SA405 – AMP

Completed

RMC

Practice Problem #11

1 The Sabre Problem

In 2015, the printer manufacturer Sabre has set up shop creating printers, copiers, and scanners to celebrate its 10th anniversary. Their machinist makes the designs each machine out of plastic and metal. To begin manufacturing each machine, Sabre must pay a significant set up cost. All relevant data can be found in the table below.

	Printers	Copiers	Scanners	Availability
Machinist Labor (Days)	2	4	5	100
Plastic (pounds)	1	1.5	1.8	30
Metal (pounds)	1	1.5	1.8	30
Profit ($ per machine)	52	30	20	–
Set-up Costs ($)	500	400	300	–

1.1 Concrete Model

Formulate Sabre's problem as a **concrete** integer programming model to maximize the total amount of profit. Define and describe all restrictions, the objective, and all decision variable(s).

$$
\begin{aligned}
\max\ & 52x_p + 30x_c + 20x_s \\
\text{s.t. } & 2x_p + 4x_c + 5s_x \leq 100 \\
& x_p + 1.5x_c + 1.8x_s \leq 30 \\
& x_p + 1.5x_c + 1.8x_s \leq 30 \\
& 30y_p \geq x_p,\ 20y_c \geq x_c,\ 16y_s \geq x_s \\
& y_p,\ y_c,\ y_s \in \{0,1\} \\
& x_p,\ x_c,\ x_s \geq 0, \text{integer}
\end{aligned}
$$

Consider the following subproblems associated with this problem above. Provide the variables and constraints needed to model these modifications.

1. Suppose that you want to restrict the number of printers you manufacture to be either 4, 8, or 13, if positive.

$v, w, z \in \{0,1\}$ equals 1 if $x_p = 4$, $x_p = 8$, $x_p = 13$, respectively

$x_p = 4v + 8w + 13z$ + $v + w + z \leq 1$

2. If you make any printers than you must make at least 20 copiers.

$$x_c \geq 20 y_p$$

3. If you make both (any number of) printers and scanners, then you receive a \$250 rebate.

New Objective $\Rightarrow$ $+250z$

$z \in \{0,1\}$ equals 1 if we make both printers and scanners

$z \leq y_p$
$z \leq y_s$
> Add these two constraints

4. If you make at least 100 scanners or printers then you cannot make any copiers.

Let $z \in \{0,1\}$ be equal to 1 if you make at least 100 scanners or printers.

$$x_s + x_p \leq 100z \quad + \quad y_c \leq 1 - z$$

or w/o any new Binary Variables

$$x_s + x_p \leq 99 + M(1 - y_c) \checkmark$$

Since x_s + x_p are non-negative integers

2 Definitions

1. In your own words, define a network *tree*.

A set of edges connecting a set of nodes without any cycles

2. In your own words, define a connected *graph*.

A connected graph is a set of nodes for which there exists a path of connected edges between every pair of nodes in the graph.

3. In your own words, define a network *path*.

A set of connected edges between two nodes.

4. In your own words, define a network *cycle*.

A set of connected edges visiting some node multiple times.

3 Phone Problem in Chauvenet Hall

You've been tasked with helping to install ethernet lines in Chauvenet Hall. You want to install lines so that each computer is able to communicate with all other computers in the building. To do so, each computer does not have to directly be able to communicate with each computer; instead, they are able to relay communications through some set of computers. The phone company charges different amounts of money to lay ethernet lines between different pairs of computers. Formal an ***abstract*** integer programming model to minimize the total cost of laying ethernet cables to solve this problem. Define all sets, parameters, and decision variables for solving this problem.

Sets

$V :=$ Set of Computers (Nodes)

$E :=$ Set of lines (i,j) between computers $i,j \in V$ in which $i<j$.

Parameters

$c_{ij}\ \forall\ (i,j) \in E \Rightarrow$ the cost of laying lines between computers i + j.

Decision Variables

$x_{ij} \in \{0,1\}$ equals 1 if we lay ethernet line between computers i + j in V.

$$\text{Minimize} \sum_{(i,j)\in E} c_{ij} x_{ij}$$

$$\text{s.t.} \sum_{j:(i,j)\in E} x_{ij} + \sum_{j:(j,i)\in E} x_{ji} \geq 1 \quad \forall i \in V$$

$$\sum_{(i,j)\in E} x_{ij} = |V| - 1$$

$$\sum_{i,j\in S:\, i<j} x_{ij} \leq |S| - 1 \quad \forall S \subset V,\ |S| \geq 3$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in E$$