SA405 – AMP COMPLETED Lesson #1

Practice Problem #1: Mathematical Modeling Review

Annapolis Coffee Roastery is trying to decide their fall coffee roasting line up. They have five different coffee roasts they can sell: Ethiopian, Colombian, Hawaiian, Kenyan, and Mexican. Their warehouse can store up to 1,800 pounds of beans, and their roasting machine roasts beans in exactly 100 lb. batches. Ethiopian beans can be sold for \$12 per pound, Colombian for \$20 per pound, Hawaiian for \$10 per pound, Kenyan for \$18 per pound, and Mexican for \$14 per pound. Annapolis Coffee Roastery wants to make sure that they roast atleast 200 lbs. and no more than 800 lbs. of coffee beans from Africa and atleast 100 lbs. and no more than 1,200 lbs. of coffee beans from North America.

to be put into storage

mainland

Also, it costs Annapolis Coffee Roastery \$100 to roast a single batch of coffee

1 Concrete Model:

Formulate the coffee roasting problem above as a **concrete** mathematical programming model to maximize Annapolis Coffee Roastery's profits. Clearly define and describe all decision variables, constrints, and the objective.

Variables

x_E, x_C, x_H, x_K, x_M

# of 100-lb batches of coffee that Annapolis Coffee Roastery roasts.

Constraints

- Must roast coffee in 100-lb batches
- Cannot exceed 1,800 total lbs of beans
- Must have at least 100 lbs and no more than 800 lbs of African coffee beans
- Must have at least 100 lbs and no more than 1,200 lbs. of North American coffee beans.

Objective

Maximize Annapolis Coffee Roastery's Total Profit

Concrete Model

$$\text{Maximize } 1200x_E + 2000x_C + 1000x_H + 1800x_K + 1400x_M - 100(x_E + x_C + x_H + x_K + x_M)$$

$$\text{s.t. } x_E + x_C + x_H + x_K + x_M \leq 18$$

$$2 \leq x_E + x_K \leq 8$$

$$1 \leq x_C + x_M \leq 12$$

$$x_E, x_C, x_H, x_K, x_M \geq 0$$

2 Abstract Model:

Formulate the coffee roasting problem as an **abstract** mathematical programming model. Clearly define and describe all sets, parameters, and decision variables.

SETS

B := Types of Coffe Beans
A := Set of African Beans
N := Set of Mainland North American Beans

Parameters

→ T := Total amount of Capacity

→ l_a, u_a := the lower and upper bounds on the # of batches of African beans to roast.

→ l_N, u_N := the lower and upper bounds on the # of batches of North American beans to roast.

→ C := Cost to roast a batch of coffee

$P_i \ \forall i \in B$:= The profit of each lb. of coffee bean $i \in B$.

→ b := the # of lbs. in a batch of roasting.

Decision Variables

$x_i \in \mathbb{Z}$:= The # of 100 lb batches of each coffee bean type to roast.

ABSTRACT MODEL

Maximize $\quad b \sum_{i \in B} P_i x_i - C \sum_{i \in B} x_i$

s.t. $\quad l_a \leq \sum_{i \in A} x_i \leq u_a$

$l_N \leq \sum_{i \in N} x_i \leq u_N$

$\sum_{i \in B} x_i \leq T$

$x_i \in \mathbb{Z}^+ \rightarrow$ Non-negative, Integer