1. (**10 points**) Convert the following linear program into canonical form.

$$\begin{array}{ll} \max & 4x_1 + 2x_2 - 7x_3 \\ \text{s.t.} & 2x_1 - x_2 + 4x_3 \leq 18 \\ & 4x_1 + 2x_2 + 5x_3 \geq 10 \\ & x_1, x_2 \geq 0, x_3 \leq 0. \end{array}$$

2. (**6 points**) Consider a canonical form LP with constraints $Ax = b$ and $x \geq 0$, where

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 2 & 1 \\ 0 & 6 & 1 & 0 & 3 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}.$$

Provide justification that $\{x_3,\ x_4,\ x_5\}$ is a valid basis for this linear program. Find the basic solution that corresponds to this basis.

3. (**2 points**) You are running the simplex method on a linear program whose objective function requires maximization. You correctly calculate the reduced costs for the nonbasic variables and find that they are given by the following: $\bar{c}_x = -6$, $\bar{c}_y = 7$, and $\bar{c}_z = -1$. Based on these results you should (circle one):

   (a) continue with the simplex method

   (b) stop the simplex method

   because (circle one):

   (a) your current solution is a global optimal solution to the linear program

   (b) the linear program is unbounded

   (c) your current solution is not a global optimal solution to the linear program

4. (**2 points**) You are running the simplex method on a linear program whose objective function requires maximization. You correctly compute an improving simplex direction, $\mathbf{d} = (-1/4, 1, 0, -11/4, -5/4)$. If the current iterate is given by $\mathbf{x} = (6, 0, 0, 18, 5)$, then the maximum step size as determined by the ratio test is (circle one):

   (a) 0

   (b) 24

   (c) 72/11

   (d) 4

   (e) There is no maximum step size because this linear program is unbounded.