Of course, the linear model makes no sense unless the Linearity Assumption is satisfied We check the Straight Enough Condition and Outlier Condition with a scatterplot, as we did for correlation, and also with a plot of residuals against either the x or the predicted values. For the standard deviation of the residuals to make sense as a summary, we have to make the Equal Variance Assumption. We check it by looking at both the original scatterplot and the residual plot for the Does the Plot Thicken? Condition.

TERMS

Model An equation or formula that simplifies and represents reality. (p. 173)

A linear model is an equation of a line. To interpret a linear model, we need to know the variables (along with their W's) and their units. (p. 173)

Predicted value The value of \hat{y} found for a given x-value in the data. A predicted value is found by substituting the x-value in the regression equation. The predicted values are the values on the fitted line; the points (x, \hat{y}) all lie exactly on the fitted line. (p. 173)

Residuals are the differences between data values and the corresponding values predicted by the regression model—or, more generally, values predicted by any model. (p. 173)

Residual = Observed value - predicted value = $e = y - \hat{y}$

The least squares criterion specifies the unique line that minimizes the variance of the residuals or, equivalently, the sum of the squared residuals. (p. 174)

Because the correlation is always less than 1.0 in magnitude, each predicted \hat{y} tends to be fewer standard deviations from its mean than its corresponding x was from its mean. This is called regression to the mean. (p. 175)

The particular linear equation

 $\hat{y} = b_0 + b_1 x$

that satisfies the least squares criterion is called the least squares regression line (LSRL). Casually, we often just call it the regression line, or the line of best fit. (p. 175)

The slope, b_1 , gives a value in "y-units per x-unit." Changes of one unit in x are associated with changes of b_1 units in predicted values of y. The slope can be found by

 $b_1 = \frac{rs_y}{s_y}$. (p. 176)

The intercept, b_0 , gives a starting value in y-units. It's the \hat{y} -value when x is 0. You can find it from $b_0 = \overline{y} - b_1 \overline{x}$. (p. 177)

The standard deviation of the residuals is found by $s_e = \sqrt{\frac{\Sigma e^2}{n-2}}$. When the assumptions

and conditions are met, the residuals can be well described by using this standard deviation and the 68-95-99.7 Rule. (p. 182)

 R^2 (the square of the correlation between y and x) gives the fraction of the variability of y accounted for by the least squares linear regression model. (p. 185)

The scatterplot or residuals plot should show consistent (vertical) spread in y-values. (p. 187)



Linear model

Residuals

Least squares

Regression to the mean

Regression line

(Line of best fit)

Intercept

Slope

Standard error (Se)

 R^2

Does the Plot Thicken? Condition