Quiz 3

The Pear Computer Company is trying to decide where to manufacture its upcoming phone. The phone will be available in three varieties, the jPhone 8, the jPhone 8 Plus, and the jPhone X. The phones can be manufactured at one of three different facilities: Closistan, Nearistan, and Faristan. There is an initial setup cost associated with using each facility. The cost for shipping the phones also varies by location. The following table summarizes the relevant data:

		C Closistan	N Nearistan	R Faristan
	Setup cost	\$400,000	\$300,000	\$200,000
8	Shipping Cost/jPhone 8	\$2	\$4	\$5
+	Shipping Cost/jPhone 8 Plus	\$3	\$6	\$9
X	Shipping Cost/jPhone X	\$7	\$8	\$10

Each month the Pear Computer Company predicts that 85,000 of the jPhone 8, 105,000 of the jPhone 8 Plus, and 150,000 of the jPhone X will need to be shipped from the manufacturing facilities. The logistics employee that you replaced did not finish the formulation, but did get it started. The LP formulation was supposed to minimize the monthly costs, while meeting the predicted shipping requirements. The following is what the logistics employee came up with so far:

Sets

P	phones, $P = \{$jPhone 8, jPhone 8 Plus, jPhone X$\}$ (8 + X)
F	Manufacturing Facilities, $F = \{$Closistan, Nearistan, Faristan$\}$ (C N R)

Data

S_f	setup cost if Manufacturing Facility f is used, for $f \in F$
$T_{p,f}$	transport cost to ship phone p from Manufacturing Facility f, for $p \in P, f \in F$
D_p	(demand) number of phones p that need to be shipped, for $p \in P$

Integer Decision Variables

$x_{p,f}$	number of phones p shipped from Manufacturing Facility f, for $p \in P, f \in F$

Binary Decision Variables

z_f	1 if Manufacturing Facility f is open, 0 otherwise, for $f \in F$

Use the start of the logistic employee's formulation to answer the complete the following elements of the model (first using no set notation, and then translating to set notation, as directed).

$x_{p,f}$

1. *(6 points)* Write an objective function that minimizes the **TOTAL** cost of meeting the monthly shipping requirements predicted by the Pear Computer Company using ONLY VARIABLES and NUMBERS (5 points). Express the objective function using sets, summations, and parameters (1 point).

$$\min\left(2x_{8,C} + 4x_{8,N} + 5x_{8,R} + 3x_{+,C} + 6x_{+,N} + 9x_{+,R} + 7x_{\mathbb{I},C} + 8x_{\mathbb{I},N} + 10x_{\mathbb{I},R}\right) + \left(400{,}000z_C + 300{,}000z_N + 200{,}000z_R\right)$$

$$\min\left[\left(\sum_{p\in P}\sum_{f\in F} T_{p,f}x_{pf}\right) + \left(\sum_{f\in F} S_f z_f\right)\right]$$

2. *(6 points)* Write a set of constraints that ensures that monthly demand requirements predicted by the Pear Computer Company are met using ONLY VARIABLES and NUMBERS (5 points). Express the same class of constraints using sets, summations, and parameters (1 point).

$$x_{8,C} + x_{8,N} + x_{8,R} \geq 85{,}000$$
$$x_{+,C} + x_{+,N} + x_{+,R} \geq 105{,}000$$
$$x_{\mathbb{I},C} + x_{\mathbb{I},N} + x_{\mathbb{I},R} \geq 150{,}000$$

$$\sum_{f\in F} x_{p,f} \geq D_p, \text{ for all } p \in P$$

Either weak or strong is correct.

3. *(6 points)* Write a set of constraints that enforces the correct behavior of the binary variables, z_f, using ONLY VARIABLES and NUMBERS (5 points). Express the same class of constraints using sets, summations, and parameters (1 point).

Weak

$$z_N \geq \frac{1}{\sum_{p \in P} D_p} \left(x_{8,N} + x_{+,N} + x_{\Sigma N} \right)$$

$$z_C \geq \frac{1}{\sum_{p \in P} D_p} \left(x_{8,C} + x_{+,C} + x_{\Sigma,C} \right)$$

$$z_R \geq \frac{1}{\sum_{p \in P} D_p} \left(x_{8,R} + x_{+,R} + x_{\Sigma,R} \right)$$

Strong

$$z_C \geq \frac{x_{8,C}}{D_8} \qquad z_N \geq \frac{x_{8,N}}{D_8} \qquad z_R \geq \frac{x_{8,R}}{D_8}$$

$$z_C \geq \frac{x_{+,C}}{D_+} \qquad z_N \geq \frac{x_{+,N}}{D_+} \qquad z_R \geq \frac{x_{+,R}}{D_+}$$

$$z_C \geq \frac{x_{\Sigma,C}}{D_\Sigma} \qquad z_N \geq \frac{x_{\Sigma,N}}{D_\Sigma} \qquad z_R \geq \frac{x_{\Sigma,R}}{D_\Sigma}$$

$$z_f \geq \frac{1}{\sum_{p \in P} D_p} \left(\sum_{p \in P} x_{p,f} \right), \text{ for all } f \in F$$

$$z_f \geq \frac{x_{p,f}}{D_p}, \text{ for all } p \in P, f \in F$$

4. *(2 points)* Is anything missing from the model? If so, what?

Yes! Nonnegativity of the x-variables.

$$x_{p,f} \geq 0, \text{ for all } p \in P, f \in F$$

(Since the z-variables are defined to be binary, we don't need to enforce nonnegativity of the z-variables.)