Tidy data analysis for demography using R

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26 June 2024

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Outline

Outline

Demographic data structures in R packages

Package	Data class
demography	demogdata
StMoMo	StMoMoData (created by converting a demogdata object)
StanMoMo	Lists of matrices
lifecontingencies	data.frame
BayesMortalityPlus	tibble (that needs to be converted to a matrix for fitting)
MortalityLaws	individual vectors
HMDHFDplus	data.frame

tibble objects

Australian Deaths 1901–2020

```
# A tibble: 145,440 x 7
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA      0.129       2511    325
 2  1901     0 male   WA      0.158       2634    416
 3  1901     1 female WA      0.0275      2219     61
 4  1901     1 male   WA      0.0391      2175     85
 5  1901     2 female WA      0.00688     2180     15
 6  1901     2 male   WA      0.0131      2208     29
 7  1901     3 female WA      0.00584     1884     11
 8  1901     3 male   WA      0.00503     1988     10
 9  1901     4 female WA      0.00290     1722      5
10  1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

tsibble objects

Australian Deaths 1901–2020

```
# A tsibble: 145,440 x 7 [1Y]
# Key:       Age, Sex, State [1,212]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA      0.129       2511    325
 2  1901     0 male   WA      0.158       2634    416
 3  1901     1 female WA      0.0275      2219     61
 4  1901     1 male   WA      0.0391      2175     85
 5  1901     2 female WA      0.00688     2180     15
 6  1901     2 male   WA      0.0131      2208     29
 7  1901     3 female WA      0.00584     1884     11
 8  1901     3 male   WA      0.00503     1988     10
 9  1901     4 female WA      0.00290     1722      5
10  1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

Variables

Index:

- `Year`

Keys:

- `Age`
- `Sex`
- `State`

Every row must have a unique combination of Index and Keys

vital objects

Australian Deaths 1901–2020

```
aus
```

```
# A vital: 145,440 x 7 [1Y]
# Key:       Age x (Sex, State) [101 x 12]
     Year   Age Sex    State Mortality Exposure Deaths
    <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1   1901     0 female WA      0.129       2511    325
 2   1901     0 male   WA      0.158       2634    416
 3   1901     1 female WA      0.0275      2219     61
 4   1901     1 male   WA      0.0391      2175     85
 5   1901     2 female WA      0.00688     2180     15
 6   1901     2 male   WA      0.0131      2208     29
 7   1901     3 female WA      0.00584     1884     11
 8   1901     3 male   WA      0.00503     1988     10
 9   1901     4 female WA      0.00290     1722      5
10   1901     4 male   WA      0.00287     1743      5
# i 145,430 more rows
```

Variables

Index:

- `Year`

Keys:

- `Age`
- `Sex`
- `State`

Every row must have a unique combination of Index and Keys

Variables denoting age, sex, deaths, births and population can also be specified.

vital objects

```
index_var(aus)
```

```
[1] "Year"
```

```
key_vars(aus)
```

```
[1] "Age"   "Sex"   "State"
```

```
vital_vars(aus)
```

```
       age         sex      deaths population 
     "Age"       "Sex"   "Deaths" "Exposure" 
```

Outline

Human Mortality Database

mortality.org

HMD

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Human Mortality Database

Reliability and Accuracy Matter

The Human Mortality Database (HMD) is the world's leading scientific data resource on mortality in developed countries. The HMD provides detailed high-quality harmonized mortality and population estimates to researchers, students, journalists, policy analysts, and others interested in human longevity. The HMD follows open data principles.

- Short-Term Mortality Fluctuations
- Cause-of-Death Data Series
- Subnational Mortality Databases
- Citing HMD

Data by country or area				
Australia	Denmark	Ireland	Norway	Switzerland
Austria	Estonia	Israel	Poland	Taiwan
Belarus	Finland	Italy	Portugal	U.K.
Belgium	France	Japan	Republic of Korea	U.S.A.

Human Fertility Database

humanfertility.org

HFD

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Human Fertility Database

The Human Fertility Database (HFD) is the leading scientific data resource on fertility in the developed countries. This open access database provides detailed and high-quality historical and recent data on period and cohort fertility by age of mother and birth order. The HFD is entirely based on official vital statistics and places a great emphasis on rigorous data checking and documentation. The HFD adopts uniform methodology to warrant data comparability across time and between countries. The database follows open data principles.

- Short-Term Fertility Fluctuations
- Human Fertility Collection
- Citing HFD
- What's new

For users who seek fast access to the most commonly used summary indicators of period and cohort fertility, we provide excel tables comprising the following indicators for all the HFD countries:

HFD summary indicators

Total fertility rate	Tempo-adjusted TFR	Mean age at birth	Mean age at first birth	Completed cohort fertility	Cohort parity

HMD imports

```r
norway <- read_hmd(
  country = "NOR",
  username = "Nora.Weigh@mymail.com",
  password = "FF!5xeEFa6"
)
norway_births <- read_hmd(
  country = "NOR",
  username = "Nora.Weigh@mymail.com",
  password = "FF!5xeEFa6",
  variables = "Births"
)
```

- Uses `HMDHFDplus` package to handle the downloads.
- Default variables: `Deaths`, `Exposures`, `Population`, `Mx`
- Only 1×1 data supported.
- `read_hmd_files()` and `read_hfd_files()` allow reading of downloaded files.

HMD imports

```r
norway_births
```

```
# A vital: 531 x 3 [1Y]
# Key:       Sex [3]
    Year Sex    Births
   <int> <chr>   <int>
 1  1846 Female  20156
 2  1846 Male    21372
 3  1846 Total   41528
 4  1847 Female  20199
 5  1847 Male    21411
 6  1847 Total   41610
 7  1848 Female  19686
 8  1848 Male    20868
 9  1848 Total   40554
10  1849 Female  21424
# i 521 more rows
```

Outline

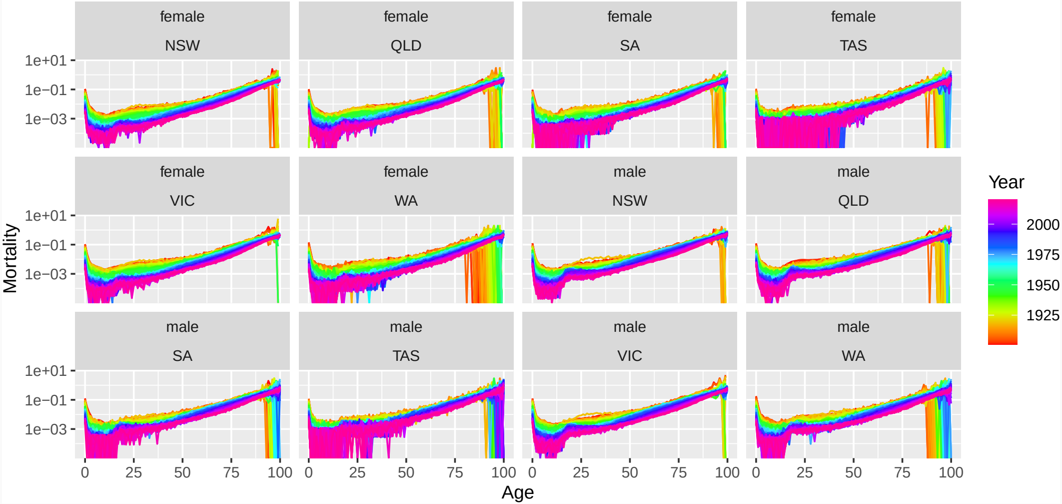
Recall: Australian mortality data

```
aus
```

```
# A vital: 145,440 x 7 [1Y]
# Key:      Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths
   <int> <int> <chr>  <chr>     <dbl>    <dbl>  <dbl>
 1  1901     0 female WA       0.129      2511    325
 2  1901     0 male   WA       0.158      2634    416
 3  1901     1 female WA       0.0275     2219     61
 4  1901     1 male   WA       0.0391     2175     85
 5  1901     2 female WA       0.00688    2180     15
 6  1901     2 male   WA       0.0131     2208     29
 7  1901     3 female WA       0.00584    1884     11
 8  1901     3 male   WA       0.00503    1988     10
 9  1901     4 female WA       0.00290    1722      5
10  1901     4 male   WA       0.00287    1743      5
# i 145,430 more rows
```

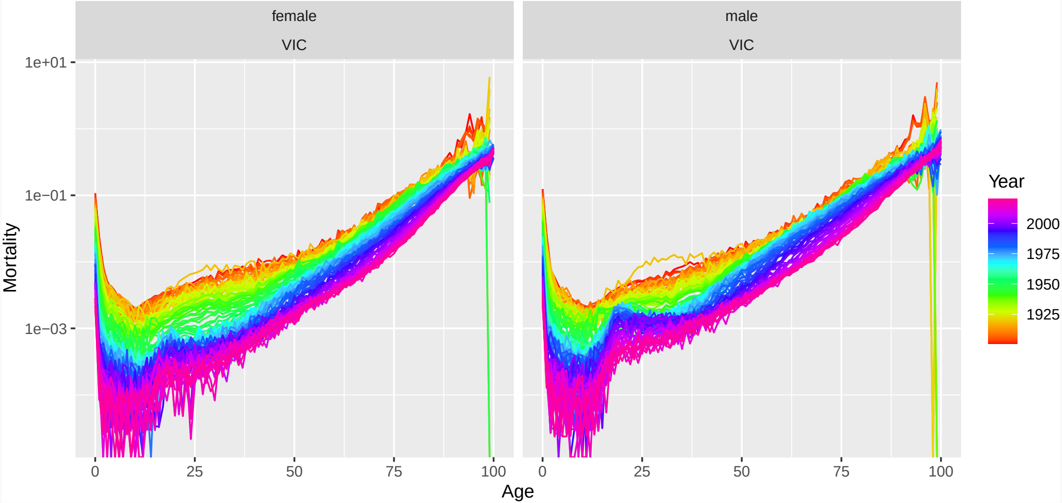
Rainbow plots

```r
aus |> autoplot(Mortality) + scale_y_log10()
```



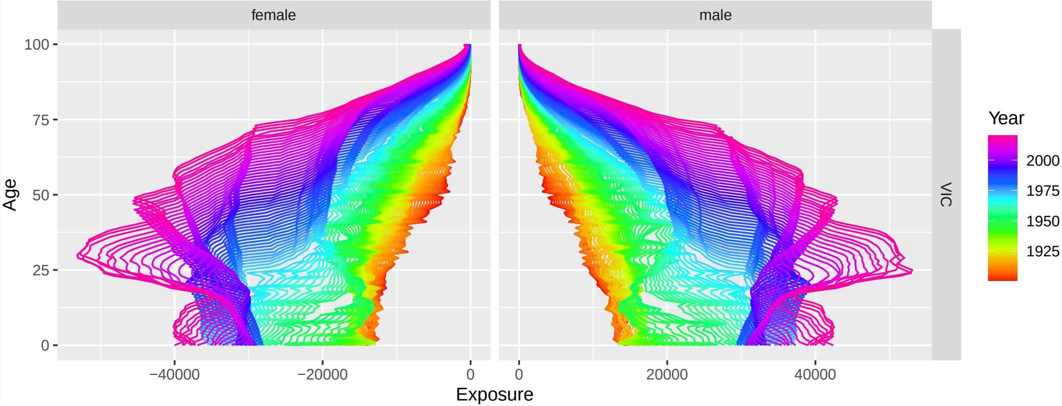
Rainbow plots

```
aus |> filter(State == "VIC") |> autoplot(Mortality) + scale_y_log10()
```



Rainbow plots

```
aus |> filter(State == "VIC") |>
  mutate(Exposure = if_else(Sex == "female", -Exposure, Exposure)) |>
  autoplot(Exposure) +
  facet_grid(State ~ Sex, scales = "free_x") + coord_flip()
```



Outline

Life tables

```r
life_table(aus)
```

```
# A vital: 145,440 x 14 [1Y]
# Key:      Age x (Sex, State) [101 x 12]
    Year   Age Sex   State      mx      qx    lx      dx    Lx    Tx    ex    rx
   <int> <int> <chr> <chr>   <dbl>   <dbl> <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 fema~ NSW   0.107   0.100   1     1.00e-1 0.935  56.2  56.2 0.935
 2  1901     1 fema~ NSW   0.0247  0.0244  0.900 2.20e-2 0.889  55.3  61.5 0.951
 3  1901     2 fema~ NSW   0.00686 0.00683 0.878 6.00e-3 0.875  54.4  62.0 0.984
 4  1901     3 fema~ NSW   0.00441 0.00441 0.872 3.84e-3 0.870  53.5  61.4 0.994
 5  1901     4 fema~ NSW   0.00374 0.00374 0.868 3.24e-3 0.867  52.7  60.7 0.996
 6  1901     5 fema~ NSW   0.00274 0.00274 0.865 2.37e-3 0.864  51.8  59.9 0.997
 7  1901     6 fema~ NSW   0.00252 0.00251 0.863 2.17e-3 0.861  50.9  59.1 0.997
 8  1901     7 fema~ NSW   0.00216 0.00216 0.860 1.86e-3 0.859  50.1  58.2 0.998
 9  1901     8 fema~ NSW   0.00169 0.00169 0.859 1.45e-3 0.858  49.2  57.3 0.998
10  1901     9 fema~ NSW   0.00109 0.00109 0.857 9.36e-4 0.857  48.4  56.4 0.999
# i 145,430 more rows
```

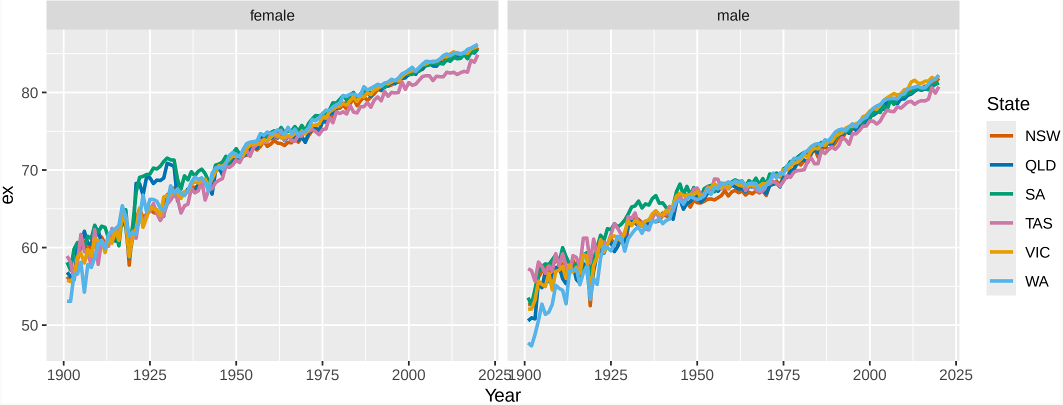
Life expectancy

```r
life_expectancy(aus)
```

```
# A vital: 1,440 x 8 [1Y]
# Key:     Age x (Sex, State) [1 x 12]
    Year   Age Sex    State    ex    rx    nx    ax
   <int> <int> <chr>  <chr> <dbl> <dbl> <dbl> <dbl>
 1  1901     0 female NSW    56.2 0.935     1 0.352
 2  1901     0 female QLD    56.8 0.937     1 0.338
 3  1901     0 female SA     58.1 0.939     1 0.324
 4  1901     0 female TAS    58.9 0.946     1 0.275
 5  1901     0 female VIC    55.8 0.937     1 0.334
 6  1901     0 female WA     53.1 0.922     1 0.35
 7  1901     0 male   NSW    52.6 0.925     1 0.33
 8  1901     0 male   QLD    50.6 0.924     1 0.33
 9  1901     0 male   SA     53.5 0.922     1 0.33
10  1901     0 male   TAS    57.3 0.930     1 0.33
# i 1,430 more rows
```

Life expectancy

```r
life_expectancy(aus) |>
  ggplot(aes(x = Year, y = ex, colour = State)) +
  geom_line(linewidth = 1) +
  facet_grid(. ~ Sex)
```



Life table calculations

- All available years and ages are included in the tables.
- $q_x = m_x/(1 + [(1 - a_x)m_x])$ as per Chiang (1984).
- The code has only been tested for data based on single-year age groups.
- Same code base as for the demography package.
- Life expectancy with `life_expectancy()` computes e_x with $x = 0$ by default, but other values are possible.

Outline

Mortality models

Let $m_{x,t}$ be the mortality rate at age x in year t.

Naive model:	$m_{x,t} = m_{x,t-1} + \varepsilon_{x,t}$
Mean model:	$m_{x,t} = \mu_x + \varepsilon_{x,t}$
Lee-Carter model:	$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$

where $\varepsilon_{x,t} \sim$ is noise term with variance σ_x^2.

Mortality models

```
fit <- aus |>
  model(
    naive = FNAIVE(Mortality),
    mean = FMEAN(Mortality),
    lc = LC(log(Mortality))
  )
fit
```

```
# A mable: 12 x 5
# Key:     Sex, State [12]
   Sex    State    naive    mean       lc
   <chr>  <chr>  <model> <model> <model>
 1 female NSW   <FNAIVE> <FMEAN>     <LC>
 2 female QLD   <FNAIVE> <FMEAN>     <LC>
 3 female SA    <FNAIVE> <FMEAN>     <LC>
 4 female TAS   <FNAIVE> <FMEAN>     <LC>
 5 female VIC   <FNAIVE> <FMEAN>     <LC>
```

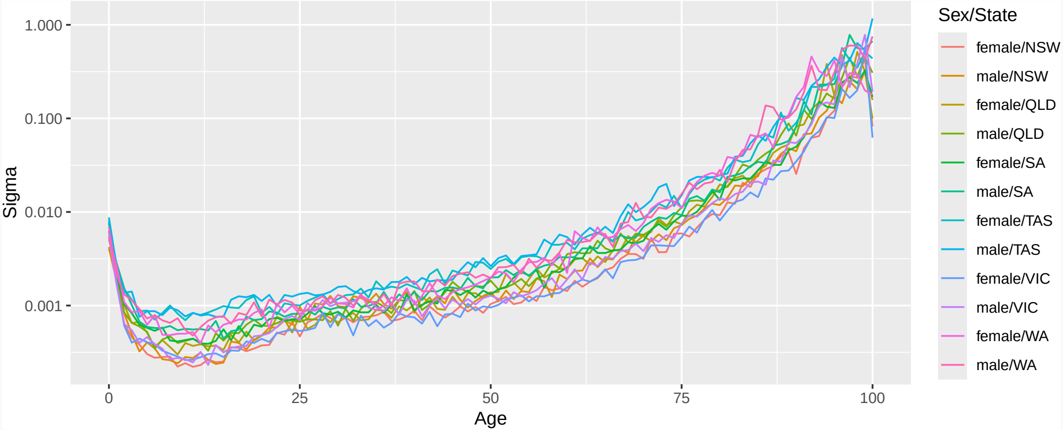
Naive model

```
fit |>
  filter(Sex == "female", State == "NSW") |>
  select(naive) |>
  report()
```

```
Series: Mortality
Model: FNAIVE

# A tibble: 101 x 2
      Age    sigma
    <int>    <dbl>
 1      0 0.00424
 2      1 0.00180
 3      2 0.000642
 4      3 0.000455
 5      4 0.000382
 6      5 0.000303
```

Naive models

```r
fit |>
  select(naive) |>
  autoplot() + scale_y_log10()
```



Lee-Carter models

Let $m_{x,t}$ be the mortality rate at age x in year t.

$$\log(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t}$$

- a_x is the mean log mortality rate at age x.
- k_t tracks mortality changes over time.
- b_x allows changes in mortality rates to vary by age.
- $\varepsilon_{x,t}$ is the error term.
- Estimation of k_t and b_x via principal component analysis.
- k_t forecast using a random walk with drift = ARIMA(0,1,0)

Lee-Carter models

```
fit |>
  filter(Sex == "female", State == "NSW") |>
  select(lc) |>
  report()
```

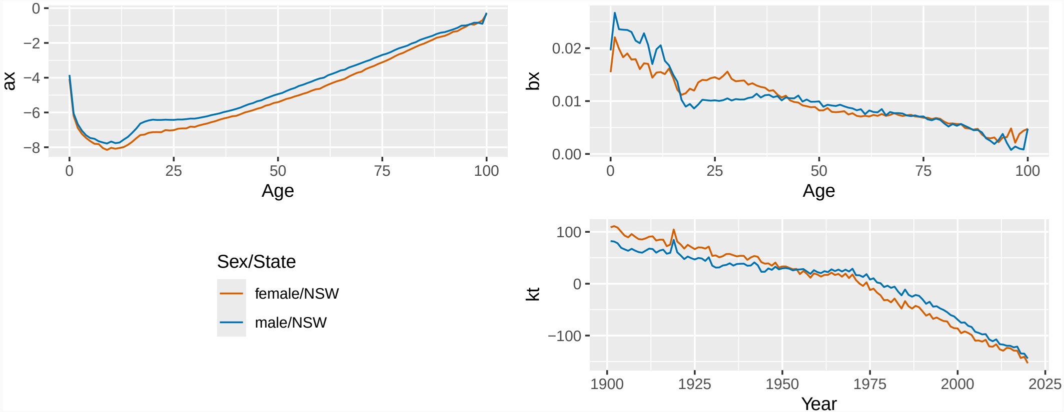
```
Series: Mortality
Model: LC
Transformation: log(Mortality)

Options:
  Adjust method: dt
  Jump choice: fit

Age functions
# A tibble: 101 x 3
    Age     ax      bx
  <int> <dbl>   <dbl>
1     0 -4.07 0.0155
2     1 -6.20 0.0221
```

Lee-Carter models

```
Age functions
# A tibble: 101 × 3
    Age    ax      bx
  <int> <dbl>   <dbl>
1     0 -4.07 0.0155
2     1 -6.20 0.0221
3     2 -6.89 0.0199
# i 98 more rows

Time coefficients
# A tsibble: 120 x 2 [1Y]
   Year    kt
  <int> <dbl>
1  1901 109.
2  1902 111.
3  1903 108.
# i 117 more rows

Time series model: RW w/ drift

Variance explained: 86.61%
```

Lee-Carter models

```
fit |>
  filter(State == "NSW") |>
  select(lc) |>
  autoplot()
```



Lee-Carter models

```
fit |> select(lc) |> age_components()
```

```
# A tibble: 1,212 x 5
   Sex    State   Age    ax     bx
   <chr>  <chr> <int> <dbl>  <dbl>
 1 female NSW       0 -4.07 0.0155
 2 female NSW       1 -6.20 0.0221
 3 female NSW       2 -6.89 0.0199
 4 female NSW       3 -7.24 0.0183
 5 female NSW       4 -7.47 0.0190
 6 female NSW       5 -7.65 0.0178
 7 female NSW       6 -7.80 0.0179
 8 female NSW       7 -7.81 0.0160
 9 female NSW       8 -8.05 0.0171
10 female NSW       9 -8.15 0.0170
# i 1,202 more rows
```

Lee-Carter models

```
fit |> select(lc) |> time_components()
```

```
# A tsibble: 1,440 x 4 [1Y]
# Key:       Sex, State [12]
   Sex    State  Year    kt
   <chr>  <chr> <int> <dbl>
 1 female NSW    1901 109.
 2 female NSW    1902 111.
 3 female NSW    1903 108.
 4 female NSW    1904 100.
 5 female NSW    1905  92.7
 6 female NSW    1906  89.5
 7 female NSW    1907  95.7
 8 female NSW    1908  90.5
 9 female NSW    1909  85.9
10 female NSW    1910  85.4
# i 1,430 more rows
```

Lee-Carter forecasts

```
fc <- fit |> forecast(h = 20)
fc
```

```
# A vital fable: 72,720 x 7 [1Y]
# Key:           Age x (Sex, State, .model) [101 x 36]
   Sex    State .model  Year   Age          Mortality   .mean
   <chr>  <chr> <chr>  <dbl> <int>             <dist>   <dbl>
 1 female NSW   naive   2021     0 N(0.0027, 1.8e-05) 0.00270
 2 female NSW   naive   2022     0 N(0.0027, 3.6e-05) 0.00270
 3 female NSW   naive   2023     0 N(0.0027, 5.4e-05) 0.00270
 4 female NSW   naive   2024     0 N(0.0027, 7.2e-05) 0.00270
 5 female NSW   naive   2025     0   N(0.0027, 9e-05) 0.00270
 6 female NSW   naive   2026     0 N(0.0027, 0.00011) 0.00270
 7 female NSW   naive   2027     0 N(0.0027, 0.00013) 0.00270
 8 female NSW   naive   2028     0 N(0.0027, 0.00014) 0.00270
 9 female NSW   naive   2029     0 N(0.0027, 0.00016) 0.00270
10 female NSW   naive   2030     0 N(0.0027, 0.00018) 0.00270
```

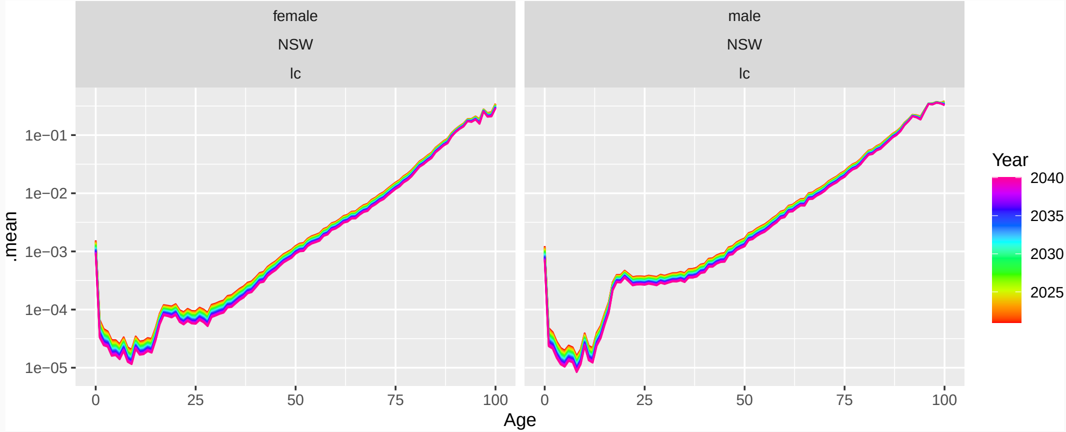
Lee-Carter forecasts

```
fc |> filter(.model == "lc")
```

```
# A vital fable: 24,240 x 7 [1Y]
# Key:           Age x (Sex, State, .model) [101 x 12]
   Sex    State .model  Year   Age         Mortality    .mean
   <chr>  <chr> <chr>  <dbl> <int>            <dist>    <dbl>
 1 female NSW   lc      2021     0 t(N(-6.5, 0.011)) 0.00155
 2 female NSW   lc      2022     0 t(N(-6.5, 0.022)) 0.00151
 3 female NSW   lc      2023     0 t(N(-6.5, 0.034)) 0.00146
 4 female NSW   lc      2024     0 t(N(-6.6, 0.046)) 0.00142
 5 female NSW   lc      2025     0 t(N(-6.6, 0.058)) 0.00138
 6 female NSW   lc      2026     0  t(N(-6.6, 0.07)) 0.00135
 7 female NSW   lc      2027     0 t(N(-6.7, 0.082)) 0.00131
 8 female NSW   lc      2028     0 t(N(-6.7, 0.094)) 0.00127
 9 female NSW   lc      2029     0  t(N(-6.7, 0.11)) 0.00124
10 female NSW   lc      2030     0  t(N(-6.8, 0.12)) 0.00120
# i 24,230 more rows
```

Lee-Carter forecasts

```r
fc |> filter(State == "NSW", .model == "lc") |>
  autoplot() + scale_y_log10()
```



Functional data models

Let $m_{x,t}$ be the mortality rate at age x in year t.

$$\log(m_{t,x}) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{t,j}\phi_j(x) + e_t(x)$$

- $s_t(x)$ = smoothed version of $y_t(x)$
- $\mu(x)$ = mean $s_t(x)$ across years.
- $\phi_j(x)$ and $\beta_{t,j}$ estimated using principal component analysis.
- $\beta_{1,j}, \dots, \beta_{T,j}$ modelled with ARIMA or ARFIMA processes.

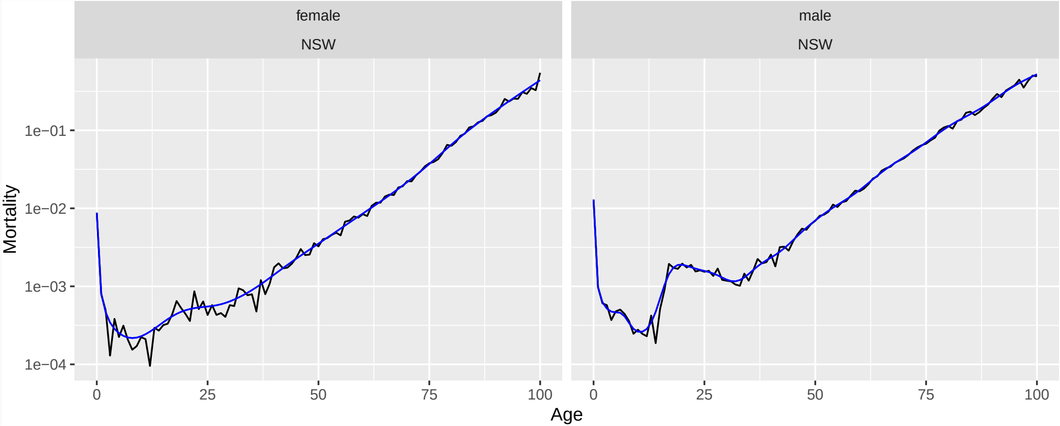
Functional data models

```
sm_aus <- aus |> smooth_mortality(Mortality)
sm_aus
```

```
# A vital: 145,440 x 9 [1Y]
# Key:      Age x (Sex, State) [101 x 12]
    Year   Age Sex    State Mortality Exposure Deaths   .smooth .smooth_se
   <int> <dbl> <chr>  <chr>     <dbl>    <dbl>  <dbl> <dbl[1d]>  <dbl[1d]>
 1  1901     0 female NSW     0.107      17143   1833   0.107     0.00295
 2  1901     1 female NSW     0.0247     15071    373   0.0237    0.00141
 3  1901     2 female NSW     0.00686    15461    106   0.00804   0.000670
 4  1901     3 female NSW     0.00441    15629     69   0.00461   0.000405
 5  1901     4 female NSW     0.00374    15762     59   0.00341   0.000305
 6  1901     5 female NSW     0.00274    16030     44   0.00275   0.000251
 7  1901     6 female NSW     0.00252    16289     41   0.00230   0.000215
 8  1901     7 female NSW     0.00216    16639     36   0.00197   0.000189
 9  1901     8 female NSW     0.00169    16554     28   0.00175   0.000173
10  1901     9 female NSW     0.00109    16468     18   0.00162   0.000163
```

Functional data models

```
sm_aus <- aus |> smooth_mortality(Mortality)
sm_aus |> filter(State == "NSW", Year == 1980) |> autoplot(Mortality) +
  geom_line(aes(y = .smooth), col = "blue") + scale_y_log10()
```



Functional data models

```r
fit <- sm_aus |> model(hu = FDM(log(.smooth)))
fit
```

```
# A mable: 12 x 3
# Key:     Sex, State [12]
   Sex    State      hu
   <chr>  <chr> <model>
 1 female NSW     <FDM>
 2 female QLD     <FDM>
 3 female SA      <FDM>
 4 female TAS     <FDM>
 5 female VIC     <FDM>
 6 female WA      <FDM>
 7 male   NSW     <FDM>
 8 male   QLD     <FDM>
 9 male   SA      <FDM>
10 male   TAS     <FDM>
```

Functional data models

```r
fit |>
  filter(Sex == "female", State == "NSW") |>
  select(hu) |>
  report()
```

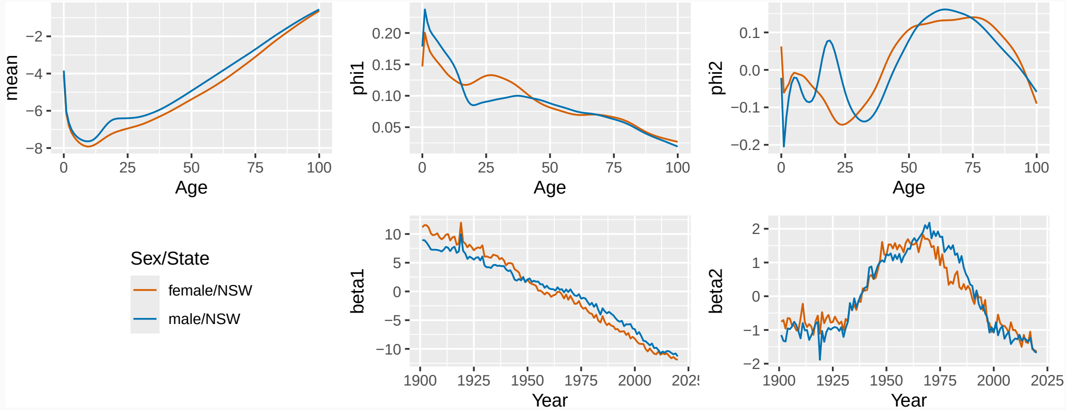
```
Series: .smooth
Model: FDM
Transformation: log(.smooth)

Basis functions
# A tibble: 101 x 8
    Age  mean  phi1    phi2    phi3    phi4       phi5    phi6
  <dbl> <dbl> <dbl>   <dbl>   <dbl>   <dbl>      <dbl>   <dbl>
1     0 -4.07 0.147  0.0625 -0.0270  0.0986  0.0112    -0.0624
2     1 -6.16 0.200 -0.0609 -0.194   0.116   0.0383    -0.238
3     2 -6.82 0.182 -0.0483 -0.157   0.0924  0.0443    -0.264
4     3 -7.17 0.170 -0.0368 -0.130   0.0362  0.000338 -0.321
5     4 -7.40 0.164 -0.0165 -0.114  -0.0154 -0.0303    -0.374
# i 96 more rows
```

Functional data models

```
Coefficients
# A tsibble: 120 x 8 [1Y]
   Year  mean beta1  beta2   beta3     beta4   beta5   beta6
  <int> <dbl> <dbl>  <dbl>   <dbl>     <dbl>   <dbl>   <dbl>
1  1901     1  11.1 -0.522 -0.0553  0.207    0.358    0.0305
2  1902     1  11.8 -0.649  0.399   0.856    0.0319   0.422
3  1903     1  11.5 -0.930 -0.485   0.398    0.399   -0.376
4  1904     1  11.1 -0.827 -0.214  -0.000305 0.00125 -0.0783
5  1905     1  10.2 -0.563 -0.105   0.324    0.122    0.0478
# i 115 more rows
# i Use `print(n = ...)` to see more rows

Time series models
   beta1 : ARIMA(0,1,1) w/ drift
   beta2 : ARIMA(0,2,2)
   beta3 : ARIMA(1,0,1)
   beta4 : ARIMA(0,0,2)
   beta5 : ARIMA(0,0,0)
   beta6 : ARIMA(2,0,2)

Variance explained
  91.38 + 1.81 + 0.58 + 0.49 + 0.42 + 0.39 = 95.06%
```

Functional data models

```
fit |>
  filter(State == "NSW") |>
  select(hu) |>
  autoplot()
```



Functional data models

```r
fit |> select(hu) |> age_components()
```

```
# A tibble: 1,212 x 10
   Sex    State   Age  mean phi1     phi2    phi3    phi4      phi5    phi6
   <chr>  <chr> <dbl> <dbl> <dbl>   <dbl>   <dbl>   <dbl>     <dbl>   <dbl>
 1 female NSW       0 -4.07 0.147  0.0625 -0.0270  0.0986  0.0112   -0.0624
 2 female NSW       1 -6.16 0.200 -0.0609 -0.194   0.116   0.0383   -0.238
 3 female NSW       2 -6.82 0.182 -0.0483 -0.157   0.0924  0.0443   -0.264
 4 female NSW       3 -7.17 0.170 -0.0368 -0.130   0.0362  0.000338 -0.321
 5 female NSW       4 -7.40 0.164 -0.0165 -0.114  -0.0154 -0.0303   -0.374
 6 female NSW       5 -7.57 0.158 -0.00759 -0.121  -0.0564  0.0247   -0.315
 7 female NSW       6 -7.71 0.153 -0.00942 -0.133  -0.0976  0.112    -0.197
 8 female NSW       7 -7.81 0.149 -0.0121 -0.143  -0.143   0.175    -0.0863
 9 female NSW       8 -7.88 0.143 -0.0141 -0.148  -0.181   0.211     0.0131
10 female NSW       9 -7.92 0.138 -0.0185 -0.142  -0.196   0.236     0.101
# i 1,202 more rows
```

Functional data models

```
fit |> select(hu) |> time_components()
```

```
# A tsibble: 1,440 x 10 [1Y]
# Key:       Sex, State [12]
   Sex    State  Year  mean beta1  beta2   beta3   beta4    beta5  beta6
   <chr>  <chr> <int> <dbl> <dbl>  <dbl>   <dbl>   <dbl>    <dbl>  <dbl>
 1 female NSW    1901     1 11.2  -0.756 -0.0301  0.269  -0.155    0.409
 2 female NSW    1902     1 11.6  -0.708  0.0899  0.207   0.0282   0.507
 3 female NSW    1903     1 11.5  -0.962  0.169  -0.103   0.366    0.323
 4 female NSW    1904     1 11.1  -0.648  0.0985 -0.433   0.131    0.270
 5 female NSW    1905     1 10.1  -0.660  0.342  -0.0910  0.0862   0.612
 6 female NSW    1906     1  9.78 -0.865  0.496  -0.147  -0.101    0.306
 7 female NSW    1907     1  9.90 -0.861  0.0530  1.33    0.278    0.181
 8 female NSW    1908     1 10.1  -1.01   0.554  -0.0198 -0.00428  0.578
 9 female NSW    1909     1  9.42 -1.02   0.293  -0.365  -0.149    0.353
10 female NSW    1910     1  9.08 -0.650  0.172  -0.559  -0.253    0.0110
# i 1,430 more rows
```

Coherent functional models

$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}$$

$$s_t(x) = \mu(x) + \sum_{j=1}^{J} \beta_{t,j}\phi_j(x) + e_t(x)$$

- $y_t(x) = \log(m^M_{x,t} m^F_{x,t})$ and $\log(m^M_{x,t}/m^F_{x,t})$
- $s_t(x)$ = smoothed version of $y_t(x)$
- $\mu(x)$ = mean $s_t(x)$ across years.
- $\phi_j(x)$ and $\beta_{t,j}$ estimated using principal component analysis.
- $\beta_{1,j}, \dots, \beta_{T,j}$ modelled with ARIMA for products and ARMA for ratios (to ensure stationary sex-ratios)

Coherent functional models

```r
pr <- sm_aus |> make_pr(.smooth)
pr
```

```
# A vital: 218,160 x 9 [1Y]
# Key:      Age x (Sex, State) [101 x 18]
    Year   Age Sex    State Mortality Exposure Deaths   .smooth .smooth_se
   <int> <dbl> <chr>  <chr>     <dbl>    <dbl>  <dbl> <dbl[1d]>  <dbl[1d]>
 1  1901     0 female NSW     0.107      17143   1833     0.939   0.00295
 2  1901     1 female NSW     0.0247     15071    373     1.03    0.00141
 3  1901     2 female NSW     0.00686    15461    106     0.965   0.000670
 4  1901     3 female NSW     0.00441    15629     69     0.982   0.000405
 5  1901     4 female NSW     0.00374    15762     59     1.02    0.000305
 6  1901     5 female NSW     0.00274    16030     44     1.04    0.000251
 7  1901     6 female NSW     0.00252    16289     41     1.04    0.000215
 8  1901     7 female NSW     0.00216    16639     36     1.01    0.000189
 9  1901     8 female NSW     0.00169    16554     28     0.972   0.000173
10  1901     9 female NSW     0.00109    16468     18     0.938   0.000163
```

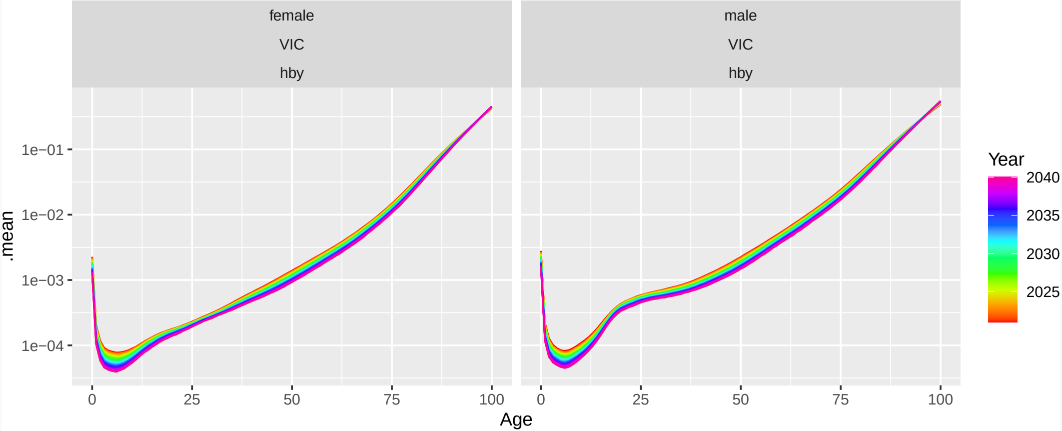
Coherent functional models

```r
pr <- sm_aus |> make_pr(.smooth)
fit <- pr |> model(hby = FDM(log(.smooth), coherent = TRUE))
fit
```

```
# A mable: 18 x 3
# Key:     Sex, State [18]
   Sex           State     hby
   <chr>         <chr> <model>
 1 female        NSW     <FDM>
 2 female        QLD     <FDM>
 3 female        SA      <FDM>
 4 female        TAS     <FDM>
 5 female        VIC     <FDM>
 6 female        WA      <FDM>
 7 geometric_mean NSW    <FDM>
 8 geometric_mean QLD    <FDM>
 9 geometric_mean SA     <FDM>
```

Coherent functional models

```
fc <- fit |> forecast(h = 20) |> undo_pr(.smooth)
fc |> filter(State == "VIC")  |> autoplot() + scale_y_log10()
```



Outline

Future plans

- Remaining tools from the `demography` package
- Stochastic population forecasting (as per Hyndman-Booth, IJF, 2008)
- All models handled by `StMoMo` package
- All methods from `MortalityLaws` package
- Suggestions from users

- **Slides**: robjhyndman.com/mpidr2024
- **Package**: pkg.robjhyndman.com/vital/