The Normal Distribution Quiz

1. The distribution of the number of transactions per day at a certain automated teller machine (ATM) is approximately normal with a mean of 80 transactions and a standard deviation of 10 transactions. Which of the following represents the parameters of the distribution?

    (A) $\bar{x} = 80; \quad s = 10$

    (B) $\bar{x} = 80; \quad s^2 = 10$

    (C) $\bar{x} = 80; \quad \sigma = 10$

    (D) $\mu = 80; \quad \sigma = 10$ ✓

    (E) $\mu = 80; \quad s = 10$

> **Answer D**
>
> Correct. The values provided in the question represent known parameter values for the distribution, and the correct symbols for these values are μ (for the population mean) and σ (for the population standard deviation).

2. At a small coffee shop, the distribution of the number of seconds it takes for a cashier to process an order is approximately normal with mean 276 seconds and standard deviation 38 seconds. Which of the following is closest to the proportion of orders that are processed in less than 240 seconds?

    (A) 0.17 ✓

    (B) 0.25

    (C) 0.36

    (D) 0.83

    (E) 0.95

> **Answer A**
>
> Correct. The correct z-score is $z = \frac{240 - 276}{38} = -0.95$. The area under the standard normal curve to the left of that value is approximately 0.17.

3. A researcher is studying a group of field mice. The distribution of the weight of field mice is approximately normal with mean 25 grams and standard deviation 4 grams. Which of the following is closest to the proportion of field mice with a weight greater than 33 grams?

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(A) 0.023	✓

(B) 0.046

(C) 0.954

(D) 0.977

(E) 1.000

> **Answer A**
>
> Correct. The z-score associated with a weight of 33 grams is 2. The area under the curve to the right of 2 in a standard normal distribution is approximately 0.023.

4. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The following frequency table summarizes the total amounts, in dollars, for 91 orders from a food truck during a certain day.

Amount	Frequency
$0 up to but not including $5	8
$5 up to but not including $10	14
$10 up to but not including $15	25
$15 up to but not including $20	27
$20 up to but not including $25	12
$25 up to but not including $30	5

(a-i) Use the data in the table to create a histogram showing the distribution of the amounts of the orders.

(a-ii) Describe the shape of the distribution of amounts.

(b) Identify a possible amount for the median of the distribution. Justify your answer.

Part A and B

The primary goals of this question are to assess a student's ability to (1) construct a histogram and comment on the shape of the distribution and (2) determine a possible value of the median for a data set grouped by intervals.

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Scoring

Parts (a) and (b) are each scored as essentially correct (E), partially correct (P), or incorrect (I).
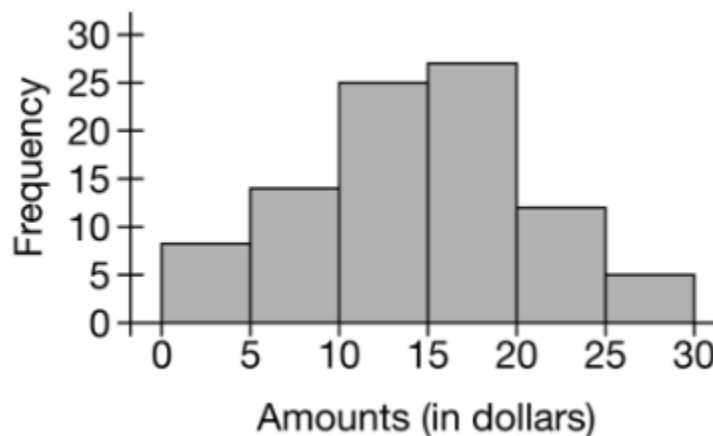
✓

0	1	2	3	4

Both parts essentially correct

- ☐ Part (a) essentially correct
- ☐ Part (a) partially correct
- ☐ Part (a) incorrect
- ☐ Part (b) essentially correct
- ☐ Part (b) partially correct
- ☐ Part (b) incorrect

Solution

Part (a-i):

Part (a-ii):

The distribution of the amounts of the orders appears to be roughly symmetric and mound shaped or approximately normal.

Scoring

Part (a) is scored as follows.

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Essentially correct (E) if the response contains all four of the following components:

· The histogram contains six bars with approximately correct heights of the bars.

· The horizontal axis is labeled with correct numbers and a correct verbal description.

· The vertical axis is labeled with correct numbers and a correct verbal description.

· The shape of the distribution is described as roughly symmetric and mound shaped or approximately normal.

Partially correct (P) if the response satisfies 3 of the 4 components.

Incorrect (I) if the response does not satisfy the criteria for E or P

Solution

Part (b): The median could be any value from $10 up to but not including $15. In a distribution of 91 values, the median value is the $46th$ value when all 91 values are ordered. The frequency of the first three bars sums to 47: $8 + 14 + 25 = 47$. The $46th$ value corresponds to a value in the third bar of the histogram

Scoring

Part (b) is scored as follows.

Essentially correct (E) if the response correctly identifies a value for the median that is contained within the interval from $10 up to but not including $15 AND provides a reasonable justification for how the median was determined

Partially correct (P) if the response correctly identifies a value for the median that is contained within the interval from $10 up to but not including $15 BUT provides weak justification for how the median was determined

Incorrect(I) if the response does not satisfy the criteria for E or P

5. Shalise competed in a jigsaw puzzle competition where participants are timed on how long they take to complete puzzles of various sizes. Shalise completed a small puzzle in 75 minutes and a large jigsaw puzzle in 140 minutes. For all participants, the distribution of completion time for the small puzzle was approximately normal with mean 60 minutes and standard deviation 15 minutes. The distribution of completion time for the large puzzle was approximately normal with mean 180 minutes and standard deviation 40 minutes.

    Approximately what percent of the participants had finishing times greater than Shalise's for each puzzle?
    (A) 16% on the small puzzle and 16% on the large puzzle
    (B) 16% on the small puzzle and 84% on the large puzzle ✓
    (C) 32% on the small puzzle and 68% on the large puzzle
    (D) 84% on the small puzzle and 84% on the large puzzle
    (E) 84% on the small puzzle and 16% on the large puzzle

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> **Answer B**
>
> Correct. According to the empirical rule, approximately 68% of the completion times are within one standard deviation of the mean of 60 minutes for the smaller puzzle. By symmetry, 16% of the remaining completion times are less than 45 minutes and 16% of the completion times are greater than 75 minutes. For the large puzzle, the empirical rule guarantees that approximately 68% of the times will be within 1 standard deviation of the mean of 180 minutes. By symmetry, 16% of the remaining times are less than 140 minutes, and 16% of the times are greater than 220 minutes. Therefore 84% of the times will be greater than Shalise's time of 140 minutes on the large puzzle.

6. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Kathy and her brother Clay recently ran in a local marathon. The distribution of finishing time for women was approximately normal with mean 259 minutes and standard deviation 32 minutes. The distribution of finishing time for men was approximately normal with mean 242 minutes and standard deviation 29 minutes.

(a) The finishing time for Clay was 289 minutes. Calculate and interpret the standardized score for Clay's marathon time. Show your work.

(b) The finishing time for Kathy was 272 minutes. What proportion of women who ran the marathon had a finishing time less than Kathy's? Show your work.

(c) The standard deviation of finishing time is greater for women than for men. What does this indicate about the finishing times of the women who ran the marathon compared to the finishing times of the men who ran the marathon?

Part A, B, and C

The primary goals of this question are to assess a student's ability to (1) calculate and interpret a $z - score$; (2) determine proportions from a normal distribution; and (3) explain the meaning of a standard deviation.

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

✓

0	1	2	3	4

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All three parts essentially correct

☐   Part (a) essentially correct

☐   Part (a) partially correct

☐   Part (a) incorrect

☐   Part (b) essentially correct

☐   Part (b) partially correct

☐   Part (b) incorrect

☐   Part (c) essentially correct

☐   Part (c) partially correct

☐   Part (c) incorrect

Solution

Part (a):

Clay's finishing time was 289 minutes and the distribution of finishing time for men had a mean of 242 minutes and a standard deviation of 29 minutes. The standardized score for Clay's marathon time is: $z = \frac{x - \mu}{\sigma} = \frac{289 - 242}{29} \approx 1.62.$

Clay's finishing time is 1.62 standard deviations greater than the men's mean finishing time.

Scoring

Part (a) is scored as follows.

Essentially correct (E) if the response satisfies the following three components for the calculation for Clay:

· Identifies parameters to be used in $z - score$ calculation

· Correct calculation of standardized score with supporting work

· Correct interpretation of the standardized score in context

Partially correct (P) if the response satisfies two of the three components

Incorrect(I) if the response does not satisfy the criteria for E or P

Note: A response that correctly substitutes the values in the $z - score$ formula satisfies component 1 without directly naming or labeling the parameters used.

Solution

Part (b):

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Kathy's finishing time was 272 minutes and the distribution of finishing time for women had a mean of 259 minutes and a standard deviation of 32 minutes. The standardized score for Kathy's marathon time is:

$$z = \frac{x-\mu}{\sigma} = \frac{272-259}{32} \approx 0.41.$$

Using technology or the standard normal table, the proportion of women who ran the marathon with a finishing time less than Kathy is approximately 0.66.

Scoring

Part (b) is scored as follows.

Essentially correct (E) if the response satisfies the following three components for the calculation for Kathy:

· Normality and parameters: Indicates use of a normal (or approximately normal) distribution and clearly identifies the correct parameter values. (Showing correct components in a $z - score$, labeling the mean and standard deviation in a normal cdf calculator statement, or drawing a normal curve with sufficient values marked to indicate the mean and standard deviation are all sufficient methods for identifying the correct parameter values.)

· Boundary and direction: Uses the correct boundary value of 272, and the correct direction ("less than").

· Calculation: Reports the correct proportion consistent with the values used in components 1 and 2.

Partially correct (P) if the response satisfies two of the three components

Incorrect(I) if the response does not satisfy the criteria for E or P

Note:If the response calculates the proportion of men who ran the marathon with a finishing time less than Clay's, the response should be lowered by one level (that is, from E to P, or P to I).

Solution

Part (c):

Since the standard deviation for women's times is greater than the standard deviation for men's times, there is more variability in the women's times than in the men's times. This indicates that, on average, women's times are further away from the women's mean marathon time than the average distance men's timesare away from the men's mean marathon time.

Scoring

Part (c) is scored as follows.

Essentially correct (E) if the response indicates that a greater standard deviation represents more variability AND correctly compares the variability in the women's times to the variability in the men's times

Partially correct (P) if the response indicates that a greater standard deviation represents more variability but fails to correctly compare the variability in the women's times to the variability in the men's times

Incorrect(I) if the response does not satisfy the criteria for E or P

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7. For a certain online store, the distribution of number of purchases per hour is approximately normal with mean 1,200 purchases and standard deviation 200 purchases. For what proportion of hours will the number of purchases at the online store exceed 1,400 ?

   (A) 68%

   (B) 32%

   (C) 16% ✓

   (D) 5%

   (E) 2.5%

Answer C

Correct. By the empirical rule, 68% of the number of purchases in an hour will be between 1,000 and 1,400, so $100\% - 68\% = 32\%$ of the number of purchases in an hour will fall outside of the interval. Since the normal distribution is symmetric around the mean, half of 32%, which is 16%, of the number of purchases in an hour will exceed 1,400.

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8. The distribution of lengths of salmon from a certain river is approximately normal with standard deviation 3.5 inches. If 10 percent of salmon are longer than 30 inches, which of the following is closest to the mean of the distribution?

   (A) 26 inches ✓

   (B) 28 inches

   (C) 30 inches

   (D) 33 inches

   (E) 34 inches

Answer A

Correct. Since 10% of the salmon are longer than 30 inches, then 90% of the salmon are less than or equal to 30 inches. From the standard normal table, approximately 90% of the area under the curve falls at or below the z-score of 1.282, so this value can be substituted into the formula for a z-score $\left(z = \frac{x-\mu}{\sigma} = \frac{30-\mu}{3.5} = 1.282 \right)$ and then solved for μ.