Revised Scaffolded Questions for Algebra 2 Assessment (Questions 5–8)

This document provides revised scaffolded questions to help students prepare for questions 5 through 8 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes four scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students.

Question 5: Finding Zeros of Polynomial Functions

The original question asks to select all x-values where the polynomial $f(x) = x^4 - 2x^3 - 29x^2 + 30x$, modeling a pelican's height, equals zero. The following questions build understanding of finding polynomial zeros.

5.1 **Understanding Zeros**: A zero of a function is an x-value where $f(x) = 0$, where the graph crosses the x-axis. Find the zeros of:

- a) $f(x) = x - 2$: Set $x - 2 = 0$, zero at $x =$ ______
- b) $f(x) = x^2 - 4 = (x - 2)(x + 2)$: Zeros at $x =$ ______, $x =$ ______
- c) What does a zero represent for a height function? ____________________

5.2 **Factoring Polynomials**: Factor each polynomial and find zeros:

- a) $f(x) = x^2 + 3x = x(x + 3)$: Zeros at $x =$ ______, $x =$ ______
- b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$: Zeros at $x =$ ______, $x =$ ______, $x =$ ______
- c) If a factor appears twice (e.g., $(x - 1)^2$), the zero has multiplicity 2, meaning the graph touches the x-axis. Why might multiplicity matter? ____________________

5.3 **Contextual Zeros**: A ball's height is modeled by $h(t) = -16t^2 + 48t$. Find when it hits the ground ($h(t) = 0$):

- a) Factor: $-16t^2 + 48t = -16t(t - 3) = 0$. Zeros at $t =$ ______, $t =$ ______
- b) Interpret: $t = 0$ is when the ball is ______; $t = 3$ is when it ______.
- c) Why ignore negative times? ____________________

5.4 **Testing Zeros**: For a polynomial $f(x) = x^4 - x^3 - 8x^2 + 8x$, test if the following are zeros by substituting:

- a) $x = -2$: Compute $f(-2) =$ ______. Is it a zero? ______
- b) $x = 1$: Compute $f(1) =$ ______. Is it a zero? ______
- c) For the original $f(x) = x^4 - 2x^3 - 29x^2 + 30x$, which of these are zeros: $-6, -5, 0, 1, 4, 6$? Test two values (e.g., $x = 0$, $x = 1$).

Question 6: Solving Quadratic Equations with Complex Numbers

The original question asks to solve $-x^2 + 5x = 7$ over complex numbers. The following questions build understanding of the quadratic formula and complex solutions.

6.1 **Complex Numbers**: The imaginary unit i satisfies $i^2 = -1$. Simplify:

a) $\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} =$ ______

b) $\sqrt{-36} =$ ______

c) Why is $\sqrt{-1} = i$? __________________

6.2 **Quadratic Formula**: For $ax^2 + bx + c = 0$, solutions are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Solve $x^2 - 2x - 3 = 0$:

a) Identify: $a =$ ______, $b =$ ______, $c =$ ______

b) Discriminant: $b^2 - 4ac =$ ______. Is it positive, negative, or zero? ______

c) Solutions: $x = \frac{\_\_\_ \pm \sqrt{\_\_\_}}{2} =$ ______

6.3 **Complex Solutions**: Solve $x^2 + 2x + 5 = 0$:

a) $a =$ ______, $b =$ ______, $c =$ ______

b) Discriminant: $b^2 - 4ac =$ ______. Since it's negative, expect complex roots.

c) Apply formula: $x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} =$ ______

d) Why does a negative discriminant mean complex roots? __________________

6.4 **Applying to the Original Problem**: Solve $-x^2 + 5x = 7$:

a) Rewrite in standard form: ______ $= 0$

b) Identify: $a =$ ______, $b =$ ______, $c =$ ______

c) Discriminant: $b^2 - 4ac =$ ______

d) Solutions: $x = \frac{\_\_\_ \pm \sqrt{\_\_\_}}{2} =$ ______. Compare to choices: $\frac{5 \pm i\sqrt{3}}{2}$, $\frac{5 \pm i\sqrt{53}}{2}$, $\frac{-5 \pm i\sqrt{53}}{2}$, $\frac{-5 \pm i\sqrt{3}}{2}$.

Question 7: Exponential Equations with Natural Logarithms

The original question asks to solve $5e^{\frac{x}{2}} = 10$. The following questions build understanding of solving exponential equations.

7.1 **Logarithm Properties**: Since $\ln(e^x) = x$ (because ln is the inverse of e^x), simplify:

a) $\ln(e^3) =$ ______

b) $e^{\ln(4)} =$ ______

c) Why does $\ln(e^x) = x$? ________________

7.2 **Simple Exponential Equations**: Solve:

a) $e^x = 6$: Take ln of both sides: $\ln(e^x) = \ln(6)$, so $x =$ ______

b) $e^x = 2$: $x =$ ______

7.3 **Coefficients in Exponents**: Solve $3e^x = 15$:

a) Isolate: $e^x = \frac{15}{3} =$ ______

b) Take ln: $\ln(e^x) = \ln($______$)$

c) Solve: $x =$ ______

7.4 **Applying to the Original Problem**: Solve $5e^{\frac{x}{2}} = 10$:

a) Isolate: $e^{\frac{x}{2}} = \frac{10}{5} =$ ______

b) Take ln: $\ln\left(e^{\frac{x}{2}}\right) = \ln($______$)$

c) Simplify: $\frac{x}{2} = \ln($______$)$

d) Solve: $x =$ ______. Write as $x = \ln($______$)$ to match the original format.

Question 8: Multiplying Complex Numbers

The original question asks to simplify $(i - 5)(3 + 2i)$. The following questions build understanding of complex number multiplication.

8.1 **Complex Number Basics**: Since $i^2 = -1$, simplify:

a) $i^2 =$ ______

b) $(2i)^2 = 4i^2 =$ ______

c) Combine: $3 + 2i - 5i =$ ______

8.2 **Simple Multiplication**: Multiply $(1 + i)(2 + i)$:

a) Use FOIL: $(1)(2) + (1)(i) + (i)(2) + (i)(i) =$ ______

b) Simplify: $2 + i + 2i + i^2 = 2 + 3i - 1 =$ ______

8.3 **Practice with Larger Numbers**: Multiply $(2 - i)(3 + 2i)$:

a) FOIL: $(2)(3) + (2)(2i) + (-i)(3) + (-i)(2i) =$ ______

b) Simplify: $6 + 4i - 3i - 2i^2 =$ ______

8.4 **Applying to the Original Problem**: Simplify $(i - 5)(3 + 2i)$:

a) FOIL: $(i)(3) + (i)(2i) + (-5)(3) + (-5)(2i) =$ ______

b) Simplify: $3i + 2i^2 - 15 - 10i =$ ______

c) Combine: ______. Compare to choices: $-7i - 13$, $13i - 17$, $-7i - 17$, $-13i - 17$.