Department of Mathematics
SA 405 - Advanced Mathematical Programming
Quiz 1

Name: _____

A medical company makes small, medium, and large heart valves. Each valve requires a certain amount of superconductors, tissue, and labor as detailed below:

Item	Superconductors (units)	Tissue (g)	Labor (hours)
Small	3	5	3
Medium	4	2	6
Large	6	12	7

The company must make at least 10 small, 10 medium, and 5 large valves. Additionally, they have at most 200 superconductors, 300 g of tissue. and 450 hours of labor. Lastly, they make a profit of \$200, \$350, and \$475 for each small, medium, and large valve, respectively. They formulate the following LP to model their situation:

Decision Variables:

Let x_s be the number of small valves made

Let x_m be the number of medium valves made

Let x_l be the number of large valves made

Objective function:

$$\text{max profit: } 200x_s + 350x_m + 475x_l$$

Constraints:

$$
\begin{aligned}
3x_s + 4x_m + 6x_l &\leq 200 \\
5x_s + 2x_m + 12x_l &\leq 300 \\
3x_s + 6x_m + 7x_l &\leq 450 \\
x_s &\geq 10 \\
x_m &\geq 10 \\
x_l &\geq 5 \\
x_s, x_m, x_l &\geq 0
\end{aligned}
$$

Part 1: (80 points) Convert this model into a parameterized model that is **as compact as possible**. *Hint: Your final model should have 3 total constraints including non-negativity.*

Part 2: (20 points) Given sets I and J, parameter u_i for all $i \in I$ and variables $x_{i,j}$ for all $i \in I$ and $j \in J$, indicate if the following parameterized constraints are valid or invalid in an LP model.

- $\sum\limits_{i \in I} \sum\limits_{j \in J} x_{i,j} \leq 10$ for all $j \in J$ VALID INVALID

- $\sum\limits_{i \in I} x_{i,j} = 4$ for all $j \in J$ VALID INVALID

- $\sum\limits_{i \in I} \sum\limits_{j \in J} u_i x_{i,j} \leq 1000$ VALID INVALID