Completed RMC

Practice Problem #9: Fixed-Charge Problem

1 The Sabre Problem

In 2015, the printer manufacturer Sabre has set up shop creating printers, copiers, and scanners to celebrate its 10th anniversary. Their machinist makes the designs each machine out of plastic and metal. To begin manufacturing each machine, Sabre must pay a significant set up cost. All relevant data can be found in the table below.

	Printers	Copiers	Scanners	Availability
Machinist Labor (Days)	2	4	5	100
Plastic (pounds)	1	1.5	1.8	30
Metal (pounds)	1	1.5	1.8	30
Profit ($ per machine)	52	30	20	–
Set-up Costs ($)	500	400	300	–

1.1 Concrete Model

Formulate Sabre's problem as a **concrete** integer programming model to maximize the total amount of profit. Define and describe all restrictions, the objective, and all decision variable(s).

Restrictions
- must pay set-up costs to manufacture a machine
- Cannot exceed the maximum availability of Labor, Plastic, and Metal.
- Non-negativity and Integers

Objectives

Maximize Total Profit

Decision Variables

- $x_p, x_c, x_s \Rightarrow$ the # of printers, copiers, and scanners manufactured.
- $y_p, y_c, y_s \in \{0,1\}$ equals 1 if Sabre manufactures any number of printers, copiers, and scanners, respectively. And 0, otherwise

$$\text{Maximize } 52x_p + 30x_c + 20x_s - 500y_p - 400y_c - 300y_s$$

s.t.

$$2x_p + 4x_c + 5x_s \le 100 \quad (L)$$

$$x_p + 1.5x_c + 1.8x_s \le 30 \quad (P)$$

$$x_p + 1.5x_c + 1.8x_s \le 30 \quad (M)$$

minimum between $\frac{100}{2}, \frac{30}{1}, \frac{30}{1}$

$$30y_p \ge x_p$$

$$20y_c \ge x_c$$

$\lceil \frac{30}{1.8} \rceil$

$$17y_s \ge x_s$$

$$y_p, y_c, y_s \in \{0,1\}$$

$$x_p, x_c, x_s \ge 0$$

1.2 Abstract Model

Sabre has come back to you for help. They found your original model to be super useful! They want to quickly scale up their operation. They want to know how to solve their problem with N different types of machines and R different types of resources (e.g., labor, metal, plastic). Formulate Sabre's problem as an **abstract** integer programming model to maximize the total amount of profit. Define and describe all sets, parameters, and all decision variable(s).

Sets

N := Set of different types of machines Sabre can manufacture

R := Set of resources used to make different types of machines

Parameters

$a_{ij} \Rightarrow$ the amount of Resource $i \in R$ used to manufacture a unit of machine $j \in N$

$m_i \Rightarrow$ Maximum available amount of resource $i \in R$.

$p_j \Rightarrow$ the profit for selling a single unit of machine $j \in N$.

$s_j \Rightarrow$ the setup cost for manufacturing machine $j \in N$.

Decision Variables

$x_j \geq 0$, Integer $\forall j \in N \Rightarrow$ the # of machine $j \in N$ manufactured.

$y_j \in \{0,1\}$ $\forall j \in N \Rightarrow$ equals 1 if any # of machine j is manufactured.

$$\text{Maximize } \sum_{j \in N} p_j x_j - \sum_{j \in N} s_j y_j$$

s.t.

$$\sum_{j \in N} a_{ij} x_j \leq m_i \quad \forall i \in R$$

$$M y_j \geq x_j \quad \forall j \in N$$

or

$$\min\left\{\left\lceil \frac{m_i}{a_{ij}} \right\rceil : i \in R\right\} y_j \geq x_j \quad \forall j \in N$$

$$y_j \in \{0,1\} \quad \forall j \in N$$

$$x_j \geq 0, \text{Integer} \quad \forall j \in N$$

2 Set-covering Models

2.1 Question:

Briefly describe the difference between set-covering, -packing, and -partitioning constraints.

→ Set-covering ensures that the sets chosen "Cover" or include every element considered (≥ 1)

→ Set-packing tries to choose as many sets (≤ 1) as possible without covering any elements more than once.

→ Set-partitioning tries to max/min some objective ($= 1$) as long as each element is included in exactly one of the selected sets.

2.2 Problem:

USNA is organizing a yard-wide athletic decathlon. Assume you have a set of athletes in your company: Jamie, Daphne, Gary, and Jackie. Each athlete excels at least one sport. Jamie swims and plays basketball. Daphne plays squash and soccer. Gary plays basketball, squash, and croquet. Finally, Jackie swims, and plays basketball and squash. Your job is to hire a #squad of athletes. You are given two requirements: (i) there has to be at least one person on the team who plays each sport (i.e., swimming, basketball, soccer, squash, and croquet), and (ii) your team should be as small as possible (maybe your team is running on a tight budget).

Use a network diagram to better visualize this problem.

a. Formulate this problem as a **concrete** integer programming model. Clearly define and describe all restrictions, the objective, and all decision variable(s).

Restrictions

- At least one person can play each sport on the team
- Binary (We can either choose a player or not)

Objectives

Minimize the # of Members on the team

Decision Variables

$x_j, x_D, x_G, x_a \in \{0,1\}$ equals 1 if we choose Jamie, Daphne, Gary, or Jackie, respectively, on the team.

Concrete Model

Minimize $x_j + x_D + x_G + x_a$

s.t.

$x_j + x_a \geq 1$ (swimming)

$x_j + x_G + x_a \geq 1$ (basketball)

$x_D \geq 1$ (soccer)

$x_D + x_G + x_a \geq 1$ (squash)

$x_G \geq 1$ (croquet)

$x_j, x_D, x_G, x_a \in \{0,1\}$

b. Formulate this problem as an **abstract** integer programming model. Clearly define and describe all restrictions, the objective, and all decision variable(s).

SETS

$M :=$ Set of Midshipmen in your company

$S :=$ Set of Sports

$S_i \; \forall i \in S :=$ Set of midshipmen that can play sport $i \in S$.

Parameters

None

Decision Variables

$x_j \in \{0,1\} \; \forall j \in M$ equals 1 if midshipman $j \in M$ is chosen, and 0, otherwise.

$$\text{Minimize} \sum_{j \in M} x_j$$

$$\text{s.t.} \sum_{j \in M : i \in S_i} x_j \geq 1 \; \forall i \in S$$

$$x_j \in \{0,1\} \; \forall j \in M$$