Scaffolded Practice Problems for Assessment Questions 1-4

Problem 1 Scaffolding: Function Transformations

Scaffold 1.1: Basic Translations

The graph of $y = |x|$ has its vertex at $(0, 0)$. What is the vertex of each transformed function?

a) $y = |x - 3|$ has vertex at $(\_\_\_\_, \_\_\_\_)$
b) $y = |x| - 5$ has vertex at $(\_\_\_\_, \_\_\_\_)$
c) $y = |x + 2| + 1$ has vertex at $(\_\_\_\_, \_\_\_\_)$

Scaffold 1.2: Identifying Transformations

Match each transformation to its effect on the graph:

Transformation	**Effect**
$f(x - h)$ where $h > 0$	A. Moves graph up
$f(x) + k$ where $k > 0$	B. Moves graph down
$f(x) - k$ where $k > 0$	C. Moves graph right
$f(x + h)$ where $h > 0$	D. Moves graph left

Scaffold 1.3: Combined Transformations

The function $f(x) = |x + 1|$ has vertex at $(-1, 0)$. Find the vertex after these transformations:

a) Translate 2 units right: vertex at $(\_\_\_\_, \_\_\_\_)$
b) Then translate 3 units down: vertex at $(\_\_\_\_, \_\_\_\_)$

What is the equation of the final transformed function? $y = \_\_\_\_\_\_\_\_\_\_$

Scaffold 1.4: Reflection and Translation

The graph of $y = -|x + 2| + 3$ is shown. If this graph is translated 5 units right and 6 units down, what is the equation of the new graph?

Original vertex: $(-2, 3)$
New vertex after translation: $(\_\_\_\_, \_\_\_\_)$
New equation: $y = \_\_\_\_\_\_\_\_\_\_$

Problem 2 Scaffolding: Vertical Asymptotes

Scaffold 2.1: Understanding Vertical Asymptotes

A vertical asymptote occurs when the denominator of a rational function equals zero but the numerator does not.

For each function, find where the denominator equals zero:

a) $f(x) = \frac{1}{x - 3}$ has vertical asymptote at $x = ____$
b) $f(x) = \frac{2}{x + 5}$ has vertical asymptote at $x = ____$
c) $f(x) = \frac{x}{x - 7}$ has vertical asymptote at $x = ____$

Scaffold 2.2: Logarithmic Functions Domain

The domain of $\ln(x)$ is $x > 0$. Find the domain of each function and identify any vertical asymptotes:

a) $f(x) = \ln(x - 2)$
Domain: $x > ____$
Vertical asymptote: $x = ____$

b) $f(x) = \ln(x + 1)$
Domain: $x > ____$
Vertical asymptote: $x = ____$

c) $f(x) = \log(x - 6)$
Domain: $x > ____$
Vertical asymptote: $x = ____$

Scaffold 2.3: Identifying Asymptotes in Transformed Logs

For each function, determine if there is a vertical asymptote at $x = 4$:

a) $f(x) = \log(x - 4)$ □ Yes □ No
b) $f(x) = \log(x + 4)$ □ Yes □ No
c) $f(x) = \log(x) - 4$ □ Yes □ No
d) $f(x) = 4\log(x)$ □ Yes □ No

Scaffold 2.4: Multiple Choice Practice

Which of these functions has a vertical asymptote at $x = 4$?

A. $f(x) = \log_2(x) - 4$
B. $f(x) = \ln(x - 4) + 1$
C. $f(x) = 3\log(x) + 4$
D. $f(x) = \log(x + 4) - 2$

Explain your reasoning: ____________________________

Problem 3 Scaffolding: Work Rate Problems

Scaffold 3.1: Understanding Rates

If a faucet can fill a tank in 6 hours, what fraction of the tank does it fill in 1 hour?

Rate = $\frac{1}{6}$ of the tank per hour

Complete these:
a) If a faucet fills a tank in 4 hours, its rate is $\frac{___}{___}$ of the tank per hour
b) If a faucet fills a tank in 8 hours, its rate is $\frac{___}{___}$ of the tank per hour
c) If a faucet fills a tank in t hours, its rate is $\frac{___}{___}$ of the tank per hour

Scaffold 3.2: Adding Rates

When two faucets work together, their rates add up.

Example: Faucet A fills $\frac{1}{6}$ of the tank per hour, Faucet B fills $\frac{1}{3}$ of the tank per hour. Together they fill: $\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$ of the tank per hour.

So together they fill the tank in 2 hours.

Practice:
a) Faucet A: 4 hours alone, rate = $\frac{___}{___}$ per hour
b) Faucet B: 12 hours alone, rate = $\frac{___}{___}$ per hour
c) Combined rate = $\frac{___}{___} + \frac{___}{___} = \frac{___}{___}$ per hour
d) Time to fill together = ____ hours

Scaffold 3.3: Setting Up the Equation

If Faucet A takes a hours and Faucet B takes b hours to fill a tank alone, the equation for the time t it takes them together is:

$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$

Set up (but don't solve) the equation for these scenarios:
a) Faucet A: 5 hours, Faucet B: 10 hours
Equation: ___________________

b) Faucet A: 6 hours, Faucet B: 9 hours
Equation: ___________________

Scaffold 3.4: Solving Work Problems

Solve: $\frac{1}{8} + \frac{1}{4} = \frac{1}{t}$

Step 1: Find common denominator for left side
$\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{___}{8} = \frac{___}{8}$

Step 2: Solve for t
$\frac{___}{8} = \frac{1}{t}$

Therefore: $t = \frac{8}{___} = ___$ hours

Convert to hours and minutes: ____ hours and ____ minutes

Problem 4 Scaffolding: Vertex Form and Transformations

Scaffold 4.1: Understanding Vertex Form

The vertex form of a quadratic is $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex.

Find the vertex of each function:
a) $f(x) = (x - 3)^2 + 5$ has vertex $(___, ___)$
b) $f(x) = (x + 2)^2 - 1$ has vertex $(___, ___)$
c) $f(x) = -2(x - 4)^2 + 7$ has vertex $(___, ___)$

Scaffold 4.2: Horizontal Translations

If $f(x)$ has vertex at $(2, -4)$, find the vertex of each transformed function:

a) $g(x) = f(x - 1)$ (shifts right 1 unit)
New vertex: $(___, ___)$

b) $h(x) = f(x + 2)$ (shifts left 2 units)
New vertex: $(___, ___)$

c) $j(x) = f(x - 5)$ (shifts right 5 units)
New vertex: $(___, ___)$

Scaffold 4.3: Vertical Translations

If $f(x)$ has vertex at $(2, -4)$, find the vertex of each transformed function:

a) $g(x) = f(x) + 3$ (shifts up 3 units)
New vertex: $(___, ___)$

b) $h(x) = f(x) - 1$ (shifts down 1 unit)

New vertex: $(____, ____)$

c) $j(x) = f(x) - 6$ (shifts down 6 units)

New vertex: $(____, ____)$

Scaffold 4.4: Combined Transformations

If $f(x)$ has vertex at $(1, 3)$, find the vertex of each transformed function:

a) $g(x) = f(x - 2) + 1$

Horizontal shift: ____ units ____

Vertical shift: ____ units ____

New vertex: $(____, ____)$

b) $h(x) = f(x + 1) - 4$

Horizontal shift: ____ units ____

Vertical shift: ____ units ____

New vertex: $(____, ____)$

c) $j(x) = f(x - 3) - 2$

New vertex: $(____, ____)$