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Robert Colson's

Lesson 4.2: **Graphing Rational Functions**

Objectives	● Graph rational functions by identifying asymptotes and end behavior. ● Rewrite simple rational expressions in different forms using long division.
Language Objective	● SWBAT use a 3-reads protocol to interpret an Algebra 2 word problem by listening to the teacher's reading and summarizing the situation in a sentence, reading it aloud to state the math question, and silently rereading to list key facts and figures.
Essential Understanding	A rational function is any function $R(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions. The domain of a rational function is all real numbers except any x-values for which $Q(x)$ equals to zero. The graph of a rational function has one or more asymptotes, which guide the end behavior of the graph.

EXPLORE & REASON *using DESMOS*

A. **Look for Relationships**. Graph each function. Determine whether the functions are linear. Find the y intercept of each function, and slope, if appropriate.

Function	Linear? (Yes/No)	y-intercept	Slope
$f(x) = x - 1$			
$g(x) = \frac{x-1}{2}$			
$h(x) = \frac{x-1}{x-2}$			

B. What is the effect on the graph of f when dividing x-1 by 2?

C. What happens to the graph of h as x approaches 2?

D. **Communicate Precisely**. What is the effect on the graph of f(x) when dividing x-1 by x-2. (Hint: Compare it to what you found in part (B)

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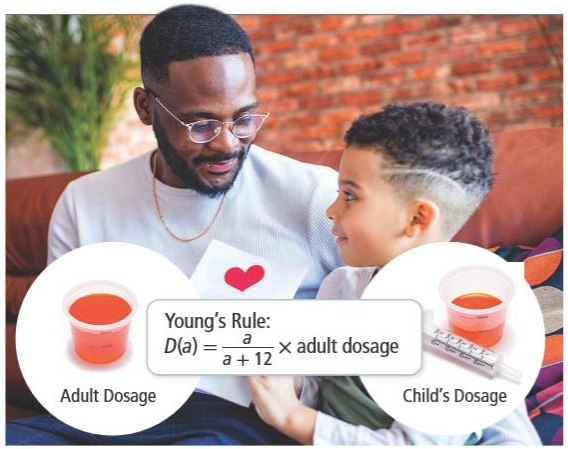
EXAMPLE 5 Use a Rational Function Model

A pediatric doctor may need to administer medication without knowing a child's weight. Young's Rule can be used to calculate a child's dosage $D(a)$ given their age a and the adult dosage.

A doctor has 60 mcg of a medication. What is the youngest a child can be to receive this dose of medication if the adult dosage is 125 mcg?



First Read - Understanding the Context	
What is the core of the problem? • I think this problem is about...	Medicine
Second Read - Interpreting the Question	
What are we trying to find out? • I know the problem is asking...because...	
Third Read - Identifying Information	
What are the important quantities, relationships, and other relevant information? • The quantities are... • I can count... • These quantities help me to answer... • The information from the situation that we need is...	
Solution (show and explain your answer)	
Interpretation of solution (interpret your solution in your own words)	

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Practice & problem solving:

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ESSENTIAL QUESTION

How can you graph a rational function?

EXAMPLE 1 Rewrite a Rational Function to Identify Asymptotes

How is the quotient $g(x) = \frac{4x}{x-3}$ related to the reciprocal function, $f(x) = \frac{1}{x}$?

Use long division to write the rational expression in the form $\frac{a}{x-h} + k$.

Check, using Desmos.

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Try It!

1. Use long division to rewrite each rational function. Find the asymptotes of *f* and sketch the graph.

a. $f(x) = \frac{6x}{2x+1}$

Try It!

1. Use long division to rewrite each rational function. Find the asymptotes of *f* and sketch the graph.

b. $f(x) = \frac{x}{x-6}$

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CONCEPT Rational Function

Just as a rational number is a number that can be expressed as the ratio of two integers, a **rational expression** is an expression that can be expressed as the ratio of two polynomials, such as $\frac{P(x)}{Q(x)}$.

A **rational function** is any function defined by a rational expression, such as $R(x) = \frac{P(X)}{Q(X)}$. The domain of $R(x)$ is all values of x for which $Q(x) \neq 0$.

The function $g(x) = \frac{4x}{x-3}$ is a rational function.

EXAMPLE 2 Find Multiple Vertical Asymptotes of a Rational Function

What are the vertical asymptotes for the graph of $f(x) = \frac{3x-2}{x^2+7x+12}$?

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Try It!

2. Find the vertical asymptotes for each function. Graph the function to check your work.

a. $g(x) = \dfrac{5x}{x^2 - x - 6}$

Try It!

2. Find the vertical asymptotes for each function. Graph the function to check your work.

b. $h(x) = \dfrac{7 - x}{(x - 5)(x + 1)(x + 3)}$

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EXAMPLE 3 Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 1:

EXAMPLE 3 Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 2:

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EXAMPLE 3 Find Types of Horizontal Asymptotes

There are 3 cases to consider, below is case 3:

EXAMPLE 3 Find Types of Horizontal Asymptotes

What are the horizontal asymptotes for the graph $f(x) = \frac{3x - 2}{x^2 + 7x + 12}$?

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Try It!

3. What are the horizontal asymptotes of the graph of each function?

a. $g(x) = \frac{2x^2 + x - 9}{2x - 8}$

b. $h(x) = \frac{x^2 + 5x + 4}{3x^2 - 12}$

c. $k(x) = \frac{x}{(2x-1)(x + 6)}$

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Practice & Problem Solving

2. What is the domain of the function $f(x) = \frac{x^2 - x - 2}{x^4 - 81}$?

Ⓐ All real numbers except 3

Ⓑ All real numbers except −1 and 3

Ⓒ All real numbers except −3 and 3

Ⓓ All real numbers except −3, 1, and 3

3. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^2 - 3x - 4}{3 - x^2}$?

Ⓐ $y = -1$; $x = \pm\sqrt{3}$

Ⓑ $y = 1$; $x = \pm\sqrt{3}$

Ⓒ $y = -1$; $x = 1$ and $x = \sqrt{3}$

Ⓓ $y = -1$; $x = 1$ and $x = -\sqrt{3}$

12. What are the horizontal and vertical asymptotes of the graph of $y = \frac{x^4 + 3}{x^4 + 2x^2 - 3}$?

Ⓐ $y = 1$; $x = \pm 1$

Ⓑ $y = 1$; $x = \pm 3$

Ⓒ $y = 0$; $x = 1$

Ⓓ $y = 0$; $x = -1$

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18. Select all the functions whose graphs have a horizontal asymptote at $y = \frac{2}{3}$.

- ☐ **A.** $y = \frac{2}{3x - 1}$
- ☐ **B.** $y = \frac{2x^2 + 1}{3x^2 - 2}$
- ☐ **C.** $y = \frac{2}{3} + \frac{1}{x}$
- ☐ **D.** $y = \frac{2x - 3}{3x^2 + 1}$
- ☐ **E.** $y = 3 + \frac{3}{2x}$

16. What are the horizontal and vertical asymptotes of the graph of $y = \frac{-x + 3}{x - 8}$?

Ⓐ $y = -1; x = 8$

Ⓑ $y = -1; x = -8$

Ⓒ $y = 1; x = 8$

Ⓓ $y = 1; x = -8$

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Graphing Rational Functions

RATIONAL FUNCTION

A function that is expressible as a fraction with polynomials in the numerator and the denominator

ASYMPTOTES

Vertical

Vertical asymptotes are guides for the behavior of a graph as it approaches a vertical line.

- The line $x = a$ is a vertical asymptote of $\frac{P(x)}{Q(x)}$, if $Q(a) = 0$ and $P(a) \neq 0$.
- The up or down behavior of the function as it approaches the asymptote can be determined by substituting values close to a on either side of the asymptote.

Horizontal

Horizontal asymptotes are guides for the end behavior of a graph as it approaches a horizontal line.

If the degree of the numerator is

- less than the degree of the denominator, the horizontal asymptote is at $y = 0$.
- greater than the denominator, there is no horizontal asymptote.
- equal to the degree of the denominator, set y equal to the ratio of the leading coefficients. The graph of this line is the horizontal asymptote.

ALGEBRA

$$f(x) = \frac{8x - 3}{4x + 1}$$

Vertical Asymptote: Let $4x + 1 = 0$ and solve.

$$x = -\frac{1}{4}$$

Horizontal Asymptote: Find the ratio of the leading coefficients $\left(\frac{8}{4}\right)$.

$$y = 2$$

GRAPH