Unit 1 Progress Check: FRQ

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Age (years)	Frequency
$20 \leq x < 30$	160
$30 \leq x < 40$	130
$40 \leq x < 50$	100
$50 \leq x < 60$	115
$60 \leq x < 70$	125
$70 \leq x < 80$	135

The manager of a vacation resort believes that the ages of adult visitors to the resort can be modeled by a normal distribution. The manager surveyed a random sample of 765 visitors and recorded their age. A summary of the responses is shown in the frequency table, where x represents the age of the visitor.

(a) Construct a histogram of the distribution of ages.

(b) Write a few sentences to describe the distribution of ages of the adult visitors to the resort.

(c) Does the histogram provide convincing evidence that the surveyed ages come from a normal distribution? Explain your answer.

Part A, B, and C

The primary goals of this question are to assess a student's ability to (1) create a histogram to represent quantitative data graphically; (2) describe a distribution of a quantitative variable; and (3) compare the resulting graph to a normal distribution.

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

0	1	2	3	4 ✓

All three parts essentially correct

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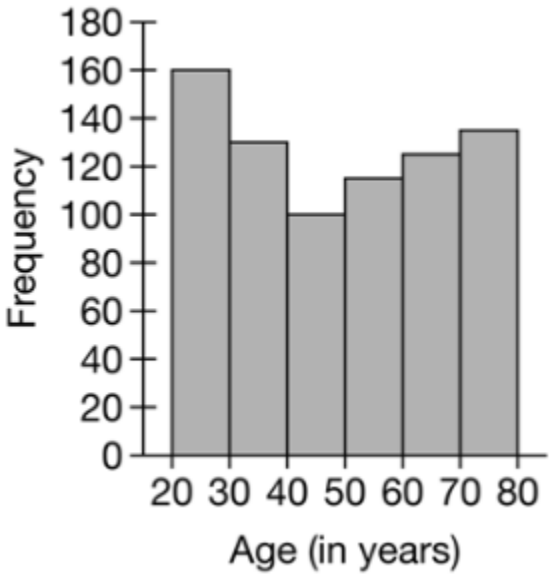
- [ ] Part (a) essentially correct
- [ ] Part (a) partially correct
- [ ] Part (a) incorrect
- [ ] Part (b) essentially correct
- [ ] Part (b) partially correct
- [ ] Part (b) incorrect
- [ ] Part (c) essentially correct
- [ ] Part (c) partially correct
- [ ] Part (c) incorrect

Solution

Part (a):



Scoring

Part (a) is scored as follows.

Essentially correct (E) if the histogram contains six bars with approximately correct values and contains all four of the following components:

· The horizontal axis is labeled with correct numbers.

· The horizontal axis is labeled with a correct verbal description.

· The vertical axis is labeled with correct numbers.

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· The vertical axis is labeled with a correct verbal description.

Partially correct (P) if the histogram contains six bars with approximately correct values and contains two or three of the following four components:

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (b): The distribution of ages of the adult visitors to the resort appears to be roughly symmetric. The distribution has a median between 40 and 50 years. The ages of adult visitors vary from a minimum value between 20 and 30 years to a maximum value between 70 and 80 years. The range in the ages of adult visitors could be between 40 and 60 years. From the frequency table and histogram, outliers and gaps cannot be identified.

Scoring

Part (b) is scored as follows.

Essentially correct (E) if the response the response includes reasonable comments on the following 4 components:

· Shape

· Center

· Variability

· Context

Partially correct (P) if the response includes 2 or 3 of the 4 components

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (c): The histogram does not suggest that the data come from a normal distribution. A normal distribution is symmetric and mound-shaped. In the histogram above, values farther from the mean are more frequent than values near the mean, which is not likely if the data had come from a normal distribution.

Scoring

Part (c) is scored as follows.

Essentially correct (E) if the response indicates that the histogram does not suggest that the data come from a normal distribution and it provides strong justification for this conclusion

Partially correct (P) if the response indicates that the histogram does not suggest that the data come from a normal distribution and it provides weak justification for this conclusion

Incorrect (I) if the response does not satisfy the criteria for E or P

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2. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A certain type of bird lives in two regions of a state. The distribution of weight for birds of this type in the northern region is approximately normal with mean 10 ounces and standard deviation 3 ounces. The distribution of weight for birds of this type in the southern region is approximately normal with mean 16 ounces and standard deviation 2.5 ounces.

(a) Calculate the z-scores for a weight of 13 ounces for a bird living in the northern region and for a weight of 13 ounces for a bird living in the southern region.

(b) Is it more likely that a bird of this type with a weight greater than 13 ounces lives in the northern region or the southern region? Justify your answer.

Part A and B

The primary goals of this question are to assess a student's ability to (1) compute z-scores and (2) compare an observed value using two distributions and determine which distribution is the more likely source of the observation.

Scoring

Parts (a) and (b) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

0	1	2	3	4 ✓

Both parts essentially correct

- [ ] Part (a) essentially correct
- [ ] Part (a) partially correct
- [ ] Part (a) incorrect
- [ ] Part (b) essentially correct
- [ ] Part (b) partially correct
- [ ] Part (b) incorrect

Solution

Part (a):

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Northern region: $z_n = \frac{x-\mu}{\sigma} = \frac{13-10}{3} = 1$

Southern region: $z_s = \frac{x-\mu}{\sigma} = \frac{13-16}{2.5} = -1.2$

Scoring

Part (a) is scored as follows.

Essentially correct (E) if the correct *z*-scores are listed with supporting calculations and appropriate labeling

Partially correct (P) if the correct *z*-scores are listed but without supporting calculations or appropriate labeling

Incorrect(I) if the response does not satisfy the criteria for E or P

Note:

Including the general formula for *z*-score $\left(z = \frac{x-\mu}{\sigma}\right)$ is not necessary to earn an E.

Solution

Part (b): A bird with a weight of 13 ounces is 1 standard deviation above the mean if the bird is from the northern region. Approximately 16% of the birds from the northern region have a weight greater than 13 ounces. A bird with a weight of 13 ounces is 1.2 standard deviations below the mean if the bird is from the southern region. Approximately 88% of the birds from the southern region have a weight greater than 13 ounces. Therefore, it is more likely that a bird of this type with a weight greater than 13 ounces lives in the southern region.

Scoring

Part (b) is scored as follows.

Essentially correct (E) if the response concludes that it is more likely that a bird of this type with a weight greater than 13 ounces lives in the southern region AND directly compares the two *z*-scores calculated in part (a) or the proportions based on the standard normal curve

Partially correct (P) if the response correctly calculates the proportions based on the standard normal curve but fails to make a comparison

Incorrect(I) if the response does not satisfy the criteria for E or P