Algebra 2 Assessment Review: Trigonometry

This document provides revised scaffolded questions to help students prepare for questions 26, 27, and 29 (Trigonometry group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 26: Cosine Functions and Midlines

The original question asks for the midline of a cosine function with period 3π, amplitude 4, and local maximum at $f(0) = 6$. The following questions build understanding of midlines.

26.1 **Cosine Properties**: For $y = A\cos(Bx) + D$, Amplitude = $|A|$, Period = $\frac{2\pi}{|B|}$, Midline = $y = D$.

- a) $y = 2\cos(x) + 1$: Amplitude = 2, period = 2π, midline = $y = 1$
- b) $y = \cos(3x)$: Amplitude = $\underline{1}$, period = $\underline{\frac{2\pi}{3}}$, midline = $\underline{y = 0}$
- c) What does the midline represent? The horizontal line halfway between the maximum and mini

26.2 **Finding Midlines from Max/Min**: Midline = $\frac{\text{Maximum Value}+\text{Minimum Value}}{2}$.

- a) Max = 7, Min = 1: Midline = $\frac{7+1}{2} = 4$, so $y = 4$
- b) Max = 5, Min = -1: Midline = $\frac{5+(-1)}{2} = \frac{4}{2} = \underline{2}$, so $y = \underline{2}$

26.3 **Amplitude and Midline Relationships**: Maximum Value = Midline Value + Amplitude Minimum Value = Midline Value - Amplitude

- a) Amplitude = 3, midline = $y = 2$ (so Midline Value = 2): Max = $2 + 3 = 5$, Min = $2 - 3 = -1$
- b) Amplitude = 5, midline = $y = 1$ (so Midline Value = 1): Max = $1 + 5 = \underline{6}$, Min = $1 - 5 = \underline{-4}$

26.4 **Applying to the Original Problem**: Given amplitude = 4, and a local maximum value is 6.

- a) We know: Maximum Value = Midline Value + Amplitude. So, 6 = Midline Value+ 4. Midline Value = $6 - 4 = \underline{2}$. So the equation of the midline is $y = \underline{2}$.
- b) Verify using minimum: If midline is $y = 2$ and amplitude is 4, then Minimum Value = Midline Value - Amplitude = $2 - 4 = -2$. Check: Midline = $\frac{\text{Max}+\text{Min}}{2} = \frac{6+(-2)}{2} = \frac{4}{2} = 2$. This matches.

c) Practice: Amplitude = 3, local maximum value = 7. Midline Value = Maximum Value - Amplitude $= 7 - 3 = \underline{4}$. Midline: $y = 4$.

(Note: The period 3π is extra information not needed to find the midline if max/min and amplitude are related.)

Scaffolded Question for Assessment Item 27: Arc Length and Radian Measure

The original question asks for the arc length on a Ferris wheel with diameter 175 feet through $\frac{\pi}{3}$ radians, rounded to the nearest foot. The following questions build understanding of arc length.

27.1 **Radian Angles**: Radians measure angles using the ratio of arc length to radius. 1 radian is the angle where arc length equals radius.

a) Angle in radians: $\frac{\pi}{6}$. Using $\pi \approx 3.14159$, $\frac{\pi}{6} \approx \frac{3.14159}{6} \approx 0.5236$ radians.

b) Angle in radians: $\frac{\pi}{4}$. $\frac{\pi}{4} \approx \frac{3.14159}{4} \approx \underline{0.7854}$ radians.

c) Why use radians for arc length formula $s = r\theta$? <u>The formula $s = r\theta$ is only valid when θ is in ra</u>

27.2 **Arc Length Formula**: $s = r\theta$, where s is arc length, r is radius, and θ is the central angle **in radians**.

a) $r = 6$ units, $\theta = \frac{\pi}{4}$ radians: $s = 6 \cdot \frac{\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2} \approx 4.71$ units.

b) $r = 10$ units, $\theta = \frac{\pi}{6}$ radians: $s = 10 \cdot \frac{\pi}{6} = \frac{10\pi}{6} = \underline{\frac{5\pi}{3}} \approx \underline{5.24}$ units.

27.3 **Diameter to Radius**: Radius (r) is half the diameter (d): $r = \frac{d}{2}$.

a) Diameter = 100 feet: $r = \frac{100}{2} = 50$ feet.

b) Diameter = 150 feet, $\theta = \frac{\pi}{4}$ radians: Radius $r = \frac{150}{2} = \underline{75}$ feet. Arc length $s = r\theta = 75 \cdot \frac{\pi}{4} = \underline{\frac{75\pi}{4}} \approx \underline{58.90}$ feet.

c) Why must we use radius (not diameter) in the arc length formula $s = r\theta$? <u>The formula is derived using the radius as the distance from the center to the arc.</u>

27.4 **Applying to the Original Problem**: Ferris wheel diameter = 175 feet, angle $\theta = \frac{\pi}{3}$ radians.

a) Calculate the radius: $r = \frac{\text{diameter}}{2} = \frac{175}{2} = \underline{87.5}$ feet.

b) Calculate the arc length: $s = r\theta = 87.5 \cdot \frac{\pi}{3} = \frac{87.5\pi}{3}$. Using $\pi \approx 3.14159$: $s \approx \frac{87.5 \times 3.14159}{3} \approx \frac{274.889}{3} \approx \underline{91.6297}$ feet.

c) Round to the nearest foot: $s \approx \underline{92}$ feet.

Scaffolded Question for Assessment Item 29: Properties of Sine Functions

The assumed question asks to evaluate statements about $y = 2\sin(x)$, including domain, vertical asymptotes, zeros, decreasing intervals, and period. The following questions build understanding of sine function properties.

29.1 **Basic Sine Properties**: For $y = \sin(x)$: Domain is all real numbers $(-\infty, \infty)$, Range is $[-1, 1]$, Period is 2π. Zeros occur at $x = n\pi$ for any integer n. It has no vertical asymptotes.

a) True/False: Domain is all real numbers: <u>True</u>

b) True/False: Zeros at $x = 0, \pi$ (within one cycle $0 \leq x < 2\pi$): <u>True</u> (also at $2\pi, 3\pi, \ldots, -\pi, -2\pi, \ldots$)

c) Why no vertical asymptotes for $y = \sin(x)$? <u>The sine function is defined for all real number inpu</u>

29.2 **Transformed Sine**: For $y = A\sin(Bx) + C$: Amplitude $= |A|$, Period $= \frac{2\pi}{|B|}$, Vertical Shift (Midline) = C. The range is $[C - |A|, C + |A|]$.

a) $y = 3\sin(x) + 1$: Amplitude = 3, Period = 2π, Range = $[1 - 3, 1 + 3] = [-2, 4]$. Midline $y = 1$.

b) $y = \sin(3x)$: Amplitude = <u>1</u>, Period = $\underline{\frac{2\pi}{3}}$. Range $[-1, 1]$. Midline $y = 0$.

29.3 **Zeros and Intervals for Transformed Sine**: For $y = 3\sin(x)$:

a) Zeros occur when $3\sin(x) = 0$, which means $\sin(x) = 0$. In the interval $[0, 2\pi]$, zeros are at $x = 0, \pi, 2\pi$.

b) Decreasing intervals: For $y = \sin(x)$, it decreases from its maximum at $x = \frac{\pi}{2}$ to its minimum at $x = \frac{3\pi}{2}$ (in the interval $[0, 2\pi]$). So, decreasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$. For $y = 3\sin(x)$, the vertical stretch by 3 does not change the x-values where it increases/decreases. So, $y = 3\sin(x)$ is decreasing on the interval: $\underline{(\frac{\pi}{2}, \frac{3\pi}{2})}$ (and other intervals shifted by $2n\pi$).

c) Practice: Zeros of $y = \sin(2x)$ in $[0, 2\pi]$. Let $2x = n\pi$, so $x = \frac{n\pi}{2}$. For $n = 0, x = 0$. For $n = 1, x = \frac{\pi}{2}$. For $n = 2, x = \pi$. For $n = 3, x = \frac{3\pi}{2}$. For $n = 4, x = 2\pi$. Zeros: $\underline{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi}$.

29.4 **Applying to the Original Problem Statements about** $y = 2\sin(x)$: (Amplitude = 2, Period = 2π, Range = $[-2, 2]$)

a) Statement A: The domain of the function is $(-\infty < x < \infty)$. Is this true for $y = 2\sin(x)$? <u>True</u>.

b) Statement B: The function has vertical asymptotes when $x = 1$. Is this true for $y = 2\sin(x)$? <u>False</u>. (Sine functions do not have vertical asymptotes).

c) Statement C: Two of the function's zeros are when $x = 0$ and $x = 2\pi$. Zeros occur when $2\sin(x) = 0 \implies \sin(x) = 0$. This happens at $x = n\pi$. So $x = 0$ is a zero.

$x = 2\pi$ is a zero. Is this statement true? True.

d) Statement D: The function is decreasing when $\frac{\pi}{2} < x < \frac{3\pi}{2}$. This is the standard interval where $\sin(x)$ decreases. Multiplying by 2 (a positive number) does not change the intervals of increase/decrease. Is this statement true? True.

e) Statement E: The period of the function is 2π. For $y = A\sin(Bx)$, period is $\frac{2\pi}{|B|}$. Here $B = 1$. Period $= \frac{2\pi}{1} = 2\pi$. Is this statement true? True.

Original Assessment Questions

Question 26

Function f is a cosine function with period 3π, amplitude 4, and a local maximum at $f(0) = 6$. Find the equation of the midline of the graph of f.

The equation of the midline of the graph of f is $y =$ $\boxed{}$

Question 27

A Ferris wheel has a diameter of about 175 feet. To the nearest foot, how far does a rider travel as the wheel rotates through $\frac{\pi}{3}$ radians?

$\boxed{}$ feet

Question 29

Select all the statements about the graph of $y = 2\sin(x)$ that are true.

- ⊠ The domain of the function is $(-\infty < x < \infty)$.
- ⊠ The function has vertical asymptotes when $x = 1$.
- ⊠ Two of the function's zeros are when $x = 0$ and $x = 2\pi$.
- ⊠ The function is decreasing when $\frac{\pi}{2} < x < \frac{3\pi}{2}$.
- ⊠ The period of the function is 2π.

(Note: Replace ⊠with □if you want empty boxes for students to fill)