Practice Problem #4: Transportation Problem

Each hour, an average of 900 cars enter a network at node 1 and seek to travel to node 6. The time it takes a car to traverse each arc is shown in Table 2. Table 1 indicates the maximum number of cars that can pass by any point on the arc during a one-hour period. If no number is listed in the table, then you can assume that the road does not exist. Formulate this problem as a mathematical programming model that minimizes the total time required for all cars to travel from node 1 to node 6.

Node	1	2	3	4	5	6
1	–	800	600	–	–	–
2	–	–	–	600	100	–
3	–	–	–	300	400	–
4	–	–	–	–	600	400
5	–	–	–	–	–	600
6	–	–	–	–	–	–

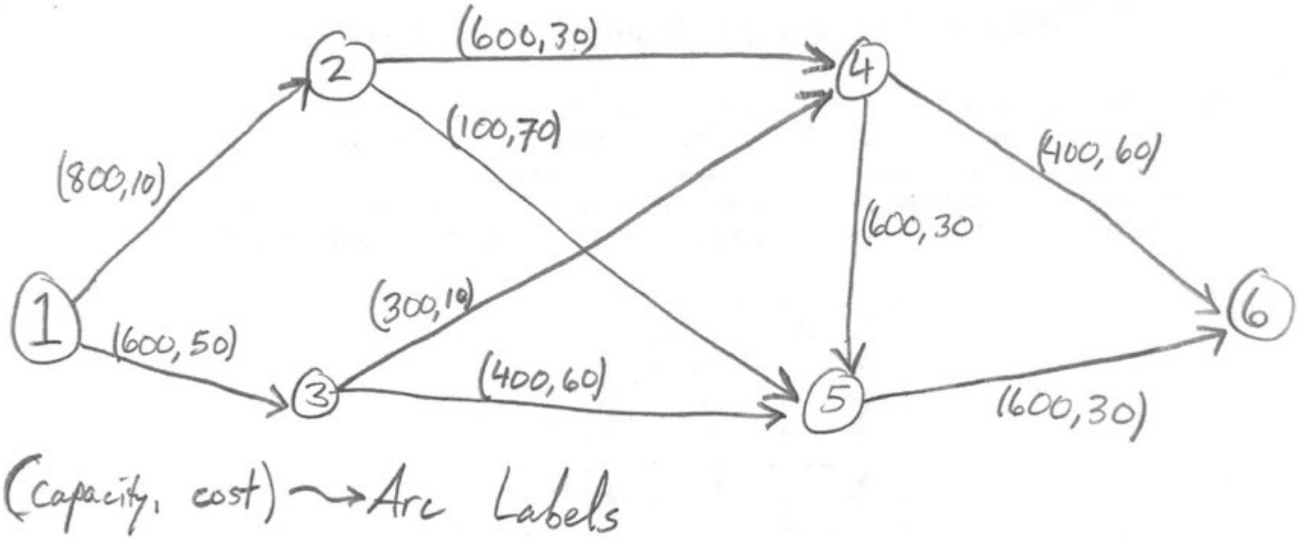
Table 1: Road Capacities

Node	1	2	3	4	5	6
1	–	10	50	–	–	–
2	–	–	–	30	70	–
3	–	–	–	10	60	–
4	–	–	–	–	30	60
5	–	–	–	–	–	30
6	–	–	–	–	–	–

Table 2: Travel Times between Locations

1 Network Representation:

Draw a network representation below. Clearly label each arc as you see appropriate.



COMPLETED

2 Concrete Model:

Formulate the problem above as a **concrete** mathematical programming model to minimize the total cost. Clearly define and describe all decision variables, constraints, and the objective.

Decision Variables

$x_{12}, x_{13}, \ldots$ the # of cars travelling from node 1 to node 2, from node 1 to node 3, and so on.

Constraints

- 900 cars must leave node 1
- 900 cars must arrive at node 6.
- Flow-Balance at node 2, 3, 4, + 5
- Cannot exceed the maximum capacity on each arc between nodes.

Objective

Minimize Total Travel Time

CONCRETE MODEL

$$\text{minimize } 10x_{12} + 50x_{13} + 30x_{24} + 70x_{25} + 10x_{34} + 60x_{35} + 30x_{45} + 60x_{46} + 30x_{56}$$

s.t.

$$x_{12} + x_{13} = 900 \quad (1)$$
$$x_{24} + x_{25} - x_{12} = 0 \quad (2)$$
$$x_{34} + x_{35} - x_{13} = 0 \quad (3)$$
$$x_{46} + x_{45} - x_{34} - x_{24} = 0 \quad (4)$$
$$+x_{56} - x_{35} - x_{25} = 0 \quad (5)$$
$$-x_{46} - x_{56} = -900 \quad (6)$$

$$800 \geq x_{12} \geq 0$$
$$600 \geq x_{13} \geq 0$$
$$600 \geq x_{24} \geq 0$$
$$100 \geq x_{25} \geq 0$$
$$300 \geq x_{34} \geq 0$$
$$400 \geq x_{35} \geq 0$$
$$600 \geq x_{45} \geq 0$$
$$400 \geq x_{46} \geq 0$$
$$600 \geq x_{56} \geq 0$$

3 Abstract Model:

Formulate the problem above as a **abstract** mathematical programming model to minimize the total cost. Clearly define and describe all sets, parameters, and decision variables.

SETS

$V :=$ Set of All Nodes $\{1,2,3,4,5,6\}$

$A :=$ Set of Arcs $\{(1,2),(1,3),(2,4),\ldots\}$

PARAMETERS

- M_{ij} $\forall (i,j)\in A :=$ the maximum capacity on arc $(i,j)\in A$.
- C_{ij} $\forall (i,j)\in A :=$ the time it takes one car to travel on arc $(i,j)\in A$.
- b_i $\forall i\in V :=$ the balance = supply − demand at node $i\in V$.

DECISION VARIABLES

$x_{ij}\geq 0$ $\forall (i,j)\in A =$ the number of cars travelling on arc $(i,j)\in A$.

ABSTRACT MODEL

$$\text{Minimize} \sum_{(i,j)\in A} C_{ij}\, x_{ij}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij} - \sum_{k:(k,i)\in A} x_{ki} = b_i$$

$$0 \leq x_{ij} \leq M_{ij} \quad \forall (i,j)\in A$$