Answer Key for Scaffolded Questions 1–16

This document provides the answer key for the scaffolded practice problems for questions 1 through 16 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each section corresponds to a problem set, with solutions for each scaffolded step, designed to support concept-naive students.

Questions 1–4: Function Transformations, Asymptotes, Work Rates, Quadratic Vertices

Question 1: Function Transformations

1.1 **Basic Vertex Shifts**:

a) $y = |x - 4|$: Vertex at $(4, 0)$

b) $y = |x| - 3$: Vertex at $(0, -3)$

c) $y = |x + 1| + 2$: Vertex at $(-1, 2)$

1.2 **Transformation Effects**:

- $f(x - h)$, $h > 0$: A (Shifts right h units)
- $f(x) + k$, $k > 0$: B (Shifts up k units)
- $-f(x)$: C (Reflects over x-axis)
- $f(x + h)$, $h > 0$: D (Shifts left h units)

1.3 **Combined Transformations**:

a) Shift 1 unit right: Vertex from $(-2, 0)$ to $(-1, 0)$

b) Then shift 4 units down: Vertex to $(-1, -4)$

c) New equation: Start with $y = |x + 2|$. Right 1: $y = |(x - 1) + 2| = |x + 1|$. Down 4: $y = |x + 1| - 4$.

1.4 **Applying to the Original Problem**:

a) Vertex from $(2, 3)$: Right 3 to $(5, 3)$, down 5 to $(5, -2)$

b) New equation: Start with $y = -|x - 2| + 3$. Right 3: $y = -|(x - 3) - 2| + 3 = -|x - 5| + 3$. Down 5: $y = -|x - 5| - 2$. Matches choice (C) $y = -|x - 1| - 2$.

Question 2: Vertical Asymptotes

2.1 **Logarithm Domain**:

a) $f(x) = \ln(x - 1)$: Domain $x > 1$, asymptote at $x = 1$

b) $f(x) = \ln(x + 3)$: Domain $x > -3$, asymptote at $x = -3$

2.2 **Transformed Logarithms**:

a) $f(x) = \log(x-5)$: Asymptote at $x = 5$

b) $f(x) = \log(x-2) + 3$: Asymptote at $x = 2$

c) The +3 shifts the graph vertically, not affecting the asymptote.

2.3 **Checking for** $x = 4$:

a) $\ln(x-4)$: Asymptote at $x = 4$

b) $\ln(x) + 4$: Asymptote at $x = 0$

c) $2\ln(x-4)$: Asymptote at $x = 4$

d) $\ln(x+4)$: Asymptote at $x = -4$

2.4 **Applying to the Original Problem**:

a) $\log_4 x - 4$: Asymptote at $x = 0$

b) $\ln(x-4)$: Asymptote at $x = 4$

c) $\log(x-4) + 4$: Asymptote at $x = 4$

d) $4\ln x - 4$: Asymptote at $x = 0$

e) $\log(x-4)$: Asymptote at $x = 4$

f) Asymptote at $x = 4$: B, C, E

Question 3: Work Rate Problems

3.1 **Understanding Rates**:

a) 5 hours: Rate $= \frac{1}{5}$ tank/hour

b) 10 hours: Rate $= \frac{1}{10}$ tank/hour

c) Rate is reciprocal because it represents the fraction of the tank filled per hour.

3.2 **Combining Rates**:

a) Combined rate: $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ tank/hour

b) Time: $t = \frac{1}{\frac{1}{4}} = 4$ hours

3.3 **Setting Up the Equation**:

a) $\frac{1}{10} + \frac{1}{5} = \frac{1}{t}$

b) Combined rate: $\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$, so $t = \frac{10}{3} \approx 3.33$ hours

3.4 **Applying to the Original Problem**:

a) Rates: A: $\frac{1}{8}$, B: $\frac{1}{4}$

b) Combined rate: $\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

c) Time: $t = \frac{1}{\frac{3}{8}} = \frac{8}{3} \approx 2.67$ hours

d) Convert: $\frac{8}{3}$ hours = 2 hours, 40 minutes (since $\frac{2}{3} \times 60 = 40$)

Question 4: Vertex Form and Transformations

4.1 **Vertex of Quadratics**:

a) $f(x) = (x-1)^2 + 4$: Vertex at $(1, 4)$

b) $f(x) = 2(x+3)^2 - 2$: Vertex at $(-3, -2)$

4.2 **Horizontal Shifts**:

a) $g(x) = f(x-2)$: Vertex from $(3, 1)$ to $(5, 1)$

b) $h(x) = f(x+1)$: Vertex to $(2, 1)$

4.3 **Combined Shifts**:

a) $g(x) = f(x-1) + 3$: Vertex from $(1, 2)$ to $(2, 5)$

b) $h(x) = f(x+2) - 1$: Vertex to $(-1, 1)$

4.4 **Applying to the Original Problem**:

a) Horizontal shift: $x - 3$ shifts 3 units right

b) Vertical shift: -2 shifts 2 units down

c) Vertex from $(2, -4)$ to $(5, -6)$

Questions 5–8: Polynomial Zeros, Complex Quadratics, Exponentials, Complex Multiplication

Question 5: Finding Zeros of Polynomial Functions

5.1 **Understanding Zeros**:

a) $x - 2 = 0$, zero at $x = 2$

b) Zeros at $x = 2$, $x = -2$

c) Zero is when height is 0 (e.g., pelican at sea level).

5.2 **Factoring Polynomials**:

a) Zeros at $x = 0$, $x = -3$

b) Zeros at $x = 0$, $x = 3$, $x = -3$

c) Multiplicity affects the graph's behavior at the zero (e.g., touches vs. crosses).

5.3 **Contextual Zeros**:

a) Zeros at $t = 0$, $t = 3$

b) $t = 0$: Ball is thrown; $t = 3$: Ball hits ground.

c) Negative times are before the event starts, so irrelevant.

5.4 **Testing Zeros**:

a) $f(-2) = (-2)^4 - (-2)^3 - 8(-2)^2 + 8(-2) = 16 + 8 - 32 - 16 = -24$. Not a zero.

b) $f(1) = 1 - 1 - 8 + 8 = 0$. Is a zero.

c) Test $x = 0$: $f(0) = 0$, zero. Test $x = 1$: $f(1) = 1 - 2 - 29 + 30 = 0$, zero.

Question 6: Solving Quadratic Equations with Complex Numbers

6.1 **Complex Numbers**:

a) $\sqrt{-16} = 4i$

b) $\sqrt{-36} = 6i$

c) $\sqrt{-1} = i$ by definition of the imaginary unit.

6.2 **Quadratic Formula**:

a) $a = 1$, $b = -2$, $c = -3$

b) Discriminant: $(-2)^2 - 4(1)(-3) = 4 + 12 = 16$, positive.

c) $x = \frac{2\pm\sqrt{16}}{2} = \frac{2\pm4}{2}$, so $x = 3$, $x = -1$

6.3 **Complex Solutions**:

a) $a = 1$, $b = 2$, $c = 5$

b) Discriminant: $2^2 - 4(1)(5) = 4 - 20 = -16$

c) $x = \frac{-2\pm4i}{2} = -1 \pm 2i$

d) Negative discriminant means no real roots, only complex.

6.4 **Applying to the Original Problem**:

a) $-x^2 + 5x - 7 = 0$

b) $a = -1$, $b = 5$, $c = -7$

c) Discriminant: $5^2 - 4(-1)(-7) = 25 - 28 = -3$

d) $x = \frac{-5\pm\sqrt{-3}}{2(-1)} = \frac{-5\pm i\sqrt{3}}{-2} = \frac{5\pm i\sqrt{3}}{2}$. Matches choice (A).

Question 7: Exponential Equations with Natural Logarithms

7.1 **Logarithm Properties**:

a) $\ln(e^3) = 3$

b) $e^{\ln(4)} = 4$

c) $\ln(e^x) = x$ because ln is the inverse of e^x.

7.2 **Simple Exponential Equations**:

a) $x = \ln(6)$

b) $x = \ln(2)$

7.3 **Coefficients in Exponents**:

a) $e^x = 5$

b) $\ln(e^x) = \ln(5)$

c) $x = \ln(5)$

7.4 **Applying to the Original Problem**:

a) $e^{\frac{x}{2}} = 2$

b) $\ln\left(e^{\frac{x}{2}}\right) = \ln(2)$

c) $\frac{x}{2} = \ln(2)$

d) $x = 2\ln(2) = \ln(2^2) = \ln(4)$

Question 8: Multiplying Complex Numbers

8.1 **Complex Number Basics**:

a) $i^2 = -1$

b) $(2i)^2 = 4(-1) = -4$

c) $3 + 2i - 5i = 3 - 3i$

8.2 **Simple Multiplication**:

a) $2 + i + 2i + i^2$

b) $2 + 3i - 1 = 1 + 3i$

8.3 **Practice with Larger Numbers**:

a) $6 + 4i - 3i - 2i^2$

b) $6 + i - 2(-1) = 6 + i + 2 = 8 + i$

8.4 **Applying to the Original Problem**:

a) $3i + 2i^2 - 15 - 10i$

b) $3i + 2(-1) - 15 - 10i = -2 - 15 + 3i - 10i$

c) $-17 - 7i$. Matches choice (C) $-7i - 17$.

Questions 9–12: Polynomial Division, Literal Equations, Inverses, Average Rate of Change

Question 9: Polynomial Long Division

9.1 **Basic Polynomial Division**:

a) $4x^2$

b) $5x^2 + 2$

c) Terms are divided separately to match powers of x.

9.2 **Simple Long Division**:

a) x, $x^2 + x$, $3x + 3$

b) 3

c) $x^2 + 4x + 3 = (x + 1)(x + 3) + 0$

9.3 **Synthetic Division**:

a)

$$\begin{array}{r|rrr} 2 & 1 & 5 & 6 \\ & & 2 & 14 \\ \hline & 1 & 7 & 20 \end{array}$$

b) Quotient: $x + 7$, Remainder: 20

c) Synthetic division is faster for linear divisors as it simplifies calculations.

9.4 **Applying to the Original Problem**:

a) Coefficients: 1, -4, 6, -2. Divisor: 1.

b)

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 6 & -2 \\ & & 1 & -3 & 3 \\ \hline & 1 & -3 & 3 & 1 \end{array}$$

c) Quotient: $x^2 - 3x + 3$, Remainder: 1. $x^3 - 4x^2 + 6x - 2 = (x - 1)(x^2 - 3x + 3) + 1$.

Question 10: Solving Literal Equations

10.1 **Simple Literal Equations**:

a) $l = \frac{A}{w}$

b) $w = \frac{P-2l}{2}$

c) To express one variable in terms of others.

10.2 **Equations with Grouping**:

a) (Given)

b) $d = \frac{C-k}{\pi}$

10.3 **Business Context**:

a) $C = R - P$

b) $R = \frac{P+SC}{S}$

10.4 **Applying to the Original Problem**:

a) (Given)

b) (Given)

c) $V = P - \frac{N+F}{S}$

Question 11: Inverse Functions

11.1 **Inverse Function Basics**:

a) $f^{-1}(9) = 4$

b) $f(5) = 2$

c) Reflects points over the line $y = x$.

11.2 **Linear Inverses**:

a) (Given)

b) $y = \frac{x-3}{2}$

c) $f^{-1}(x) = \frac{x-3}{2}$

11.3 **Square Root Inverses**:

a) (Given)

b) $y = x^2 + 4$

c) Restriction ensures the inverse is a function (one-to-one).

11.4 **Applying to the Original Problem**:

a) $x^2 = y - 10$, $y = x^2 + 10$

b) $f^{-1}(x) = x^2 + 10$, years as a function of profit. Matches choice (C).

Question 12: Average Rate of Change

12.1 **Basic Average Rate of Change**:

a) $f(1) = 4$, $f(3) = 10$

b) $\frac{10-4}{3-1} = 3$

12.2 **Quadratic Functions**:

a) $f(-1) = 1$, $f(1) = 1$

b) $\frac{1-1}{2} = 0$

12.3 **Negative and Decimal Intervals**:

a) $f(-2.5) = -6.25 + 4 = -2.25$

b) $f(0) = 4$

c) $\frac{4-(-2.25)}{2.5} = \frac{6.25}{2.5} = 2.5$

12.4 **Applying to the Original Problem**:

a) $f(-3.5) = -2(12.25) + 5 = -24.5 + 5 = -19.5$

b) $f(0) = 5$

c) $\frac{5-(-19.5)}{3.5} = \frac{24.5}{3.5} = 7$. Matches choice (B).

Questions 13–16: Population Density, Radicals/Exponents, Inverse Variation, Logarithmic Equations

Question 13: Population Density and Radius

13.1 **Area of a Circle**: Area $= \pi(3)^2 = 9\pi \approx 9 \times 3.14 = 28.26$ square miles

13.2 **Population from Density**: Population $= 1000 \times 4 = 4000$ people

13.3 **Solving for Radius**:

a) Area $= \frac{12000}{1500} = 8$ square miles

b) $8 = \pi r^2$, $r^2 = \frac{8}{3.14} \approx 2.55$, $r \approx \sqrt{2.55} \approx 1.6$ miles

13.4 **Applying to the Original Problem**:

a) Area $= \frac{30000}{1200} = 25$ square miles

b) $25 = \pi r^2$, $r^2 = \frac{25}{3.14} \approx 7.96$, $r \approx \sqrt{7.96} \approx 2.82 \approx 2.8$ miles. Matches choice (A).

Question 14: Simplifying Radicals and Exponents

14.1 **Simplifying a Single Radical**: $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

14.2 **Combining Like Radicals**: $\sqrt{12} + \sqrt{48} = \sqrt{4 \cdot 3} + \sqrt{16 \cdot 3} = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

14.3 **Understanding Exponents**: $3^{\frac{3}{2}} = \sqrt{3^3} = \sqrt{27} = 3\sqrt{3}$

14.4 **Applying to the Original Expression**:

a) $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$, $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$, $2^{\frac{3}{2}} = \sqrt{2^3} = \sqrt{8} = 2\sqrt{2}$

b) $2\sqrt{2} + 4\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$. Matches choice (C).

Question 15: Inverse Variation

15.1 **Understanding Inverse Variation**: $y = \frac{k}{x}$, $6 = \frac{k}{4}$, $k = 24$

15.2 **Finding a New Value**: $y = \frac{12}{3} = 4$

15.3 **Setting Up the Equation**: $M = 5$, $x = 8$: $5 = \frac{k}{8}$, $k = 40$. For $x = 4$: $M = \frac{40}{4} = 10$

15.4 **Applying to the Original Problem**: $M = 2$, $x = 10$: $2 = \frac{k}{10}$, $k = 20$. For $x = 5$: $M = \frac{20}{5} = 4$

Question 16: Solving Logarithmic Equations

16.1 **Understanding Logarithms**: $\ln(y) = 2$, $y = e^2$

16.2 **Solving a Simple Log Equation**: $\ln(x) = 3$, $x = e^3$

16.3 **Handling Coefficients**: $2\ln(x) = 4$, $\ln(x) = 2$, $x = e^2$

16.4 **Applying to the Original Equation**:

a) $-2\ln(3x) = 5$, $\ln(3x) = -\frac{5}{2}$

b) $3x = e^{-\frac{5}{2}}$, $x = \frac{e^{-\frac{5}{2}}}{3} \approx \frac{0.082}{3} \approx 0.027$. Matches choice (B).