Algebra 2 Assessment Review: Probability & Statistics

This document provides revised scaffolded questions to help students prepare for questions 28, 36, 37, and 38 (Probability & Statistics group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 28: Statistics Terminology

The original question asks whether 45 (average points for the first 3 games) is a variable, parameter, sample, or statistic, given a season average of 42. The following questions build understanding of statistical terms.

28.1 **Population vs. Sample**: A **population** is the entire group of individuals or items that we are interested in studying. A **sample** is a subset of the population that is selected for study.

a) Identify population and sample: Population: All basketball games in a season. Sample: The first 5 games of the season.

b) Your example: Population: All students in a particular high school. Sample: All 10th-grade students in that high school (or, 50 randomly selected students from the school.

28.2 **Parameter vs. Statistic**: A **parameter** is a numerical measure that describes a characteristic of the entire *population*. A **statistic** is a numerical measure that describes a characteristic of a *sample*. (Hint: **P**arameter for **P**opulation; **S**tatistic for **S**ample)

a) Average score of all games played by a team in a season: 50 points. (This describes all games - the population). This is a parameter. Average score of a sample of 10 games played by the team: 52 points. (This describes a sample). This is a statistic.

b) Average height of all students in a university: 5'6". This is a parameter. Average height of a sample of 30 students from that university: 5'7". This is a statistic.

28.3 **Identifying Terms in Context**: Classify the given numbers:

a) A town's mayor wants to know the average income of all households in the town. The true average income of all 2,500 households is $55,000. This $55,000 is a parameter. The mayor surveys 100 households and finds their average income is $52,000. This $52,000 is a statistic.

b) A basketball coach calculates the average points scored by the team in the first 3 games of the season as 55 points. This 55 points is a statistic (since it's based on a sample of games, not all games).

28.4 **Applying to the Original Problem**: A high school basketball team had a season average of 42 points per game (this describes the entire season's games - the population of games for that season). For the first 3 games of the season (this is a sample of games), they averaged 45 points per game.

a) Population: All games played by the team in that season. Sample: The first 3 games of that se

b) The value 42 (season average) describes the population, so it is a parameter. The value 45 (average of first 3 games) describes the sample, so it is a statistic.

c) Practice: A company wants to know the average age of its 3000 employees. The true average age is 38 years. A researcher selects 100 employees and finds their average age is 37.5 years. The 37.5 years is a statistic.

Scaffolded Question for Assessment Item 36: Set Operations and Probability

The original question asks whether the winning outcomes (odd number or 6) are the union, intersection, or complement of $A = \{1, 2, 3, 5, 6\}$ and $B = \{1, 3, 4, 5, 6\}$ when rolling a number cube. (The sample space for a number cube is $S = \{1, 2, 3, 4, 5, 6\}$).

36.1 **Set Operations Review**: Let $U = \{1, 2, 3, 4, 5, 6\}$ be the universal set. Let $A = \{1, 3, 5\}$ (set of odd numbers). Let $B = \{2, 4, 6\}$ (set of even numbers). Let $C = \{4, 5, 6\}$.

a) Union ($\cup$): $A \cup C = \{x \mid x \in A \text{ OR } x \in C \text{ (or both)}\} = \{1, 3, 5\} \cup \{4, 5, 6\} = \underline{\{1, 3, 4, 5, 6\}}$.

b) Intersection ($\cap$): $A \cap C = \{x \mid x \in A \text{ AND } x \in C\} = \{1, 3, 5\} \cap \{4, 5, 6\} = \underline{\{5\}}$.

c) Complement (A^c or A'): $A^c = \{x \in U \mid x \notin A\} = \{1, 2, 3, 4, 5, 6\} - \{1, 3, 5\} = \underline{\{2, 4, 6\}} = B$.

d) Why use union in probability for "OR" events? Union includes all outcomes that satisfy one eve

36.2 **Probability Context - "OR"**: A standard six-sided die is rolled. You win if you roll a 1 OR a 2. Let event E_1 be rolling a 1: $E_1 = \{1\}$. Let event E_2 be rolling a 2: $E_2 = \{2\}$.

a) The set of winning outcomes is $E_1 \cup E_2 = \{1\} \cup \{2\} = \underline{\{1, 2\}}$.

b) If you win on an "odd number OR a multiple of 3": Odd numbers: $O = \{1, 3, 5\}$. Multiples of 3: $M_3 = \{3, 6\}$. Winning outcomes $= O \cup M_3 = \{1, 3, 5\} \cup \{3, 6\} = \underline{\{1, 3, 5, 6\}}$.

36.3 **Analyzing Specific Sets for "OR" Events**: Given Set $X = \{1, 3, 5\}$ and Set $Y = \{1, 2, 5\}$.

a) Union: $X \cup Y = \{1, 3, 5\} \cup \{1, 2, 5\} = \underline{\{1, 2, 3, 5\}}$.

b) Intersection: $X \cap Y = \{1,3,5\} \cap \{1,2,5\} = \underline{\{1,5\}}$.

c) Practice: If a game is won by an outcome in set X OR an outcome in set Y, which operation represents the winning outcomes? Union ($\cup$).

36.4 **Applying to the Original Problem**: Winning outcomes for Milianna: "an odd number OR 6". Let $S = \{1,2,3,4,5,6\}$ be the sample space for rolling a number cube. Set of odd numbers: $O = \{1,3,5\}$. Set containing 6: $S_6 = \{6\}$.

a) The set of Milianna's winning outcomes is $W = O \cup S_6 = \{1,3,5\} \cup \{6\} = \underline{\{1,3,5,6\}}$.

b) The question provides two sets: $G_1 = \{1,2,3,5,6\}$ and $G_2 = \{1,3,4,5,6\}$. We need to determine if Milianna's winning set $W = \{1,3,5,6\}$ is the union, intersection, or complement of G_1 and G_2.

c) Calculate $G_1 \cup G_2$: $\{1,2,3,5,6\} \cup \{1,3,4,5,6\} = \{1,2,3,4,5,6\}$. Is this W? No.

d) Calculate $G_1 \cap G_2$: $\{1,2,3,5,6\} \cap \{1,3,4,5,6\} = \underline{\{1,3,5,6\}}$. Is this W? Yes.

e) (Complement usually needs a universal set defined by the problem context. If $U = \{1,2,3,4,5,6\}$, $G_1^c = \{4\}$, $G_2^c = \{2\}$. Neither is W.)

f) Answer: Milianna's winning outcomes $W = \{1,3,5,6\}$ are the intersection of G_1 and G_2.

Scaffolded Question for Assessment Items 37-38: Conditional Probability and Data Analysis

The original questions involve calculating P(heavy metal — 12th grade) and comparing P(10th grade — rock) vs. P(rock — 10th grade) using a two-way table. The following questions build understanding of conditional probability.

Music Preference Data

	Rock	Hip-Hop	Heavy Metal	Total
10th Grade	16	12	4	32
11th Grade	18	10	12	40
12th Grade	16	8	6	30
Total	50	30	22	102

37-38.1 **Reading Two-Way Tables**:

a) How many 11th graders prefer hip-hop? Look at the intersection of "11th Grade" row and "Hip-Hop" column: 10.

b) What is the total number of 10th graders surveyed? Look at the total for the "10th Grade" row: 32.

c) What is the total number of students who prefer Rock? 50.

d) What is the grand total of students surveyed? 102.

e) Why are totals important in probability? They often serve as the denominator (sample space si

37-38.2 **Basic Probability from a Table**: $P(\text{Event}) = \frac{\text{Number of favorable outcomes for Event}}{\text{Total number of outcomes}}$.

a) Probability a randomly selected student is in 11th grade: $P(\text{11th grade}) = \frac{\text{Total 11th graders}}{\text{Grand Total}} = \frac{40}{102} = \underline{\frac{20}{51}}$.

b) Probability a randomly selected student prefers Heavy Metal: $P(\text{Heavy Metal}) = \frac{\text{Total Heavy Metal}}{\text{Grand Total}} = \frac{22}{102} = \underline{\frac{11}{51}}$.

37-38.3 **Conditional Probability** $P(A|B)$: The probability of event A occurring GIVEN that event B has already occurred. Formula: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\text{Number of outcomes in both A and B}}{\text{Number of outcomes in B}}$. When using a table, the condition B restricts our "new" sample space to only those outcomes in B.

a) P(prefers Hip-Hop — student is in 10th grade): Our sample space is now only the 10th graders (Total = 32). Out of these 10th graders, how many prefer Hip-Hop? 12. So, $P(\text{Hip-Hop} \mid \text{10th grade}) = \frac{12}{32} = \underline{\frac{3}{8}}$.

b) P(is in 10th grade — student prefers Hip-Hop): Our sample space is now only students who prefer Hip-Hop (Total = 30). Out of these Hip-Hop preferrers, how many are in 10th grade? 12. So, $P(\text{10th grade} \mid \text{Hip-Hop}) = \frac{12}{30} = \underline{\frac{2}{5}}$.

c) Compare $P(\text{Hip-Hop} \mid \text{10th grade}) = \frac{3}{8} = 0.375$ and $P(\text{10th grade} \mid \text{Hip-Hop}) = \frac{2}{5} = 0.4$. Is $P(A|B)$ generally equal to $P(B|A)$? <u>No</u>.

37-38.4 **Applying to the Original Problems**:

a) **Question 37**: What is the probability that a randomly selected 12th grade student at the school favors heavy metal? This is $P(\text{Heavy Metal} \mid \text{12th grade})$. Condition: Student is in 12th grade. Total 12th graders = <u>30</u>. (This is our new denominator). Out of these 12th graders, how many favor Heavy Metal? <u>6</u>. $P(\text{Heavy Metal} \mid \text{12th grade}) = \frac{6}{30} = \frac{1}{5} = 0.20$. As a percentage: $0.20 \times 100\% = \underline{20\%}$.

b) **Question 38**: Compare $P(\text{10th grade} \mid \text{Rock})$ and $P(\text{Rock} \mid \text{10th grade})$. Calculate $P(\text{10th grade} \mid \text{Rock})$: Condition: Student chose Rock. Total Rock preferrers = <u>50</u>. Number of 10th graders who chose Rock = <u>16</u>. $P(\text{10th grade} \mid \text{Rock}) = \frac{16}{50} = \frac{8}{25} = \underline{0.32}$.

Calculate $P(\text{Rock} \mid \text{10th grade})$: Condition: Student is in 10th grade. Total 10th graders = <u>32</u>. Number of 10th graders who chose Rock = <u>16</u>. $P(\text{Rock} \mid \text{10th grade}) = \frac{16}{32} = \frac{1}{2} = \underline{0.5}$.

Compare: 0.32 versus 0.5. Since $0.32 < 0.5$, then $P(\text{10th grade} \mid \text{Rock})$ is <u>less than</u> $P(\text{Rock} \mid \text{10th grade})$.

Original Assessment Questions

Question 28

A high school basketball team had a season average of 42 points per game. For the first 3 games of the season, they averaged 45 points per game. Which word best describes the number 45?

A. variable

B. sample

C. parameter

D. statistic

Question 36

Milianna rolls a number cube and will win a game with an outcome of an odd number or 6. Complete the statement. The winning outcomes are the

- ⊠ union
- ⊠ intersection
- ⊠ complement
- ⊠ event

of $\{1, 2, 3, 5, 6\}$ and $\{1, 3, 4, 5, 6\}$. (Note: Replace ⊠with □if you want empty boxes for students to fill)

Use the data in Items 37 and 38.

The data show the favorite music of a random sample of students.

	Rock	Hip-Hop	Heavy Metal	Total
10th Grade	16	12	4	32
11th Grade	18	10	12	40
12th Grade	16	8	6	30
Total	50	30	22	102

Question 37

What is the probability that a randomly selected 12th grade student at the school favors heavy metal?

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Question 38

Complete the following to make a true statement. The probability of randomly selecting a 10th grade student given the student chose rock is

- ☒ greater than
- ☒ less than
- ☒ equal to

selecting a student who chose rock given the student is in 10th grade. (Note: Replace ☒with ☐if you want empty boxes for students to fill)