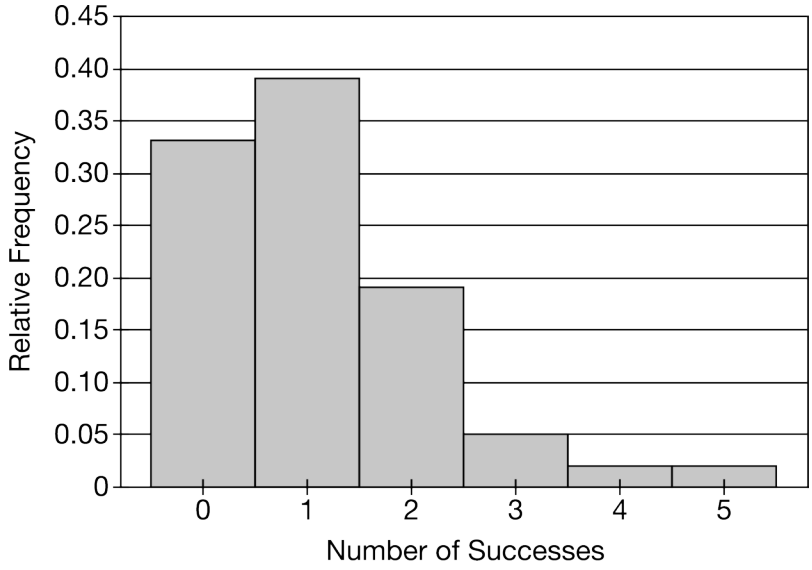
Unit 4 Progress Check: MCQ Part C

1.



A thumbtack that is tossed can land point up or point down. The probability of a tack landing point up is 0.2. A simulation was conducted in which a trial consisted of tossing 5 thumbtacks and recording the number of thumbtacks that land point up. Many trials of the simulation were conducted and the results are shown in the histogram.

Based on the results of the simulation, which of the following is closest to the probability that at least 2 thumbtacks land pointing up when 5 thumbtacks are tossed?

(A) 0.09

(B) 0.19

(C) 0.28 ✓

(D) 0.72

(E) 0.91

Answer C

Correct. The bars corresponding to 2, 3, 4, and 5 represent at least two successes. The sum of the heights of the bars corresponds to a probability of approximately 0.28.

2. According to a 2018 survey, 74 percent of employed young adults expect to bring work on a vacation trip. A random sample of 20 employed young adults will be selected. What is the probability that 8 of the selected young adults will expect to bring work on a vacation trip?

Unit 4 Progress Check: MCQ Part C

(A) $\binom{20}{8}(0.26)^8(0.74)^{12}$

(B) $\binom{20}{8}(0.74)^8(0.26)^{12}$ ✓

(C) $\binom{12}{8}(0.26)^8(0.74)^{12}$

(D) $\binom{12}{8}(0.74)^8(0.26)^{12}$

(E) $\binom{28}{8}(0.26)^8(0.74)^{12}$

Answer B

Correct. Let the random variable X represent the number of young adults in a sample of 20 who expect to bring work on a vacation trip. Random variable X has a binomial distribution with success defined as selecting one who expects to take work, probability 0.74, and failure as selecting one who does not, probability 0.26. The probability of selecting 8 young adults who expect to take work on vacation is $P(X = 8) = \binom{20}{8}(0.74)^8(0.26)^{12}$.

3. In the United States, 75 percent of adults wear glasses or contact lenses. A random sample of 10 adults in the United States will be selected. Which of the following is closest to the probability that fewer than 8 of the selected adults wear glasses or contact lenses?

(A) 0.10

(B) 0.28

(C) 0.47 ✓

(D) 0.53

(E) 0.76

Answer C

Correct. Let the random variable X represent the number of adults out of 10 who wear glasses or contact lenses. The random variable has a binomial distribution. Define success as wearing glasses or contact lenses with probability 0.75, and define failure as not wearing them with probability 0.25. The probability that fewer than 8 adults wear glasses or contact lenses is

Unit 4 Progress Check: MCQ Part C

$$1 - P(X \geq 8) = 1 - [P(X = 8) + P(X = 9) + P(X = 10)]$$
$$= 1 - \left[\binom{10}{8}(0.75)^8(0.25)^2 + \binom{10}{9}(0.75)^9(0.25)^1 + \binom{10}{10}(0.75)^{10}(0.25)^0\right] \approx 0.47 .$$

4. According to a 2016 survey, 6 percent of workers arrive to work between 6:45 A.M. and 7:00 A.M. Suppose 300 workers will be selected at random from all workers in 2016. Let the random variable W represent the number of workers in the sample who arrive to work between 6:45 A.M. and 7:00 A.M. Assuming the arrival times of workers are independent, which of the following is closest to the standard deviation of W ?

(A) 0.24

(B) 4.11 ✓

(C) 4.24

(D) 16.79

(E) 16.92

Answer B

Correct. Random variable W has a binomial distribution. The standard deviation for random variable W is $\sqrt{np(1-p)} = \sqrt{300(0.06)(0.94)}$.

5. In the United States, the generation of people born between 1946 and 1964 are known as baby boomers, and the generation of people born between 1981 and 1996 are known as millennials. Currently, 18 percent of the population are baby boomers and 27 percent of the population are millennials. A random sample of 500 people will be selected. Let the random variable B represent the number of baby boomers in the sample, and let the random variable M represent the number of millennials in the sample. By how much will the mean of M exceed the mean of B ?

(A) 9

(B) 45 ✓

(C) 90

(D) 135

(E) 225

Answer B

Correct. The mean of B is $np = 500(0.18) = 90$. The mean of M is $np = 500(0.27) = 135$. The difference of the means is $135 - 90 = 45$.

Unit 4 Progress Check: MCQ Part C

6. According to a 2015 Census Bureau survey, 75,511 of the 822,959 residents of Baltimore County, Maryland, were enrolled in college. Consider a sample of 800 residents of Baltimore County, Maryland in 2015 selected at random. Which of the following is closest to the expected value of the number in the sample enrolled in college?

(A) 8.17

(B) 28.3

(C) 66.7

(D) 73.4 ✓

(E) 94.4

Answer D

Correct. The distribution can be modeled with a binomial variable. The expected value is the mean for a binomial distribution given by $np = 800\left(\frac{75,511}{822,959}\right)$.

7. During a severe storm, electrical transformers that function independently are expected to operate 85 percent of the time. Suppose 20 electrical transformers are randomly selected from the population. Let the random variable T represent the number of electrical transformers operating during a severe storm. Which of the following is the best interpretation of the random variable T ?

(A) It is a binomial variable with mean 17 transformers and standard deviation $\sqrt{2.55}$ transformers. ✓

(B) It is a binomial variable with mean 17 severe storms and standard deviation $\sqrt{2.55}$ severe storms.

(C) It is a binomial variable with mean 0.85 transformer and standard deviation 20 transformers.

(D) It is a variable that is not binomial with mean 17 transformers and standard deviation $\sqrt{2.55}$ transformers.

(E) It is a variable that is not binomial with mean 0.85 severe storm and standard deviation 20 severe storms.

Answer A

Correct. The variable is binomial because the selected transformer is either operating or not, there are 20 fixed trials, the probability of success (operating) is constant, and the transformers function independently. The mean is $20(0.85) = 17$ and the standard deviation is $\sqrt{20(0.85)(0.15)} = \sqrt{2.55}$. The unit associated with the mean and standard deviation is transformers.

Unit 4 Progress Check: MCQ Part C

8. A random sample of n people selected from a large population will be asked whether they have read a novel in the past year. Let the random variable R represent the number of people from the sample who answer yes. The variance of random variable R is 6. Assume the responses are independent of each other. If the proportion of people from the population who read a novel in the past year is 0.40, which of the following is the best interpretation of random variable R ?

(A) A binomial variable with 15 independent trials

(B) A binomial variable with 25 independent trials ✓

(C) A variable that is not binomial with 25 independent trials

(D) A binomial variable with 40 independent trials

(E) A variable that is not binomial with 40 independent trials

Answer B

Correct. The variable is binomial because the selected person will answer yes or no, there are n fixed trials, the probability of success (responding yes) is constant, and responses are independent. If $n = 25$, the variance is $np(1-p) = 25(0.4)(0.6) = 6$, which is the correct variance.

9. A local department store estimates that 10 percent of its customers return the merchandise they purchase. Let the random variable R represent the number of returns for a random sample of 40 customers. Assume that random variable R follows a binomial distribution. What is described by the value of $\binom{40}{8}(0.1)^8(0.9)^{32}$?

(A) The mean of the random variable

(B) The variance of the random variable

(C) The standard deviation of the random variable

(D) The probability that 8 customers in the sample will return merchandise ✓

(E) The probability of selecting a sample of 40 customers who will all return merchandise

Answer D

Correct. The value represents the probability for a binomial random variable of seeing 8 successes in 40 trials, where the probability of success is 0.1.

10. Past records indicate that 15 percent of the flights for a certain airline are delayed. Suppose flights are randomly selected one at a time from all flights. Assume each selection is independent of another. Which of the following is closest to the probability that it will take 5 selections to find one flight that is delayed?

Unit 4 Progress Check: MCQ Part C

(A) 0.0783 ✓

(B) 0.0921

(C) 0.4780

(D) 0.5220

(E) 0.5563

Answer A

Correct. Let random variable X represent the number chosen to find a delayed flight. The variable can be modeled with a geometric distribution. Thus, $P(X = 5) = 0.15\left(0.85^4\right) \approx 0.0783$.

11. A representative from a company that manufactures items for left-handed people will attend a large convention. The representative hopes to find a left-handed person at the convention to try out the items. The representative will select an attendee at random until a left-handed person is found. Assume each selection is independent of another. If 10 percent of the convention attendees are left-handed, what is the probability that the representative must select 4 attendees to find one who is left-handed?

(A) $0.1\left(0.9^3\right)$ ✓

(B) $0.1\left(0.9^4\right)$

(C) $0.1^2\left(0.9^2\right)$

(D) $0.9\left(0.1^3\right)$

(E) $0.9\left(0.1^4\right)$

Answer A

Correct. Let the discrete random variable X represent the number of people chosen to find a left-handed person. The random variable has a geometric distribution and $P(X = 4) = 0.1\left(0.9^3\right)$.

12. Approximately 9 percent of the residents of a large city have seen a certain theater production that is currently playing in the city. A marketing researcher will randomly select residents until one is found who has seen the production. What is the expected number of residents the researcher will need to ask to find someone who has seen the production?

Unit 4 Progress Check: MCQ Part C

(A) 0.09
(B) 0.30
(C) 10.60
(D) 11.00
(E) 11.11 ✓

Answer E

Correct. Let random variable X represent the number of residents the researcher will need to ask to find someone who has seen the production. Random variable X has a geometric distribution. The expected value of random variable X is $E[X] = \frac{1}{\text{p}} = \frac{1}{.09} \approx 11.11$ people.

13. The random variable K has a geometric distribution with mean 16. Which of the following is closest to the standard deviation of random variable K ?

(A) 0.0625
(B) 0.9375
(C) 4
(D) 15.49 ✓
(E) 240

Answer D

Correct. Since the formula for mean $= \frac{1}{p}$, rearranging the formula gives $p = \frac{1}{\mu} = \frac{1}{16} = 0.0625$.
Thus, the standard deviation of random variable K is $\sigma_K = \frac{\sqrt{(1-0.0625)}}{0.0625} \approx 15.49$.

14. In a certain region, 10 percent of the homes have solar panels. A city official is investigating energy consumption for homes within the region. Each week, the city official selects a random sample of homes from the region. Let random variable Y represent the number of homes selected at random from the region until a home that has solar panels is selected. The random variable Y has a geometric distribution with a mean of 10. Which of the following is the best interpretation of the mean?

(A) Each week, the number of homes with solar panels increases by 10.
(B) For a randomly selected week, it will take 10 homes before a home with solar panels is selected.
(C) The average number of solar panels per home is equal to 10.
(D) Over many weeks, the average number of homes with solar panels is 10.
(E) Over many weeks, it takes 10 homes, on average, before a home with solar panels is selected. ✓

Unit 4 Progress Check: MCQ Part C

Answer E

Correct. The mean is the long-run average over repeated trials. Over time, the average number of homes that need to be selected before a home with solar panels is selected is 10.

15.

At a certain restaurant, 35 percent of the customers order the daily special each day. Assume that each day the customers arrive randomly and order independently. Let the random variable X represent the number of orders placed until the first daily special is ordered. The distribution of X is geometric and has an expected value of approximately 2.86. Which of the following is the best interpretation of the expected value?

(A) Each day, 3 customers will order the daily special.

(B) Over many days, the average number of customers ordering the daily special is approximately 2.86.

(C) Over many days, it takes about 2.86 orders, on average, to be placed until the first daily special is ordered. ✓

(D) For a random sample of the days, the average number of orders of the daily special will be 2.86.

(E) Each day, the ratio of the number of orders of the daily special to the number of orders of other menu items is about 2.86 to 1.

Answer C

Correct. The mean of a probability distribution is the expected value—that is, the long-run average over repeated trials. Over time, the average number of orders placed until the daily special is ordered is approximately 2.86.