Algebra 2 Assessment Review: Functions & Transformations

This document provides revised scaffolded questions to help students prepare for questions 1, 2, 4, 11, and 12 (Functions Transformations group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 1: Function Transformations

The original question involves translating a graph of an absolute value function 3 units right and 5 units down to find the new equation. The following questions build understanding of transformations.

1.1 **Basic Vertex Shifts**: The graph of $y = |x|$ has a vertex at $(0, 0)$. A horizontal shift right by h units changes the equation to $y = |x - h|$, and a vertical shift down by k units adds $-k$. Find the vertex of each:

a) $y = |x - 4|$: Vertex at (_____, _____)

b) $y = |x| - 3$: Vertex at (_____, _____)

c) $y = |x + 1| + 2$: Vertex at (_____, _____)

1.2 **Transformation Effects**: Match each transformation to its effect on the graph of $y = f(x)$:

- $f(x - h)$, $h > 0$: _____ (A. Shifts right h units)
- $f(x) + k$, $k > 0$: _____ (B. Shifts up k units)
- $-f(x)$: _____ (C. Reflects over x-axis)
- $f(x + h)$, $h > 0$: _____ (D. Shifts left h units)

1.3 **Combined Transformations**: Start with $y = |x + 2|$, vertex at $(-2, 0)$. Apply these transformations:

a) Shift 1 unit right: New vertex at (_____, _____)

b) Then shift 4 units down: New vertex at (_____, _____)

c) Write the equation: Start with $y = |x + 2|$. A right shift by 1 replaces x with $(x - 1)$, and a down shift by 4 subtracts 4. New equation: $y =$ _____

1.4 **Applying to the Original Problem**: Suppose the original graph is $y = -|x-2|+3$, with vertex at $(2, 3)$. Translate it 3 units right and 5 units down:

a) New vertex: Right 3 units adds 3 to x-coordinate; down 5 units subtracts 5 from y-coordinate. Vertex at (_____, _____)

b) New equation: Start with $y = -|x - 2| + 3$. Right 3 units replaces $x - 2$ with $(x - 3) - 2 = x - 5$; down 5 units subtracts 5 from the constant. New equation: $y =$ _____

c) Compare to choices: $y = -|x + 1| - 2$, $y = -|x + 1| + 2$, $y = -|x - 1| - 2$, $y = -|x - 1| + 2$.

Scaffolded Question for Assessment Item 2: Vertical Asymptotes

The original question asks to identify functions with a vertical asymptote at $x = 4$. The following questions build understanding of asymptotes in logarithmic functions.

2.1 **Logarithm Domain**: The function $\ln(x)$ is defined for $x > 0$, with a vertical asymptote at $x = 0$. Find the domain and asymptote for:

a) $f(x) = \ln(x - 1)$: Domain $x >$ _____, asymptote at $x =$ _____

b) $f(x) = \ln(x + 3)$: Domain $x >$ _____, asymptote at $x =$ _____

2.2 **Transformed Logarithms**: For $f(x) = \log(x - a)$, the asymptote is at $x = a$. Determine the asymptote for:

a) $f(x) = \log(x - 5)$: Asymptote at $x =$ _____

b) $f(x) = \log(x - 2) + 3$: Asymptote at $x =$ _____

c) Why does the $+3$ in part b not affect the asymptote? _____

2.3 **Checking for** $x = 4$: Determine if each function has a vertical asymptote at $x = 4$. Write the asymptote equation or "None."

a) $f(x) = \ln(x - 4)$: _____

b) $f(x) = \ln(x) + 4$: _____

c) $f(x) = 2\ln(x - 4)$: _____

d) $f(x) = \ln(x + 4)$: _____

2.4 **Applying to the Original Problem**: Select all functions with a vertical asymptote at $x = 4$. For each, find the argument of the logarithm (e.g., $\ln(u)$) and set $u = 0$ to find the asymptote:

a) $f(x) = \log_4 x - 4$: Asymptote at _____

b) $f(x) = \ln(x - 4)$: Asymptote at _____

c) $f(x) = \log(x - 4) + 4$: Asymptote at _____

d) $f(x) = 4\ln x - 4$: Asymptote at _____

e) $f(x) = \log(x - 4)$: Asymptote at _____

f) Which have asymptote at $x = 4$? _____

Scaffolded Question for Assessment Item 4: Vertex Form and Transformations

The original question involves finding the vertex of $g(x) = f(x-3)-2$, given $f(x)$ has vertex at $(2, -4)$. The following questions build understanding of quadratic transformations.

4.1 **Vertex of Quadratics**: For a quadratic $f(x) = a(x-h)^2 + k$, the vertex is (h, k). Find the vertex:

a) $f(x) = (x-1)^2 + 4$: Vertex at (_____, _____)

b) $f(x) = 2(x+3)^2 - 2$: Vertex at (_____, _____)

4.2 **Horizontal Shifts**: If $f(x)$ has vertex at $(3, 1)$, find the vertex after:

a) $g(x) = f(x-2)$: Shift right 2 units, vertex at (_____, _____)

b) $h(x) = f(x+1)$: Shift left 1 unit, vertex at (_____, _____)

4.3 **Combined Shifts**: If $f(x)$ has vertex at $(1, 2)$, find the vertex of:

a) $g(x) = f(x-1)+3$: Shift right 1 unit, up 3 units, vertex at (_____, _____)

b) $h(x) = f(x+2)-1$: Shift left 2 units, down 1 unit, vertex at (_____, _____)

4.4 **Applying to the Original Problem**: Given $f(x)$ has vertex at $(2, -4)$, find the vertex of $g(x) = f(x-3)-2$:

a) Horizontal shift: $x - 3$ shifts _____ units _____

b) Vertical shift: -2 shifts _____ units _____

c) New vertex: (_____, _____)

Scaffolded Question for Assessment Item 11: Inverse Functions

The original question asks for the inverse of $f(x) = \sqrt{x-10}$, representing years as a function of profits. The following questions build understanding of inverse functions.

11.1 **Inverse Function Basics**: If $f(a) = b$, then $f^{-1}(b) = a$. The inverse swaps x and y-coordinates:

a) If $f(4) = 9$, then $f^{-1}(9) =$ _____

b) If $f^{-1}(2) = 5$, then $f(5) =$ _____

c) Why swap x and y? _____

11.2 **Linear Inverses**: Find the inverse of $f(x) = 2x + 3$:

a) Set $y = 2x + 3$, switch: $x = 2y + 3$

b) Solve: $x - 3 = 2y$, so $y =$ _____

c) Inverse: $f^{-1}(x) =$ _____

11.3 **Square Root Inverses**: Find the inverse of $f(x) = \sqrt{x-4}$, $x \geq 4$:

a) Set $y = \sqrt{x-4}$, switch: $x = \sqrt{y-4}$

b) Solve: Square both sides: $x^2 = y - 4$, so $y =$ _____

c) Inverse: $f^{-1}(x) = x^2 + 4$, for $x \geq 0$ (since $y \geq 0$). Why the restriction? _____

11.4 **Applying to the Original Problem**: For $f(x) = \sqrt{x-10}$, representing profit after x years:

a) Find inverse: Set $y = \sqrt{x-10}$, switch: $x = \sqrt{y-10}$, solve: $y =$ _____

b) Inverse: $f^{-1}(x) =$ _____, for $x \geq 0$. What does $f^{-1}(x)$ represent? _____

c) Compare to choices: $(x-10)^2$, $x^2 + 10$, with domains $x \geq 0$ or $x \geq -10$.

Scaffolded Question for Assessment Item 12: Average Rate of Change

The original question asks for the average rate of change of $f(x) = -2x^2 + 5$ over $-3.5 \leq x \leq 0$. The following questions build understanding of average rate of change.

12.1 **Basic Average Rate of Change**: The average rate of change is the slope of the secant line: $\frac{f(b)-f(a)}{b-a}$. For $f(x) = 3x + 1$, find from $x = 1$ to $x = 3$:

a) $f(1) =$ _____, $f(3) =$ _____

b) Rate: $\frac{f(3)-f(1)}{3-1} =$ _____

12.2 **Quadratic Functions**: For $f(x) = -x^2 + 2$, find from $x = -1$ to $x = 1$:

a) $f(-1) =$ _____, $f(1) =$ _____

b) Rate: $\frac{f(1)-f(-1)}{1-(-1)} =$ _____

12.3 **Negative and Decimal Intervals**: For $f(x) = -x^2 + 4$, find from $x = -2.5$ to $x = 0$:

a) $f(-2.5) = -(-2.5)^2 + 4 =$ _____

b) $f(0) =$ _____

c) Rate: $\frac{f(0)-f(-2.5)}{0-(-2.5)} =$ _____

12.4 **Applying to the Original Problem**: For $f(x) = -2x^2 + 5$, find from $x = -3.5$ to $x = 0$:

a) $f(-3.5) = -2(-3.5)^2 + 5 =$ _____

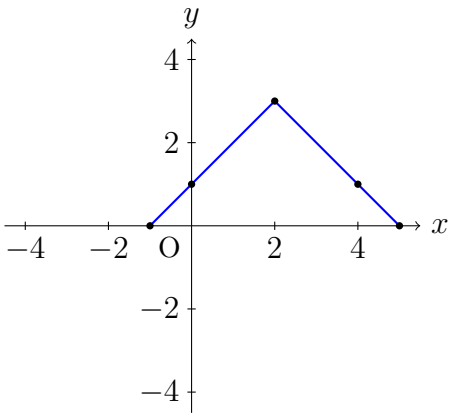
b) $f(0) =$ _____

c) Rate: $\frac{f(0)-f(-3.5)}{0-(-3.5)} =$ _____. Compare to choices: 19.5, 7, -7, -19.5.

Original Assessment Questions

Question 1

The graph below is translated 3 units right, and 5 units down. What is the equation of the new graph?



A. $y = -|x + 1| + 2$

B. $y = -|x + 1| - 2$

C. $y = -|x - 1| - 2$

D. $y = -|x - 1| + 2$

Question 2

Select all functions whose graph has a vertical asymptote at $x = 4$.

☒ $f(x) = \log_4 x - 4$

☒ $f(x) = \ln(x - 4)$

☒ $f(x) = \log(x - 4) + 4$

☒ $f(x) = 4 \ln x - 4$

☒ $f(x) = \log(x - 4)$

(Note: Replace ☒with ☐if you want empty boxes for students to fill)

Question 4

The graph of a quadratic function $f(x)$ has a vertex at $(2, -4)$. What is the vertex of $g(x)$ if $g(x) = f(x - 3) - 2$?

(_____ , _____)

Question 11

The function $f(x) = \sqrt{x - 10}$ represents the profits of a company after x years in business. Which function represents the number of years as a function of the profits?

A. $f^{-1}(x) = (x - 10)^2$, for $x \geq 0$

B. $f^{-1}(x) = (x - 10)^2$, for $x \geq -10$

C. $f^{-1}(x) = x^2 + 10$, for $x \geq 0$

D. $f^{-1}(x) = x^2 + 10$, for $x \geq -10$

(Note: The original test image shows $f(x) = \sqrt{x} - 10$. The scaffold and options align better with $f(x) = \sqrt{x - 10}$, which is used here.)

Question 12

What is the average rate of change for the function $f(x) = -2x^2 + 5$ over the interval $-3.5 \leq x \leq 0$?

A. 19.5

B. 7

C. -7

D. -19.5