Lesson 13. Facility Location

1 Today...

- Facility Location Introduction
- Two Variations of the Facility Location Problem
  - Set Covering Location Problem
  - Maximal Covering Location Problem

2 Facility Location Introduction

In these problems, the **input data** is...

- a network of *customers* (demand nodes),
- a set of **possible** *facilities* (supply nodes),
- a set of edges between customers and facilities that **could** serve them
- distances on the edges (which could represent distance, time, cost, or some combination of these factors)

The **goal** is to choose a set of supply facilities to serve the customers' demand based on some metric(s).

- For example: minimize the number of supply facilities opened while requiring that all customers are served.
- Real world problems of this type include locating
  - military installations,
  - fire/police stations,
  - cell phone towers,
  - retail distribution centers and stores,
  - schools.

3 *General* Facility Location Problems

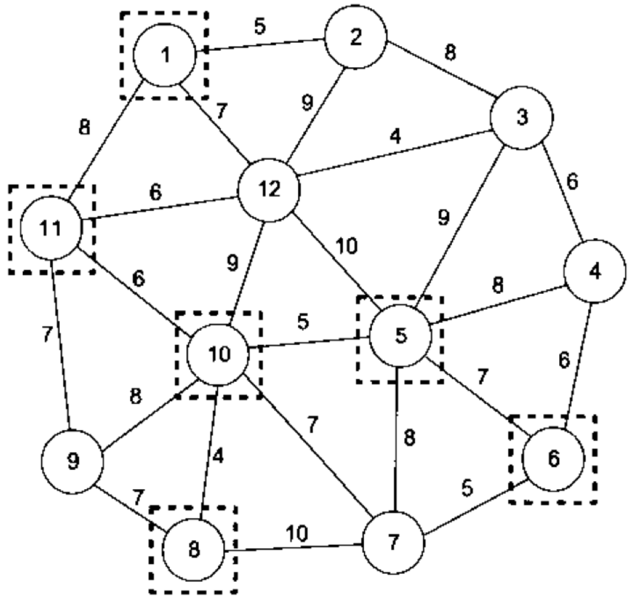

FIGURE 4.1 Example problem used for facility location models.

General Facility Location Problem goal:

Choose a set of supply facilities to meet customer demand according to some metric.

Notation:

Sets:

C = set of customer nodes

S = set of possible supply nodes

E = edges (c, s) connecting a customer c with a supply facility s that *could* serve the customer

Parameters:

d_{cs} = distance (or cost or time) between customer c and supply location s, for $(c, s) \in E$

h_c = demand of customer c, for $c \in C$

Decision Variables:

$$x_s = \begin{cases} 1 \text{ if } \boxed{} \\ 0 \text{ otherwise} \end{cases}, \text{ for all } s \in S$$

Problem 1. We will use the network and data on the following page for all of our example problems.

(a) *All* vertices represent customers. *Boxed* vertices represent possible supply locations. Use set notation to list the elements of the sets C and S.

(b) The distance between a customer c and a supplier s is the length of the shortest path between them. Find $d_{1,1}$, $d_{4,1}$, and $d_{8,5}$. (Note that the edges in the model do not correspond directly to the edges in the graph.)

(c) How should the columns and rows in the distance matrix be labeled? Do your answers in part (a) agree with the corresponding values in the distance matrix?

DATA for FACILITY LOCATION EXAMPLES:

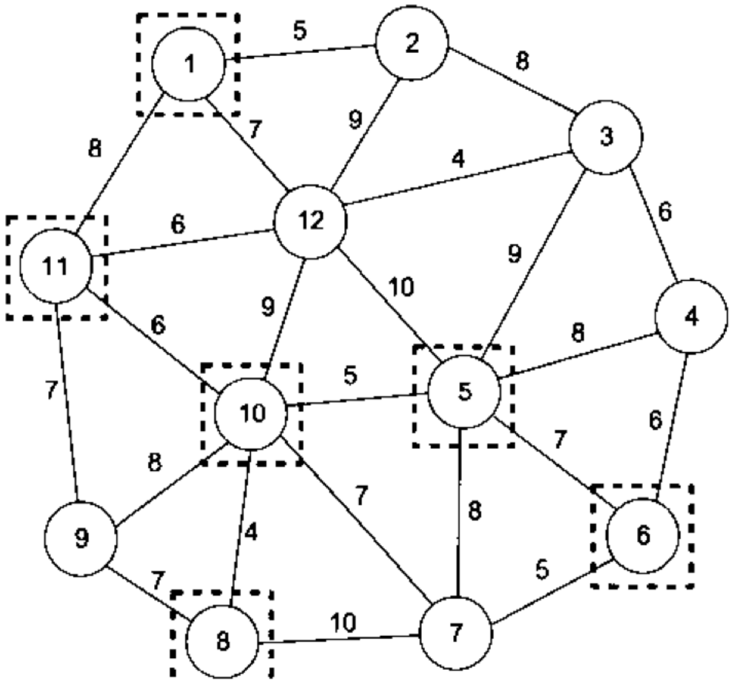

FIGURE 4.1 Example problem used for facility location models.

$$\mathbf{d} = \begin{bmatrix} 0 & 17 & 23 & 18 & 14 & 8 \\ 5 & 17 & 20 & 22 & 18 & 13 \\ 11 & 9 & 12 & 17 & 13 & 10 \\ 17 & 8 & 6 & 17 & 13 & 16 \\ 17 & 0 & 7 & 9 & 5 & 11 \\ 23 & 7 & 0 & 15 & 12 & 18 \\ 21 & 8 & 5 & 10 & 7 & 13 \\ 18 & 9 & 15 & 0 & 4 & 10 \\ 15 & 13 & 20 & 7 & 8 & 7 \\ 14 & 5 & 12 & 4 & 0 & 6 \\ 8 & 11 & 18 & 10 & 6 & 0 \\ 7 & 10 & 16 & 13 & 9 & 6 \end{bmatrix}.$$

$\mathbf{h} = (100,\ 90,\ 110,\ 120,\ 80,\ 100,\ 95,\ 75,\ 110,\ 90,\ 120,\ 85).$

4 *Set Covering* Facility Location Problem

Goal of the **set covering facility location problem**:

> Open the fewest number of facilities so that every customer is "covered" by some facility.

Problem 2. Complete the set covering facility location formulation below by adding the objective function description (1) and the missing constraint (2).

Set covering facility location model

***New* Sets:**

$N_c =$ the facilities in S that *could* serve customer $\forall c \in C$

($N_c \subseteq S$, for all $c \in C$. N_c is called the "neighborhood" of c.)

Objective and constraint descriptions:

(1) $\boxed{}$

(2) Ensure that customer c is covered by an open facility, for all $c \in C$.

$$\begin{aligned} \text{minimize} \quad & \sum_{s \in S} x_s && (1) \\ \text{subject to} \quad & \boxed{}, \ \text{for all } \boxed{} && (2) \\ & x_s \in \{0,1\}, \ \text{for } s \in S \end{aligned}$$

4.1 Example: Set Covering Facility Location (Neighborhoods determined by distance.)

Problem 3. Find the minimum number of facilities required to serve all customers. A facility must be within $D = 9$ miles of a customer in order to serve the customer.

(a) For each customer c, the neighborhood of N_c is the set of facilities that can cover customer c:

$$N_c = \left\{ s \in S : \boxed{} \right\}, \text{ for all } c \in C.$$

(b) Complete the missing neighborhoods.

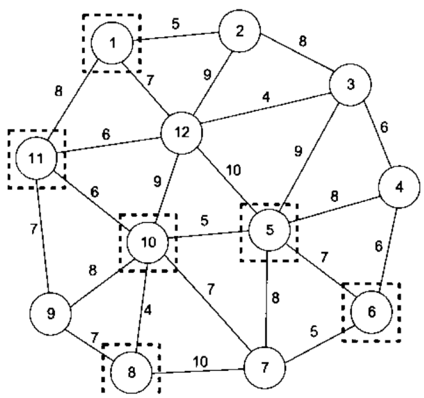

FIGURE 4.1 Example problem used for facility location models.

$N_1 = \{1, 11\}$

$N_2 = \{1\}$

$N_3 =$ $\boxed{}$

$N_4 = \{5, 6\}$

$N_5 = \{5, 6, 8, 10\}$

$N_6 = \{5, 6\}$

$N_7 = \{5, 6, 10\}$

$N_8 =$ $\boxed{}$

$N_9 = \{8, 10, 11\}$

$N_{10} =$ $\boxed{}$

$N_{11} = \{1, 10, 11\}$

$N_{12} =$ $\boxed{}$

(c) How many facilities are required? List the values of the decision variables that correspond to an optimal feasible solution.

(d) Write an abbreviated version of the concrete model.

5 *Maximal Covering* Location Problem

The goal of the **maximal covering location problem**:

> Select the p facilities to open in order to maximize the customer demand that is covered. (A customer, c, can only be covered by a supply facility, s, in its neighborhood: $s \in N_c$.)

Problem 4. Complete the maximal covering facility location formulation below by adding the objective function (3), the missing constraint (4), and the description for constraint (5).

Maximal covering facility location model

***New* Parameters:**

$h_c =$ the demand at customer c, for all $c \in C$

$p =$ the number of facilities to open

***New* Decision Variables:**

$$y_c = \begin{cases} 1 \text{ if a facility } s \text{ in the neighborhood of } c \text{ has been selected} \\ 0 \text{ otherwise} \end{cases}, \text{ for all } c \in C$$

Objective and constraint descriptions:

(3) Maximize the total customer demand that is covered.

(4) Ensure that exactly p facilities are opened.

(5) $\boxed{}$

$$\begin{aligned} \text{maximize} \quad & \boxed{} && (3) \\ \text{subject to} \quad & \boxed{} && (4) \\ & \sum_{s \in N_c} x_s \geq y_c, \text{ for } c \in C && (5) \\ & x_s \in \{0,1\}, \text{ for } s \in S \\ & y_c \in \{0,1\}, \text{ for } c \in C \end{aligned}$$

5.1 Example: Maximal Covering Facility Location Problem

Problem 5. We saw in the last example that every customer's demand can be met by three facilities. Suppose that we can only afford to build and maintain $p = 2$ facilities, and the demand values for the customers (in order) are

$$\mathbf{h} = (100, 90, 110, 120, 80, 100, 95, 75, 110, 90, 120, 85).$$

Opening which two facilities will allow us to cover the most total customer demand?

(a) The optimal solution is to choose facilities 5 and 11. List the values of the decision variables x_s and y_c in the optimal solution. Illustrate the solution on the graph of the network.

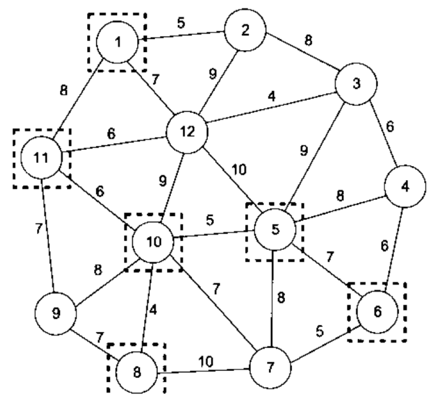

FIGURE 4.1 Example problem used for facility location models.

(b) Find the optimal objective function value. What does it mean?

(c) Write concrete versions of the following:

i objective function:

ii constraint (4):

iii constraints (5) for customers 4 and 8: