Algebra 2 Assessment Review: Quadratics & Complex Numbers

This document provides revised scaffolded questions to help students prepare for questions 6, 8, 17, 20, 21, and 25 (Quadratics & Complex Numbers group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 6: Solving Quadratic Equations with Complex Numbers

The original question asks to solve $-x^2 + 5x = 7$ over complex numbers. The following questions build understanding of the quadratic formula and complex solutions.

6.1 **Complex Numbers**: The imaginary unit i satisfies $i^2 = -1$. Simplify:

a) $\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1} =$ _____

b) $\sqrt{-36} =$ _____

c) Why is $\sqrt{-1} = i$? ______________________________

6.2 **Quadratic Formula**: For $ax^2 + bx + c = 0$, solutions are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Solve $x^2 - 2x - 3 = 0$:

a) Identify: $a =$ ___, $b =$ ___, $c =$ ___

b) Discriminant: $b^2 - 4ac =$ _____. Is it positive, negative, or zero? _____

c) Solutions: $x = \frac{\_\_ \pm \sqrt{\_\_}}{2} =$ _______

6.3 **Complex Solutions**: Solve $x^2 + 2x + 5 = 0$:

a) $a =$ ___, $b =$ ___, $c =$ ___

b) Discriminant: $b^2 - 4ac =$ _____. Since it's negative, expect complex roots.

c) Apply formula: $x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} =$ _______

d) Why does a negative discriminant mean complex roots? ____________________

6.4 **Applying to the Original Problem**: Solve $-x^2 + 5x = 7$:

a) Rewrite in standard form: ____________ $= 0$

b) Identify: $a =$ ___, $b =$ ___, $c =$ ___

c) Discriminant: $b^2 - 4ac =$ _____

d) Solutions: $x = \frac{\_\_ \pm \sqrt{\_\_}}{\_\_} =$ ____________. Compare to choices.

Scaffolded Question for Assessment Item 8: Multiplying Complex Numbers

The original question asks to simplify $(i-5)(3+2i)$. The following questions build understanding of complex number multiplication.

8.1 **Complex Number Basics**: Since $i^2 = -1$, simplify:

a) $i^2 =$ _____

b) $(2i)^2 = 4i^2 =$ _____

c) Combine: $3 + 2i - 5i =$ _______

8.2 **Simple Multiplication**: Multiply $(1+i)(2+i)$:

a) Use FOIL: $(1)(2) + (1)(i) + (i)(2) + (i)(i) =$ ______________

b) Simplify: $2 + i + 2i + i^2 = 2 + 3i - 1 =$ _______

8.3 **Practice with Larger Numbers**: Multiply $(2-i)(3+2i)$:

a) FOIL: $(2)(3) + (2)(2i) + (-i)(3) + (-i)(2i) =$ ______________

b) Simplify: $6 + 4i - 3i - 2i^2 =$ _______

8.4 **Applying to the Original Problem**: Simplify $(i-5)(3+2i)$:

a) FOIL: $(i)(3) + (i)(2i) + (-5)(3) + (-5)(2i) =$ ______________

b) Simplify: $3i + 2i^2 - 15 - 10i =$ ______________

c) Combine: _______. Compare to choices.

Scaffolded Question for Assessment Item 17: Factoring Quadratics and Finding Zeros

The original question asks to factor $x^2 - 33x + 32$ to find the zeros of $f(x) = x^2 - 33x + 32$. The following questions build understanding of factoring quadratics.

17.1 **Basic Factoring**: Factor by finding two numbers that multiply to the constant term and add to the middle coefficient:

a) $x^2 + 7x + 10$: Numbers multiply to 10, add to 7: 2, 5.
Factored: $(x+2)(x+5)$

b) $x^2 - 9x + 20$: Numbers multiply to 20, add to -9: -4, -5.
Factored: $(x-4)(x-5)$

c) Why does factoring find zeros? ____________________________

17.2 **Finding Zeros**: Find zeros by setting factors to zero:

a) $f(x) = (x-3)(x+6)$: Zeros: $x = 3$, $x = -6$

b) $f(x) = (x-2)(x-8)$: Zeros: $x =$ _____, $x =$ _____

c) Verify one zero: For $x = 2$, compute $f(2) = (2-2)(2-8) =$ _____.

17.3 **Larger Coefficients**: Factor $x^2 - 14x + 45$:

a) Factors of 45: 1×45, 3×15, 5×9.
Add to -14: -5, -9.
Factored: $(x-5)(x-9)$

b) Zeros: $x = 5$, $x = 9$

c) Practice: Factor $x^2 - 16x + 60$: Numbers: ___, ___.
Factored: __________. Zeros: ___, ___.

17.4 **Applying to the Original Problem**: Factor $x^2 - 33x + 32$:

a) Factors of 32: 1×32, 2×16, 4×8.
Add to -33: _____, _____.

b) Factored: $(x$_____$)(x$_____$)$

c) Zeros: $x =$ _____, $x =$ _____

Scaffolded Question for Assessment Item 20: Completing the Square

The original question asks for the constant to add to both sides of $3x^2 + 4x = 5$ to complete the square. The following questions build understanding of completing the square.

20.1 **Perfect Square Trinomials**: Complete to form a perfect square:

a) $x^2 + 10x +$ _____ $= (x+5)^2$: Half of 10: 5, squared: 25.

b) $x^2 - 6x +$ _____ $= (x -$ ___$)^2$: Half of -6: -3, squared: 9.

c) $x^2 + 12x +$ _____ $= (x +$ ___$)^2$: _____, ___.

20.2 **Completing the Square ($a = 1$)**: For $x^2 + 8x = 3$:

a) Half of 8: 4, squared: 16.

b) Add: $x^2 + 8x + 16 = 3 + 16$.

c) Factor: $(x+4)^2 = 19$.

d) Practice: For $x^2 + 10x = 6$: Constant: _____. Result: _____________.

20.3 **Completing the Square ($a \neq 1$)**: For $2x^2 + 12x = 8$:

a) Factor: $2(x^2 + 6x) = 8$.

b) Complete inside: Half of 6: 3, squared: 9.
$2(x^2 + 6x + 9) = 8 + 2 \cdot 9 = 26$.

c) Simplify: $2(x+3)^2 = 26$.

d) Constant added to right: $2 \cdot 9 = 18$.

e) Practice: For $4x^2 + 8x = 12$: Constant: _____. Result: ____________.

20.4 **Applying to the Original Problem**: For $3x^2 + 4x = 5$:

a) Factor: $3(x^2 + \frac{4}{3}x) = 5$.

b) Complete: Half of $\frac{4}{3}$: $\frac{2}{3}$, squared: $\frac{4}{9}$.
$3\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) = 5 + 3 \cdot \frac{4}{9}$.

c) Simplify: $3\left(x + \frac{2}{3}\right)^2 = 5 + \frac{12}{9} = 5 + \frac{4}{3} = \frac{19}{3}$.

d) Constant added: $3 \cdot \frac{4}{9} = \frac{4}{3}$. Matches choice (B).

Scaffolded Question for Assessment Item 21: Complex Solutions to Quadratic Equations

The original question asks to select the solutions to $x^2 = -64$. The following questions build understanding of complex solutions.

21.1 **Square Roots of Negative Numbers**: Since $\sqrt{-1} = i$, $\sqrt{-a} = i\sqrt{a}$ for positive a. Simplify:

a) $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$

b) $\sqrt{-36} =$ _____

c) $\sqrt{-100} =$ _____

d) Why is $\sqrt{-1} = i$? ______________________________

21.2 **Simple Complex Solutions**: Solve:

a) $x^2 = -16$: $x = \pm\sqrt{-16} = \pm 4i$

b) $x^2 = -49$: $x = \pm$_____

c) Verify: For $x = 7i$, compute $(7i)^2 =$ _____.

21.3 **Quadratic Formula**: Solve $x^2 + 9 = 0$:

a) Rewrite: $x^2 = -9$

b) Quadratic formula: $a = 1$, $b = 0$, $c = 9$.
$x = \frac{0 \pm \sqrt{0 - 4(1)(9)}}{2} = \frac{\pm\sqrt{-36}}{2} = \frac{\pm 6i}{2} = \pm 3i$

c) Practice: Solve $x^2 + 25 = 0$: $x = \pm$_____.

21.4 **Applying to the Original Problem**: Solve $x^2 = -64$:

a) Direct: $x = \pm\sqrt{-64} = \pm 8i$

b) Quadratic formula: $x^2 + 64 = 0$, $a = 1$, $b = 0$, $c = 64$.
$x = \frac{\pm\sqrt{-256}}{2} = \frac{\pm 16i}{2} = \pm 8i$

c) Verify: $(8i)^2 = 64i^2 = -64$, $(-8i)^2 = 64i^2 = -64$.

d) Select solutions from: 8, $-8i$, -8, $32i$, $8i$, $-32i$: _____, _____.

Scaffolded Question for Assessment Item 25: Quadratic Formula and Simplifying Radicals

The original question asks to solve $x^2 + 10x + 6 = 0$ using the quadratic formula. The following questions build understanding of the quadratic formula and radical simplification.

25.1 **Identifying Coefficients**: For $ax^2+bx+c = 0$, identify a, b, c to use in $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

a) $x^2 + 3x + 2 = 0$: $a = 1$, $b = 3$, $c = 2$

b) $2x^2 - 5x + 1 = 0$: $a =$ ___, $b =$ ___, $c =$ ___

c) Why identify coefficients? ______________________________

25.2 **Calculating Discriminant**: The discriminant $b^2 - 4ac$ determines the number of roots (positive: two real, zero: one, negative: complex):

a) $x^2 + 4x + 3 = 0$: $b^2 - 4ac = 4^2 - 4(1)(3) = 16 - 12 = 4$

b) $x^2 + 6x + 2 = 0$: $b^2 - 4ac =$ _____ $-$ _____ $=$ _____

c) What does a positive discriminant mean? ______________________________

25.3 **Simplifying Radicals**: Simplify square roots for the quadratic formula:

a) $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}$

b) $\sqrt{76} = \sqrt{4 \cdot 19} =$ _____

c) Practice: $\sqrt{80} =$ _____. Why simplify radicals? _________________________

25.4 **Applying to the Original Problem**: Solve $x^2 + 10x + 6 = 0$:

a) Coefficients: $a = 1$, $b = 10$, $c = 6$

b) Discriminant: $b^2 - 4ac = 10^2 - 4(1)(6) = 100 - 24 = 76$

c) Simplify: $\sqrt{76} = \sqrt{4 \cdot 19} = 2\sqrt{19}$

d) Solve: $x = \frac{-10\pm\sqrt{76}}{2} = \frac{-10\pm 2\sqrt{19}}{2} = -5 \pm \sqrt{19}$

Original Assessment Questions

Question 6

Solve $-x^2 + 5x = 7$ over the set of complex numbers.

A. $\frac{5 \pm i\sqrt{3}}{2}$, $\frac{5 - i\sqrt{3}}{2}$

B. $\frac{5 \pm i\sqrt{53}}{2}$, $\frac{5 - i\sqrt{53}}{2}$

C. $\frac{-5 \pm i\sqrt{53}}{2}$, $\frac{-5 - i\sqrt{53}}{2}$

D. $\frac{-5 \pm i\sqrt{3}}{2}$, $\frac{-5 - i\sqrt{3}}{2}$

(Note: The options in the image are presented as single fractions. For clarity in LaTeX and to match the scaffold process, I've kept them as two separate roots for each choice if distinct, or as $\pm$ if combined. The original options seemed to list $\frac{A+B}{C}, \frac{A-B}{C}$. Option B in the image is the one where the discriminant would be -3, leading to $5 \pm i\sqrt{3}$ over -2, which is $\frac{-5 \mp i\sqrt{3}}{2}$. The actual answer is $\frac{5 \pm i\sqrt{-3}}{-2} = \frac{5 \pm i\sqrt{3}}{-2} = \frac{-5 \mp i\sqrt{3}}{2}$. The closest choice, if there was a typo in the question and it was $x^2 - 5x + 7 = 0$, would be $\frac{5 \pm i\sqrt{3}}{2}$ (Option A from image). For $-x^2 + 5x - 7 = 0$, $x = \frac{-5 \pm \sqrt{25-28}}{-2} = \frac{-5 \pm \sqrt{-3}}{-2} = \frac{-5 \pm i\sqrt{3}}{-2} = \frac{5 \mp i\sqrt{3}}{2}$. This matches Option A in the image.)

Question 8

Which of the following is equivalent to the expression $(i - 5)(3 + 2i)$?

A. $-7i - 13$

B. $13i - 17$

C. $-7i - 17$

D. $-13i - 17$

Question 17

Factor the expression $x^2 - 33x + 32$ to reveal the zeros of the function defined by $f(x) = x^2 - 33x + 32$.

- The factored expression is $(x+$ ☐$)$ $(x+$ ☐$)$
- The zeros of the function are ☐ and ☐

Question 20

What constant do you add to each side of the equation to solve by completing the square?

$$3x^2 + 4x = 5$$

A. $\frac{9}{16}$ C. $\frac{3}{2}$

B. $\frac{4}{3}$ D. 6

Question 21

Select the solutions of the equation $x^2 = -64$.

⊠ 8 D. [⊠] $32i$

⊠ $-8i$ E. [⊠] $8i$

⊠ -8 F. [⊠] $-32i$

(Note: Replace ⊠with □if you want empty boxes for students to fill)

Question 25

Solve $x^2 + 10x + 6 = 0$. Use the choices provided to complete the solution.

10	-10	5	-5
$\sqrt{19}$	$\sqrt{6}$	$\sqrt{10}$	

$x = \boxed{} \pm \boxed{}$