

## Sampling Distributions for Differences in Sample Means Quiz

1. A fair six-sided die, with sides numbered 1 through 6, will be rolled a total of 15 times. Let  $\bar{x}_1$  represent the average of the *first* ten rolls, and let  $\bar{x}_2$  represent the average of the *remaining* five rolls. What is the mean  $\mu_{(\bar{x}_1 - \bar{x}_2)}$  of the sampling distribution of the difference in sample means  $\bar{x}_1 - \bar{x}_2$ ?
- (A)  $\frac{3.5}{10} - \frac{3.5}{5} = -0.35$
- (B)  $3.5 - 3.5 = 0$  ✓
- (C)  $10 - 5 = 5$
- (D)  $10(3.5) - 5(3.5) = 17.5$
- (E)  $6(10 - 5) = 30$

## Answer B

Correct. If  $X_1$  and  $X_2$  are random variables representing the values that occur on a roll for the first ten rolls and the remaining five rolls, respectively, then the mean of each discrete random variable is  $\mu_X = \Sigma[(n)(P(n))] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = 3.5$ . The random variables  $X_1$  and  $X_2$  thus have identical population means of 3.5, and the mean of the sampling distribution of  $\bar{x}_1 - \bar{x}_2$  is the difference in population means, which is 0.

2. At a large university, the division of computing services surveyed a random sample of 45 biology majors and 55 business majors from populations of over 1,000 biology and 1,000 business majors. The sampled students were asked how many hours they spend per week using any university computer lab. Let  $\bar{x}_1$  represent the average hours per week spent in any university computer lab by the 45 biology majors, and let  $\bar{x}_2$  represent the average hours per week spent in any university computer lab by the 55 business majors.

Which of the following is the best explanation for why the sampling distribution of  $\bar{x}_1 - \bar{x}_2$  can be modeled with a normal distribution?

- (A) The two sample standard deviations are assumed to be equal.
- (B) The sample sizes are both sufficiently large. ✓
- (C) The distributions of the population are normal.
- (D) The population standard deviations are assumed equal.
- (E) There are at least 30 students in each of the two populations.

## Answer B

Correct. No information is given about whether the population of biology majors or the population of business majors are approximately normal, but because both sample sizes are at least 30, the sampling

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distribution of  $\bar{x}_1 - \bar{x}_2$  can be modeled with a normal distribution.

3. Suppose the variance in trunk diameter of the giant sequoia tree species is  $15.7 \text{ m}^2$ , while the variance in trunk diameter of the California redwood tree species is  $10.6 \text{ m}^2$ . Let  $\bar{x}_1$  represent the average trunk diameter of four randomly sampled giant sequoia trees, and let  $\bar{x}_2$  represent the average trunk diameter of three randomly sampled California redwood trees. If the random sampling is done with replacement, what is the standard deviation  $\sigma_{(\bar{x}_1 - \bar{x}_2)}$  of the sampling distribution of the difference in sample means  $\bar{x}_1 - \bar{x}_2$ ?
- (A)  $\sqrt{\frac{15.7}{4} - \frac{10.6}{3}}$
- (B)  $\sqrt{15.7 - 10.6}$
- (C)  $\sqrt{\frac{15.7}{4} + \frac{10.6}{3}}$  ✓
- (D)  $\sqrt{\frac{15.7}{4}} + \sqrt{\frac{10.6}{3}}$
- (E)  $\frac{15.7}{4} + \frac{10.6}{3}$

## Answer C

Correct. The standard deviation of the sampling distribution of  $\bar{x}_1 - \bar{x}_2$  is  $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ , with  $\sigma^2$  being the variance of the respective distribution.

4. Two non-profit organizations, L and M, accept donations from people. In a certain month, 140 people donated to organization L, with an average donation amount of  $\bar{x}_L = \$113$ , and 42 people donated to organization M, with an average donation amount of  $\bar{x}_M = \$390$ .

What is the correct unit of measure for the mean of the sampling distribution of  $\bar{x}_L - \bar{x}_M$ ?

- (A) Donations
- (B) Months
- (C) People
- (D) Organizations
- (E) Dollars ✓

## Answer E

Correct. The unit for the mean of the sampling distribution of  $\bar{x}_L - \bar{x}_M$  is the same as the unit for the individual sample means, which is dollars.

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5. A large school district held a district-wide track meet for all high school students. For the 2-mile run, the population of female students participating had a mean running time of 8.8 minutes with standard deviation of 3.3 minutes, and the population of male students participating had a mean running time 7.3 minutes with standard deviation of 2.9 minutes. Suppose 8 female students and 8 male students who participated in the 2-mile run are selected at random from each population. Let  $\bar{x}_F$  represent the sample mean running time for the female students, and let  $\bar{x}_M$  represent the sample mean running time for the male students.

What are the mean and standard deviation of the sampling distribution of the difference in sample means  $\bar{x}_F - \bar{x}_M$ ?

- (A) The mean is 0.4, and the standard deviation is  $\sqrt{\frac{8.8}{8} + \frac{7.3}{8}}$ .
- (B) The mean is 0.4, and the standard deviation is  $\sqrt{\frac{8.8^2}{8} + \frac{7.3^2}{8}}$ .
- (C) The mean is 1.5, and the standard deviation is  $\sqrt{\frac{3.3^2}{8} - \frac{2.9^2}{8}}$ .
- (D) The mean is 1.5, and the standard deviation is  $\sqrt{\frac{3.3}{8} + \frac{2.9}{8}}$ .
- (E) The mean is 1.5, and the standard deviation is  $\sqrt{\frac{3.3^2}{8} + \frac{2.9^2}{8}}$ . ✓

## Answer E

Correct. The mean is equal to the difference in the population means  $8.8 - 7.3 = 1.5$ , and the standard

deviation of the sampling distribution of  $\bar{x}_F - \bar{x}_M$  is  $\sqrt{\frac{\sigma_F^2}{n_F} + \frac{\sigma_M^2}{n_M}} = \sqrt{\frac{3.3^2}{8} + \frac{2.9^2}{8}}$ .

6. Cheryl practices hitting a softball in an indoor stadium by using both an aluminum bat and a composite bat made of carbon fiber and graphite. She records the distance traveled by the ball for each hit. Let  $\bar{x}_1$  represent the average distance traveled by balls hit with the aluminum bat, and let  $\bar{x}_2$  represent the average distance traveled by balls hit with the composite bat. Assume Cheryl's batting practice hits are independent.

Which of the following conditions are sufficient to model the sampling distribution of  $\bar{x}_1 - \bar{x}_2$  with a normal distribution?

- I. There are at least 30 recorded distances traveled for each type of bat.
- II. The distribution of distance traveled by the ball is approximately normal for each bat.
- III. The total number of distances traveled is at least 60 for the two bats combined.

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- (A) I only  
(B) II only  
(C) III only  
(D) I and II only  
(E) II and III only



## Answer D

Correct. The sampling distribution of  $\bar{x}_1 - \bar{x}_2$  can be modeled with a normal distribution if population 1 and population 2 can each be modeled with a normal distribution OR can be modeled approximately with a normal distribution if the sample sizes for each sample,  $n_1$  and  $n_2$ , are at least 30.

7. For a population of leghorn chickens, the mean number of eggs laid per chicken is 0.70 with standard deviation 0.20 egg. For a population of Sussex chickens, the mean number of eggs laid per chicken is 0.50 with standard deviation 0.10 egg. Two independent random samples of chickens were taken from the populations. The following table shows the sample statistics.

	$n$	$\bar{x}$	$s$
Leghorn	36	0.75	0.30
Sussex	36	0.45	0.12

Mike claims that for all samples of size 36 from the population of leghorn chickens and all samples of size 36 from the population of Sussex chickens, the mean of all possible differences in sample means (leghorn minus Sussex) is 0.30 eggs per chicken. Is Mike's claim correct?

- (A) Yes. The mean is  $0.75 - 0.45 = 0.30$  egg per chicken.  
(B) No. The mean is  $0.30 - 0.12 = 0.18$  egg per chicken.  
(C) No. The mean is  $0.70 - 0.50 = 0.20$  egg per chicken.  
(D) No. The mean is  $0.20 - 0.10 = 0.10$  egg per chicken.  
(E) No. The mean is  $36(0.75) - 36(0.45) = 10.8$  eggs per chicken.



## Answer C

Correct. Mike's claim is not correct. The mean of all differences in sample means from all possible samples of size 36 of leghorn chickens and of size 36 of Sussex chickens is equal to the difference in the population means.

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8. The distribution of ocean wave height at a certain California beach is approximately normal with mean 7.2 feet. The distribution of ocean wave height at a certain Florida beach is approximately normal with mean 6.6 feet. Six waves from each beach will be selected at random and the heights will be recorded. Let  $\bar{x}_C$  represent the sample mean height of the 6 California waves, and let  $\bar{x}_F$  represent the sample mean height of the 6 Florida waves.

Which of the following is the best interpretation of  $P(\bar{x}_C - \bar{x}_F > 0.5) = 0.55$  ?

- (A) The probability that the heights for all 6 California waves will exceed the heights for all 6 Florida waves by more than 0.55 feet is 0.5.
- (B) The probability that the heights for all 6 California waves will exceed the heights for all 6 Florida waves by more than 0.5 feet is 0.55.
- (C) The probability of observing a difference (California minus Florida) greater than 0.5 feet between the mean height of 6 California waves and the mean height of 6 Florida waves is 0.55. ✓
- (D) The probability of observing a difference greater than 0.5 feet between the height of one wave in California and the height of one wave in Florida is 0.55.
- (E) The probability of observing a difference greater than 0.55 feet between the height of one wave in California and the height of one wave in Florida is 0.5.

**Answer C**

Correct. The statement gives a probability of 0.55 that the sample mean for California exceeds the sample mean for Florida by more than 0.5 feet.

9. Researchers are studying two populations of wild horses living in the western regions of a country. In a random sample of 32 horses taken from the first population, the mean age of the sample was 21 years. In a random sample of 41 horses from the second population, the mean age of the sample was 19 years.

Is the sampling distribution of the difference in sample mean ages approximately normal?

- (A) Yes, because the two populations of wild horses can be modeled by a normal distribution.
- (B) Yes, because the samples were selected at random.
- (C) Yes, because the sample sizes are both greater than 30. ✓
- (D) No, because the populations are not normal.
- (E) No, because the difference in sample mean ages was not 0.

**Answer C**

Correct. Both sample sizes are at least 30, which indicates that the sampling distribution of the difference in sample mean ages can be modeled approximately by a normal distribution.