SA405, Practice Test 1 September 21, 2017

INSTRUCTOR: PROF. TRAVES

Name (please print): ________________________

Instructions:

- Do **not** write your name on each page, only write your name above.
- No books or notes are allowed.
- You may use your calculator on this test.
- Show all work clearly. (Little or no credit will be given for a numerical answer without the correct accompanying work. Partial credit is given where appropriate.)
- If you need more space than is provided, use the back of the previous page.
- Please read the question carefully. If you are not sure what a question is asking, ask for clarification.
- If you start over on a problem, please CLEARLY indicate what your final answer is, along with its accompanying work.
- All formulations must have descriptions of any indices, parameters, and decision variables used. All constraints must be described.

Problem	Points	Score
1	15	
2	20	
3	20	
Total	55	

1. (15 points) Consider the following **MINIMIZING**, canonical form linear program, labeled (P):

$$
\begin{array}{lll}
\min & 2x_1 \qquad - 3x_3 + 18x_4 & \\
\text{s.t.} & x_1 - x_2 + 2x_3 + x_4 & = 4 \\
& x_1 + x_2 \qquad + 3x_4 & = 2 \\
& x_1, x_2, x_3, x_4 \geq 0 &
\end{array} \tag{P}
$$

(a) (*6 points*) Assume that x_1 and x_3 are basic. Solve for the current basic feasible solution.

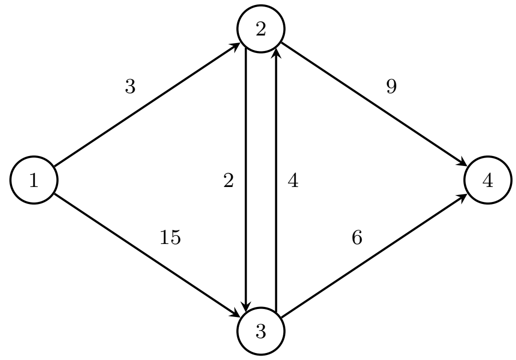
(b) (*3 points*) Given the feasible direction $d^{x_2} = [-1, 1, 1, 0]^T$ associated with the basis $\mathcal{B} = \{x_1, x_3\}$, determine whether or not it is an improving direction. Based on your answer about whether the direction is improving, state whether the simplex algorithm would continue or terminate at this point. **Briefly** explain your answer for full credit. You may find it helpful to recall that $\bar{c}_k = c^T d^k = c_k + \sum_{i \in \mathcal{B}} c_i d_i^k$.

(c) (*3 points*) For the simplex direction $d^{x_2} = [-1, 1, 1, 0]^T$ associated with the basis $\mathcal{B} = \{x_1, x_3\}$, use the ratio test to find the maximum step size, λ. Based on your answer about λ, which variable will leave the basis and become nonbasic? You may find it helpful to recall that

$$\lambda_{max} = \min \left\{ \frac{x_j}{-d_j^k} : d_j^k < 0 \right\}$$

(d) (*3 points*) Use your answers from parts a and c above to compute the new solution generated by this iteration of the Simplex Method.

2. (20 points) Consider the directed network shown below, where the numbers on the arcs represent cost, c_{ij}, to send one unit of flow along the arc. Use the start of a formulation given below to answer the following questions.



<u>Indices</u>

$i \in N$	nodes, $N = \{1, 2, 3, 4\}$
$(i, j) \in A$	arcs

<u>Data</u>

c_{ij}	cost to flow one unit (defined on figure above)

<u>Decision Variables</u>

X_{ij}	Amount of flow on arc (i, j)

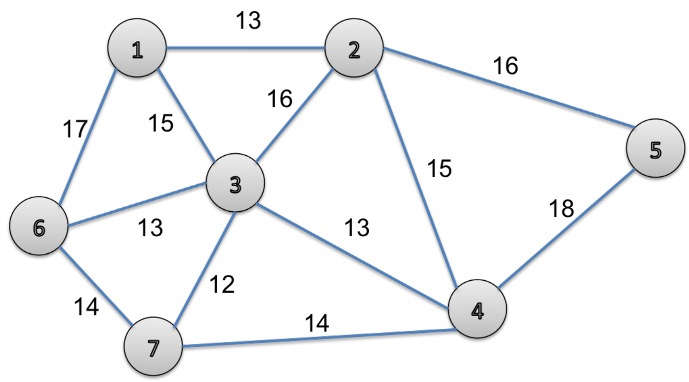
(a) (*6 points*) We place one unit of supply at node 1, 1 unit of demand at node 4 and zero units of supply at all other nodes. Formulate an objective function to compute the minimum cost network flow.

(b) (*4 points*) Label all four nodes in the network diagram above with their "net supply" or b_i value.

(c) (*6 points*) Write the balance of flow constraint for node 3.

(d) (*4 points*) Find the optimal objective value by inspection, and write the corresponding values of the decision variables for this solution.

3. (20 points) US NorthWest plans to install fiber in a metro area network that is expected to experience increased demand due to the opening of a large manufacturing facility. The Central Offices (COs) in the network are represented by vertices in the graph below. The edges in the graph represent the possible fiber paths linking the COs. The number on edge (i, j) represents the cost c_{ij} (in \$1000) of installing fiber between COs i and j.



Network planners developed the following integer program to solve for the collection of edges that will minimize the cost of connecting all the central offices in the network via a fiber spanning tree. (Let V represent the set of vertices in the graph. Let E represent the set of edges in the graph.)

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E} c_{ij} X_{ij} & & \\
\text{s.t.} \quad & \sum_{j:(i,j)\in E} X_{ij} + \sum_{j:(j,i)\in E} X_{ji} \geq 1, \quad \forall i \in V & & \text{(a)} \\
& \sum_{(i,j)\in E} X_{ij} = \ ? & & \text{(b)} \\
& \sum_{\substack{i,j\in V' \\ (i,j)\in E}} X_{ij} \leq |V'| - 1 \qquad \forall V' \subseteq V & & \text{(c)} \\
& X_{ij} \in \{0,1\} \qquad \forall (i,j) \in E. & &
\end{aligned}
\qquad \text{(MST)}
$$

(a) (*4 points*) Write down the constraint of type (a) from (MST) for vertex 4.

(b) (*2 points*) What GUSEK code would implement the objective function? Assume that c, X and E have already been defined for you.

(c) (*6 points*) Architect Ima Klutz spilled cappuccino on the formulation rendering the right hand side of constraint (b) illegible. What is the purpose of constraint (b), and what should the missing number be?

(d) (*4 points*) The modeling team learns that city ordinances require that if link $(3, 7)$ is built, then neither $(1, 2)$ nor $(3, 4)$ may be built. Write a constraint to model this new requirement.

(e) (*4 points*) The Central Office represented by vertex 3 is centrally located, but the equipment there is outdated. If vertex 3 is used as a hub, meaning three or more fiber paths meet at vertex 3, the Central Office there will require a $25,000 upgrade. The modeling team adds a new binary variable Y that indicates whether or not vertex 3 is used as a hub in the network design. They modify the objective function by adding the term $25 * Y$. Write a constraint that forces Y to be 1 if three or more selected edges connect to vertex 3.