Revised Scaffolded Questions for Algebra 2 Assessment (Questions 1–4)

This document provides revised scaffolded questions to help students prepare for questions 1 through 4 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes four scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students.

Question 1: Function Transformations

The original question involves translating a graph of an absolute value function 3 units right and 5 units down to find the new equation. The following questions build understanding of transformations.

1.1 **Basic Vertex Shifts**: The graph of $y = |x|$ has a vertex at $(0, 0)$. A horizontal shift right by h units changes the equation to $y = |x - h|$, and a vertical shift down by k units adds $-k$. Find the vertex of each:

a) $y = |x - 4|$: Vertex at (_____, _____)

b) $y = |x| - 3$: Vertex at (_____, _____)

c) $y = |x + 1| + 2$: Vertex at (_____, _____)

1.2 **Transformation Effects**: Match each transformation to its effect on the graph of $y = f(x)$:

- $f(x - h)$, $h > 0$: _____ (A. Shifts right h units)
- $f(x) + k$, $k > 0$: _____ (B. Shifts up k units)
- $-f(x)$: _____ (C. Reflects over x-axis)
- $f(x + h)$, $h > 0$: _____ (D. Shifts left h units)

1.3 **Combined Transformations**: Start with $y = |x + 2|$, vertex at $(-2, 0)$. Apply these transformations:

a) Shift 1 unit right: New vertex at (_____, _____)

b) Then shift 4 units down: New vertex at (_____, _____)

c) Write the equation: Start with $y = |x + 2|$. A right shift by 1 replaces x with $(x - 1)$, and a down shift by 4 subtracts 4. New equation: $y =$ _____

1.4 **Applying to the Original Problem**: Suppose the original graph is $y = -|x-2|+3$, with vertex at $(2, 3)$. Translate it 3 units right and 5 units down:

a) New vertex: Right 3 units adds 3 to x-coordinate; down 5 units subtracts 5 from y-coordinate. Vertex at (_____, _____)

b) New equation: Start with $y = -|x - 2| + 3$. Right 3 units replaces $x - 2$ with $(x - 3) - 2 = x - 5$; down 5 units subtracts 5 from the constant. New equation: $y =$ _____

c) Compare to choices: $y = -|x+1| - 2$, $y = -|x+1| + 2$, $y = -|x-1| - 2$, $y = -|x-1| + 2$.

Question 2: Vertical Asymptotes

The original question asks to identify functions with a vertical asymptote at $x = 4$. The following questions build understanding of asymptotes in logarithmic functions.

2.1 **Logarithm Domain**: The function $\ln(x)$ is defined for $x > 0$, with a vertical asymptote at $x = 0$. Find the domain and asymptote for:

a) $f(x) = \ln(x-1)$: Domain $x >$ _____, asymptote at $x =$ _____

b) $f(x) = \ln(x+3)$: Domain $x >$ _____, asymptote at $x =$ _____

2.2 **Transformed Logarithms**: For $f(x) = \log(x-a)$, the asymptote is at $x = a$. Determine the asymptote for:

a) $f(x) = \log(x-5)$: Asymptote at $x =$ _____

b) $f(x) = \log(x-2) + 3$: Asymptote at $x =$ _____

c) Why does the +3 in part b not affect the asymptote? _____

2.3 **Checking for** $x = 4$: Determine if each function has a vertical asymptote at $x = 4$. Write the asymptote equation or "None."

a) $f(x) = \ln(x-4)$: _____

b) $f(x) = \ln(x) + 4$: _____

c) $f(x) = 2\ln(x-4)$: _____

d) $f(x) = \ln(x+4)$: _____

2.4 **Applying to the Original Problem**: Select all functions with a vertical asymptote at $x = 4$. For each, find the argument of the logarithm (e.g., $\ln(u)$) and set $u = 0$ to find the asymptote:

a) $f(x) = \log_4 x - 4$: Asymptote at _____

b) $f(x) = \ln(x-4)$: Asymptote at _____

c) $f(x) = \log(x-4) + 4$: Asymptote at _____

d) $f(x) = 4\ln x - 4$: Asymptote at _____

e) $f(x) = \log(x-4)$: Asymptote at _____

f) Which have asymptote at $x = 4$? _____

Question 3: Work Rate Problems

The original question involves two faucets filling a tank together, one taking 8 hours and the other 4 hours. The following questions build understanding of work rates.

3.1 **Understanding Rates**: If a faucet fills a tank in t hours, its rate is $\frac{1}{t}$ tanks per hour. Calculate:

a) Faucet takes 5 hours: Rate = _____ tank/hour

b) Faucet takes 10 hours: Rate = _____ tank/hour

c) Why is the rate the reciprocal of time? _____

3.2 **Combining Rates**: Two faucets work together. Faucet A takes 6 hours ($\frac{1}{6}$ tank/hour), Faucet B takes 12 hours ($\frac{1}{12}$ tank/hour).

a) Combined rate: $\frac{1}{6} + \frac{1}{12} = \frac{\quad}{12} + \frac{\quad}{12} = \frac{\quad}{12}$ tank/hour

b) Time to fill: $t = \frac{1}{\text{combined rate}} =$ _____ hours

3.3 **Setting Up the Equation**: For Faucet A (takes a hours) and Faucet B (takes b hours), the combined time t satisfies:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$$

Write the equation for:

a) Faucet A: 10 hours, Faucet B: 5 hours: _____

b) Solve the equation from part a: Combined rate = _____, so $t =$ _____ hours

3.4 **Applying to the Original Problem**: Faucet A takes 8 hours, Faucet B takes 4 hours.

a) Rates: Faucet A: _____ tank/hour, Faucet B: _____ tank/hour

b) Combined rate: $\frac{1}{8} + \frac{1}{4} = \frac{\quad}{8} + \frac{\quad}{8} = \frac{\quad}{8}$

c) Time to fill: $t = \frac{1}{\text{combined rate}} =$ _____ hours

d) Convert to hours and minutes: _____ hours, _____ minutes

Question 4: Vertex Form and Transformations

The original question involves finding the vertex of $g(x) = f(x-3)-2$, given $f(x)$ has vertex at $(2, -4)$. The following questions build understanding of quadratic transformations.

4.1 **Vertex of Quadratics**: For a quadratic $f(x) = a(x-h)^2 + k$, the vertex is (h, k). Find the vertex:

a) $f(x) = (x-1)^2 + 4$: Vertex at (_____, _____)

b) $f(x) = 2(x+3)^2 - 2$: Vertex at (_____, _____)

4.2 **Horizontal Shifts**: If $f(x)$ has vertex at $(3, 1)$, find the vertex after:

a) $g(x) = f(x-2)$: Shift right 2 units, vertex at (_____, _____)

b) $h(x) = f(x+1)$: Shift left 1 unit, vertex at (_____, _____)

4.3 **Combined Shifts**: If $f(x)$ has vertex at $(1, 2)$, find the vertex of:

a) $g(x) = f(x-1) + 3$: Shift right 1 unit, up 3 units, vertex at (_____, _____)

b) $h(x) = f(x+2) - 1$: Shift left 2 units, down 1 unit, vertex at (_____, _____)

4.4 **Applying to the Original Problem**: Given $f(x)$ has vertex at $(2, -4)$, find the vertex of $g(x) = f(x-3) - 2$:

a) Horizontal shift: $x - 3$ shifts _____ units _____

b) Vertical shift: -2 shifts _____ units _____

c) New vertex: (_____, _____)