Revised Scaffolded Questions for Algebra 2 Assessment (Questions 21–24)

This document provides revised scaffolded questions to help students prepare for questions 21 through 24 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes four scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students.

Question 21: Complex Solutions to Quadratic Equations

The original question asks to select the solutions to $x^2 = -64$. The following questions build understanding of complex solutions.

21.1 **Square Roots of Negative Numbers**: Since $\sqrt{-1} = i$, $\sqrt{-a} = i\sqrt{a}$ for positive a. Simplify:

  a) $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$

  b) $\sqrt{-36} =$ ______

  c) $\sqrt{-100} =$ ______

  d) Why is $\sqrt{-1} = i$? __________________

21.2 **Simple Complex Solutions**: Solve:

  a) $x^2 = -16$: $x = \pm\sqrt{-16} = \pm 4i$

  b) $x^2 = -49$: $x = \pm$______

  c) Verify: For $x = 7i$, compute $(7i)^2 =$ ______.

21.3 **Quadratic Formula**: Solve $x^2 + 9 = 0$:

  a) Rewrite: $x^2 = -9$

  b) Quadratic formula: $a = 1$, $b = 0$, $c = 9$.
  $x = \frac{0\pm\sqrt{0-4(1)(9)}}{2} = \frac{\pm\sqrt{-36}}{2} = \frac{\pm 6i}{2} = \pm 3i$

  c) Practice: Solve $x^2 + 25 = 0$: $x = \pm$______.

21.4 **Applying to the Original Problem**: Solve $x^2 = -64$:

  a) Direct: $x = \pm\sqrt{-64} = \pm 8i$

  b) Quadratic formula: $x^2 + 64 = 0$, $a = 1$, $b = 0$, $c = 64$.
  $x = \frac{\pm\sqrt{-256}}{2} = \frac{\pm 16i}{2} = \pm 8i$

  c) Verify: $(8i)^2 = 64i^2 = -64$, $(-8i)^2 = 64i^2 = -64$.

  d) Select solutions from: 8, $-8i$, -8, $32i$, $8i$, $-32i$: ______, ______.

Question 22: Multiplying Polynomials

The original question asks to simplify $(x^2 + 4x)(x^2 + x + 2)$. The following questions build understanding of polynomial multiplication.

22.1 **Monomial Distribution**: Multiply by distributing each term:

a) $x(x + 5) = x^2 + 5x$

b) $3x(x^2 + 2) =$ ______ + ______

c) Why distribute each term? ____________________

22.2 **Binomial Multiplication**: Use FOIL:

a) $(x + 3)(x + 2)$: First: x^2, Outer: $3x$, Inner: $2x$, Last: 6.
Result: $x^2 + 5x + 6$

b) $(x - 1)(x + 4)$: ______ + ______ + ______ + ______ = ______.

22.3 **Quadratic by Binomial**: Multiply:

a) $(x^2 + 1)(x + 3) = x^2(x + 3) + 1(x + 3) = x^3 + 3x^2 + x + 3$

b) $(x^2 + 2x)(x + 1) =$ ______ + ______ + ______ + ______ = ______

22.4 **Applying to the Original Problem**: Multiply $(x^2 + 4x)(x^2 + x + 2)$:

a) Distribute: $x^2(x^2 + x + 2) + 4x(x^2 + x + 2)$

b) Compute: $x^4 + x^3 + 2x^2 + 4x^3 + 4x^2 + 8x$

c) Combine: $x^4 + (1 + 4)x^3 + (2 + 4)x^2 + 8x =$ ______.
Compare to choices: $x^4 + 5x^3 + 6x^2 + 8x$.

Question 23: Polynomial Function Behavior

The original question asks to analyze $f(x) = x^3 + 3x^2$, finding zeros and describing end behavior. The following questions build understanding of polynomial analysis.

23.1 **Finding Zeros**: Factor to find zeros:

a) $f(x) = x(x - 4)$: Zeros: $x = 0$, $x = 4$

b) $f(x) = x^2(x + 2)$: Zeros: $x =$ ______ (multiplicity ______), $x =$ ______

c) What does multiplicity mean graphically? ____________________

23.2 **End Behavior**: End behavior depends on the leading term:

a) $f(x) = 2x^3$: As $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$.

b) $f(x) = -x^3 + x$: Leading term: $-x^3$.
As $x \to -\infty$, $f(x) \to$ ______; as $x \to +\infty$, $f(x) \to$ ______.

23.3 **Graphing Cubics**: For $f(x) = x^3 - x^2 = x^2(x-1)$:

a) Zeros: $x = 0$ (multiplicity 2), $x = 1$

b) Test points: $f(-1) = (-1)^3 - (-1)^2 = -1 - 1 = -2$; $f(2) = 8 - 4 = 4$.

c) Multiplicity 2 at $x = 0$: Graph ______ the x-axis.

23.4 **Applying to the Original Problem**: For $f(x) = x^3 + 3x^2$:

a) Factor: $f(x) = x^2(x+3)$. Zeros: ______, ______.

b) End behavior: Leading term x^3.
As $x \to -\infty$, $f(x) \to$ ______; as $x \to +\infty$, $f(x) \to$ ______$. Atx = 0$ (multiplicity 2): Graph ______ the x-axis.

Question 24: Properties of Logarithms

The original question asks to explain steps to solve $\log x + \log x^4 = 10$ using logarithm properties. The following questions build understanding of logarithm properties.

24c1) **Logarithm Properties**: Use properties to rewrite:

a) $\log(3 \cdot 4) = \log 3 + \log 4$ (Product Property)

b) $\log(x^2) =$ ______ (Power Property)

c) $\log\left(\frac{x}{y}\right) =$ ______ (Quotient Property)

d) Why do these properties work? ____________________

24.2 **Combining Logarithms**: Combine using properties:

a) $\log 2 + \log 5 = \log(2 \cdot 5) = \log 10$

b) $\log x + \log x^2 = \log(x \cdot x^2) = \log x^3$

c) Practice: $\log 3 + \log x^3 =$ ______.

24.3 **Solving Logarithmic Equations**: Solve:

a) $\log x + \log x^3 = 8$:
Combine: $\log(x \cdot x^3) = \log x^4 = 8$.
Power: $4 \log x = 8$, so $\log x = 2$.
$x = 10^2 = 100$.

b) Practice: $\log x + \log x^2 = 6$: $x =$ ______.

24.4 **Applying to the Original Problem**: Solve $\log x + \log x^4 = 10$:

a) Combine: $\log(x \cdot x^4) = \log x^5 = 10$ (Product Property).

b) Simplify: $5 \log x = 10$ (Power Property).

c) Solve: $\log x = 2$, $x = 10^2 = 100$.

d) Verify: $\log 100 + \log 100^4 = \log 100 + \log 10^8 = 2 + 8 = 10$.