Rader 3.16
SA405

3.16: NOTE: The problem is INFEASIBLE for the data given in the text. Use due dates of 10, 15, 42, 31, 5, 22, and 32 for jobs 1 through 7, respectively. Let $\mathcal{J} = \{1, \ldots, 7\}$ denote the set of jobs. For a job $j \in \mathcal{J}$, let r_j denote the release date, d_j the due date, and p_j the duration (or process time). Finally, let $D = \max_j d_j$. We give two formulations

In this formulation, we use the variables:

$$S_j = \text{the start time of job } j, \text{ for all } j \in \mathcal{J} \text{ and} \qquad z = \text{the maximum completion time.}$$

We also use the binary variables:

$$x_{ij} = \begin{cases} 1 & \text{job } i \text{ starts before job } j \\ 0 & \text{otherwise.} \end{cases}$$

Then, the formulation is:

$$\begin{array}{lll l}
\min & z & & \\
\text{s.t.} & z \geq S_j + p_j & \forall j \in \mathcal{J} & \text{(a)} \\
& x_{ij} + x_{ji} = 1 & \forall i, j \in \mathcal{J}, i \neq j & \text{(b)} \\
& r_j \leq S_j \leq d_j - p_j & \forall j \in \mathcal{J} & \text{(c)} \\
& S_i + p_i - S_j \leq D(1 - x_{ij}) & \forall i, j \in \mathcal{J}, i \neq j & \text{(d)} \\
& x_{ij} \in \{0, 1\} & \forall i, j \in \mathcal{J}, i \neq j &
\end{array}$$

Constraints (a) ensure that z is the maximum completion time. Constraints (b) ensure that either job i completes before job j or vice versa (but not both). Constraints (c) ensure that the start time of a job is between the release date and the due date minus its duration. Constraints (d) ensure that if job i starts before job j, then the job j does not start until enough time has passed to complete job i.