Lesson 6: Fixed-Charge Facility Location

1 Today...

- We extend a min cost network flow model to a *fixed-charge facility location* model.
- This will require the use of *binary decision variables*.
- There are two well-known formulations for modeling the fixed-charge forcing constraints: the so-called *weak* and *strong* formulations.

2 Gotit Grocery

Gotit Grocery Company is considering 3 locations for new distribution centers to serve its customers in Maryland. The following table shows the fixed cost (in millions of dollars) of opening each potential center, the number (in thousands) of truckloads forecasted to be demanded at each city over the next 5 years, and the transporation cost (in millions of dollars) per thousand truckloads moved from each center location to each city.

	Fixed Cost	Transport Costs Annapolis	Ocean City	Laurel	Baltimore
Distribution Center 1	200	6	5	9	3
Distribution Center 2	400	4	3	5	6
Distribution Center 3	225	5	8	2	4
Demand	—	11	18	15	25

According to management, if Gotit does open a new distribution center, that center must send at least 10 thousand truckloads in order to be a worthwhile investment. Gotit seeks a minimum cost distribution system assuming any distribution center can meet any or all demands.

3 Concrete Model

Ignoring the facility opening costs/variables write a concrete model for this transportation problem. For your variables, let $x_{i,j}$ be the number of truckloads sent from distribution center i to city j.

4 Binary Decision Variables

The use of binary $\{0, 1\}$ variables to model yes/no decisions is very common. In this model, we use a binary decision variable for each potential distribution center that indicates whether or not it is opened:

a value of $\boxed{}$ indicates that the facility is used, and we must pay the "fixed-cost" to open it;

a value of $\boxed{}$ indicates that the facility is not used, and therefore we don't have to pay for it.

Whenever we use binary decision variables, we must include $\boxed{}$ that enforce the correct behavior of the variable in the context of the model. **This can require some thought, careful logic, and even creativity.**

1. Define three new decision variables, z_1, z_2, z_3, which encapsulate the logic described above.

2. Using these new decision variables, modify the objective function of your formulation to incorporate the fixed costs of opening the distribution centers.

5 Fixed-Charge Forcing Constraints

Explicitly, using the binary variables above, if $z_1 = 1$ then we are opening distribution center 1. However, implicitly, we must force the logic that if $z_1 = 0$ then:

Constraints that enforce this logic are called **forcing constraints**. There are two options for this, single variable OR multiple variable.

3. Write a constraint that enforces the logic that if $z_1 = 0$ then $x_{1,1}$ must also be zero.

This type of constraint is often referred to as a **strong forcing constraint**.

4. Write all of the strong forcing constraints needed for this model.

The next type of constraint is generally referred to as **weak forcing constraints**.

5. Write an inequality that enforces the logic that, if $z_1 = 0$ then $x_{1,1} = x_{1,2} = x_{1,3} = x_{1,4} = 0$.

6. Write all of the weak forcing constraints for this model.

Why are these referred to as weak and strong constraints?

6 Parameterized Model: Fixed-Charge Facility Location

$$\text{maximize} \quad \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \boxed{}$$

$$\text{subject to} \quad \sum_{i \in I} x_{ij} \geq d_j, \text{ for } j \in J$$

weak forcing constraints: (see next page)

$$\boxed{}$$

OR

strong forcing constraints: (see next page)

$$\boxed{}$$

lower bound constraints:

$$\boxed{}$$

$x_{ij} \geq 0$, integer, $\forall i \in I, j \in J$

$$\boxed{}$$