Unit 8 Progress Check: FRQ

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A marketing director for a beverage company conducted a study to investigate people's soda preferences in two regions of the country. The director selected a random sample of 100 people from the east coast and a random sample of 100 people from the west coast to survey. The responses are summarized in the following table.

	East Coast	West Coast
Regular soda	44	37
Diet soda	39	42
No preference	17	21
Total	100	100

Do the data provide convincing statistical evidence, at the level of $\alpha = 0.05$, that the preferences are different between the two regions of the country? Complete the appropriate inference procedure to support your answer.

3-part Inference scoring

The primary goal of this question is to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals are to assess a student's ability to (1) identify the appropriate procedure and state appropriate hypotheses; (2) check appropriate conditions, calculate a test statistic, and calculate a *p*-value; and (3) draw an appropriate conclusion, with justification, in the context of the study.

Scoring

Scoring steps 1, 2, and 3 are each scored as essentially correct (E), partially correct (P), or incorrect (I).

0	1	2	3	4 ✓

All three scoring parts essentially correct

- ☐ Scoring Step 1 essentially correct
- ☐ Scoring Step 1 partially correct
- ☐ Scoring Step 1 incorrect
- ☐ Scoring Step 2 essentially correct
- ☐ Scoring Step 2 partially correct
- ☐ Scoring Step 2 incorrect

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- [ ] Scoring Step 3 essentially correct
- [ ] Scoring Step 3 partially correct
- [ ] Scoring Step 3 incorrect

Solution

Scoring step 1: Appropriate test and hypotheses

The appropriate test is the chi-square test for homogeneity. The hypotheses are as follows:

H_0: There is no difference in soda preference between people from the east coast and people from the west coast

H_a: There is a difference in soda preference between people from the east coast and people from the west coast.

Scoring

Scoring step 1 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· The correct test is identified.

· The null and alternative hypotheses are stated correctly.

· Context is explicitly stated or is implied through labeling of variables.

Partially correct (P) if the response satisfies only two of the three components

Incorrect (I) if the response does not meet the criteria for E or P

Note: Component 1 can be satisfied in scoring step 2 if the correct formula is shown.

Solution

Scoring step 2: Conditions and calculations

Conditions:

1) Independence

Two random samples were taken, one from the population of people on the east coast and one from the population of people on the west coast.

The sample size of 100 is less than 10% of the population size for both the east and west coast.

2) Large counts should be used.

All expected counts should be greater than 5

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	East Coast	West Coast
Regular soda	40.5	40.5
Diet soda	40.5	40.5
No preference	19	19

Calculations:

The test statistic is $\chi^2 \approx 1.137$ with 2 degrees of freedom. The *p*-value is 0.5663.

Scoring

Scoring step 2 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· The three conditions are correctly checked.

· The correct test statistic is computed.

· A *p*-value consistent with the computed test statistic.

Partially correct (P) if the response satisfies only two of the three components

Incorrect (I) if the response does not meet the criteria for E or P

Note:

A response that indicates that a single stratified random sample was selected, with coast as strata, satisfies the first of the three conditions.

Solution

Scoring step 3: Justification of conclusion

The *p*-value of 0.5663 is greater than the significance level of 0.05, so the null hypothesis is not rejected. There is not sufficient statistical evidence to support a claim that the soda preferences are different between people on the east and west coasts of the country.

Scoring

Scoring step 3 is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· Explicitly compares the *p*-value to the significance level

· Provides a correct decision about the null hypothesis that is consistent with the computed *p*-value

· Gives statement of conclusion in context

Partially correct (P) if the response includes only two of the three conditions

Incorrect (I) if the response does not meet the criteria for E or P

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Note: Any response that implies accepting the null hypothesis should be scored an I.

2. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Fingerprint analysis and blood grouping are features that do not change through the lifetime of an individual. Fingerprint features appear early in the development of a fetus, and blood types are determined by genetics. Therefore, each is considered an effective tool for identification of individuals. These characteristics are also of interest in the discipline of biological anthropology—a scientific discipline concerned with the biological and behavioral aspects of human beings.

The relationship between these characteristics was the subject of a study conducted by biological anthropologists with a simple random sample of male students from a certain region with a large student population. Fingerprint patterns are generally classified as loops, whorls, and arches. The four principal blood types are designated as A, B, AB, and O. The table shows the distribution of fingerprint patterns and blood types for the sample. Expected counts are listed in parentheses. The anthropologists were interested in the possible association between the variables.

		Blood Type			
	A	**B**	**AB**	**O**	**Total**
Loops	66 (71.69)	99 (112.19)	35 (32.29)	101 (84.83)	301
Whorls	51 (47.16)	91 (73.80)	15 (21.24)	41 (55.80)	198
Arches	14 (12.15)	15 (19.01)	9 (5.47)	13 (14.37)	51
Total	131	205	59	155	550

(a) Is the test for an association in this case a chi-square test of independence, or a chi-square test of homogeneity? Justify your choice.

(b) Identify the conditions for the chi-square inference procedure selected in part (a), and indicate whether the conditions are met.

(c) The resulting chi-square test statistic from the appropriate test is approximately 18.930. What are the degrees of freedom and p-value of the test?

(d) Biological anthropology is concerned with the comparative study of human origin, evolution and diversity. Considering the sampling design in this study, to what population is it reasonable for the researchers to generalize their results?

Part A, B, C, and D

The primary goals of this question are to (1) assess a student's ability to identify whether a test for association is a test for independence or a test for homogeneity; (2) identify and check the conditions for a chi-square inference procedure; (3) find the p-value for a chi-square test; and (4) identify the population to which the results of a chi-square test can be generalized.

Scoring

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Parts (a), (b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Each essentially correct (E) part counts as 1 point.

Each partially correct (P) part counts as ½ point.

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

Reasons to score up:

· *All notation is correct and clearly marked*

· *All explanations are clear*

· *No wrong information is included that was not part of the scoring (for example, saying sample size must be greater than 30 when that has nothing to do with the problem)*

· *No minor calculation errors are made, if they are not part of the scoring*

· *Interpretation parts are especially strong*

Reasons to score down:

· *Notation is not wrong, but is spotty and not clearly marked*

· *Explanations are not wrong, but are hard to follow*

· *Wrong or extraneous information is included but not part of scoring*

· *Minor calculation errors that are not part of the scoring are made*

· *Interpretation parts are scored an E but are considered a weak E*

0	1	2	3	4 ✓

Parts (a) through (d) sum to 4 points

OR

Parts (a) through (d) sum to 3½ points AND a holistic approach is used to decide to score up

- [ ] Part (a) essentially correct
- [ ] Part (a) partially correct
- [ ] Part (a) incorrect
- [ ] Part (b) essentially correct
- [ ] Part (b) partially correct

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- [ ] Part (b) incorrect
- [ ] Part (c) essentially correct
- [ ] Part (c) partially correct
- [ ] Part (c) incorrect
- [ ] Part (d) essentially correct
- [ ] Part (d) partially correct
- [ ] Part (d) incorrect

Solution

Part (a)

This is a chi-square test of independence; there were two variables measured from a single random sample selected from one population.

Scoring

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· Correctly indicates “Independence”

· Justifies the choice by referencing the sampling process

· Indicates there was one population of interest OR indicates data was collected using a single random sample

Partially correct (P) if the response satisfies two of the three components

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (b)

The conditions that must be checked are the following.

1) Independence

· Data were collected using a simple random sample.

· The sample size is at most 10 percent of the population size when sampling without replacement.

2) Large counts should be used.

· All expected counts should be greater than 5.

The independence condition is met since a random sample was used.

The population of male students from the region is likely greater than 10 times the 550 male students in the sample.

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All expected counts are greater than 5.

Scoring

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

· States that a simple random sample was selected from one population

· States the sample size is likely less than 10 percent of the population size

· States that the condition about expected counts has been met

Partially correct (P) if the response includes only two of the three components

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (c)

The degrees of freedom are equal to the number of rows minus 1 multiplied by the number of columns minus 1. In this case, the degrees of freedom are equal to $(3-1)(4-1)=6$. The *p*-value for a chi-square test statistic of 18.930 with 6 degrees of freedom is approximately 0.004.

Or

Using the chi-square table, a chi-square test statistic of 18.930 with 6 degrees of freedom has a *p*-value between 0.0025 and 0.005.

Scoring

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components.

· The correct number of degrees of freedom is indicated

· A *p*-value between 0.0025 and 0.005 is indicated

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Solution

Part (d)

The results can be generalized to male students from the region.

Scoring

Part (d) is scored as follows:

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Essentially correct (E) if the response states that the results can be generalized to male students from the region

Partially correct (P) if the response states that the results can be generalized to students from the region, but does not mention males specifically

Incorrect (I) if the response does not satisfy the criteria for E or P