Setting Up a Chi-Square Test for Goodness of Fit Quiz

1. A veterinarian keeps track of the types of animals treated by an animal clinic. The following distribution represents the percentages of animals the clinic has historically encountered.

Animal type	Dogs	Cats	Livestock	Birds	Other
Percent	61%	22%	8%	6%	3%

If the animal clinic treats 230 animals in a month, how many of each animal type would be expected?

(A)

Animal type	Dogs	Cats	Livestock	Birds	Other
Expected	61	22	8	6	3

(B)

Animal type	Dogs	Cats	Livestock	Birds	Other
Expected	122	44	16	12	6

(C)

Animal type	Dogs	Cats	Livestock	Birds	Other
Expected	140	51	18	14	7

(D)

Animal type	Dogs	Cats	Livestock	Birds	Other
Expected	46	46	46	46	46

(E) ✓

Animal type	Dogs	Cats	Livestock	Birds	Other
Expected	140.3	50.6	18.4	13.8	6.9

Answer E

Correct. To determine the expected counts for each category, the sample size of 230 is multiplied by the percentage for the corresponding category.

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2. A private lake sells boating memberships and currently has 600 members. During the application process the potential members are asked which recreational activity they do the most. Their choices are fishing, skiing, boarding, swimming, or tubing. The lake manager chooses clients according to their interests to maximize the use of all areas of the lake. Every month, the lake rangers randomly sample the boats on the lake and categorize them according to the activity they are doing. The lake manager performs a chi-square goodness-of-fit test using the following null hypothesis to see whether their samples differ significantly from what the original applications claim.

$H_0 : p_{fish} = 0.26, p_{ski} = 0.21, p_{board} = 0.30, p_{swim} = 0.12, p_{tube} = 0.11$

In order to meet the conditions for independence and large counts for a chi-square goodness-of-fit test, which of the following represents all possible sizes of the monthly samples?

(A) $n \geq 30$

(B) $30 \leq n \leq 50$

(C) $46 \leq n \leq 60$ ✓

(D) $n \geq 46$

(E) $n \leq 60$

Answer C

Correct. To meet the independence condition that $n \leq 0.10N$, the sample sizes must be 60 or less $(n \leq 0.10 \cdot 600)$. To meet the large counts condition that each expected value must be at least 5, the sample sizes must be 46 or greater (i.e., check the smallest proportion [0.11] to make sure $0.11(n) \geq 5$).

3. Which of the following is an appropriate description of the chi-square distribution?

(A) A chi-square distribution will only contain positive values and will be skewed right, with the skew becoming less pronounced with increasing degrees of freedom. ✓

(B) A chi-square distribution will only contain positive values and will be skewed left, with the skew becoming less pronounced with increasing degrees of freedom.

(C) A chi-square distribution will only contain positive values and will be skewed right, with the skew becoming more pronounced with increasing degrees of freedom.

(D) A chi-square distribution will contain positive and negative values, and will be skewed right, with the skew becoming less pronounced with increasing degrees of freedom.

(E) A chi-square distribution will contain positive and negative values and will be skewed left, with the skew becoming less pronounced with increasing degrees of freedom.

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> ### Answer A
>
> Correct. Chi-square distributions have positive values and are skewed right. The skew becomes less pronounced with increasing degrees of freedom.

4. How are the expected counts calculated when a chi-square goodness-of-fit test is conducted?

   (A) The expected counts are calculated by multiplying each proportion in the null hypothesis by 100.

   (B) The expected counts are calculated by multiplying each proportion in the alternative hypothesis by 100.

   (C) The values observed from the sample are the expected counts.

   (D) The expected counts are calculated by multiplying each proportion in the null hypothesis by the sample size. ✓

   (E) The expected counts are calculated by multiplying each proportion in the alternative hypothesis by the sample size.

> ### Answer D
>
> Correct. The expected count for a category is found by multiplying the sample size by the hypothesized value of the population proportion that corresponds to that category. The hypothesized values of the population proportions are the proportions in the null hypothesis.

5. A fisheries biologist collected a random sample of fish from a lake and conducted a chi-square goodness-of-fit test to see if the distribution of fish changed over time. The table below shows the distribution of fish that were put into the lake when it was originally stocked.

Fish Type	Trout	Bass	Perch	Sunfish	Catfish
Percent	25%	25%	20%	15%	15%

The biologist found evidence to reject the null hypothesis in favor of the alternative hypothesis. Which of the following represents the alternative hypothesis of the test?

   (A) At least one of the fish proportions is different than the corresponding proportion when the lake was originally stocked. ✓

   (B) The proportions for the different fish types are the same as the corresponding proportions when the lake was originally stocked.

   (C) The proportions are evenly distributed among fish types.

   (D) At least one of the fish proportions is the same as the corresponding proportion when the lake was stocked.

   (E) All of the fish proportions are different than the corresponding proportions when the lake was stocked.

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> ### Answer A
>
> Correct. The alternative hypothesis should state that at least one of the proportions is different. Rejecting the null hypothesis in favor of the alternative hypothesis means there is evidence that at least one of the proportions is different from the proportions when the lake was originally stocked.

6. Which of the following is <u>not</u> a condition for a chi-square goodness-of-fit test?

   (A) Data should be collected using a random sample or randomized experiment.

   (B) When sampling without replacement, the sample size cannot be greater than 10 percent of the population size.

   (C) All expected counts should be greater than 5.

   (D) The distribution of the sample should be approximately normal. ✓

   (E) During the sampling process, each individual chosen should be independent of the next.

> ### Answer D
>
> Correct. The sample is not required to have a normal distribution to meet the conditions for a chi-square goodness-of-fit test. Normality is not a condition because chi-square distributions are naturally right skewed.

7. For which of the following is a chi-square goodness-of-fit test most appropriate?

   (A) Estimating a difference between two population means

   (B) Estimating a difference between two population proportions

   (C) Finding the expected value of a probability distribution

   (D) Determining whether a categorical variable has a significantly different distribution of proportions than the expected distribution ✓

   (E) Determining the best shape for a set of data

> ### Answer D
>
> Correct. The goodness-of-fit test is used to investigate whether or not the distribution of proportions generated by a sample differs from a predetermined, expected distribution.

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8.   A company claims that each bag of grass seed that it sells contains the following distribution of grass-seed types.

Type of Grass Seed	Percent
Fescue (F)	55%
Buffalo grass (B)	22%
Blue grama (BG)	10%
Indian grass (I)	7%
Green needlegrass (NG)	6%

A quality control specialist tests samples of the seed being packaged and uses a chi-square goodness-of-fit test to see whether the proportions in the samples match what is claimed by the company. Which of the following best describes the null hypothesis and the alternative hypothesis for the test?

(A) $H_0 : p = 0.20$
       $H_a : p \neq 0.20$

(B) $H_0 : p_F = 0.20, p_B = 0.20, p_{BG} = 0.20, p_I = 0.20, p_{NG} = 0.20$
       $H_a :$ At least one of the proportions is different.

(C) $H_0 : p_F = 0.55, p_B = 0.22, p_{BG} = 0.10, p_I = 0.07, p_{NG} = 0.06$
       $H_a :$ At least one of the proportions is different. ✓

(D) $H_0 :$ All proportions are equally likely.
       $H_a :$ All of the proportions are different.

(E) $H_0 :$ There is no association between the grass-seed types.
       $H_a :$ There is an association between the grass-seed types.

Answer C

Correct. The null hypothesis represents the hypothesized distribution of seed within the bag, and this is the correct alternative hypothesis.

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9. A human resources manager selected a random sample of 200 workers who donate to charity. The following table shows the distribution of the 200 workers.

Type of worker	Count
Management	96
Other white collar	50
Blue collar	54

The manager conducts a goodness-of-fit test to determine whether the proportions of workers of these types are identical to the population proportions of workers donating to charity, which are 50 percent for management, 30 percent for other white-collar workers, and 20 percent for blue-collar workers. Which of the following statements must be true about the sample?

(A) The expected number of blue-collar workers donating to charity is less than 30.

(B) The expected number of management workers donating to charity is 100. ✓

(C) The expected numbers of other white-collar and blue-collar workers donating to charity are the same.

(D) The expected number of other white-collar workers donating to charity is 50.

(E) The combined expected numbers of other white-collar and blue-collar workers donating to charity is greater than the expected number of management workers donating to charity.

Answer B

Correct. The expected number of management workers donating to charity is 50 percent of 200, which is 100.

10. Jimmy likes to listen to a variety of music. His library has the following distribution of music genres.

Music Type	Country	R&B	Pop	Retro	Rock
Percent	22%	10%	12%	20%	36%

Jimmy believes that the shuffle feature on his music player is malfunctioning by not playing songs that meet this distribution of music types. To test this, he listens to 100 songs randomly chosen when his player is in shuffle mode and records the number of songs in each category. Which inference procedure should he use to test whether or not the shuffle feature is working correctly?

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(A)   A one-sample z-test for a population proportion

(B)   A two-sample z-test for a difference between population proportions

(C)   A chi-square goodness-of-fit test ✓

(D)   A chi-square test for homogeneity

(E)   A matched pairs t-test for a mean difference

Answer C

Correct. The chi-square goodness-of-fit test is used to investigate whether or not the distribution of proportions generated by a sample differs from a predetermined, expected distribution for one categorical variable.

11.   A statistician is conducting a chi-square goodness-of-fit test and is limited by the cost, per individual, to conduct the study. The statistician selects a sample of size 39, which is the smallest sample possible that will meet the condition for large expected counts. Which of the following could not be the null hypothesis for the study?

(A) $H_0 : p_1 = 0.20, p_2 = 0.20, p_3 = 0.20, p_4 = 0.20, p_5 = 0.20$

(B) $H_0 : p_1 = 0.15, p_2 = 0.35, p_3 = 0.22, p_4 = 0.15, p_5 = 0.13$

(C) $H_0 : p_1 = 0.24, p_2 = 0.23, p_3 = 0.21, p_4 = 0.18, p_5 = 0.14$

(D) $H_0 : p_1 = 0.34, p_2 = 0.21, p_3 = 0.14, p_4 = 0.15, p_5 = 0.16$

(E) $H_0 : p_1 = 0.43, p_2 = 0.23, p_3 = 0.17, p_4 = 0.09, p_5 = 0.08$ ✓

Answer E

Correct. To meet the condition for large expected counts, each expected count needs to be greater than 5. The least proportion value is 0.08, and the expected value is $39(0.08) = 3.12$, so at least one of the expected counts is less than 5.

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12. A factory produces bags of rubber bands. A bag of rubber bands has five different sizes: extra large (XL), large (L), medium (M), small (S), and extra small (XS). A quality control specialist collects a random sample of 450 rubber bands from the bagging machine and calculates a chi-square goodness-of-fit test to see if the frequencies for each size in the sample match the hypothesized distribution. The quality control specialist will test his sample against the following null hypothesis.
$H_0 : p_{XL} = 0.10, p_L = 0.20, p_M = 0.40, p_S = 0.20, p_{XS} = 0.10$

How many medium rubber bands are expected in the random sample of 450 rubber bands?

(A) 20

(B) 40

(C) 90

(D) 180 ✓

(E) 1,125

Answer D

Correct. The expected count is determined by multiplying the hypothesized proportion by the sample size. In this case the hypothesized proportion is $p_M = 0.40$ and the sample size is 450. So the expected count is $0.40 \times 450 = 180$.

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13. A forester reported the following distribution of tree types to a local logging company.

Tree Type	Spruce	Pine	Fir	Deciduous	Other
Percent	52%	22%	8%	10%	8%

The logging company generated a random sample of 100 trees and observed the following distribution of trees in each of the categories.

Tree Type	Spruce	Pine	Fir	Deciduous	Other
Observed Count	51	21	10	9	9

The logging company would like to use its sample to provide convincing statistical evidence that over 50 percent of the trees in the forest are spruce trees. The logging company has decided to use a chi-square goodness-of-fit test to justify its claim. Why is the chi-square goodness-of-fit test <u>not</u> an appropriate procedure for the logging company to use?

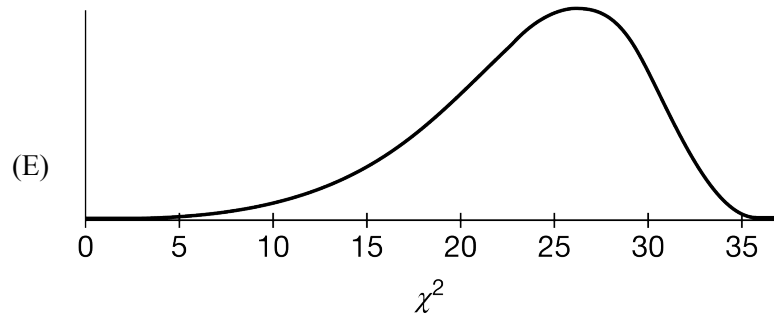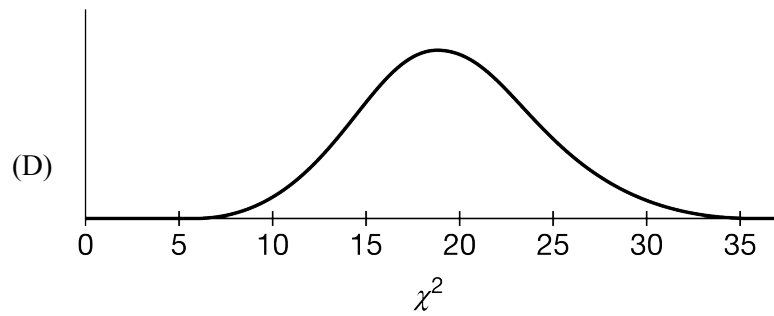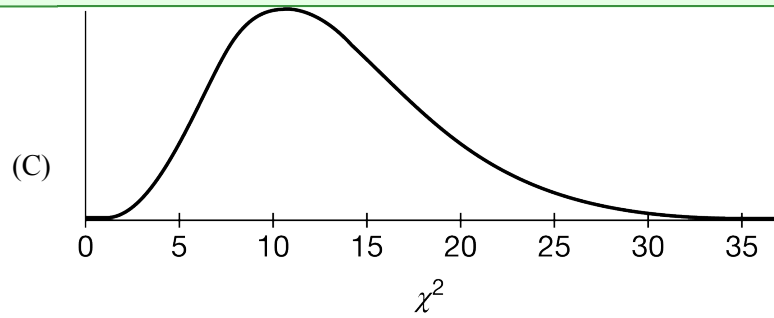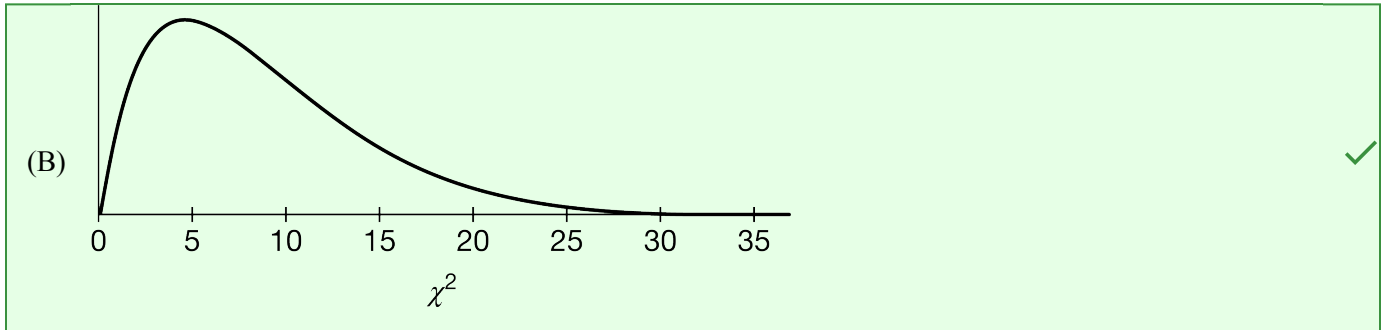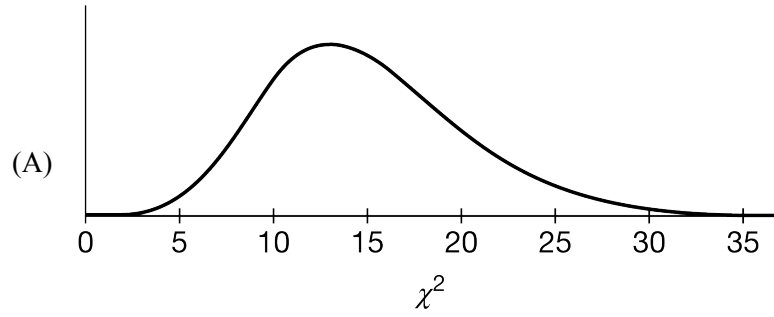## Setting Up a Chi-Square Test for Goodness of Fit Quiz

(A) A chi-square goodness-of-fit test would be used to show that the entire distribution of trees in the forest is different than what the forester reported, not necessarily the individual proportion representing the spruce trees. ✓

(B) The logging company should find the average number of spruce trees using several samples and then construct a confidence interval for a difference in population means to show that there are more spruce trees in the forest than reported.

(C) The logging company does not need to complete an inference procedure; there are more than 50 percent spruce trees in the sample.

(D) In order to perform a chi-square test, the logging company needs expected counts, not percentages. The logging company should declare its current sample as expected values and then generate a new sample of observed values to compute the test statistic.

(E) The sample does not meet the minimum requirements needed for a chi-square goodness-of-fit test.

Answer A

Correct. The logging company inappropriately used a chi-square goodness-of-fit test. If the company is interested in comparing just the proportion of spruce trees, it should use a one-sample z-test for a population proportion.

14. Which of the following chi-square distributions has the smallest number of degrees of freedom?

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(A)

χ^2

(B) ✓

χ^2

(C)

χ^2

(D)

χ^2

(E)

χ^2

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> **Answer B**
>
> Correct. Within a family of density curves for χ^2, the skew becomes less pronounced with increasing degrees of freedom. Since this curve has the most skew of the curves provided, this chi-square distribution has the smallest number of degrees of freedom.

15. A spinner made for a game of chance has 8 equally likely spaces. Alfonso records the result of a sample of 400 spins. Alfonso decides to calculate a chi-square test statistic for a goodness-of-fit test to see whether the spinner is fair. Which of the following is the appropriate null hypothesis?

(A) $H_0 : p_1 = 0.125, p_2 = 0.125, p_3 = 0.125, p_4 = 0.125, p_5 = 0.125, p_6 = 0.125, p_7 = 0.125, p_8 = 0.125$

(B) H_0 : At least one proportion is different.

(C) $H_0 : p = 0.125$

(D) $H_0 : p_1 \neq 0.125, p_2 \neq 0.125, p_3 \neq 0.125, p_4 \neq 0.125, p_5 \neq 0.125, p_6 \neq 0.125, p_7 \neq 0.125, p_8 \neq 0.125$

(E) $H_0 : p_1 = 0.08, p_2 = 0.08, p_3 = 0.08, p_4 = 0.08, p_5 = 0.08, p_6 = 0.08, p_7 = 0.08, p_8 = 0.08$

> **Answer A**
>
> Correct. There is a $\frac{1}{8}$ or 0.125 chance of the spinner landing on any of the eight spaces if the spinner is fair. The null hypothesis is that the proportion of times the spinner lands on a space will be the same for all eight spaces.