Scaffolded Practice Problems for Assessment Questions 5-8

Problem 5 Scaffolding: Finding Zeros of Polynomial Functions

Scaffold 5.1: Understanding Zeros

A zero of a function is an x-value where $f(x) = 0$. Find the zeros of these simple functions:

a) $f(x) = x - 3$
Set equal to zero: $x - 3 = 0$
Zero: $x = ____$

b) $f(x) = x + 5$
Zero: $x = ____$

c) $f(x) = 2x - 8$
Zero: $x = ____$

d) $f(x) = x^2 - 9 = (x-3)(x+3)$
Zeros: $x = ____$ and $x = ____$

Scaffold 5.2: Factoring to Find Zeros

Factor each polynomial and find all zeros:

a) $f(x) = x(x - 4)$
Already factored. Set each factor to zero:
$x = 0$ or $x - 4 = 0$
Zeros: $x = ____$ and $x = ____$

b) $f(x) = x^2 + 5x = x(x + 5)$
Zeros: $x = ____$ and $x = ____$

c) $f(x) = x^3 - 4x^2 = x^2(x - 4)$
Zeros: $x = ____$ (with multiplicity 2) and $x = ____$

Scaffold 5.3: Context Problems

A ball is thrown upward. Its height is modeled by $h(t) = -16t^2 + 32t$ where t is time in seconds.

a) When does the ball hit the ground? (When is $h(t) = 0$?)
$-16t^2 + 32t = 0$
Factor: $-16t(t - 2) = 0$
Solutions: $t = ____$ or $t = ____$

b) What do these solutions mean in context?
$t = 0$: ________________________
$t = 2$: ________________________

Scaffold 5.4: Higher Degree Polynomials

For $f(x) = x^3 - 6x^2 + 9x$, find all zeros:

Step 1: Factor out the GCF
$f(x) = x(x^2 - 6x + 9)$

Step 2: Factor the quadratic
$x^2 - 6x + 9 = (x - ___)^2$

Step 3: Complete factorization
$f(x) = x(x - ___)^2$

Step 4: Find zeros
Set each factor to zero: $x = ___$ and $x = ___$ (multiplicity 2)

Problem 6 Scaffolding: Solving Quadratic Equations with Complex Numbers

Scaffold 6.1: Review of Complex Numbers

Recall: $i = \sqrt{-1}$, so $i^2 = -1$

Simplify:
a) $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = 2i$
b) $\sqrt{-9} = ___$
c) $\sqrt{-25} = ___$
d) $\sqrt{-7} = ___$

Scaffold 6.2: Quadratic Formula Review

For $ax^2 + bx + c = 0$, the solutions are: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Identify a, b, and c:
a) $x^2 + 3x + 2 = 0$: $a = ___$, $b = ___$, $c = ___$
b) $2x^2 - 5x + 1 = 0$: $a = ___$, $b = ___$, $c = ___$
c) $-x^2 + 5x - 7 = 0$: $a = ___$, $b = ___$, $c = ___$

Scaffold 6.3: Discriminant and Complex Solutions

The discriminant is $b^2 - 4ac$. When it's negative, solutions are complex.

For $x^2 + 2x + 5 = 0$:

a) $a = ____$, $b = ____$, $c = ____$

b) Discriminant = $b^2 - 4ac = (____)^2 - 4(____)(____) = ____$

c) Since the discriminant is negative, solutions will be __________

Apply quadratic formula:

$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

Scaffold 6.4: Practice with Standard Form

Solve $x^2 - 4x + 8 = 0$:

Step 1: Identify coefficients

$a = ____$, $b = ____$, $c = ____$

Step 2: Calculate discriminant

$b^2 - 4ac = (____)^2 - 4(____)(____) = ____$

Step 3: Apply quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{____ \pm \sqrt{____}}{____}$

Step 4: Simplify

$x = \frac{____ \pm ____i}{____} = ____ \pm ____i$

Problem 7 Scaffolding: Exponential Equations with Natural Logarithms

Scaffold 7.1: Properties of Logarithms and Exponentials

Remember: $\ln(e^x) = x$ and $e^{\ln(x)} = x$

Simplify:

a) $\ln(e^5) = ____$

b) $\ln(e^{-2}) = ____$

c) $e^{\ln(7)} = ____$

d) $e^{\ln(x+1)} = ____$

Scaffold 7.2: Solving Simple Exponential Equations

Solve each equation:

a) $e^x = 10$

Take natural log of both sides: $\ln(e^x) = \ln(10)$

Simplify: $x = \ln(10)$

b) $e^x = 5$

Solution: $x = \_\_\_\_$

c) $e^{2x} = 8$

Take natural log: $\ln(e^{2x}) = \ln(8)$

Simplify: $2x = \ln(8)$

Solve for x: $x = \frac{\ln(8)}{2}$

Scaffold 7.3: Equations with Coefficients

Solve: $3e^x = 12$

Step 1: Isolate the exponential

Divide both sides by 3: $e^x = \frac{12}{3} = \_\_\_\_$

Step 2: Take natural logarithm

$\ln(e^x) = \ln(\_\_\_\_)$

Step 3: Simplify

$x = \ln(\_\_\_\_)$

Practice: Solve $2e^x = 20$

$x = \ln(\_\_\_\_)$

Scaffold 7.4: Equations with Fractional Exponents

Solve: $4e^{\frac{x}{3}} = 16$

Step 1: Isolate exponential term

$e^{\frac{x}{3}} = \frac{16}{4} = \_\_\_\_$

Step 2: Take natural logarithm

$\ln(e^{\frac{x}{3}}) = \ln(\_\_\_\_)$

Step 3: Simplify left side

$\frac{x}{3} = \ln(\_\_\_\_)$

Step 4: Solve for x

$x = 3 \cdot \ln(\_\_\_\_) = \ln(\_\_\_\_)$

Note: $3\ln(4) = \ln(4^3) = \ln(64)$

Problem 8 Scaffolding: Multiplying Complex Numbers

Scaffold 8.1: FOIL with Complex Numbers

Remember: $i^2 = -1$

Multiply: $(2 + 3i)(1 + i)$
$= 2(1) + 2(i) + 3i(1) + 3i(i)$
$= 2 + 2i + 3i + 3i^2$
$= 2 + 5i + 3(-1)$
$= 2 + 5i - 3$
$= -1 + 5i$

Practice: $(1 + 2i)(3 + i)$
$= ____ + ____ + ____ + ____$
$= ____ + ____i + ____i^2$
$= ____ + ____i + ____(-1)$
$= ____ + ____i$

Scaffold 8.2: Multiplying with Negative Real Parts

Multiply: $(3 - 2i)(1 + 4i)$

Using FOIL:
First: $3 \cdot 1 = ____$
Outer: $3 \cdot 4i = ____$
Inner: $(-2i) \cdot 1 = ____$
Last: $(-2i) \cdot 4i = ____i^2 = ____(-1) = ____$

Combine: $____ + ____i + ____i + ____ = ____ + ____i$

Scaffold 8.3: Pure Imaginary Times Complex

Multiply: $2i(3 + 5i)$

Distribute: $2i \cdot 3 + 2i \cdot 5i$
$= 6i + 10i^2$
$= 6i + 10(-1)$
$= 6i - 10$
$= -10 + 6i$

Practice: $3i(2 - 4i) = ____i - ____i^2 = ____i - ____(-1) = ____ + ____i$

Scaffold 8.4: Complex Numbers with Negative Leading Terms

Multiply: $(i - 5)(3 + 2i)$

Method 1 - FOIL:

$(i - 5)(3 + 2i)$

$= i(3) + i(2i) + (-5)(3) + (-5)(2i)$

$= 3i + 2i^2 - 15 - 10i$

$= 3i + 2(-1) - 15 - 10i$

$= 3i - 2 - 15 - 10i$

$= -17 + (3i - 10i)$

$= -17 - 7i$

Method 2 - Rearrange first:

$(-5 + i)(3 + 2i)$

$= -5(3) + (-5)(2i) + i(3) + i(2i)$

$= -15 - 10i + 3i + 2i^2$

$= -15 - 7i - 2$

$= -17 - 7i$

Check: Both methods give the same answer.