ILEMT calibration accuracy report:

* Sensitivity of the pose solution to coupling error? Possible short paper in itself.

Project description, medical application, general open technology. Paper is somewhat general and tutorial. Not only can ILEMT hardware and software be used outside of the medical application, with different source and sensor configurations and workspaces, there is also a lack of specific and comprehensive literature on EMT calibration methods. Open source, pointer to repository.

Specific performance goals: speed, noise, workspace, accuracy

Evaluation: methods and results. Linearity and cross-coupling

Project summary:

The ILEMT (In-Loop ElectroMagnetic Tracker) has the specific aim of developing a tracker with adequate speed, resolution and accuracy to be used within a feedback system that stabilizes handheld surgical instruments. For this particular use, the primary focus is on measurement speed and low noise which is primarily due the (open) hardware design.

A magnetic position tracker measures the *pose* (position and orientation) of a *sensor* with respect to a magnetic *source*. [2D single axis drawing]

A single field measurement is not sufficient to determine the pose, so the source incorporates distinctly modulated electromagnet source coils, and the sensor often measures the local magnetic field along independent axes. Frequently , but other combinations are possible. For each pose, each source field experiences a characteristic magnetic coupling into each sensor axis, giving distinct couplings, which can be represented as the coupling matrix , where is the coupling from source axis into sensor axis .

We notate a pose as the linear transform which maps the source coordinate frame onto the sensor coordinates. [picture] This can be represented as a 4x4 linear homogenous transform matrix. Let be the *measurement function*: the actual coupling measured in pose . In practice, depends on magnetic interactions that are intractable to model, and kinematic constants which are not precisely known. Instead we resort to a *measurement model* This is a physical model (such as the ideal dipole) which is parameterized by . Calibration is the process of determining

During calibration we measure for many different known poses spread across the position and orientation workspace. Then a general nonlinear optimizer is used to determine the that minimizes:

Where the matrix 2-norm is the sum of element-wise squares:

This least-squares formulation permits us to use a specialized least squares solver (here Matlab lsqnonlin). These solvers give a much more robust convergence than general function minimization, especially when the derivatives of are unknown and the initial value for is inaccurate.

Magnetic tracking is an *inverse problem*. Once we have chosen a measurement model that we expect to approximate the true source/sensor coupling, it is straightforward to apply the physics to predict what coupling we should expect to see in a given sensor pose. But the physics does not tell us how to solve the inverse problem: given a measurement , what is ? Some measurement models permit a closed-form pose solution using linear algebra [cite], but in general it is again necessary to use nonlinear optimization, this time finding which minimizes .

One way to appreciate the challenge of measurement modelling is that we expect to achieve an accuracy which is much smaller than the size of the source and sensor. In the tested ILEMT configuration, the source is ??? mm, and the sensor ??? mm, while the specified accuracy is 200 µm. Due to the physics of magnetism, this is not as problematic is it seems. The magnetic fields drops off rapidly, , and with increasing the field also rapidly converges toward the dipole field of an ideal magnetic point source. At source to sensor distances greater than 10x the source size the dipole deviation is ???. In ILEMT, to minimize noise, the sensor distance is around 5x the source size, which makes higher order models more useful.

Some EMTs use other sensing principles and source modulations [cite Ascension, rotating field thing, etc], but we are concerned with the most common approach where the source coil has a sinusoidal drive voltage, and the sensor uses inductive pickup coils. Sometimes the source and sensor are identical, but the common practice is to make the source much larger, with a correspondingly increased drive current. Then the sensor can be made much smaller while still maintaining the same SNR. Since the source is much larger than the sensor, the dipole deviation of the source tends to dominate the error of the measurement model.

It may help intuitive understanding of the problem to realize that the source and sensor are magnetically interchangeable. Our actual measurement is the coupling between the source and sensor coils (mutual inductance), and this is a symmetrical relation. If it were possible to drive the sensor coil at high current, then the same small voltage would be induced in the source coil as was previously measured in the sensor. With this view, we can model the sensor as though it were a source, and in particular, use multi-dipole configurations such as the magnetic quadrupole. [Given weak value of high-order sensor models, not clear how important it is to have this in the paper. But it is part of what we have implemented, and lack of usefulness is somewhat interesting.]

Concentricity and orthogonality:

Frequently the source and sensor coils are wound around a common center (*concentric*), in a cube or sphere configuration. [picture] As well as being more compact than multiple separate coils, this also simplifies the pose solution problem [cite Kim18, and everyone back]. It might additionally be assumed that the coils are orthogonal, precisely aligned on the source and sensor coordinate axes.

But even if the coils are fabricated to approach the concentric orthogonal ideal, there is always some deviation, so accuracy can be improved by adding calibration parameters to model the coil position and orientation error. This is in part a tradeoff between making the coupling measurement obey a simple measurement model, versus adopting a more complex measurement model. But highly non-concentric coil arrangements may also be useful [cite planar source work], in which case we must model the actual coil configuration.

Ad-hoc pose correction:

When high accuracy is required over a well-defined workspace, it is possible to measure the error at many poses, and then apply a correction to the pose solution. This can be done via an interpolated lookup table. [cite Polhemus, VR work] One advantage of this is that it is possible to correct for environmental magnetic interference, but it can also correct intrinsic errors in the tracker calibration. The main shortcoming of the lookup table approach is that it requires a very large number measurements, and also can’t be extrapolated beyond the calibration volume.

In microsurgery the accuracy requirement is high, but the workspace is inherently constrained. We have found that the Direct Linear Transformation [cite] on the position significantly improves position accuracy.[reference result table] [Equation for transform here?] Since this is implemented by a 4x4 transform matrix, it is fairly low order, so does not require additional calibration data. The linear correction also has a stable extrapolation, and remains available beyond edge of the calibrated volume, degrading gradually.

Practical difficulties:

Pose representation:

In both the pose solution and in the calibration optimization the optimizer state includes 6DOF poses. The three translation DOF cause no problems, but for the orientation, many representations for 3DOF rotations (such as the rotation matrix and quaternions) have more than three elements. These excess degrees of freedom are normally eliminated by invariants such as orthonormality of the rotation matrix, but the optimizer does not know how to do this. Euler three-angle representations do not have excess degrees of freedom, but suffer from gimbal lock and numerical instability. We have had good results using a *rotation vector* representation during optimization. This is closely related to the axis-angle representation , where is the unit vector for the axis of rotation, and is the rotation in radians. The corresponding rotation vector is . This representation is then converted to a linear homogenous matrix when the linear transform must be applied.

Position and rotation, error and noise:

EMT performance has often been specified and evaluated primarily by the position error [cite], with fewer attempts to consider sensor angular error. While an EMT could be used in ways where rotation error would become particularly important, the most common application is measurement a tool tip position where the tool tip and sensor may be at opposite ends of the tool. The tool length creates a moment which converts angular error and noise into tip position error. With optical trackers, angular error can often dominate the tip position error [cite ASAP]. But an EMT measures position using a polar principle. For the sensor position with respect to the source, the tangential position error is larger than the radial error, but the angular error at the sensor is a similar magnitude to the source-referred angular error. This means when the tip offset moment is well less than the source to sensor distance, the tip position error due to sensor angle error is small compared to the sensor position error alone. This is a rarely mentioned benefit of EMTs, and should be considered when comparing to other technologies. [use moment in position error reports?] [picture of moment effect]

Positioning for calibration:

Calibration and accuracy testing require placing the sensor in many known poses, in both translation and rotation. This is our ground truth, and must have significantly higher accuracy than the expected tracker performance. The ILEMT microsurgical application is particularly demanding. For ILEMT, the accuracy as a fraction of the source to sensor distance specification is similar to other EMTs, but the workspace is smaller, so the absolute accuracy required for calibration is also more demanding. To characterize an accuracy of 200 µm we require a ground truth position accuracy of 20—70 µm. Expressed as an angular error at 200 mm, this is 100—350 microradians, or 0.006—0.020 degrees.

Compounding the difficulty, the sensor positioner must also be non-metallic to avoid interfering with the measurement. Specifically, any magnetic or highly conductive material must be sufficiently far away from the workspace. Fortunately this metallic interference drops as , so if the closest interferer is at 3x the source/sensor distance, then the interference will be reduced to .

One approach has been to use a nonmetallic manual positioning fixture [cite duplo, medical paper]. This procedure is tedious, which tends to limit the number of poses used. Rotation measurement has often been ignored, perhaps in part because of the combinatorial explosion created when rotation and translation are both varied.

For a credible performance evaluation, it is necessary to position the sensor in multiple rotations, as well as across the translation workspace. It can be possible to calibrate a low order measurement model using only translation data, but it then remains unclear what accuracy can be expected with the sensor is rotated. Unsurprisingly, we have found that calibrations done with translation only do perform worse when there is also angular motion.

For 3D translation and a single rotation axis, magnetic isolation can be achieved by placing the sensor on a nonmetal pole extending away from the large metal components in the positioning stage. [cite Polaris, Johnson & Johnson] The use of this pole does create stronger demands on the ideality of the stage translational motion, with minimal incidental rotation (stage runout).

We use this approach, and then achieve additional rotations by using manual fixtures to get multi-axis 90 degree rotations of the source and sensor. Given the primary concern for sensor position error (compared to rotation), the accuracy of the source rotation fixture is particularly important, since source rotation error is converted to sensor position error by the source to sensor moment. Broadly, rotation of the sensor helps to identify the sensor response, while rotations of the source identify the pattern of the source field. The source and sensor responses are independent, so it is not necessary to evaluate the full cross product of source and sensor rotations.

Fixture transforms:

When calibrating to a high accuracy, one challenge is that the calibration setup itself has unknown kinematic parameters. What exactly is the pose of the source and sensor with respect to the coordinate systems of our calibration fixtures? And what is the pose of the source fixture with respect to the sensor fixture?

EMTs have mainly been applied for measurement of relative motion, and neither the source nor the sensor have a precise mechanically defined coordinate system. That is, the accuracy is not defined with respect to any directly measurable marks or edges on the source and sensor. We might assume that the coordinates are nominally centered within the source or sensor, and aligned with mounting holes or other features, but we can’t mechanically measure to a point inside an object.

Fortunately we can evade this problem because any centering or alignment inaccuracy actually matters very little if our concern is purely with relative motion. We introduce additional *fixture transforms* which absorb the unknowns of the calibration setup. [picture of the full pose diagram] The calibration parameters are augmented with the fixture transforms, and the calibration optimization solves for these together with the parameters needed for the measurement model

[kinematic equation for desired pose given stage position]

Specific measurement model and parameters:

[Could include quadrupole if Sayan comes out first]

Our calibration code only supports configurations with three source axes and three sensor axes. In the dipole model, each coil has a 3DOF position and a 3DOF moment, giving 6 parameters per coil. The moment is a vector representing the orientation and magnitude of the axis response. [consistently use “coil” or “axis”?] The field is modelled as axially symmetric, so the coil pose is only characterized in 5DOF. The extra degree of freedom out of 6 is in the moment magnitude, which is a magnetic and electronic sensitivity parameter, not kinematics.

[dipole model equation and discussion here?]

[forward kinematics equations?]

It is convenient to combine the model parameters into four 3x3 matrices: source positions, source moments, sensor positions, sensor moments. Conceptually there are 36 parameters in these matrices, but we only implement ??? 29? of these. The fixture transforms add 12 parameters, giving 41 optimization variables in the general case. Less general measurement models can be calibrated by forcing fixed parameter values. For example, concentric calibration forces the coil positions to zero.

Augmenting the calibration state with the fixture transforms results in redundant degrees of freedom. Also, the source and sensor gains (moment magnitudes) are not independently measurable; we only observe the product of the two. We resolve this by forcing some parameters to fixed values. [Z moment and position, X moment Y component.]

Conclusion:

We have described in detail EMT calibration methods that can achieve [blah performance over blah workspace]. We expect that most of these methods have been used before; the point is to gather them in one place, establish that they work, and give some understanding of why they work. The primary research innovation is the characterization of tracker performance with greater precision and comprehensiveness than seen before. Characterization of tracker linearity and the rotation/translation cross coupling are particularly important for understanding the performance achievable during small motions.

[might go in related work] A huge number of magnetic trackers have been developed in the nearly 50 years that this technology has existed. EMTs have been a niche technology, with optical and computer vision techniques being more easily applied, often more accurate, and increasingly much less expensive due to the high production volume of cameras and wide availability of computer vision libraries. The niche of EMTs remains those uses where clear sightlines cannot be guaranteed.

But the use of EMTs is often not considered even when they might be appropriate because of the need to largely reinvent the technology each time. While the ILEMT tracker has specific aims for medical application, we hope that our open-source signal processing code and hardware designs will help more engineers succeed in applying EMTs across a broader scope. It is only the small size of the market that causes standalone EMTs to be so expensive.

[estimate of number of trackers, number of papers on technology development, number of application papers] it is a reasonable graduate student project to implement an EMT, and then develop refinements of signal processing or source and sensor configuration.