COMPLETED NOTES

Practice Problem #5: Maximum Flow Problem

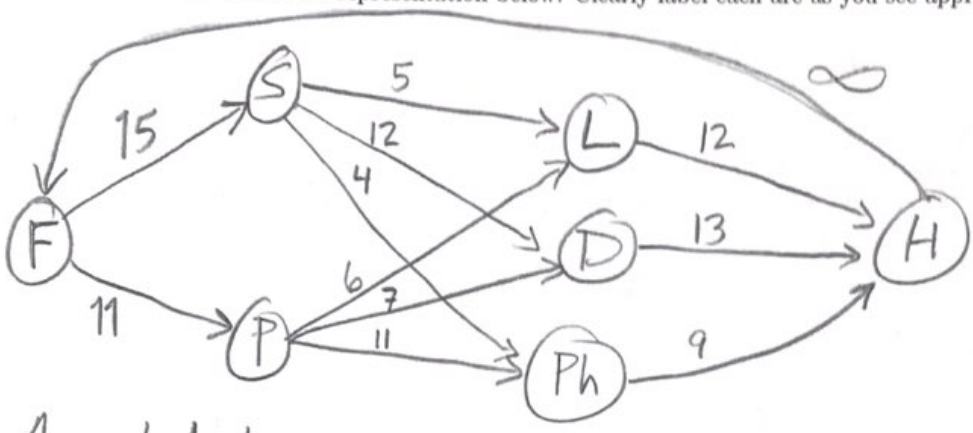
Naval Airlines must determine how many connecting flights daily can be arranged between Fairbanks, Alaska, and Houston, Texas. Connecting flows must stop in Seattle or Portland and then stop in eitehr Los Angeles, Denver, or Phoenix. Because of limited landing space, Naval Airlines is limited to making the number of daily flights between pairs of cities shown in the table below.

Flight	Max # of Flights
Fairbanks-Seattle	15
Fairbanks-Portland	11
Seattle-LA	5
Seattle-Denver	12
Seattle-Phoenix	4
Portland-LA	6
Portland-Denver	7
Portland-Phoenix	11
LA-Houston	12
Denver-Houston	13
Phoenix-Houston	9

Table 1: Flight Capacities

1 Network Representation:

Draw a network representation below. Clearly label each arc as you see appropriate.



- Arc Labels correspond to the Maximum # of Flights
- Nodes correspond to each city.

2 Concrete Model:

Formulate the problem above as a **concrete** mathematical programming model to maximize the total number of flights from Fairbanks to Houston. Clearly define and describe all decision variables, constraints, and the objective.

Max $x_{H,F}$

s.t.

$$x_{FS} + x_{FP} - x_{HF} = 0 \quad (F)$$
$$x_{SL} + x_{SD} + x_{S,Ph} - x_{FS} = 0 \quad (S)$$
$$x_{PL} + x_{PD} + x_{P,Ph} - x_{FP} = 0 \quad (P)$$
$$x_{LH} - x_{SL} - x_{PL} = 0 \quad (L)$$
$$x_{DH} - x_{SD} - x_{PD} = 0 \quad (D)$$
$$x_{Ph,H} - x_{S,Ph} - x_{P,Ph} = 0 \quad (Ph)$$
$$x_{HF} - x_{LH} - x_{DH} - x_{Ph,H} = 0 \quad (H)$$

$0 \le x_{FS} \le 15$, $0 \le x_{FP} \le 11$, $0 \le x_{SL} \le 5$, $0 \le x_{SD} \le 12$, $0 \le x_{S,Ph} \le 4$, $0 \le x_{PL} \le 6$, $0 \le x_{PD} \le 7$, $0 \le x_{P,Ph} \le 11$, $0 \le x_{LH,} \le 12$, $0 \le x_{DH} \le 13$, $0 \le x_{Ph,H} \le 9$, $x_{HF} \ge 0$, INTEGER

Decision Variables

$x_{FS}, x_{FP}, \ldots$ – the # of Flights from One City to Another

Constraints

- Flow Balance at each node/city
- Cannot exceed the maximum # of Flights from one city to another.
- Integer # of Flights

Objective

Maximize the # of Flights from Fairbanks to Houston

3 Abstract Model:

Formulate the problem above as a **abstract** mathematical programming model to maximize the total number of flights from Fairbanks to Houston. Clearly define and describe all sets, parameters, and decision variables.

SETS

$V :=$ Set of All nodes ~ Including origin node $\underline{s}$ and termination node $\underline{t}$.

$A :=$ Set of All arcs

Parameters

→ U_{ij} $\forall (i,j) \in A \Rightarrow$ the maximum number of flights on arc $(i,j) \in A$.

Decision Variables

$x_{ij} \in \mathbb{Z}^+ =$ the # of flights on arc $(i,j) \in A$.

ABSTRACT MODEL

$$\text{Max} \quad x_{TS}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij} - \sum_{k:(k,i)\in A} x_{ki} = 0 \quad \forall\, i \in V$$

$$U_{ij} \geq x_{ij} \geq 0 \quad \forall (i,j) \in A + \underline{\text{Integer}}$$