

# Week 1 Homework Probability Problems

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**a)**  $P(S \geq 7) = \frac{7}{36}$

Let  $o$  be the ordinary die. Let  $w$  be the weird die.

$$P(S \geq 7) = 2[P(o_6) \cdot P(w_1)] + [P(o_6) \cdot P(w_2)] + [P(o_5) \cdot P(w_2)] + [P(o_6) \cdot P(w_3)] + [P(o_5) \cdot P(w_3)] + [P(o_4) \cdot P(w_3)]$$

$$P(S \geq 7) = (\frac{1}{6} \cdot \frac{1}{3}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6})$$

$$P(S \geq 7) = \frac{7}{36}$$

**b)**  $\frac{4}{36}$

Let  $o$  be the ordinary die. Let  $w$  be the weird die. Let  $v$  be the value of a die.

$$P(o_v == w_v) = [P(o_1) \cdot P(w_1)] + [P(o_2) \cdot P(w_2)] + [P(o_3) \cdot P(w_3)]$$

$$P(o_v == w_v) = [\frac{1}{6} \cdot \frac{1}{3}] + [\frac{1}{6} \cdot \frac{1}{6}] + [\frac{1}{6} \cdot \frac{1}{6}]$$

$$P(o_v == w_v) = \frac{1}{18} + \frac{1}{36} + \frac{1}{36}$$

$$P(o_v == w_v) = \frac{4}{36}$$

**c)**  $P(w) = \frac{1}{20}$

**d)**  $P(w|0) = 1$

$$P(w|0) = \frac{P(0|w) \cdot P(w)}{P(0|w) \cdot P(w) + P(0|\neg w) \cdot P(\neg w)}$$

$$P(w|0) = \frac{\frac{1}{60}}{\frac{1}{60}+0}$$

$$P(w|0) = 1$$

$$\mathbf{e)} \ P(w|6) = 0$$

$$P(w|6) = \frac{P(6|w) \cdot P(w)}{P(6|w) \cdot P(w) + P(6|\neg w) \cdot P(\neg w)}$$

$$P(w|0) = \frac{\frac{1}{3} \cdot \frac{1}{20}}{\frac{1}{3} \cdot \frac{1}{20} + 0 \cdot \frac{19}{20}}$$

$$P(w|0) = \frac{0}{0 + \frac{19}{20}}$$

$$P(w|0) = 0$$

$$\mathbf{f)} \ P(w|3) = \frac{12}{50}$$

$$P(w|3) = \frac{P(3|w) \cdot P(w)}{P(3|w) \cdot P(w) + P(3|\neg w) \cdot P(\neg w)}$$

$$P(w|3) = \frac{\frac{1}{6} \cdot \frac{1}{20}}{\frac{1}{6} \cdot \frac{1}{20} + \frac{1}{6} \cdot \frac{19}{120}}$$

$$P(w|3) = \frac{\frac{1}{120}}{\frac{1}{120} + \frac{19}{720}}$$

$$P(w|3) = \frac{1}{1 + \frac{2280}{720}}$$

$$P(w|3) = \frac{720}{2280}$$

$$P(w|3) = \frac{720}{3000}$$

$$P(w|3) = \frac{12}{50}$$

$$\mathbf{g)} \ P(w|1) = \frac{2}{21}$$

$$P(w|1) = \frac{P(1|w) \cdot P(w)}{P(1|w) \cdot P(w) + P(1|\neg w) \cdot P(\neg w)}$$

$$P(w|1) = \frac{\frac{1}{3} \cdot \frac{1}{20}}{\frac{1}{3} \cdot \frac{1}{20} + \frac{1}{6} \cdot \frac{19}{20}}$$

$$P(w|1) = \frac{\frac{1}{60}}{\frac{1}{60} + \frac{19}{120}}$$

$$P(w|1) = \frac{1}{1 + \frac{1140}{120}}$$

$$P(w|1) = \frac{2}{21}$$

$$\mathbf{h)} \quad P(w) = \frac{1}{10}$$

We should use combinatorics here.

$$P(w) = \frac{\# \text{ of valid combinations}}{\# \text{ of total combinations}}$$

$$P(w) = \frac{19}{\frac{20!}{2! \cdot 18!}}$$

$$P(w) = \frac{19}{\frac{20 \cdot 19}{2}}$$

$$P(w) = \frac{38}{380}$$

$$P(w) = \frac{1}{10}$$

$$\mathbf{i)} \quad P(w|e) = \frac{2}{29}$$

Let  $w$  be the weird die. Let  $e$  be the event that the dice had the exact value exactly once in ten rolls.

We know that the probability of pulling out a weird die if you pull two at the same time is  $\frac{1}{10}$  from (h).

We also know from (b) that that if you have one regular die and one weird die the probability of rolling the same value once is  $\frac{4}{36}$ .

$$P(w|e) = \frac{\frac{4}{36} \cdot \frac{1}{10}}{\frac{4}{36} \cdot \frac{1}{10} + \frac{1}{6} \cdot \frac{9}{10}}$$

$$P(w|e) = \frac{\frac{4}{36} \cdot \frac{1}{10}}{\frac{4}{36} \cdot \frac{1}{10} + \frac{1}{6} \cdot \frac{9}{10}}$$

$$P(w|e) = \frac{\frac{4}{360}}{\frac{4}{360} + \frac{9}{60}}$$

$$P(w|e) = \frac{4}{4+54}$$

$$P(w|e) = \frac{2}{29}$$