Lectures 1: Mathematical Modeling Review

1 Goals

- Formulate a concrete linear programming model.
- Introduce an integrality requirement on variables.
- Convert the linear program to parameterized form.

2 Review of SA305 Formulations

The four components of formulating an optimization model are:

1.

2.

3.

4.

What are the 3 assumptions/characteristics that make an optimization model a linear program?

1.

2.

3.

How does an **integer program** differ from a linear program?

This is the key difference between SA405 and SA305. It seems like a very simple difference, but integer programming allows you to model a much wider variety of problems than LP does. Moreover, because of this seemingly small difference, the math behind integer programming is significantly more complicated than the math behind LP (indeed, solving a general LP is known to be a "easy" problem while solving an IP in general is known to be a "hard" problem).

3 Concrete Model

Chelsea is heading out on a camping trip, and she wants to carry only one pack that has 5.3 ft^3 of volumetric space. To keep from hurting her back, she needs to make sure that the contents of her backpack weighs no more than 12.5 lbs. You can assume the backpack weight is negligible. See the list of items that she is able to bring:

ID	Item	Volume (ft^3)	Usefulness Factor	Weight (lbs.)
1	Rope	2	1	3
2	Matches	0.01	5	0.1
3	Tent	3	7	10
4	Sleeping bag	2	6	4
5	Hammock	0.4	4.5	4
6	Granola bars	0.67	8	2

Problem 1. Write a concrete linear program whose solution maximizes the usefulness of the contents of Chelsea's bag given volume and weight requirements.

a) Define decision variables and then describe (in words) the objective function and the role of each constraint.

b) Write the concrete model.

Parameters

Decision Variables

Objective Function

Constraints

4 Understanding Integrality Restrictions

- **Continuous Linear Program (LP):** Suppose that Chelsea is allowed to bring fractional amounts of each item, so that *variables can take on any nonnegative values.* Let z be the optimal objective function value to this problem.

- **Integer Linear Program (IP):** Suppose that Chelsea can either bring the entire item or not, so that *variable values are restricted to 0 or 1.* Let $\bar{z}$ be the optimal objective function value to this problem.

Problem 2. How does z compare to $\bar{z}$? Provide justification for your response.

5 Convert to Parameterized Models

Problem 3. Assuming integrality restrictions, convert your model to a parameterized model. Clearly define all sets, parameters, and decision variables.

Parameters

Decision Variables

Objective Function

Constraints