                                             2208
                                                      29
    1901
              3 female WA
                                0.00584
                                             1884
                                                       11
              3 male
                                0.00503
                                             1988
    1901
                       WA
                                                       10
             4 female WA
                                0.00290
                                             1722
    1901
    1901
              4 male
                       WΑ
                                0.00287
                                             1743
```

Variables

Index:

Year

Keys:

Age

Sex

State

Every row must have a unique combination of Index and Keys

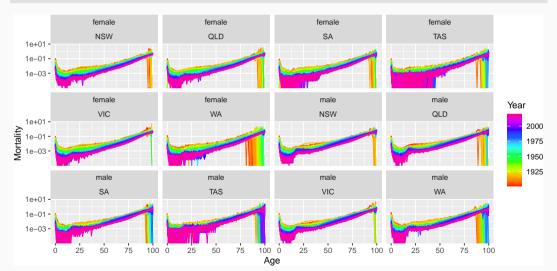
Variables denoting age, sex, deaths, births and population can also be specified as attributes.

vital objects

```
index_var(aus)
[1] "Year"
key_vars(aus)
[1] "Age" "Sex" "State"
vital_vars(aus)
                          deaths population
                  sex
       age
     "Age"
                "Sex"
                        "Deaths" "Exposure"
```

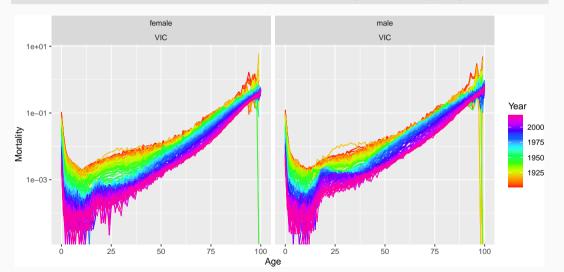
Rainbow plots

aus ▷ autoplot(Mortality) + scale_y_log10()



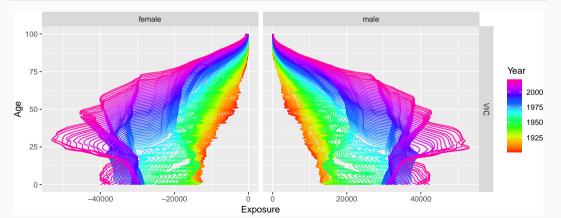
Rainbow plots

aus ▷ filter(State == "VIC") ▷ autoplot(Mortality) + scale_y_log10()



Rainbow plots

```
aus ▷ filter(State == "VIC") ▷
mutate(Exposure = if_else(Sex == "female", -Exposure, Exposure)) ▷
autoplot(Exposure) +
facet_grid(State ~ Sex, scales = "free_x") + coord_flip()
```



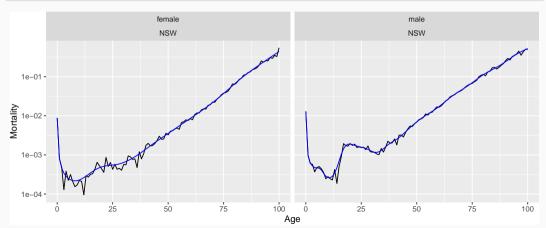
Smoothing

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
# Key:
           Age x (Sex, State) [101 \times 12]
                      State Mortality Exposure Deaths
    Year
           Age Sex
                                                          .smooth .smooth_se
                                 <dbl>
                                          <dbl> <dbl> <dbl[1d]>
   <int> <dhl> <chr> <chr>
                                                                    <dbl[1d]>
    1901
             0 female NSW
                               0.107
                                           17143
                                                   1833
                                                          0.107
                                                                     0.00295
    1901
             1 female NSW
                               0.0247
                                           15071
                                                    373
                                                          0.0237
                                                                     0.00141
    1901
             2 female NSW
                               0.00686
                                           15461
                                                    106
                                                          0.00804
                                                                     0.000670
    1901
             3 female NSW
                               0.00441
                                           15629
                                                     69
                                                          0.00461
                                                                     0.000405
             4 female NSW
                                           15762
                                                          0.00341
    1901
                               0.00374
                                                     59
                                                                     0.000305
 6
    1901
             5 female NSW
                               0.00274
                                           16030
                                                     44
                                                          0.00275
                                                                     0.000251
             6 female NSW
                                           16289
    1901
                               0.00252
                                                     41
                                                          0.00230
                                                                     0.000215
             7 female NSW
                               0.00216
                                           16639
                                                          0.00197
    1901
                                                     36
                                                                     0.000189
    1901
             8 female NSW
                               0.00169
                                           16554
                                                     28
                                                          0.00175
                                                                     0.000173
10
    1901
             9 female NSW
                               0.00109
                                           16468
                                                     18
                                                          0.00162
                                                                     0.000163
```

Smoothing

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus ▷ filter(State == "NSW", Year == 1980) ▷ autoplot(Mortality) +
geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



Life tables

life_table(aus)

```
# A vital: 145,440 x 14 [1Y]
           Age x (Sex, State) [101 \times 12]
# Kev:
    Year
           Age Sex
                     State
                                 mx
                                               lχ
                                                       dx
                                                              Lx
                                                                    Tx
                                         qx
                                                                          ex
                                                                                rx
   <int> <int> <chr> <chr>
                            <dbl>
                                      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
                                           1
 1 1901
             0 fema∼ NSW
                           0.107
                                    0.100
                                                  1.00e-1 0.935
                                                                 56.2
                                                                        56.2 0.935
    1901
             1 fema~ NSW
                           0.0247
                                    0.0244
                                            0.900 2.20e-2 0.889
                                                                 55.3
                                                                        61.5 0.951
                           0.00686 0.00683 0.878 6.00e-3 0.875 54.4
   1901
             2 fema~ NSW
                                                                        62.0 0.984
             3 fema~ NSW
                           0.00441 0.00441 0.872 3.84e-3 0.870
                                                                 53.5
    1901
                                                                        61.4 0.994
    1901
             4 fema~ NSW
                            0.00374 0.00374 0.868 3.24e-3 0.867
                                                                 52.7
                                                                        60.7 0.996
                           0.00274 0.00274 0.865 2.37e-3 0.864
    1901
             5 fema~ NSW
                                                                 51.8
                                                                        59.9 0.997
                            0.00252 0.00251 0.863 2.17e-3 0.861
    1901
             6 fema~ NSW
                                                                 50.9
                                                                        59.1 0.997
    1901
             7 fema~ NSW
                            0.00216 0.00216 0.860 1.86e-3 0.859
                                                                 50.1
                                                                        58.2 0.998
    1901
             8 fema~ NSW
                           0.00169 0.00169 0.859 1.45e-3 0.858
                                                                 49.2
                                                                        57.3 0.998
10
    1901
             9 fema~ NSW
                           0.00109 0.00109 0.857 9.36e-4 0.857
                                                                 48.4
                                                                        56.4 0.999
# i 145,430
            more rows
```

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Life expectancy

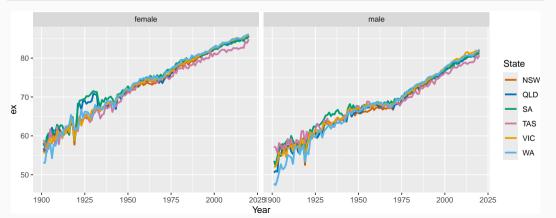
life_expectancy(aus)

```
# A vital: 1,440 x 8 [1Y]
# Kev:
           Age x (Sex, State) [1 \times 12]
           Age Sex State
    Year
                               ex
                                     rx
                                           nx
                                                 ax
   <int> <int> <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <</pre>
 1 1901
             0 female NSW
                             56.2 0.935
                                            1 0.352
             0 female OLD 56.8 0.937
   1901
                                            1 0.338
   1901
             0 female SA 58.1 0.939
                                            1 0.324
   1901
             0 female TAS 58.9 0.946
                                            1 0.275
   1901
             0 female VIC
                             55.8 0.937
                                            1 0.334
             O female WA
   1901
                             53.1 0.922
                                            1 0.35
    1901
             0 male NSW
                             52.6 0.925
                                            1 0.33
                    QLD
                             50.6 0.924
   1901
             0 male
                                            1 0.33
   1901
             0 male
                     SA
                             53.5 0.922
                                            1 0.33
   1901
                      TAS
                             57.3 0.930
10
             0 male
                                            1 0.33
# i 1,430 more rows
```

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Life expectancy

```
life_expectancy(aus) D
ggplot(aes(x = Year, y = ex, colour = State)) +
geom_line(linewidth = 1) +
facet_grid(. ~ Sex)
```



Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

Naive:
$$m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$$

Lee-Carter:
$$log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

$$\varepsilon_{x,t}$$
 = noise term with variance σ_x^2 .

Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

```
Naive: m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}

Lee-Carter: \log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}

\varepsilon_{x,t} = noise term with variance \sigma_x^2.
```

```
fit ← aus ▷
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
)
```

Mortality models

 $m_{x,t}$ = mortality rate at age x in year t.

```
Naive: m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}

Lee-Carter: \log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}

\varepsilon_{x,t} = noise term with variance \sigma_x^2.
```

```
fit ← aus ▷
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
)
```

```
fit
# A mable: 12 x 4
# Kev: Sex, State [12]
  Sex State
                  naive
                             10
   <chr> <chr> <model> <model>
 1 female NSW
               <FNATVF>
                           <I C>
 2 female OLD
                           <LC>
               <FNAIVE>
 3 female SA
               <FNAIVE>
                           <LC>
 4 female TAS
               <FNAIVE>
                           <LC>
                           <I C>
 5 female VIC
                <FNATVF>
 6 female WA
                <FNAIVE>
                           <LC>
 7 male
         NSW
                <FNATVE>
                           <LC>
                           <LC>
 8 male
         0LD
                <FNATVE>
 9 male
         SA
                <FNAIVE>
                           <LC>
10 male
         TAS
                           <LC>
               <FNATVE>
                           <LC>
11 male
         VIC
                <FNATVE>
12 male
         WΑ
                <FNATVF>
                           <LC>
```

Lee-Carter models

```
\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}
```

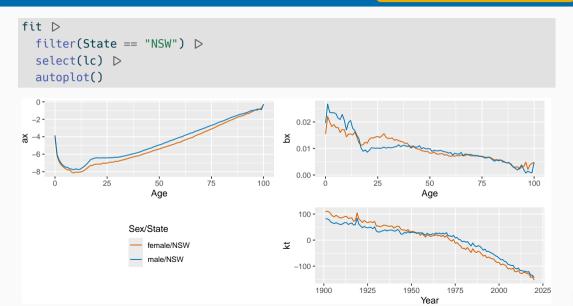
```
fit ▷
  filter(Sex == "female",
         State == "NSW") >
  select(lc) ▷
  report()
Series: Mortality
Model: LC
Transformation: log(Mortality)
Options:
  Adjust method: dt
  Jump choice: fit
```

```
Age functions
# A tibble: 101 \times 3
   Age ax bx
  <int> <dbl> <dbl>
     0 -4.07 0.0155
   1 -6.20 0.0221
     2 -6.89 0.0199
# i 98 more rows
Time coefficients
# A tsibble: 120 x 2 [1Y]
  Year kt
  <int> <dbl>
1 1901 109.
2 1902 111.
3 1903 108.
# i 117 more rows
```

Time series model: RW w/ drift Variance explained: 86.61%

Lee-Carter models

 $\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$



Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit ▷ select(lc) ▷ age components()
# A tibble: 1,212 x 5
  Sex
        State
               Age ax
                           bx
  <chr> <chr> <int> <dbl> <dbl>
                 0 -4.07 0.0155
 1 female NSW
 2 female NSW 1 -6.20 0.0221
 3 female NSW
                 2 -6.89 0.0199
4 female NSW
                 3 -7.24 0.0183
 5 female NSW 4 -7.47 0.0190
6 female NSW
                 5 -7.65 0.0178
 7 female NSW
                 6 -7.80 0.0179
 8 female NSW
                 7 -7.81 0.0160
 9 female NSW
                 8 -8.05 0.0171
10 female NSW
                 9 -8.15 0.0170
# i 1,202 more rows
```

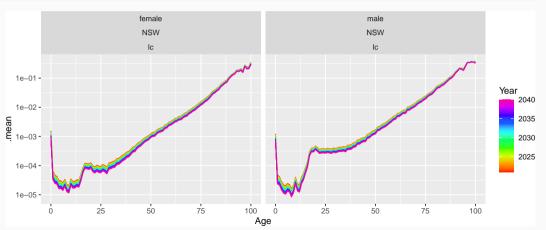
```
fit D select(lc) D time components()
# A tsibble: 1,440 x 4 [1Y]
# Key: Sex, State [12]
  Sex State Year kt
  <chr> <chr> <int> <dbl>
1 female NSW
               1901 109.
2 female NSW 1902 111.
3 female NSW 1903 108.
4 female NSW
               1904 100.
5 female NSW
                1905 92.7
6 female NSW
                1906 89.5
7 female NSW
                1907
                    95.7
8 female NSW
               1908
                    90.5
9 female NSW
               1909
                    85.9
10 female NSW
               1910 85.4
# i 1.430 more rows
```

Forecasts

```
fc \leftarrow fit \triangleright forecast(h = 20)
fc
# A vital fable: 72,720 x 7 [1Y]
                Age x (Sex, State, .model) [101 \times 36]
# Key:
   Sex State .model Year Age
                                            Mortality
                                                        .mean
   <chr> <chr> <chr> <dbl> <int>
                                               <dist>
                                                        <dbl>
 1 female NSW
               naive
                      2021
                                 0 N(0.0027, 1.8e-05) 0.00270
 2 female NSW
               naive
                      2022
                                 0 N(0.0027, 3.6e-05) 0.00270
 3 female NSW
                naive
                      2023
                                 0 N(0.0027, 5.4e-05) 0.00270
 4 female NSW
                naive
                       2024
                                 0 N(0.0027, 7.2e-05) 0.00270
                                     N(0.0027, 9e-05) 0.00270
 5 female NSW
                naive
                        2025
                        2026
                                 0 N(0.0027, 0.00011) 0.00270
 6 female NSW
                naive
 7 female NSW
                naive
                        2027
                                 0 N(0.0027, 0.00013) 0.00270
 8 female NSW
                naive
                        2028
                                 0 N(0.0027, 0.00014) 0.00270
 9 female NSW
                                 0 N(0.0027, 0.00016) 0.00270
                naive
                        2029
10 female NSW
                naive
                        2030
                                 0 N(0.0027, 0.00018) 0.00270
```

NSW forecasts using Lee-Carter method

```
fc > filter(State == "NSW", .model == "lc") >
  autoplot() + scale_y_log10()
```



Let $m_{x,t}$ be the mortality rate at age x in year t.

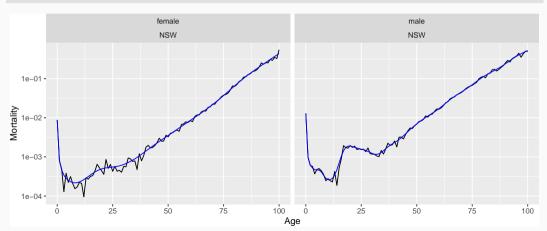
$$\log(m_{t,x}) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$
$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

- $s_{\star}(x)$ = smoothed version of $y_{\star}(x)$
- $\mu(x) = \text{mean } s_t(x) \text{ across years.}$
- $\phi_i(x)$ and β_{ti} estimated using principal component analysis.
- $\beta'_{1j},...,\beta_{Tj}$ modelled with ARIMA or ARFIMA processes.

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
# Key:
           Age x (Sex, State) [101 \times 12]
                      State Mortality Exposure Deaths
    Year
           Age Sex
                                                         .smooth .smooth_se
                                 <dbl>
                                          <dbl> <dbl> <dbl[1d]>
   <int> <dhl> <chr> <chr>
                                                                   <dbl[1d]>
    1901
             0 female NSW
                              0.107
                                          17143
                                                  1833
                                                         0.107
                                                                    0.00295
    1901
             1 female NSW
                              0.0247
                                          15071
                                                   373
                                                         0.0237
                                                                    0.00141
    1901
             2 female NSW
                              0.00686
                                          15461
                                                   106
                                                         0.00804
                                                                    0.000670
    1901
             3 female NSW
                              0.00441
                                          15629
                                                    69
                                                         0.00461
                                                                    0.000405
             4 female NSW
                                          15762
                                                          0.00341
    1901
                               0.00374
                                                    59
                                                                    0.000305
    1901
             5 female NSW
                               0.00274
                                          16030
                                                    44
                                                         0.00275
                                                                    0.000251
             6 female NSW
                                          16289
    1901
                               0.00252
                                                    41
                                                          0.00230
                                                                    0.000215
             7 female NSW
                               0.00216
                                          16639
                                                          0.00197
    1901
                                                    36
                                                                    0.000189
    1901
             8 female NSW
                               0.00169
                                          16554
                                                    28
                                                         0.00175
                                                                    0.000173
10
    1901
             9 female NSW
                               0.00109
                                          16468
                                                    18
                                                          0.00162
                                                                    0.000163
```

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus ▷ filter(State == "NSW", Year == 1980) ▷ autoplot(Mortality) +
geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



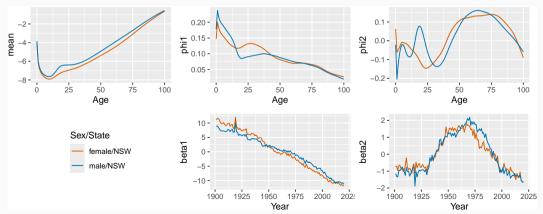
```
fit ← sm_aus ▷ model(hu = FDM(log(.smooth)))
fit
# A mable: 12 x 3
# Key: Sex, State [12]
  Sex State
                    hu
  <chr> <chr> <model>
 1 female NSW
                 <FDM>
2 female QLD
                 <FDM>
3 female SA
                 <FDM>
4 female TAS
                 <FDM>
5 female VIC
                 <FDM>
6 female WA
                 <FDM>
 7 male
         NSW
                 <FDM>
8 male OLD
                 <FDM>
9 male
        SA
                 <FDM>
10 male
         TAS
                 <FDM>
```

```
fit ▷
 filter(Sex == "female", State == "NSW") >
 select(hu) ▷
  report()
Series: .smooth
Model: FDM
Transformation: log(.smooth)
Basis functions
# A tibble: 101 \times 8
   Age mean phi1 phi2 phi3 phi4 phi5 phi6
  <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
     0 -4.07 0.147 0.0625 -0.0270
                                  0.0986 0.0112 -0.0624
2
     1 -6.16 0.200 -0.0609 -0.194
                                  0.116 0.0383
                                                  -0.238
     2 -6.82 0.182 -0.0483 -0.157 0.0924 0.0443 -0.264
     3 -7.17 0.170 -0.0368 -0.130 0.0362 0.000338 -0.321
                                                 -0.374
     4 -7.40 0.164 -0.0165 -0.114 -0.0154 -0.0303
# i 96 more rows
```

```
Coefficients
# A tsibble: 120 x 8 [1Y]
  Year mean beta1 beta2
                         beta3
                                   beta4
                                          beta5
                                                  beta6
  <int> <dbl> <dbl> <dbl> <dbl>
                                   <dbl>
                                          <dbl>
                                                  <dbl>
1 1901
          1 11.1 -0.522 -0.0553
                                0.207
                                       0.358
                                                 0.0305
  1902 1 11.8 -0.649 0.399
                                0.856
                                        0.0319
2
                                                 0.422
 1903 1 11.5 -0.930 -0.485 0.398 0.399
                                                -0.376
4 1904 1 11.1 -0.827 -0.214 -0.000305 0.00125 -0.0783
  1905
          1 10.2 -0.563 -0.105 0.324 0.122
                                                 0.0478
# i 115 more rows
# i Use `print(n = ...)` to see more rows
Time series models
  beta1 : ARIMA(0,1,1) w/ drift
  beta2 : ARIMA(0,2,2)
  beta3 : ARIMA(1,0,1)
  beta4 : ARIMA(0,0,2)
  beta5 : ARIMA(0,0,0)
  beta6 : ARIMA(2,0,2)
```

Variance explained 91.38 + 1.81 + 0.58 + 0.49 + 0.42 + 0.39 = 95.06%

```
fit D
  filter(State == "NSW") D
  select(hu) D
  autoplot()
```



```
fit ▷ select(hu) ▷ age_components()
```

```
# A tibble: 1,212 x 10
         State
                                  phi2
                                             phi3
                                                     phi4
                                                              phi5
                                                                       phi6
   Sex
                 Age mean
                            phi1
   <chr> <chr> <dbl> <dbl> <dbl>
                                    <dbl>
                                            <dbl>
                                                    <dbl>
                                                              <dbl>
                                                                      <dbl>
 1 female NSW
                   0 -4.07 0.147
                                  0.0625
                                          -0.0270
                                                   0.0986
                                                           0.0112
                                                                     -0.0624
 2 female NSW
                   1 -6.16 0.200 -0.0609 -0.194
                                                   0.116
                                                           0.0383
                                                                    -0.238
 3 female NSW
                   2 -6.82 0.182 -0.0483 -0.157
                                                   0.0924
                                                           0.0443
                                                                     -0.264
 4 female NSW
                   3 -7.17 0.170 -0.0368 -0.130
                                                   0.0362
                                                           0.000338
                                                                    -0.321
 5 female NSW
                   4 -7.40 0.164 -0.0165
                                         -0.114
                                                   -0.0154 - 0.0303
                                                                     -0.374
 6 female NSW
                   5 -7.57 0.158 -0.00759
                                          -0.121
                                                   -0.0564
                                                           0.0247
                                                                    -0.315
 7 female NSW
                   6 -7.71 0.153 -0.00942
                                          -0.133
                                                   -0.0976
                                                           0.112
                                                                     -0.197
 8 female NSW
                   7 -7.81 0.149 -0.0121
                                          -0.143
                                                   -0.143
                                                           0.175
                                                                     -0.0863
                   8 -7.88 0.143 -0.0141 -0.148
 9 female NSW
                                                   -0.181
                                                           0.211
                                                                     0.0131
10 female NSW
                    9 -7.92 0.138 -0.0185 -0.142
                                                           0.236
                                                                     0.101
                                                   -0.196
# i 1.202 more rows
```

28

fit ▷ select(hu) ▷ time_components()

```
# A tsibble: 1.440 x 10 [1Y]
# Kev:
             Sex, State [12]
         State Year mean beta1
                                                           beta5
                                                                  beta6
   Sex
                                  beta2
                                          beta3
                                                  beta4
   <chr> <chr> <chr> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
                                                           <dbl> <dbl>
 1 female NSW
                 1901
                         1 11.2 -0.756 -0.0301 0.269
                                                         -0.155
                                                                 0.409
 2 female NSW
                         1 11.6 -0.708
                1902
                                         0.0899 0.207
                                                         0.0282
                                                                 0.507
 3 female NSW
                1903
                         1 11.5 -0.962
                                         0.169
                                                         0.366
                                                                 0.323
                                                 -0.103
 4 female NSW
                 1904
                          1 11.1
                                 -0.648
                                         0.0985
                                                -0.433
                                                         0.131
                                                                 0.270
 5 female NSW
                 1905
                          1 10.1
                                 -0.660
                                         0.342
                                                 -0.0910
                                                         0.0862
                                                                 0.612
 6 female NSW
                 1906
                          1 9.78 -0.865
                                         0.496
                                                 -0.147
                                                         -0.101
                                                                 0.306
 7 female NSW
                 1907
                            9.90 - 0.861
                                         0.0530 1.33
                                                         0.278
                                                                 0.181
 8 female NSW
                                         0.554
                 1908
                          1 10.1 -1.01
                                                -0.0198 -0.00428 0.578
 9 female NSW
                 1909
                          1 9.42 -1.02
                                         0.293
                                                -0.365
                                                         -0.149
                                                                 0.353
10 female NSW
                                                                 0.0110
                 1910
                            9.08 - 0.650
                                         0.172
                                                 -0.559
                                                         -0.253
# i 1.430 more rows
```

2

$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

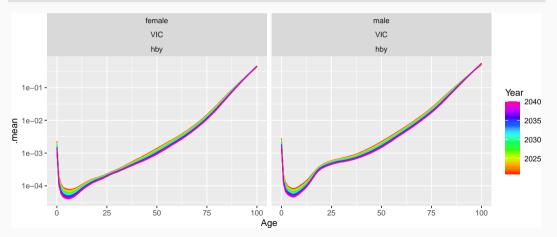
- $y_t(x) = \log(m_{x,t}^M m_{x,t}^F)$ and $\log(m_{x,t}^M / m_{x,t}^F)$
- $s_t(x)$ = smoothed version of $y_t(x)$
- $\mu(x)$ = mean $s_{t}(x)$ across years.
- $\phi_i(x)$ and β_{ti} estimated using principal component analysis.
- $\beta_{1j},...,\beta_{Tj}$ modelled with ARIMA for products and ARMA for ratios (to ensure stationary sex-ratios)

 $pr \leftarrow sm_aus \triangleright make_pr(.smooth)$

```
pr
# A vital: 218,160 x 9 [1Y]
# Key:
           Age x (Sex, State) [101 \times 18]
           Age Sex
                      State Mortality Exposure Deaths
                                                          .smooth .smooth se
    Year
   <int> <dbl> <chr> <chr>
                                 <dbl>
                                          <dbl> <dbl> <dbl[1d]>
                                                                    <dbl[1d]>
    1901
             0 female NSW
                               0.107
                                           17143
                                                   1833
                                                            0.939
                                                                     0.00295
    1901
             1 female NSW
                               0.0247
                                           15071
                                                    373
                                                            1.03
                                                                     0.00141
    1901
             2 female NSW
                               0.00686
                                           15461
                                                    106
                                                            0.965
                                                                     0.000670
    1901
             3 female NSW
                               0.00441
                                           15629
                                                     69
                                                            0.982
                                                                     0.000405
    1901
             4 female NSW
                               0.00374
                                           15762
                                                     59
                                                            1.02
                                                                     0.000305
    1901
             5 female NSW
                               0.00274
                                           16030
                                                     44
                                                            1.04
                                                                     0.000251
             6 female NSW
                                           16289
    1901
                               0.00252
                                                     41
                                                             1.04
                                                                     0.000215
    1901
             7 female NSW
                               0.00216
                                           16639
                                                     36
                                                             1.01
                                                                     0.000189
             8 female NSW
                                           16554
    1901
                               0.00169
                                                     28
                                                            0.972
                                                                     0.000173
10
             9 female NSW
                                                     18
    1901
                               0.00109
                                           16468
                                                            0.938
                                                                     0.000163
```

```
pr ← sm_aus ▷ make_pr(.smooth)
fit \leftarrow pr \triangleright model(hby = FDM(log(.smooth), coherent = TRUE))
fit
# A mable: 18 x 3
# Key: Sex, State [18]
   Sex
                   State
                             hbv
   <chr>
                   <chr> <model>
 1 female
                   NSW
                           <FDM>
 2 female
                   0LD
                           <FDM>
 3 female
                   SA
                           <FDM>
 4 female
                   TAS
                           <FDM>
 5 female
                   VIC
                           <FDM>
 6 female
                   WA
                           <FDM>
 7 geometric_mean NSW
                           <FDM>
 8 geometric_mean QLD
                           <FDM>
 9 geometric_mean SA
                           <FDM>
```

```
fc ← fit ▷ forecast(h = 20) ▷ undo_pr(.smooth)
fc ▷ filter(State == "VIC") ▷ autoplot() + scale_y_log10()
```



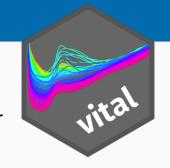
Other functionality

- Import data from Human Mortality Database and Human Fertility Database
- Convert demogdata, tsibble & data.frame objects to vital.
- Compute net migration from population, births and deaths.
- Compute total fertility rates from age-specific fertility rates.
- Various smoothing functions



Future plans

- Remaining tools from the demography package
- Stochastic population forecasting (as per Hyndman-Booth, IJF, 2008)
- All models handled by StMoMo package
- All methods from MortalityLaws package
- Suggestions from users



Future plans

- Remaining tools from the demography package
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- Suggestions from users
- **Slides**: robjhyndman.com/user2024
- **Package**: pkg.robjhyndman.com/vital/

