**Algebra 2 Assessment Review: Polynomials**

This document provides revised scaffolded questions to help students prepare for questions 5, 9, 22, and 23 (Polynomials group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

# Scaffolded Review Questions

## Scaffolded Question for Assessment Item 5: Finding Zeros of Polynomial Functions

The original question asks to select all $x$-values where the polynomial $f(x) = x^4 - 2x^3 - 29x^2 + 30x$, modeling a pelican's height, equals zero. The following questions build understanding of finding polynomial zeros.

5.1 **Understanding Zeros**: A zero of a function is an $x$-value where $f(x) = 0$, where the graph crosses the x-axis. Find the zeros of:

- a) $f(x) = x - 2$: Set $x - 2 = 0$, zero at $x =$ _____
- b) $f(x) = x^2 - 4 = (x - 2)(x + 2)$: Zeros at $x =$ _____, $x =$ _____
- c) What does a zero represent for a height function? ______________________________

5.2 **Factoring Polynomials**: Factor each polynomial and find zeros:

- a) $f(x) = x^2 + 3x = x(x + 3)$: Zeros at $x =$ _____, $x =$ _____
- b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$: Zeros at $x =$ _____, $x =$ _____, $x =$ _____
- c) If a factor appears twice (e.g., $(x - 1)^2$), the zero has multiplicity 2, meaning the graph touches the x-axis. Why might multiplicity matter? ____________________

5.3 **Contextual Zeros**: A ball's height is modeled by $h(t) = -16t^2 + 48t$. Find when it hits the ground ($h(t) = 0$):

- a) Factor: $-16t^2 + 48t = -16t(t - 3) = 0$. Zeros at $t =$ _____, $t =$ _____
- b) Interpret: $t = 0$ is when the ball is __________; $t = 3$ is when it __________.
- c) Why ignore negative times? ______________________________

5.4 **Testing Zeros**: For a polynomial $f(x) = x^4 - x^3 - 8x^2 + 8x$, test if the following are zeros by substituting:

- a) $x = -2$: Compute $f(-2) =$ _____. Is it a zero? _____
- b) $x = 1$: Compute $f(1) =$ _____. Is it a zero? _____

c) For the original $f(x) = x^4 - 2x^3 - 29x^2 + 30x$, which of these are zeros: $-6, -5, 0, 1, 4, 6$? Test two values (e.g., $x = 0$, $x = 1$).
$f(0) =$ _____ (Zero? ___) $f(1) =$ _____ (Zero? ___)
Selected Zeros: ________________________

## Scaffolded Question for Assessment Item 9: Polynomial Long Division

The original question asks to divide $x^3 - 4x^2 + 6x - 2$ by $x - 1$ and complete the quotient. The following questions build understanding of polynomial division.

9.1 **Basic Polynomial Division**: Divide each term by the divisor, matching powers of $x$:

a) $\frac{8x^3}{2x} =$ _____

b) $\frac{10x^4+4x^2}{2x^2} = \frac{10x^4}{2x^2} + \frac{4x^2}{2x^2} =$ _____ + _____

c) Why divide term by term? ______________________________

9.2 **Simple Long Division**: Divide $x^2 + 4x + 3$ by $x + 1$:

a) $x^2 \div x =$ _____, multiply: $x(x+1) =$ _______, subtract: $(x^2+4x+3)-(x^2+x) =$ _______

b) Continue: $3x \div x =$ _____, multiply, subtract to get remainder 0.

c) Result: $x^2 + 4x + 3 = (x + 1)($_______$) +$ ___

9.3 **Synthetic Division**: Use synthetic division for $x^2 + 5x + 6$ by $x - 2$:

a) Divisor $x - 2$, so use 2. Coefficients: 1, 5, 6. Setup:

$$\begin{array}{c|ccc} 2 & 1 & 5 & 6 \\ & & \_\_ & \_\_ \\ \hline & \_\_ & \_\_ & \_\_ \end{array}$$

b) Quotient: _______, Remainder: ___

c) Why is synthetic division faster for linear divisors? ____________________

9.4 **Applying to the Original Problem**: Divide $x^3 - 4x^2 + 6x - 2$ by $x - 1$ using synthetic division:

a) Coefficients: ___, ___, ___, ___. Divisor: $x - 1$, so use ___.

b) Perform synthetic division:

$$\begin{array}{c|cccc} 1 & 1 & -4 & 6 & -2 \\ & & \_\_ & \_\_ & \_\_ \\ \hline & \_\_ & \_\_ & \_\_ & \_\_ \end{array}$$

c) Quotient: ____________, Remainder: ___. Write as: $x^3 - 4x^2 + 6x - 2 = (x-1)(\_\_\_\_\_\_\_\_\_\_\_\_) + \_\_\_$.

## Scaffolded Question for Assessment Item 22: Multiplying Polynomials

The original question asks to simplify $(x^2 + 4x)(x^2 + x + 2)$. The following questions build understanding of polynomial multiplication.

22.1 **Monomial Distribution**: Multiply by distributing each term:

a) $x(x + 5) = x^2 + 5x$

b) $3x(x^2 + 2) =$ _______ + _______

c) Why distribute each term? ____________________________

22.2 **Binomial Multiplication**: Use FOIL:

a) $(x + 3)(x + 2)$: First: $x^2$, Outer: $3x$, Inner: $2x$, Last: 6.
Result: $x^2 + 5x + 6$

b) $(x - 1)(x + 4)$: _____ + _____ + _____ + _____ = ____________.

22.3 **Quadratic by Binomial**: Multiply:

a) $(x^2 + 1)(x + 3) = x^2(x + 3) + 1(x + 3) = x^3 + 3x^2 + x + 3$

b) $(x^2 + 2x)(x + 1) =$ _____ + _____ + _____ + _____ = ____________

22.4 **Applying to the Original Problem**: Multiply $(x^2 + 4x)(x^2 + x + 2)$:

a) Distribute: $x^2(x^2 + x + 2) + 4x(x^2 + x + 2)$

b) Compute: $x^4 + x^3 + 2x^2 + 4x^3 + 4x^2 + 8x$

c) Combine: $x^4 + (1 + 4)x^3 + (2 + 4)x^2 + 8x =$ _______________.
Compare to choices: $x^4 + 5x^3 + 6x^2 + 8x$.

## Scaffolded Question for Assessment Item 23: Polynomial Function Behavior

The original question asks to analyze $f(x) = x^3 + 3x^2$, finding zeros and describing end behavior. The following questions build understanding of polynomial analysis.

23.1 **Finding Zeros**: Factor to find zeros:

a) $f(x) = x(x - 4)$: Zeros: $x = 0$, $x = 4$

b) $f(x) = x^2(x + 2)$: Zeros: $x =$ _____ (multiplicity ___), $x =$ _____

c) What does multiplicity mean graphically? ____________________________

23.2 **End Behavior**: End behavior depends on the leading term:

a) $f(x) = 2x^3$: As $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$.

b) $f(x) = -x^3 + x$: Leading term: $-x^3$.
As $x \to -\infty$, $f(x) \to$ _____; as $x \to +\infty$, $f(x) \to$ _____.

23.3 **Graphing Cubics**: For $f(x) = x^3 - x^2 = x^2(x-1)$:

a) Zeros: $x = 0$ (multiplicity 2), $x = 1$

b) Test points: $f(-1) = (-1)^3 - (-1)^2 = -1 - 1 = -2$; $f(2) = 8 - 4 = 4$.

c) Multiplicity 2 at $x = 0$: Graph __________ the x-axis.

23.4 **Applying to the Original Problem**: For $f(x) = x^3 + 3x^2$:

a) Factor: $f(x) = x^2(x+3)$. Zeros: _____, _____.

b) End behavior: Leading term $x^3$.
As $x \to -\infty$, $f(x) \to$ _____; as $x \to +\infty$, $f(x) \to$ _____$.At x = 0$ (multiplicity 2):
Graph __________ the x-axis.

# Original Assessment Questions

## Question 5

The height above sea level of a pelican diving for fish is modeled by $f(x) = x^4 - 2x^3 - 29x^2 + 30x$. Select all the x-values where the pelican enters or exits the water.

⊠ -6 D. [⊠] 1

⊠ -5 E. [⊠] 4

⊠ 0 F. [⊠] 6

(Note: Replace ⊠with □if you want empty boxes for students to fill)

## Question 9

Divide $x^3 - 4x^2 + 6x - 2$ by $x - 1$. Complete the quotient using the choices provided.

| | | | |
|---|---|---|---|
| 3x | -5x | -3x | 3 |
| 11 | -3 | $\frac{9}{x-1}$ | $\frac{1}{x-1}$ |

$x^2+$ ☐ $+$ ☐ $+$ ☐

## Question 22

Simplify $(x^2 + 4x)(x^2 + x + 2)$.

A. $8x^2 + 5x^3 + 8x$

B. $x^4 + 5x^3 + 6x^2 + 8x + 2$

C. $x^4 + 5x^3 + 6x^2 + 8x$

D. $4x^5 + 4x^4 + 8x^3$

## Question 23

Use a graph of the polynomial function $f(x) = x^3 + 3x^2$ to complete the following:

- The zeros of $f$ are $\boxed{\phantom{xx}}$ and $\boxed{\phantom{xx}}$.
- As $x$ decreases, $f(x)$ [⊠] increases. [⊠] decreases.
- As $x$ increases, $f(x)$ [⊠] increases. [⊠] decreases.

(Note: Replace ⊠with □if you want empty boxes for students to fill)