Justifying a Claim Based on a Confidence Interval for a Population Proportion Quiz

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The manufacturer of a certain type of new cell phone battery claims that the average life span of the batteries is 500 charges; that is, the battery can be charged at least 500 times before failing. To investigate the claim, a consumer group will select a random sample of cell phones with the new battery and use the phones through 500 charges of the battery. The proportion of batteries that fail to last through 500 charges will be recorded. The results will be used to construct a 95 percent confidence interval to estimate the proportion of all such batteries that fail to last through 500 charges.

(a) Explain in context what it means to be 95 percent confident.

(b) Suppose the consumer group conducts its investigation with a random sample of 5 cell phones with the new battery, and 1 battery out of the 5 fails to last through 500 charges. Verify all conditions for inference for a 95 percent confidence interval for a population proportion. Indicate whether any condition has not been met. Do not construct the interval.

In some studies, large sample sizes are not feasible because of the time and expense involved. However, small sample sizes can create issues with inference procedures. A simulation can be used to investigate what might happen with intervals constructed from small sample sizes. Consider a population in which 30 percent of the population displays a certain characteristic. For each trial of the simulation, 5 observations are selected from the population and the sample proportion $\hat{p}$ is calculated, where $\hat{p}$ represents the proportion in the sample that display the characteristic. The following table shows the frequency distribution of $\hat{p}$ in the $1{,}000$ trials. Also shown are the upper and lower endpoints of a 95 percent confidence created from the value of $\hat{p}$, using the formula $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. For example, the sample proportion of 0.4 occurred 309 times in the $1{,}000$ trials and produced a confidence interval of $(-0.029, 0.829)$.

$\hat{p}$	Frequency	Lower Endpoint	Upper Endpoint
0	168	0	0
0.2	360	-0.151	0.551
0.4	309	-0.029	0.829
0.6	133	0.171	1.029
0.8	28	0.449	1.151
1.0	2	1	1

(c) Based on the simulation, what proportion of the 95 percent confidence intervals capture the population proportion of 0.3 ? Explain how you determined your answer.

(d) For small sample sizes, an alternate method of constructing a confidence interval is available. To implement this method, first, include an additional 4 observations in the sample, 2 of which are successes and 2 of which are failures. Second, calculate a new sample proportion $\hat{p}_{\text{new}}$ for the new sample. Finally, construct a confidence interval using $\hat{p}_{\text{new}}$, with the formula $\hat{p}_{\text{new}} \pm 1.96\sqrt{\frac{\hat{p}_{\text{new}}(1-\hat{p}_{\text{new}})}{n+4}}$.

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(i) For the cell phone batteries, consider a sample of 5 in which 1 battery fails to last through 500 charges. Using the alternate method described, what is the value of $\hat{p}_{\text{new}}$? Show your work.

The following table shows the results of the original simulation with revised 95 percent confidence intervals constructed with the alternate method.

Original $\hat{p}$	Frequency	Revised Lower Endpoint	Revised Upper Endpoint
0	168	-0.049	0.494
0.2	360	0.025	0.641
0.4	309	0.120	0.769
0.6	133	0.231	0.880
0.8	28	0.359	0.975
1.0	2	0.506	1.049

(ii) Based on the results of the simulation, is the alternate method better than the traditional method described in part (b) to construct a 95 percent confidence interval with a small sample size? Explain your reasoning.

2. From a random sample of potential voters in an upcoming election, 47% indicated they intended to vote for Candidate R. A 95 percent confidence interval was constructed from the sample, and the margin of error for the estimate was 5%.

Which of the following is the best interpretation of the interval?

(A) We are 95% confident that the proportion who intend to vote for Candidate R from the random sample is between 42% and 52%.

(B) We are 95% confident that the proportion who intend to vote for Candidate R from the population is between 42% and 52%.

(C) We are 95% confident that the proportion who intend to vote for Candidate R from the random sample is 47%.

(D) We are 95% confident that the proportion who intend to vote for Candidate R from the population is 47%.

(E) We are confident that 95% of the population intend to vote for Candidate R.

3. A random sample of residents in city J were surveyed about whether they supported raising taxes to increase bus service for the city. From the results, a 95 percent confidence interval was constructed to estimate the proportion of people in the city who support the increase. The interval was $(0.46, 0.52)$.

Based on the confidence interval, which of the following claims is supported?

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(A) More than 90 percent of the residents support the increase.

(B) More than 60 percent of the residents support the increase.

(C) More than 40 percent of the residents support the increase.

(D) Fewer than 10 percent of the residents support the increase.

(E) Fewer than 25 percent of the residents support the increase.

4. A random sample of 1,175 people in a certain country were asked whether they thought climate change was a problem. The sample proportion of those who think climate change is a problem was calculated, and a 95 percent confidence interval was constructed as $(0.146, 0.214)$.

Which of the following is a correct interpretation of the interval?

(A) We are 95 percent confident that any sample of 1,175 people will produce a sample proportion between 0.146 and 0.214.

(B) We are 95 percent confident that the proportion of all people in the country who think climate change is a problem is between 0.146 and 0.214.

(C) We are 95 percent confident that the proportion of people in the sample who think climate change is a problem is between 0.146 and 0.214.

(D) The probability that 95 percent of all people in the country who think climate change is a problem is between 0.146 and 0.214.

(E) The probability is 0.95 that the proportion of all people in the country who think climate change is a problem is between 0.146 and 0.214.

5. Elly and Drew work together to collect data to estimate the percentage of their classmates who own a particular brand of shoe. Using the same data, Elly will construct a 90 percent confidence interval and Drew will construct a 99 percent confidence interval. Which of the following statements is true?

(A) The midpoint of Elly's interval will be greater than the midpoint of Drew's interval.

(B) The midpoint of Elly's interval will be less than the midpoint of Drew's interval.

(C) The width of Elly's interval will be greater than the width of Drew's interval.

(D) The width of Elly's interval will be less than the width of Drew's interval.

(E) The width of Elly's interval will be equal to the width of Drew's interval.

6. Consider a 90 percent confidence interval to estimate a population proportion that is constructed from a sample proportion of 66 percent. If the width of the interval is 10 percent, what is the margin of error?

(A) 2.5 percent

(B) 5 percent

(C) 10 percent

(D) 20 percent

(E) 45 percent

7. Based on findings from a recent study on women's health, researchers created a 90 percent confidence interval of $(0.42, 0.48)$ to estimate the percent of all women who do not find time to focus on their own health. Based on the confidence interval, which of the following claims is not supported?

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(A) Less than half of all women do not find time to focus on their own health.

(B) More than 40 percent of all women do not find time to focus on their own health.

(C) Approximately 45 percent of all women do not find time to focus on their own health.

(D) More than 45 percent of all women do not find time to focus on their own health.

(E) More than 25 percent of all women do not find time to focus on their own health.

8. Consider a 90 percent confidence interval for a population proportion p. Which of the following is a correct interpretation of the confidence level 90 percent?

(A) There is approximately a 90 percent chance that p is contained in the interval.

(B) There is approximately a 90 percent chance that a randomly selected proportion $\hat{p}$ will be contained in the interval.

(C) Approximately 90 percent of all possible sample proportions $\hat{p}$ will be contained in the interval.

(D) In repeated samplings with the same sample size, approximately 90 percent of the intervals created will capture the population proportion p.

(E) In repeated samplings with the same sample size, approximately 90 percent of the intervals created will capture the sample proportion $\hat{p}$.

9. A recent study on the way that people talk indicated, with 95 percent confidence, that between 35 percent and 41 percent of all adults find the word "whatever" to be the most annoying word in conversation.

Based on the confidence interval, which of the following claims is supported?

(A) Less than 25 percent of all adults find the word "whatever" to be the most annoying word in conversation.

(B) More than 30 percent of all adults find the word "whatever" to be the most annoying word in conversation.

(C) More than 45 percent of all adults find the word "whatever" to be the most annoying word in conversation.

(D) More than half of all adults find the word "whatever" to be the most annoying word in conversation.

(E) At least 95 percent of all adults find the word "whatever" to be the most annoying word in conversation.

10. Lila and Robert attend different high schools. They will estimate the population percentage of students at their respective schools who have seen a certain movie. Lila and Robert each select a random sample of students from their respective schools and use the data to create a 95 percent confidence interval. Lila's interval is $(0.30, 0.35)$, and Robert's interval is $(0.27, 0.34)$. Which of the following statements can be concluded from the intervals?

(A) Lila's sample size is most likely greater than Robert's sample size.

(B) Robert's sample size is mostly likely greater than Lila's sample size.

(C) Lila and Robert will both find the same sample proportion of students who have seen the movie.

(D) Lila's interval has a greater degree of confidence than that of Robert.

(E) Robert's interval has a greater degree of confidence than that of Lila.