Scaffolded Practice Problems for Assessment Questions 9-12

Problem 9 Scaffolding: Polynomial Long Division

Scaffold 9.1: Basic Division Review

Complete these basic polynomial divisions:

a) $\frac{6x^2}{2x} = ____$

b) $\frac{15x^3}{3x} = ____$

c) $\frac{8x^4}{4x^2} = ____$

d) $\frac{12x^3 + 6x^2}{3x} = \frac{12x^3}{3x} + \frac{6x^2}{3x} = ____ + ____$

Scaffold 9.2: Simple Long Division

Divide: $(x^2 + 5x + 6) \div (x + 2)$

Step 1: How many times does x go into x^2? $____$
Write x above the division bar.

Step 2: Multiply: $x(x + 2) = x^2 + 2x$

Step 3: Subtract: $(x^2 + 5x + 6) - (x^2 + 2x) = ____x + 6$

Step 4: How many times does x go into $3x$? $____$
Write 3 above the division bar.

Step 5: Multiply: $3(x + 2) = 3x + 6$

Step 6: Subtract: $(3x + 6) - (3x + 6) = ____$

Answer: $x^2 + 5x + 6 = (x + 2)(____) + ____$

Scaffold 9.3: Division with Remainder

Divide: $(x^2 + 3x + 1) \div (x + 1)$

$\begin{array}{c|cc cc}
& x & +2 \
\hline
x+1 & x^2 & +3x & +1 \
& x^2 & +x & \

\hline
& & 2x & +1 \
& & 2x & +2 \
\hline
& & & -1
\end{array}$

Complete the division:
Quotient: $x + 2$
Remainder: -1

Write the answer: $(x^2 + 3x + 1) = (x + 1)(x + 2) + (-1)$

Or: $\frac{x^2 + 3x + 1}{x + 1} = x + 2 + \frac{-1}{x + 1}$

Scaffold 9.4: Longer Polynomial Division

Divide: $(x^3 + 2x^2 - x - 2) \div (x + 2)$

Set up the division:
$\begin{array}{c|cc cc}
& & & \
\hline
x+2 & x^3 & +2x^2 & -x & -2 \
\end{array}$

Step 1: $x^3 \div x = ____$ (write above)
Multiply: $x^2(x + 2) = __________$
Subtract to get: $__________$

Step 2: Continue the process...
Final answer: $x^3 + 2x^2 - x - 2 = (x + 2)(__________) + ____$

Problem 10 Scaffolding: Solving Literal Equations

Scaffold 10.1: Basic Literal Equations

Solve for the indicated variable:

a) $y = mx + b$, solve for m
$y - b = mx$
$m = \frac{y - b}{x}$

b) $A = lw$, solve for l
$l = ____$

c) $P = 2l + 2w$, solve for w
$P - 2l = 2w$
$w = ____$

d) $C = 2\pi r$, solve for r
$r = ____$

Scaffold 10.2: Equations with Fractions

Solve for the indicated variable:

a) $\frac{x}{y} = z$, solve for x
Multiply both sides by y: $x = ____$

b) $\frac{a}{b} = \frac{c}{d}$, solve for a
$a = ____$

c) $\frac{P}{V} = \frac{nRT}{V}$, solve for T
Cross multiply or multiply both sides by V:
$P = nRT$
$T = ____$

Scaffold 10.3: More Complex Literal Equations

Solve for the indicated variable:

a) $A = P(1 + rt)$, solve for t
$\frac{A}{P} = 1 + rt$
$\frac{A}{P} - 1 = rt$
$t = \frac{\frac{A}{P} - 1}{r} = \frac{A - P}{Pr}$

b) $S = \frac{n(a + l)}{2}$, solve for a
$2S = n(a + l)$
$\frac{2S}{n} = a + l$
$a = \frac{2S}{n} - l$

c) $V = \frac{1}{3}\pi r^2 h$, solve for h
$h = ____$

Scaffold 10.4: Business Formula Practice

Given: $P = R - C$ where P = profit, R = revenue, C = cost

a) Solve for R: $R = ____$

b) If $R = px$ (price × quantity) and $C = F + vx$ (fixed costs + variable costs), then:
$P = px - (F + vx) = px - F - vx$

Solve for v (variable cost per unit):
$P = px - F - vx$
$P + F = px - vx$
$P + F = x(p - v)$
$\frac{P + F}{x} = p - v$
$v = p - \frac{P + F}{x}$

Practice with the assessment formula $N = S(P - V) - F$:
Solve for V: $V = ____$

Problem 11 Scaffolding: Inverse Functions

Scaffold 11.1: Understanding Inverse Functions

If $f(2) = 5$, then $f^{-1}(5) = 2$. The inverse function "undoes" the original function.

Complete:
a) If $f(3) = 7$, then $f^{-1}(7) = ____$
b) If $f(-1) = 4$, then $f^{-1}(4) = ____$
c) If $f^{-1}(0) = 6$, then $f(6) = ____$

Scaffold 11.2: Finding Inverses Algebraically

To find the inverse of $y = f(x)$:

1. Switch x and y
2. Solve for y
3. The result is $f^{-1}(x)$

Find the inverse of $f(x) = 2x + 1$:
Step 1: $y = 2x + 1$
Step 2: Switch variables: $x = 2y + 1$
Step 3: Solve for y:
$x - 1 = 2y$
$y = \frac{x - 1}{2}$

Therefore: $f^{-1}(x) = \frac{x - 1}{2}$

Practice: Find the inverse of $f(x) = 3x - 5$
$f^{-1}(x) = ____$

Scaffold 11.3: Square Root Functions and Their Inverses

Find the inverse of $f(x) = \sqrt{x + 2}$ for $x \geq -2$:

Step 1: $y = \sqrt{x + 2}$
Step 2: Switch: $x = \sqrt{y + 2}$
Step 3: Solve for y:
Square both sides: $x^2 = y + 2$
$y = x^2 - 2$

So $f^{-1}(x) = x^2 - 2$

What should the domain restriction be?
Since the original function had range $y \geq 0$, the inverse should have domain $x \geq 0$.

Practice: Find the inverse of $f(x) = \sqrt{x - 5}$ for $x \geq 5$
$f^{-1}(x) = ____$ for $x \geq ____$

Scaffold 11.4: Context and Domain Restrictions

A function represents profit P (in thousands) after t years: $P = \sqrt{t - 3}$ for $t \geq 3$.

To find years as a function of profit:
Step 1: $P = \sqrt{t - 3}$
Step 2: Switch: $t = \sqrt{P - 3}$ (This doesn't make sense)
Let me restart: $P = \sqrt{t - 3}$
Step 2: Switch: $t = \sqrt{P - 3}$ (No, this is wrong)

Correct approach:
Step 1: $P = \sqrt{t - 3}$
Step 2: Switch variables: $t = \sqrt{P - 3}$ (Wrong again)

Let me be more careful:
$P = \sqrt{t - 3}$
Switch variables: $t = \sqrt{P - 3}$ (This is backwards)

Correct:
$P = \sqrt{t - 3}$

To find inverse, switch P and t: $t = \sqrt{P - 3}$
Solve for P: $t^2 = P - 3$, so $P = t^2 + 3$

Wait, let me restart completely:
$f(t) = \sqrt{t - 3}$ gives profit as function of time
For inverse: $y = \sqrt{x - 3}$
Switch: $x = \sqrt{y - 3}$
Square: $x^2 = y - 3$
Solve: $y = x^2 + 3$

So time as function of profit: $f^{-1}(P) = P^2 + 3$ for $P \geq 0$

Problem 12 Scaffolding: Average Rate of Change

Scaffold 12.1: Understanding Average Rate of Change

Average rate of change = $\frac{\text{change in y}}{\text{change in x}} = \frac{f(b) - f(a)}{b - a}$

For $f(x) = x^2$, find the average rate of change from $x = 1$ to $x = 3$:
$f(1) = 1^2 = 1$
$f(3) = 3^2 = 9$
Average rate of change = $\frac{9 - 1}{3 - 1} = \frac{8}{2} = 4$

Practice: For $f(x) = 2x + 1$, find average rate of change from $x = 0$ to $x = 4$:
$f(0) = ____$
$f(4) = ____$
Average rate of change = $\frac{____ - ____}{4 - 0} = ____$

Scaffold 12.2: Negative Intervals

For $f(x) = x^2$, find the average rate of change from $x = -2$ to $x = 1$:

$f(-2) = (-2)^2 = ____$
$f(1) = 1^2 = ____$

Average rate of change = $\frac{f(1) - f(-2)}{1 - (-2)} = \frac{____ - ____}{____} = ____$

Scaffold 12.3: Quadratic Functions

For $f(x) = -x^2 + 3$, find the average rate of change from $x = -1$ to $x = 2$:

$f(-1) = -(-1)^2 + 3 = -1 + 3 = 2$
$f(2) = -(2)^2 + 3 = -4 + 3 = -1$

Average rate of change = $\frac{-1 - 2}{2 - (-1)} = \frac{-3}{3} = -1$

Practice: For $f(x) = -x^2 + 4$, find average rate of change from $x = -2$ to $x = 1$:

$f(-2) = ____$

$f(1) = ____$

Average rate of change = $____$

Scaffold 12.4: Decimal Intervals

For $f(x) = -2x^2 + 5$, find the average rate of change from $x = -1.5$ to $x = 0.5$:

$f(-1.5) = -2(-1.5)^2 + 5 = -2(2.25) + 5 = -4.5 + 5 = 0.5$

$f(0.5) = -2(0.5)^2 + 5 = -2(0.25) + 5 = -0.5 + 5 = 4.5$

Average rate of change = $\frac{4.5 - 0.5}{0.5 - (-1.5)} = \frac{4}{2} = 2$

Now try the assessment problem setup:

For $f(x) = -2x^2 + 5$, find average rate of change from $x = -3.5$ to $x = 0$:

$f(-3.5) = -2(-3.5)^2 + 5 = -2(12.25) + 5 = ____$

$f(0) = -2(0)^2 + 5 = ____$

Average rate of change = $\frac{____ - ____}{0 - (-3.5)} = \frac{____}{3.5} = ____$