RADER 3.2 AND 3.3
SA405

3.2

Indices

$m \in M$	machines, $M = \{1, 2, 3, 4, 5\}$
$j \in J$	jobs, $J = \{1, 2, 3, 4, 5\}$
$(m, j) \in P$	$P = \{(m, j) \vert m \text{ can do job } j\}$

Data

$setup_m$	setup time (in minutes) if machine m is used
$work_{m,j}$	time to complete job j on machine m (if applicable)
additional	extra 20 minutes needed if machines 1 and 2 are used

Binary Decision Variables

USE_m	1 if machine m is used, 0 otherwise
$OPERATE_{m,j}$	1 if machine m is used to do job j, 0 otherwise
$BOTH$	1 if both machines 1 and 2 are used, 0 otherwise

Formulation

$$
\begin{array}{llll}
\displaystyle\min_{USE, OPERATE, BOTH} & \displaystyle\sum_{(m,j)\in P} work_{m,j} OPERATE_{m,j} & \displaystyle + \sum_{m \in M} setup_m USE_m & + additional BOTH \\
\text{s.t.} & \displaystyle\sum_{\{m \mid (m,j)\in P\}} OPERATE_{m,j} \geq 1 & \forall j \in J & \text{(Job Completion)} \\
& \displaystyle\sum_{\{j \mid (m,j)\in P\}} OPERATE_{m,j} \leq 2 & \forall m \in M & \text{(Machine Utilization)} \\
& \displaystyle\sum_{\{j \mid (m,j)\in P\}} OPERATE_{m,j} \leq 5 USE_m & \forall m \in M & \text{(Can't operate unless do setup)} \\
& BOTH \leq USE_1 & & \text{(Logical Constraints on } BOTH \\
& BOTH \leq USE_2 & & \text{to ensure } BOTH = 1 \text{ if use} \\
& BOTH + 1 \geq USE_1 + USE_2 & & \text{machines 1 and 2)} \\
& BOTH, OPERATE_{m,j}, USE_m \in \{0, 1\} & \forall m \in M, \forall j \in J &
\end{array}
$$

3.3

Indices

$p \in P$	products, $P = \{1, 2, 3\}$
$l \in L$	lines, $L = \{1, 2, 3\}$
$\mathbb{N}$	natural numbers (including zero)

Data

$setup\_cost_l$	setup cost if line l used in given week
$worker\_cost_l$	pay per worker on line l
$production_{p,l}$	production per worker for product p on line l
$max\_workers$	maximum number of workers, 20 in this instance
$demand_p$	number of product p that must be produced

Integer Decision Variables

$WORKERS_{p,l}$	number of workers assigned to product p on line l

Binary Decision Variables

$OPEN_l$	1 if line l is open, 0 otherwise

Formulation

$$
\begin{array}{llll}
\min\limits_{WORKERS, OPEN} & \sum\limits_{l \in L, p \in P} worker\_cost_l WORKERS_{p,l} & + \sum\limits_{l \in L} setup\_cost_l OPEN_l & \\
\text{s.t.} & \sum\limits_{p \in P} WORKERS_{p,l} \leq max\_workers OPEN_l & \forall l \in L & \text{(No workers unless line open)} \\
& \sum\limits_{l \in L} production_{p,l} WORKERS_{p,l} \geq demand_p & \forall p \in P & \text{(Demand constraint)} \\
& \sum\limits_{l \in L, p \in P} WORKERS_{p,l} \leq max\_workers & & \text{(Total workers constraint)} \\
& WORKERS_{p,l} \in \mathbb{N} & \forall p \in P, l \in L & \text{(integer constraint)} \\
& OPEN_l \in \{0, 1\} & \forall l \in L & \text{(binary constraint)}
\end{array}
$$