SA 405 **Final Exam**
Fall 2018 **Dec 14, 2018**

Midshipmen are persons of integrity. **Name:** ______________________

- Do **not** write your name on each page, only write your name above.
- No books or notes are allowed.
- Show all work clearly. (Little or no credit will be given for a numerical answer without the correct accompanying work. Partial credit is given where appropriate.)
- Please read the question carefully. If you are not sure what a question is asking, ask for clarification.
- If you start over on a problem, please CLEARLY indicate what your final answer is, along with its accompanying work.
- All formulations must have descriptions of any indices, parameters, and decision variables used. All constraints must be described.

Grade Table (for teacher use only)

Question	Points	Score
Total:	0	

All possible modeling topics

- Network flow problems
  - Shortest path problem

    The Superintendent asks his most trusted Ops Research majors (you all) to give him directions from the Yard to Los Angeles, CA. Of course, he wants to find the path the minimizes his total travel time. Assume that he can travel through a set of cities $\mathcal{C}$ along a set of roads $\mathcal{A}$ between each city. He assumes that traveling along some arc $(i, j) \in \mathcal{A}$ will require $t_{ij} > 0$ minutes.

    (a) Define your starting node s and your destination/termination node t. Using these definitions, formulate an abstract mathematical programming model to help the superintendent solve his problem.

    (b) The superintendent now wants to consider traffic delays in each city. To account for this added time it takes to travel through each city, he assumes that it will take $d_i > 0$ extra units of time to travel through city $i \in \mathcal{C}$. Identify and define any additional variables and/or constrains, and modify your mathematical programming model above to account for the extra traffic-related time delays.

  - Maximum Flow

    Ad-hoc networks are made up of low-powered wireless devices. These networks can be used on battlefields, in regions that have recently suffered from natural disasters, and in other hard-to-reach areas. The idea is that a large collection of cheap, simple devices could be distributed through the area of interest (for example, by dropping them from an airplane); the devices would then automatically configure themselves into a functioning wireless network. These devices are used to gather as much information about a network as possible. Define your destination/collection node t where all gathered information is to be analyzed. Additionally, these networks adhere to multi-hop communication, in which each device can relay information originating at another node along to the sink t.

    Definitions:

    * Set $\mathcal{V}$ of devices.
    * Set $\mathcal{A}$ of connections $(i, j)\mathcal{A}$ between devices $i,\ j \in \mathcal{V}$.
    * Maximum amount of information that can be sent on connection $(i, j) \in \mathcal{A}$ is c_{ij}.

    (a) Formulate an abstract mathematical programming model to maximize the total amount of information gathered by all devices. *Hint:* Consider adding *dummy* node(s).

- Fixed-charge problem
  - weak-forcing
  - strong-forcing

    I feel like my mids have been tested over this pretty thoroughly.

- Set-covering, packing, and partitioning constraints—-I think that we can test them over these on the facility location problem.
- Logical Constraints
  - These can be added throughout as subproblems.
- Minimum Spanning Tree
  - They've been tested over this and had a quiz. Maybe we could just ask them to identify a problem of this type and run Prims?
- Travelling salesperson problem
  - I say we go with a vehicle routing problem here? What do you think?
- Vehicle Routing problem

  Your regional manager, Michael Scott, has asked you to solve a Vehicle Routing Problem (VRP) to determine an optimal set of 3 paper-delivery routes that begin and end at their warehouse in Scranton, PA. At least one of these routes must visit each one of the 12 customers found in the Paper Belt. Each vehicle must be used and can transport no more than 250 cases of paper, and must visit at least two customers. He also tells you that his goal is to minimize the total distance traveled among all vehicles.

  Answer the following questions related to Michael Scott's request.

  (a) Please provide the **concrete** integer programming model for solving Mr. Scott's problem above. Clearly define your objective function, constraints, and any variables and parameters required.

  (b) You solve the problem, and your solution contains a cycle of customers $1, 4, and 5$. Please provide the appropriate set of subtour-elimination constraints.

  (c) You solve the problem again, and your solution thankfully contains no cycles. In this solution, vehicle 1 visits the sequence of customers $(1, 4, 6, 7)$, vehicle 2 visits the sequence of customers $(3, 8, 9, 12)$ and vehicle 3 visits the sequence of customers $(2, 5, 10, 11)$. Is this solution feasible? If not, provide the appropriate set of route-splitting constraints.

(a) We could give a solution, and ask them if it is feasible. (it won't be), then we could ask them to provide the appropriate constraint to eliminate the infeasible solution?

- Facillity Location Problem
  - Ebola virus outbreak There is currently an outbreak of the Ebola virus in the Democratic Republic of Congo. Once contracted with the Ebola virus, a person does not show any symptoms of the virus until two weeks after being infected. Thus, it is vital to identify those who have infected the virus to quarantine them before anyone else is infected. To do so, the World Health Organization (WHO) must build temporary testing facilities that will be assigned to one or more cities to identify who has been infected in those cities. The population of each city is given as p_c. Now, WHO's task is determine which of the three possible testing facilities should be builet to maximize the total population tested for Ebola.

Maybe we could draw out a picture here?

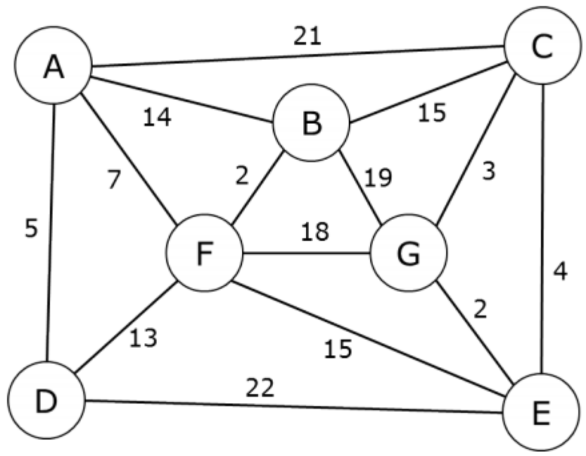
Present an integer programming model to maximize the total number of people tested for Ebola in the Democratic Republic of Congo. Clearly present an objective function, variables, and all required constraints.

(a) Assume that each facility can test no more than P_f number of people. Additionally, we assume that, if assigned, a testing facility test a city's entire population. Please provide all additional constraint(s) needed to accomodate this additional constraint.

(b) Now, assume the cost of building testing facility f is given by c_f. The WHO was just told that they only have one million dollars for building temporary facilities. Please provide all additional constraint(s) needed to accomodate this additional constraint.

Algorithms

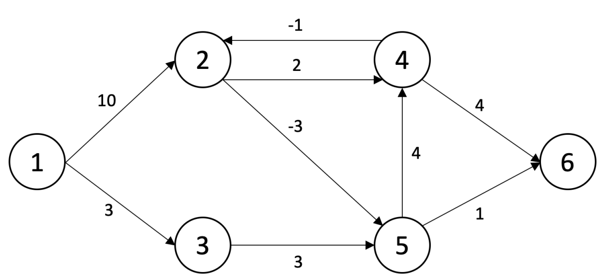
- Nearest Neighbor
  - They've already been tested over this?
- Prims–We could ask them some questions here about Prims? Total number of iterations?



Execute Prim's algorithm to calculate a minimum spanning tree starting at vertex A. Show your steps in the table below. As the algorithm iterates, cross out old distance labels and predecessors and update with the new values. Break any ties by choosing the letter coming first in the alphabet. Also, list the verticies in the order which Prim's algorithm finalizes their distance label. Finally, let T contain the set of nodes forming the minimum spanning tree.

Iteration #	Set S	Cut-set $C_{S,\bar{S}}$	Tree Edges
1			
2			
3			
4			
5			

- Bellman-Ford



Integer Programming Theory

- Total Unimodularity and Bounds

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

Vertex	Distance Label	Predecessor
A		
B		
C		
D		
E		
F		
G		

  - Can we ask them to show whether or not a matrix is TU?
  - I think that we could just ask them some questions about relaxations and the bounds, etc.

- Formulations

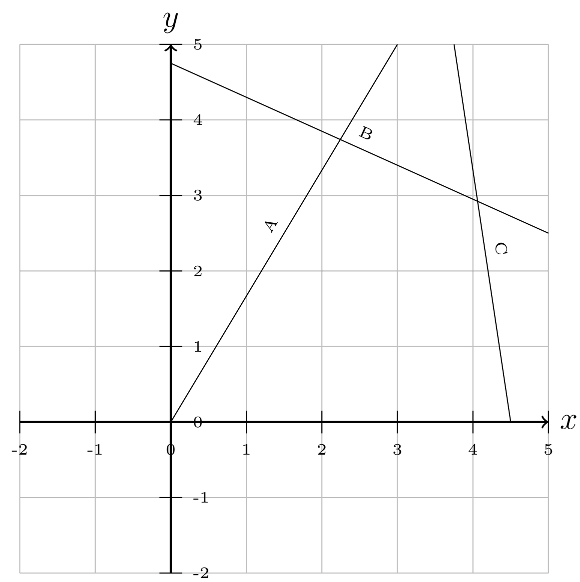
  - We could ask them to draw the convex hull for the IP below?

    Given the integer programming model:

$$\begin{aligned} \max\ & c^T x \\ \text{s.t. } & Ax = b \\ & x \geq 0, \text{integer} \end{aligned}$$

    (a) What two conditions must be true for the IP above to be deemed "easy"?

    (b) If a solution to the relaxation of the above model is integer, then A is totally unimodular? (TRUE or FALSE)

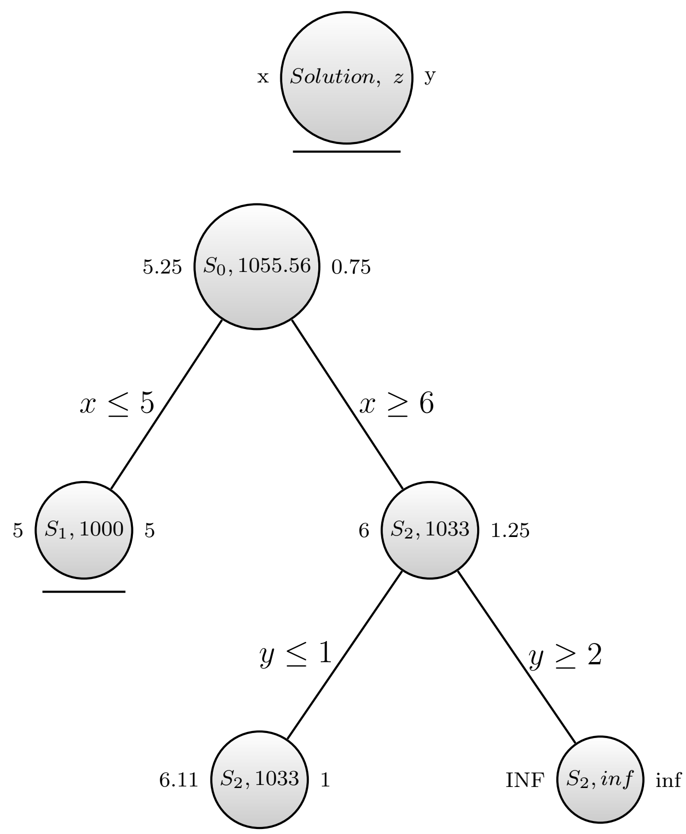


    (a) On the graph above, draw the convex full that constitutes an ideal formulation.

  - We could also ask them why the convex hull is *ideal*?

- We could ask them a graph of the LP-relaxation. We could then ask them add constraints that provide a *tighter* formulation but does not provide the convex hull

- Branch and Bound
  - Here, we could given them a partial B&B tree and ask them to tell us whether or not each node should be pruned, is active, or incumbent. We could ask them to provide the current upper and lower bounds to the IP.
  - And, we could ask them to determine the next step in the B&B algorithm.



(a) What is the current incumbent solution?

(b) Clearly label each node as *incumbent*, *active*, or *fathomed*.

(c) On which active node should we branch next? Draw the next set of branches on the graph along with the appropriate inequalities.