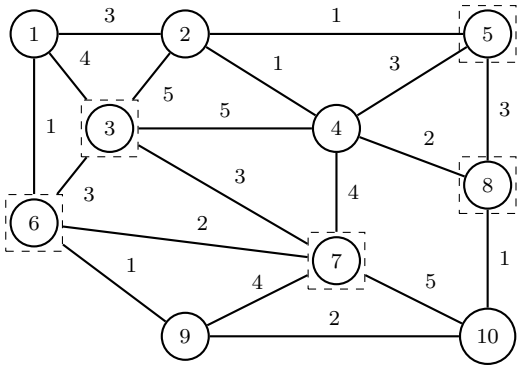
1. You have been tasked to work with Naval Supply Systems Command to analyze a proposed supply chain management plan. The commander has asked for your assistance in deciding where to place **exactly** four supply facilities. The figure below will be used to help make the decision, where each of the $n = 10$ nodes $I = \{1, 2, ..., 10\}$ correspond to the fleet units who need supplies and the nodes $J = \{3, 5, 6, 7, 8\}$ in dashed boxes are the possible supply facility locations. The weight on each edge (i, j) corresponds to direct distances between i and j. Let the distance d_{ij} be the length of the shortest path in this graph between nodes i and j.



Based on historical order data, the demand (in thousands) h_i for the different fleet units have been projected for future orders. In addition, the estimated capacity (in thousands) C_j of each supply facility has been determined. Finally, the fixed charge f_j associated with opening a facility at location $j \in J$ has been calculated. The data are summarized in the table below.

Fleet Unit	1	2	3	4	5	6	7	8	9	10
h_i	45	90	110	35	60	105	80	100	75	55
C_j	-	-	150	-	250	200	350	100	-	-
f_j	-	-	400	-	300	500	200	350	-	-

In what follows, use the binary decision variables

$$X_j = \begin{cases} 1 & \text{if node } j \text{ is the location of a supply facility} \\ 0 & \text{otherwise.} \end{cases} , j = \{3, 5, 6, 7, 8\}.$$

and

$$Y_{ij} = \begin{cases} 1 & \text{if node } i \text{ is served by supply facility } j \\ 0 & \text{otherwise.} \end{cases} , i \in I,\ j \in J$$

(a) Write an objective function which minimizes the TOTAL cost of opening facilities AND supplying the required materials to fleet units. You may assume a cost of \$15 per thousand units of demand per unit distance when including the supply costs.

(b) Write a set of constraints which ensures each fleet unit's demand is met by exactly one supply facility.

(c) Write a single constraint which ensures exactly four supply facilities are opened.

(d) Write a single constraint which ensures supply facility 7 provides NO supplies if it is closed, while allowing it to supply fleet units with as much as its upper bound if it is open.

2. You are employed by the Speedy Package Delivery Company. They want to figure out the best way to deliver packages to eight customer locations numbered $1, \ldots, 8$. The owner of the company seeks routes for the drivers that minimize the total distance traveled. There are three drivers that will be used to deliver packages on a daily basis to the customers. The drivers always start and end their deliveries at the depot, designated as node 0. Each delivery truck can carry a maximum of 4 packages at a time. The daily package delivery requirements for each customer location is given in the table below.

Customer Location	1	2	3	4	5	6	7	8
Number of Packages to Deliver	2	3	2	1	1	1	1	1

You have determined that the distances between the depot and the customers are given by a symmetric matrix. An entry in this matrix d_{ij} gives the distance between location i and location j, where i and j range from 0 to 8.

You copied down part of the model for the Vehicle Routing problem from a copy of the course text:

$$\begin{aligned}
\min \quad & \sum_{(i,j)\in\mathcal{E}} d_{ij}X_{ij} \\
\text{s.t.} \quad & \sum_{j|i<j} X_{ij} + \sum_{j|j<i} X_{ji} = 2 \quad i = 1, \ldots, 8 \quad \text{(a)} \\
& \sum_{j=1}^{8} X_{0j} = 2m \qquad \text{(b)} \\
& X_{ij} \in \{0, 1\} \qquad \forall (i,j) \in \mathcal{E}.
\end{aligned}$$

You have figured out that the graph should be $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{0, 1, \ldots, 8\}$ and $\mathcal{E} = \{(i,j) | i < j, i, j \in \mathcal{V}\}$.

(a) Explain in complete sentences what behavior constraints (a) and (b) enforce in the solution.

(b) You use an integer programming solver to find a solution to the formulation as described. The solver provides the following solution: $X_{01} = X_{02} = X_{03} = X_{06} = X_{07} = X_{08} = X_{12} = X_{34} = X_{48} = X_{56} = X_{57} = 1$. Draw the vehicle routes that this solution describes.

(c) Is the solution provided by the solver in part (b) above feasible? Explain why or why not.

(d) Suppose the solver returns the routes 0-1-8-0, 0-2-7-0, and 0-3-4-5-6-0. Write any route-splitting constraints that you would add to the model based on this collection of vehicle routes.

(e) If you added multiple constraints in part (d), choose one (indicate your choice). Explain the behavior that this constraint enforces on the next collection of routes obtained by the solver.

(f) You and your friends decided to take a BIG trip to see all 30 of the Major League Baseball (MLB) stadiums. We assume that you all are only taking a single car, and you must start and finish in St. Louis (Go Cardinals!). Since you are an OR major, the other midshipmen ask you to plan the trip so as to minimize the total distance traveled to visit every single MLB stadium. We assume that each stadium is only visited once. Present an integer programming model for solving this problem. Clearly define and explain all sets, variables, and parameters needed to do so. Additionally, briefly explain all constraints included in your model.

3. Textbook Problems:

    - 3.22 (Concrete and Abstract)
    - 4.8 and 4.9 (Concrete and Abstract)