1. A chi-square test for homogeneity was conducted to investigate whether the four high schools in a school district have different absentee rates for each of four grade levels. The chi-square test statistic and p-value of the test were 19.02 and 0.025, respectively. Which of the following is the correct interpretation of the p-value in the context of the test?

(A) Assuming that each high school has the same absentee rate for each grade level, there is a 2.5 percent chance of finding a test statistic 19.02 or smaller.

(B) Assuming that each high school has the same absentee rate for each grade level, there is a 2.5 percent chance of finding a test statistic 19.02 or larger. ✓

(C) Assuming that each high school has a different absentee rate for each grade level, there is a 2.5 percent chance of finding a test statistic 19.02 or larger.

(D) There is a 2.5 percent chance that the absentee rate for each grade level at the four schools is the same.

(E) There is a 2.5 percent chance that the absentee rate for each grade level at the four schools is different.

Answer B

Correct. The null hypothesis is that there is no difference in the absentee rates for the four high schools. Under the assumption that the null hypothesis is true, a p-value of 0.025 means that there is a 2.5 percent probability of obtaining a test statistic value as extreme as or more extreme than a test statistic value of 19.02.

2. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A large national bank determines if each of its branches is profitable or not profitable. Additionally, the location of each branch is classified as urban, suburban, or rural. A summary of the profitability and location type of a random sample of 175 of the bank's branches is shown in the table.

	Profitable	**Not Profitable**
Urban	64	17
Suburban	39	22
Rural	18	15

(a) From the sample of 175 branches summarized above:

(i) What proportion of branches are profitable?
(ii) What proportion of branches from an urban location are profitable?
(iii) What proportion of branches from a rural location are profitable?

(b) Assuming the conditions for inference are met, does the data provide convincing statistical evidence that profitability and location type are independent for the bank at a 5 percent level of significance? Complete the appropriate inference procedure to support your answer.

4-part Inference scoring

The primary goals of this question are to assess a student's ability to (1) compute conditional relative frequencies from a two-way table and (2) identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question.

Scoring

Part (b) has three scoring steps. Those scoring steps 1, 2, 3 and part (a) are each scored as essentially correct (E), partially correct (P), or incorrect (I). So, part (b) is worth 3 Es and part (a) is worth 1 E.

Each essentially correct (E) part counts as 1 point.

Each partially correct (P) part counts as ½ point.

If a response is between two scores (for example, 2½ points, use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

Reasons to score up:

· *All notation is correct and clearly marked*

· *All explanations are clear*

· No wrong information is included that was not part of the scoring (for example, saying sample size must be greater than 30 when that has nothing to do with the problem)

· No minor calculation errors are made, if they are not part of the scoring

· Interpretation parts are especially strong

Reasons to score down:

· Notation is not wrong, but is spotty and not clearly marked

· Explanations are not wrong, but are hard to follow

· Wrong or extraneous information is included but not part of scoring

· Minor calculation errors that are not part of the scoring are made

· Interpretation parts are scored an E but are considered a weak E

0	1	2	3	4 ✓

Scoring steps 1, 2, 3 and part (a) sum to 4 points

OR

Scoring steps 1, 2, 3 and part (a) sum to 3½ points AND a holistic approach is used to decide to score up

- ☐ Part (a) essentially correct
- ☐ Part (a) partially correct
- ☐ Part (a) incorrect
- ☐ Scoring Step 1 Part (b) essentially correct
- ☐ Scoring Step 1 Part (b) partially correct
- ☐ Scoring Step 1 Part (b) incorrect
- ☐ Scoring Step 2 Part (b) essentially correct
- ☐ Scoring Step 2 Part (b) partially correct
- ☐ Scoring Step 2 Part (b) incorrect
- ☐ Scoring Step 3 Part (b) essentially correct
- ☐ Scoring Step 3 Part (b) partially correct
- ☐ Scoring Step 3 Part (b) incorrect

Solution

Part (a):

(a-i) The proportion of branches that are profitable is equal to $\frac{(64+39+18)}{175} \approx 0.6914.$

(a-ii)The proportion of branches from an urban location that are profitable is equal to $\frac{64}{(64+17)} \approx 0.7901$

(a-iii)The proportion of branches from a rural location that are profitable is equal to $\frac{18}{(18+15)} \approx 0.5455$

Scoring

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components

The numeric answer to part (a-i) is correct.

The numeric answers to parts (a-ii) and (a-iii) are correct.

The response indicates that a conditional probability is asked for in parts (a-ii) and (a-iii) either through the use of conditional notation when describing the probability, or work showing a fraction with a number in the denominator counting a proper subset of the branches.

Partially correct (P) if the response satisfies only two of the three components

Incorrect (I) if the response does not meet the criteria for E or P

Solution

Part (b)

Scoring step 1: Identification and hypotheses

The appropriate test is a chi-square test for independence. The hypotheses are:

H_0 : Profitability and location type are independent

H_a : Profitability and location type are not independent

OR

H_0 : There is no association between profitability and location

H_a : There is an association between profitability and location type

Scoring

Part (b)

Scoring step (1) is scored as follows:

Essentially correct (E) if the response satisfies the following three components.

· Correctly states, either in words or in a formula, the chi-square test for independence

· Indicates correct variables (profitability and location type)

· Correct hypotheses

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Solution

Part (b)

Scoring step 2: Calculation

The expected counts for each cell in the table are as follows.

	Profitable	Not Profitable
Urban	$\frac{(81)(121)}{175} \approx 56.01$	$\frac{(81)(54)}{175} \approx 24.99$
Suburban	$\frac{(61)(121)}{175} \approx 42.18$	$\frac{(61)(54)}{175} \approx 18.82$
Rural	$\frac{(33)(121)}{175} \approx 22.82$	$\frac{(33)(54)}{175} \approx 10.18$

The number of degrees of freedom is $(3-1)(2-1)=2$

The χ^2 test statistic is

$$\chi^2 = \frac{(64-56.01)^2}{56.01} + \frac{(17-24.99)^2}{24.99} + \frac{(39-42.18)^2}{42.18} + \frac{(22-18.82)^2}{18.82} + \frac{(18-22.82)^2}{22.82} + \frac{(15-10.18)^2}{10.18} \approx 7.77$$

The corresponding *p*-value, found using technology, is approximately 0.021.

Scoring

Part (b)

Scoring step (2) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

· The correct test statistic

· The correct *p*-value *or* correct critical value (5.99)

Partially correct (P) if the response satisfies only one of the two components

Incorrect (I) if the response does not meet the criteria for E or P

Note: The *p*-value in component 2 is satisfied if it follows correctly from an incorrectly calculated value of the test

statistic.

Solution

Scoring step 3: Conclusion with justification

Since the *p*-value is less than the significance level of 0.05, the null hypothesis is rejected. There is sufficient evidence to conclude that profitability and location type are not independent.

Scoring

Part (b)

Scoring step (3) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· Comparison of *p*-value to alpha (or test statistic to critical value)

· Correct decision to reject the null

· States conclusion in context

Partially correct (P) if the response satisfies only two of the three components

Incorrect (I) if the response does not meet the criteria for E or P

3. The manager of the cafeteria at a local high school wanted to see if there was an association between a student's grade level and whether the student approves of the food choices in the cafeteria. The manager selected a random sample of students and asked if they approved of the food choices and also recorded the grade levels of the students. If a student was in ninth or tenth grade, he or she was labeled as an underclassman, and if the student was in eleventh or twelfth grade, he or she was labeled as an upperclassman. The table shows the results of the survey.

	Underclassman	**Upperclassman**	**Total**
Approves of the Cafeteria Food	50	20	70
Does Not Approve of the Cafeteria Food	30	60	90
Total	80	80	160

Assuming that all conditions for inference have been met, which of the following equations gives the appropriate chi-square test statistic and the correct number of degrees of freedom to determine if there is an association between grade level and whether a student approves of the food choices in the cafeteria?

(A) $\chi^2 = \frac{(50-40)^2}{40} + \frac{(20-40)^2}{40} + \frac{(30-40)^2}{40} + \frac{(60-40)^2}{40}$ with 4 degrees of freedom

(B) $\chi^2 = \frac{(50-35)^2}{50} + \frac{(20-35)^2}{20} + \frac{(30-45)^2}{30} + \frac{(60-45)^2}{60}$ with 4 degrees of freedom

(C) $\chi^2 = \frac{(50-35)^2}{35} + \frac{(20-35)^2}{35} + \frac{(30-45)^2}{45} + \frac{(60-45)^2}{45}$ with 4 degrees of freedom

(D) $\chi^2 = \frac{(50-40)^2}{40} + \frac{(20-40)^2}{40} + \frac{(30-40)^2}{40} + \frac{(60-40)^2}{40}$ with 1 degree of freedom

(E) $\chi^2 = \frac{(50-35)^2}{35} + \frac{(20-35)^2}{35} + \frac{(30-45)^2}{45} + \frac{(60-45)^2}{45}$ with 1 degree of freedom ✓

Answer E

Correct. The correct formula for the test statistic of the chi-square test for independence is $\chi^2 = \Sigma\frac{(\text{observed count}-\text{expected count})^2}{\text{expected count}}$, with degrees of freedom equal to (number of rows minus 1)(number of columns minus 1).

4. Students in a high school statistics class wanted to see if the distribution of the colors of a popular candy was different in the bags for different types of candies the company manufactures. The students purchased several large bags of regular candies, tropical-flavored candies, and sour-flavored candies. For each type of candy, the students took a random sample of 100 candies and recorded how many of each color (red, green, yellow, or blue) were in the sample. The students verified the conditions for inference and calculated a chi-square test statistic of 12.59 with a corresponding p-value of 0.05. Which of the following is the correct interpretation of the p-value in the context of the test?

(A) The hypothesis test has a significance level of $\alpha = 0.05$.

(B) There is a 5 percent chance that the distribution of colors is different for the different types of candies.

(C) There is a 5 percent chance that the distribution of colors is the same for the different types of candies.

(D) Assuming that the distribution of colors for the different types of candies is the same, there is a 5 percent chance of finding a test statistic of 12.59 or larger. ✓

(E) Assuming that the distribution of colors for the different types of candies is different, there is a 5 percent chance of finding a test statistic of 12.59 or larger.

Answer D

Correct. The null hypothesis is that there is no difference in the distribution of colors. Under the assumption that the null hypothesis is true, a p-value of 0.05 means that there is a 5 percent probability of obtaining a test statistic value as extreme as, or more extreme than, 12.59.

5. The district manager of four different restaurants wanted to investigate whether the four restaurants differed with respect to customers ordering dessert or not based on family classification (with children or without children). Independent random samples of 100 customers who ordered dessert were selected from each restaurant, and the customers were identified as either being with children or without children. After verifying the conditions for the appropriate hypothesis test, the manager calculated a chi-square test statistic of 6.45 with an associated p-value of 0.092. Based on the p-value and $\alpha = 0.05$, what conclusion should the manager make regarding the proportion of customers who order dessert at each restaurant and the customers' family classification?

(A) There is convincing statistical evidence to suggest that the proportion of customers who order dessert at each restaurant is the same based on family classification.

(B) There is convincing statistical evidence to suggest that the proportion of customers who order dessert at each restaurant is not the same based on family classification.

(C) There is not convincing statistical evidence to prove that the proportion of customers who order dessert at each restaurant is not the same based on family classification.

(D) There is not convincing statistical evidence to suggest that the proportion of customers who order dessert at each restaurant is the same based on family classification.

(E) There is not convincing statistical evidence to suggest that the proportion of customers who order dessert at each restaurant is not the same based on family classification. ✓

Answer E

Correct. The null hypothesis is that the proportion of customers who order dessert at the restaurants is the same based on family classification. Since the p-value is greater than α, the correct decision is to fail to reject the null hypothesis. This means that there is insufficient evidence to reject the null hypothesis and conclude the alternative hypothesis.

6. A state highway commission is considering removing the lane that allows people to pay cash for a toll on a toll road and requiring all people who use the toll road to pay with an electronic transponder that is connected to their car. The commission wants to know whether the proportion of people who live in the northern part of the state and are in favor of removing the cash lane is different from the proportion of people who live in the southern part of the state and are in favor of removing the cash lane. Independent random samples are selected from the northern and southern parts of the state. The table summarizes the responses of those surveyed.

	Northern	**Southern**	**Total**
Remove Cash Lanes	112	98	210
Keep Cash Lanes	89	105	194
Total	201	203	404

Which of the following is closest to the p-value of the appropriate test to investigate whether the proportion of people living in the northern part of the state who are in favor of removing the cash lane is different from the proportion of people living in the southern part of the state who are in favor of removing the cash lane?

(A) 0.0671

(B) 0.1342 ✓

(C) 0.5235

(D) 0.6912

(E) 0.9329

Answer B

Correct. The correct test to use is the chi-square test for homogeneity. The expected counts are $\frac{(210)(201)}{404} = 104.48$, $\frac{(210)(203)}{404} = 105.52$, $\frac{(194)(201)}{404} = 96.52$, and $\frac{(194)(203)}{404} = 97.48$. The chi-square test statistic is $\chi^2 = \frac{(112-104.48)^2}{104.48} + \frac{(98-105.52)^2}{105.52} + \frac{(89-96.52)^2}{96.52} + \frac{(105-97.48)^2}{97.48} \approx 2.243$ with degrees of freedom of $(2-1)(2-1) = 1$. The p-value is $P(\chi^2 \geq 2.243) = 0.1342$.

7. An administrator at a local high school wants to investigate whether there is an association between the grade level of a student (either ninth, tenth, eleventh, or twelfth) and how the student commutes to school (either walks, bikes, takes the bus, receives a ride, or drives). After a chi-square test for association was conducted, the results indicated that the chi-square test statistic was 14.63 with a p-value of 0.2623. Which of the following is the correct interpretation of the p-value in the context of the test?

(A) There is a 26.23 percent chance that grade level and how a student commutes to school are independent.

(B) There is a 26.23 percent chance that there is no association between grade level and how a student commutes to school.

(C) Assuming there is no association between a student's grade level and how the student commutes to school, there is a 26.23 percent chance of finding a test statistic that is 14.63 or larger. ✓

(D) Assuming that a student's grade level and the way the student commutes to school are dependent, there is a 26.23 percent chance of finding a test statistic that is 14.63 or larger.

(E) Assuming that a student's grade level and the way the student commutes to school are dependent, there is a 26.23 percent chance of finding a test statistic that is 14.63 or smaller.

Answer C

Correct. The null hypothesis is that there is no association between grade level and how a student commutes to school. Under the assumption that the null hypothesis is true, a p-value of 0.2623 means that there is a 26.23 percent probability of obtaining a test statistic value as extreme as or more extreme than a test statistic value of 14.63.

8. A chi-square test was conducted to investigate whether there is an association between a person's favorite flavor of ice cream and their favorite toppings. Each of 200 randomly selected customers at an ice cream parlor was asked to pick their favorite flavor from vanilla, chocolate, chocolate chip, or none of these. They were also asked to pick their favorite topping from chocolate sauce, peanuts, crumbled cookies, crushed candies, or none of these. The hypothesis test had a test statistic of 24.97 with an associated p-value of 0.015. If the significance level of the test was $\alpha = 0.05$, which of the following is the correct decision for this hypothesis test?

(A) There is not convincing statistical evidence to suggest an association between favorite ice cream flavor and favorite topping.

(B) There is convincing statistical evidence to suggest an association between favorite ice cream flavor and favorite topping. ✓

(C) There is convincing statistical evidence to suggest there is not an association between favorite ice cream flavor and favorite topping.

(D) There is proof that a person's favorite ice cream topping is dependent on the person's favorite ice cream flavor.

(E) There is proof that a person's favorite ice cream topping is independent of the person's favorite ice cream flavor.

Answer B

Correct. The null hypothesis that there is no association between a person's favorite flavor of ice cream and that person's favorite topping should be rejected since the p-value is less than α. There is convincing statistical evidence to suggest an association.

9. A survey was conducted to investigate whether there is an association between a person's age and their thoughts on what a state senate should do about state parks. Participants selected from "leave state parks the way they are," "increase funding to state parks," or "completely overhaul state parks," and their age was categorized as under 30 years old, between 30 and 50 years old, and over 50 years old. The hypothesis test statistic was calculated to be 15.01. Which of the following is closest to the p-value of the test?

(A) 0.0047 ✓

(B) 0.0103

(C) 0.0202

(D) 0.9897

(E) 0.9953

Answer A

Correct. The value 0.0047 is the probability that χ^2 is greater than 15.01 in an χ^2 distribution with $(3-1)(3-1)=4$ degrees of freedom.

10. An administrator at a local high school wanted to investigate whether there is an association between the amount of time a student studies for a test and the type of extracurricular activity the student is involved in. Three hundred students selected at random were asked how long they had studied for the last math test and how many extracurricular activities they are involved in. The times they had studied were recorded as either not at all, less than 30 minutes, or more than 30 minutes. Each student also identified which extracurricular activity (out of a total of 5 extra curricular activities) they were involved in. The calculated chi-square test statistic was 7.53 with a corresponding p-value of 0.4807. Based on this p-value, which of the following is the correct decision for the appropriate hypothesis test at the $\alpha = 0.05$ significance level?

(A) Reject the null hypothesis. The test is statistically significant because a p-value of 0.4807 is greater than a significance level of 0.05.

(B) Reject the null hypothesis. The test is statistically significant because a p-value of 0.4807 is less than the test statistic of 7.53.

(C) Fail to reject the null hypothesis. The test is not statistically significant because a p-value of 0.4807 is greater than a significance level of 0.05. ✓

(D) Fail to reject the null hypothesis. The test is not statistically significant because a p-value of 0.4807 is less than the test statistic of 7.53.

(E) Accept the null hypothesis. The test is not statistically significant because a p-value of 0.4807 is greater than a significance level of 0.05.

Answer C

Correct. The null hypothesis is that the amount of time a student studies for a test and type of extracurricular activity are independent (no association). The correct decision is to fail to reject the null

hypothesis since the p-value is greater than α $(0.4807 > 0.05)$.