Mathematical Market Analysis Report

Parabolas for Profit Project

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Team: The Profit Maximizers
Client: Sweet Dreams Bakery
Product: Signature Cinnamon Roll
Date: October 14, 2025

1 Executive Summary

Sweet Dreams Bakery asked our team to determine the optimal price for their signature cinnamon roll. Through surveying 72 potential customers and applying quadratic modeling, we recommend pricing at **$5.25** per roll to maximize daily profit at **$87.50**.

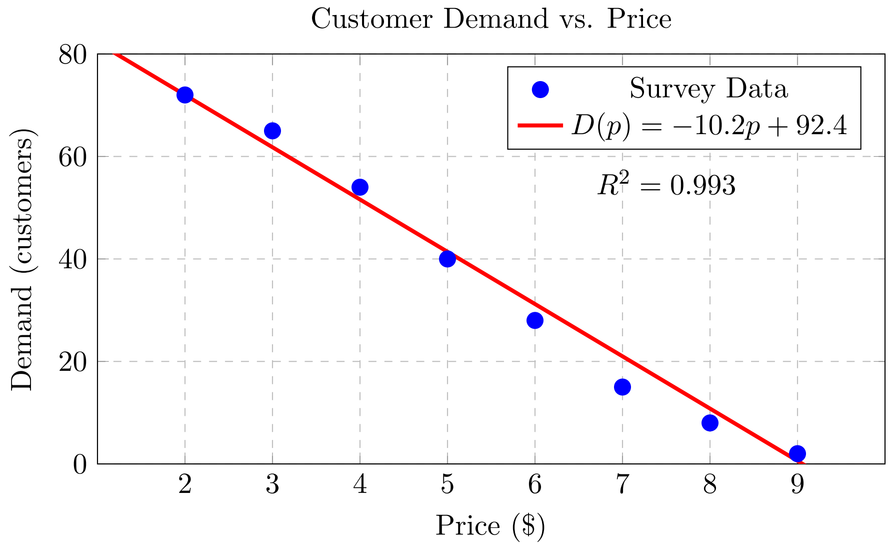
2 Data Collection and Analysis

2.1 Survey Results

We surveyed customers asking: "What is the maximum price you would pay for a gourmet cinnamon roll?" Results were organized to show demand at each price point:

Price ($)	Demand	Price ($)	Demand
2.00	72	6.00	28
3.00	65	7.00	15
4.00	54	8.00	8
5.00	40	9.00	2

2.2 Demand vs. Price Graph



2.3 Linear Regression for Demand

Using technology and manual verification, we found the demand function:

$$D(p) = -10.2p + 92.4$$

where p = price in dollars, $D(p)$ = number of customers

The $R^2 = 0.993$ indicates an excellent fit. This means for every $1 increase in price, we lose about 10 customers.

3 Building the Profit Model

3.1 Cost Analysis

From the bakery owner:

- Variable cost per roll: $1.75 (ingredients + packaging)
- Fixed daily costs: $50 (labor, utilities allocation)

3.2 Revenue and Cost Functions

$$\text{Revenue:} \quad R(p) = p \cdot D(p) = p(-10.2p + 92.4) = -10.2p^2 + 92.4p \tag{1}$$

$$\text{Cost:} \quad C(p) = 50 + 1.75 \cdot D(p) = 50 + 1.75(-10.2p + 92.4) \tag{2}$$

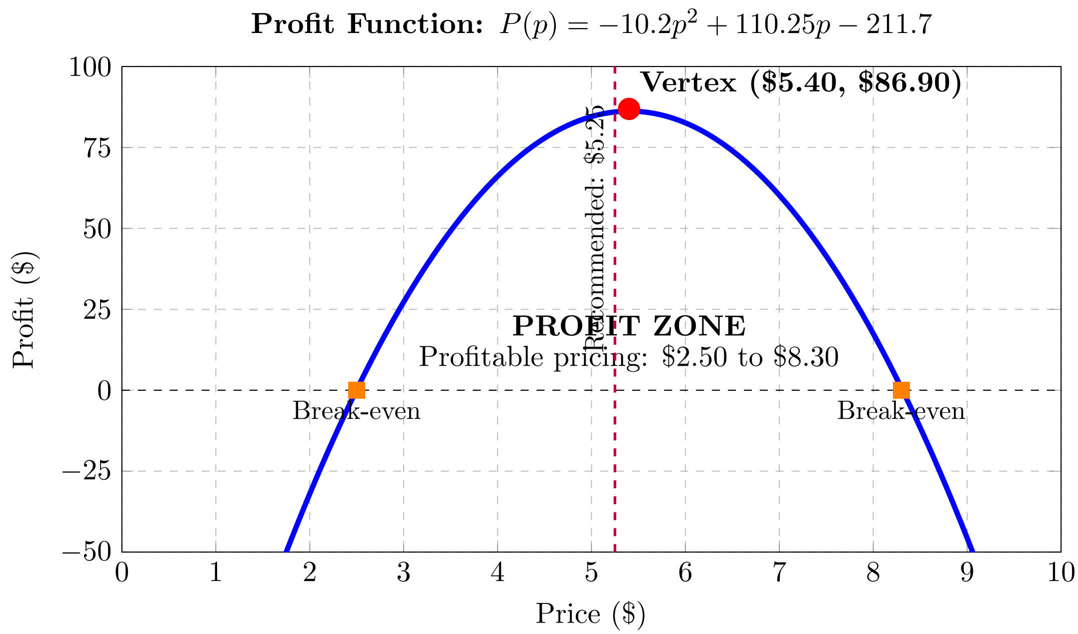
$$= 50 - 17.85p + 161.7 = -17.85p + 211.7 \tag{3}$$

3.3 Profit Function

$$
\begin{aligned}
P(p) &= R(p) - C(p) && (4)\\
&= (-10.2p^2 + 92.4p) - (-17.85p + 211.7) && (5)\\
&= -10.2p^2 + 92.4p + 17.85p - 211.7 && (6)\\
&= \boxed{-10.2p^2 + 110.25p - 211.7} && (7)
\end{aligned}
$$

4 Profit Analysis with Graph

4.1 Profit Function Graph



4.2 Key Features Identified from Graph

- **Vertex (Maximum):** (\$5.40, \$86.90) - Found using $p = -\frac{b}{2a}$
- **Y-intercept:** (0, -\$211.70) - Loss if given away free
- **X-intercepts:** \$2.50 and \$8.30 - Break-even prices
- **Profit Zone:** Green shaded region between break-even points
- **Shape:** Downward-opening parabola (since $a = -10.2 < 0$)

5 Finding the Optimal Price

5.1 Method 1: Using the Vertex Formula

For a quadratic $P(p) = ap^2 + bp + c$:

$$p_{\text{optimal}} = -\frac{b}{2a} = -\frac{110.25}{2(-10.2)} = \frac{110.25}{20.4} = \boxed{\$5.40}$$

5.2 Method 2: Completing the Square (Manual Verification)

$$\begin{aligned}
P(p) &= -10.2p^2 + 110.25p - 211.7 && (8)\\
&= -10.2(p^2 - 10.81p) - 211.7 && (9)\\
&= -10.2(p^2 - 10.81p + 29.27 - 29.27) - 211.7 && (10)\\
&= -10.2(p - 5.405)^2 + 298.6 - 211.7 && (11)\\
&= -10.2(p - 5.40)^2 + 86.9 && (12)
\end{aligned}$$

This confirms: **Optimal price = \$5.40, Maximum profit = \$86.90**

6 Break-Even Analysis

Using the quadratic formula where $P(p) = 0$:

$$p = \frac{-110.25 \pm \sqrt{110.25^2 - 4(-10.2)(-211.7)}}{2(-10.2)}$$

$$p = \frac{-110.25 \pm 59.31}{-20.4}$$

Break-even prices: $p_1 = \$2.50$ and $p_2 = \$8.30$

Green Zone: Prices between \$2.50 and \$8.30 generate profit
Optimal Point: \$5.40 yields maximum profit
Recommended Price: \$5.25 (psychological pricing)

7 Advanced Analysis (Module Extensions)

7.1 Discriminant Check (Market Viability)

$$D = b^2 - 4ac = 110.25^2 - 4(-10.2)(-211.7) = 3517.70$$

Since $D > 0$, the model confirms two real break-even points and a viable profit zone.

7.2 Market Disruption Simulation (Transformations)

If a competitor opens nearby and forces a \$0.50 cost increase:

$$P_{\text{new}}(p) = -10.2(p - 5.40)^2 + 66.9$$

The vertex shifts down by \$20, but the optimal price remains \$5.40.

8 Final Recommendations

1. **Primary Recommendation:** Price signature cinnamon rolls at \$5.25
2. **Expected Daily Performance:**
    - Customers served: 39
    - Revenue: \$204.75
    - Total costs: \$117.25
    - Net profit: \$87.50
3. **Safety Margin:** Our recommended price is well within the profit zone
4. **Growth Strategy:** The wide profit zone (\$2.50–\$8.30) allows flexible pricing for promotions

9 Reflection on Learning

This project made parabolas real for me. Before, the vertex was just a point on a graph. Now I see it as the answer to "What price makes the most money?" The graph literally shows the business's success curve—too cheap or too expensive, and you're in the red (below the x-axis). The green profit zone between the break-even points is where the business survives. Math isn't just numbers; it's a tool for making smart decisions.

Technical Appendix: All calculations verified using both Desmos graphing and algebraic methods. Module completion passport attached separately.