RMC

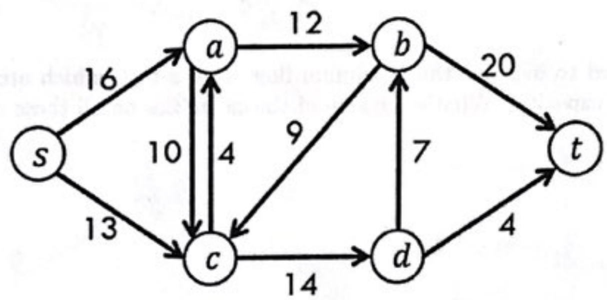
Completed

Practice Problem #6: Maximum Flow Problem

1 Solve the following Maximum Flow Problems via an Augmenting-path Algorithm

1.1 Network #1

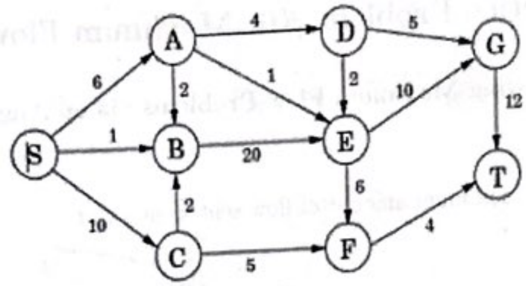
a Determine the maximum amount of flow sent from s to t.



b If you wanted to increase the maximum flow from s to t, which arcs would you choose to increase the capacity? What's the sum of the capacities on all these arcs?

1.2 Network #2

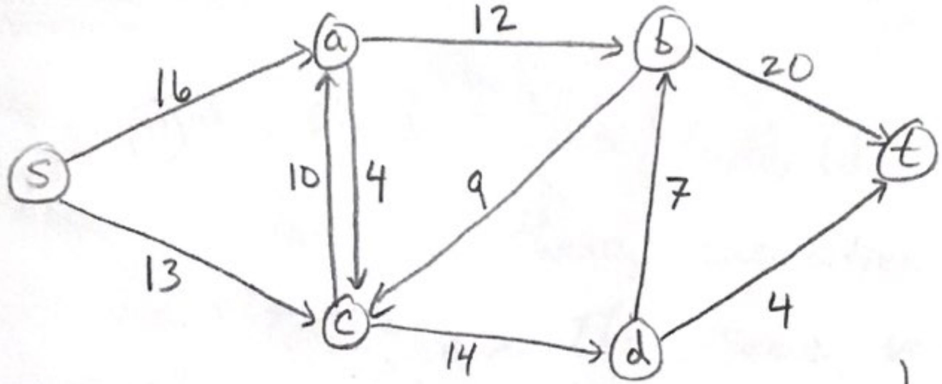
a Determine the maximum amount of flow sent from s to t.



b If you wanted to increase the maximum flow from s to t, which arcs would you choose to increase the capacity? What's the sum of the capacities on all these arcs?

PRACTICE PROBLEM #6

Problem 1a

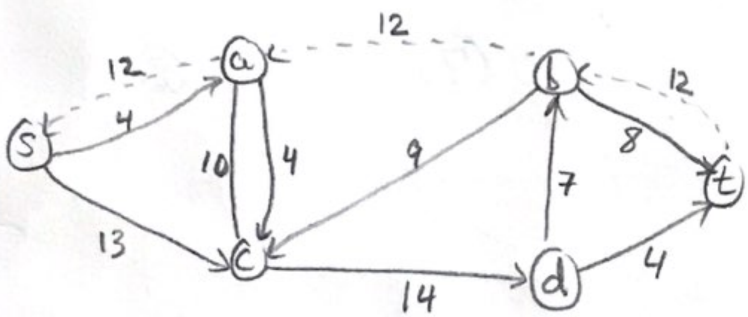


Path #1

s → a → b → t

12 units

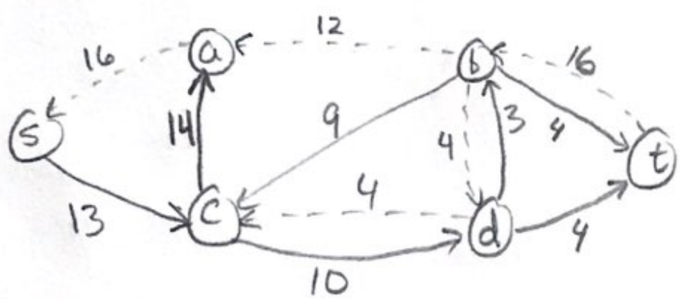
↓



Path #2

s → a → d → b → t

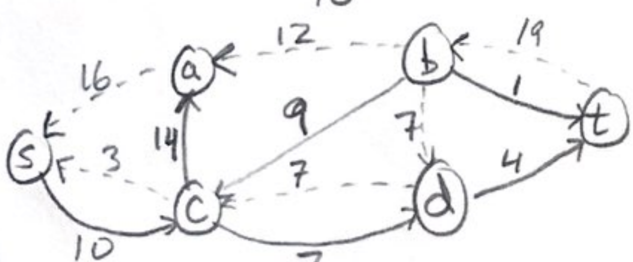
4 units



Path #3

s → c → d → b → t

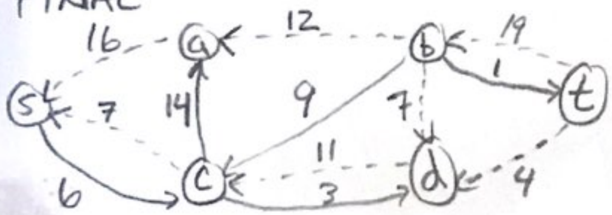
3 units



Path #4

s → c → d → t 4 units

FINAL



No more paths exist. Therefore, we are optimal with maximum flow equal to 23.

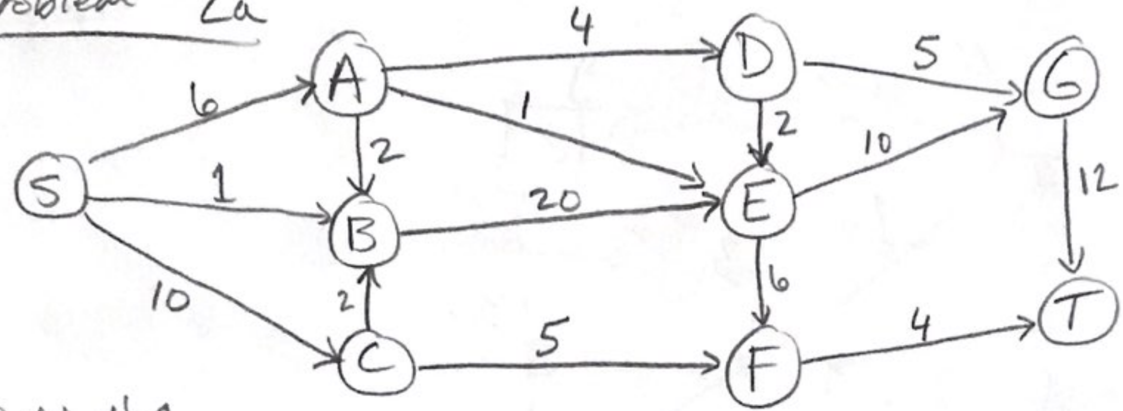
PRACTICE PROBLEM #6

Problem 1b

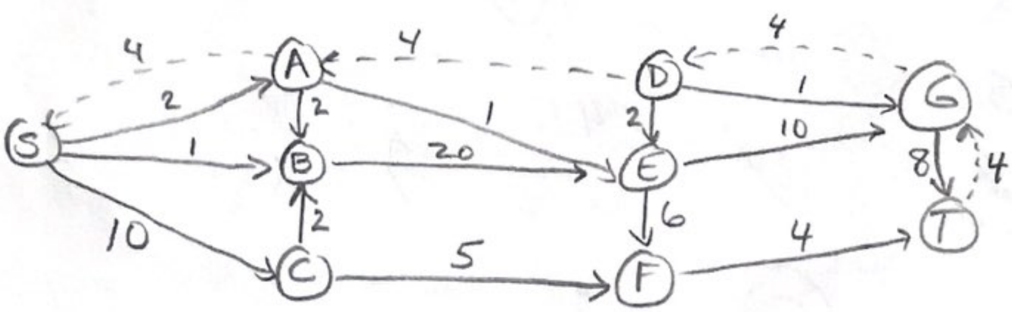
We would choose arcs $\{(a,b), (d,b), (d,t)\}$.
The sum of their capacities is
$12 + 7 + 4 = 23$. The same value as the maximum flow.

PRACTICE PROBLEM #6

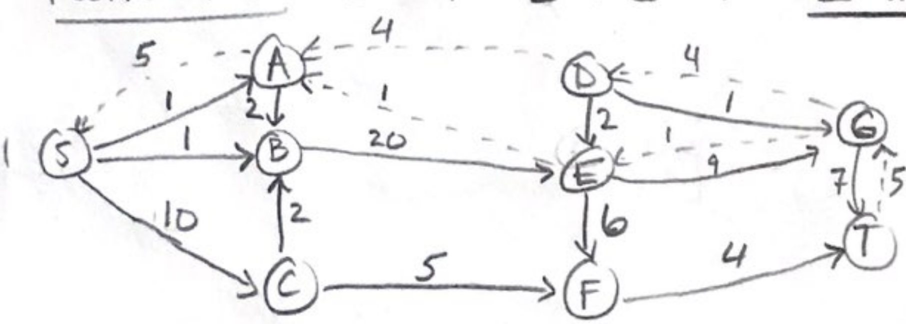
Problem 2a



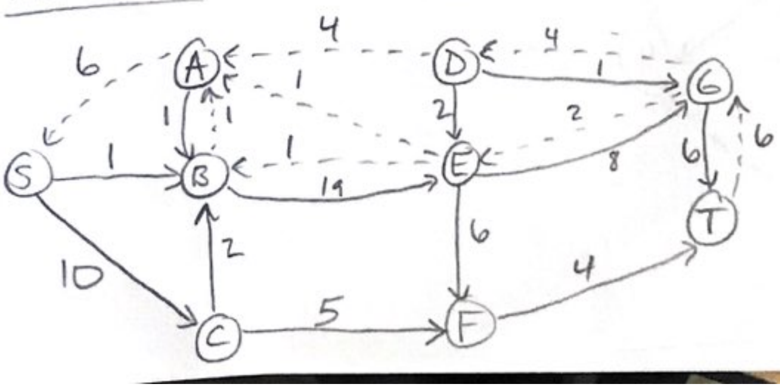
Path #1 S → A → D → G → T <u>4 units</u>



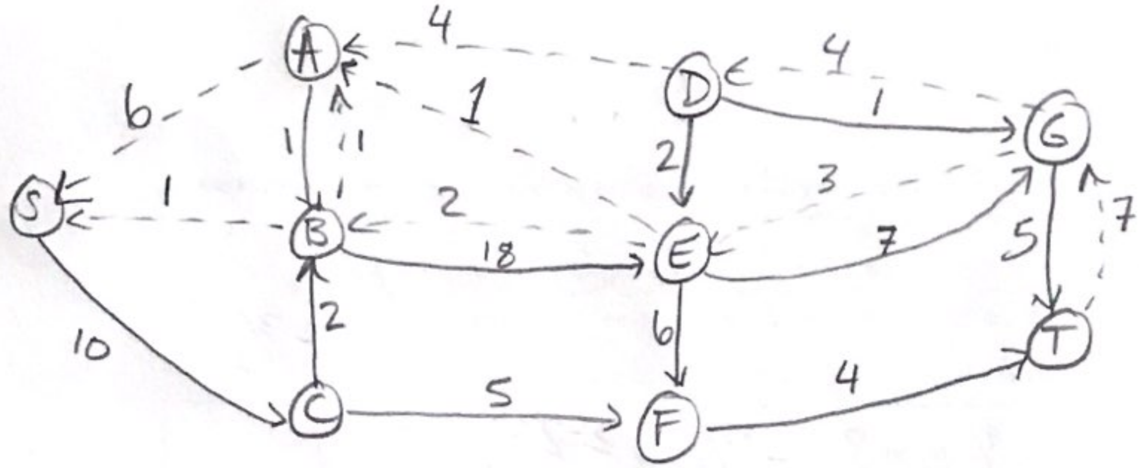
Path #2 S → A → E → G → T <u>1 unit</u>



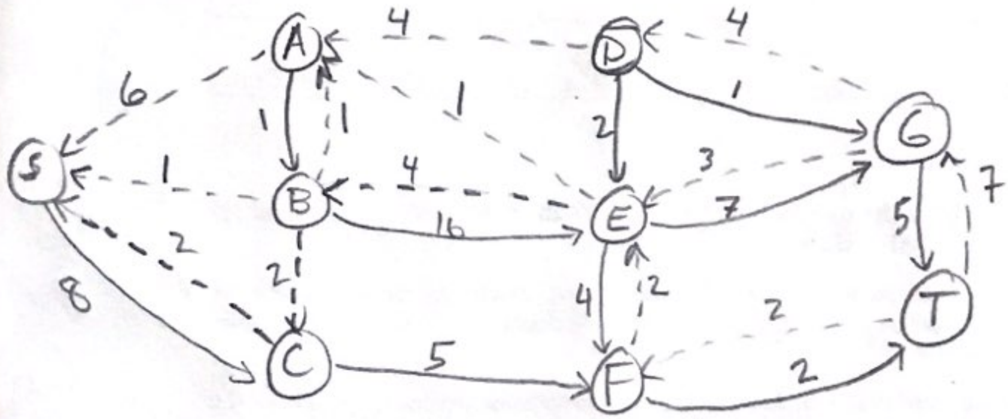
Path #3 S → A → B → E → G → T <u>1 unit</u>



Path #4
S → B → E → G → T
1 unit



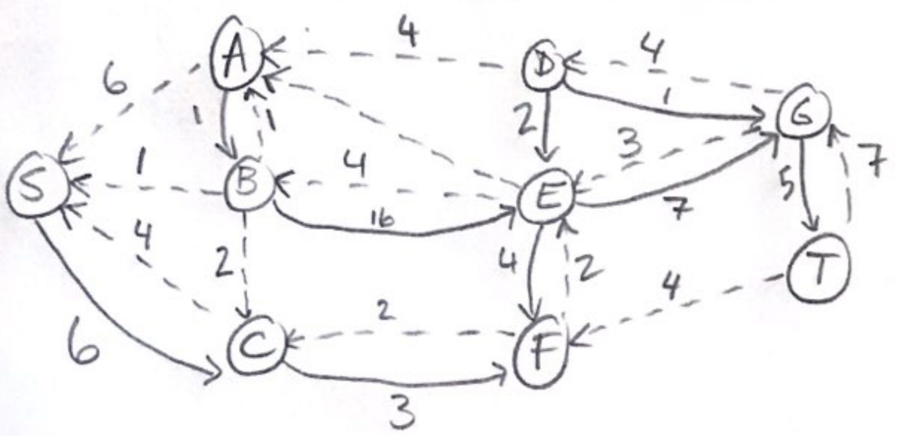
Path #5 S → C → B → E → F → T 2 units



Path #6

S → C → F → T

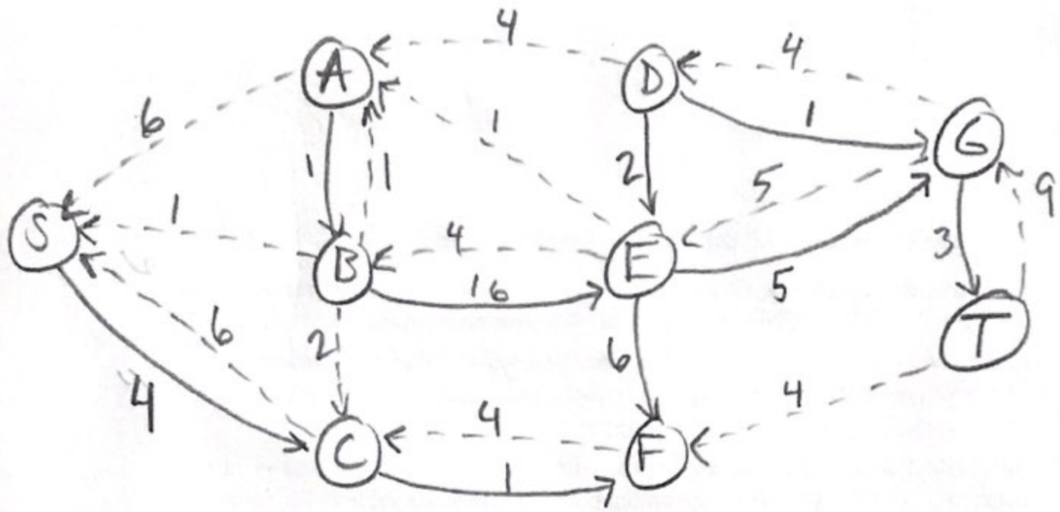
2 units



Path #7

S → C → F → E → G → T

2 units



No more paths exist, so we are optimal with maximum flow value equal to $\underline{\underline{13}}$.

$\bar{x}_{SA} = 6$ $\bar{x}_{SB} = 1$ $\bar{x}_{SC} = 6$ $\bar{x}_{AB} = 1$ $\bar{x}_{AD} = 4$ $\bar{x}_{AE} = 1$

$\bar{x}_{BC} = 2$ $\bar{x}_{BE} = 4$ $\bar{x}_{CF} = 4$ $\bar{x}_{DE} = 0$ $\bar{x}_{DG} = 4$ $\bar{x}_{EF} = 0$

$\bar{x}_{EG} = 5$ $\bar{x}_{FT} = 4$ $\bar{x}_{GT} = 9$

Part 2b

We would choose arcs $\{(S,A), (S,B), (C,B), (F,T)\}$

the sum of their capacities is $6 + 1 + 2 + 4 = \underline{\underline{13}}$