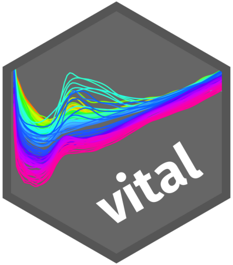

vital: Tidy data analysis for demography

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9 July 2024

Demographic data structures in R packages

Package	Data class
demography	demogdata
StMoMo	StMoMoData (created by converting a demogdata object)
StanMoMo	Lists of matrices
lifecontingencies	data.frame
BayesMortalityPlus	tibble (that needs to be converted to a matrix for fitting)
MortalityLaws	individual vectors
HMDHFDplus	data.frame

tibble objects

Australian Deaths 1901–2020

```
# A tibble: 145,440 x 7
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA      0.129       2511    325
 2  1901     0 male   WA      0.158       2634    416
 3  1901     1 female WA      0.0275      2219     61
 4  1901     1 male   WA      0.0391      2175     85
 5  1901     2 female WA      0.00688     2180     15
 6  1901     2 male   WA      0.0131      2208     29
 7  1901     3 female WA      0.00584     1884     11
 8  1901     3 male   WA      0.00503     1988     10
 9  1901     4 female WA      0.00290     1722      5
10  1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

tsibble objects

Australian Deaths 1901–2020

```
# A tsibble: 145,440 x 7 [1Y]
# Key:       Age, Sex, State [1,212]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA      0.129       2511    325
 2  1901     0 male   WA      0.158       2634    416
 3  1901     1 female WA      0.0275      2219     61
 4  1901     1 male   WA      0.0391      2175     85
 5  1901     2 female WA      0.00688     2180     15
 6  1901     2 male   WA      0.0131      2208     29
 7  1901     3 female WA      0.00584     1884     11
 8  1901     3 male   WA      0.00503     1988     10
 9  1901     4 female WA      0.00290     1722      5
10  1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

Variables

Index:

- Year

Keys:

- Age
- Sex
- State

Every row must have a unique combination of Index and Keys

vital objects

Australian Deaths 1901–2020

```
aus
```

```
# A vital: 145,440 x 7 [1Y]
# Key:      Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA      0.129       2511    325
 2  1901     0 male   WA      0.158       2634    416
 3  1901     1 female WA      0.0275      2219     61
 4  1901     1 male   WA      0.0391      2175     85
 5  1901     2 female WA      0.00688     2180     15
 6  1901     2 male   WA      0.0131      2208     29
 7  1901     3 female WA      0.00584     1884     11
 8  1901     3 male   WA      0.00503     1988     10
 9  1901     4 female WA      0.00290     1722      5
10  1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

Variables

Index:

- Year

Keys:

- Age
- Sex
- State

Every row must have a unique combination of Index and Keys

Variables denoting age, sex, deaths, births and population can also be specified as attributes.

vital objects

```
index_var(aus)
```

```
[1] ”Year”
```

```
key_vars(aus)
```

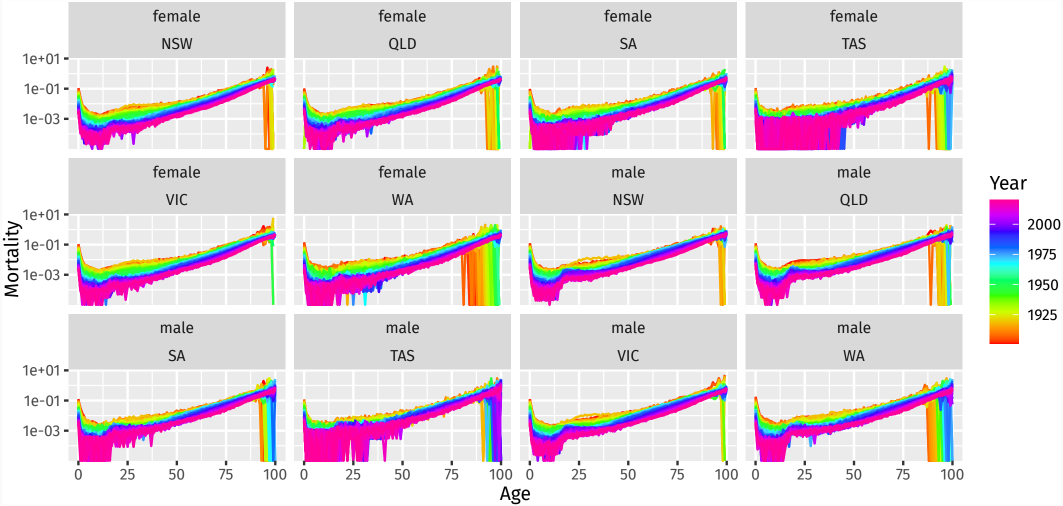
```
[1] ”Age”   ”Sex”   ”State”
```

```
vital_vars(aus)
```

```
       age        sex     deaths population
     ”Age”      ”Sex”   ”Deaths” ”Exposure”
```

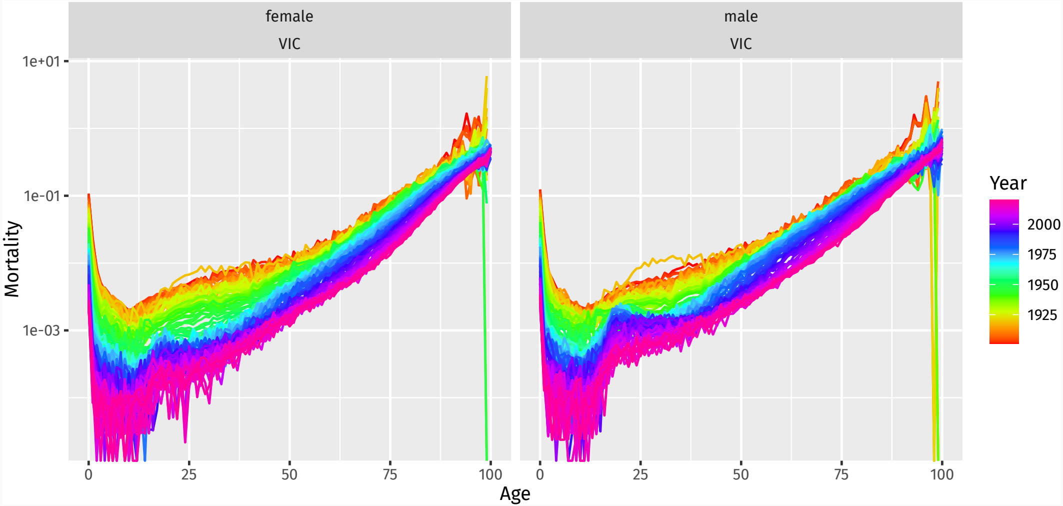
Rainbow plots

```
aus ▷ autoplot(Mortality) + scale_y_log10()
```



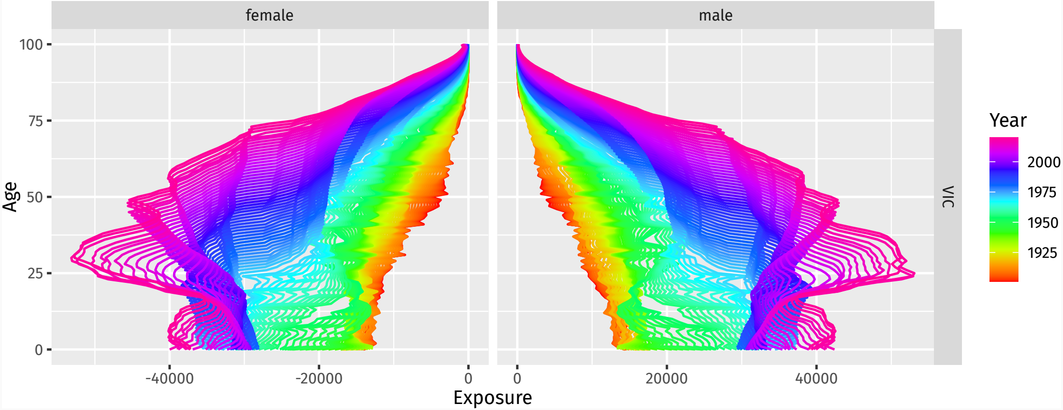
Rainbow plots

```
aus ▷ filter(State == "VIC") ▷ autoplot(Mortality) + scale_y_log10()
```



Rainbow plots

```
aus ▷ filter(State == ”VIC”) ▷
  mutate(Exposure = if_else(Sex == ”female”, -Exposure, Exposure)) ▷
  autoplot(Exposure) +
  facet_grid(State ~ Sex, scales = ”free_x”) + coord_flip()
```



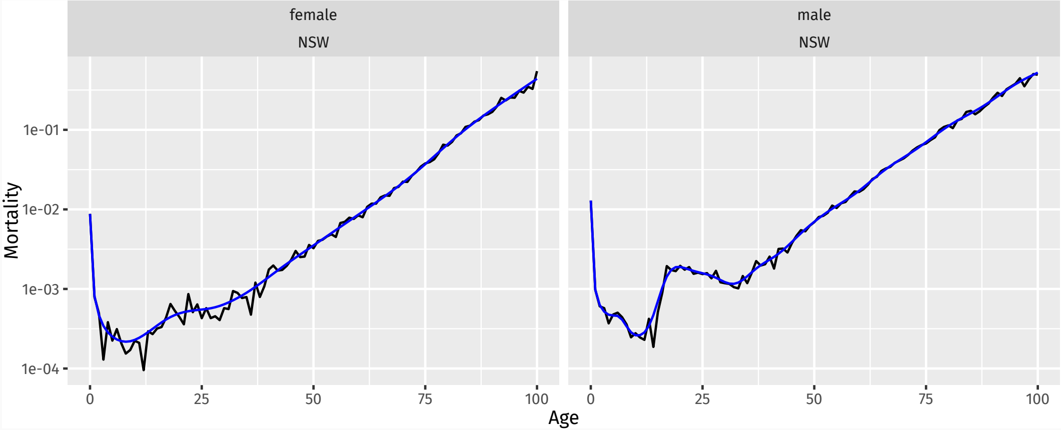
Smoothing

```
sm_aus <- aus |> smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
# Key:     Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths   .smooth .smooth_se
   <int> <dbl> <chr>  <chr>     <dbl>    <dbl>  <dbl> <dbl[1d]>  <dbl[1d]>
 1  1901     0 female NSW     0.107      17143   1833   0.107     0.00295
 2  1901     1 female NSW     0.0247     15071    373   0.0237    0.00141
 3  1901     2 female NSW     0.00686    15461    106   0.00804   0.000670
 4  1901     3 female NSW     0.00441    15629     69   0.00461   0.000405
 5  1901     4 female NSW     0.00374    15762     59   0.00341   0.000305
 6  1901     5 female NSW     0.00274    16030     44   0.00275   0.000251
 7  1901     6 female NSW     0.00252    16289     41   0.00230   0.000215
 8  1901     7 female NSW     0.00216    16639     36   0.00197   0.000189
 9  1901     8 female NSW     0.00169    16554     28   0.00175   0.000173
10  1901     9 female NSW     0.00109    16468     18   0.00162   0.000163
# i 145,430 more rows
```

Smoothing

```
sm_aus ← aus ▷ smooth_mortality(Mortality)
sm_aus ▷ filter(State == "NSW", Year == 1980) ▷ autoplot(Mortality) +
  geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



Life tables

```r
life_table(aus)
```

```
# A vital: 145,440 x 14 [1Y]
# Key:     Age x (Sex, State) [101 x 12]
    Year   Age Sex   State      mx      qx    lx      dx    Lx    Tx    ex    rx
   <int> <int> <chr> <chr>   <dbl>   <dbl> <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 fema~ NSW   0.107   0.100   1     1.00e-1 0.935  56.2  56.2 0.935
 2  1901     1 fema~ NSW   0.0247  0.0244  0.900 2.20e-2 0.889  55.3  61.5 0.951
 3  1901     2 fema~ NSW   0.00686 0.00683 0.878 6.00e-3 0.875  54.4  62.0 0.984
 4  1901     3 fema~ NSW   0.00441 0.00441 0.872 3.84e-3 0.870  53.5  61.4 0.994
 5  1901     4 fema~ NSW   0.00374 0.00374 0.868 3.24e-3 0.867  52.7  60.7 0.996
 6  1901     5 fema~ NSW   0.00274 0.00274 0.865 2.37e-3 0.864  51.8  59.9 0.997
 7  1901     6 fema~ NSW   0.00252 0.00251 0.863 2.17e-3 0.861  50.9  59.1 0.997
 8  1901     7 fema~ NSW   0.00216 0.00216 0.860 1.86e-3 0.859  50.1  58.2 0.998
 9  1901     8 fema~ NSW   0.00169 0.00169 0.859 1.45e-3 0.858  49.2  57.3 0.998
10  1901     9 fema~ NSW   0.00109 0.00109 0.857 9.36e-4 0.857  48.4  56.4 0.999
# i 145,430 more rows
# i 2 more variables: nx <dbl>, ax <dbl>
```

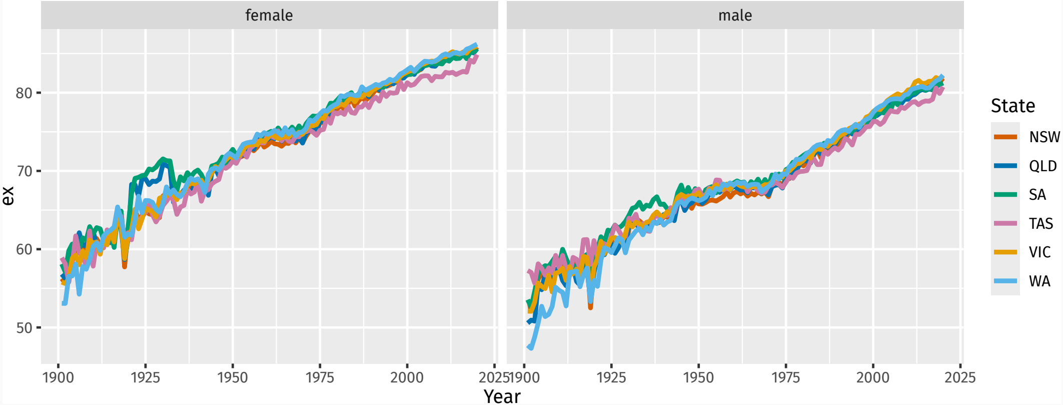
Life expectancy

```
life_expectancy(aus)
```

```
# A vital: 1,440 x 8 [1Y]
# Key:     Age x (Sex, State) [1 x 12]
    Year   Age Sex    State    ex    rx    nx    ax
   <int> <int> <chr>  <chr> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 female NSW    56.2 0.935     1 0.352
 2  1901     0 female QLD    56.8 0.937     1 0.338
 3  1901     0 female SA     58.1 0.939     1 0.324
 4  1901     0 female TAS    58.9 0.946     1 0.275
 5  1901     0 female VIC    55.8 0.937     1 0.334
 6  1901     0 female WA     53.1 0.922     1 0.35
 7  1901     0 male   NSW    52.6 0.925     1 0.33
 8  1901     0 male   QLD    50.6 0.924     1 0.33
 9  1901     0 male   SA     53.5 0.922     1 0.33
10  1901     0 male   TAS    57.3 0.930     1 0.33
# i 1,430 more rows
```

Life expectancy

```r
life_expectancy(aus) |>
  ggplot(aes(x = Year, y = ex, colour = State)) +
  geom_line(linewidth = 1) +
  facet_grid(. ~ Sex)
```



Mortality models

$m_{x,t}$ = mortality rate at age x in year t.

Naive: $m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$

Lee-Carter: $\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$

$\varepsilon_{x,t}$ = noise term with variance σ_x^2.

Mortality models

$m_{x,t}$ = mortality rate at age x in year t.

Naive: $$m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$$

Lee-Carter: $$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

$\varepsilon_{x,t}$ = noise term with variance σ_x^2.

Lee-Carter variations

- Lee & Carter (*JASA* 1992)
- Lee & Miller (*Demography* 2001)
- Booth, Maindonald & Smith (*Population Studies* 2002)

Mortality models

$m_{x,t}$ = mortality rate at age x in year t.

Naive: $m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$

Lee-Carter: $\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$

$\varepsilon_{x,t}$ = noise term with variance σ_x^2.

```
fit <- aus |>
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
  )
```

Mortality models

$m_{x,t}$ = mortality rate at age x in year t.

Naive: $$m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$$

Lee-Carter: $$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

$\varepsilon_{x,t}$ = noise term with variance σ_x^2.

```
fit ← aus ▷
  model(
    naive = FNAIVE(Mortality),
    lc = LC(log(Mortality))
  )
```

```
fit
```

```
# A mable: 12 x 4
# Key:     Sex, State [12]
   Sex    State     naive      lc
   <chr>  <chr>   <model> <model>
 1 female NSW    <FNAIVE>    <LC>
 2 female QLD    <FNAIVE>    <LC>
 3 female SA     <FNAIVE>    <LC>
 4 female TAS    <FNAIVE>    <LC>
 5 female VIC    <FNAIVE>    <LC>
 6 female WA     <FNAIVE>    <LC>
 7 male   NSW    <FNAIVE>    <LC>
 8 male   QLD    <FNAIVE>    <LC>
 9 male   SA     <FNAIVE>    <LC>
10 male   TAS    <FNAIVE>    <LC>
11 male   VIC    <FNAIVE>    <LC>
12 male   WA     <FNAIVE>    <LC>
```

Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit ▷
  filter(Sex == "female",
         State == "NSW") ▷
  select(lc) ▷
  report()
```

```
Series: Mortality
Model: LC
Transformation: log(Mortality)

Options:
  Adjust method: dt
  Jump choice: fit
```

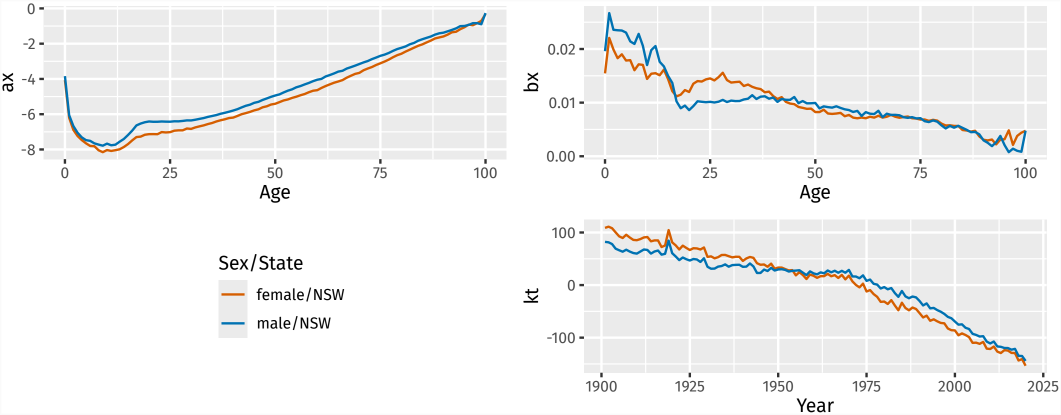
```
Age functions
# A tibble: 101 × 3
    Age    ax     bx
  <int> <dbl>  <dbl>
1     0 -4.07 0.0155
2     1 -6.20 0.0221
3     2 -6.89 0.0199
# i 98 more rows

Time coefficients
# A tsibble: 120 x 2 [1Y]
   Year    kt
  <int> <dbl>
1  1901 109. 
2  1902 111. 
3  1903 108. 
# i 117 more rows
```

Lee-Carter models

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

```
fit |>
  filter(State == ”NSW”) |>
  select(lc) |>
  autoplot()
```



Lee-Carter models

$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$

```
fit ▷ select(lc) ▷ age_components()
```

```
# A tibble: 1,212 x 5
   Sex    State   Age    ax     bx
   <chr>  <chr> <int> <dbl>  <dbl>
 1 female NSW       0 -4.07 0.0155
 2 female NSW       1 -6.20 0.0221
 3 female NSW       2 -6.89 0.0199
 4 female NSW       3 -7.24 0.0183
 5 female NSW       4 -7.47 0.0190
 6 female NSW       5 -7.65 0.0178
 7 female NSW       6 -7.80 0.0179
 8 female NSW       7 -7.81 0.0160
 9 female NSW       8 -8.05 0.0171
10 female NSW       9 -8.15 0.0170
# i 1,202 more rows
```

```
fit ▷ select(lc) ▷ time_components()
```

```
# A tsibble: 1,440 x 4 [1Y]
# Key:       Sex, State [12]
   Sex    State  Year    kt
   <chr>  <chr> <int> <dbl>
 1 female NSW    1901 109.
 2 female NSW    1902 111.
 3 female NSW    1903 108.
 4 female NSW    1904 100.
 5 female NSW    1905  92.7
 6 female NSW    1906  89.5
 7 female NSW    1907  95.7
 8 female NSW    1908  90.5
 9 female NSW    1909  85.9
10 female NSW    1910  85.4
# i 1,430 more rows
```

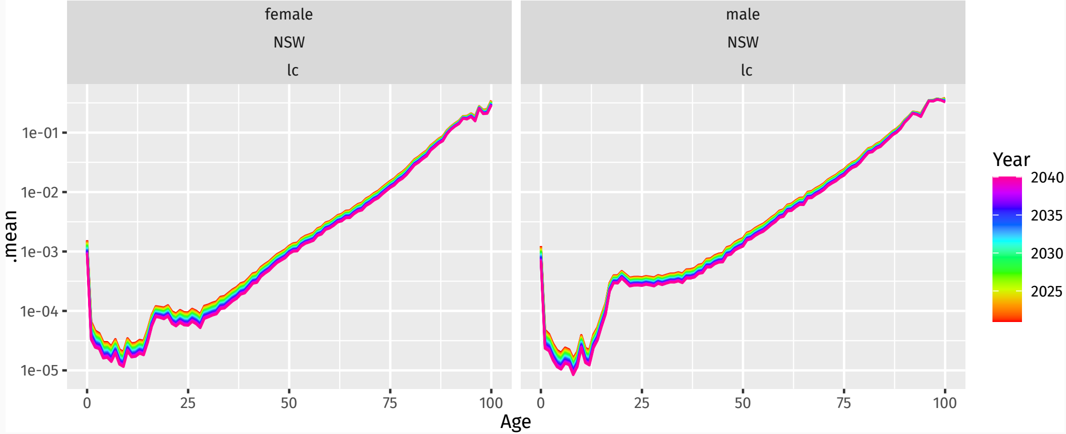
Forecasts

```r
fc <- fit |> forecast(h = 20)
fc
```

```
# A vital fable: 48,480 x 7 [1Y]
# Key:           Age x (Sex, State, .model) [101 x 24]
   Sex    State .model  Year   Age          Mortality   .mean
   <chr>  <chr> <chr>  <dbl> <int>             <dist>   <dbl>
 1 female NSW   naive   2021     0 N(0.0027, 1.8e-05) 0.00270
 2 female NSW   naive   2022     0 N(0.0027, 3.6e-05) 0.00270
 3 female NSW   naive   2023     0 N(0.0027, 5.4e-05) 0.00270
 4 female NSW   naive   2024     0 N(0.0027, 7.2e-05) 0.00270
 5 female NSW   naive   2025     0   N(0.0027, 9e-05) 0.00270
 6 female NSW   naive   2026     0 N(0.0027, 0.00011) 0.00270
 7 female NSW   naive   2027     0 N(0.0027, 0.00013) 0.00270
 8 female NSW   naive   2028     0 N(0.0027, 0.00014) 0.00270
 9 female NSW   naive   2029     0 N(0.0027, 0.00016) 0.00270
10 female NSW   naive   2030     0 N(0.0027, 0.00018) 0.00270
# i 48,470 more rows
```

NSW forecasts using Lee-Carter method

```
fc ▷ filter(State == ”NSW”, .model == ”lc”) ▷
  autoplot() + scale_y_log10()
```



Functional data models

Hyndman & Ullah (*CSDA*, 2007)

Let $m_{x,t}$ be the mortality rate at age x in year t.

$$\log(m_{t,x}) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

- $s_t(x)$ = smoothed version of $y_t(x)$
- $\mu(x)$ = mean $s_t(x)$ across years.
- $\phi_j(x)$ and β_{tj} estimated using principal component analysis.
- $\beta_{1j}, \dots, \beta_{Tj}$ modelled with ARIMA or ARFIMA processes.

Functional data models

```
fit ← aus ▷
  smooth_mortality(Mortality) ▷
  model(hu = FDM(log(.smooth)))
fit
```

```
# A mable: 12 x 3
# Key:     Sex, State [12]
   Sex    State     hu
   <chr>  <chr> <model>
 1 female NSW     <FDM>
 2 female QLD     <FDM>
 3 female SA      <FDM>
 4 female TAS     <FDM>
 5 female VIC     <FDM>
 6 female WA      <FDM>
 7 male   NSW     <FDM>
 8 male   QLD     <FDM>
 9 male   SA      <FDM>
10 male   TAS     <FDM>
```

Functional data models

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

```r
fit |>
  filter(Sex == "female", State == "NSW") |>
  report()
```

```
Series: .smooth
Model: FDM
Transformation: log(.smooth)

Basis functions
# A tibble: 101 x 8
    Age  mean  phi1    phi2    phi3    phi4      phi5    phi6
  <dbl> <dbl> <dbl>   <dbl>   <dbl>   <dbl>     <dbl>   <dbl>
1     0 -4.07 0.147  0.0625 -0.0270  0.0986  0.0112   -0.0624
2     1 -6.16 0.200 -0.0609 -0.194   0.116   0.0383   -0.238
3     2 -6.82 0.182 -0.0483 -0.157   0.0924  0.0443   -0.264
4     3 -7.17 0.170 -0.0368 -0.130   0.0362  0.000338 -0.321
5     4 -7.40 0.164 -0.0165 -0.114  -0.0154 -0.0303   -0.374
# i 96 more rows
```

Functional data models

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

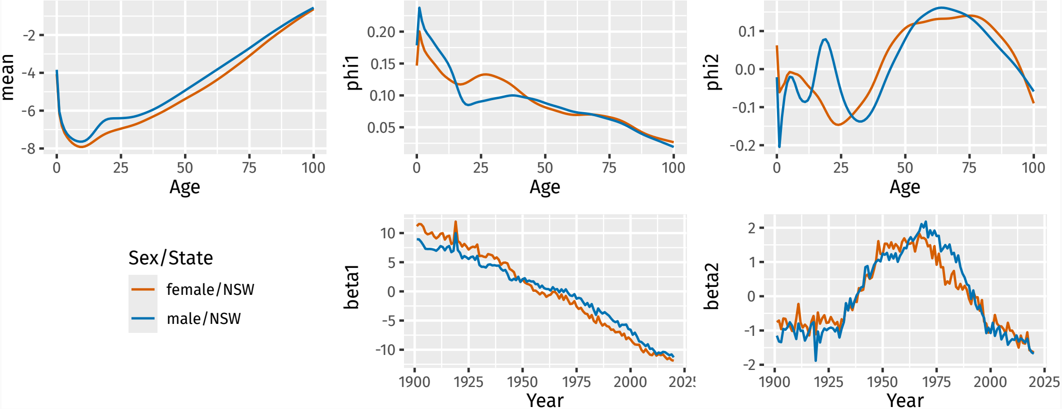
```
Coefficients
# A tsibble: 120 x 8 [1Y]
   Year  mean beta1  beta2   beta3     beta4   beta5   beta6
  <int> <dbl> <dbl>  <dbl>   <dbl>     <dbl>   <dbl>   <dbl>
1  1901     1  11.1 -0.522 -0.0553  0.207    0.358    0.0305
2  1902     1  11.8 -0.649  0.399   0.856    0.0319   0.422
3  1903     1  11.5 -0.930 -0.485   0.398    0.399   -0.376
4  1904     1  11.1 -0.827 -0.214  -0.000305 0.00125 -0.0783
5  1905     1  10.2 -0.563 -0.105   0.324    0.122    0.0478
# i 115 more rows
# i Use ‘print(n = ...)‘ to see more rows

Time series models
   beta1 : ARIMA(0,1,1) w/ drift
   beta2 : ARIMA(0,2,2)
   beta3 : ARIMA(1,0,1)
   beta4 : ARIMA(0,0,2)
   beta5 : ARIMA(0,0,0)
   beta6 : ARIMA(2,0,2)
```

Functional data models

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

```
fit ▷
  filter(State == "NSW") ▷
  autoplot()
```



Functional data models

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

```
fit |> age_components()
```

```
# A tibble: 1,212 x 10
   Sex    State   Age  mean  phi1     phi2    phi3    phi4      phi5    phi6
   <chr>  <chr> <dbl> <dbl> <dbl>    <dbl>   <dbl>   <dbl>     <dbl>   <dbl>
 1 female NSW       0 -4.07 0.147  0.0625  -0.0270  0.0986  0.0112   -0.0624
 2 female NSW       1 -6.16 0.200 -0.0609  -0.194   0.116   0.0383   -0.238
 3 female NSW       2 -6.82 0.182 -0.0483  -0.157   0.0924  0.0443   -0.264
 4 female NSW       3 -7.17 0.170 -0.0368  -0.130   0.0362  0.000338 -0.321
 5 female NSW       4 -7.40 0.164 -0.0165  -0.114  -0.0154 -0.0303   -0.374
 6 female NSW       5 -7.57 0.158 -0.00759 -0.121  -0.0564  0.0247   -0.315
 7 female NSW       6 -7.71 0.153 -0.00942 -0.133  -0.0976  0.112    -0.197
 8 female NSW       7 -7.81 0.149 -0.0121  -0.143  -0.143   0.175    -0.0863
 9 female NSW       8 -7.88 0.143 -0.0141  -0.148  -0.181   0.211     0.0131
10 female NSW       9 -7.92 0.138 -0.0185  -0.142  -0.196   0.236     0.101
# i 1,202 more rows
```

Functional data models

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{tj}\phi_j(x) + e_t(x)$$

```r
fit |> time_components()
```

```
# A tsibble: 1,440 x 10 [1Y]
# Key:       Sex, State [12]
   Sex    State  Year  mean beta1  beta2   beta3   beta4    beta5  beta6
   <chr>  <chr> <int> <dbl> <dbl>  <dbl>   <dbl>   <dbl>    <dbl>  <dbl>
 1 female NSW    1901     1 11.2  -0.756 -0.0301  0.269  -0.155   0.409
 2 female NSW    1902     1 11.6  -0.708  0.0899  0.207   0.0282  0.507
 3 female NSW    1903     1 11.5  -0.962  0.169  -0.103   0.366   0.323
 4 female NSW    1904     1 11.1  -0.648  0.0985 -0.433   0.131   0.270
 5 female NSW    1905     1 10.1  -0.660  0.342  -0.0910  0.0862  0.612
 6 female NSW    1906     1  9.78 -0.865  0.496  -0.147  -0.101   0.306
 7 female NSW    1907     1  9.90 -0.861  0.0530  1.33    0.278   0.181
 8 female NSW    1908     1 10.1  -1.01   0.554  -0.0198 -0.00428 0.578
 9 female NSW    1909     1  9.42 -1.02   0.293  -0.365  -0.149   0.353
10 female NSW    1910     1  9.08 -0.650  0.172  -0.559  -0.253   0.0110
# i 1,430 more rows
```

Other functionality

- Import data from Human Mortality Database and Human Fertility Database
- Convert demogdata, tsibble & data.frame objects to vital.
- Compute net migration from population, births and deaths.
- Compute total fertility rates from age-specific fertility rates.
- Various smoothing functions
- Coherent functional data models

Future plans

- Remaining tools from the `demography` package
- Stochastic population forecasting (as per Hyndman & Booth, *IJF*, 2008)
- All models handled by `StMoMo` package
- All methods from `MortalityLaws` package
- Suggestions from users

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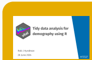

Tidy data analysis for demography using R

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26 June 2024

robjhyndman.com/user2024

pkg.robjhyndman.com/vital

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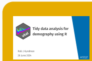

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