

Name_____

Date_____

Period_____

Robert Colson's

Lesson 4.1: Inverse Variation and the Reciprocal Function

Objectives	<ul style="list-style-type: none">● Use inverse variation to write and graph reciprocal functions.● Identify the effect of transformations on the graph of the parent reciprocal function and define the effects of h and k on the function $f(x) = 1/(x-h) + k$.
Language Objective	<ul style="list-style-type: none">● Students will write their own definition of key vocabulary.● Students will do a 3 reads on pg. 2● Students will complete sentence frames with a word bank on pg. 8
Essential Understanding	A reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases, the other decreases.

define the following Vocabulary:

Asymptote: _____

Constant of Variation: _____

Inverse variation: _____

reciprocal function: _____

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13. It takes 4 h for Faucet A to fill a tank, and it takes Faucet B 6 h. How many hours will it take the two faucets to fill the tank together?

First Read - Understanding the Context
What is the core of the problem?

- I think this problem is about...
- This problem is about...
- In this problem, I think that...is happening.
- I agree with you that...
- I disagree because...

Second Read - Interpreting the Question
What are we trying to find out?

- The question is...
- I know the problem is asking...because...

Third Read - Identifying Information
What are the important quantities, relationships, and other relevant information?

- The quantities are...
- I can count...
- I notice the quantities are...
- These quantities help me to answer...
- The information from the situation that we need is...

EXAMPLE 1 Identify Inverse Variation

How do you determine if a relationship represents an inverse variation?

A. Does the table of values represent an inverse variation?

<i>x</i>	1	2	3	4	6	12
<i>y</i>	12	6	4	3	2	1

Find the product, xy , for each column in the table.

B. Does the table of values represent an inverse variation?

<i>x</i>	1	2	3	4	5	6
<i>y</i>	20	17	14	11	8	5

Find the product, xy , for each column in the table.

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Try It!

1. Determine if each table of values represents an inverse variation.

a.

x	1	2	3	5	6	15
y	25.5	12.75	8.50	5.10	4.25	1.70

b.

x	6.6	5.5	4.4	3.3	2.2	1.1
y	3	5	7	9	11	13

EXAMPLE 2 Use Inverse Variation

In an inverse variation, $x = 10$ when $y = 3$. Write an equation to represent the inverse variation. Then find the value of y when $x = -6$.

Step 1 Write the equation for an inverse variation and solve for k .

Step 2 Substitute $k = 30$ in the inverse variation equation and then find y .

Try It!

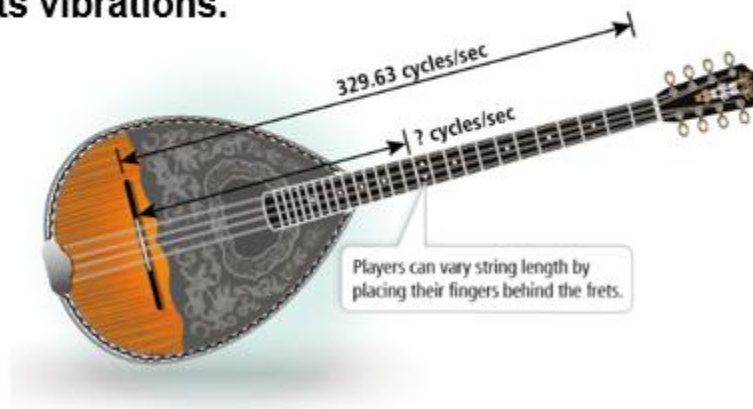
2. In an inverse variation, $x = 6$ and $y = \frac{1}{2}$.

a. What is the equation that represents the inverse variation?

b. What is the value of y when $x = 15$?

EXAMPLE 3 Use an Inverse Variation Model

On a Greek bouzouki, the string length s varies inversely with the frequency f of its vibrations.



The frequency of a 26-inch E-string is 329.63 cycles per second. What is the frequency when the string length is 13 inches?

Step 1 Write the equation for an inverse variation and solve for k .

Step 2 Substitute $k = 8,570.38$ in the inverse variation equation and then find f .

Try It!

3. The amount of time it takes for an ice cube to melt varies inversely to the air temperature, in degrees. At 20° Celsius, the ice will melt in 20 minutes. How long will it take the ice to melt if the temperature is 30° Celsius?

EXAMPLE 4 Understand the Graph of the Reciprocal Function

What are the key features of the reciprocal function $f(x) = \frac{1}{x}$?

The **reciprocal function** $f(x) = \frac{1}{x}$ maps every non-zero real number to its reciprocal. Use technology to graph the function.



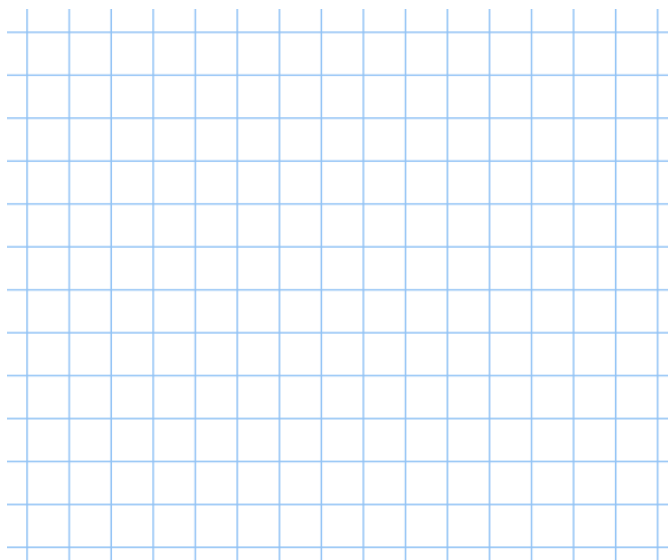
EXAMPLE 4 Understand the Graph of the Reciprocal Function

What are the key features of the reciprocal function $f(x) = \frac{1}{x}$?

Fill in the following sentences with terms from the word bank ->>	Word Bank
The graph of f has a _____ asymptote $y = 0$.	Vertical
An _____ is a line that a graph approaches.	
For x -values close to 0:	\neq
- As x approaches 0 from _____ values, $f(x)$ goes to ∞	x
- As x approaches 0 from _____ values, $f(x)$ goes to $-\infty$	Negative
The graph of f has a _____ asymptote $x = 0$.	0
	Horizontal
The domain of $f(x) = 1/x$ is $\{ \text{_____} \mid x \neq 0 \}$.	Positive
The range is $\{y \mid y \text{ _____ } 0\}$.	asymptote
The end behavior is $f(x) \rightarrow \text{_____}$ as $x \rightarrow \pm \infty$.	

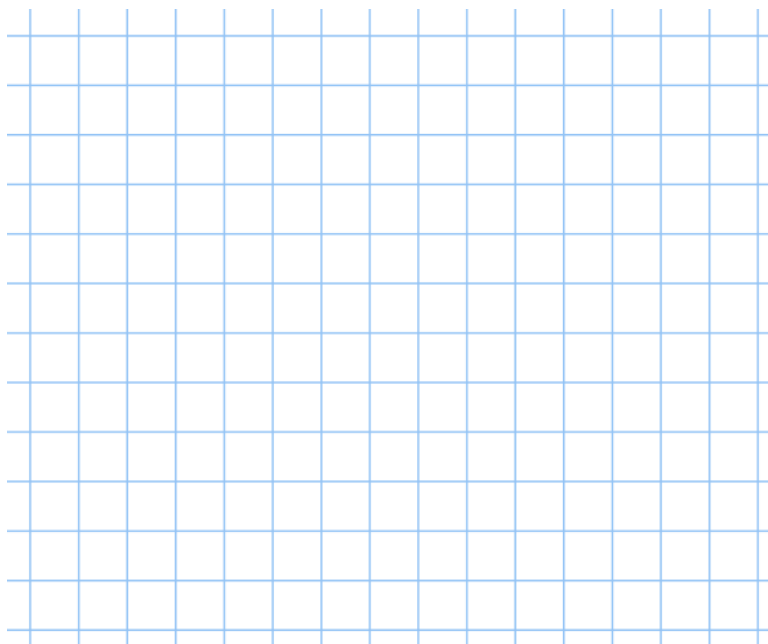
Try It!

4. Graph the function $g(x) = \frac{10}{x}$. What are the domain, range, and asymptotes of the function?

**EXAMPLE 5** Graph Translations of the Reciprocal Function

Graph $g(x) = \frac{1}{x-3} + 2$. What are the equations of the asymptotes?
What are the domain and range?

Use technology to graph the parent function, $f(x) = \frac{1}{x}$.



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In terms of $f(x)$, you can write $g(x)$ as $g(x) = \frac{1}{x-3} + 2 = f(x-3) + 2$.

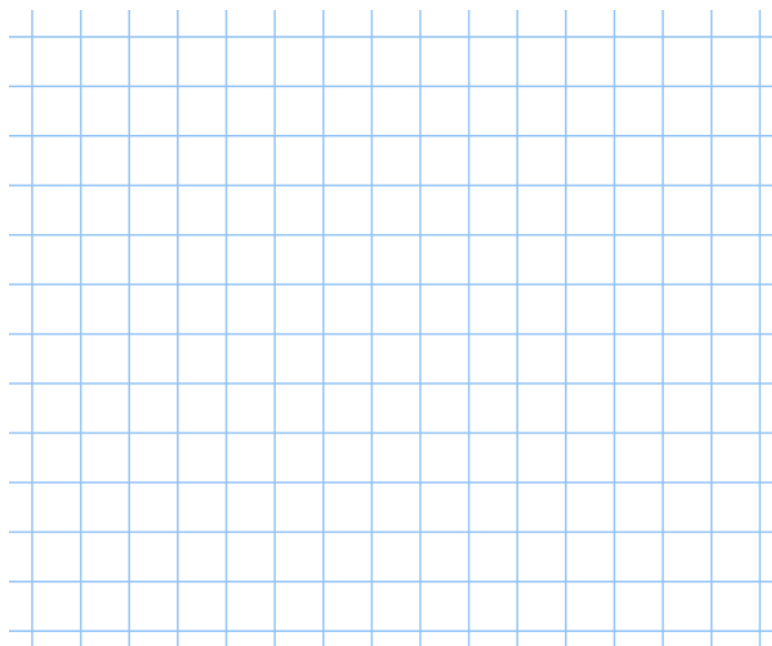
Therefore, the graph of g is the graph of f translated 3 units right and 2 units up.

Sketch $g(x)$, and then check with DESMOS.com.



Try It!

5. Graph $g(x) = \frac{1}{x+2} - 4$. What are the equations of the asymptotes?
What are the domain and range?



Practice & Problem Solving**Ex 1,2,3**

7. Two robots can do a task in 5 min, working together. The first robot, working alone, can do the task in 15 min. How many minutes will it take the second robot, working alone, to do the task?

(A) 10 (B) 7.5 (C) 5 (D) 2

Ex 3

3. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^2 - 3x - 4}{3 - x^2}$?

(A) $y = -1$; $x = \pm\sqrt{3}$
(B) $y = 1$; $x = \pm\sqrt{3}$
(C) $y = -1$; $x = 1$ and $x = \sqrt{3}$
(D) $y = -1$; $x = 1$ and $x = -\sqrt{3}$

Ex 4

2. What is the domain of the function $f(x) = \frac{x^2 - x - 2}{x^4 - 81}$?

(A) All real numbers except 3
(B) All real numbers except -1 and 3
(C) All real numbers except -3 and 3
(D) All real numbers except -3 , 1, and 3

Ex 4

3. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^2 - 3x - 4}{3 - x^2}$?

(A) $y = -1$; $x = \pm\sqrt{3}$
(B) $y = 1$; $x = \pm\sqrt{3}$
(C) $y = -1$; $x = 1$ and $x = \sqrt{3}$
(D) $y = -1$; $x = 1$ and $x = -\sqrt{3}$

Ex 4

18. Select all the functions whose graphs have a horizontal asymptote at $y = \frac{2}{3}$.

- ☐ A. $y = \frac{2}{3x-1}$
☐ B. $y = \frac{2x^2+1}{3x^2-2}$
☐ C. $y = \frac{2}{3} + \frac{1}{x}$
☐ D. $y = \frac{2x-3}{3x^2+1}$
☐ E. $y = 3 + \frac{3}{2x}$

13. It takes 4 h for Faucet A to fill a tank, and it takes Faucet B 6 h. How many hours will it take the two faucets to fill the tank together?

- (A) 1.4 (B) 2.4 (C) 2.0 (D) 5.0

Ex 5

5. Describe the transformations needed to translate the graph of $y = \frac{1}{x}$ to the graph of $y = 2 + \frac{1}{x-5}$.

- (A) to the left 5 and up 2
 (B) to the left 2 and down 5
 (C) to the right 2 and down 5
 (D) to the right 5 and up 2

Ex 5

12. What are the horizontal and vertical asymptotes of the graph of

$$y = \frac{x^4+3}{x^4+2x^2-3}?$$

- (A) $y = 1; x = \pm 1$
 (B) $y = 1; x = \pm 3$
 (C) $y = 0; x = 1$
 (D) $y = 0; x = -1$

Ex 5

14. The graph of $xy = 6$ is translated up 2 units and to the left 2 units. Select all the possible equations for the translated graph.

- ☐ A. $y = 2 + \frac{6}{x+2}$
☐ B. $\frac{y}{2} = \frac{x+5}{x+2}$
☐ C. $y = \frac{2x+10}{x+2}$
☐ D. $y = \frac{6x+10}{x-2}$
☐ E. $y = 4 + \frac{4}{x-2}$

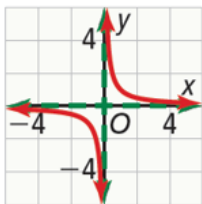
Ex 5

16. What are the horizontal and vertical asymptotes of the graph of

$$y = \frac{-x+3}{x-8}?$$

- (A) $y = -1; x = 8$
 (B) $y = -1; x = -8$
 (C) $y = 1; x = 8$
 (D) $y = 1; x = -8$

CONCEPT SUMMARY**Inverse Variation and the Reciprocal Function**

	Inverse Variation	Transformations of the Reciprocal Function
WORDS	An inverse variation is a relation between two variables such that as one variable increases, the other decreases proportionally.	The reciprocal function models the inverse variation, $y = \frac{1}{x}$. Like other functions, it can be transformed.
ALGEBRA	$y = \frac{k}{x}$, where $k \neq 0$	$y = \frac{a}{x-h} + k$
EXAMPLES	$y = \frac{1}{x}$ asymptotes: $x = 0$ $y = 0$ 	$y = \frac{1}{x-4} - 2$ $h = 4$ $k = -2$ Parent is transformed down 2 and right 4. asymptotes: $x = 4$ $y = -2$ 