Lesson 3: Network Models: Transportation & Minimum Cost Network Flow Models

1 A Transportation Problem: Bakeries

A local baked goods company has two bakeries where they bake their goods, which they then ship to three different area stores to sell. Each bakery can produce up to 50 truckloads of baked goods per week, and each bakery can supply any of the stores. The weekly demands (in truckloads) anticipated at each store along with the transportation costs (per truckload) are provided in the tables below. Note that partial truckloads cost just as much as full truckloads. How many truckloads should be sent from each bakery to each store in order to minimize total shipping cost?

	Demand
Store 1	35
Store 2	25
Store 3	40

	Store 1	Store 2	Store 3
Bakery a	\$20	\$45	\$35
Bakery b	\$35	\$35	\$50

1.1 Graphical representation of the network.

1. Draw a directed graph to represent the problem. Bakeries and stores are represented by *nodes* (or *vertices*). Directed *arcs* (or *edges*) represent flow of goods. Label each node (a, b, 1, 2, 3), and beside each node indicate its corresponding supply or demand.

2. Find a feasible (not necessarily optimal) solution.

3. Define decision variables. (Hint: what information must a solution provide?)

4. Add decision variables to your graphical network diagram.

1.2 Concrete model

Problem 1. Using the usual format, write a concrete model to find a feasible transportation.

1.3 Parameterized model

Problem 2. Convert your model to a parameterized model.

1.4 Assumptions and Special Scenarios

Notice that the transportation problem has a very important assumption: namely that the total supply is equal to the total demand.

- If the total supply exceeds the total demand or vice versa there's a problem with the network.

1. What if bakery 1 could produce 100 units instead of just 50? How would that modify our network diagram/formulation?

2. What if store 1 needed 75 units instead of 35? How would that modify our network diagram/formulation?

2 Minimum Cost Network Flow Models

Consider the bakery again, only this time there are two warehouses that baked goods must be delivered to before they are taken to their final destinations. The warehouses ($w1$ and $w2$) have no supply nor demand. These are the transportation costs:

	Warehouse 1	Warehouse 2
Bakery 1	\$10	\$15
Bakery 2	\$20	\$25

	Store 1	Store 2	Store 3
Warehouse 1	\$20	\$45	\$35
Warehouse 2	\$35	\$35	\$50

Also, recall that the store demands are $d_{s1} = 35$, $d_{s2} = 25$, and $d_{s3} = 40$, so that *total supply* $=$ *total demand* $= 100$.

- The warehouses are called **transshipment nodes**.
- This is called a **transshipment problem** or a **minimum cost network flow problem**.

Problem 3. Draw the new network diagram. What's the major difference this time that doesn't make it a simple transportation problem?

2.1 Concrete model

Problem 4. Using the usual format, write a concrete model to find a feasible transportation.

2.2 Balance of Flow

Before writing the parameterized model, it will be helpful to rearrange the constraints so that they all have the same form. First, verify that you have one constraint per node, and that all of your constraints have one of the following forms:

$$flow\ out \ \leq\ supply \qquad \text{or} \qquad demand\ \leq\ flow\ in.$$

We can write all of our constraints using the same form by adding these two forms:

$$demand + flow\ out\ =\ supply + flow\ in.$$

These are called **flow balence** constraints. (Recall this is an assumption)

- Why does it make sense for *supply* at a node to be combinded with $flow\ in$ and *demand* at a node to be combined with flow out?

- Some like to combine supply and demand into a single parameter, $b = supply - demand$, for each node. Write a general form for the **balance of flow constraint** using b:

2.3 Parameterized model

Problem 5. Convert your model to a parameterized model.