Algebra 2 Assessment Review: Exponentials & Logarithms

This document provides revised scaffolded questions to help students prepare for questions 7, 16, 24, 30, 31, and the exponential part of 33 (Exponentials & Logarithms group) of the enVision Algebra 2 Progress Monitoring Assessment Form C. Each question includes scaffolded steps to build understanding from basic concepts to the level required by the assessment, with clear guidance for concept-naive students. This is followed by the original assessment questions.

Scaffolded Review Questions

Scaffolded Question for Assessment Item 7: Exponential Equations with Natural Logarithms

The original question asks to solve $5e^{\frac{x}{2}} = 10$. The following questions build understanding of solving exponential equations.

7.1 **Logarithm Properties**: Since $\ln(e^y) = y$ (because ln is the inverse of e^y), simplify:

a) $\ln(e^3) =$ _____

b) $e^{\ln(4)} =$ _____

c) Why does $\ln(e^y) = y$? ______________________________

7.2 **Simple Exponential Equations**: Solve:

a) $e^x = 6$: Take ln of both sides: $\ln(e^x) = \ln(6)$, so $x =$ _______

b) $e^x = 2$: $x =$ _______

7.3 **Coefficients in Exponents**: Solve $3e^x = 15$:

a) Isolate: $e^x = \frac{15}{3} =$ _____

b) Take ln: $\ln(e^x) = \ln($_____$)$

c) Solve: $x =$ _______

7.4 **Applying to the Original Problem**: Solve $5e^{\frac{x}{2}} = 10$:

a) Isolate: $e^{\frac{x}{2}} = \frac{10}{5} =$ _____

b) Take ln: $\ln\left(e^{\frac{x}{2}}\right) = \ln($_____$)$

c) Simplify: $\frac{x}{2} = \ln($_____$)$

d) Solve: $x =$ _______. Write as $x = \ln($_______$)$ to match the original format if needed (or $x = 2\ln(\text{value})$ then $x = \ln(\text{value}^2)$).

Scaffolded Question for Assessment Item 16: Solving Logarithmic Equations

The original question asks to solve $-2\ln(3x) = 5$. The following questions build skills in solving equations involving natural logarithms.

16.1 **Understanding Logarithms**: If $\ln(y) = 2$, find y. Use the fact that $\ln(y) = c$ means $y = e^c$. $y =$ ________

16.2 **Solving a Simple Log Equation**: Solve the equation $\ln(x) = 3$. Write the equation in exponential form and compute x. $x =$ ________

16.3 **Handling Coefficients**: Solve the equation $2\ln(x) = 4$. First, isolate the logarithm by dividing both sides, then convert to exponential form to find x. $\ln(x) =$ _____, so $x =$ ________

16.4 **Applying to the Original Equation**: Solve $-2\ln(3x) = 5$.

a) Divide both sides to isolate the logarithm: $\ln(3x) =$ ________

b) Convert to exponential form: $3x = e^{\_\_\_\_\_\_}$

c) Solve for x: $x = \frac{e^{\_\_\_\_\_\_}}{3} \approx$ ________. Compare to the choices.

Scaffolded Question for Assessment Item 24: Properties of Logarithms

The original question asks to explain steps to solve $\log x + \log x^4 = 10$ using logarithm properties. The following questions build understanding of logarithm properties.

24.1 **Logarithm Properties**: Use properties to rewrite:

a) $\log(3 \cdot 4) = \log 3 + \log 4$ (Product Property)

b) $\log(x^2) =$ ________ (Power Property)

c) $\log\left(\frac{x}{y}\right) =$ ______________ (Quotient Property)

d) Why do these properties work? ________________________________ (Hint: Relate to exponent rules)

24.2 **Combining Logarithms**: Combine using properties:

a) $\log 2 + \log 5 = \log(2 \cdot 5) = \log 10$

b) $\log x + \log x^2 = \log(x \cdot x^2) = \log x^3$

c) Practice: $\log 3 + \log x^3 =$ ___________.

24.3 **Solving Logarithmic Equations**: Solve:

a) $\log x + \log x^3 = 8$:
Combine: $\log(x \cdot x^3) = \log x^4 = 8$.

Power (alternative after combining): If $\log M = N$, then $M = 10^N$. So $x^4 = 10^8$. Solve for x: $x = (10^8)^{1/4} = 10^2 = 100$.
Or using power property first: $4 \log x = 8$, so $\log x = 2$.
$x = 10^2 = 100$.

b) Practice: $\log x + \log x^2 = 6$: $x =$ _____.

24.4 **Applying to the Original Problem**: Solve $\log x + \log x^4 = 10$:

a) Combine: $\log(x \cdot x^4) = \log x^5 = 10$ (Property used: __________).

b) Simplify: $5 \log x = 10$ (Property used: __________).

c) Solve: $\log x = 2$, $x = 10^2 = 100$.

d) Verify: $\log 100 + \log 100^4 = \log 100 + \log(10^2)^4 = \log 10^2 + \log 10^8 = 2 + 8 = 10$.

Scaffolded Question for Assessment Item 30: Exponential Functions and Growth Factors (Hypothetical based on graph)

The assumed question asks to compare the growth factor of f (points $(0, 4)$, $(1, 12)$, $(-1, \frac{4}{3})$) to other functions. The following questions build understanding of growth factors.

30.1 **Growth Factors**: For $f(x) = ab^x$, b is the growth factor:

a) $f(x) = 2 \cdot 4^x$: $b = 4$

b) $f(x) = 5 \cdot (0.8)^x$: $b =$ _____

c) Why does $b > 1$ mean growth? ______________________________

30.2 **Finding Growth Factors**: For points $(0, 3)$, $(1, 9)$:

a) $f(0) = a \cdot b^0 = a = 3$

b) $f(1) = ab^1 = 3b = 9$, $b = 3$

c) Verify: If another point is $(2, 27)$, check: $f(2) = 3 \cdot 3^2 = 3 \cdot 9 = 27$. (Matches? ___)

30.3 **Comparing Growth Factors**: Compare:

a) $f(x) = 2 \cdot 5^x$, $g(x) = 3 \cdot 2^x$: 5 ¿ 2, so $f(x)$ grows faster.

b) $f(x) = 4^x$, $g(x) = 1.5^x$: _______ grows faster.

30.4 **Applying to the Original Problem**: Points $(0, 4)$, $(1, 12)$:

a) $a = 4$ (from $f(0) = 4$), $ab = 4b = 12$, $b = 3$. So $f(x) = 4 \cdot 3^x$.

b) Verify: $(-1, \frac{4}{3})$: $f(-1) = 4 \cdot 3^{-1} = 4 \cdot \frac{1}{3} = \frac{4}{3}$. (Matches? ___)

c) Compare its growth factor $b = 3$ to other functions' growth factors: For A: $a(x) = 3 \cdot 4^x \implies b = 4$. Greater than 3? ___ For B: $b(x) = 1.25^x \implies b = 1.25$. Greater than 3? ___ For C: $c(x) = (\frac{1}{12}) \cdot 12^x \implies b = 12$. Greater than 3?

___ For D: $d(x) = 12 \cdot (\frac{4}{3})^x \implies b = \frac{4}{3} \approx 1.33$. Greater than 3? ___ For E: $e(x) = (\frac{9}{16})^x \implies b = \frac{9}{16} \approx 0.56$. Greater than 3? ___

d) Select functions with growth factor greater than 3: __________.

Scaffolded Question for Assessment Item 31: Fractional Exponents and Radicals

The original question asks to complete a statement about $81^{\frac{1}{3}}$. The following questions build understanding of fractional exponents.

31.1 **Fractional Exponents**: $a^{\frac{1}{n}} = \sqrt[n]{a}$:

a) $16^{\frac{1}{2}} = \sqrt{16} = 4$

b) $64^{\frac{1}{3}} = \sqrt[3]{64} =$ _____

c) Why does $a^{\frac{1}{3}} = \sqrt[3]{a}$? ______________________________ (Hint: $(a^{1/3})^3 = a^1$)

31.2 **Exploring Bases**: For 64:

a) $64 = 4^3$, so $64^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = 4^{(3 \cdot \frac{1}{3})} = 4^1 = 4$

b) $64^{\frac{1}{2}} = \sqrt{64} =$ _____

31.3 **Verifying Exponents**: Verify $64^{\frac{1}{3}} = 4$:

a) $(64^{\frac{1}{3}})^3 = 64$, so $4^3 = 64$. (Is this true? ___)

b) Practice: Verify $16^{\frac{1}{2}} = 4$: $(16^{1/2})^2 = 16$, so $(\_\_\_)^2 = 16$. (Is this true? ___)

31.4 **Applying to the Original Problem**: For $81^{\frac{1}{3}}$:

a) $81 = 3^4$, so $81^{\frac{1}{3}} = \sqrt[3]{81} = \sqrt[3]{3^3 \cdot 3} = 3\sqrt[3]{3}$.

b) The question asks what $81^{1/3}$ is equivalent to, and what is the reason. Equivalent to (from choices): _______ Because (from choices): __________

Scaffolded Question for Assessment Item 33: Exponential Growth Models

The original question involves modeling Lucia's linear (12 residents/day) and Caleb's exponential (4 people, each contacting 4 more daily) growth. This focuses on the exponential part for Caleb.

33-Exp.1 **Exponential Models**: Exponential functions $f(x) = ab^x$ model multiplicative growth:

a) Triples daily, starts at 5: $f(x) = 5 \cdot 3^x$

b) Starts at 2, each contacts 3 more daily (meaning total becomes 4 times previous - original 1 + 3 more): Day 0: 2 (initial) Day 1: $2 \cdot 4 = 8$ Day 2: $8 \cdot 4 = 32$ This means the base $b = 4$. So $f(x) = 2 \cdot 4^x$. The question states "Caleb contacts 4 people on the first day. Those people will then contact 4 people the next day." This phrasing is a bit ambiguous. Interpretation 1: Caleb contacts 4 unique people on day 1. On day 2, THOSE 4 people each contact 4 MORE people (16 new people). Day 1 (x=1): 4 people contacted by Caleb. Total contacted = 4. Day 2 (x=2): The 4 from Day 1 each contact 4 more. $4 \times 4 = 16$ new people. Total contacted $= 4 + 16 = 20$. (This is not simple ab^x).

Interpretation 2 (More standard for these problems): Caleb's initial group is 4. Each person in the group then contacts 4 *new* people each day, and those new people become part of the group for the next day's contacting. Let $g(x)$ be the number of people contacted on day x. Day 1 ($x = 1$): Caleb contacts 4 people. Day 2 ($x = 2$): Those 4 people each contact 4 people. So $4 \times 4 = 16$ people are contacted on day 2. Day 3 ($x = 3$): Those 16 people each contact 4 people. So $16 \times 4 = 64$ people are contacted on day 3. This means $g(x) = 4^x$ is the number of people contacted *on day x*. The total number of people contacted *by* day x would be a geometric sum $4 + 16 + 64 + ... = \sum_{i=1}^{x} 4^i$. However, the problem says "Write a function that models the number of people contacted by both Lucia and Caleb after x days." This usually implies the *cumulative* number of people in Caleb's network (or people he has *caused* to be contacted).

Let's re-read "Caleb uses a different strategy. He contacts 4 people on the first day. Those people will then contact 4 people the next day. This pattern continues each day." If $g(x)$ is the *total number of people in Caleb's network* who have been contacted: End of Day 1 ($x = 1$): Caleb contacts 4. $g(1) = 4$. End of Day 2 ($x = 2$): The 4 from Day 1 each contact 4 people. $4 \times 4 = 16$ new. Total in network = 4(from day 1) + 16(new on day 2) = 20. This is not 4^x.

Let's consider the wording "a function that models the number of people contacted by ... Caleb after x days." If Caleb himself contacts 4 people (on day 1), and those 4 people contact 4 people (on day 2), etc., the number of *new* people contacted on day x is 4^x. The problem asks for $g(x)$ in $g(x) = \text{Lucia}(x) + \text{Caleb}(x)$ for "number of residents she/he contacts after x days". This seems to imply for Caleb $C(x)$ should be the total number of people reached by his method. If it means the number of *newly* contacted people on day x by Caleb's method, it's 4^x. If $g(x)$ in the question means $f(x)$ = Lucia's contribution + Caleb's contribution, and Caleb's contribution is the *number of people reached by his method by day x*, this is complicated. The fill-in-the-blank for $g(x)$ looks like

Original Assessment Questions

Question 7

Find the exact solution to $5e^{\frac{x}{2}} = 10$.

$$x = \ln(\boxed{})$$

Question 16

Solve the equation $-2\ln(3x) = 5$.

A. 0.082

B. 0.027

C. 4.061

D. 36.547

Question 24

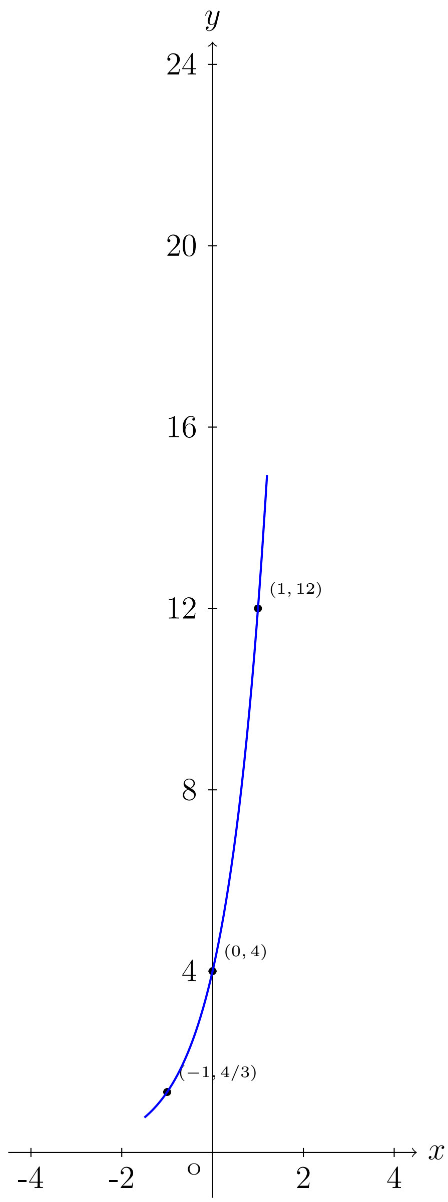
Explain each step used to solve the equation using the properties of logarithms.

$$\begin{aligned} \log x + \log x^4 &= 10 \quad (\boxed{\qquad\qquad} \text{ Property}) \\ \log x^5 &= 10 \quad (\boxed{\qquad\qquad} \text{ Property, or definition of log if } x^5 = 10^{10}) \\ 5\log x &= 10 \quad (\boxed{\qquad\qquad} \text{ Property}) \\ \log x &= 2 \\ x &= 100 \end{aligned}$$

(Students would typically drag/drop "Product", "Quotient", "Power" into the boxes. For the step $\log x^5 = 10 \rightarrow 5\log x = 10$, the property is Power. For $\log x + \log x^4 = 10 \rightarrow \log x^5 = 10$, the property is Product.)

Question 30

Function f is graphed below.



Select all the functions with a greater growth factor than f.

- ☒ $a(x) = 3 \cdot 4^x$
- ☒ $b(x) = 1.25^x$
- ☒ $c(x) = \left(\frac{1}{12}\right) \cdot 12^x$
- ☒ $d(x) = 12 \cdot \left(\frac{4}{3}\right)^x$
- ☒ $e(x) = \left(\frac{9}{16}\right)^x$

(Note: Replace ☒with ☐if you want empty boxes for students to fill)

Question 31

Complete the following sentence to make a true statement about the expression $81^{\frac{1}{3}}$.

- $81^{\frac{1}{3}}$ is equivalent to [⊠] $\sqrt[3]{81}$ [⊠] 3 [⊠] $\sqrt{81^3}$ [⊠] 2
- because [⊠] $9 = \sqrt{81}$ [⊠] $(\sqrt[3]{81})^3 = 81$ [⊠] $9^2 = 81$ [⊠] $\sqrt{81^3} = 1$

(Note: Replace ⊠with □if you want empty boxes for students to fill. The options here are presented as selectable items from the test image.)

Question 33 (Relevant Parts A and B)

Two community activists plan to contact local residents to urge them to vote for their preferred candidate for county sheriff.

Part A Lucía plans to contact 12 residents per day. Write a function that models the number of residents she contacts after x days. $f(x) = \boxed{}\,x$

Caleb uses a different strategy. He contacts 4 people on the first day. Those people will then contact 4 people the next day. This pattern continues each day. Write a function that models the number of people contacted by both Lucía and Caleb after x days. (This refers to the total number of people newly reached by their combined efforts on day x, or cumulative. Given the scaffold, 4^x is Caleb's contribution for newly contacted on day x). Let's assume the question implies $g(x)$ is the total new contacts on day x. $g(x) = \boxed{}\,x + \boxed{}^{\boxed{}}$ (The assessment form shows: $g(x) = \underline{\text{Space}}x + \underline{\text{Space}}^{\underline{\text{Space}}}$. Lucia's is $12x$. Caleb's is 4^x. So $g(x) = 12x + 4^x$.)

Part B Past experience shows that only 35% of people contacted will actually vote for their preferred candidate. Write a function that models the number of votes Lucía and Caleb can expect to gain for their candidate after x days. $h(x) = \boxed{}(\boxed{}\,x + \boxed{}^{\boxed{}})$ (This would be $h(x) = 0.35(12x + 4^x)$.)

If Lucía and Caleb start contacting people 7 days before the election, how many additional votes does the model predict they will gain for their candidate? Round to the nearest whole number. $\boxed{}$