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**Please note: Some of the questions in this former practice exam may no longer perfectly align with the AP exam. Even though these questions do not fully represent the 2020 exam, teachers indicate that imperfectly aligned questions still provide instructional value. Teachers can consult the Question Bank to determine the degree to which these questions align to the 2020 Exam.**

**This exam may not be posted on school or personal websites, nor electronically redistributed for any reason.** This exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

**Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing.** Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II.

**When all Section I materials have been collected and accounted for and you are ready for the break, say:**

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on your desk during the break. You are not allowed to consult teachers, other students, notes, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you may never discuss the multiple-choice exam content with anyone, and if you disclose the content through any means, your AP Exam score will be canceled. Are there any questions? . . .



You may begin your break. Testing will resume at \_\_\_\_\_ .

## SECTION II: Free Response

**After the break, say:**

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and, using your pen, complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

If this is your last AP Exam, you may keep your Student Pack. Place it under your chair for now. Otherwise I will collect all Student Packs. . . .

Read the information on the back cover of the exam booklet. Do not open the booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs.

**Then say:**

Are there any questions? . . .

Section II has two parts. You have 1 hour and 30 minutes to complete all of Section II. You are responsible for pacing yourself and may proceed freely from one part to the next. You must write your answers in the exam booklet using a pen with black or dark blue ink or a No. 2 pencil. If you use a pencil, be sure that your writing is dark enough to be easily read. If you need more paper during the exam, raise your hand. At the top of each extra sheet of paper you use, write only your AP number and the question number you are working on. Do not write your name. Are there any questions? . . .

You may begin Section II.



**Note Start Time** \_\_\_\_ . **Note Stop Time** \_\_\_\_ .

You should also make sure that Hewlett-Packard calculators' infrared ports are not facing each other and that students are not sharing calculators.

**After 1 hour and 5 minutes, say:**

There are 25 minutes remaining and you may want to move on to Part B, if you have not already started answering that question.

**After 15 minutes, say:**

There are 10 minutes remaining.

**After 10 minutes, say:**

**Stop working and close your exam booklet. Place it on your desk, faceup. . . .**

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their exam booklets. Complete an Incident Report after the exam (see page 67 of the *2017-18 AP Coordinator's Manual* for complete details).

**Then say:**

**Remain in your seat, without talking, while the exam materials are collected. . . .**

Collect a Section II booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

**If you are giving the regularly scheduled exam, say:**

You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.

**If you are giving the alternate exam for late testing, say:**

None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

## After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

**NOTE:** If you administered exams to students with accommodations, review the *2017-18 AP Coordinator's Manual* and the *2017-18 AP SSD Guidelines* for information about completing the NAR form, and returning these exams.

The exam proctor should complete the following tasks if asked to do so by the AP coordinator. Otherwise, the AP coordinator must complete these tasks:

- Complete an Incident Report for any students who used extra paper for the free-response section. (Incident Report forms are provided in the coordinator packets sent with the exam shipments.) **These forms must be completed with a No. 2 pencil.** It is best to complete a single Incident Report for multiple students per exam subject, per administration (regular or late testing), as long as all required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 67 of the *2017-18 AP Coordinator's Manual* for complete details).
- Return all exam materials to secure storage until they are shipped back to the AP Program. (See page 26 of the *2017-18 AP Coordinator's Manual* for more information about secure storage.) Before storing materials, check the “School Use Only” section on page 1 of the answer sheet and:
  - ◆ Fill in the appropriate section number circle in order to access a separate AP Instructional Planning Report (for regularly scheduled exams only) or subject score roster at the class section or teacher level. See “Post-Exam Activities” in the *2017-18 AP Coordinator's Manual*.
  - ◆ Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

# AP<sup>®</sup> Statistics Exam

## SECTION II: Free Response

2018

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

### At a Glance

**Total Time**

1 hour and 30 minutes

**Number of Questions**

6

**Percent of Total Score**

50%

**Writing Instrument**

Either pencil or pen with black or dark blue ink

**Electronic Device**

Graphing calculator expected

### Part A

**Number of Questions**

5

**Suggested Time**

1 hour and 5 minutes

**Percent of Section II Score**

75%

### Part B

**Number of Questions**

1

**Suggested Time**

25 minutes

**Percent of Section II Score**

25%

### IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

### Instructions

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I

Form Code 40BP4-S

90

Formulas begin on page 3.  
Questions begin on page 6.  
Tables begin on page 20.

## Formulas

### (I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If  $X$  has a binomial distribution with parameters  $n$  and  $p$ , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If  $\bar{x}$  is the mean of a random sample of size  $n$  from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



(III) Inferential Statistics

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval:  $\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$  Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$  Special case when $p_1 = p_2$ $\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

# STATISTICS

## SECTION II

### Part A

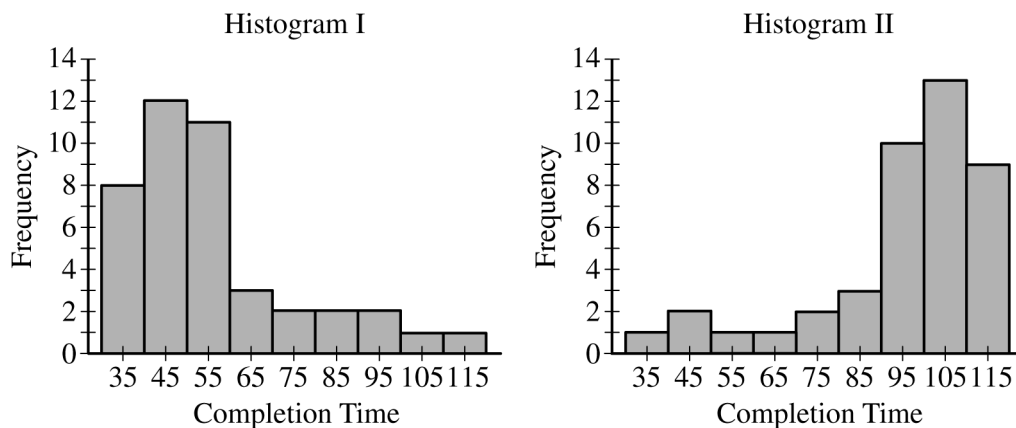
#### Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The students enrolled in honors biology at a high school were given the task of using a spreadsheet program to investigate a topic in genetics. All students in the class had similar background knowledge of the topic. Some students in the class had no spreadsheet experience, Group R, and needed time to learn the program to complete the task. The rest of the students, Group S, had previous spreadsheet experience and typically took less time to complete the task. Each of the histograms below show the distribution of completion times, in minutes, for one of the two groups.



- (a) Of the two histograms shown, I and II, which is more likely to be the distribution of completion times for the students in Group R? Justify your answer.

(b) Describe the shape of a histogram created from the data of the two groups of students combined.

(c) Consider the population of all students in honors biology classes in the high school's state who are given the task of using the spreadsheet program to investigate the topic in genetics. The distribution of the completion times has a shape similar to the combined histogram of students at the high school, with mean 70 minutes and standard deviation 26.5 minutes. For random samples of 50 students taken from the population, describe the sampling distribution of the sample mean completion time.

2. Researchers are designing an experiment to compare two different types of running shoes, A and B, to investigate which type is better for minimizing running time for a one-mile run. The experiment will consist of distributing the shoes to runners who are classified as either professional or recreational. A randomized block design is planned, with blocking by classification of runner. Random samples of 50 professional runners and 50 recreational runners will be selected. Each runner within each classification will be randomly assigned to wear either the type A shoe or the type B shoe, and their running times will be recorded for a one-mile run.

(a) What is a statistical advantage of blocking by the classification of runner?

(b) Why is it important to randomize the type of shoe the runner will wear instead of allowing the runner to choose the shoe?

(c) Explain how the design of the experiment will address replication. What is the benefit of the replication?

3. A large university offers STEM (science, technology, engineering, and mathematics) internships to women in STEM majors at the university. A woman must be 20 years or older to meet the age requirement for the internships. The table shows the probability distribution of the ages of the women in STEM majors at the university.

Age (years)	17	18	19	20	21	22	23 or older
Probability	0.005	0.107	0.111	0.252	0.249	0.213	0.063

- (a) Suppose one woman is selected at random from the women in STEM majors at the university. What is the probability that the woman selected will not meet the age requirement for the internships?

The university will select a sample of 100 women in STEM majors to participate in a focus group about the internships.

- (b) Suppose a simple random sampling process is used to select the sample of 100 women. What is the probability that at least 30 percent of the women in the sample will not meet the age requirement for the internships?

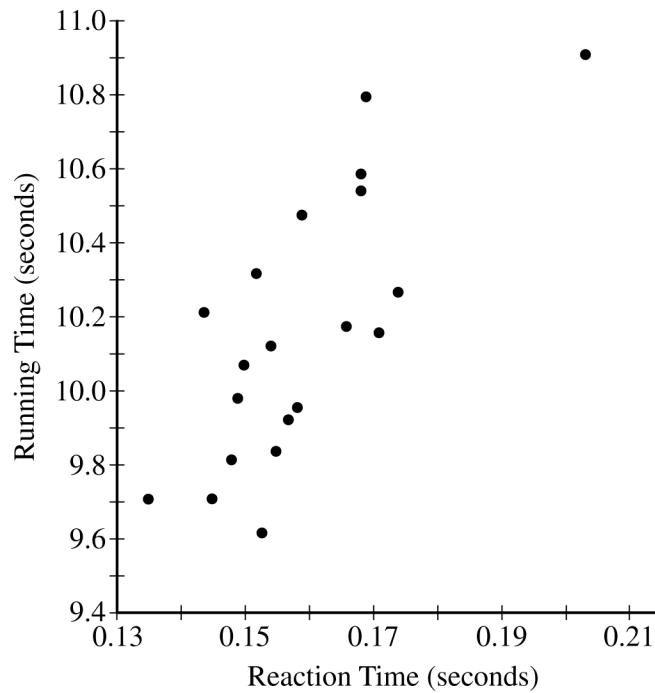
- (c) Suppose a stratified random sampling design is used to select a sample of 30 women who do not meet the age requirement and a sample of 70 women who do meet the age requirement. Based on the probability distribution, is a woman who does not meet the age requirement more likely, less likely, or equally likely to be selected with a stratified random sample than with a simple random sample? Justify your answer.

4. Activity trackers are electronic devices that people wear to record physical activity. Researchers wanted to estimate the mean number of steps taken on a typical workday for people working in New York City who wear such trackers. A random sample of 61 people working in New York City who wear an activity tracker was selected. The number of steps taken on a typical workday for each person in the sample was recorded. The mean was 9,797 steps and the standard deviation was 2,313 steps.
- (a) Construct and interpret a 99 percent confidence interval for the mean number of steps taken on a typical workday for all people working in New York City who wear an activity tracker.



- (b) A wellness director at a company in New York City wants to investigate whether it is unusual for one person working in the city who wears an activity tracker to record approximately 8,500 steps on a typical workday. Is it appropriate to use the confidence interval found in part (a) to conduct the investigation? Explain your answer.

5. The total race time for a 100-meter dash can be considered the sum of two variables: the reaction time to the starting signal and the running time for the 100 meters. The scatterplot shows reaction times and running times for 20 runners in a certain race. The winner was the runner with the least total race time.



- (a) Circle the point on the graph that represents the runner who won the race and approximate the total race time for that runner.

- (b) Based on the graph, is it reasonable to assume that reaction time and running time are independent? Justify your answer.
- (c) Based on the least-squares regression model created from the data, explain why the use of extrapolation to predict the running time for a runner whose reaction time is 0.30 second might not be appropriate.

## STATISTICS

### SECTION II

#### Part B

#### Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A large company produces an equal number of brand-name lightbulbs and generic lightbulbs. The director of quality control sets guidelines that production will be stopped if there is evidence that the proportion of all lightbulbs that are defective is greater than 0.10. The director also believes that the proportion of brand-name lightbulbs that are defective is not equal to the proportion of generic lightbulbs that are defective. Therefore, the director wants to estimate the average of the two proportions.

To estimate the proportion of brand-name lightbulbs that are defective, a simple random sample of 400 brand-name lightbulbs is taken and 44 are found to be defective. Let  $X$  represent the number of brand-name lightbulbs that are defective in a sample of 400, and let  $p_X$  represent the proportion of all brand-name lightbulbs that are defective. It is reasonable to assume that  $X$  is a binomial random variable.

- (a) One condition for obtaining an interval estimate for  $p_X$  is that the distribution of  $\hat{p}_X$  is approximately normal. Is it reasonable to assume that the condition is met? Justify your answer.

- (b) The standard error of  $\hat{p}_X$  is approximately 0.0156. Show how the value of the standard error is calculated.

(c) How many standard errors is the observed value of  $\hat{p}_X$  from 0.10 ?

To estimate the proportion of generic lightbulbs that are defective, a simple random sample of 400 generic lightbulbs is taken and 104 are found to be defective. Let  $Y$  represent the number of generic lightbulbs that are defective in a sample of 400. It is reasonable to assume that  $Y$  is a binomial random variable and the distribution of  $\hat{p}_Y$  is approximately normal, with an approximate standard error of 0.0219. It is also reasonable to assume that  $X$  and  $Y$  are independent.

The parameter of interest for the manager of quality control is  $D$ , the average proportion of defective lightbulbs for the brand-name and the generic lightbulbs.  $D$  is defined as  $D = \frac{p_X + p_Y}{2}$ .

(d) Consider  $\hat{D}$ , the point estimate of  $D$ .

(i) Calculate  $\hat{D}$  using data from the sample of brand-name lightbulbs and the sample of generic lightbulbs.

(ii) Calculate  $s_{\hat{D}}$ , the standard error of  $\hat{D}$ .

Consider the following hypotheses.

$H_0$  : The average proportion of all lightbulbs that are defective is 0.10. ( $D = 0.10$ )

$H_a$  : The average proportion of all lightbulbs that are defective is greater than 0.10. ( $D > 0.10$ )

A reasonable test statistic for the hypotheses is  $W$ , defined as  $W = \frac{\hat{D} - 0.10}{s_{\hat{D}}}$ .

(e) Calculate  $W$  using your answer to part (d).

(f) Chebyshev's inequality states that the proportion of any distribution that lies within  $k$  standard errors of the mean is at least

$$1 - \frac{1}{k^2}.$$

Use Chebyshev's inequality and the value of  $W$  to decide whether there is statistical evidence, at the significance level of  $\alpha = 0.05$ , that  $D$ , the average proportion of all lightbulbs that are defective, is greater than 0.10.

**STOP**

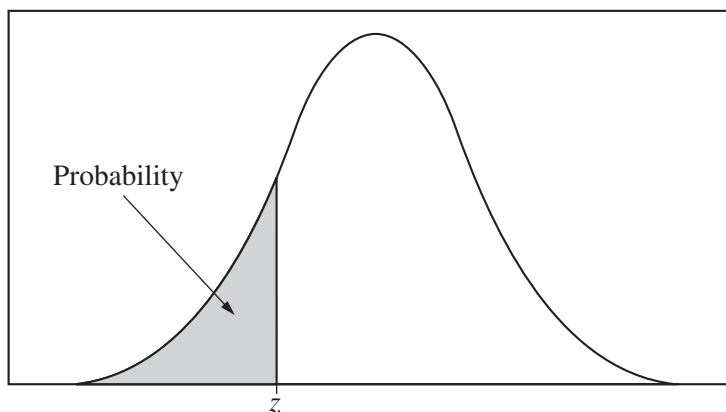
**END OF EXAM**

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**THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.**

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Table entry for  $z$  is the probability lying below  $z$ .



**Table A** Standard normal probabilities

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



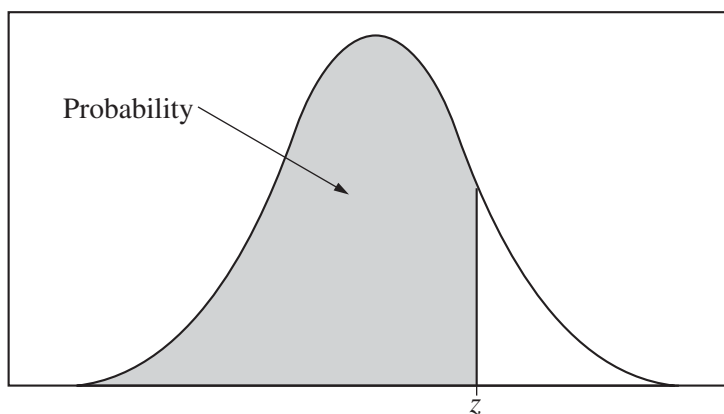
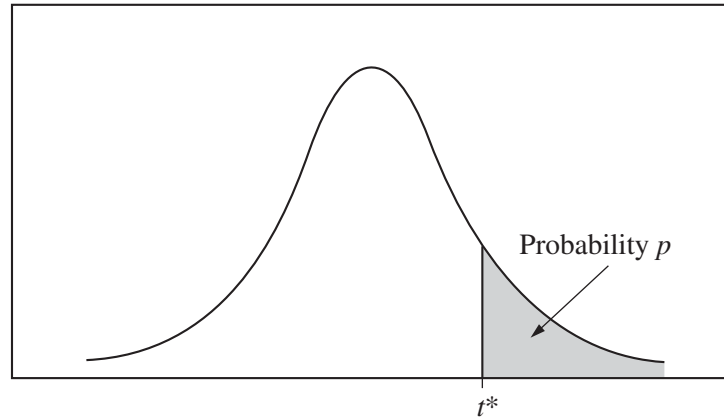


Table entry for  $z$  is the probability lying below  $z$ .

**Table A** (Continued)

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

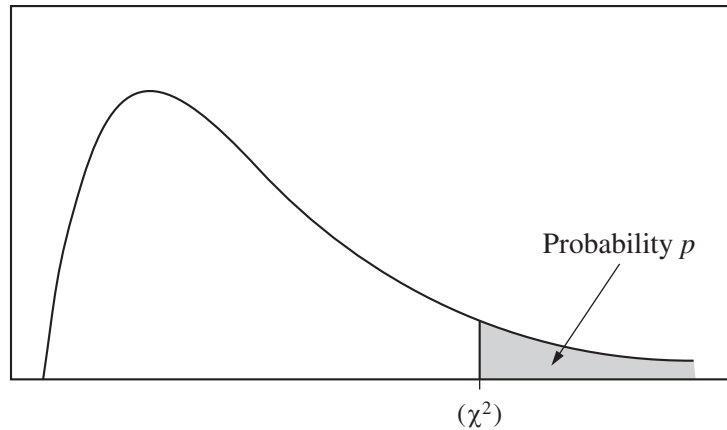
Table entry for  $p$  and  $C$  is the point  $t^*$  with probability  $p$  lying above it and probability  $C$  lying between  $-t^*$  and  $t^*$ .



**Table B**  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level $C$												

Table entry for  $p$  is the point ( $\chi^2$ ) with probability  $p$  lying above it.



**Table C**  $\chi^2$  critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2