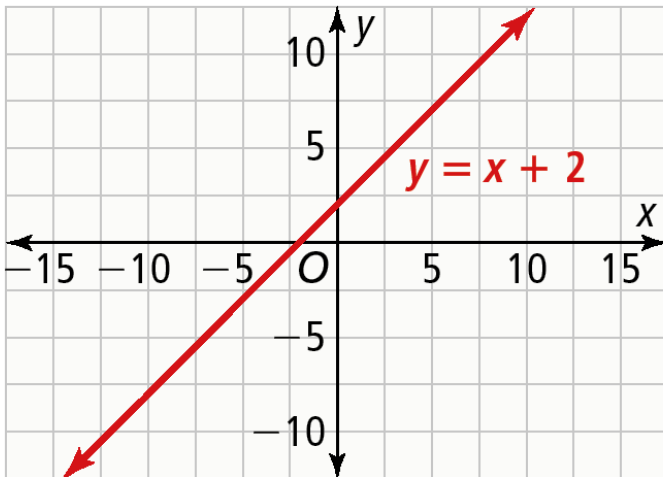


Lesson 4.3: Multiplying and Dividing Rational Expressions

Objectives	<ul style="list-style-type: none"> ● Use the structure of rational expressions to rewrite simple rational expressions in different forms. ● Understand that rational expressions form a system analogous to the system of rational numbers and use that understanding to multiply and divide rational expressions.
Language Objective	<ul style="list-style-type: none"> ● SWBAT use a 3-reads protocol to interpret an Algebra 2 word problem by listening to the teacher's reading and summarizing the situation in a sentence, reading it aloud to state the math question, and silently rereading to list key facts and figures.
Essential Understanding	Rational expressions form a system similar to the system of rational numbers and can be multiplied and divided by applying the properties of operations as they apply to rational expressions.

EXPLORE & REASON

Consider the following graph of the function $y = x + 2$.



A. What is the domain of this function?

B. Sketch a function that resembles the graph, but restrict its domain to exclude 2.

C. Use Structure Consider the function you have sketched.

What kind of function might have a graph like this? Explain.

EXAMPLE 1 Write Equivalent Rational Expressions

Write an expression that is equivalent to $\frac{x+3}{x+9}$. For what domain are the expressions equivalent?

$$\frac{4}{5} = \frac{4}{5} \cdot 1 = \frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10}$$

You can multiply by factors of 1 in any form 1 to write equivalent rational expressions. In this Example, multiply by $\frac{x+6}{x+6}$ and $\frac{x}{x}$.

CONCEPTUAL UNDERSTANDING**EXAMPLE 1** Write Equivalent Rational Expressions

Expressions are equivalent for all values of x that are in both domains.

Try It!

1. Write an expression equivalent to $\frac{x-4}{x}$ over the domain $\{x \mid x \neq 0 \text{ or } -2\}$.

EXAMPLE 2 Simplify a Rational Expression

What is the simplified form of the rational expression? What is the domain for which the identity between the two expressions is valid?

$$\frac{4 - x^2}{x^2 + 3x - 10}$$

Try It!

2. Simplify each expression and state the domain.

a. $\frac{x^2 + 2x + 1}{x^3 - 2x^2 - 3x}$

b. $\frac{x^3 + 4x^2 - x - 4}{x^2 + 3x - 4}$

EXAMPLE 3 Multiply Rational Expressions

A. What is the product of $\frac{2xy}{z}$ and $\frac{3x^2}{4yz}$?

To multiply rational expressions, follow a similar method to that for multiplying two numerical fractions.

B. What is the product of $\frac{5x}{x+3} \cdot \frac{x^2+x-6}{x^2+2x+1} \cdot \frac{x^2+x}{5x-10}$ in simplified form?

Try It!

3. Find the simplified form of each product, and state the domain.

a. $\frac{x^2 - 16}{9 - x} \cdot \frac{x^2 + x - 90}{x^2 + 14x + 40}$

b. $\frac{x + 3}{4x} \cdot \frac{3x - 18}{6x + 18} \cdot \frac{x^2}{4x + 12}$

EXAMPLE 4 Multiply a Rational Expression by a Polynomial

What is the product of $\frac{x+2}{x^4-16}$ and x^3+4x^2-12x ?

Try It!

4. Find the simplified form of each product and the domain

a. $\frac{x^3-4x}{6x^2-13x-5} \cdot (2x^3-3x^2-5x)$

b. $\frac{3x^2 + 6x}{x^2 - 49} \cdot (x^2 + 9x + 14)$

EXAMPLE 5 Divide Rational Expressions

What is the quotient of $\frac{x^3 + 3x^2 + 3x + 1}{1 - x^2}$ and $\frac{x^2 + 5x + 4}{x^2 + 3x - 4}$?

Try It!

5. Find the simplified quotient and the domain of each expression.

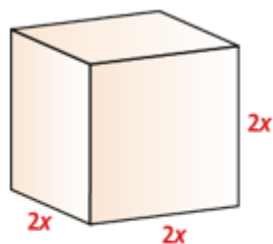
a. $\frac{1}{x^2 + 9x} \div \left(\frac{6 - x}{3x^2 - 18x} \right)$

b. $\frac{2x^2 - 12x}{x + 5} \div \left(\frac{x - 6}{x + 5} \right)$

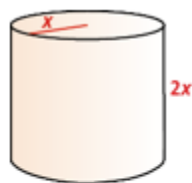
APPLICATION

EXAMPLE 6 Use Division of Rational Expressions

A company is evaluating two packaging options for its product line. The more efficient design will have the lesser ratio of surface area to volume. Should the company use packages that are cylinders or rectangular prisms?



Surface Area: $2(2x)^2 + 4(2x)^2$
Volume: $(2x)^3$



Surface Area: $2\pi x^2 + 2\pi x(2x)$
Volume: $\pi x^2(2x)$

<p>6. <u>The company compares the ratios of surface area to volume for two more containers. One is a rectangular prism with a square base. The other is a rectangular prism with a rectangular base. One side of the base is equal to the side length of the first container, and the other side is twice as long. The surface area of this second container is $4x^2 + 6xh$. The heights of the two containers are equal. Which has the smaller surface area-to-volume ratio?</u></p>	First Read - Understanding the Context	
	<i>What is the core of the problem?</i> <ul style="list-style-type: none"> I think this problem is about... 	
	Second Read - Interpreting the Question	
	<i>What are we trying to find out?</i> <ul style="list-style-type: none"> I know the problem is asking...because... 	
	Third Read - Identifying Information	
	<i>What are the important quantities, relationships, and other relevant information?</i> <ul style="list-style-type: none"> The quantities are... I can count... These quantities help me to answer... The information from the situation that we need is... 	
	Solution (show and explain your answer)	
	Interpretation of solution (interpret your solution in your own words)	

CONCEPT SUMMARY

Products and Quotients of Rational Expressions

	Multiply	Multiply an Integer or a Polynomial	Divide
RATIONAL EXPRESSIONS	$\frac{3x}{x+1} \cdot \frac{x^2+x}{3x-6}$ <p>The domain is $x \neq -1$ or 2.</p>	$\frac{x+2}{x^2-4} \cdot (x^2-2x)$ $= \frac{x+2}{x^2-4} \cdot \frac{x^2-2x}{1}$ <p>The domain is $x \neq -2$ or 2.</p>	$\frac{1-x^2}{x^2+3x-4} \div \frac{x+1}{x+4}$ $= \frac{1-x^2}{x^2+3x-4} \cdot \frac{x+4}{x+1}$ <p>The domain is $x \neq -4,$ $-1,$ or 1.</p>
WORDS	Identify common factors and simplify.	Write the polynomial as a rational expression with 1 in the denominator. Then <u>multiply</u> .	Multiply by the reciprocal of the <u>divisor</u> .