Constructing a Confidence Interval for a Population Mean Quiz

1. The local ranger station tracked and tagged 2,844 adult female black bears in a national park. A random sample of 9 adult female black bears from those tagged had an average body weight of 203 pounds with standard deviation 25 pounds. Which of the following is a point estimate for the population mean weight of all female black bears that are tagged?

   (A) 9

   (B) $\frac{25}{\sqrt{9}}$

   (C) 25

   (D) 203 ✓

   (E) 2,844

> **Answer D**
>
> Correct. The mean weight of the sample acts as a point estimate for the mean weight of the population.

2. Researchers at a medical center studied the amount of caffeine, in milligrams (mg), contained in a 16-ounce cup of coffee made at one machine at the center's cafeteria. They selected a random sample of 40 16-ounce cups of coffee made at different times of the day during a one-month period. The mean and standard deviation of the amount of caffeine in the sample were 159.88 mg and 36.72 mg, respectively. A graph of the sample data revealed a right skew with one outlier. The researchers will construct a confidence interval to estimate the amount of caffeine for all 16-ounce cups made at the machine.

   Which of the following conditions is <u>not</u> needed for the inference?

   (A) The samples were selected at random.

   (B) The observations are independent of one another.

   (C) The sample size of 40 is less than 10% of the population size.

   (D) The graph of the sample data is symmetric with no outliers. ✓

   (E) The sample size is large enough to assume that the sampling distribution of sample means is approximately normal.

> **Answer D**
>
> Correct. Because the sample size is large enough $(40 > 30),$ the assumption of normality of sample means holds, regardless of the shape of the distribution of the sample data.

Constructing a Confidence Interval for a Population Mean Quiz

3.   A random sample of 10 employees of a company was selected to estimate the mean one-way commute time for all employees at the company. The mean and standard deviation of the sample were 38 minutes and 6 minutes, respectively.

Assuming all conditions for inference are met, which of the following is the margin of error, in minutes, for a 95 percent confidence interval for the population mean one-way commute time?

(A) $1.812\left(\frac{6}{\sqrt{10}}\right)$

(B) $1.833\left(\frac{6}{\sqrt{10}}\right)$

(C) $1.96\left(\frac{6}{\sqrt{10}}\right)$

(D) $2.228\left(\frac{6}{\sqrt{10}}\right)$

(E) $2.262\left(\frac{6}{\sqrt{10}}\right)$ ✔

Answer E

Correct. The standard error is the sample standard deviation divided by the square root of the sample size, or $\frac{6}{\sqrt{10}}$. With a sample size of 10, the number of degrees of freedom is equal to $n - 1 = 10 - 1 = 9$, and the correct critical value is $t^* = 2.262$.

4.   A marine biologist wants to estimate the average weight of a population of dolphins living in a certain region of the ocean. The biologist will collect a random sample of dolphins and use the sample weights to create the estimate.

Which of the following is an appropriate method for the biologist to use for inference to the population?

(A)   A one-sample t-interval for a population mean ✔

(B)   A one-sample t-interval for a sample mean

(C)   A one-sample z-interval for a population proportion

(D)   A matched-pairs t-interval for a mean difference

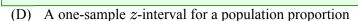(E)   A two-sample t-interval for a difference between means

Answer A

Correct. Weight is a quantitative variable. The appropriate procedure for estimation is the one-sample t-interval for a population mean.

Constructing a Confidence Interval for a Population Mean Quiz

5. To study learned behavior in mice, researchers used a sample of mice in a maze experiment. Each mouse had to find its way through a maze to reach food at the end. The mouse was timed on its first run through the maze and again on its tenth run through the maze. The difference in the times was recorded for each mouse.

Which of the following is the most appropriate inference procedure for the researchers to use?

(A) A two-sample z-interval for a difference between proportions

(B) A two-sample t-interval for a difference between means

(C) A matched-pairs t-interval for a mean difference ✓

(D) A one-sample z-interval for a population proportion

(E) A one-sample t-interval for a sample mean difference
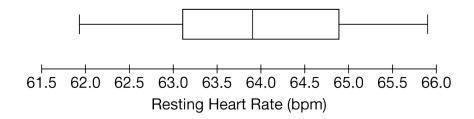
> **Answer C**
>
> Correct. Two observations were recorded for each mouse, the first-run time and the tenth-run time. Since observations are paired for each mouse, the best method is a matched-pairs t-interval.

6. A university researcher wants to estimate the mean number of novels that seniors read during their time in college. An exit survey was conducted with a random sample of 9 seniors. The sample mean was 7 novels with standard deviation 2.29 novels. Assuming that all conditions for conducting inference have been met, which of the following is a 95 percent confidence interval for the population mean number of novels read by all seniors?

(A) $7 \pm 1.960\left(\frac{2.29}{\sqrt{8}}\right)$

(B) $7 \pm 1.960\left(\frac{2.29}{\sqrt{9}}\right)$

(C) $7 \pm 2.262\left(\frac{2.29}{\sqrt{9}}\right)$

(D) $7 \pm 2.306\left(\frac{2.29}{\sqrt{8}}\right)$

(E) $7 \pm 2.306\left(\frac{2.29}{\sqrt{9}}\right)$ ✓

> **Answer E**
>
> Correct. The point estimate is 7. The standard error is the sample standard deviation 2.29 divided by the square root of the sample size 9. There are $n - 1 = 9 - 1 = 8$ degrees of freedom, and the correct value of t is 2.306.

Constructing a Confidence Interval for a Population Mean Quiz

7. The director of fitness for a large corporation with over 5,000 employees recorded the resting heart rate, in beats per minute (bpm), for 35 employees who were known to wear activity trackers. The following boxplot summarizes the results.

61.5 62.0 62.5 63.0 63.5 64.0 64.5 65.0 65.5 66.0
Resting Heart Rate (bpm)

The director wants to estimate the resting heart rate for all employees with a confidence interval. Have all conditions for inference been met?

(A) Yes, all conditions have been met.

(B) No, the distribution of the sample data is not approximately symmetric.

(C) No, the sample size is greater than 10 percent of the population size.

(D) No, the distribution of resting heart rate in the population cannot be assumed to be approximately normal.

(E) No, the sample was not selected at random. ✓

Answer E

Correct. Only employees wearing activity trackers were used in the sample. It is not reasonable to believe that these employees are representative of all 5,000 employees, because there will be some employees who do not wear activity trackers.

8. A team of ecologists will select a random sample of nesting robins in a certain region to estimate the average number of eggs per nest for all robins in the region. Which of the following is a correct inference procedure for the ecologists to use?

(A) A one-sample t-interval for a sample mean

(B) A one-sample t-interval for a population mean ✓

(C) A one-sample z-interval for a population proportion

(D) A two-sample t-interval for a difference between means

(E) A two-sample z-interval for a difference between proportions

Constructing a Confidence Interval for a Population Mean Quiz

> **Answer B**
>
> Correct. The researchers are interested in estimating the <u>average</u> number of eggs per nest, not a proportion. The correct procedure to estimate the average number in the population is the one-sample t -interval.

9. A national travel association with over 3,000 members selected a random sample of 100 members of the association. The selected members were asked to report the number of miles they traveled on their last vacation. The mean and standard deviation of the responses were 150 miles and 40 miles, respectively. A graph of the sample data displayed a right skew. The association will construct a confidence interval to estimate the mean number of miles traveled for all members.

    Have the conditions for inference been met?

    (A) Yes. All conditions for inference have been met. ✓

    (B) No. The distribution of sample data was not approximately normal.

    (C) No. The sample size was greater than 10% of the population size.

    (D) No. The sample was not selected at random.

    (E) No. The sample consisted only of members of the travel association.

> **Answer A**
>
> Correct. All conditions have been met. The sample was selected at random, the sample size was less than 10% of the population size, and although the sample data had right skew, based on the central limit theorem, the sample size was large enough to assume normality of the sample means $(100 > 30)$.
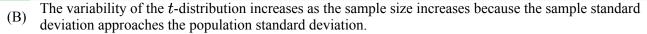
10. Which of the following correctly compares the t-distribution and z-distribution?

    (A) For small sample sizes, the density curve of the t-distribution is not symmetric, but the density curve of the z-distribution is symmetric.

    (B) For large sample sizes, the density curve of the t-distribution is not symmetric, but the density curve of the z-distribution is symmetric.

    (C) The curves of both distributions are symmetric, but the height of the density curve of the t-distribution is taller than the height of the density curve of the z-distribution.

    (D) The area under the density curve of the t-distribution is greater than the area under the density curve of the z-distribution, especially for small sample sizes.

    (E) The density curve of the t-distribution is more spread out than the density curve of the z-distribution, especially for small sample sizes. ✓

Constructing a Confidence Interval for a Population Mean Quiz

> **Answer E**
>
> Correct. The t-curve tends to be lower and wider than the z-curve, especially when the sample size is small.

11. Which of the following statements correctly explains what happens to the variability of a t-distribution as the sample size increases?

   (A) The variability of the t-distribution decreases as the sample size increases because the sample standard deviation approaches the population standard deviation. ✓

   (B) The variability of the t-distribution increases as the sample size increases because the sample standard deviation approaches the population standard deviation.

   (C) The variability of the t-distribution increases as the sample size increases because the mean of the distribution increases.

   (D) The variability of the t-distribution decreases as the sample size increases because the mean of the distribution decreases.

   (E) The variability of the t-distribution remains constant as the sample size increases because the t-statistic is defined by the sample standard deviation.

> **Answer A**
>
> Correct. As sample size increases, s approaches σ, and the t-distribution approaches the z-distribution. There are more observations closer to the center of a z-distribution than in any t-distribution, so the variability in a t-distribution decreases as the sample size increases because the t-distribution approaches a z-distribution.

12. For a t-distribution with sample size 10, $P(t \geq 1.96) \approx 0.0408$ and $P(t \leq -1.96) \approx 0.0408$. Which of the following is a property of the t-distribution illustrated by the probabilities?

   (A) With sample size 10, the tails of the curve of the t-distribution have more area than the tails of the curve of the z-distribution. ✓

   (B) With sample size 10, the tails of the curve of the t-distribution have less area than the tails of the curve of the z-distribution.

   (C) With sample size 10, the middle of the curve of the t-distribution has more area than the middle of the curve of the z-distribution.

   (D) With sample size 10, the mean of the t-distribution is greater than the mean of the z-distribution.

   (E) With sample size 10, the mean of the t-distribution is less than the mean of the z-distribution.

Constructing a Confidence Interval for a Population Mean Quiz

Answer A

Correct. The area under the z-curve is 0.025 to the right of 1.96 and to the left of -1.96. Because 0.0408 is greater than 0.025, we can conclude that the t-curve has more area in its tails.