

The Geometric Distribution Quiz

1. In a certain board game, a player rolls two fair six-sided dice until the player rolls doubles (where the value on each die is the same). The probability of rolling doubles with one roll of two fair six-sided dice is $\frac{1}{6}$.

What is the probability that it takes three rolls until the player rolls doubles?

- (A) $\left(\frac{1}{6}\right)^3$
(B) $\left(\frac{5}{6}\right)^3$
(C) $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3$
(D) $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$
(E) $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2$



Answer D

Correct. Let the discrete random variable X represent the number of rolls it takes until the player rolls doubles. The random variable has a geometric distribution, where $P(X = 3) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$.

2. The transaction history at an electronic goods store indicates that 21 percent of customers purchase the extended warranty when they buy an eligible item. Suppose customers who buy eligible items are chosen at random, one at a time, until one is found who purchased the extended warranty. Let the random variable X represent the number of customers it takes to find one who purchased the extended warranty. Assume customers' decisions on whether to purchase the extended warranty are independent. Which of the following is closest to the probability that $X > 3$; that is, the probability that it takes more than 3 customers who buy an eligible item to find one who purchased the extended warranty?

- (A) 0.131
(B) 0.390
(C) 0.493
(D) 0.507
(E) 0.624



Answer C

Correct. The random variable is geometric with $p = 0.21$. Therefore,
 $P(X > 3) = 1 - P(X \leq 3) = 1 - [0.21 + 0.79(0.21) + 0.79^2(0.21)] \approx 0.493$.

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3. Of all the fish in a certain river, 20 percent are salmon. Once a year, people can purchase a fishing license that allows them to catch up to 8 fish. Assume each catch is independent. Which of the following represents the probability of needing to catch 8 fish to get the first salmon?
- (A) 0.2
(B) $\frac{1}{0.2}$
(C) 0.2^8
(D) $0.2(0.8)^7$ ✓
(E) $0.8(0.2)^7$

Answer D

Correct. Let random variable X represent the number of fish caught to get the first salmon. Random variable X has a geometric distribution with $p = 0.2$. Thus, $P(X = 8) = 0.2(0.8)^7$.

4. According to a recent survey, 47 percent of the people living in a certain region carry a certain genetic trait. People from the region will be selected at random one at a time until someone is found who carries the genetic trait. Let the random variable G represent the number of people selected to find one person who carries the genetic trait. On average, how many people from the region will need to be selected to find one person who carries the genetic trait?
- (A) 0.47
(B) 0.69
(C) 1.55
(D) 2.00
(E) 2.13 ✓

Answer E

Correct. Random variable G has a geometric distribution. The expected value, or mean, of random variable G is $\mu_G = \frac{1}{p} = \frac{1}{0.47} \approx 2.13$ people.

5. Let S represent the number of randomly selected adults in a community surveyed to find someone with a certain genetic trait.

The random variable S follows a geometric distribution with mean 4.66. Which of the following is a correct interpretation of the mean?

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- (A) A value randomly selected from the distribution of S is expected to be 4.66.
- (B) In repeated sampling from the distribution of S , the average of the values will approach 4.66. ✓
- (C) For a sample of values randomly selected from the distribution of S , the average of the sample will be 4.66.
- (D) The probability is 0.66 that a value randomly selected from the distribution of S will be close to the mean.
- (E) For a sample of values randomly selected from the distribution of S , the average of the sample will vary from the population mean by no more than 4.66.

Answer B

Correct. The mean of a probability distribution is the long-run average of the values selected from many repetitions.

6. Let W represent the number of attempted experiments to get one experiment that is not successful.

The random variable W has a geometric distribution with mean 4 and standard deviation 3.5. Which of the following is the best interpretation of the standard deviation?

- (A) A single value randomly selected from the distribution of W will vary from 4 by 3.5 attempted experiments.
- (B) A single value randomly selected from the distribution of W will vary from 3.5 by 4 attempted experiments.
- (C) Values of W typically vary from 4 by about 3.5 attempted experiments, on average. ✓
- (D) Values of W typically vary from 3.5 by about 4 attempted experiments, on average.
- (E) The standard deviation of a random sample of values selected from the distribution of W will be 3.5 attempted experiments.

Answer C

Correct. The standard deviation is a measure of typical variability about the mean, on average. In this case, the values of W typically vary by about 3.5, on average, from the mean of 4 attempted experiments.

7. In which of the following should the random variable X not be modeled with a geometric distribution?

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- (A) According to a recent study, approximately 8% of adults in the country have a master's degree. Let X represent the number of randomly selected adults in the country surveyed to find someone with a master's degree.
- (B) Suppose it is known that 5% of the light bulbs manufactured at a particular company are defective. Let X represent the number of randomly selected light bulbs that are inspected to find one defective light bulb.
- (C) A particular basketball player is known to consistently make 90% of her free throws, and the outcomes of her free-throw attempts are independent. Let X represent the number of attempted free-throws to get one missed free-throw.
- (D) In a bag of 30 different colored candies, about 20% are red. One candy will be selected one at a time without replacement, and its color will be recorded. Let X represent the number of candies selected before red is selected. ✓
- (E) It is believed that about 40% of people in the country have purchased a certain product. Let X represent the number of people randomly selected to find the first one who has purchased the product.

Answer D

Correct. This is not a geometric setting. The selections are not independent because of sampling without replacement from a small population. Therefore, the probability, p , will change with each selection.

8. The random variable W has a geometric distribution with $p = 0.25$. Approximately how far do the values of W typically vary, on average, from the mean of the distribution?
- (A) 1.73
- (B) 2.00
- (C) 3.00
- (D) 3.46 ✓
- (E) 4.00

Answer D

Correct. The standard deviation of random variable W is $\sigma_W = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{0.75}}{0.25} \approx 3.46$.

9. A recent report indicated that 22 percent of the households in a certain community speak a language other than English at home. A reporter will randomly select a household from the community until the first household that speaks a language other than English at home is selected. Let random variable Q represent the number of attempts needed until the first household that speaks a language other than English at home is selected. The random variable Q has a geometric distribution with $p = 0.22$. Which of the following is closest to the variance of the random variable?

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- (A) 0.0484
- (B) 3.5454
- (C) 4.0144
- (D) 4.5455

(E) 16.1157

**Answer E**

Correct. The variance of random variable Q is $(\sigma_Q)^2 = \left[\frac{(\sqrt{1-p})}{p} \right]^2 = \left[\frac{(\sqrt{0.78})}{0.22} \right]^2 \approx 16.1157$.