

**Carrying Out a Chi-Square Test for Goodness of Fit Quiz**

1. A company claims they produce their mixed bag of candies so that, of the candies in the bag, 20 percent are dark chocolate, 60 percent are milk chocolate, and 20 percent are white chocolate. In a random sample of candies of size 50, the counts are as follows: 6 dark, 32 milk, and 12 white. Assuming the conditions for inference are met, what is the test statistic for a chi-square goodness-of-fit test to investigate whether the distribution of the sample is consistent with the company's claim?
- (A)  $\chi^2 = 6^2 + 32^2 + 12^2$
- (B)  $\chi^2 = 10^2 + 30^2 + 10^2$
- (C)  $\chi^2 = (6 - 10)^2 + (32 - 30)^2 + (12 - 10)^2$
- (D)  $\chi^2 = \frac{(6-10)^2}{10} + \frac{(32-30)^2}{30} + \frac{(12-10)^2}{10}$
- (E)  $\chi^2 = \frac{(10-6)^2}{6} + \frac{(30-32)^2}{32} + \frac{(10-12)^2}{12}$
2. A local restaurant claims that it gets 45 percent of its customers from Monday through Thursday, 20 percent on Friday, 20 percent on Saturday, and 15 percent on Sunday. How many degrees of freedom should be used to conduct a chi-square goodness-of-fit test of the claim?
- (A) 3
- (B) 4
- (C) 6
- (D) 7
- (E) It is not possible to determine the degrees of freedom without knowing the sample size.

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3. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A small coffee shop sells freshly squeezed juices in a refrigerated unit with slots where juice is displayed. These slots are called facings. The manager of the coffee shop suspects that the distribution of juice sales is different than the distribution of facings for each type of juice, so the manager records the sales of each juice over a two-week period. The proportion of facings and the sales for each type of juice are shown in the tables.

Juice	Mango	Orange	Apple	Pineapple	Grapefruit	Grape
Proportion of Facings	0.1875	0.250	0.250	0.125	0.125	0.0625

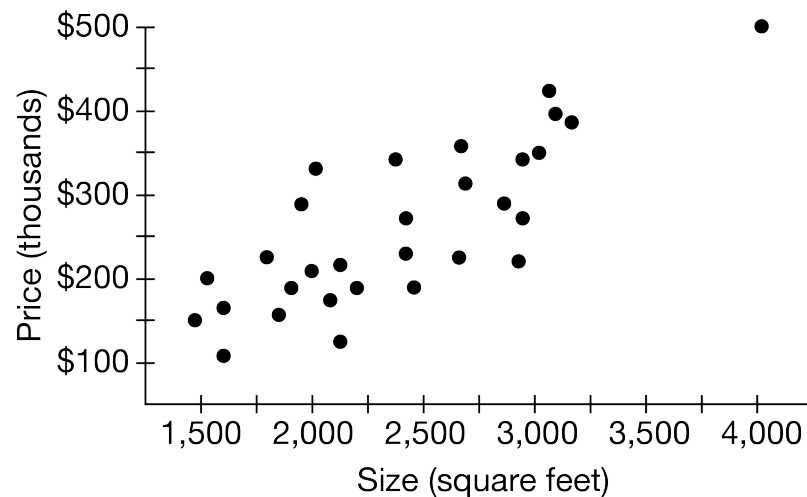
Juice	Mango	Orange	Apple	Pineapple	Grapefruit	Grape
Observed Number of Sales	23	35	46	12	10	5

- (a) Construct a single bar chart that contains both the expected proportion of sales based on the proportion of facings and the observed proportion of sales for each type of juice.
- (b) Assuming the conditions for inference have been met, does the coffee shop owner have sufficient evidence to conclude that the distribution of sales is proportional to the number of facings at a 5 percent level of significance? Conduct the appropriate statistical test to support your conclusion.
4. A chi-square goodness-of-fit test where all assumptions were met yielded the test statistic  $\chi^2 = 12.4$ . Henry claims the corresponding  $p$ -value of 0.03 means that the probability of observing a test statistic of  $\chi^2 = 12.4$  is 0.03, assuming the null hypothesis is true. Which of the following is a valid criticism of this interpretation of the  $p$ -value?
- (A) The null hypothesis can never be assumed to be true.
- (B) The null hypothesis is not stated.
- (C) The  $p$ -value is not the probability of observing 12.4 exactly.
- (D) The significance level is not stated.
- (E) The degrees of freedom are not stated.

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5. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The following scatterplot shows the size, in square feet, and the selling price, in thousands of dollars, for a sample of 30 houses for sale in a certain area.



- (a) Describe the association shown in the scatterplot.
- (b) In the region, houses are considered large if they are greater than 2,500 square feet and expensive if the selling price is greater than \$300,000. The following two-way table summarizes the houses in the sample.

	Large	Not Large	Total
Expensive	8	2	10
Not Expensive	4	16	20
Total	12	18	30

- (i) Use the information in the table to construct a graphical display of the data.
- (ii) Assume there is no association between size (large, not large) and price (expensive, not expensive). Use the given totals to complete the following table with the expected number of houses for each classification if there was no association.

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	Large	Not Large	Total
Expensive			10
Not Expensive			20
Total	12	18	30

For associations displayed in the scatterplot, the strength of linear association is measured by the correlation coefficient. For the scatterplot of houses,  $r = 0.82$ .

For associations that are summarized in two-way tables, the strength of association is measured by the chi-square statistic. The formula for the chi-square statistic is  $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ , where expected is the count assuming no association and observed is the count shown by the data. Greater values of  $\chi^2$  indicate stronger association. For the table of counts in part (b),  $\chi^2 = 10$ .

(c) Suppose the selling price for the most expensive house in the sample is decreased from \$489,000 to \$325,000.

(i) What effect would the decrease have on the value of  $r$ ? Explain your reasoning.

(ii) What effect would the decrease have on the value of  $\chi^2$ ? Explain your reasoning.

(d) Based on your answer to part (c), explain one benefit and one drawback of using  $\chi^2$  rather than  $r$  to measure the strength of an association.

6. A company claims it audits its employees' transactions based on their job level. For entry-level positions, the company claims that 50 percent get a basic audit, 30 percent get an enhanced audit, and 20 percent get a complete audit. The company tests this hypothesis using a random sample and finds  $\chi^2 = 0.771$  with a corresponding  $p$ -value of 0.68. Assuming conditions for inference were met, which of the following is the correct interpretation of the  $p$ -value?
- (A) There is a 68 percent chance of obtaining a chi-square value of at least 0.771.
  - (B) There is a 68 percent chance that the company's claim is correct.
  - (C) If the null hypothesis were true, there would be a 68 percent chance that the company's claim is correct.
  - (D) If the null hypothesis were true, there would be a 68 percent chance of obtaining a chi-square value of 0.771.
  - (E) If the null hypothesis were true, there would be a 68 percent chance of obtaining a chi-square value of at least 0.771.
7. A  $\chi^2$  goodness-of-fit test was used to test the hypothesis that students at a local university select majors in the same proportions as other universities in the state. A chi-square test statistic of  $\chi^2 = 45.6$  was calculated with a corresponding  $p$ -value of 0.005. Which of the following is correct?

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- (A) There is sufficient evidence to conclude that students at the local university do not select majors in the same proportions as do students in the rest of the state.
- (B) There is sufficient evidence to conclude that students at the local university select majors in the same proportions as do students in the rest of the state.
- (C) There is insufficient evidence to conclude that students at the local university do not select majors in the same proportions as do students in the rest of the state.
- (D) There is insufficient evidence to conclude that students at the local university select majors in the same proportions as do students in the rest of the state.
- (E) Students at the local university select majors in the same proportions as do students in the rest of the state.
8. A newspaper article indicated that 43 percent of cars with black seats are white, 46 percent of cars with black seats are blue, 7 percent of cars with black seats are red, and 4 percent of cars with black seats are black. A test was conducted to investigate whether the color of cars with black seats was consistent with the newspaper article. A random sample of cars of these colors was selected, and the value of the chi-square test statistic was  $\chi^2 = 8.2$ . Which of the following represents the  $p$ -value for the test?
- (A)  $P(\chi^2 \geq 8.2) = 0.08$
- (B)  $P(\chi^2 \geq 8.2) = 0.04$
- (C)  $P(\chi^2 \leq 8.2) = 0.96$
- (D)  $P(\chi^2 = 8.2) = 0.00$
- (E) The  $p$ -value cannot be calculated because the sample size is not given.
9. Ms. Harper knows that her students in a computing course can choose from one of three operating systems for the semester: Doors, Banana, or Duix. Ms. Harper wants to test the hypothesis that her students will select the operating systems in the same proportion as students in other computing courses at the university. She conducts a  $\chi^2$  goodness-of-fit test and calculates  $\chi^2 = 3.79$  with a corresponding  $p$ -value of 0.15. Which of the following is correct at a 5-percent level of significance?
- (A) Reject the null hypothesis, since  $3.79 > 2$ .
- (B) Fail to reject the null hypothesis, since  $3.79 > 2$ .
- (C) Reject the null hypothesis, since  $0.15 > 0.05$ .
- (D) Fail to reject the null hypothesis, since  $0.15 > 0.05$ .
- (E) Reject the null hypothesis, since  $0.15 < 3.79$ .
10. A certain type of legal proceeding has three possible outcomes: in favor of party A, in favor of party B, or not in favor of either party. The outcomes are expected to be 40 percent, 20 percent, and 40 percent, respectively. A random sample of 40 cases is selected from a certain judge to investigate whether the judge's outcomes are consistent with the expected outcomes. A chi-square goodness-of-fit test is conducted, and the value of the chi-square test statistic is  $\chi^2 = 9.19$  with a corresponding  $p$ -value of 0.01. Assuming the conditions for inference were met, which of the following is the correct interpretation of the  $p$ -value?
- (A) There is a 1 percent chance that the company's claim is correct.
- (B) If the null hypothesis is true, there is a 1 percent chance that the company's claim is correct.
- (C) If the null hypothesis is true, there is a 1 percent chance of obtaining a chi-square value of 9.19.
- (D) If the null hypothesis is true, there is a 1 percent chance of obtaining a chi-square value of at least 9.19.
- (E) There is a 1 percent chance of obtaining a chi-square value of at least 9.19.

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11. A regional highway uses 8 tollbooths that are open to all vehicles. A chi-square goodness-of-fit test using a significance level of  $\alpha = 0.05$  was conducted to determine whether the tollbooths are all used in equal proportions. A chi-square value of  $\chi^2 = 19.1$  was calculated with a corresponding  $p$ -value of 0.008. Which of the following is correct?
- (A) There is insufficient evidence to suggest that the tollbooths are not used in equal proportions.
  - (B) There is insufficient evidence to suggest that the tollbooths are used in equal proportions.
  - (C) There is sufficient evidence to suggest that the tollbooths are not used in equal proportions.
  - (D) There is sufficient evidence to suggest that the tollbooths are used in equal proportions.
  - (E) The tollbooths are used in equal proportions.