Sampling Distributions for Differences in Sample Proportions Quiz

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A polling agency is investigating the voter support for a ballot measure in an upcoming city election. The agency will select a random sample of 500 voters from one region, Region A, of the city. Assume that the population proportion of voters who would support the ballot measure in Region A is 0.47.

(a) What is the probability that the proportion of voters in the sample of Region A who support the ballot measure is greater than 0.50 ?

The polling agency will take another sample from a different region, Region B, of the city. The agency plans to select a random sample of 400 voters. Assume that the population proportion of voters who would support the ballot measure in Region B is 0.51.

(b) Describe the sampling distribution of the difference in sample proportions (Region B minus Region A).

(c) What is the probability that the two sample proportions will differ by more than 0.05 ?

A, B, and C

The primary goals of this question are to assess a student's ability to (1) calculate a probability of interest for a sample proportion; (2) describe the sampling distribution for the difference between sample proportions; (3) calculate a probability for the difference in two proportions.

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

0	1	2	3	4 ✓

All three parts essentially correct

- ☐ · Part (a) essentially correct
- ☐ · Part (a) partially correct
- ☐ · Part (a) incorrect
- ☐ · Part (b) essentially correct
- ☐ · Part (b) partially correct
- ☐ · Part (b) incorrect
- ☐ · Part (c) essentially correct

- · Part (c) partially correct
- · Part (c) incorrect

Solution

Part (a)

Normal Approximation approach:

Since $np = 500(0.47) = 235$ and $n(1-p) = 500(0.53) = 265$ are both greater than or equal to 10, the sample size is large enough to ensure that the sampling distribution of the sample proportion, $\hat{p}$ will be approximately normal with mean $\mu_{\hat{p}} = 0.47$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{(0.47)(0.53)}{500}} \approx 0.0223$ Therefore,
$P(\hat{p} > 0.5) = P\left(z > \frac{0.5-0.47}{0.0223}\right) = P(z > 1.35) \approx 0.089$

The probability that the proportion of voters who support the ballot measure is greater than 0.50 is approximately equal to 0.089.

Binomial approach:

Let the random variable X be the number of voters out of 500 who support the ballot measure in the upcoming election. Random variable X is modeled by a binomial distribution with $n = 500$ and $p = 0.47$

The probability that more than 250 voters (proportion of 0.5) in the sample of Region A who support the ballot measure is $P(X > 250) \approx 0.0825$

Scoring

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components

· indicates that a normal distribution is being used

· verifies that the sample size is large enough for the use of the normal distribution to be appropriate

· correctly computes the probability

OR

Essentially correct (E) if the response satisfies the following three components

· indicates that a binomial distribution is being used

· states the values of *n* and *p*

· correctly computes the probability

Partially correct (P) if the response satisfies only two of the three components

Sampling Distributions for Differences in Sample Proportions Quiz

Incorrect (I) if the response does not meet the criteria for E or *P*.

Solution

Part (b):

Because the samples are independent (selected from different regions) with large sample sizes*, the sampling distribution of $\hat{p}_B - \hat{p}_A$ is approximately normal with mean $\mu_{\hat{p}_B-\hat{p}_A} = 0.51 - 0.47 = 0.04$ and standard deviation $\sigma_{\hat{p}_B-\hat{p}_A} = \sqrt{\frac{(0.47)(0.53)}{500} + \frac{(0.51)(0.49)}{400}} \approx 0.0335$

*Check for large sample size of Region B

$n_B p_B = 400(0.51) = 204$ and $n_B(1-p_B) = 400(0.49) = 196$ are greater than 10.

Scoring

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components.

· states that the sampling distribution is approximately normal and verifies that the sample sizes are large enough

· correctly determines the mean of the sampling distribution

· correctly determines the standard deviation of the sampling distribution

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Solution

Part (c):

$$P(|\hat{p}_B - \hat{p}_A| > 0.05) = P\left(z < \frac{-0.05-0.04}{0.0335}\right) + P\left(z > \frac{0.05-0.04}{0.0335}\right) =$$
$$P(z < -2.69) + P(z > 0.30) \approx 0.0038 + 0.3821 \approx 0.3859$$

Scoring

Essentially correct (E) if the response satisfies the following three components.

· indicates the probability that is being computed with either a correct probability statement or a picture of the normal distribution with shaded region.

· correctly calculates the probability consistent with the values for the mean and standard deviation given in part (b).

· shows work

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Sampling Distributions for Differences in Sample Proportions Quiz

2. In two common species of flowers, A and B, the proportions of flowers that are blue are p_a and p_b , respectively. Suppose that independent random samples of 50 species-A flowers and 100 species-B flowers are selected. Let $\hat{p}_a$ be the sample proportion of blue species-A flowers and $\hat{p}_b$ be the sample proportion of blue species-B flowers. What is the mean of the sampling distribution of $\hat{p}_a - \hat{p}_b$?

(A) $p_a - p_b$ ✓

(B) $\frac{p_a}{50} - \frac{p_b}{100}$

(C) $\hat{p}_a - \hat{p}_b$

(D) $\frac{p_a(1-p_a)}{50} + \frac{p_b(1-p_b)}{100}$

(E) $\sqrt{\frac{p_a(1-p_a)}{50} + \frac{p_b(1-p_b)}{100}}$

Answer A

Correct. The mean $\mu_{\hat{p}_a - \hat{p}_b}$ of the sampling distribution of the difference in sample proportions is $p_a - p_b$
.

3. On a given day, the proportion of workers from Company B who purchase a coffee from the company cafeteria is 0.62 and the proportion of workers from Company C who purchase a coffee from the company cafeteria is 0.71. A random sample of40 workers was selected from Company B and another random sample of 40 workers was selected from Company C. The proportion of workers from Company B who purchased coffee was $\hat{p}_{\mathrm{B}} = 0.70$ and the proportion of workers from Company C who purchased coffee was $\hat{p}_{\mathrm{C}} = 0.75$.

What is the correct unit of measure for the mean of the sampling distribution of $\hat{p}_{\mathrm{B}} - \hat{p}_{\mathrm{C}}$?

(A) Days

(B) Dollars

(C) Companies

(D) Workers

(E) There are no units associated with the mean of the sampling distribution. ✓

Answer E

Correct. The unit for the mean of the sampling distribution of $\hat{p}_{\mathrm{B}} - \hat{p}_{\mathrm{C}}$ is the same as the unit for the individual sample proportions. Sample proportions are unitless.

Sampling Distributions for Differences in Sample Proportions Quiz

4. Researchers are studying populations of two squirrels, the eastern gray and the western gray. For the eastern gray squirrel, about 22 percent of the population weighs over 0.5 kilogram (kg). For the western gray squirrel, about 36 percent of the population weighs over 0.5 kg. A random sample of 60 squirrels will be selected from the population of eastern gray squirrels, and a random sample of 120 squirrels will be selected from the population of western gray squirrels. What is the mean of the sampling distribution of the difference in sample proportions (eastern minus western) ?

(A) $\sqrt{\frac{0.22(0.78)}{120}+\frac{0.36(0.64)}{60}}$

(B) $\sqrt{\frac{0.22(0.78)}{60}+\frac{0.36(0.64)}{120}}$

(C) $60(0.22)-120(0.36)$

(D) $0.36-0.22$

(E) $0.22-0.36$

Answer E

Correct. The mean of the sampling distribution of the difference in sample proportions for eastern minus western is equal to the difference in the population proportions, which equals $0.22-0.36$.

5. A manufacturing company uses two different machines, A and B, each of which produces a certain item part. The number of defective parts produced by each machine is about 1 percent. Suppose two independent random samples, each of size 100, are selected, where one is a sample of parts produced by machine A and the other is a sample of parts produced by machine B. Which of the following is true about the sampling distribution of the difference in the sample proportions of defective parts?

(A) The mean is 0 and the distribution is approximately normal.

(B) The mean is 0 and the distribution will not be approximately normal.

(C) The mean is 0.01 and the distribution is approximately normal.

(D) The mean is 0.01 and the distribution will not be approximately normal.

(E) The mean is 1 and the distribution is approximately normal.

Answer B

Correct. The mean is the difference in the population proportions, $0.01-0.01=0$. Normality cannot be assumed because $100(0.01)<10$, meaning the sample sizes are not large enough.

Sampling Distributions for Differences in Sample Proportions Quiz

6. A consumer group is investigating two brands of popcorn, R and S. The population proportion of kernels that will pop for Brand R is 0.90. The population proportion of kernels that will pop for Brand S is 0.85. Two independent random samples were taken from the population. The following table shows the sample statistics.

	Number of Kernels in Samples	**Proportion from Sample that Popped**
Brand R	100	0.92
Brand S	200	0.89

The consumer group claims that for all samples of size 100 kernels from Brand R and 200 kernels from Brand S, the mean of all possible differences in sample proportions (Brand R minus Brand S) is 0.03. Is the consumer group's claim correct?

(A) Yes. The mean is $0.92 - 0.89 = 0.03$.

(B) No. The mean is $\frac{0.92+0.89}{2} = 0.905$.

(C) No. The mean is $\frac{0.92-0.89}{2} = 0.015$.

(D) No. The mean is $\frac{0.90+0.85}{2} = 0.875$.

(E) No. The mean is $0.90 - 0.85 = 0.05$. ✓

Answer E

Correct. The consumer group's claim is not correct. The mean of all differences in sample proportions from all possible samples of size 100 kernels from Brand R and 200 kernels from Brand S is equal to the difference in the population proportions.

7. City R is a large city with 4 million residents, and City S is a smaller city with 0.25 million residents. Researchers believe that the proportion of City S residents who regularly ride bicycles is between 10 percent and 25 percent and the proportion of City R residents who regularly ride bicycles is between 20 percent and 50 percent.

Suppose two independent random samples of residents from each city will be taken and the proportion of residents who ride bicycles is recorded. Based on the population proportions of residents who regularly ride bicycles, which of the following sample sizes is large enough to guarantee that the sampling distribution of the difference in sample proportions will be approximately normal?

(A) 30 in City R and 30 in City S

(B) 30 in City R and 60 in City S

(C) 60 in City R and 30 in City S

(D) 50 in City R and 100 in City S ✓

(E) 100 in City R and 50 in City S

Sampling Distributions for Differences in Sample Proportions Quiz

Answer D

Correct. For City S, using the smallest value of the population proportion $(p = 0.1)$, a sample size of 100 is required for $n_{\mathrm{S}}p_{\mathrm{S}} \geq 10$ and $n_{\mathrm{S}}(1 - p_{\mathrm{S}}) \geq 10$. For city R, using the smallest value of the population proportion $(p = 0.2)$, a sample size of 50 is required for $n_{\mathrm{R}}p_{\mathrm{R}} \geq 10$ and $n_{\mathrm{R}}(1 - p_{\mathrm{R}}) \geq 10$.

8. Two different drugs, X and Y, are currently in use to treat a certain condition. About 7 percent of the people using either drug experience side effects. A random sample of 75 people using drug X and a random sample of 150 people using drug Y are selected. The proportion of people in each sample who experience side effects is recorded.

Are the sample sizes large enough to assume that the sampling distribution of the difference in sample proportions is approximately normal?

(A) Yes. Both sample sizes are large enough.

(B) No. The sample size for drug X is large enough, but the sample size for drug Y is not.

(C) No. The sample size for drug Y is large enough, but the sample size for drug X is not. ✓

(D) No. Neither sample size is large enough.

(E) There is not enough information provided to determine whether the sampling distribution is approximately normal.

Answer C

Correct. The sampling distribution of the difference in sample proportions will have an approximately normal distribution provided the sample sizes are large enough. For drug X, $75(0.07) < 10$, which is not large enough. For drug Y, both conditions are greater than 10.

9. The manager of a symphony in a large city wants to investigate music preferences for adults and students in the city. Let p_A represent the population proportion of adults who live in the city who prefer pop music. Let p_S represent the population proportion of students who live in the city who prefer pop music. Random samples of 200 adults from the city and 200 students from the city will be selected.

Which of the following is the best interpretation of $P(\hat{p}_A - \hat{p}_S > 0) \approx 0.022$?

Sampling Distributions for Differences in Sample Proportions Quiz

(A) The difference between the population proportion of adults who prefer pop music and the population proportion of students who prefer pop music is approximately 0.022.

(B) The difference between the population proportion of adults who prefer pop music and the population proportion of students who prefer pop music is 0.

(C) For all random samples of 200 adults from the city and 200 students from the city, the difference between the sample proportion of adults who prefer pop music and the sample proportion of students who prefer pop music will be 0.022.

(D) For all random samples of 200 adults from the city and 200 students from the city, the difference between the sample proportion of adults who prefer pop music and the sample proportion of students who prefer pop music will be 0.

(E) For all random samples of 200 adults from the city and 200 students from the city, the sample proportion of adults who prefer pop music will be greater than the sample proportion of students who prefer pop music in about 2.2% of samples. ✓

Answer E

Correct. The statement gives a probability of approximately 0.022 for the difference in sample proportions, $\hat{p}_A - \hat{p}_S$, being greater than 0.

10. Two coins, A and B, each have a side for heads and a side for tails. When coin A is tossed, the probability it will land tails-side up is 0.5. When coin B is tossed, the probability it will land tails-side up is 0.8. Both coins will be tossed 20 times. Let $\hat{p}_{\mathrm{A}}$ represent the proportion of times coin A lands tails-side up, and let $\hat{p}_{\mathrm{B}}$ represent the proportion of times coin B lands tails-side up.

Is the number of tosses for each coin enough for the sampling distribution of the difference in sample proportions $\hat{p}_{\mathrm{A}} - \hat{p}_{\mathrm{B}}$ to be approximately normal?

(A) Yes, 20 tosses for each coin is enough.

(B) No, 20 tosses for coin A is enough, but 20 tosses for coin B is not enough. ✓

(C) No, 20 tosses for coin A is not enough, but 20 tosses for coin B is enough.

(D) No, 20 tosses is not enough for either coin.

(E) There is not enough information given to determine if 20 tosses is enough.

Answer B

Correct. For coin A, $20(0.5) \geq 10$, so 20 tosses is enough. However, for coin B, $20(0.8) \geq 10$, but $20(0.2) < 10$, so 20 tosses is not enough.