

Lesson 4.4: Adding and Subtracting Rational Expressions

Objectives	<ul style="list-style-type: none"> Understand that rational expressions form a system analogous to the system of rational numbers and use that understanding to add and subtract rational expressions.
Language Objective	<ul style="list-style-type: none"> SWBAT use a 3-reads protocol to interpret an Algebra 2 word problem by listening to the teacher's reading and summarizing the situation in a sentence, reading it aloud to state the math question, and silently rereading to list key facts and figures.
Essential Understanding	The properties of operations used to add and subtract rational numbers can be applied to adding and subtracting rational expressions.

By the end of this lesson, you should feel comfortable solving these two questions. Go ahead, try them (Show all work!!!)

4. Simplify $\frac{1}{x-y} - \frac{-6}{y-x}$. What are the any restrictions on the domain of the expression?

The domain is $\{x|x \neq \text{ } \}$.

10. What is the sum $\frac{1}{x-4} + \frac{-8}{x^2-16}$?

(A) $\frac{-8}{x^2 + x - 20}$

(B) $\frac{-7}{x+4}$

(C) $\frac{1}{x^2 + x + 2}$

(D) $\frac{1}{x+4}$

Warmup #1

Set 1: Same Denominators

Add the following rational numbers. All denominators within each problem are the same.

1. $\frac{3}{8} + \frac{2}{8}$

2. $\frac{1}{5} + (-\frac{4}{5})$

3. $\frac{7}{10} + \frac{1}{10}$

4. $\frac{1}{6} + \frac{1}{(2 \cdot 3)}$

5. $\frac{2}{4 \cdot 3} + \frac{5}{12}$

6. $\frac{3}{9} + \frac{(-1)}{(3 \cdot 3)}$

EXAMPLE 1 Add Rational Expressions With Like Denominators

What is the sum?

A. $\frac{x}{x+4} + \frac{5}{x+4}$

What is the sum?

B. $\frac{2x+1}{x^2+3x} + \frac{3x-8}{x(x+3)}$

Try It

a. $\frac{10x-5}{2x+3} + \frac{8-4x}{2x+3}$

b. $\frac{x-5}{x+5} + \frac{3x-21}{x+5}$

Warm up #2

Find the Least Common Multiple (LCM) of the following pairs of numbers by factoring them and using each factor the greatest number of times it appears in either number.

Example: LCM of 18 and 24

- Factor: $18 = 2 \cdot 3^2$ $24 = 2^3 \cdot 3$
- Highest powers: 2^3 , 3^2
- LCM: $2^3 \cdot 3^2 = 8 \cdot 9 = 72$

1. 12 and 18

2. 20 and 25

3. 8 and 12

4. 15 and 36

5. 24 and 30

EXAMPLE 2 Identify the Least Common Multiple of Polynomials

How can you find the least common multiple (LCM) of polynomials?

A. $(x + 2)^2$, $x^2 + 5x + 6$

B. $x^3 - 9x$, $x^2 - 2x - 15$, $x^2 - 5x$

Try It!

2. Find the LCM for each set of expressions.

a. $x^3 + 9x^2 + 27x + 27$, $x^2 - 4x - 21$

b. $10x^2 - 10y^2$, $15x^2 - 30xy + 15y^2$, $x^2 + 3xy + 2y^2$

Warm up #3

Adding Rational Numbers with Unlike Denominators

Example: $\frac{2}{3} + \frac{1}{4}$

- LCM of 3 and 4: $3 = 3$, $4 = 2^2$, $\text{LCM} = 3 \cdot 2^2 = 12$
- Rewrite fractions: $\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$ $\frac{1}{4} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}$
- Add: $\frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$

Practice Problems

1. $\frac{1}{6} + \frac{2}{9}$

2. $\frac{3}{10} + \frac{1}{5}$

3. $\frac{5}{12} + \frac{1}{8}$

4. $\frac{2}{15} + \frac{3}{20}$

5. $\frac{1}{4} + \frac{2}{6}$

EXAMPLE 3 Add Rational Expressions With Unlike Denominators

What is the sum of $\frac{x+3}{x^2-1}$ and $\frac{2}{x^2-3x+2}$?

$$\frac{x+3}{x^2-1} + \frac{2}{x^2-3x+2} = \frac{x+3}{(x+1)(x-1)} + \frac{2}{(x-1)(x-2)}$$

Factor each denominator.

$$= \frac{(x+3)(x-2)}{(x+1)(x-1)(x-2)} + \frac{2(x+1)}{(x+1)(x-1)(x-2)}$$

Use the LCM as the least common denominator (LCD).

$$= \frac{(x+3)(x-2) + 2(x+1)}{(x+1)(x-1)(x-2)}$$

$$= \frac{(x^2 + x - 6) + (2x + 2)}{(x+1)(x-1)(x-2)}$$

In the numerator only, multiply factors, add like terms and factor again.

$$= \frac{x^2 + 3x - 4}{(x+1)(x-1)(x-2)}$$

$$= \frac{(x+4)(x-1)}{(x+1)(x-1)(x-2)}$$

$$= \frac{(x+4)}{(x+1)(x-2)} \cdot \frac{(x-1)}{(x-1)}$$

Simplify and state the domain.

$$= \frac{x+4}{(x+1)(x-2)} \text{ for } x \neq -1, 1, \text{ and } 2$$

The sum of $\frac{x+3}{x^2-1}$ and $\frac{2}{x^2-3x+2}$ is $\frac{x+4}{(x+1)(x-2)}$ for $x \neq -1, 1, \text{ and } 2$.

Try It!

3. Find the sum.

a. $\frac{x+6}{x^2-4} + \frac{2}{x^2-5x+6}$

Try It!

3. Find the sum.

b. $\frac{2x}{3x+4} + \frac{4x^2 - 11x - 12}{6x^2 + 5x - 4}$

Warm up #4

Subtracting Rational Numbers with Unlike Denominators

Example: $\frac{3}{4} - \frac{2}{5}$

- LCM of 4 and 5: $4 = 2^2$, $5 = 5$, $\text{LCM} = 2^2 \cdot 5 = 20$
- Rewrite fractions: $\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$ $\frac{2}{5} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$
- Subtract: $\frac{15}{20} - \frac{8}{20} = \frac{15-8}{20} = \frac{7}{20}$

1. $5/6 - 1/4$

2. $2/3 - 1/9$

5. $3/8 - 1/12$

EXAMPLE 4 Subtract Rational Expressions

Try It!

4. Simplify.

a. $\frac{1}{3x} + \frac{1}{6x} - \frac{1}{x^2}$

b. $\frac{3x - 5}{x^2 - 25} - \frac{2}{x + 5}$



Jorge drives his car to the mechanic, then he takes the commuter rail train back to his neighborhood. The average speed for the 10-mile trip is 15 miles per hour faster on the train. Find an expression for Jorge's total travel time. If he drove 30 mph, how long did this take?

First Read - Understanding the Context	
<i>What is the core of the problem?</i> <ul style="list-style-type: none"> I think this problem is about... 	
Second Read - Interpreting the Question	
<i>What are we trying to find out?</i> <ul style="list-style-type: none"> I know the problem is asking...because... 	
Third Read - Identifying Information	
<i>What are the important quantities, relationships, and other relevant information?</i> <ul style="list-style-type: none"> The quantities are... I can count... These quantities help me to answer... The information from the situation that we need is... 	
Solution (show and explain your answer)	
Interpretation of solution (interpret your solution in your own words)	

CONCEPT SUMMARY

Find Sums and Differences of Rational Expressions

WORDS	To add or subtract rational expressions with common denominators, add the numerators and keep the denominator the same.	To add or subtract rational expressions with different denominators, rewrite each expression so that its denominator is the LCD, then add or subtract the numerators.
NUMBERS	$\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$	$\begin{aligned}\frac{1}{6} + \frac{1}{15} &= \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 5} \\ &= \frac{1 \cdot 5 + 1 \cdot 2}{2 \cdot 3 \cdot 5}\end{aligned}$
ALGEBRA	$\frac{x}{x+4} + \frac{5}{x+4} = \frac{x+5}{x+4}$	$\begin{aligned}\frac{x+3}{x^2-1} + \frac{2}{x^2-3x+2} \\ &= \frac{x+3}{(x+1)(x-1)} + \frac{2}{(x-1)(x-2)} \\ &= \frac{(x+3)(x-2)}{(x+1)(x-1)(x-2)} + \frac{2(x+1)}{(x+1)(x-1)(x-2)}\end{aligned}$ <div>Rewrite the rational expressions using the LCD.</div>