

Revised Scaffolded Questions for Algebra 2 Assessment (Questions 33–38)

This document provides revised scaffolded questions to help students prepare for questions 33 through 38 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes four scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naïve students.

Question 33: Linear vs Exponential Growth Models

The original question involves modeling Lucia’s linear (12 residents/day) and Caleb’s exponential (4 people, each contacting 4 more daily) growth, combining them, applying a

35

33.1 Linear Models: Linear functions $f(x) = mx + b$ have a constant rate m :

- a) 10 contacts/day: $f(x) = 10x$
- b) 5 units/week, initial 20: $f(x) = \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$
- c) Why constant rate? $\underline{\hspace{2cm}}$

33.2 Exponential Models: Exponential functions $f(x) = ab^x$ model multiplicative growth:

- a) Triples daily, starts at 5: $f(x) = 5 \cdot 3^x$
- b) Starts at 2, each contacts 3 more daily: Day 1: 2, Day 2: 6, Day 3: 18 \rightarrow
 $f(x) = \underline{\hspace{1cm}}$
- c) Why cumulative growth? $\underline{\hspace{2cm}}$

33.3 Combining Models: Add linear and exponential contributions:

- a) Linear: $8x$, Exponential: 2^x : $g(x) = 8x + 2^x$
- b) Linear: $10x$, Exponential: 3^x , 30
- c) For $x = 2$: $h(2) = \underline{\hspace{1cm}}$

33.4 Applying to the Original Problem: Lucia: $12x$, Caleb: 4^x , 35

- a) Combined: $g(x) = 12x + 4^x$
- b) Votes: $h(x) = 0.35(12x + 4^x)$
- c) For $x = 7$: $12 \cdot 7 = 84$, $4^7 = 16384$, $h(7) = 0.35(84 + 16384) \approx \underline{\hspace{1cm}}$

Question 34: Rational Equations and Extraneous Solutions

The original question asks to solve $\frac{x^2+4}{x-1} = \frac{5}{x-1}$ and identify extraneous solutions. The following questions build understanding of rational equations.

34.1 Basic Rational Equations: If denominators are equal, equate numerators:

a) $\frac{x+2}{x-3} = \frac{5}{x-3}$: $x + 2 = 5$, $x = 3$.
Check: $x = 3$ makes denominator zero (extraneous).

b) $\frac{2x}{x+1} = \frac{4}{x+1}$: $x = \underline{\hspace{2cm}}$. Check: $\underline{\hspace{2cm}}$.

34.2 Extraneous Solutions: Solutions making denominators zero are extraneous:

a) $\frac{x}{x-4} = \frac{2}{x-4}$: Extraneous: $x = \underline{\hspace{2cm}}$

b) Solve: $x = 2$. Check: $\underline{\hspace{2cm}}$.

34.3 Solving Equations: Solve $\frac{x^2+1}{x-2} = \frac{3}{x-2}$:

a) Restriction: $x \neq 2$

b) Equate: $x^2 + 1 = 3$, $x^2 = 2$, $x = \pm\sqrt{2}$

c) Check: Neither $\sqrt{2}$ nor $-\sqrt{2}$ equals 2 (valid).

34.4 Applying to the Original Problem: Solve $\frac{x^2+4}{x-1} = \frac{5}{x-1}$:

a) Restriction: $x \neq 1$

b) Equate: $x^2 + 4 = 5$, $x^2 = 1$, $x = \pm 1$

c) Check: $x = 1$ is extraneous, $x = -1$ is valid.

d) Answer: $x = -1$, extraneous: $x = 1$.

Question 35: Discontinuities in Rational Functions

The original question asks where discontinuities occur in $f(x) = \frac{x^2+5x}{x^2-2x-35}$. The following questions build understanding of discontinuities.

35.1 Factoring Quadratics: Factor to find zeros:

a) $x^2 - 6x + 8 = (x - 2)(x - 4)$: Zeros: $x = 2, 4$

b) $x^2 + 3x - 10$: Zeros: $\underline{\hspace{2cm}}, \underline{\hspace{2cm}}$

35.2 Finding Discontinuities: Discontinuities occur where denominator = 0:

a) $f(x) = \frac{x}{x-5}$: Discontinuity: $x = 5$

b) $f(x) = \frac{1}{x^2-4}$: Discontinuities: $\underline{\hspace{2cm}}, \underline{\hspace{2cm}}$

c) Why discontinuities? $\underline{\hspace{4cm}}$

35.3 Removable Discontinuities: Common factors may cancel:

a) $f(x) = \frac{x-1}{x^2-x} = \frac{x-1}{x(x-1)}$: Cancel $x - 1$, discontinuity at $x = 0$

b) $f(x) = \frac{x+2}{x^2+5x+6}$: Discontinuities: $\underline{\hspace{2cm}}, \underline{\hspace{2cm}}$.

35.4 Applying to the Original Problem: For $f(x) = \frac{x^2+5x}{x^2-2x-35}$:

- a) Numerator: $x(x+5)$, zeros: $x = 0, -5$
- b) Denominator: $(x-7)(x+5)$, zeros: $x = 7, -5$
- c) Discontinuities: $x = 7, -5$. At $x = -5$, cancel $x+5$, but original function undefined. Answer: $x = -5, 7$.

Question 36: Set Operations and Probability

The original question asks whether the winning outcomes (odd number or 6) are the union, intersection, or complement of $\{1, 2, 3, 5, 6\}$ and $\{1, 3, 4, 5, 6\}$. The following questions build understanding of set operations.

36.1 Set Operations: For $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$:

- a) Union: $A \cup B = \{1, 2, 3, 4, 5, 6\}$
- b) Intersection: $A \cap B = \emptyset$
- c) Why use union in probability? _____

36.2 Probability Context: Dice roll, win on 1 or 2:

- a) Set: $\{1, 2\}$
- b) Outcomes: $\{1, 2, 3, 4, 5, 6\}$. Win = $\{1, 2\}$.

36.3 Analyzing Sets: For $\{1, 3, 5\}$, $\{1, 2, 5\}$:

- a) Union: $\{1, 2, 3, 5\}$
- b) Intersection: $\{1, 5\}$
- c) Practice: If win on 1 or 5, which operation? _____.

36.4 Applying to the Original Problem: Win on odd or 6:

- a) Winning: $\{1, 3, 5, 6\}$
- b) Sets: $\{1, 2, 3, 5, 6\}$, $\{1, 3, 4, 5, 6\}$
- c) Intersection: $\{1, 3, 5, 6\}$. Answer: Intersection.

Questions 37–38: Conditional Probability and Data Analysis

The original questions involve calculating $P(\text{heavy metal} \mid \text{12th grade})$ and comparing $P(\text{10th grade} \mid \text{rock})$ vs. $P(\text{rock} \mid \text{10th grade})$ using a two-way table. The following questions build understanding of conditional probability.

37.1 Reading Tables:

	Rock	Hip-Hop	Heavy Metal	Total
10th	16	12	4	32
11th	18	10	12	40
12th	16	8	6	30
Total	50	30	22	102

- a) 11th graders, hip-hop: 10
- b) Total 10th graders: _____
- c) Why use totals? _____

37.2 Basic Probability: $P(\text{event}) = \frac{\text{favorable}}{\text{total}}$:

- a) P(11th grade): $\frac{40}{102}$
- b) P(rock | 11th grade): $\frac{18}{40} = \frac{9}{20}$

37.3 Conditional Probability: $P(A|B) = \frac{A \text{ and } B}{B}$:

- a) P(hip-hop | 10th grade): $\frac{12}{32} = \frac{3}{8}$
- b) P(10th grade | hip-hop): $\frac{12}{30} = \frac{2}{5}$. Compare: _____.

37.4 Applying to the Original Problems:

- a) Q37: P(heavy metal | 12th grade): $\frac{6}{30} = \frac{1}{5} = 20\%$
- b) Q38: P(10th grade | rock): $\frac{16}{50} = 0.32$.
 P(rock | 10th grade): $\frac{16}{32} = 0.5$.
 $0.32 < 0.5$, so P(10th grade | rock) is _____ P(rock | 10th grade).