

2.41: The linear program uses the arc flow variable x_{ij} . Denote the node for Hillbilly, MO by s and the node for Beverly Hills, CA by t . To make the problem a circulation, we add the dummy arc, (t, s) to the network. Note that the capacity on the dummy arc, (t, s) is unbounded. In the LP formulation below we are effectively maximizing the flow on the dummy return arc (t, s) :

$$\begin{array}{ll}
 \min & -x_{ts} \\
 \text{s.t.} & x_{s1} + x_{s3} - x_{ts} = 0 \\
 & x_{12} + x_{14} - x_{s1} = 0 \\
 & x_{2t} - x_{12} - x_{32} = 0 \\
 & x_{32} + x_{34} - x_{s3} = 0 \\
 & x_{4t} - x_{14} - x_{34} = 0 \\
 & x_{ts} - x_{2t} - x_{4t} = 0 \\
 & 0 \leq x_{s1} \leq 9 \\
 & 0 \leq x_{s3} \leq 8 \\
 & 0 \leq x_{12} \leq 5 \\
 & 0 \leq x_{14} \leq 7 \\
 & 0 \leq x_{32} \leq 10 \\
 & 0 \leq x_{34} \leq 7 \\
 & 0 \leq x_{2t} \leq 10 \\
 & 0 \leq x_{4t} \leq 12 \\
 & x_{ts} \geq 0
 \end{array}$$

2.44: For the case when $\sum_i s_i > \sum_j d_j$, add a dummy node, t , plus arcs from each supply node s_i to the dummy node t . The arcs have no capacity and no cost. The demand at t is $b_t = -(\sum_i s_i - \sum_j d_j)$. For the case when $\sum_i s_i < \sum_j d_j$, add a dummy node, t , plus arcs from each demand node d_j to the dummy node t . The arcs have no capacity and no cost. The supply at t is $b_t = (\sum_j d_j - \sum_i s_i)$.