

1. A hospital ER needs to keep doctors on call, so that a qualified individual is available to perform every medical operation that might be required (there is an official list of such procedures). For each of the five doctors available for on-call duty, the additional salary they need to be paid (in thousands of \$), and which operations they can perform, is known and provided below. The goal is to choose doctors so that each operation is covered, at a minimum cost.

	Doc 1 (\$1)	Doc 2 (\$3)	Doc 3 (\$2)	Doc 4 (\$3)	Doc 5 (\$4)	Doc 6 (\$2)
Op 1	✓			✓		
Op 2	✓				✓	
Op 3		✓	✓			
Op 4	✓					✓
Op 5		✓	✓			✓
Op 6		✓				

- (a) Clearly define all variables, parameters, and the objective function. Formulate the above problem as a concrete integer programming model.
- (b) Does this integer programming model include set-covering, set-packing, or set-partitioning constraints?

2. Vance Refrigeration has a factory located in Dallas, Texas, producing small refrigerators bound for their outlet store in Baltimore. All refrigerators are shipped from the company to Baltimore via train. The train operates trains daily on the national rail network. The train cars are packed in large boxes, there exists a limit on the number of boxes that can be shipped from one city to another. The table below shows these limits. If a “–” is contained within a cell below, then we assume that refrigerators cannot be shipped along that arc.

	To				
From	Dallas	Little Rock	Richmond	Atlanta	Baltimore
Dallas	–	20	–	12	–
Little Rock	–	–	18	16	–
Richmond	–	–	–	8	13
Atlanta	–	–	18	–	17
Baltimore	–	–	11	–	–

- (a) Draw a picture of this network with the nodes labeled as the name of each city and the arcs labeled with the maximum capacity on that arc.

- (b) Formulate the concrete model for maximizing the number of refrigerators shipped from Dallas to Baltimore. Clearly define your parameters, variables, constraints, and objective function.

(c) The CEO of Vance Refrigeration told you that, if you send refrigerators from Dallas to Little Rock, then you must send at least 10 units but no more than 20 units. What variables and constraints should you add to your concrete model to enforce this constraint?

(d) The CEO of Vance Refrigeration keeps changing their mind. She realizes that their warehouse in the city of Little Rock must now pay taxes to ship refrigerators out of the city. This warehouse currently has \$20 million for paying these taxes, and these taxes will cost them \$0.5 million and \$1 million to ship a single refrigerator to Richmond and Atlanta, respectively. What parameters and constraints should you add to your concrete model to enforce this constraint?

3. The Superintendent asks his most trusted Ops Research majors (you all) to give him directions from the Yard to Los Angeles, CA. Of course, he wants to find the path the minimizes his total travel time. Assume that he can travel through a set of cities \mathcal{C} along a set of roads \mathcal{A} between each city. He assumes that traveling along some arc $(i, j) \in \mathcal{A}$ will require $t_{ij} > 0$ minutes.
- (a) Formulate an abstract mathematical programming model to help the superintendent solve his problem.
 - (b) The superintendent now wants to consider traffic delays in each city. To account for this added time it takes to travel through each city, he assumes that it will take $d_i > 0$ extra units of time to travel through city $i \in \mathcal{C}$. Modify your mathematical programming model above to account for the extra traffic-related time delays.

4. Trader Bill's Clothing Company is capable of manufacturing three types of attire: shirts, pants, and jackets. The machinery needed to manufacture each type of clothing must be rented at the following rates: shirt machinery, \$200 per week; pants machinery, \$150 per week; and jacket machinery, \$100 per week. The manufacturer of each type of clothing also requires the amounts of cloth and labor shown in Table 4. Additionally, Table 4 includes the maximum number of each type of clothing that Trader Bill's can manufacture. Each week, 150 hours of labor and 160 sq yd of cloth are available. The variable unit cost and selling price for each type of clothing are shown in Table 4.

Clothing Type	Labor in Hours	Cloth in Sq. Yards	Maximum Allowable
shirt	3	4	100
pants	2	3	220
jacket	6	4	180

Clothing Type	Sales Price in \$	Variable Cost in \$
shirt	12	6
shorts	8	4
sweater	15	8

- (a) Formulate a concrete integer programming model whose solution will maximize Trader Bill's weekly profits.

- (b) Trader Bill now tells you that if you are able sell 50 shirts then he will give you an extra bonus of \$100. Modify your integer programming model to include this bonus.

Maximum-flow Formulation for a given source/origin node s and a termination/destination node t .