Algebra 2 Assessment Review: Rational Functions & Equations

This document provides revised scaffolded questions to help students prepare for questions 34 and 35 (Rational Functions/Equations group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 34: Rational Equations and Extraneous Solutions

The original question asks to solve $\frac{x^2+4}{x-1} = \frac{5}{x-1}$ and identify extraneous solutions. The following questions build understanding of rational equations.

34.1 **Basic Rational Equations**: If denominators are equal, equate numerators:

a) $\frac{x+2}{x-3} = \frac{5}{x-3}$: $x + 2 = 5$, $x = 3$.
Check: $x = 3$ makes denominator zero (extraneous). No solution.

b) $\frac{2x}{x+1} = \frac{4}{x+1}$: $2x = 4$, so $x =$ _____. Check: Does $x = 2$ make the denominator zero? ___. So, is $x = 2$ a valid solution? ___.

34.2 **Extraneous Solutions**: Solutions making denominators zero are extraneous:

a) For the equation $\frac{x}{x-4} = \frac{2}{x-4}$: What value of x would make the denominator zero (and thus be an extraneous solution if it arises)? Extraneous if $x =$ _____.

b) Solve $\frac{x}{x-4} = \frac{2}{x-4}$: $x = 2$. Check: Is $x = 2$ the value that makes the denominator zero? ___. So, is $x = 2$ a valid solution or extraneous? _______.

34.3 **Solving Equations**: Solve $\frac{x^2+1}{x-2} = \frac{3}{x-2}$:

a) Restriction: Denominator cannot be zero, so $x - 2 \neq 0$, which means $x \neq 2$.

b) Equate numerators: $x^2 + 1 = 3$. Solve for x: $x^2 = 2$, so $x = \pm\sqrt{2}$.

c) Check against restriction: Is $\sqrt{2} = 2$? ___. Is $-\sqrt{2} = 2$? ___. Are the solutions valid? ___.

34.4 **Applying to the Original Problem**: Solve $\frac{x^2+4}{x-1} = \frac{5}{x-1}$:

a) Restriction: $x - 1 \neq 0$, so $x \neq$ _____.

b) Equate numerators: $x^2 + 4 = 5$. Solve for x: $x^2 =$ _____, so $x = \pm$_____.

c) Check against restriction: One potential solution is $x = 1$. Is this the restricted value? ___. So, $x = 1$ is _______. The other potential solution is $x = -1$. Is this the restricted value? ___. So, $x = -1$ is _______.

d) Answer: Valid solution is $x =$ _____. Extraneous solution is $x =$ _____.

Scaffolded Question for Assessment Item 35: Discontinuities in Rational Functions

The original question asks where discontinuities occur in $f(x) = \frac{x^2+5x}{x^2-2x-35}$. The following questions build understanding of discontinuities.

35.1 **Factoring Quadratics**: Factor to find zeros (roots):

a) $x^2 - 6x + 8 = (x-2)(x-4)$: Zeros: $x = 2, 4$

b) $x^2 + 3x - 10$: Find two numbers that multiply to -10 and add to 3. Numbers: 5, -2. Factored form: $(x + \_\_)(x - \_\_)$. Zeros: _____, _____.

35.2 **Finding Discontinuities**: Discontinuities occur where the denominator of a rational function equals 0 (because division by zero is undefined):

a) $f(x) = \frac{x}{x-5}$: Denominator is $x - 5$. Set $x - 5 = 0$. Discontinuity at $x = 5$.

b) $f(x) = \frac{1}{x^2-4}$: Denominator is $x^2 - 4$. Factor it: $(x-2)(x+2)$. Set denominator to zero: $(x-2)(x+2) = 0$. Discontinuities at $x =$ _____, $x =$ _____.

c) Why do discontinuities occur where the denominator is zero? ______________________________

35.3 **Removable Discontinuities (Holes) vs. Vertical Asymptotes**: If a factor $(x - c)$ in the denominator cancels with a factor in the numerator, there is a removable discontinuity (a hole) at $x = c$. If it doesn't cancel, it's usually a vertical asymptote. However, both are types of discontinuities.

a) $f(x) = \frac{x-1}{x^2-x} = \frac{x-1}{x(x-1)}$. Denominator zeros at $x = 0$ and $x = 1$. The factor $(x-1)$ cancels. Discontinuity at $x = 0$ (vertical asymptote). Discontinuity at $x = 1$ (removable discontinuity/hole).

b) $f(x) = \frac{x+2}{x^2+5x+6}$. Factor denominator: $x^2 + 5x + 6 = (x+2)(x+3)$. So, $f(x) = \frac{x+2}{(x+2)(x+3)}$. Denominator zeros at $x = -2$ and $x = -3$. These are the locations of discontinuities. At $x = -2$, the factor $(x+2)$ cancels, so it's a hole (removable). At $x = -3$, the factor $(x+3)$ does not cancel, so it's a vertical asymptote. Discontinuities at: _____, _____.

35.4 **Applying to the Original Problem**: For $f(x) = \frac{x^2+5x}{x^2-2x-35}$:

a) Factor the numerator: $x^2 + 5x = x(x+5)$. Zeros of numerator: $x = 0, -5$.

b) Factor the denominator: $x^2 - 2x - 35$. Find two numbers that multiply to -35 and add to -2. Numbers: -7, 5. Denominator: $(x-7)(x+5)$. Zeros of denominator: $x =$ _____, $x =$ _____.

c) Discontinuities occur where the original denominator is zero. So, discontinuities are at $x = 7$ and $x = -5$. At $x = -5$, the factor $(x+5)$ cancels from numerator and denominator. This means there is a hole (removable discontinuity) at $x = -5$. At $x = 7$, the factor $(x-7)$ does not cancel. This means there is a vertical asymptote at $x = 7$. The question asks "Where will the discontinuities occur". Both types are discontinuities. Answer: $x = \underline{-5}$, $x = \underline{7}$.

Original Assessment Questions

Question 34

Use the equation $\frac{x^2+4}{x-1} = \frac{5}{x-1}$ to answer the questions.

Part A Solve the equation for x.

$$x = \boxed{}$$

Part B Are there any extraneous solutions? Explain why or why not.

A. There are no extraneous solutions because all solutions are real numbers.

B. $x = 1$ is an extraneous solution because it makes a denominator equal to 0.

C. $x = -1$ is an extraneous solution because it makes a denominator equal to 0.

D. $x = 0$ is an extraneous solution because zero can not be a solution.

Question 35

Where will the discontinuities occur in the graph of the rational function?

$$f(x) = \frac{x^2 + 5x}{x^2 - 2x - 35}$$

A. at $x = -5$

B. at $x = 7$

C. at $x = 0$, $x = -5$ and $x = 7$

D. at $x = -5$ and $x = 7$