Week 1 Homework Probability Problems

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a) $P(S \geq 7) = \frac{7}{36}$

Let o be the ordinary die. Let w be the weird die.

$P(S \geq 7) = 2[P(o_6) \cdot P(w_1)] + [P(o_6) \cdot P(w_2)] + [P(o_5) \cdot P(w_2)] + [P(o_6) \cdot P(w_3)] + [P(o_5) \cdot P(w_3)] + [P(o_4) \cdot P(w_3)]$

$P(S \geq 7) = (\frac{1}{6} \cdot \frac{1}{3}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6}) + (\frac{1}{6} \cdot \frac{1}{6})$

$P(S \geq 7) = \frac{7}{36}$

b) $\frac{4}{36}$

Let o be the ordinary die. Let w be the weird die. Let v be the value of a die.

$P(o_v == w_v) = [P(o_1) \cdot P(w_1)] + [P(o_2) \cdot P(w_2)] + [P(o_3) \cdot P(w_3)]$

$P(o_v == w_v) = [\frac{1}{6} \cdot \frac{1}{3}] + [\frac{1}{6} \cdot \frac{1}{6}] + [\frac{1}{6} \cdot \frac{1}{6}]$

$P(o_v == w_v) = \frac{1}{18} + \frac{1}{36} + \frac{1}{36}$

$P(o_v == w_v) = \frac{4}{36}$

c) $P(w) = \frac{1}{20}$

d) $P(w|0) = 1$

$P(w|0) = \frac{P(0|w) \cdot P(w)}{P(0|w \cdot P(w) + P(0|\neg w) \cdot P(\neg w)}$

$P(w|0) = \frac{\frac{1}{60}}{\frac{1}{60}+0}$

$P(w|0) = 1$

e) $P(w|6) = 0$

$P(w|6) = \frac{P(6|w) \cdot P(w)}{P(6|w) \cdot P(w) + P(6|\neg w \cdot P(\neg w)}$

$P(w|0) = \frac{\frac{1}{3} \cdot \frac{1}{20}}{\frac{1}{3} \cdot \frac{1}{20} + 0 \cdot \frac{19}{20}}$

$P(w|0) = \frac{0}{0+\frac{19}{20}}$

$P(w|0) = 0$

f) $P(w|3) = \frac{12}{50}$

$P(w|3) = \frac{P(3|w) \cdot P(w)}{P(3|w) \cdot P(w) + P(3|\neg w) \cdot P(\neg w)}$

$P(w|3) = \frac{\frac{1}{6} \cdot \frac{1}{20}}{\frac{1}{6} \cdot \frac{1}{20} + \frac{1}{6} \cdot \frac{19}{120}}$

$P(w|3) = \frac{\frac{1}{120}}{\frac{1}{120} + \frac{19}{720}}$

$P(w|3) = \frac{1}{1+\frac{2280}{720}}$

$P(w|3) = \frac{720}{\frac{720}{2280}}$

$P(w|3) = \frac{720}{3000}$

$P(w|3) = \frac{12}{50}$

g) $P(w|1) = \frac{2}{21}$

$P(w|1) = \frac{P(1|w) \cdot P(w)}{P(1|w) \cdot P(w) + P(1|\neg w) \cdot P(\neg w)}$

$P(w|1) = \frac{\frac{1}{3} \cdot \frac{1}{20}}{\frac{1}{3} \cdot \frac{1}{20} + \frac{1}{6} \cdot \frac{19}{20}}$

$P(w|1) = \frac{\frac{1}{60}}{\frac{1}{60} + \frac{19}{120}}$

$P(w|1) = \frac{1}{1+\frac{1140}{120}}$

$P(w|1) = \frac{2}{21}$

h) $P(w) = \frac{1}{10}$

We should use combinatorics here.

$P(w) = \frac{\#\ of\ valid\ combinations}{\#\ of\ total\ combinations}$

$P(w) = \frac{19}{\frac{20!}{2!\cdot 18!}}$

$P(w) = \frac{19}{\frac{20\cdot 19}{2}}$

$P(w) = \frac{38}{380}$

$P(w) = \frac{1}{10}$

i) $P(w|e) = \frac{2}{29}$

Let w be the weird die. Let e be the event that the dice had the exact value exactly once in ten rolls.

We know that the probability of pulling out a weird die if you pull two at the same time is $\frac{1}{10}$ from (h).

We also know from (b) that that if you have one regular die and one weird die the probability of rolling the same value once is $\frac{4}{36}$.

$P(w|e) = \frac{\frac{4}{36}\cdot\frac{1}{10}}{\frac{4}{36}\cdot\frac{1}{10}+\frac{1}{6}\cdot\frac{9}{10}}$

$P(w|e) = \frac{\frac{4}{36}\cdot\frac{1}{10}}{\frac{4}{36}\cdot\frac{1}{10}+\frac{1}{6}\cdot\frac{9}{10}}$

$P(w|e) = \frac{\frac{4}{360}}{\frac{4}{360}+\frac{9}{60}}$

$P(w|e) = \frac{4}{4+54}$

$P(w|e) = \frac{2}{29}$