

Independent Events and Unions of Events Quiz

1. For which of the following probability assignments are events A and B independent?

(A) $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.12$, and $P(A^c \cap B) = 0.4$.

(B) $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.3$, and $P(A^c \cap B) = 0.3$.

(C) $P(A \cap B^c) = 0.1$, $P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$. ✓

(D) $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.0$, and $P(A^c \cap B) = 0.2$.

(E) $P(A \cap B^c) = 0.5$, $P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$.

Answer C

Correct. Events A and B are independent if $P(A \cap B) = [P(A)][P(B)]$. Here,
 $P(A) = P(A \cap B) + P(A \cap B^c) = 0.1 + 0.1 = 0.2$ and
 $P(B) = P(B \cap A) + P(B \cap A^c) = 0.1 + 0.4 = 0.5$. Therefore,
 $[P(A)][P(B)] = (0.2)(0.5) = 0.1$ and $0.1 = P(A \cap B)$.

2. A company is considering purchasing the mineral rights to two different mountains. The probability that it will purchase the mineral rights to the first mountain is 0.55. The probability that it will purchase the mineral rights to the second mountain is 0.4. Assuming the decisions to purchase the mineral rights to each mountain are made independently, what is the probability that it will purchase the mineral rights to exactly one of the two mountains?

(A) 0.18

(B) 0.22

(C) 0.33

(D) 0.51 ✓

(E) 0.95

Answer D

Correct. If F represents the event that the mineral rights to the first mountain are purchased and S represents the event that the mineral rights to the second mountain are purchased, then
 $P(F \cap S^c) + P(F^c \cap S) = [P(F)][P(S^c)] + [P(F^c)][P(S)] = (0.55)(0.6) + (0.45)(0.4) = 0.51$.

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3. A student is applying to two different agencies for scholarships. Based on the student's academic record, the probability that the student will be awarded a scholarship from Agency A is 0.55 and the probability that the student will be awarded a scholarship from Agency B is 0.40. Furthermore, if the student is awarded a scholarship from Agency A, the probability that the student will be awarded a scholarship from Agency B is 0.60. What is the probability that the student will be awarded at least one of the two scholarships?
- (A) 0.60
- (B) 0.62
- (C) 0.71
- (D) 0.73
- (E) 0.95

Answer B

Correct. The probability that a student will be awarded at least one of the two scholarships is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. To find $P(A \cap B)$, use the formula for conditional probability, $P(B|A) = \frac{P(A \cap B)}{P(A)}$, then solve $P(A \cap B) = [P(A) \cdot P(B|A)] = (0.55)(0.6) = 0.33$. Then $P(A \cup B) = 0.55 + 0.40 - 0.33 = 0.62$.