

Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 1

Name: _____

A medical company makes small, medium, and large heart valves. Each valve requires a certain amount of superconductors, tissue, and labor as detailed below:

Item	Superconductors (units)	Tissue (g)	Labor (hours)
Small	3	5	3
Medium	4	2	6
Large	6	12	7

The company must make at least 10 small, 10 medium, and 5 large valves. Additionally, they have at most 200 superconductors, 300 g of tissue, and 450 hours of labor. Lastly, they make a profit of \$200, \$350, and \$475 for each small, medium, and large valve, respectively. They formulate the following LP to model their situation:

Decision Variables:

Let x_s be the number of small valves made

Let x_m be the number of medium valves made

Let x_l be the number of large valves made

Objective function:

$$\text{max profit: } 200x_s + 350x_m + 475x_l$$

Constraints:

$$3x_s + 4x_m + 6x_l \leq 200$$

$$5x_s + 2x_m + 12x_l \leq 300$$

$$3x_s + 6x_m + 7x_l \leq 450$$

$$x_s \geq 10$$

$$x_m \geq 10$$

$$x_l \geq 5$$

$$x_s, x_m, x_l \geq 0$$

Part 1: (80 points) Convert this model into a parameterized model that is **as compact as possible**. *Hint: Your final model should have 3 total constraints including non-negativity.*

Sets

Let V be the set of valves $V = \{s, m, l\}$

Let R be the set of resource $R = \{\text{superconductor, tissue, labor}\}$

Variables

Let x_i be the number of valves of type i made for all $i \in V$

Parameters

Let p_i be the profit of valve i for all $i \in V$

Let d_i be the demand of valve i for all $i \in V$

Let $a_{i,j}$ be the amount of resource j that valve i uses for all $i \in V$ and $j \in R$

Let u_j be the max amount of resource j available for all $j \in R$

Objective

$$\text{max profit: } \sum_{i \in V} c_i x_i$$

Constraints

$$\begin{aligned} \sum_{i \in V} a_{i,j} x_i &\leq u_j && \text{for all } j \in R \\ x_i &\geq d_i && \text{for all } i \in V \\ x_i &\geq 0 && \text{for all } i \in V \end{aligned}$$

Part 2: (20 points) Given sets I and J , parameter u_i for all $i \in I$ and variables $x_{i,j}$ for all $i \in I$ and $j \in J$, indicate if the following parameterized constraints are valid or invalid in an LP model.

- $\sum_{i \in I} \sum_{j \in J} x_{i,j} \leq 10$ for all $j \in J$ VALID INVALID

Invalid, j is defined twice

- $\sum_{i \in I} x_{i,j} = 4$ for all $j \in J$ VALID INVALID

Valid

- $\sum_{i \in I} \sum_{j \in J} u_i x_{i,j} \leq 1000$ VALID INVALID

Valid