Homework 3
SA405, Fall 2018
Instructor: Foraker

1. 3.2

Indices

$m \in M$	machines, $M = \{1, 2, 3, 4, 5\}$
$j \in J$	jobs, $J = \{1, 2, 3, 4, 5\}$
$(m, j) \in P$	$P = \{(m, j) \mid m \text{ can do job } j\}$

Data

$setup_m$	setup time (in minutes) if machine m is used
$work_{m,j}$	time to complete job j on machine m (if applicable)
$additional$	extra 20 minutes needed if machines 1 and 2 are used

Binary Decision Variables

USE_m	1 if machine m is used, 0 otherwise
$OPERATE_{m,j}$	1 if machine m is used to do job j, 0 otherwise
$BOTH$	1 if both machines 1 and 2 are used, 0 otherwise

Formulation

$$\min_{USE,OPERATE,BOTH} \quad \sum_{(m,j)\in P} work_{m,j} OPERATE_{m,j} \quad + \sum_{m \in M} setup_m USE_m \quad + additional BOTH$$

$$\begin{array}{llll}
\text{s.t.} & \displaystyle\sum_{\{m|(m,j)\in P\}} OPERATE_{m,j} \geq 1 & \forall j \in J & \text{(Job Completion)} \\
& \displaystyle\sum_{\{j|(m,j)\in P\}} OPERATE_{m,j} \leq 2 & \forall m \in M & \text{(Machine Utilization)} \\
& \displaystyle\sum_{\{j|(m,j)\in P\}} OPERATE_{m,j} \leq 5USE_m & \forall m \in M & \text{(Can't operate unless do setup)} \\
& BOTH \leq USE_1 & & \text{(Logical Constraints on } BOTH \\
& BOTH \leq USE_2 & & \text{to ensure } BOTH = 1 \text{ if use} \\
& BOTH + 1 \geq USE_1 + USE_2 & & \text{machines 1 and 2)} \\
& BOTH, OPERATE_{m,j}, USE_m \in \{0, 1\} & \forall m \in M, \forall j \in J &
\end{array}$$

2. 3.3

Indices

$p \in P$	products, $P = \{1, 2, 3\}$
$l \in L$	lines, $L = \{1, 2, 3\}$
$\mathbb{N}$	natural numbers (including zero)

Data

$setup\_cost_l$	setup cost if line l used in given week
$worker\_cost_l$	pay per worker on line l
$production_{p,l}$	production per worker for product p on line l
$max\_workers$	maximum number of workers, 20 in this instance
$demand_p$	number of product p that must be produced

Integer Decision Variables

$WORKERS_{p,l}$	number of workers assigned to product p on line l

Binary Decision Variables

$OPEN_l$	1 if line l is open, 0 otherwise

Formulation

$$
\begin{array}{llll}
\min\limits_{WORKERS, OPEN} & \sum\limits_{l \in L, p \in P} worker\_cost_l WORKERS_{p,l} & + \sum\limits_{l \in L} setup\_cost_l OPEN_l & \\
\text{s.t.} & \sum\limits_{p \in P} WORKERS_{p,l} \leq max\_workers OPEN_l & \forall l \in L & \text{(No workers unless line open)} \\
& \sum\limits_{l \in L} production_{p,l} WORKERS_{p,l} \geq demand_p & \forall p \in P & \text{(Demand constraint)} \\
& \sum\limits_{l \in L, p \in P} WORKERS_{p,l} \leq max\_workers & & \text{(Total workers constraint)} \\
& WORKERS_{p,l} \in \mathbb{N} & \forall p \in P, l \in L & \text{(integer constraint)} \\
& OPEN_l \in \{0, 1\} & \forall l \in L & \text{(binary constraint)}
\end{array}
$$

Not to be handed in.

1. 3.5:

   Indices

$i \in I$	cities (alias j), $I = \{AT, BO, CH, DA, LA, NY, OC, PI, RI, SL, SF, SE\}$

   Data

$served_{i,j}$	1 if city i is within 1200 miles of city j, 0 otherwise

   Binary Decision Variables

HUB_i	1 if hub is to be located in city i, 0 otherwise

   Formulation

$$
\begin{array}{lll}
\min\limits_{HUB} & \sum\limits_{i \in I} HUB_i & \\
\text{s.t.} & \sum\limits_{i \in I} served_{i,j} HUB_i \geq 1 \quad \forall j \in I & \text{(Must service all locations)} \\
& HUB_i \in \{0, 1\} \qquad\qquad \forall i \in I & \text{(binary constraint)}
\end{array}
$$

2. 3.8 The formulation below assumes a maximum of 1 TV per location b and s.

Indices

$b \in B$	locations for 100 inch TV's, $B = \{A, B, C, D, E, F\}$
$s \in S$	locations for 32 inch TV's, $S = \{1, 2, ..., 22\}$
$z \in Z$	zones, $Z = \{1, 2, ..., 12\}$
$(b, z) \in P$	$P = \{(b, z) \mid \text{can see 100 inch TV at location } b \text{ in zone } z\}$
$(s, z) \in Q$	$Q = \{(s, z) \mid \text{can see 32 inch TV at location } s \text{ in zone } z\}$

Data

cost_big	cost of 100 inch TV's, \$5000 in this instance
cost_small	cost of 32 inch TV's, \$750 in this instance
min_big	minimum number of 100 inch TV's to buy, 2 in this instance
min_small	minimum number of 32 inch TV's to buy, 8 in this instance

Binary Decision Variables

TV_BIG_b	1 if 100 inch TV at location b, 0 otherwise
TV_SMALL_s	1 if 32 inch TV at location s, 0 otherwise
CAN_SEE_z	1 if can see 100 inch TV in zone z, 0 otherwise

Formulation

$$
\begin{array}{lll}
\min & \sum_{b \in B} cost\_big TV\_BIG_b + \sum_{s \in S} cost\_small TV\_SMALL_s & \\
\text{s.t.} & \sum_{\{b \mid (b,z) \in P\}} TV\_BIG_b + \sum_{\{s \mid (s,z) \in Q\}} TV\_SMALL_s \geq 2CAN\_SEE_z + 3\left(1 - CAN\_SEE_z\right) & \forall z \in Z \\
& CAN\_SEE_z \leq \sum_{\{b \mid (b,z) \in P\}} TV\_BIG_b & \forall z \in Z \\
& CAN\_SEE_z \geq TV\_BIG_b & \forall z \in Z, \forall b \mid (b, z) \in P \\
& \sum_{s \in S} TV\_SMALL_s \geq min\_small & \\
& \sum_{b \in B} TV\_BIG_b \geq min\_big & \\
& TV\_BIG_b, TV\_SMALL_s, CAN\_SEE_z \in \{0, 1\} & \forall b \in B, \forall s \in S, \forall z \in Z
\end{array}
$$

The first constraint ensures that at least 2 TV's can be seen or at least 3 TV's can be seen if the 100 inch TV is not visible. The second and third constraints are logical constraints to ensure that *CAN_SEE* takes on a value of 1 only when appropriate. The fourth constraint ensures that at least 8 of the 32 inch TV's are purchased. The fifth constraint ensures that at least 2 of the 100 inch TV's are purchased.

3. 3.8 The formulation below assumes a maximum of 1 TV per location b and s. This is an alternate, but equivalent, form of the formulation provided above.

Indices

$b \in B$	locations for 100 inch TV's, $B = \{A, B, C, D, E, F\}$
$s \in S$	locations for 32 inch TV's, $S = \{1, 2, ..., 22\}$
$z \in Z$	zones, $Z = \{1, 2, ..., 12\}$

Data

$view\_big_{b,z}$	1 if can see 100 inch TV at location b in zone z, 0 otherwise
$view\_small_{s,z}$	1 if can see 32 inch TV at location s in zone z, 0 otherwise
$cost\_big$	cost of 100 inch TV's, \$5000 in this instance
$cost\_small$	cost of 32 inch TV's, \$750 in this instance
min_big	minimum number of 100 inch TV's to buy, 2 in this instance
min_small	minimum number of 32 inch TV's to buy, 8 in this instance

Binary Decision Variables

TV_BIG_b	1 if 100 inch TV at location b, 0 otherwise
TV_SMALL_s	1 if 32 inch TV at location s, 0 otherwise
CAN_SEE_z	1 if can see 100 inch TV in zone z, 0 otherwise

Formulation

$$
\begin{array}{lll}
\min & \sum_{b \in B} cost\_big TV\_BIG_b + \sum_{s \in S} cost\_small TV\_SMALL_s & \\
\text{s.t.} & \sum_{b \in B} view\_big_{b,z} TV\_BIG_b + \sum_{s \in S} view\_small_{s,z} TV\_SMALL_s \geq 2CAN\_SEE_z + 3\,(1 - CAN\_SEE_z) & \forall z \in Z \\
 & CAN\_SEE_z \leq \sum_{b \in B} view\_big_{b,z} TV\_BIG_b & \forall z \in Z \\
 & CAN\_SEE_z \geq view\_big_{b,z} TV\_BIG_b & \forall z \in Z, \forall b \in B \\
 & \sum_{s \in S} TV\_SMALL_s \geq min\_small & \\
 & \sum_{b \in B} TV\_BIG_b \geq min\_big & \\
 & TV\_BIG_b, TV\_SMALL_s, CAN\_SEE_z \in \{0, 1\} & \forall b \in B, \forall s \in S, \\
 & & \forall z \in Z
\end{array}
$$

The first constraint ensures that at least 2 TV's can be seen or at least 3 TV's can be seen if the 100 inch TV is not visible. The second and third constraints are logical constraints to ensure that CAN_SEE takes on a value of 1 only when appropriate. The fourth constraint ensures that at least 8 of the 32 inch TV's are purchased. The fifth constraint ensures that at least 2 of the 100 inch TV's are purchased.