Potential Errors When Performing Tests Quiz

1. Which of the following gives the probability of making a Type I error?

(A) The sample size

(B) The power

(C) The significance level ✓

(D) The standard error

(E) The p-value

Answer C

Correct. The significance level α chosen for the test defines the probability of making a Type I error.

2. Consider the results of a hypothesis test, which indicate there is not enough evidence to reject the null hypothesis. Which of the following statements about error is correct?

(A) A Type I error could have been made, but not a Type II error.

(B) A Type II error could have been made, but not a Type I error. ✓

(C) Both types of error could have been made, but the probability of a Type I error is greater than the probability of a Type II error.

(D) Both types of error could have been made, but the probability of a Type I error is less than the probability of a Type II error.

(E) The type of error that could have been made is not possible to determine without knowing the statement of the null hypothesis.

Answer B

Correct. A Type II error occurs when the null hypothesis is false but has not been rejected. A Type II error can be made only when we fail to reject the null hypothesis; in other words, a Type II error is possible only when we do not find enough evidence for an alternative hypothesis.

3. At a manufacturing company for medical supplies, machines produce parts used in highly specialized lasers. Company researchers are testing a new machine designed to improve the precision of the parts. The null hypothesis is that the new machine does not improve the precision. For the researchers, the more consequential error would be that the new machine actually improves the precision, but the test does not detect the improvement.

Which of the following should the researchers do to avoid the more consequential error?

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(A) Increase the significance level to increase the probability of a Type I error. ✓

(B) Increase the significance level to decrease the probability of a Type I error.

(C) Decrease the significance level to increase the probability of a Type I error.

(D) Decrease the significance level to decrease the probability of a Type I error.

(E) Decrease the significance level to decrease the standard error.

Answer A

Correct. The researchers would like to decrease the chances of a Type II error; that is, the new machine is effective, but the test fails to detect it. To decrease the probability of a Type II error, the researchers can increase the probability of a Type I error by increasing the significance level.

4. Machines at a bottling plant are set to fill bottles to 12 ounces. The quality control officer at the plant periodically tests the machines to be sure that the bottles are filled to an appropriate amount. The null hypothesis of the test is that the mean is at least 12 ounces. The alternative hypothesis is that the mean is less than 12 ounces.

Which of the following describes a Type I error that could result from the test?

(A) The test does not provide convincing evidence that the mean is less than 12 ounces, but the actual mean is at least 12 ounces.

(B) The test does not provide convincing evidence that the mean is less than 12 ounces, but the actual mean is less than 12 ounces.

(C) The test does not provide convincing evidence that the mean is less than 12 ounces, but the actual mean is 12 ounces.

(D) The test provides convincing evidence that the mean is less than 12 ounces, but the actual mean is at least 12 ounces. ✓

(E) The test provides convincing evidence that the mean is less than 12 ounces, but the actual mean is 11 ounces.

Answer D

Correct. A Type I error occurs if the test finds evidence for an alternative hypothesis that is actually false. In this case, the actual mean fill amount is at least 12 ounces, but the test indicates the mean is less than 12 ounces.

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5. At a large company, employees can take a course to become certified to perform certain tasks. There is an exam at the end of the course that needs to be passed for certification. The current pass rate is 0.7, but a new program is being tested to help increase the pass rate. The null hypothesis of the test is that the pass rate for the new program is 0.7. The alternative is that the pass rate for the new program is greater than 0.7.

Which of the following describes a Type II error that could result from the test?

(A) The test does not provide convincing evidence that the pass rate is greater than 0.7, but the actual pass rate is 0.8. ✓

(B) The test does not provide convincing evidence that the pass rate is greater than 0.7, but the actual pass rate is 0.7.

(C) The test does not provide convincing evidence that the pass rate is greater than 0.7, but the actual pass rate is 0.6.

(D) The test provides convincing evidence that the pass rate is greater than 0.7, but the actual pass rate is 0.8.

(E) The test provides convincing evidence that the pass rate is greater than 0.7, but the actual pass rate is 0.6.

Answer A

Correct. A Type II error occurs if a false null hypothesis is not rejected; in other words, the test fails to find evidence for a true alternative hypothesis. In this case, the program is effective in raising the pass rate (the rate is 0.8), but the test fails to detect it (not finding evidence that the program is greater than 0.7).

6. Consider a hypothesis test in which the significance level is $\alpha = 0.05$ and the power of the test is 0.65. What is the probability of making a Type II error?

(A) 0.95

(B) 0.65

(C) 0.60

(D) 0.35 ✓

(E) 0.05

Answer D

Correct. The probability of making a Type II error is equal to 1 minus the power of the test: $1 - 0.65 = 0.35$.

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7. Educators are testing a new program designed to help children improve their reading skills. The null hypothesis of the test is that the program does not help children improve their reading skills. For the educators, the more consequential error would be that the program does not help children improve their reading skills but the test indicated that it does help.

Which of the following should the researchers do to avoid the more consequential error?

(A) Increase the significance level to increase the probability of Type I error.

(B) Increase the significance level to decrease the probability of Type I error.

(C) Decrease the significance level to increase the probability of Type I error.

(D) Decrease the significance level to decrease the probability of Type I error. ✓

(E) Decrease the significance level to decrease the standard error.

Answer D

Correct. The educators would like to decrease the chances of a Type I error; that is, the program is really not effective but the test gives evidence that it is effective. To decrease the probability of a Type I error, the researchers can decrease the significance level.

8. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The distribution of pH levels for all community swimming pools in a large county is approximately normal with mean 7.5 and standard deviation 0.2. According to swimming pool studies, the safest pH levels for water in swimming pools are between 7.2 and 7.8.

(a) One community swimming pool in the county will be selected at random. What is the probability that the selected pool has a pH level that is not considered safe?

The county health inspector will select a random sample of 4 community swimming pools in the county to investigate the pH levels.

(b) Describe the sampling distribution of the sample mean for samples of size 4.

(c) Consider the situation in which the health inspector finds the sample mean of the 4 pools to be outside the safe pH levels. As a result, the inspector declares that the population mean is not 7.5. However, if the population mean really is 7.5, the inspector will have made an error. Such an error is called a Type I error. Find the probability that the inspector will make a Type I error with the sample of 4 pools. Show your work.

Part A, B, and C

The primary goals of this question are to assess a student's ability to (1) calculate a probability from a normal distribution;

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(2) describe a sampling distribution of a sample mean; and (3) determine the probability of a Type I error.

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

0	1	2	3	4 ✓

All three parts essentially correct

- [ ] Part (a) essentially correct
- [ ] Part (a) partially correct
- [ ] Part (a) incorrect
- [ ] Part (b) essentially correct
- [ ] Part (b) partially correct
- [ ] Part (b) incorrect
- [ ] Part (c) essentially correct
- [ ] Part (c) partially correct
- [ ] Part (c) incorrect

Solution

Part (a):

The distribution of pH levels for the swimming pools in a certain region is approximately normal with mean 7.5 and standard deviation of 0.2. The z-score for a pH level of 7.2 is $z = \frac{7.2-7.5}{0.2} = -1.5$. The z-score for a pH level of 7.8 is $z = \frac{7.8-7.5}{0.2} = 1.5$.

Based on the standard normal table, $P(z > 1.5) + P(z < -1.5) \approx 0.0668 + 0.0668 \approx 0.1336$

The probability that the randomly selected swimming pool is not considered safe is 0.1336.

Scoring

Part (a) is scored as follows.

Essentially correct (E) if the response correctly includes the following four components:

· Indicates use of a normal distribution and clearly identifies the correct parameter values (using a z-score is sufficient);

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· Uses the correct boundary value;

· Reports the correct normal probability consistent with components 1 and 2;

· Includes context.

Partially correct (P) if the response correctly includes three of the four components.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Solution

Part (b):

The sampling distribution of the sample mean will be approximately normal with mean pH level of $\mu_{\bar{x}} = 7.5$ and standard deviation $\sigma_{\bar{x}} = \frac{0.2}{\sqrt{4}} = 0.1$.

Scoring

Part (b) is scored as follows.

Essentially correct (E) if the response correctly includes the following three components:

Identifies the mean of the sampling distribution as 7.5;

Identifies the standard deviation of the sampling distribution as 0.1 AND includes work;

Identifies that the shape of the sampling distribution is approximately normal;

Partially correct (P) if the response correctly includes two of the three components.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Solution

Part (c):

The sampling distribution is normally distributed with mean 7.5 and standard deviation 0.1.

A Type I error will occur for sample means less than 7.2 and greater than 7.8.

The *z*-score for a sample mean pH level of 7.2 is $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{7.2 - 7.5}{0.1} = -3.$

The *z*-score for a pH level of 7.8 is $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{7.8 - 7.5}{0.1} = 3.$

Based on the standard normal table, $P(z > 3) + P(z < -3) \approx 0.00135 + 0.00135 \approx 0.0027.$

The probability of a Type I error occurring in this situation is 0.0027.

Scoring

Part (c) is scored as follows.

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Essentially correct (E) if the response correctly includes the following five components:

· Indicates use of a normal distribution

· Clearly identifies parameter values that are consistent with the answer in part (b);

· Uses the correct boundary values for determining a Type I error;

· Reports the correct normal probability consistent with components 1, 2 and 3;

· Includes context.

Partially correct (P) if the response correctly includes three or four of the five components.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

9. Medical researchers are testing a new surgical procedure designed to minimize the side effects of surgery. The null hypothesis is that the procedure is not effective in minimizing side effects. For the researchers, the more consequential error would be that the procedure actually is effective in minimizing the side effects, but the test does not detect the effectiveness of the procedure.

Which of the following should the researchers do to avoid the more consequential error?

(A) Increase the significance level to increase the probability of a Type I error. ✓

(B) Increase the significance level to decrease the probability of a Type I error.

(C) Decrease the significance level to increase the probability of a Type I error.

(D) Decrease the significance level to decrease the probability of a Type I error.

(E) Decrease the significance level to decrease the standard error.

Answer A

Correct. The researchers are seeking to decrease a Type II error—that is, the procedure is effective but the test does not detect it. To decrease the probability of a Type II error, the researchers can increase the probability of a Type I error by increasing the significance level.

10. If all else is constant, which of the following would result in a decrease of the probability of a Type II error?

(A) The true parameter is closer to the null.

(B) The sample size is increased. ✓

(C) The significance level is decreased.

(D) The standard error is increased.

(E) Type II error cannot be decreased, only increased.

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Answer B

Correct. An increase in sample size will result in a decrease in the variability of the sampling distribution, making differences easier to detect. The decrease in variability of the sampling distribution will result in an increase in the power of the test and decrease the probability of Type II error.

11. The following list shows three factors that can either increase or decrease the probability of a Type II error.

I. The sample size is increased.
II. The significance level is increased.
III. The standard error is increased.

Which factors alone will cause the probability of a Type II error to increase?

(A) I only
(B) II only
(C) III only ✓
(D) I and II only
(E) I, II, and III

Answer C

Correct. An increase in standard error increases variability of the sampling distribution. Due to the increase in variability of the sampling distribution, it is more difficult to detect a true difference, and the probability of a Type II error is increased.

12. If all else is constant, which of the following results in an increase in the probability of a Type II error?

(A) The significance level is increased.
(B) The standard error is decreased.
(C) The probability of a Type II error cannot be increased, only decreased.
(D) The true parameter is farther from the null hypothesis.
(E) The sample size is decreased. ✓

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Answer E

Correct. A decrease in sample size will result in an increase in the variability of the sampling distribution, making differences more difficult to detect. The increase in variability of the sampling distribution will result in a decrease in the power of the test and increase the probability of Type II error.

13. Which of the following is the best interpretation of the power of a significance test?

(A) Power is the probability of detecting an effect if no effect exists.

(B) Power is the probability of detecting an effect if an effect exists. ✓

(C) Power is the probability of detecting an effect whether or not an effect exists.

(D) Power is the probability of not detecting an effect if no effect exists.

(E) Power is the probability of not detecting an effect if an effect exists.

Answer B

Correct. In general, a null hypothesis is that there is no effect (or no difference related to a treatment). Power is the probability that the test will correctly reject a false null hypothesis; in other words, power is the probability of finding evidence for an effect that truly exists.