Confidence Intervals for the Difference of Two Means Quiz

1. Alicia would like to know if there is a difference in the average price between two brands of shoes. She selected and analyzed a random sample of 40 different types of Brand A shoes and 33 different types of Brand B shoes. Alicia observes that the boxplot of the sample of Brand A shoe prices shows two outliers. Alicia wants to construct a confidence interval to estimate the difference in population means.

Is the sampling distribution of the difference in sample means approximately normal?

(A) Yes, because Alicia selected a random sample.

(B) Yes, because for each brand it is reasonable to assume that the population size is greater than ten times its sample size.

(C) Yes, because the size of each sample is at least 30.

(D) No, because the distribution of Brand A shoes has outliers.

(E) No, because the shape of the population distribution is unknown.

Answer C

Correct. Although the boxplot shows outliers, both sample sizes (40 and 33) are greater than 30, so based on the central limit theorem, the distribution of the difference in sample means is approximately normal.

2. A sports equipment researcher investigated how different types of wood used to make baseball bats might affect batting. The researcher selected a sample of 80 batters from summer baseball leagues and randomly assigned the batters to one of two groups: the ash bat group or the maple bat group. The mean number of hits for each group was recorded at the end of the season, and the difference in the sample means was calculated.

Which of the following is the appropriate inference procedure for analyzing the results of the investigation?

(A) A one-sample t-interval for a population mean

(B) A one-sample t-interval for a sample mean

(C) A matched pairs t-interval for a mean difference

(D) A two-sample t-interval for a difference between sample means

(E) A two-sample t-interval for a difference between population means

Answer E

Correct. There were two independent groups in the experiment and data was collected on a single quantitative variable (number of hits). The correct procedure is the two-sample t-interval for a difference in population means.

Confidence Intervals for the Difference of Two Means Quiz

3. A national consumer agency selected independent random samples of 45 owners of newer cars (less than five years old) and 40 owners of older cars (more than five years old) to estimate the difference in mean dollar cost of yearly routine maintenance, such as oil changes, tire rotations, filters, and wiper blades. The agency found the mean dollar cost per year for newer cars was $195 with a standard deviation of $46. For older cars, the mean was $286 with a standard deviation of $58.

Which of the following represents the 95 percent confidence interval to estimate the difference (newer minus older) in the mean dollar cost of routine maintenance between newer and older cars?

(A) $(195-286)\pm 1.992\sqrt{\frac{46}{45}+\frac{58}{40}}$

(B) $(195-286)\pm 1.992\sqrt{\frac{46^2+58^2}{45+40}}$

(C) $(195-286)\pm 1.992\sqrt{\frac{46^2}{45}+\frac{58^2}{40}}$ ✓

(D) $(286-195)\pm 1.992\sqrt{\frac{46^2}{45}+\frac{58^2}{40}}$

(E) $(286-195)\pm 1.992\sqrt{\frac{46^2+58^2}{45+40}}$

Answer C

Correct. The confidence interval is correctly given by $(\bar{x}_{\mathrm{N}}-\bar{x}_{\mathrm{O}})\pm t^*\sqrt{\frac{s_{\mathrm{N}}^2}{n_{\mathrm{N}}}+\frac{s_{\mathrm{O}}^2}{n_{\mathrm{O}}}}$, where $\bar{x}_{\mathrm{N}}=195$, $\bar{x}_{\mathrm{O}}=286$, $s_{\mathrm{N}}=46$, $s_{\mathrm{O}}=58$, $n_{\mathrm{N}}=45$, and $n_{\mathrm{O}}=40$.

4. An ecologist is examining whether the number of fish caught in a large river basin has changed since a fire burned some of the surrounding forested area and vegetation along the river. Data in the form of fishing reports was available for a five-year period before the fire. From a random sample of 195 fishing reports before the fire, the mean catch was 6.3 fish with a standard deviation of 1.6. In a random sample of 143 reports three years after the fire, the mean catch was 7.1 fish with a standard deviation of 2.1.

Which of the following represents the standard error of the difference in the mean number of fish caught before and after the fire?

(A) $\sqrt{\frac{1.6}{195}+\frac{2.1}{143}}$

(B) $\sqrt{\frac{1.6^2}{195}+\frac{2.1^2}{143}}$ ✓

(C) $\frac{1.6^2}{\sqrt{195}}+\frac{2.1^2}{\sqrt{143}}$

(D) $\sqrt{\frac{1.6^2}{195}}+\sqrt{\frac{2.1^2}{143}}$

(E) $\sqrt{\frac{1.6^2+2.1^2}{195+143}}$

Confidence Intervals for the Difference of Two Means Quiz

Answer B

Correct. The standard error is the square root of the sum of two fractions. Each fraction is the respective sample variance divided by the sample size. The standard error can be found using the formula $\sqrt{\frac{(s_1)^2}{n_1}+\frac{(s_2)^2}{n_2}}$.

5. Researchers investigated whether there is a difference between two headache medications, R and S. Researchers measured the mean times required to obtain relief from a headache for patients taking one of the medications. From a random sample of 75 people with chronic headaches, 38 were randomly assigned to medication R and the remaining 37 were assigned to medication S. The time, in minutes, until each person experienced relief from a headache was recorded. The sample mean times were calculated for each medication.

Have the conditions been met for inference with a confidence interval for the difference in population means?

(A) Yes, all conditions have been met. ✓

(B) No, because the data were not collected using a random sample.

(C) No, because cause and effect cannot be inferred since there is a random sample.

(D) No, because the sample sizes are not large enough to assume the distribution of the difference in sample means is approximately normal.

(E) No, because the sample sizes are not the same.

Answer A

Correct. The medications were randomly assigned to treatments, so the independence condition is met. Each sample size is greater than 30, so the condition that the distribution of sample means is approximately normal has also been met.

6. Health programs routinely study the number of days that patients stay in hospitals. In one study, a random sample of 12 men had a mean of 7.95 days and a standard deviation of 6.2 days, and a random sample of 19 women had a mean of 7.1 days and a standard deviation of 5.0 days. The sample data will be used to construct a 95 percent confidence interval to estimate the difference between men and women in the mean number of days for the length of stay at a hospital.

Have the conditions been met for inference with a confidence interval?

Confidence Intervals for the Difference of Two Means Quiz

(A) Yes. All conditions have been met.

(B) No. The data were not collected using a random method.

(C) No. The size of at least one of the samples is greater than 10 percent of the population.

(D) No. The sample sizes are not large enough to assume that the sampling distribution of the difference in sample means is approximately normal.

(E) No. The sample sizes are not the same.

Answer D

Correct. Each sample size is less than 30, and information about the distributions of the populations is not known. Therefore, normality cannot be assumed for the sampling distribution of the difference in sample means.

7. A snowboarding competition site is using a new design for the parallel giant slalom. The designer of the slalom is investigating whether there is a difference in the mean times taken to complete a run for men and women competitors. As part of the investigation, independent random samples of men and women who will use the slalom run are selected and their times to complete a run are recorded. Which of the following is the appropriate inference procedure by which the designer can estimate the difference in the mean completion times for men and women?

(A) A one-sample t-interval for a sample mean

(B) A one-sample t-interval for a population mean

(C) A matched pairs t-interval for a mean difference

(D) A two-sample t-interval for a difference between population means

(E) A two-sample t-interval for a difference between sample means

Answer D

Correct. With two independent random samples from the two populations (men and women snowboarders), the correct procedure is the two-sample t-interval for a difference in population means.

8. The management of a large hardware store is interested in estimating the difference between the mean dollar amount of purchases made by customers who use the store's credit card and the mean dollar amount of purchases made by customers who use a different credit card. A random sample of 74 customers who used the store's credit card showed a mean purchase of $107 with a standard deviation of $12. A separate random sample of 58 customers who used a different credit card showed a mean purchase of $132 with a standard deviation of $9. Technology was used to calculate that the correct number of degrees of freedom is 129.78.

Which of the following represents the margin of error for a 98 percent confidence interval to estimate the difference in the mean purchase amount for the two types of credit cards?

Confidence Intervals for the Difference of Two Means Quiz

(A) $2.355\sqrt{\frac{12^2}{74}+\frac{9^2}{58}}$

(B) $2.355\sqrt{\frac{12^2+9^2}{74+58}}$

(C) $2.326\sqrt{\frac{12}{74}+\frac{9}{58}}$

(D) $2.326\sqrt{\frac{12^2+9^2}{74+58}}$

(E) $2.326\sqrt{\frac{12^2}{74}+\frac{9^2}{58}}$

Answer A

Correct. The margin of error is given by $t^*\sqrt{\frac{(s_1)^2}{n_1}+\frac{(s_2)^2}{n_2}}$, where $t^*=2.355$, $s_1=12$, $n_1=74$, $s_2=9$, and $n_2=58$.

9. A researcher in sports equipment is investigating the design of racing swimsuits for women. The researcher selected a sample of 40 women swimmers from high school swim teams in the state and randomly assigned each swimmer to one of two groups: suit A or suit B. The women will wear the assigned suits for a certain race, and the mean swim times for each group will be recorded. The difference in the sample mean swim times will be calculated.

Which of the following is the appropriate inference procedure for analyzing the results?

(A) A two-sample t-interval for a difference between sample means

(B) A two-sample t-interval for a difference between population means

(C) A one-sample t-interval for a sample mean

(D) A one-sample t-interval for a population mean

(E) A matched-pairs t-interval for a mean difference

Answer B

Correct. There were two independent groups in the experiment and data were collected on a single quantitative variable (swim times). The correct procedure is the two-sample t-interval for a difference in population means.