Name_____

Robert Colson's

Lesson 4.2: **Graphing Rational Functions**

Objectives	● Graph rational functions by identifying asymptotes and end behavior. ● Rewrite simple rational expressions in different forms using long division.
Language Objective	● SWBAT use a 3-reads protocol to interpret an Algebra 2 word problem by listening to the teacher's reading and summarizing the situation in a sentence, reading it aloud to state the math question, and silently rereading to list key facts and figures.
Essential Understanding	A rational function is any function $R(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions. The domain of a rational function is all real numbers except any x-values for which $Q(x)$ equals to zero. The graph of a rational function has one or more asymptotes, which guide the end behavior of the graph.

EXPLORE & REASON *using DESMOS*

A. **Look for Relationships**. Graph each function. Determine whether the functions are linear. Find the y intercept of each function, and slope, if appropriate.

Function	Linear? (Yes/No)	y-intercept	Slope
$f(x) = x - 1$			
$g(x) = \dfrac{x - 1}{2}$			
$h(x) = \dfrac{x - 1}{x - 2}$			

B. What is the effect on the graph of f when dividing x-1 by 2?
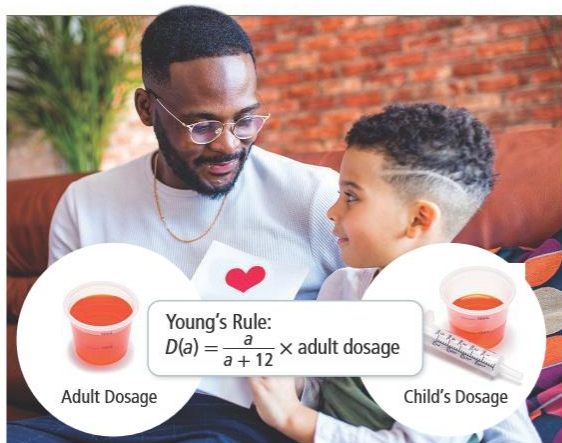
C. What happens to the graph of h as x approaches 2?

D. **Communicate Precisely**. What is the effect on the graph of f(x) when dividing x-1 by x-2. (Hint: Compare it to what you found in part (B)

Name_____ Date_____ Period_____

	First Read - Understanding the Context	
EXAMPLE 5 Use a Rational Function Model		
A pediatric doctor may need to administer medication without knowing a child's weight. Young's Rule can be used to calculate a child's dosage $D(a)$ given their age a and the adult dosage.	*What is the core of the problem?*	
	• I think this problem is about...	
A doctor has 60 mcg of a medication. What is the youngest a child can be to receive this dose of medication if the adult dosage is 125 mcg?		
	Second Read - Interpreting the Question	
	What are we trying to find out?	
	• I know the problem is asking...because...	
Young's Rule: $D(a) = \dfrac{a}{a+12} \times$ adult dosage		
Adult Dosage Child's Dosage	**Third Read - Identifying Information**	
	What are the important quantities, relationships, and other relevant information?	
	• The quantities are...	
	• I can count...	
	• These quantities help me to answer...	
	• The information from the situation that we need is...	
	Solution (show and explain your answer)	
	Interpretation of solution (interpret your solution in your own words)	

ESSENTIAL QUESTION

How can you graph a rational function?

EXAMPLE 1 Rewrite a Rational Function to Identify Asymptotes

How is the quotient $g(x) = \frac{4x}{x-3}$ related to the reciprocal function, $f(x) = \frac{1}{x}$?

Use long division to write the rational expression
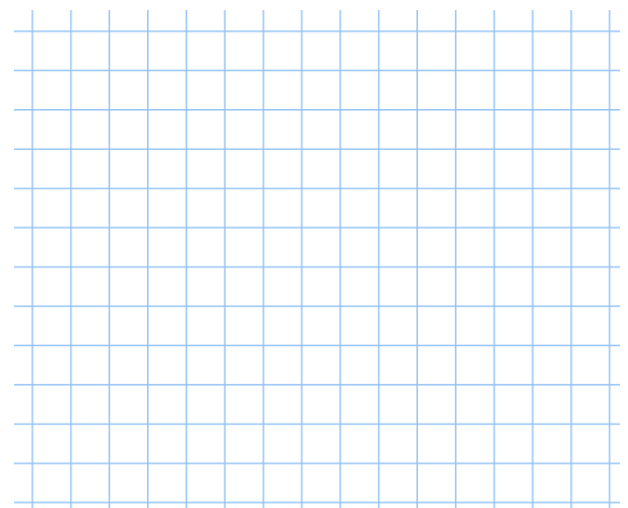in the form $\frac{a}{x-h} + k$.
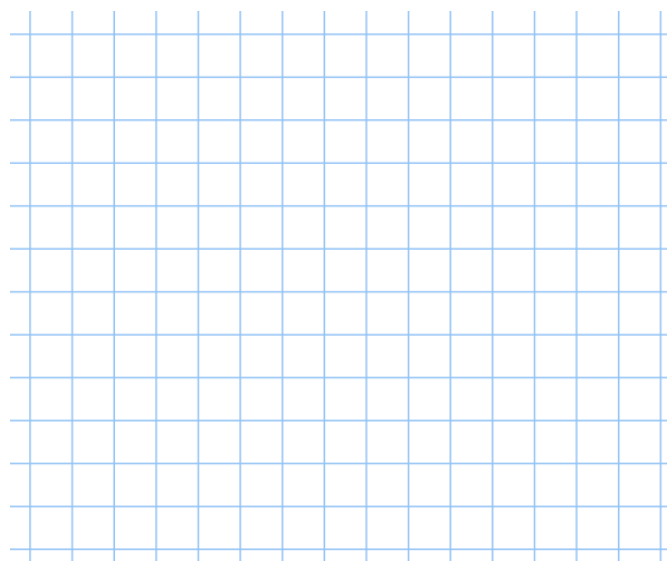
Check, using Desmos.

Try It!

1. Use long division to rewrite each rational function. Find the asymptotes of *f* and sketch the graph.

a. $f(x) = \dfrac{6x}{2x + 1}$

- -

Try It!

1. Use long division to rewrite each rational function. Find the asymptotes of *f* and sketch the graph.

b. $f(x) = \dfrac{x}{x - 6}$

CONCEPT Rational Function

Just as a rational number is a number that can be expressed as the ratio of two integers, a **rational expression** is an expression that can be expressed as the ratio of two polynomials, such as $\frac{P(x)}{Q(x)}$.

A **rational function** is any function defined by a rational expression, such as $R(x) = \frac{P(X)}{Q(X)}$. The domain of $R(x)$ is all values of x for which $Q(x) \neq 0$.

The function $g(x) = \frac{4x}{x-3}$ is a rational function.

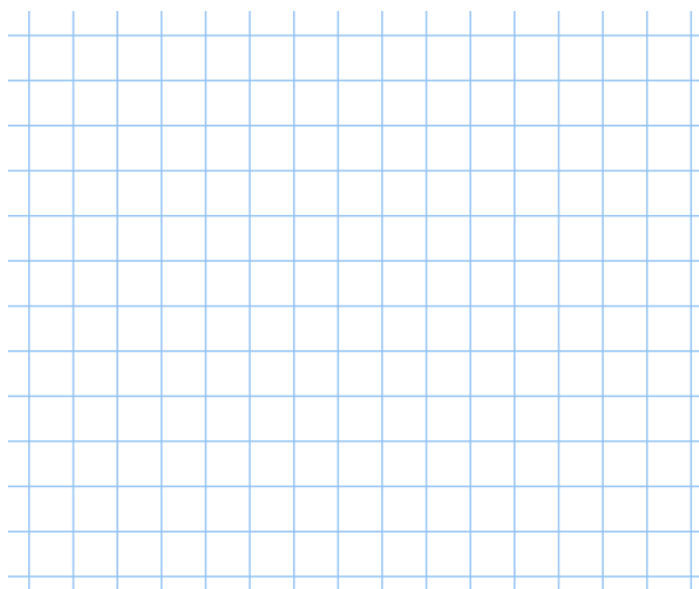## EXAMPLE 2 Find Multiple Vertical Asymptotes of a Rational Function

What are the vertical asymptotes for the graph of $f(x) = \dfrac{3x - 2}{x^2 + 7x + 12}$?
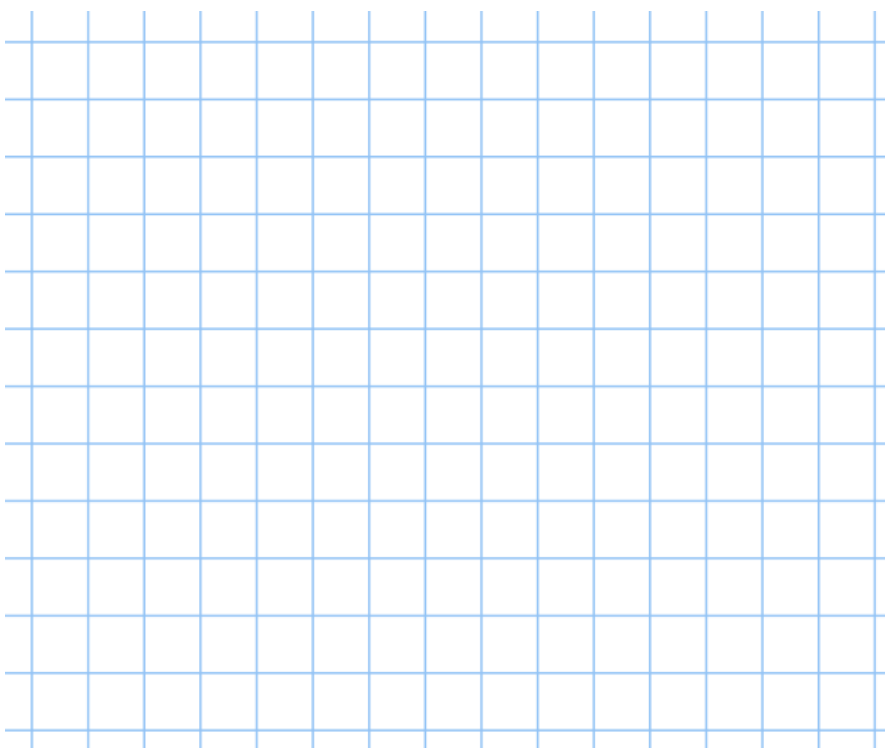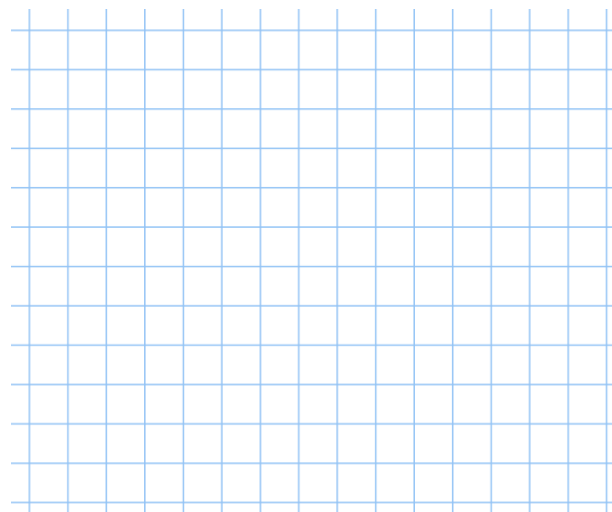
Try It!

2. Find the vertical asymptotes for each function. Graph the function to check your work.

   a. $g(x) = \dfrac{5x}{x^2 - x - 6}$

Try It!

2. Find the vertical asymptotes for each function. Graph the function to check your work.

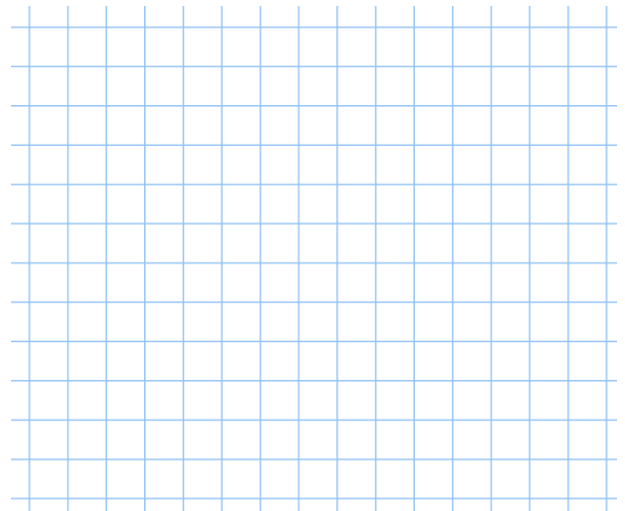   **b.** $h(x) = \dfrac{7 - x}{(x - 5)(x + 1)(x + 3)}$

EXAMPLE 3   Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 1:

EXAMPLE 3   Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 2:

EXAMPLE 3 Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 3:

EXAMPLE 3 Find Types of Horizontal Asymptotes

What are the horizontal asymptotes for the graph $f(x) = \dfrac{3x - 2}{x^2 + 7x + 12}$?

Try It!

3. What are the horizontal asymptotes of the graph of each function?

a. $g(x) = \dfrac{2x^2 + x - 9}{2x - 8}$

b. $h(x) = \dfrac{x^2 + 5x + 4}{3x^2 - 12}$

c. $k(x) = \dfrac{x}{(2x-1)(x+6)}$

Practice & Problem Solving

2. What is the domain of the f

$f(x) = \dfrac{x^2 - x - 2}{x^4 - 81}$?

Ⓐ All real numbers except

Ⓑ All real numbers except –

Ⓒ All real numbers except –

Ⓓ All real numbers except and 3

12. What are the horizontal a vertical asymptotes of the

$y = \dfrac{x^4 + 3}{x^4 + 2x^2 - 3}$?

Ⓐ $y = 1; x = \pm 1$

Ⓑ $y = 1; x = \pm 3$

Ⓒ $y = 0; x = 1$

Ⓓ $y = 0; x = -1$

3. What are the horizontal and vertical asymptotes of the g

$y = \dfrac{x^2 - 3x - 4}{3 - x^2}$?

Ⓐ $y = -1; x = \pm\sqrt{3}$

Ⓑ $y = 1; x = \pm\sqrt{3}$

Ⓒ $y = -1; x = 1$ and $x = \sqrt{3}$

Ⓓ $y = -1; x = 1$ and $x = -\sqrt{}$

18. Select all the functions who
   graphs have a horizontal
   asymptote at $y = \frac{2}{3}$.

   ☐ **A.** $y = \dfrac{2}{3x - 1}$

   ☐ **B.** $y = \dfrac{2x^2 + 1}{3x^2 - 2}$

   ☐ **C.** $y = \dfrac{2}{3} + \dfrac{1}{x}$

   ☐ **D.** $y = \dfrac{2x - 3}{3x^2 + 1}$

   ☐ **E.** $y = 3 + \dfrac{3}{2x}$

16. What are the horizontal an
   vertical asymptotes of the g
   $y = \dfrac{-x + 3}{x - 8}$?

   Ⓐ $y = -1; x = 8$

   Ⓑ $y = -1; x = -8$

   Ⓒ $y = 1; x = 8$

   Ⓓ $y = 1; x = -8$

Graphing Rational Functions

RATIONAL FUNCTION	A function that is expressible as a fraction with polynomials in the numerator and the denominator

ASYMPTOTES	**Vertical**	**Horizontal**
	Vertical asymptotes are guides for the behavior of a graph as it approaches a vertical line.	Horizontal asymptotes are guides for the end behavior of a graph as it approaches a horizontal line.
	• The line $x = a$ is a vertical asymptote of $\frac{P(x)}{Q(x)}$, if $Q(a) = 0$ and $P(a) \neq 0$.	If the degree of the numerator is
	• The up or down behavior of the function as it approaches the asymptote can be determined by substituting values close to a on either side of the asymptote.	• less than the degree of the denominator, the horizontal asymptote is at $y = 0$.
		• greater than the denominator, there is no horizontal asymptote.
		• equal to the degree of the denominator, set y equal to the ratio of the leading coefficients. The graph of this line is the horizontal asymptote.

ALGEBRA	**GRAPH**
$f(x) = \dfrac{8x - 3}{4x + 1}$	
Vertical Asymptote: Let $4x + 1 = 0$ and solve.	
$x = -\dfrac{1}{4}$	
Horizontal Asymptote: Find the ratio of the leading coefficients $\left(\frac{8}{4}\right)$.	
$y = 2$	