Return vs. Risk

Investments are about ① Return and ② Risk:

Return = r

Risk = σ = standard deviation

Expected Value

- Can't know *actual* r until its too late to choose
- Can calculate $E(r)$ beforehand based on our probabilistic beliefs

Expected Return and Standard Deviation

- Expected returns

$$E(r) = \sum_s p(s)r(s)$$

  - $p(s)$ = Probability of a state
  - $r(s)$ = Return if a state occurs
  - s = State

"$-r_F$"

- $r - r_F$ is *Excess Return*
- $E(r) - r_F$ is Risk Premium
- Subtract r_F to see if a risky asset beats a risk free asset

The Reward-to-Volatility (Sharpe) Ratio

- *Excess Return*
  - The difference in any particular period between the actual rate of return on a risky asset and the actual risk-free rate
- *Risk Premium*
  - The difference between the expected HPR on a risky asset and the risk-free rate
- *Sharpe Ratio* $\dfrac{\text{Risk premium}}{\text{SD of excess returns}}$

Notation

- Complete portfolio is a mix -
  - % optimized risky portfolio: y
  - % risk-free T-bills: $(1-y)$
- **C**omplete portfolio = r_C, σ_C
- risky **P**ortfolio = r_P, σ_P
- risk **F**ree t-bills = $r_F, \sigma_F = 0$

See also: 🧭 Big Picture

Capital Allocation Decision

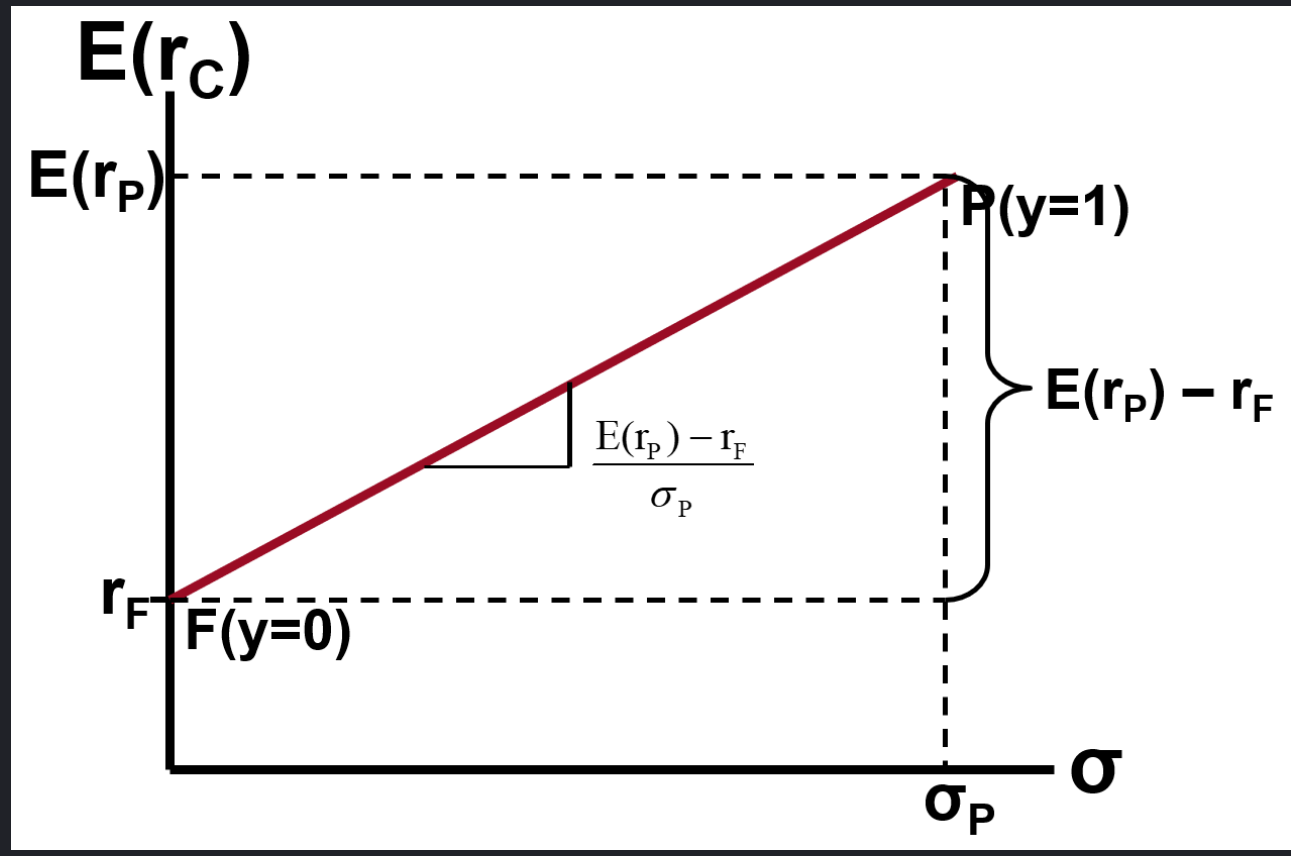
- This means choosing y and (1-y)
- Notation: **C**omplete, risky **P**ortfolio (y), risk-**F**ree (1-y)
- **Return**: $E(r_C) = r_F + y(E(r_P) - r_F)$
- **Risk**: $\sigma_C = y\sigma_P$

✅ Do you understand *each letter* of these equations?

See also: 🧭 Big Picture

Visually

- **Return** on y axis and **Risk** on x axis
- y determines location on red line.
- What y does investor prefer?



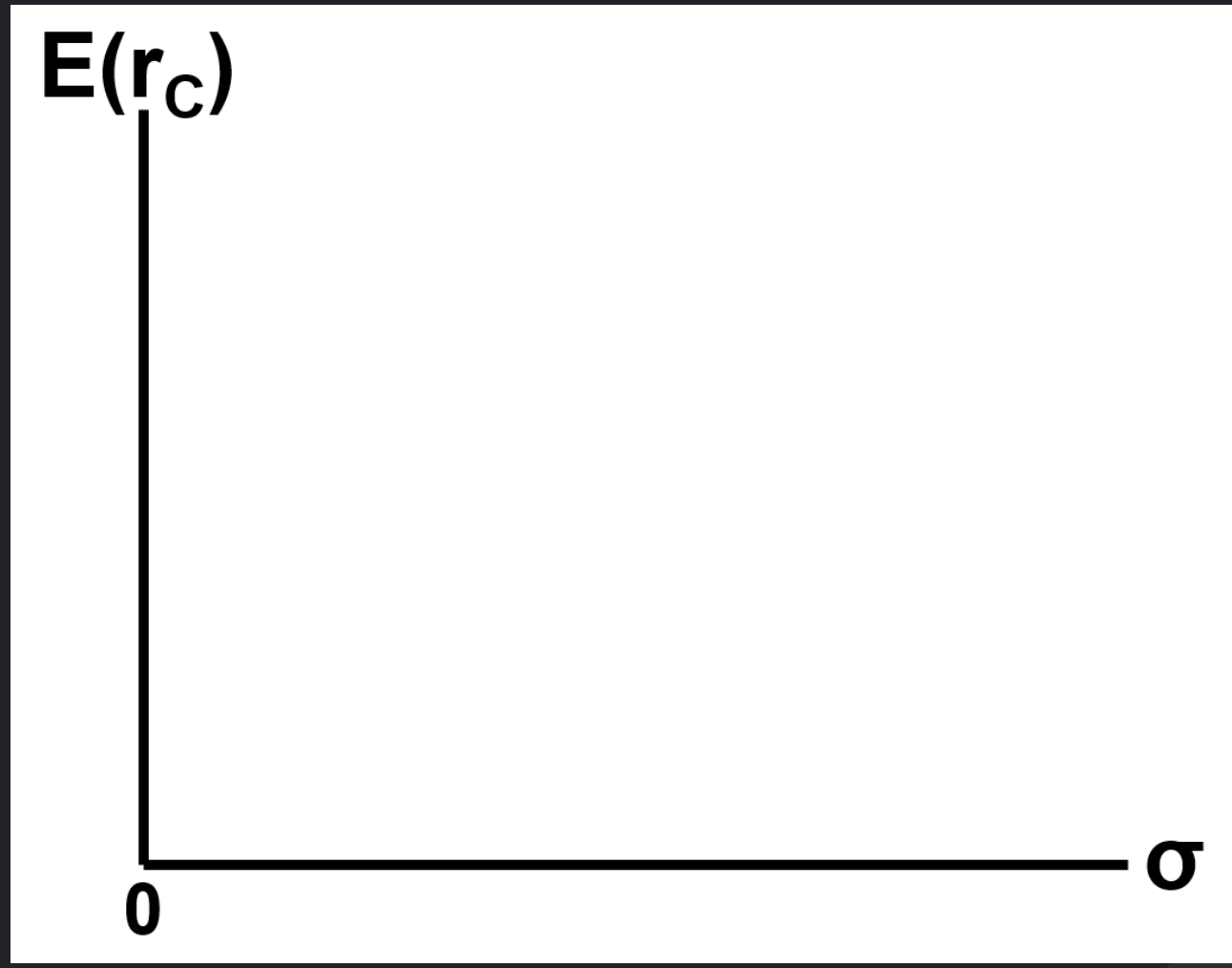
See also: 🧭 Big Picture

Replicating the diagram

- Steps:
  1. Calculate $E(r_C)$ and σ_C for $y = 0$ and $y = 1$.
  2. Plot the points.
  3. Calculate the slope of the line connecting them.

See also: 🧭 Big Picture

✍️ Replicate diagram



See robmunger.com/1920sectionslides for full solution.

✅ Try it on your own

See robmunger.com/1920sectionslides for full solution.

Roots in probability theory

Return: $E(aX + bY) = aE(X) + bE(Y)$

Risk:

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$$

Next Week we will apply these two two formulas to get new formulas for Return and Risk

See also 🔎 Where did our two formulas come from? for full solution.