Quiz 4

1. A special operations unit is looking to buy new weapons for use in upcoming deployments. There are four weapons available that can effectively engage targets at different ranges. The different weapons, their costs, and capabilities are summarized in the table below. A YES in the column for a weapon means that it can effectively engage targets at that range.

Target Range	Weapon 1	Weapon 2	Weapon 3	Weapon 4
Short	YES	YES	YES	YES
Medium	NO	YES	YES	NO
Long	NO	NO	YES	YES
Cost	$ 6300	$ 5400	$ 4600	$ 3100

Use the variable x_j defined below, to answer the following questions. Remember to define **ANY** other terms you use in your answers to the questions below.

$$x_j = \begin{cases} 1 & \text{if Weapon } j \text{ is selected} \\ 0 & \text{otherwise.} \end{cases}$$

(a) *(5 Points)* Write an objective function that finds a minimum cost collection of weapons. (4 points for no-sets version + 1 point for sets version.)

$$\min \quad 6300x_1 + 5400x_2 + 4600x_3 + 3100x_4$$

Sets: W = set of weapons ($W = \{1,2,3,4\}$

$Cost_i$ = purchase cost of weapon i, for $i \in W$

$$\min \sum_{i \in W} Cost_i x_i$$

(b) *(5 Points)* Write the set covering constraints which ensure the unit will have **AT LEAST ONE** weapon that can effectively engage targets at each range. (4 points for no-sets version + 1 point for sets version.)

$$X_1 + X_2 + X_3 + X_4 \geq 1 \quad \text{(short range covered)}$$

$$X_2 + X_3 \geq 1 \quad \text{(Med. range covered)}$$

$$X_3 + X_4 \geq 1 \quad \text{(long range covered)}$$

R = set of ranges

Parameter $\text{covers}_{i,r} = 1$ if weapon i covers range r, 0 otherwise, for $i \in W$, $r \in R$

$$\sum_{i \in W} \text{covers}_{i,r} X_i \geq 1, \text{ for } r \in R$$

(c) *(5 Points)* Write the set partitioning constraints which ensure the unit will have **EXACTLY ONE** weapon that can effectively engage targets at each range. (4 points for no-sets version + 1 point for sets version.)

$$X_1 + X_2 + X_3 + X_4 = 1 \quad \text{(short)}$$

$$X_2 + X_3 = 1 \quad \text{(medium)}$$

$$X_3 + X_4 = 1 \quad \text{(long)}$$

$$\sum_{i \in W} \text{covers}_{i,r} X_i = 1, \text{ for } r \in R$$

See next page for another way to handle sets notation.

R = set of ranges $\left(R = \{S, M, L\} \right)$

R_i = set of ranges covered by weapon i, $i \in W$

(e.g. $R_1 = \{S\}$, $R_2 = \{S, M\}$, etc. $\}$

$$\sum_{\{i \in W : r \in R_i\}} x_i \geq 1, \text{ for } r \in R$$

$$\sum_{\{i \in W : r \in R_i\}} x_i = 1, \text{ for } r \in R$$

include the weapons that have range r in their set of covered ranges.

OR, Yet another way

R = set of ranges

W_r = set of weapons that cover rang $r, \forall r \in R$

(e.g. $W_S = \{1, 2, 3, 4\}$, $W_M = \{2, 3\}$, $W_L = \{3, 4\}$)

$$\sum_{i \in W_r} x_i \geq 1, \text{ for } r \in R$$

$$\sum_{i \in W_r} x_i = 1, \text{ for } r \in R$$