Revised Scaffolded Questions for Algebra 2 Assessment (Questions 9–12)

This document provides revised scaffolded questions to help students prepare for questions 9 through 12 of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes four scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students.

Question 9: Polynomial Long Division

The original question asks to divide $x^3 - 4x^2 + 6x - 2$ by $x - 1$ and complete the quotient. The following questions build understanding of polynomial division.

9.1 **Basic Polynomial Division**: Divide each term by the divisor, matching powers of x:

- a) $\frac{8x^3}{2x} =$ ______
- b) $\frac{10x^4+4x^2}{2x^2} = \frac{10x^4}{2x^2} + \frac{4x^2}{2x^2} =$ ______ + ______
- c) Why divide term by term? __________________

9.2 **Simple Long Division**: Divide $x^2 + 4x + 3$ by $x + 1$:

- a) $x^2 \div x =$ ______, multiply: $x(x+1) =$ ______, subtract: $(x^2+4x+3)-(x^2+x) =$ ______
- b) Continue: $3x \div x =$ ______, multiply, subtract to get remainder 0.
- c) Result: $x^2 + 4x + 3 = (x + 1)($______$) +$ ______

9.3 **Synthetic Division**: Use synthetic division for $x^2 + 5x + 6$ by $x - 2$:

- a) Divisor $x - 2$, so use 2. Coefficients: 1, 5, 6. Setup:

$$\begin{array}{r|rrr} 2 & 1 & 5 & 6 \\ & & \underline{\quad} & \underline{\quad} \\ \hline & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{array}$$

- b) Quotient: ______, Remainder: ______
- c) Why is synthetic division faster for linear divisors? __________________

9.4 **Applying to the Original Problem**: Divide $x^3 - 4x^2 + 6x - 2$ by $x - 1$ using synthetic division:

- a) Coefficients: ______, ______, ______, ______. Divisor: $x - 1$, so use ______.
- b) Perform synthetic division:

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 6 & -2 \\ & & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \hline & \underline{\quad} & \underline{\quad} & \underline{\quad} & \underline{\quad} \end{array}$$

c) Quotient: ______, Remainder: ______. Write as: $x^3 - 4x^2 + 6x - 2 = (x - 1)(\_\_\_\_\_\_) + \_\_\_\_\_\_$.

Question 10: Solving Literal Equations

The original question asks to solve $N = S(P - V) - F$ for the variable cost per unit V. The following questions build understanding of solving literal equations.

10.1 **Simple Literal Equations**: Solve for the indicated variable, isolating it like solving for x:

a) $A = lw$, for l: $l =$ ______

b) $P = 2l + 2w$, for w: $w =$ ______

c) Why isolate variables? ____________________

10.2 **Equations with Grouping**: Solve:

a) $y = m(x + b)$, for m: $m = \frac{y}{x+b}$

b) $C = \pi d + k$, for d: $d =$ ______

10.3 **Business Context**: Solve profit-related formulas:

a) $P = R - C$, for C: $C =$ ______

b) $P = S(R - C)$, for R: $P = SR - SC$, so $R =$ ______

10.4 **Applying to the Original Problem**: Given $N = S(P - V) - F$, solve for V:

a) Isolate the term with V: $N + F = S(P - V)$

b) Divide: $\frac{N+F}{S} = P - V$

c) Solve: $V =$ ______

Question 11: Inverse Functions

The original question asks for the inverse of $f(x) = \sqrt{x - 10}$, representing years as a function of profits. The following questions build understanding of inverse functions.

11.1 **Inverse Function Basics**: If $f(a) = b$, then $f^{-1}(b) = a$. The inverse swaps x and y-coordinates:

a) If $f(4) = 9$, then $f^{-1}(9) =$ ______

b) If $f^{-1}(2) = 5$, then $f(5) =$ ______

c) Why swap x and y? ____________________

11.2 **Linear Inverses**: Find the inverse of $f(x) = 2x + 3$:

a) Set $y = 2x + 3$, switch: $x = 2y + 3$

b) Solve: $x - 3 = 2y$, so $y =$ ______

c) Inverse: $f^{-1}(x) =$ ______

11.3 **Square Root Inverses**: Find the inverse of $f(x) = \sqrt{x - 4}$, $x \geq 4$:

a) Set $y = \sqrt{x - 4}$, switch: $x = \sqrt{y - 4}$

b) Solve: Square both sides: $x^2 = y - 4$, so $y =$ ______

c) Inverse: $f^{-1}(x) = x^2 + 4$, for $x \geq 0$ (since $y \geq 0$). Why the restriction? ____________

11.4 **Applying to the Original Problem**: For $f(x) = \sqrt{x - 10}$, representing profit after x years:

a) Find inverse: Set $y = \sqrt{x - 10}$, switch: $x = \sqrt{y - 10}$, solve: $y =$ ______

b) Inverse: $f^{-1}(x) =$ ______, for $x \geq 0$. What does $f^{-1}(x)$ represent? ______

c) Compare to choices: $(x - 10)^2$, $x^2 + 10$, with domains $x \geq 0$ or $x \geq -10$.

Question 12: Average Rate of Change

The original question asks for the average rate of change of $f(x) = -2x^2 + 5$ over $-3.5 \leq x \leq 0$. The following questions build understanding of average rate of change.

12.1 **Basic Average Rate of Change**: The average rate of change is the slope of the secant line: $\frac{f(b)-f(a)}{b-a}$. For $f(x) = 3x + 1$, find from $x = 1$ to $x = 3$:

a) $f(1) =$ ______, $f(3) =$ ______

b) Rate: $\frac{f(3)-f(1)}{3-1} =$ ______

12.2 **Quadratic Functions**: For $f(x) = -x^2 + 2$, find from $x = -1$ to $x = 1$:

a) $f(-1) =$ ______, $f(1) =$ ______

b) Rate: $\frac{f(1)-f(-1)}{1-(-1)} =$ ______

12.3 **Negative and Decimal Intervals**: For $f(x) = -x^2 + 4$, find from $x = -2.5$ to $x = 0$:

a) $f(-2.5) = -(-2.5)^2 + 4 =$ ______

b) $f(0) =$ ______

c) Rate: $\frac{f(0)-f(-2.5)}{0-(-2.5)} =$ ______

12.4 **Applying to the Original Problem**: For $f(x) = -2x^2 + 5$, find from $x = -3.5$ to $x = 0$:

a) $f(-3.5) = -2(-3.5)^2 + 5 =$ ______

b) $f(0) =$ ______

c) Rate: $\frac{f(0)-f(-3.5)}{0-(-3.5)} =$ ______. Compare to choices: 19.5, 7, -7, -19.5.