1. (**10 points**) Convert the following linear program into canonical form.

$$
\begin{array}{ll}
\max & 4x_1 + 2x_2 - 7x_3 \\
\text{s.t.} & 2x_1 - x_2 + 4x_3 \le 18 \\
 & 4x_1 + 2x_2 + 5x_3 \ge 10 \\
 & x_1, x_2 \ge 0, x_3 \le 0.
\end{array}
$$

$$\boxed{x_3' = -x_3 \Leftrightarrow x_3 = -x_3'}$$

$$
\begin{array}{ll}
\max & 4x_1 + 2x_2 + 7x_3' \\
\text{st.} & 2x_1 - x_2 - 4x_3' - s_1 = 18 \\
 & 4x_1 + 2x_2 + 5x_3 + S_2 = 10 \\
 & x_1, x_2, x_3', s_1, s_2 \ge 0
\end{array}
$$

2. (**6 points**) Consider a canonical form LP with constraints $Ax = b$ and $x \ge 0$, where

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 2 & 1 \\ 0 & 6 & 1 & 0 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}.$$

B

Provide justification that $\{x_3, x_4, x_5\}$ is a valid basis for this linear program. Find the basic solution that corresponds to this basis.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

det $B = 6 \Rightarrow$ the columns of B are linearly indep.

$\therefore \{x_3, x_4, x_5\}$ is a valid basis.

Set $x_1 = x_2 = 0$ & solve for x_3, x_4, x_5:

$A\vec{x} = \vec{b} \Leftrightarrow \boxed{x_3 = 1}$

$2x_4 + x_5 = 1$

$x_3 + 3x_5 = 4$

$1 + 3x_5 = 4$

$\boxed{x_5 = 1}$

$2x_4 + 1 = 1$

$\boxed{x_4 = 0}$

So the bfs is $(0, 0, 1, 0, 1)$.

3. (**2 points**) You are running the simplex method on a linear program whose objective function requires maximization. You correctly calculate the reduced costs for the nonbasic variables and find that they are given by the following: $\bar{c}_x = -6$, $\bar{c}_y = 7$, and $\bar{c}_z = -1$. Based on these results you should (circle one):

   (a) continue with the simplex method

   (b) stop the simplex method

   $\vec{d}_y$ is an improving, feasible direction.

   because (circle one):

   (a) your current solution is a global optimal solution to the linear program

   (b) the linear program is unbounded

   (c) your current solution is not a global optimal solution to the linear program

4. (**2 points**) You are running the simplex method on a linear program whose objective function requires maximization. You correctly compute an improving simplex direction, $\mathbf{d} = (-1/4, 1, 0, -11/4, -5/4)$. If the current iterate is given by $\mathbf{x} = (6, 0, 0, 18, 5)$, then the maximum step size as determined by the ratio test is (circle one):

   (a) 0

   (b) 24

   (c) 72/11

   (d) 4

   (e) There is no maximum step size because this linear program is unbounded.

$$\vec{x} + \lambda \vec{d} = \begin{pmatrix} 6 \\ 0 \\ 0 \\ 18 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{1}{4} \\ 1 \\ 0 \\ -11/4 \\ -5/4 \end{pmatrix} \geq 0$$

$$\Leftrightarrow \quad 6 - \tfrac{1}{4}\lambda \geq 0 \quad \Leftrightarrow \quad \lambda \leq 24$$

$$18 - \tfrac{11}{4}\lambda \geq 0 \quad \Leftrightarrow \quad \lambda \leq$$

$$5 - \tfrac{5}{4}\lambda \geq 0$$