# Lesson 4: Network Flows: Shortest Path

## 1   Print Shop – Copier Purchase Plan

In Scranton, PA, Dunder Mifflin prints high volumes of photocopying to meet their high demand. The office manager, Michael Scott, is interested in determining when to purchase a new high-speed copier over the next 5 years. During the years that a copier is not purchased, maintenance must be performed. The maintenance cost depends on the age of the copier. The table below provides estimated maintenance cost per age of machine.

| Age at Beginning of Year | Maintenance Cost for the Coming Year |
|:---:|:---:|
| 0 | $2000 |
| 1 | $3500 |
| 2 | $6000 |
| 3 | $9500 |

The cost (in today's dollars) of purchasing copiers at the beginning of each year is given below.

| Year | Purchase Cost |
|:---:|:---:|
| 1 | $10,000 |
| 2 | $13,000 |
| 3 | $16,500 |
| 4 | $20,000 |

Determine the years in which a new copier should be purchased in order to minimize the cost (purchase + maintenance) of having a machine for 5 years.

- Draw a node for each year, 1 through 5, from left to right. Draw every possible directed arc from a year to a later year; e.g., $(1, 2)$ and $(1, 3)$, but not $(3, 1)$.

- Arc $(i, j)$ represents the cost of purchasing a copier at the beginning of year $i$ and maintaining it until the beginning of year $j$. For example, the cost incurred by selecting arc $(1, 4)$ (in thousands) is $\$10 + \$2 + \$3.5 + \$6 = \$21.5$: which is the cost of purchasing a new copier in year 1, then maintaining it through years 1, 2, and 3. Add arc costs to the network diagram.

## 2   How is this a shortest path problem?

> A **path** is an ordered sequence of connected arcs such that any node is "visited" at most once.

1. In this problem, the minimum cost strategy corresponds to the minimum cost *path* from where to where?

2. What are the decision variables for this problem?

3. Write the concrete model for this problem

4. Write the parameterized model for this problem