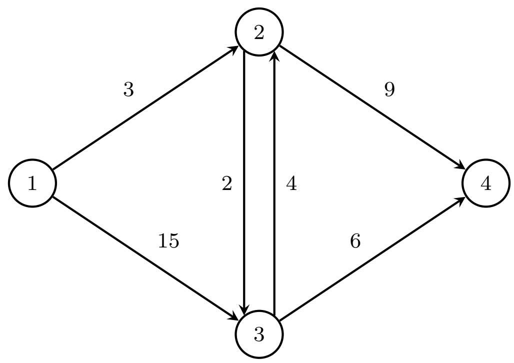
SA 405 **Name:** ____________________

Fall 2018

Test 1 Sample Questions

1. (20 points) Consider the directed network shown below, where the numbers on the arcs represent cost, c_{ij}, to send one unit of flow along the arc. Use the start of a formulation given below to answer the following questions.



Indices

$N :=$ set of nodes, i

$A :=$ set of directed arcs (i, j) from node i to node j, for some i and j in N

Data

$c_{ij} :=$ cost to flow one unit from node i to node j, for $(i, j) \in A$

Decision Variables

$x_{ij} :=$ number of units of flow on arc (i, j), for $(i, j) \in A$

(a) We place one unit of supply at node 1, 1 unit of demand at node 4 and zero units of supply at all other nodes. Formulate an objective function to compute the minimum cost strategy for meeting the demand at node 4.

(b) What two classes of network problems does this one belong to?

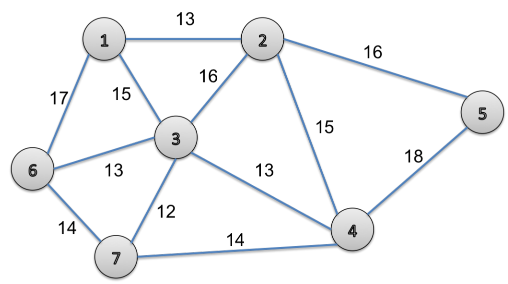
(c) Label all four nodes in the network diagram above with their "net supply" or b_i value.

(d) Write the constraints for this problem using concrete form.

(e) Write the constraints for this problem using abstract form.

(f) Find the optimal objective value by inspection, and write the corresponding values of the decision variables for this solution.

2. US NorthWest plans to install fiber in a metro area network that is expected to experience increased demand due to the opening of a large manufacturing facility. The Central Offices (COs) in the network are represented by vertices in the graph below. The edges in the graph represent the possible fiber paths linking the COs. The number on edge (i, j) represents the cost c_{ij} (in \$1000) of installing fiber between COs i and j.



Network planners developed the following partial integer program to solve for the collection of edges that will minimize the cost of connecting all the central offices in the network via a fiber spanning tree. (Let V represent the set of vertices in the graph. Let E represent the set of edges in the graph.)

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E} c_{ij} X_{ij} & & \\
\text{s.t.} \quad & \textstyle\sum_{j:(i,j)\in E} X_{ij} + \sum_{j:(j,i)\in E} X_{ji} \geq 1, \quad \forall i \in V & & \text{(a)} \\
& \textstyle\sum_{(i,j)\in E} X_{ij} = \ ? & & \text{(b)} \qquad \text{(MST)} \\
& t \textstyle\sum_{\substack{i,j\in V' \\ (i,j)\in E}} X_{ij} \leq |V'| - 1 \qquad \forall V' \subseteq V & & \text{(c)} \\
& X_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E. & &
\end{aligned}
$$

(a) Write the concrete form of constraint type (a) from (MST) for vertex 4. What is the purpose of this constraint?

(b) Architect Ima Klutz spilled cappuccino on the formulation rendering the right hand side of constraint (b) illegible. What is the purpose of constraint (b), and what should the missing number be?

(c) The modeling team learns that city ordinances require that if link $(3, 7)$ is built, then neither $(1, 2)$ nor $(3, 4)$ may be built. Write a constraint to model this new requirement.

(d) The Central Office represented by vertex 3 is centrally located, but the equipment there is outdated. If vertex 3 is used as a hub, meaning three or more fiber paths meet at vertex 3, the Central Office there will require a \$25,000 upgrade. The modeling team adds a new binary variable Y that indicates whether or not vertex 3 is used as a hub in the network design. They modify the objective function by adding the term $25 * Y$. Write a constraint that forces Y to be 1 if three or more selected edges connect to vertex 3.