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Lesson 4.1: **Inverse Variation and the Reciprocal Function**

Objectives	● Use inverse variation to write and graph reciprocal functions. ● Identify the effect of transformations on the graph of the parent reciprocal function and define the effects of h and k on the function f(x) =1/(x-h) + k..
Language Objective	● Students will write their own definition of key vocabulary. ●
Essential Understanding	A reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases, the other decreases.

Using words or pictures, define the following Vocabulary:

Asymptote:_____

_____

Constant of Variation:_____

_____

Inverse variation:_____

_____

reciprocal function:_____

_____

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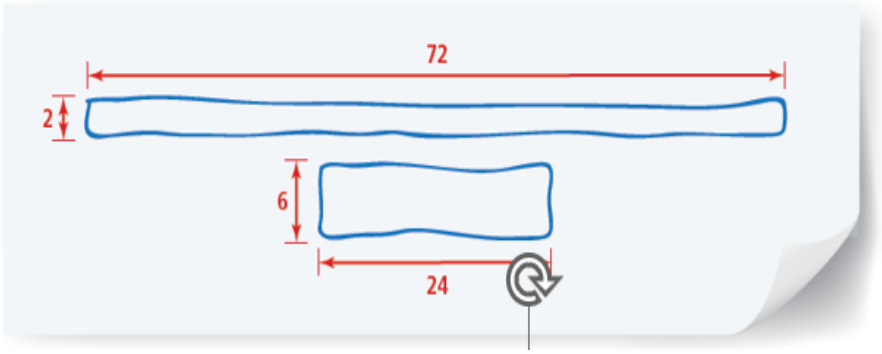
MODEL & DISCUSS

The two rectangles shown both have an area of 144 square units.



a. Sketch as many other rectangles as you can that have the same area on a piece of graph paper. organize and record your data for the lengths and widths of the rectangles.

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b. Considering rectangles with an area of 144 square units, what happens to the width of the rectangle as the length increases? Show your reasoning below, use pictures, words, or mathematical expressions.

c. Examine at least five other pairs of rectangles, each pair sharing the same area. How would you describe the relationship between the lengths and widths? Show your reasoning below, use pictures, words, or mathematical expressions.

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EXAMPLE 1 Identify Inverse Variation

How do you determine if a relationship represents an inverse variation?

A. Does the table of values represent an inverse variation?

How do you determine if a relationship represents an inverse variation?

B. Does the table of values represent an inverse variation?

Try It!

1. Determine if each table of values represents an inverse variation.

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EXAMPLE 2 Use Inverse Variation

In an inverse variation, $x = 10$ when $y = 3$. Write an equation to represent the inverse variation. Then find the value of y when $x = -6$.

Try It!

2. In an inverse variation, $x = 6$ and $y = \frac{1}{2}$.

a. What is the equation that represents the inverse variation?

b. What is the value of y when $x = 15$?

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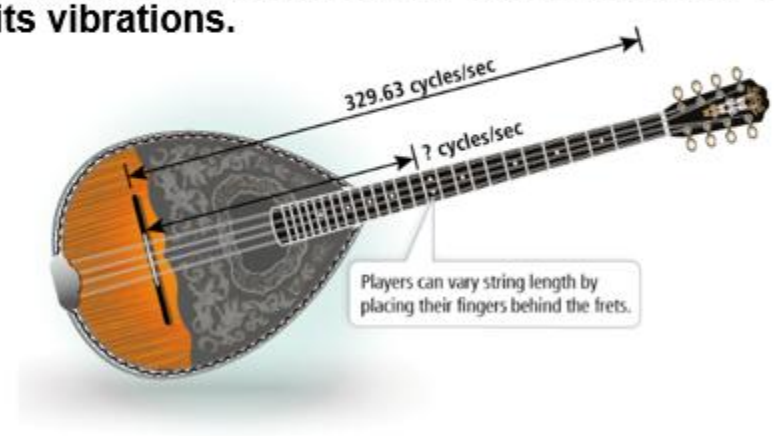
EXAMPLE 3 Use an Inverse Variation Model

On a Greek bouzouki, the string length *s* varies inversely with the frequency *f* of its vibrations.



The frequency of a 26-inch E-string is 329.63 cycles per second. What is the frequency when the string length is 13 inches?

Step 1 Write the equation for an inverse variation and solve for *k*.

Step 2 Substitute $k = 8{,}570.38$ in the inverse variation equation and then find *f*.

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Try It!

3. The amount of time it takes for an ice cube to melt varies inversely to the air temperature, in degrees. At 20° Celsius, the ice will melt in 20 minutes. How long will it take the ice to melt if the temperature is 30° Celsius?

EXAMPLE 4 Understand the Graph of the Reciprocal Function

What are the key features of the reciprocal function $f(x) = \frac{1}{x}$?

Practice & problem solving: 7, 3, 2, 18

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Try It!

4. Graph the function $g(x) = \frac{10}{x}$. What are the domain, range, and asymptotes of the function?

EXAMPLE 5 Graph Translations of the Reciprocal Function

Graph $g(x) = \frac{1}{x-3} + 2$. What are the equations of the asymptotes? What are the domain and range?

Use technology to graph the parent function, $f(x) = \frac{1}{x}$.

In terms of $f(x)$, you can write $g(x)$ as $g(x) = \frac{1}{x-3} + 2 = f(x-3) + 2$.

Practice & problem solving: 2, 18, 5, 12, 16, 14

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Try It!

5. Graph $g(x) = \frac{1}{x+2} - 4$. What are the equations of the asymptotes? What are the domain and range?

Practice & problem solving: 5, 12, 16, 14

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Practice & Problem Solving

Ex 1,2,3

7. Two robots can do a task in 5 min, working together. The first robot, working alone, can do the task in 15 min. How many minutes will it take the second robot, working alone, to do the task?

Ⓐ 10 Ⓑ 7.5 Ⓒ 5 Ⓓ 2

Ex 3

3. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^2 - 3x - 4}{3 - x^2}$?

Ⓐ $y = -1; x = \pm\sqrt{3}$

Ⓑ $y = 1; x = \pm\sqrt{3}$

Ⓒ $y = -1; x = 1$ and $x = \sqrt{3}$

Ⓓ $y = -1; x = 1$ and $x = -\sqrt{3}$

Ex 4

2. What is the domain of the function $f(x) = \frac{x^2 - x - 2}{x^4 - 81}$?

Ⓐ All real numbers except 3

Ⓑ All real numbers except −1 and 3

Ⓒ All real numbers except −3 and 3

Ⓓ All real numbers except −3, 1, and 3

Ex 4

3. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^2 - 3x - 4}{3 - x^2}$?

Ⓐ $y = -1; x = \pm\sqrt{3}$

Ⓑ $y = 1; x = \pm\sqrt{3}$

Ⓒ $y = -1; x = 1$ and $x = \sqrt{3}$

Ⓓ $y = -1; x = 1$ and $x = -\sqrt{3}$

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Ex 4

18. Select all the functions whose graphs have a horizontal asymptote at $y = \frac{2}{3}$.

- ☐ A. $y = \frac{2}{3x - 1}$
- ☐ B. $y = \frac{2x^2 + 1}{3x^2 - 2}$
- ☐ C. $y = \frac{2}{3} + \frac{1}{x}$
- ☐ D. $y = \frac{2x - 3}{3x^2 + 1}$
- ☐ E. $y = 3 + \frac{3}{2x}$

Ex 5

5. Describe the transformations needed to translate the graph of $y = \frac{1}{x}$ to the graph of $y = 2 + \frac{1}{x - 5}$.

Ⓐ to the left 5 and up 2

Ⓑ to the left 2 and down 5

Ⓒ to the right 2 and down 5

Ⓓ to the right 5 and up 2

Ex 5

12. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^4 + 3}{x^4 + 2x^2 - 3}$?

Ⓐ $y = 1; x = \pm 1$

Ⓑ $y = 1; x = \pm 3$

Ⓒ $y = 0; x = 1$

Ⓓ $y = 0; x = -1$

Ex 5

14. The graph of $xy = 6$ is translated up 2 units and to the left 2 units. Select all the possible equations for the translated graph.

- ☐ A. $y = 2 + \frac{6}{x + 2}$
- ☐ B. $\frac{y}{2} = \frac{x + 5}{x + 2}$
- ☐ C. $y = \frac{2x + 10}{x + 2}$
- ☐ D. $y = \frac{6x + 10}{x - 2}$
- ☐ E. $y = 4 + \frac{4}{x - 2}$

Ex 5

16. What are the horizontal and vertical asymptotes of the graph of $y = \frac{-x + 3}{x - 8}$?

Ⓐ $y = -1; x = 8$

Ⓑ $y = -1; x = -8$

Ⓒ $y = 1; x = 8$

Ⓓ $y = 1; x = -8$

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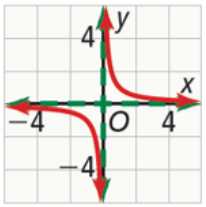
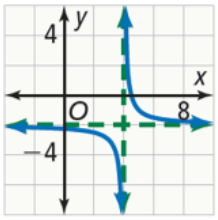
CONCEPT SUMMARY

Inverse Variation and the Reciprocal Function

	Inverse Variation	Transformations of the Reciprocal Function
WORDS	An inverse variation is a relation between two variables such that as one variable increases, the other decreases proportionally.	The reciprocal function models the inverse variation, $y = \frac{1}{x}$. Like other functions, it can be transformed.
ALGEBRA	$y = \frac{k}{x}$, where $k \neq 0$	$y = \frac{a}{x - h} + k$
EXAMPLES	$y = \frac{1}{x}$ asymptotes: $x = 0$ $y = 0$ 	$y = \frac{1}{x - 4} - 2$ $h = 4$ $k = -2$ Parent is transformed down 2 and right 4. asymptotes: $x = 4$ $y = -2$ 

Practice & problem solving: