Homework 6 – solutions
SA405, Fall 2018
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1. 4.4: This is the Set Covering Location Model (Integer Program 4.2 in the book).

Indices and Sets

$i \in I$	customers, $I = \{1, 2, ..., 10\}$
$j \in J$	possible facility locations, $J = \{3, 5, 6, 7, 8\}$

Data

d_{ij}	the length of the shortest path between nodes i and j
D	maximum allowable distance between a customer and the facility it utilizes
$N_i = \{j \in J : d_{ij} \leq D\}$	set of facilities j that can serve node i

Decision Variables [units]

x_j	1 if node j is the location of a facility, 0 otherwise [binary]

Formulation

$$\min_x \quad \sum_{j \in J} x_j \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N_i} x_j \geq 1 \qquad \forall i \in I \tag{2}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J \tag{3}$$

Discussion

Objective (1) minimizes the number of facilities. Constraint (2) is a covering constraint which ensures at least one of the facilities in the neighborhood N_i must be selected. Note that $D = 4$ in this particular problem. We find that 2 facilities are necessary. Facility 5 can service customers $\{1, 2, 4, 5, 8, 10\}$, and facility 6 can service customers $\{1, 2, 3, 6, 7, 8, 9, 10\}$. Another solution is facility 7 can service customers $\{1, 3, 4, 6, 7, 9\}$, and facility 8 can service customers $\{2, 4, 5, 6, 8, 9, 10\}$.

2. 4.5: This is the Maximal Covering Location Model (Integer Program 4.3 in the book).

Indices and Sets

$i \in I$	customers, $I = \{1, 2, ..., 10\}$
$j \in J$	possible facility locations, $J = \{3, 5, 6, 7, 8\}$

Data

d_{ij}	the length of the shortest path between nodes i and j
D	maximum allowable distance between a customer and the facility it utilizes
$N_i = \{j \in J : d_{ij} \leq D\}$	set of facilities j that can serve node i
h_i	customer demand at node i
p	number of facilities we can afford to use

Decision Variables [units]

x_j	1 if node j is the location of a facility, 0 otherwise [binary]
y_i	1 if node i has its demand satisfied by some facility, 0 otherwise [binary]

Formulation

$$\max_{x,y} \quad \sum_{i \in I} h_i y_i \tag{4}$$

$$\text{s.t.} \quad \sum_{j \in N_i} x_j \geq y_i \qquad \forall i \in I \tag{5}$$

$$\sum_{j \in J} x_j = p \tag{6}$$

$$x_j \in \{0, 1\} \qquad \forall j \in J \tag{7}$$

$$y_i \in \{0, 1\} \qquad \forall i \in I \tag{8}$$

Discussion

Objective (4) maximizes the demand covered by open facilities. Constraint (5) ensures node i can only have its demand satisfied if one of the facilities in its neighborhood, N_i, is open. Constraint (6) ensures that the total number of open facilities is equal to p. Note that in this particular problem $D = 4$ and $p = 1$. We find that the one facility should be located at 6, which leaves customers $\{4, 5\}$ unsatisfied and 790 units of demand satisfied.