

**Combining Random Variables Quiz**

1. At a large regional collegiate women's swim meet, an official records the time it takes each swimmer to swim 100 meters for all swimmers who compete in only one stroke category. The following table shows the mean times and corresponding standard deviations for the collegiate women at the swim meet for each of the four stroke categories.

Stroke Category	Mean 100 meter Time	Standard Deviation
Backstroke	55.6 seconds	0.70 seconds
Breaststroke	63.3 seconds	0.92 seconds
Butterfly	54.4 seconds	0.94 seconds
Freestyle	50.2 seconds	0.76 seconds

For each of the 4 stroke categories, consider a random variable representing the time of a randomly selected swimmer in that category. What is the standard deviation of the sum of the 4 random variables?

- (A) 0.83 seconds  
(B) 1.67 seconds  
(C) 2.80 seconds  
(D) 3.32 seconds  
(E) 3.76 seconds

**Answer B**

Correct. To find the standard deviation the variances are added.

$$\sigma_{X+Y+Z+W}^2 = \sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + \sigma_W^2 = 0.70^2 + 0.92^2 + 0.94^2 + 0.76^2 = 2.7976, \text{ so}$$
$$\sigma = \sqrt{2.7976} \approx 1.67.$$

2. A player pays \$15 to play a game in which a chip is randomly selected from a bag of chips. The bag contains 10 red chips, 4 blue chips, and 6 yellow chips. The player wins \$5 if a red chip is selected, \$10 if a blue chip is selected, and \$20 if a yellow chip is selected. Let the random variable  $X$  represent the amount won from the selection of the chip, and let the random variable  $W$  represent the total amount won, where  $W = X - 15$ . What is the mean of  $W$ ?

- (A) \$10.50  
(B) \$4.50  
(C) -\$4.50  
(D) -\$6.50  
(E) -\$10.50



## Combining Random Variables Quiz

### Answer C

Correct. The expected payout is  $E(X) = \frac{10}{20}(\$5) + \frac{4}{20}(\$10) + \frac{6}{20}(\$20) = \$10.50$ . After subtracting the \$15 to play the game, the result is  $-\$4.50$  expected winnings.

3. The distribution of random variable  $R$  has mean 10 and standard deviation 4. The distribution of random variable  $S$  has mean 7 and standard deviation 3. If  $R$  and  $S$  are independent, what are the mean and standard deviation of the distribution of  $R - S$ ?
- (A) Mean 3 and standard deviation 1
  - (B) Mean 3 and standard deviation 5
  - (C) Mean 3 and standard deviation 7
  - (D) Mean 17 and standard deviation 1
  - (E) Mean 17 and standard deviation 5

### Answer B

Correct. When subtracting two independent random variables, the means are subtracted and the variances are added. Thus,  $\mu_{R-S} = \mu_R - \mu_S = 10 - 7 = 3$ , and  $\sigma_{R-S}^2 = \sigma_R^2 + \sigma_S^2 = 4^2 + 3^2 = 25$ , so  $\sigma_{R-S} = \sqrt{25} = 5$ .

## Combining Random Variables Quiz

4. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Company F sells fabrics known as fat quarters, which are rectangles of fabric created by cutting a yard of fabric into four pieces. Occasionally the manufacturing process results in a fabric defect. Let the random variable  $X$  represent the number of defects on a fat quarter created by Company F. The following table shows the probability distribution of  $X$ .

$X$	0	1	2	3	4 or more
Probability	0.58	0.23	0.11	0.05	0.03

If a fat quarter has more than 2 defects, it cannot be sold and is discarded. Let the random variable  $Y$  represent the number of defects on a fat quarter that can be sold by Company F.

- (a) Construct the probability distribution of the random variable  $Y$ .
- (b) Determine the mean and standard deviation of  $Y$ . Show your work.

Company G also sells fat quarters. The mean and standard deviation of the number of defects on a fat quarter that can be sold by Company G are 0.40 and 0.66, respectively. The fat quarters sell for \$5.00 each, but are discounted by \$1.50 for each defect found.

- (c) What are the mean and standard deviation of the selling price for the fat quarters sold by Company G?

### Part A, B, and C

The primary goals of this question are to assess a student's ability to (1) calculate conditional probabilities; (2) compute the mean and standard deviation of a random variable; and (3) describe the effect of a linear transformation on the parameters of random variables.

## Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).



0	1	2	3	4
---	---	---	---	---

All three parts essentially correct

☐

Part (a) essentially correct

## Combining Random Variables Quiz

- ☐ Part (a) partially correct
- ☐ Part (a) incorrect
- ☐ Part (b) essentially correct
- ☐ Part (b) partially correct
- ☐ Part (b) incorrect
- ☐ Part (c) essentially correct
- ☐ Part (c) partially correct
- ☐ Part (c) incorrect

## Solution

**Part (a):**

$Y$	Probability
0	0.630
1	0.250
2	0.120

The probability that a fat quarter can be sold is equal to 0.92, or

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

or

$$P(Y = 0) = P(X = 0 | X \leq 2) = \frac{P(X=0 \cap X \leq 2)}{P(X \leq 2)} = \frac{0.58}{0.92} \approx 0.630$$

$$P(Y = 1) = P(X = 1 | X \leq 2) = \frac{P(X=1 \cap X \leq 2)}{P(X \leq 2)} = \frac{0.23}{0.92} = 0.250$$

$$P(Y = 2) = P(X = 2 | X \leq 2) = \frac{P(X=2 \cap X \leq 2)}{P(X \leq 2)} = \frac{0.11}{0.92} \approx 0.120$$

## Scoring

**Part (a)** is scored as follows:

Essentially correct (E) if the response satisfies the following two components.

Identifies the three values of  $Y$  as 0, 1, and 2.

Gives correct conditional probabilities for the three values of  $Y$ .

Partially correct (P) if only one of the two components is satisfied.

Incorrect (I) if the response does not satisfy the criteria for E or P.

## Combining Random Variables Quiz

### Solution

#### **Part (b):**

The mean number of defects is  $\mu_Y = 0(0.63) + 1(0.25) + 2(0.12) = 0.49$ .

The standard deviation in the number of defects is

$$\sigma_Y = \sqrt{0.63(0 - 0.49)^2 + 0.25(1 - 0.49)^2 + 0.12(2 - 0.49)^2} \approx 0.70.$$

### Scoring

**Part (b)** is scored as follows:

Essentially correct (E) if the response satisfies the following three components.

- Correct mean.
- Correct standard deviation.
- The mean and standard deviation are clearly labeled.

*Note:* The components can be satisfied if they follow correctly from an incorrect, but reasonable, answer in part (a). To be reasonable, the probabilities in part (a) must sum to 1 and be nonnegative.

Partially correct (P) if the response satisfies only one or two of the three components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

### Solution

#### **Part (c):**

The mean selling price is  $5 - 1.5(0.40) = 4.40$  or \$4.40.

The standard deviation in the selling price is  $|-1.5|(0.66) = 0.99$  or \$0.99.

### Scoring

**Part (c)** is scored as follows:

Essentially correct (E) if the response satisfies the following three components.

- Correct mean.
- Correct standard deviation.
- The mean and standard deviation are clearly labeled.

Partially correct (P) if the response satisfies only one or two of the three components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

## Combining Random Variables Quiz

5. At a certain company, loan agents are paid based on the number of loans they close in a day. Based on company records, the number of loans  $X$  that a randomly selected loan agent closes on a randomly selected day has the probability distribution below.

$x$	1	2	3	4	5	6	7
$P(x)$	0.05	0.10	0.22	0.30	0.18	0.12	0.03

At the company, the daily salary of a loan agent is \$150 plus \$50 per loan closed. Let  $Y$  represent the amount of money made by a randomly selected loan agent on a randomly selected day. Which of the following statements is NOT true?

- (A) The mean of  $X$  is less than the mean of  $Y$ .
- (B) The standard deviation of  $Y$  is approximately \$71.
- (C) The mean daily salary is greater than \$350 per day. ✓
- (D) The standard deviation of  $X$  is less than the standard deviation of  $Y$ .
- (E) The shape of the probability distribution of  $Y$  is unimodal and roughly symmetric.

## Answer C

Correct. Since the mean of  $X$  is

$\mu_x = 1(0.05) + 2(0.10) + 3(0.22) + 4(0.30) + 5(0.18) + 6(0.12) + 7(0.03) = 3.94$ , then the mean of  $Y$  is  $\mu_Y = a + b\mu_X = 150 + 50(3.94) = \$347$ . Since  $\$347 < \$350$ , the statement is false.

6. Data were collected on the ages, in years, of the men and women enrolled in a large sociology course. Let the random variables  $M$  and  $W$  represent the ages of the men and women, respectively. The distribution of  $M$  has mean 20.7 years and standard deviation 1.73 years. The distribution of  $W$  has mean 20.2 years and standard deviation 1.60 years. Of all of those enrolled in the course, 54 percent are men and 46 percent are women. What is the mean age of the combined distribution of both men and women in the course?
- (A) 20.2 years
- (B) 20.43 years
- (C) 20.45 years
- (D) 20.47 years ✓
- (E) 40.9 years

## Combining Random Variables Quiz

### Answer D

Correct. Since 54% of the population are men and 46% are women, the mean of the combined distribution of men and women is  $0.54M + 0.46W$  which is  $0.54(20.7) + 0.46(20.2) = 20.47$ .

7. The random variable  $X$  has mean 12 and standard deviation 3. The random variable  $W$  is defined as  $W = 7 + 2X$ . What are the mean and standard deviation of  $W$ ?
- (A) The mean is 24, and the standard deviation is 6.
  - (B) The mean is 24, and the standard deviation is 13.
  - (C) The mean is 31, and the standard deviation is 3.
  - (D) The mean is 31, and the standard deviation is 6. ✓
  - (E) The mean is 31, and the standard deviation is 13.

### Answer D

Correct. The mean and standard deviation of  $W$  are  $\mu_W = a + b\mu_X = 7 + 2(12) = 31$  and  $\sigma_W = |b|\sigma_X = |2|(3) = 6$ , respectively. Mean is affected by the multiplication and addition of a constant, but standard deviation is affected only by the multiplication of a constant.