STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A large company produces an equal number of brand-name lightbulbs and generic lightbulbs. The director of quality control sets guidelines that production will be stopped if there is evidence that the proportion of all lightbulbs that are defective is greater than 0.10. The director also believes that the proportion of brand-name lightbulbs that are defective is not equal to the proportion of generic lightbulbs that are defective. Therefore, the director wants to estimate the average of the two proportions.

   To estimate the proportion of brand-name lightbulbs that are defective, a simple random sample of 400 brand-name lightbulbs is taken and 44 are found to be defective. Let X represent the number of brand-name lightbulbs that are defective in a sample of 400, and let p_X represent the proportion of all brand-name lightbulbs that are defective. It is reasonable to assume that X is a binomial random variable.

   (a) One condition for obtaining an interval estimate for p_X is that the distribution of $\hat{p}_X$ is approximately normal. Is it reasonable to assume that the condition is met? Justify your answer.

   (b) The standard error of $\hat{p}_X$ is approximately 0.0156. Show how the value of the standard error is calculated.

(c) How many standard errors is the observed value of $\hat{p}_X$ from 0.10 ?

To estimate the proportion of generic lightbulbs that are defective, a simple random sample of 400 generic lightbulbs is taken and 104 are found to be defective. Let Y represent the number of generic lightbulbs that are defective in a sample of 400. It is reasonable to assume that Y is a binomial random variable and the distribution of $\hat{p}_Y$ is approximately normal, with an approximate standard error of 0.0219. It is also reasonable to assume that X and Y are independent.

The parameter of interest for the manager of quality control is D, the average proportion of defective lightbulbs for the brand-name and the generic lightbulbs. D is defined as $D = \frac{p_X + p_Y}{2}$.

(d) Consider $\hat{D}$, the point estimate of D.

(i) Calculate $\hat{D}$ using data from the sample of brand-name lightbulbs and the sample of generic lightbulbs.

(ii) Calculate $s_{\hat{D}}$, the standard error of $\hat{D}$.

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Consider the following hypotheses.

H_0 : The average proportion of all lightbulbs that are defective is 0.10. $(D = 0.10)$

H_a : The average proportion of all lightbulbs that are defective is greater than 0.10. $(D > 0.10)$

A reasonable test statistic for the hypotheses is W, defined as $W = \dfrac{\hat{D} - 0.10}{s_{\hat{D}}}$.

(e) Calculate W using your answer to part (d).

(f) Chebyshev's inequality states that the proportion of any distribution that lies within k standard errors of the mean is at least

$$1 - \frac{1}{k^2}.$$

Use Chebyshev's inequality and the value of W to decide whether there is statistical evidence, at the significance level of $\alpha = 0.05$, that D, the average proportion of all lightbulbs that are defective, is greater than 0.10.

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